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Elasticity of attention and optimal monetary policy

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ABSTRACT

Optimal monetary policy depends on whether agents have exogenous, endogenous–inelastic, or endogenous–elastic attention. Under elastic attention, optimal monetary policy induces equilibria that are not possible under the other two settings: no attention to any shocks that generate inefficient economic fluctuations.

1. Introduction

There are two forms of endogenous attention in the rational inattention literature. When attention is inelastic, decision makers have a fixed information-processing capacity. When attention is elastic, they face a fixed marginal cost of processing information. Sims (2010), Coibion and Gorodnichenko (2015) and Li et al. (2017) argue that the elastic attention model is more convincing that the inelastic one, as information rigidities vary with macroeconomic conditions.

This paper presents a new result: depending on whether attention is inelastic, elastic, or simply exogenous, the optimal monetary policy is drastically different. Under elastic attention, optimal monetary policy may induce decision makers to pay no attention at all to shocks that generates inefficient fluctuations under perfect information, an outcome that is not allowed under exogenous or inelastic attention.

The result is driven by the choice decision makers face between learning the aggregate and idiosyncratic shocks that cause inefficient economic fluctuations. Under both elastic and inelastic attention, optimal monetary policy always makes sure that decision makers pay no attention to the aggregate condition. When attention is inelastic, decision makers are induced to pay all attention on the idiosyncratic condition. When attention is elastic, it is possible that decision makers choose not to pay attention to the idiosyncratic condition as well. Such equilibria of no attention allow welfare loss to be at the minimum.

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This paper belongs to the literature on rational inattention (e.g. Sims, 2010; Mackowiak and Wiederholt, 2009; Luo and Young, 2014, etc.) and on the optimal monetary policy under information friction, in particular Adam (2007) and Paciello and Wiederholt (2014). While Adam (2007) and Paciello and Wiederholt (2014) study the optimal monetary policy under imperfect information on the aggregate condition only, this paper introduces information friction on both aggregate and idiosyncratic conditions. Thus, this paper differs from them that the attention allocation among the two conditions plays a central role. Adam (2007) discusses the optimal policy under purely exogenous and inelastic attention, while Paciello and Wiederholt (2014) emphasizes that the optimal policy depends on whether information is exogenous or endogenous. This paper shows that not only does the optimal policy depends on whether the attention is endogenous, it also depends on the form of endogeneity, i.e., elastic or inelastic.

2. Model setup

The economy has a representative household, monopolistically competitive firms, and a central bank. Prices are flexible. Firms face aggregate and idiosyncratic shocks, and due to information friction they have to choose how much attention to devote to them. The central bank controls nominal aggregate demand, and it has perfect information on the aggregate condition. Optimal policy depends on the attention allocation decision of firms. Derivation details are provided in the Online Appendix.

2.1. The representative household

It solves the following problem:

$$\max_{Y,L} \quad U(Y) - V(L)$$

$$s.t. \quad PY = WL + \Pi - T,$$

where Y denotes aggregate consumption, L denotes aggregate labour supply, W is the wage rate, P is the price index, Π represents profit transfer from firms, T denotes nominal transfers.

2.2. Firms

There are a continuum of monopolistically competitive firms $i \in [0, 1]$, and they produce intermediate goods with the production function

$$Y_i = AL_i,$$

where A represents technology.

Final goods come from a Dixit–Stiglitz aggregator,

$$Y = \left(\int_0^1 (Y_i)^{\eta-1/\eta} \, di \right)^{\eta/\eta-1},$$

where η is the elasticity of substitution between varieties, but η is also a stochastic mark-up shock defined as $\eta = \eta^{ss} + \tilde{\eta}$, with $\tilde{\eta} \sim \mathcal{N}(0, \sigma_{\tilde{\eta}}^2)$.

Denote P_i as the price of firm i, the profit maximization problem of firm i is

$$\max_{P_i} E_i \left[(1 - t_i) P^i Y^i \left(P^i \right) - WY_i \left(P_i \right) / A \right]$$

where $t_i = (1 - \eta^{ss})^{-1} + u_i$ and u_i represents an idiosyncratic revenue shock (e.g. liquidity or taxation shock). It is an exogenous stochastic process and pairwise independent across agents. u_i is a random i.i.d. drawn from a normal distribution, $u_i \sim \mathcal{N}(0, \phi^2)$.

The log-linearized version of the problem is

$$\min_{p_i} E_i \left[p_i - (1 - \alpha) p - \alpha(\pi - y^*) - \tilde{\theta} - u_i \right]^2. \tag{1}$$

Lowercase letters represent the percentage deviation of the corresponding uppercase variable from its steady state. We consider a symmetric and deterministic steady state (denoted by the upper bar) with $P^i = \bar{P}$ and $Y^i = \bar{Y}$, where $\bar{P}$ is some price level chosen by the central bank. $\bar{A} = 1$ at steady state. Y^* represents the efficient level of output such that $V'(Y^*)/U'(Y^*) = A$. y^* represents the percentage deviation of Y^* from $\bar{Y}$. $\pi \equiv p + y$ denotes the aggregate nominal expenditure.

We rewrite the markup shock as $\tilde{\theta} \equiv \tilde{\eta}/[\eta^{ss}(1 - \eta^{ss})]$ and $\tilde{\theta} \in \mathcal{N}(0, \sigma_\theta^2)$. The parameter $\alpha \equiv \left[(V''(\bar{Y})U'(\bar{Y}) - V'(\bar{Y})U''(\bar{Y}))/ (U'(\bar{Y}))^2 \right] \bar{Y}$ measures strategic complementarity.

2.3. Central bank

It chooses the optimal level of the nominal demand π to minimize loss:

$$\min_{\pi} \underbrace{\left(\pi - p - y^* \right)^2}_{\text{output gap}} + \underbrace{\lambda \int (p_i - p)^2 \, dj}_{\text{price dispersion}}, \tag{2}$$

with $\lambda = \eta^{ss}/\alpha$.

We focus on linear Markovian strategies such that policy is an affine function of state variables $\tilde{\theta}$, i.e.

$$\pi = \beta_0 + \beta_1 \tilde{\theta}, \tag{3}$$

where β_0 and β_1 are the policy choice variables. The Online Appendix presents that the optimal level of $\beta_0 = y^*$, i.e. the monetary policy perfectly offsets the aggregate technology shock. For ease of exposition, we assume that firms have perfect information on the aggregate technology shock and focus on the optimal β_1 and the learning decision on the markup shock.

3. Information processing

3.1. Learning the shocks

Firms have imperfect information on both the aggregate markup shock $\tilde{\theta}$ and the idiosyncratic revenue shock u_i. They get a private signal about the aggregate shock $\theta_i \sim \mathcal{N}(\tilde{\theta}, \sigma_i^2)$ where the magnitude of σ_i^2 depends on their learning effort. Denote x as the logarithm of the reduction in variance from devoting attention to learn about $\tilde{\theta}$, with $e^{-x} = \sigma_i^2/(\sigma_i^2 + \sigma_\theta^2)$. The expected $\tilde{\theta}$ for each firm with learning effort x is,

$$E_i(\tilde{\theta}) = (1 - e^{-x})\theta_i,$$

and the posterior variance is $\sigma_\theta^2 e^{-x}$.

Firms may also pay attention to the idiosyncratic shock u_i and obtain a private signal $e_i \sim \mathcal{N}\{u_i, \sigma_e^2\}$. The posterior belief of the idiosyncratic shock follows

$$E_i(u_i) = (1 - e^{-z})e_i,$$

where z denotes the reduction in variance from paying attention to u_i. The posterior variance is $\phi^2 e^{-z}$.

3.2. Inelastic attention

Under inelastic attention, we assume

$$x + z \leq 2k, \tag{4}$$

where k represents the attention capacity. Firms face a trade-off dividing their limited attention between the two shocks, which related to the discussion in Mackowiak and Wiederholt (2009).

To find the optimal learning decision $\{x, z\}$, first guess a solution for p. With complete information, it is straightforward to show that $p = \pi - y^* + \tilde{\theta}/\alpha$. With incomplete information, the following starting guess is made:

$$p = \beta_0 + \left(\beta_1 + \frac{1}{\alpha} \right) \gamma \tilde{\theta} - y^*. \tag{5}$$

γ is a variable governed by the learning effort, the value of which is to be determined later. It can be interpreted as the level of "coordinated total learning", as γ is affected by the strategic complementarity parameter α.

Substitute Eqs. (5) and (3) to (1) delivers the optimal learning condition,

$$x = \max \left\{ \ln \left(\frac{\left(\beta_1 + \frac{1}{\alpha} \right) \left[(1 - \alpha)\gamma + \alpha \right] \sigma_\theta e^k}{\phi} \right), 0 \right\}.$$

The remaining amount of $2k$ will be spent on learning the idiosyncratic shock with $z = 2k - x$. Learning effort on both shocks depend on monetary policy and the relative noisiness of the two shocks, and there is a learning trade-off between them.

3.3. Elastic attention

Denote the marginal cost of attention as μ. Firm i solves

$$\min \left[p_i - (1 - \alpha) p - \alpha(\pi - y^*) - \tilde{\theta} - u_i \right]^2 + \mu k, \tag{6}$$

where $I(\tilde{\theta}, \theta_i) + I(u_i, e_i) = k$. Substitute Eqs. (5) and (3) to firm's problem (6) delivers the optimal learning decision

$$x = \min\left\{\ln\left(\frac{\left(\beta_1 + \frac{1}{\alpha}\right)^2 [(1-\alpha)\gamma + \alpha]^2 \sigma_\theta^2}{0.5\mu}\right), 0\right\}$$

and

$$z = \min\left\{\ln\left(\frac{\phi^2}{0.5\mu}\right), 0\right\}.$$

If $\mu > 2\phi^2$, or when reducing the imprecision is not worth the cost, we have $z = 0$.

Unlike the case of inelastic attention, with elastic attention learning effort on the idiosyncratic shock (i.e. z) is independent of monetary policy. It only depends on the cost of learning μ and the precision of that shock ϕ^2. The choice of x does not depend on ϕ^2 anymore. The learning trade-off between the two shocks disappears.

4. Optimal monetary policy

If information is perfect, the following results immediately follow. Intermediate good prices are $p_i = \pi + \tilde{\theta}/\alpha - y^* + u_i$, and the welfare loss equals $\sigma_\theta^2/\alpha^2 + \lambda\phi^2$, which is independent of monetary policy. An optimal monetary policy that stabilizes price level would choose

$$\pi = y^* - \frac{1}{\alpha}\tilde{\theta}.$$

It is consistent with the standard result that nominal variables have no impact on the real activities in a frictionless economy.

When information is imperfect, the optimal policy depends on the elasticity of attention.[1]

4.1. Under inelastic attention

We need to consider two separate cases, as a corner solution of no learning on the aggregate shock is possible.

When $x > 0$: welfare loss becomes

$$\left(\pi - p - y^*\right)^2 + \lambda \int (p_i - p)^2 di$$
$$= \left(-\frac{1}{\alpha} + \frac{\phi}{\alpha\sigma_\theta e^k}\right)^2 \sigma_\theta^2 + \lambda\phi^2\left(1 - e^{-2k}\right).$$

Welfare loss is independent of x and hence also of monetary policy.

When $x = 0$: welfare loss becomes

$$(\pi - p - y^*)^2 + \lambda \int (p_i - p)^2 di = \beta_1^2\sigma_\theta^2 + \lambda\phi^2(1 - e^{-2k}).$$

Price dispersion is again independent of monetary policy. Output gap now depends on monetary policy and the loss is minimized when the response to the aggregate shock β_1 is zero.

Optimal policy: the central bank needs to choose among the two scenarios above. That is, by picking β_1, the central bank determines whether there is learning on the aggregate shock. The optimal monetary policy follows

$$\beta_1 = \begin{cases} 0, & \text{if } \phi \geq \sigma_\theta e^k; \\ \frac{\phi}{\alpha\sigma_\theta e^k} - \frac{1}{\alpha}, & \text{otherwise.} \end{cases} \tag{7}$$

Given that price dispersion is always independent of monetary policy, the central bank only needs to "dissuade" firms from learning the aggregate shock. Thus policymakers minimize output gap by choosing optimal β_1 which depends on the condition $\phi \geq \sigma_\theta e^k$.

[1] In the Online Appendix, we show that the results reduce to those in Adam (2007) and Paciello and Wiederholt (2014) when there is no idiosyncratic condition.

4.2. Under elastic attention

When $x > 0$: welfare loss becomes

$$(\pi - p - y^*)^2 + \lambda \int (p_i - p)^2 \, di$$
$$= \left(\frac{\sqrt{0.5\mu e^{-x}} - \sigma_\theta}{\alpha}\right)^2 + \lambda 0.5\mu(1 - e^{-x}) + \lambda\phi^2(1 - e^{-z}).$$

As discussed in Section 3, firms' choice of x depends on β_1 while the choice of z does not. It means that we can think of the central bank as choosing β_1 in a way that the resulting x minimizes the loss function above.

First, unlike the case with inelastic attention, both output gap and price dispersion depend on the level of learning on the aggregate condition, and monetary policy plays a different role here.

When $x = 0$: welfare loss becomes

$$(\pi - p - y^*)^2 + \lambda \int (p_i - p)^2 di = \beta_1^2\sigma_\theta^2 + \lambda\phi^2(1 - e^{-z}).$$

It is the same as under inelastic condition, and the optimal policy has β_1 being zero.

Optimal policy: again the central bank needs to choose among the two scenarios above. The optimal policy follows

$$\beta_1 = \begin{cases} \frac{\sqrt{0.5\mu}}{\sigma_\theta\alpha} - \frac{1}{\alpha}, & \text{if } \mu \leq 2\sigma_\theta^2, \\ 0, & \text{if } \mu > 2\sigma_\theta^2. \end{cases} \tag{8}$$

4.3. Comparing the results

In both cases, the optimal monetary policy incentivizes no learning on the markup shock. However, learning on the idiosyncratic condition z depends on the monetary policy in the inelastic case, but not in the elastic case. In addition, if there is learning on the aggregate condition, welfare loss depends on the monetary policy in the elastic attention case, but not in the inelastic attention case.

To make sure that the two cases are comparable, we calculate the total learning effort $k = 0.5(x+z)$ under the optimal monetary policy with elastic attention for any given positive value of attention cost μ. We then take this level of k as the learning capacity for the inelastic case. That is, we compare the optimal monetary policy under the two cases for each μ (which implies a specific k). Clearly, μ and k are negatively correlated as presented in Fig. 1(a). Given that $x = 0$ at the optimal equilibrium, k comoves with z which is a decreasing function of μ.

Fig. 1(b) presents the optimal β_1 for the three cases, where the fixed or exogenous attention has $x = 2k$ (see the Online Appendix for detail). As μ increases, policy response increases towards zero. In all three cases, firms only pay attention to the idiosyncratic condition. When μ is small, z is positive and it does not matter whether firms have elastic or inelastic attention. As μ goes up further, attention to the idiosyncratic condition also reaches zero in the elastic case, implying $k = 0$, which is not allowed in the exogenous and inelastic cases. The optimal monetary policy still requires a negative coefficient β_1 to keep attention to the aggregate condition at zero. As μ increases, the optimal coefficient β_1 eventually reaches zero and there is no need for the central bank to discourage firms from learning.

We can look at the results from the perspective of welfare loss in Fig. 1(c) as well. When there is no friction, price dispersion is at the maximum and welfare loss is at the highest. When z is positive, elastic and inelastic attention produces the same welfare loss. When z hits zero, as long as the central bank is responding to the aggregate condition in the elastic case, welfare loss is positive. But as β_1 approaches zero as μ increases, welfare loss reaches the minimum of zero.

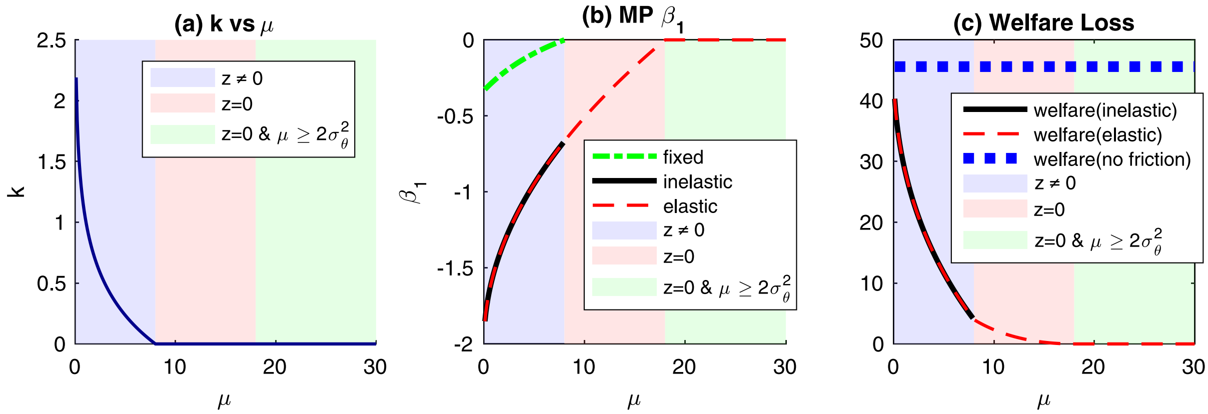

Fig. 1. Attention Parameters, Optimal Policy Coefficient, and Welfare Loss. Note: We set $\sigma_\theta = 3$, $\alpha = 0.5$, $\eta^{ss} = 1.2$, and $\phi = 2$. The patterns are not sensitive to these parameters values we choose.

5. Conclusion

One possible extension to this paper is on the timing of the optimal policy. The policymaker will face an additional choice of whether to commit after firms have made learning decision but before setting price. Exogenous attention can also be interpreted as a predetermined information acquisition process.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.econlet.2020.109393.

References

Adam, Klaus, 2007. Optimal monetary policy with imperfect common knowledge. J. Monetary Econ. 54, 276–301.
Coibion, Oliver, Gorodnichenko, Yuriy, 2015. Information rigidity and the expectations formation process: A simple framework and new facts. Amer. Econ. Rev. 105 (8), 2644–2678.
Li, Wei, Luo, Yulei, Nei, Jun, 2017. Elastic attention, risk sharing, and interntional comovements. J. Econom. Dynam. Control 79, 1–20.
Luo, Yulei, Young, Eric R., 2014. Signal extraction and rational inattention. Econ. Inq. 52, 811–829.
Mackowiak, Bartosz, Wiederholt, Mirko, 2009. Optimal sticky prices under rational inattention. Amer. Econ. Rev. 99, 769–803.
Paciello, Luigi, Wiederholt, Mirko, 2014. Exogenous information, endogenous information, and optimal monetary policy. Rev. Econom. Stud. 81, 356–388.
Sims, Christopher, 2010. Rational inattention and monetary economics. In: Friedman, Benjamin M., Woodford, Michael (Eds.), Handbook of Monetary Economics. Elsevier, Amsterdam.