Xiaojin Sun[1] / Kwok Ping Tsang[2]

What cycles? Data detrending in DSGE models

[1] University of Texas at El Paso, Economics and Finance, El Paso, TX, USA
[2] Virginia Polytechnic Institute and State University, Economics, Blacksburg, VA 24061-0131, USA, E-mail: byront@vt.edu

Abstract:
It is widely-known that different methods of detrending data yield different business cycle features. The choice of the detrending method, however, is usually arbitrarily made. This paper aims at revealing potential pitfalls of different detrending methods for the estimation of a standard medium-scale DSGE model. By comparing nine popular detrending methods, we find that model parameter estimates, variance decompositions, optimal monetary policies, and out-of-sample forecasting performances of the model are all sensitive to how the data are detrended. We also discuss some possible criteria to choose among different methods.

Keywords: DSGE models, Data detrending, Monetary policy, Forecasting

JEL classification: E32, E47, E52

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1 Introduction

Business cycle fluctuations are usually defined as deviations from some trend, and most empirical studies of business cycles involve detrending. The explicit and implicit implications of using different detrending methods to produce business cycle facts have caught researchers' attention since the 1980s. For example, Singleton (1988) points out that "stylized facts motivating recent specifications of business cycle models may have been distorted by the prefiltering procedures." Canova (1998) also observes that "within the empirical literature, there is fundamental disagreement on the properties of the trend and on its relationship with the cyclical component of a series." In a nutshell, choice matters.[1] In emerging economies, this trend-cycle decomposition is an even more influential issue [see Aguiar and Gopinath (2007) and Andrle (2008)].

The dynamic stochastic general equilibrium (DSGE) models, which aim at explaining business cycle fluctuations, are no exceptions. One way to connect DSGE models with data is by building a trend component into the model and fitting it to the raw data using a model-driven transformation. For example, Iacoviello and Neri (2010) build a deterministic trend into the model and Justiniano, Primiceri, and Tambalotti (2011) specify a stochastic trend through a unit root in the investment-specific technology. In a recent study, Slanicay (2016) proposes a flexible trend specification in the form of AR(1) processes. An alternative way is to remove the trend from the raw data by an *ad hoc* filtering method and then fit the stationary cyclical DSGE models to the prefiltered data. For example, Smets and Wouters (2007) use the first difference filter (applied to real variables only), Ireland (2001) and Bouakez, Cardia, and Ruge-Murcia (2005) linearly detrend the data, Baele et al. (2015) use a quadratic trend to measure potential GDP, Lubik and Schorfheide (2004) use the Hodrick and Prescott (1997) filter, and Döpke et al. (2008) choose the Christiano and Fitzgerald (2003) bandpass filter. The first approach suffers from the limitation that it puts a strong faith on the trend specification (e.g. a deterministic trend or a stochastic trend of the unit-root form) of the model. The second approach is also problematic that the choice of the filtering method is arbitrarily made. In addition, as Canova (2014a) points out, as two-sided moving averages of the raw data, most statistical filters change the timing of information in the data.[2]

Since Smets and Wouters (2004) show that these structural models forecast well comparing to vector autoregressions, modern sticky-price DSGE models have become an important component of central banks' toolboxes for policy analysis and macroeconomic forecasting. Examples include the small open economy model at the Sveriges Riksbank [see Adolfson, Lindé, and Villani (2007)], the New Area-Wide Model (NAWM) at the European Central Bank [see Coenen, McAdam, and Straub (2008)], and the Federal Reserve Board's new Estimated, Dynamic, Optimization-based (Edo) model [see Edge, Kiley, and Laforte (2010)].[3] While it is well-known that alternative detrending methods produce different moments in the cyclical components of the data [see Canova (1998) and Nelson (2008)] and different estimates of the structural parameters [see Gorodnichenko and Ng (2010), Canova and Ferroni (2011), and Canova (2014a)], there is little evidence concerning the effects of various detrending methods on the policy implications and forecasting performance of DSGE models. As DSGE models become widely used for forecasting [see Diebold, Schorfheide, and Shin (2016)], it is important to

Kwok Ping Tsang is the corresponding author.

understand to what extent the forecasting performance of a given DSGE model can be affected by the arbitrary choice of the detrending method.

The approach of this paper is straightforward. We fit the stationary and cyclical version of the standard medium-scale DSGE model developed by Smets and Wouters (2007) to data prefiltered by nine widely-used detrending methods – the first difference filter, the linear and quadratic deterministic trends, the two-sided and one-sided versions of the Hodrick-Prescott and Christiano-Fitzgerald filters, and two versions of the Beveridge and Nelson (1981) filter. As in Canova and Ferroni (2011), we work with cyclical models, under an implicit assumption that the driving forces of cyclical and noncyclical fluctuations are distinct and orthogonal, for the reason that jointly accounting for both cyclical and noncyclical fluctuations in the DSGE framework is challenging. After all, there is still little consensus on how cyclical shocks can be propagated at longer horizons or how permanent shocks can produce important implications at business cycle frequencies. We find that model parameter estimates, variance decomposition results, and optimal monetary policy implications from the model are all rather sensitive to the choice of the detrending method. With regard to the out-of-sample forecasting performance, while we do not find any particular method dominating the others, we notice that, among the methods we consider, the Christiano and Fitzgerald (2003) filter has the best performance for the cyclical component of model variables and the quadratic and first-difference detrending methods have the best performance for the raw data. In terms of reproducing the correlation properties of the estimation data, the first difference filter applied to all variables, including both real and nominal variables, has the best performance.[4]

The issue of data detrending in the estimation of DSGE models is well-known in the literature. Several methods have been proposed to overcome this problem. Canova and Ferroni (2011) share Boivin and Giannoni (2006)'s idea that the model variables do not have an exact observable counterpart and treat data filtered with alternative procedures as contaminated proxies of the relevant model variables. The structural parameters of the model are then estimated from multiple sources of cyclical information. They emphasize that "in the estimation it is important to use filters which overestimate the low frequency contribution of the cyclical components" because standard filters resemble high pass filters and tend to underestimate the low frequency contribution. We apply their method to the model of Smets and Wouters (2007) and find that results strongly depend on the selection of the filter that overestimates the low frequency contribution; see the Appendix. Canova (2014a) proposes a promising method of estimating both a DSGE model and the trends in some variables. The trends are specified to be flexible random walks with drifts, and only weak priors are imposed on the parameters. While the method aims at "letting the data speak," it results in the flexible trend components explaining most of the fluctuations in the data, leaving little for the DSGE model to explain. In addition, estimating with simulated data [see the working paper version Canova (2014b)], the method seems to give precise but biased estimates for the structural parameters, while other detrending methods give more biased estimates with much wider confidence intervals. It is unclear to us that the flexible method resolves the problem of making a choice in detrending.

While the methods proposed by Canova and Ferroni (2011) and Canova (2014a) are both useful to practitioners, neither method is immune to the central issue we point out in this paper: choice still matters, and one needs to be clear about what is being optimized with a particular choice.[5] Though this paper does not provide one solution that works for all purposes (and we tend not to think that there is one), we believe that it tells a "cautionary tale" to users of DSGE models (e.g. policymakers) when they are evaluating certain policies or forecasting. While it is important to have a DSGE model that is suitable for the problem at hand, it is equivalently important to know, for example, that which policy is optimal depends on how one deals with trends in the data. According to our results, if good forecasting performance is the main goal, one might consider the Christiano and Fitzgerald (2003) filter for forecasting the cyclical components and the quadratic or first-differnce detrending method for forecasting the raw data. Regardless of what is the main goal, we do recommend model users to be explicit about why a detrending method is chosen and what it is trying to acheive.

2 The smets and wouters model

To compare and contrast alternative detrending methods, we choose to work with the most popular medium-scale New Keynesian model in the DSGE literature developed by Smets and Wouters (2007), based on their previous work – Smets and Wouters (2003) – and the work of Christiano, Eichenbaum, and Evans (2005). The model features households that make consumption and working decisions to maximize their life-time utility and firms that combine capital and labor inputs to produce differentiated goods and set prices according to the Calvo (1983) model. We briefly explain the main model equations in this section and refer the reader to Smets and Wouters (2007) for more details.

Aggregate resource constraint:

$$y_t = c_y c_t + i_y i_t + z_y z_t + \epsilon_t^g. \tag{1}$$

Consumption Euler equation:

$$c_t = c_1 c_{t-1} + (1 - c_1)\mathbb{E}_t c_{t+1} + c_2\left(l_t - \mathbb{E}_t l_{t+1}\right) - c_3\left(r_t - \mathbb{E}_t \pi_{t+1} + \epsilon_t^b\right). \tag{2}$$

Investment Euler equation:

$$i_t = i_1 i_{t-1} + (1 - i_1)\mathbb{E}_t i_{t+1} + i_2 q_t + \epsilon_t^i, \tag{3}$$

Tobin's Q:

$$q_t = q_1 \mathbb{E}_t q_{t+1} + (1 - q_1)\mathbb{E}_t r_{t+1}^k - \left(r_t - \mathbb{E}_t \pi_{t+1} + \epsilon_t^b\right). \tag{4}$$

Production function:

$$y_t = \phi_p\left(\alpha k_t^s + (1 - \alpha) l_t + \epsilon_t^a\right). \tag{5}$$

Price markup:

$$\mu_t^p = \alpha(k_t^s - l_t) + \epsilon_t^a - w_t. \tag{6}$$

Price Phillips curve:

$$\pi_t = \pi_1 \pi_{t-1} + \pi_2 \mathbb{E}_t \pi_{t+1} - \pi_3 \mu_t^p + \epsilon_t^p. \tag{7}$$

Wage markup:

$$\mu_t^w = w_t - \left(\sigma_l l_t + \frac{1}{1 - \lambda/\gamma}(c_t - \lambda/\gamma c_{t-1})\right). \tag{8}$$

Wage Phillips curve:

$$w_t = w_1 w_{t-1} + (1 - w_1)(\mathbb{E}_t w_{t+1} + \mathbb{E}_t \pi_{t+1}) - w_2 \pi_t + w_3 \pi_{t-1} - w_4 \mu_t^w + \epsilon_t^w. \tag{9}$$

Monetary policy:

$$r_t = \rho r_{t-1} + (1 - \rho)[r_\pi \pi_t + r_y(y_t - y_t^p)] + r_{\Delta y}[(y_t - y_t^p) - (y_{t-1} - y_{t-1}^p)] + \epsilon_t^r. \tag{10}$$

Government spending shock:

$$\epsilon_t^g = \rho_g \epsilon_{t-1}^g + \eta_t^g + \rho_{ga} \eta_t^a. \tag{11}$$

Risk premium shock:

$$\epsilon_t^b = \rho_b \epsilon_{t-1}^b + \eta_t^b. \tag{12}$$

Investment-specific technology shock:

$$\epsilon_t^i = \rho_i \epsilon_{t-1}^i + \eta_t^i. \tag{13}$$

TFP shock:

$$\epsilon_t^a = \rho_a \epsilon_{t-1}^a + \eta_t^a. \tag{14}$$

Price markup shock:

$$\epsilon_t^p = \rho_p \epsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p. \tag{15}$$

Wage markup shock:

$$\epsilon_t^w = \rho_w \epsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w. \tag{16}$$

Monetary policy shock:

$$\epsilon_t^r = \rho_r \epsilon_{t-1}^r + \eta_t^r. \tag{17}$$

The variables y_t, c_t, i_t, z_t, and ϵ_t^g are output, consumption, investment, capital-utilization rate, and exogenous spending, respectively. l_t is labor input (hours worked), r_t is the nominal interest rate, π_t is the inflation rate, and ϵ_t^b represents a wedge between the interest rate controlled by the central bank and the return on assets held by the households (a risk premium shock). q_t is the real value of the existing capital stock and ϵ_t^i is an investment-specific technology shock. r_t^k is the real rental rate on capital, k_t^s is capital input, and $k_t^s = k_{t-1} + z_t$, the sum of capital installed in the previous period and the degree of capital utilization. The term ϵ_t^a captures a total factor productivity (TFP) shock. w_t denotes real wage. μ_t^p and μ_t^w are price and wage markups. ϵ_t^p and ϵ_t^w are price and wage markup shocks. ϵ_t^r denotes the monetary policy shock and y_t^p is the potential output.

The parameters c_y and i_y stand for the consumption-output ratio and the investment-output ratio in the steady state. z_y is the product of the steady-state rental rate of capital and the steady-state capital-output ratio. In the consumption dynamics equation, $c_1 = (\lambda/\gamma)(1 + \lambda/\gamma)$, $c_2 = [(\sigma_c - 1)(W_*^h L_*/C_*)]/[\sigma_c(1 + \lambda/\gamma)]$, and $c_3 = (1 - \lambda/\gamma)/[(1 + \lambda/\gamma)\sigma_c]$. Here, λ is the degree of external habit formation, γ is the steady-state growth rate, σ_c is the inverse of the elasticity of intertemporal substitution, and $W_*^h L_*/C_*$ is households' wage income/consumption ratio in the steady state. In the investment dynamics equation, $i_1 = 1/\left(1 + \beta\gamma^{(1-\sigma_c)}\right)$ and $i_2 = 1/\left(1 + \beta\gamma^{(1-\sigma_c)}\right)\gamma^2\varphi$, where φ is the steady-state elasticity of the capital adjustment cost function and β is the discount factor applied by households. In the Tobin's Q equation, $q_1 = \beta\gamma^{-\sigma_c}(1 - \delta)$ and δ is the depreciation rate of capital. The parameter α is the share of capital in production and the parameter ϕ_p is one plus the share of fixed costs in production. In the price Phillips curve, $\pi_1 = \iota_p/\left(1 + \beta\gamma^{(1-\sigma_c)}\iota_p\right)$, $\pi_2 = \beta\gamma^{(1-\sigma_c)}/\left(1 + \beta\gamma^{(1-\sigma_c)}\iota_p\right)$, and $\pi_3 = 1/\left(1 + \beta\gamma^{(1-\sigma_c)}\iota_p\right)[(1 - \beta\gamma^{(1-\sigma_c)}\xi_p)(1 - \xi_p)/\xi_p((\phi_p - 1)\varepsilon_p + 1)]$, where ξ_p is the degree of price stickiness, ι_p is the degree of indexation to past inflation, and ε_p is the curvature of the Kimball goods market aggregator. The parameter σ_l is the elasticity of labor supply with respect to the real wage. In the wage Phillips curve, $w_1 = 1/(1 + \beta\gamma^{1-\sigma_c})$, $w_2 = (1 + \beta\gamma^{1-\sigma_c}\iota_w)/(1 + \beta\gamma^{1-\sigma_c})$, $w_3 = \iota_w/(1 + \beta\gamma^{1-\sigma_c})$, and $w_4 = 1/(1 + \beta\gamma^{1-\sigma_c})[(1 - \beta\gamma^{1-\sigma_c}\xi_w)(1 - \xi_w)/(\xi_w((\phi_w - 1)\varepsilon_w + 1))]$, where ξ_w is the degree of wage stickiness, ι_w is the degree of wage indexation to past inflation, and ε_w is the curvature of the Kimball labor market aggregator. The parameter ρ captures the smoothness of interest rate. r_π, r_y, and $r_{\Delta y}$ capture the response of the nominal interest rate to inflation, output gap, and output gap growth, respectively.

The structural parameters of the model and their definitions are presented in Table 1. The first set of parameters are calibrated as in Smets and Wouters (2007) (including δ, ϕ_w, g_y, ε_w, and ε_p) or to match the steady-state values of the model variables with the sample means of the observed series (including $\bar{l}$, $\bar{\pi}$, and β). We do not incorporate growth rate in the model and hence set the steady-state growth rate γ to one, so that the quarterly trend growth rate, $100(\gamma - 1)$, is zero. Instead, following the usual practice, we detrend the data prior to estimation and match the detrended data with this cyclical model.

Table 1: Model parameters and definitions.

Parameter	Definition
Calibration	
δ	Capital depreciation rate
ϕ_w	Steady-state labor market markup
g_y	Steady-state exogenous spending-output ratio
ε_w	Curvature of the Kimball labor market aggregator

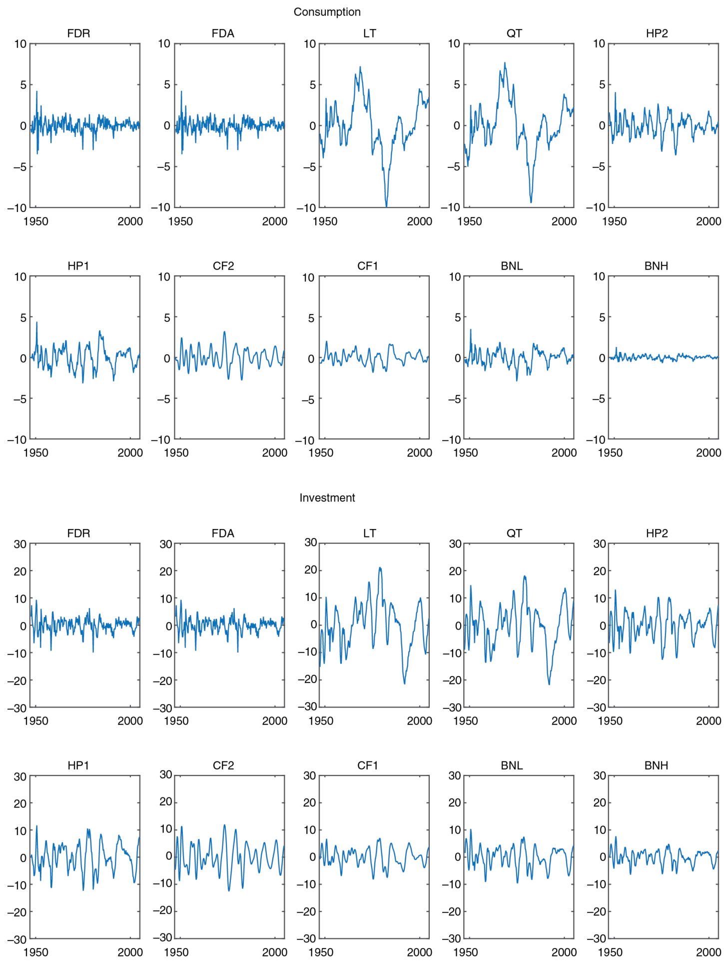
ε_p	Curvature of the Kimball goods market aggregator
γ	Steady-state growth rate
$\bar{l}$	Steady-state hours worked
$\bar{\pi}$	Steady-state inflation rate
β	Discount factor
Estimation	
φ	Steady-state elasticity of the capital adjustment cost function
σ_c	Inverse elasticity of intertemporal substitution
h	External habit formation
ξ_w	Degree of price stickiness
σ_l	Elasticity of labor supply with respect to the real wage
ξ_p	Degree of wage stickiness
ι_w	Degree of price indexation to past inflation
ι_p	Degree of wage indexation to past inflation
ψ	Elasticity of the capital utilization adjustment cost function
Φ	Share of fixed costs in production plus one
r_π	Response of interest rate to inflation
ρ	Degree of interest rate smoothing
r_y	Response of interest rate to output gap
$r_{\Delta y}$	Response of interest rate to the change in output gap
α	Share of capital in production
σ_a	Standard error of the total factor productivity shock
σ_b	Standard error of the risk premium shock
σ_g	Standard error of the exogenous spending shock
σ_I	Standard error of the investment-specific technology shock
σ_r	Standard error of the monetary policy shock
σ_p	Standard error of the price markup shock
σ_w	Standard error of the wage markup shock
ρ_a	Persistence of total factor productivity
ρ_b	Persistence of risk premium
ρ_g	Persistence of exogenous spending
ρ_I	Persistence of investment-specific technology
ρ_r	Persistence of monetary policy
ρ_p	Persistence of price markup
ρ_w	Persistence of wage markup
μ_p	Response of price markup to previous period price markup shock
μ_w	Response of wage markup to previous period wage markup shock
ρ_{ga}	Response of exogenous spending to the TFP shock

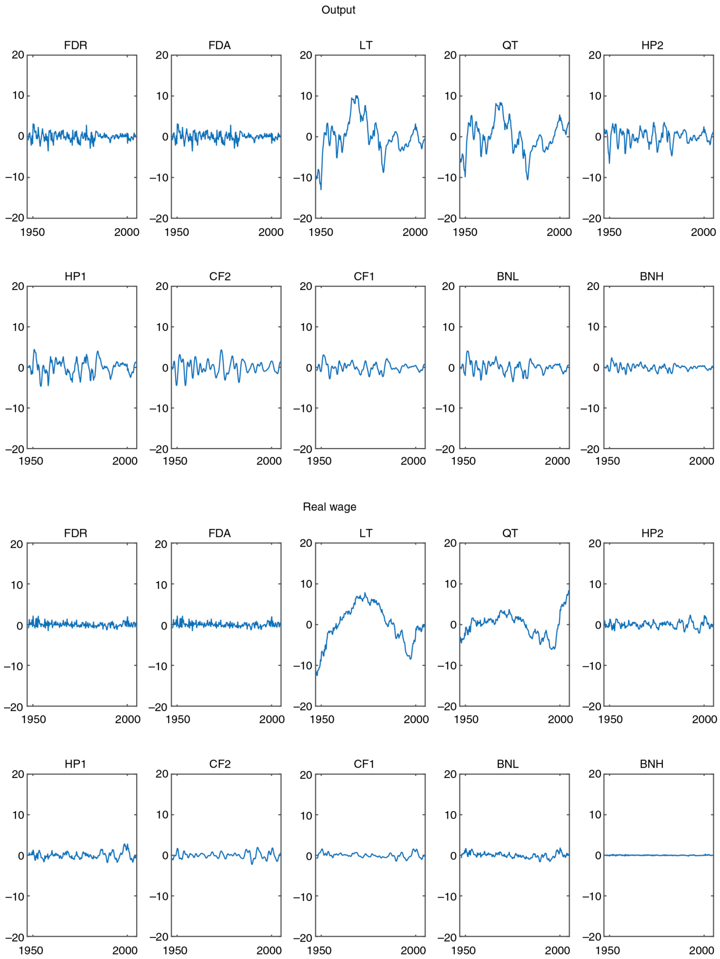
3 Data and detrending methods

We use the raw data directly taken from Smets and Wouters (2007), which range from 1947:Q2 to 2004:Q4 and include seven key US macroeconomic time series: real GDP, real consumption, real investment, real wage, and hours worked in their logarithmic form, the log difference of the GDP deflator, and the federal funds rate. All data series are seasonally adjusted.[6]

We consider nine detrending methods – the first difference filter (denoted “FD”), the linear trend filter (denoted “LT”), the quadratic trend filter (denoted “QT”), the two-sided and one-sided versions of the Hodrick and Prescott (1997) filter (denoted “HP2” and “HP1”, respectively), the two-sided and one-sided versions of the Christiano and Fitzgerald (2003) filter (denoted “CF2” and “CF1”, respectively), and two versions of the Beveridge and Nelson (1981) filter (denoted “BNL” and “BNH”, respectively).[7]

We set the smoothing parameter for the HP filters to 1600, as suggested by Hodrick and Prescott (1997), and choose the cutoff lengths to be 6 and 32 quarters for the CF filters, in order to be consistent with the widely-accepted understanding of business cycle frequencies. When using the FD filter, we consider two scenarios, where we apply the filter to real variables only (denoted “FDR”) and to all variables (denoted “FDA”). The former scenario is consistent with Smets and Wouters (2007), what we use as a benchmark. Other filters are applied to all seven time series, both real and nominal variables.[8] The first version of the Beveridge-Nelson filter imposes a low signal-to-noise ratio that maximizes the implied amplitude-to-noise ratio as proposed by Kamber, Morley, and Wong (2018) and the second version sets the ratio to one, a relatively high value.[9]


Consumption
FDR
FDA
LT
QT
HP2
HP1
CF2
CF1
BNL
BNH
Investment
FDR
FDA
LT
QT
HP2
HP1
CF2
CF1
BNL
BNH


Output
FDR
FDA
LT
QT
HP2
HP1
CF2
CF1
BNL
BNH
Real wage
FDR
FDA
LT
QT
HP2
HP1
CF2
CF1
BNL
BNH
20
10
0
−10
−20
1950
2000

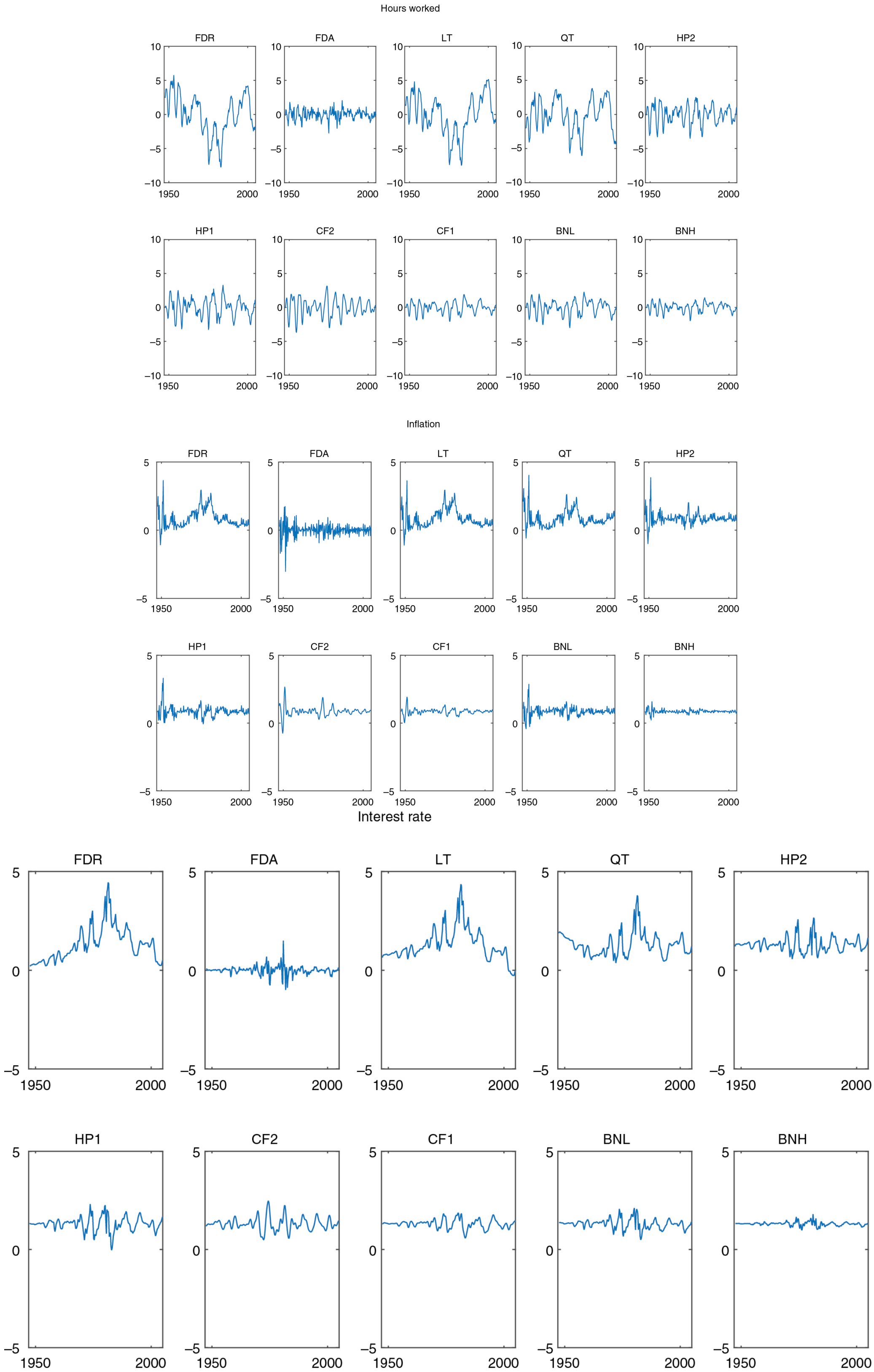


Figure 1: Detrended data series.

Figure 1 illustrates the difference in the cyclical components of the data series generated by alternative detrending methods. The FD filter applied to real variables converts these variables into their growth rates that look more like white noise. Other variables, i.e. hours works, inflation, and interest rate, that are not filtered can hardly be considered stationary however. The second version of the FD filter also applies to these nominal variables. The LT- and QT-filtered data still contain the low frequency, the high frequency, and the medium frequency (or the business cycle frequency) components. Since regressing hours, inflation, and interest rate on time and its quadratic term gives almost zero coefficient estimates, the LT- and QT-filtered nominal variables are similar to their respective original series. From the frequency perspective, the HP filters are highpass filters and they remove the low frequency component only. The CF filters remove both the low frequency and the high frequency components, leaving the filtered series, at business cycle frequencies, relatively smooth. The distinction between the two-sided and one-sided versions of the HP and CF filters is primarily represented by the different timing of information in the data. For example, HP2 and CF2 show that there is a business cycle peak in consumption around 1980 while their one-sided counterparts both suggest that the peak arrives a few quarters early. A traditional BN filter (e.g. BNH) imposes a relatively high value on the signal-to-noise ratio, leaving little in the business cycle. For example, the BNH-filtered real wage is almost a flat line. The BNL filter proposed by Kamber, Morley, and Wong (2018) uses a much smaller signal-to-noise ratio and attributes much more variations to the cycle.

We report the standard deviations and the autocorrelation coefficients of the filtered data series in Table 2. The highest cyclical variations are generated by the LT and QT filters, followed by HP and CF filters, and then FD and BN filters. The BNH filter leaves the least variations for the model to explain. While we don't observe a significant difference between the two-sided and one-sided versions of the HP filter, we notice that the one-sided CF filter generates less variations than does its two-sided counterpart.

Table 2: Moments of the cyclical components, standard deviations and autocorrelation coefficients.

										Standard deviations
Variable	**FDR**	**FDA**	**LT**	**QT**	**HP2**	**HP1**	**CF2**	**CF1**	**BNL**	**BNH**
Consumption (C)	0.84	0.84	3.21	3.16	1.20	1.26	1.08	0.73	0.89	0.25
Investment (I)	2.61	2.61	8.49	7.99	5.00	4.94	4.96	3.22	3.83	2.49
Output (Y)	1.01	1.01	4.39	3.94	1.74	1.77	1.64	1.06	1.34	0.68
Real wage (W)	0.62	0.62	4.84	2.97	0.81	0.84	0.75	0.57	0.62	0.09
Hours worked (H)	2.90	0.71	2.82	2.37	1.34	1.33	1.30	0.79	1.03	0.65
Inflation (π)	0.67	0.47	0.67	0.61	0.44	0.39	0.36	0.21	0.35	0.14
Interest rate (R)	0.87	0.22	0.81	0.59	0.36	0.35	0.33	0.23	0.25	0.10
										Autocorrelation Coefficients
Consumption (C)	−0.03	−0.03	0.97	0.96	0.77	0.84	0.91	0.93	0.80	0.49
Investment (I)	0.49	0.49	0.95	0.95	0.88	0.90	0.92	0.93	0.91	0.87
Output (Y)	0.34	0.34	0.97	0.97	0.85	0.89	0.91	0.92	0.90	0.87
Real wage (W)	−0.04	−0.04	0.99	0.98	0.74	0.81	0.91	0.94	0.78	−0.26
Hours worked (H)	0.97	0.46	0.97	0.95	0.87	0.90	0.92	0.92	0.91	0.89
Inflation (π)	0.75	−0.32	0.75	0.69	0.42	0.48	0.88	0.84	0.34	0.29
Interest rate (R)	0.97	0.23	0.96	0.93	0.81	0.85	0.92	0.93	0.82	0.60

FDR and FDA are two versions of the first difference filter applied to real variables and all variables, respectively. LT and QT stand for linear and quadratic detrends. HP2 and HP1 are the two-sided and one-sided versions of the Hodrick and Prescott (1997) filter. CF2 and CF1 are the two-sided and one-sided versions of the Christiano and Fitzgerald (2003) filter. BNL and BNH are two versions of the Beveridge and Nelson (1981) filter with low and high signal-to-noise ratios, respectively.

The highest persistence is also generated by the LT and QT filters. Under these specifications, most variables are highly persistent. The only exception is inflation, with an autocorrelation coefficient of about 0.7. The CF filtered data are less persistent in general. The HP filters yield even lower persistence in the cycles. The BNL filter produces similar autocorrelation structure to the HP1 filter. The BNH filter is the only one (besides FD that converts the level of a variable into its growth rate) that implies negative autocorrelation in real wage.

Table 3 reports the cross-correlation coefficients of the seven observable variables under alternative detrending methods. All real variables are positively correlated with one another under alternative detrending specifications, except that the traditional BN filter with a relatively high signal-to-noise ratio implies a negative correlation between real wage and consumption and investment. The unfiltered inflation and interest rate are positively correlated. This positive correlation retains under all detrending methods but the BNH filter.

Table 3: Moments of the cyclical components, cross-correlation coefficients.

							FDR								**FDA**
	C	**I**	**Y**	**W**	**H**	π	**R**		**C**	**I**	**Y**	**W**	**H**	π	**R**
C	1.00							C	1.00						
I	0.54	1.00						I	0.54	1.00					
Y	0.57	0.65	1.00					Y	0.57	0.65	1.00				
W	0.19	0.15	0.22	1.00				W	0.19	0.15	0.22	1.00			

H	0.05	0.05	0.14	0.18	1.00		
π	−0.27	−0.13	−0.16	−0.34	−0.34	1.00	
R	−0.17	−0.20	−0.21	−0.21	−0.42	0.56	1.00
							LT

H	0.42	0.62	0.65	−0.16	1.00		
π	0.07	0.04	−0.07	−0.33	0.11	1.00	
R	0.18	0.32	0.28	0.01	0.38	0.05	1.00
							QT

C	1.00						
I	0.26	1.00					
Y	0.74	0.53	1.00				
W	0.14	0.45	0.61	1.00			
H	0.59	0.07	0.25	−0.56	1.00		
π	−0.16	0.44	0.14	0.40	−0.35	1.00	
R	−0.38	0.51	0.15	0.47	−0.38	0.59	1.00
							HP2

C	1.00						
I	0.34	1.00					
Y	0.93	0.45	1.00				
W	0.47	0.31	0.48	1.00			
H	0.60	0.32	0.65	−0.25	1.00		
π	−0.10	0.34	−0.06	0.11	−0.15	1.00	
R	−0.37	0.41	−0.24	−0.17	−0.01	0.45	1.00
							HP1

C	1.00						
I	0.77	1.00					
Y	0.79	0.77	1.00				
W	0.20	0.07	0.08	1.00			
H	0.73	0.80	0.88	−0.10	1.00		
π	0.20	0.26	0.24	−0.10	0.31	1.00	
R	0.18	0.35	0.32	−0.04	0.46	0.31	1.00
							CF2

C	1.00						
I	0.74	1.00					
Y	0.83	0.76	1.00				
W	0.24	0.07	0.13	1.00			
H	0.75	0.81	0.88	−0.07	1.00		
π	0.17	0.24	0.22	−0.05	0.29	1.00	
R	0.12	0.35	0.27	−0.03	0.42	0.34	1.00
							CF1

C	1.00						
I	0.86	1.00					
Y	0.81	0.80	1.00				
W	0.24	0.20	0.06	1.00			
H	0.76	0.80	0.90	−0.08	1.00		
π	0.26	0.33	0.37	−0.07	0.41	1.00	
R	0.21	0.28	0.35	−0.07	0.47	0.45	1.00
							BNL

C	1.00						
I	0.69	1.00					
Y	0.83	0.66	1.00				
W	0.29	0.04	0.21	1.00			
H	0.74	0.78	0.85	−0.10	1.00		
π	0.17	0.27	0.26	−0.08	0.34	1.00	
R	0.03	0.31	0.22	−0.15	0.40	0.45	1.00
							BNH

C	1.00						
I	0.69	1.00					
Y	0.83	0.66	1.00				
W	0.29	0.04	0.21	1.00			
H	0.74	0.78	0.85	−0.10	1.00		
π	0.17	0.27	0.26	−0.08	0.34	1.00	
R	0.03	0.31	0.22	−0.15	0.40	0.45	1.00

C	1.00						
I	0.54	1.00					
Y	0.69	0.73	1.00				
W	−0.02	−0.03	0.01	1.00			
H	0.57	0.73	0.79	−0.01	1.00		
π	−0.21	−0.23	−0.12	−0.25	−0.19	1.00	
R	0.07	0.33	0.18	0.06	0.33	−0.20	1.00

4 Business cycle properties

In this section, we compare and contrast the nine detrending methods in three aspects – parameter estimates, variance decompositions, and the model-implied optimal monetary policy.

4.1 Parameter estimates

Estimation is conducted using Bayesian methods. We refer to Smets and Wouters (2007) for all prior distributions and calibrated parameter values, except that we set the steady-state growth rate γ to one so that the DSGE model itself becomes cyclical. Then we fit the stationary cyclical model to the detrended data. The parameter estimate results are presented in Table 4.

Table 4: Model parameter estimates.

	FDR	FDA	LT	QT	HP2	HP1	CF2	CF1	BNL	BNH
φ	4.47	5.90	5.89	5.85	2.18	2.46	4.52	4.24	2.39	4.57
	(1.99)	(2.47)	(1.83)	(1.87)	(0.24)	(0.54)	(1.16)	(1.26)	(0.51)	(1.81)
σ_c	1.57	1.55	1.69	1.54	1.09	1.01	1.02	1.03	1.05	1.47
	(0.55)	(0.44)	(0.43)	(0.38)	(0.21)	(0.18)	(0.08)	(0.10)	(0.24)	(0.21)
h	0.51	0.67	0.67	0.69	0.52	0.55	0.83	0.81	0.35	0.42
	(0.23)	(0.19)	(0.12)	(0.14)	(0.13)	(0.12)	(0.04)	(0.05)	(0.11)	(0.09)

ξ_w	0.76	0.79	0.77	0.76	0.79	0.75	0.86	0.84	0.77	0.84
	(0.07)	(0.08)	(0.08)	(0.10)	(0.08)	(0.09)	(0.04)	(0.05)	(0.09)	(0.07)
σ_l	1.93	2.04	1.85	1.54	1.32	1.42	2.36	2.16	1.40	0.72
	(1.19)	(1.17)	(1.19)	(1.11)	(0.94)	(1.02)	(1.00)	(1.05)	(0.80)	(0.65)
ξ_p	0.56	0.58	0.56	0.56	0.56	0.53	0.58	0.59	0.52	0.75
	(0.07)	(0.08)	(0.07)	(0.07)	(0.07)	(0.04)	(0.07)	(0.08)	(0.02)	(0.07)
ι_w	0.50	0.48	0.47	0.50	0.50	0.49	0.41	0.37	0.36	0.85
	(0.19)	(0.20)	(0.18)	(0.18)	(0.19)	(0.16)	(0.13)	(0.11)	(0.16)	(0.10)
ι_p	0.20	0.21	0.18	0.20	0.62	0.26	0.87	0.75	0.24	0.16
	(0.13)	(0.14)	(0.13)	(0.14)	(0.18)	(0.19)	(0.08)	(0.12)	(0.17)	(0.09)
ψ	0.54	0.47	0.51	0.51	0.56	0.56	0.47	0.48	0.59	0.80
	(0.22)	(0.20)	(0.19)	(0.20)	(0.18)	(0.22)	(0.21)	(0.21)	(0.16)	(0.13)
Φ	1.62	1.71	1.72	1.69	1.64	1.58	1.55	1.51	1.64	1.63
	(0.15)	(0.15)	(0.14)	(0.15)	(0.15)	(0.14)	(0.15)	(0.11)	(0.15)	(0.14)
r_π	1.99	1.79	1.83	1.76	1.19	1.32	1.45	1.60	1.18	1.49
	(0.33)	(0.36)	(0.30)	(0.35)	(0.26)	(0.35)	(0.28)	(0.43)	(0.23)	(0.18)
ρ	0.88	0.89	0.88	0.88	0.86	0.85	0.88	0.85	0.85	0.76
	(0.03)	(0.03)	(0.03)	(0.03)	(0.04)	(0.04)	(0.03)	(0.03)	(0.04)	(0.05)
r_y	0.13	0.14	0.13	0.12	0.24	0.21	0.25	0.20	0.23	0.23
	(0.05)	(0.06)	(0.05)	(0.06)	(0.08)	(0.09)	(0.07)	(0.07)	(0.07)	(0.07)
$r_{\Delta y}$	0.18	0.16	0.15	0.15	0.18	0.18	0.20	0.21	0.22	0.11
	(0.05)	(0.05)	(0.04)	(0.04)	(0.05)	(0.05)	(0.04)	(0.04)	(0.06)	(0.05)
α	0.19	0.22	0.22	0.21	0.20	0.19	0.20	0.19	0.22	0.20
	(0.04)	(0.04)	(0.04)	(0.03)	(0.04)	(0.04)	(0.03)	(0.03)	(0.04)	(0.02)
σ_a	0.51	0.49	0.49	0.49	0.44	0.42	0.24	0.15	0.28	0.20
	(0.06)	(0.06)	(0.05)	(0.05)	(0.04)	(0.05)	(0.02)	(0.02)	(0.03)	(0.02)
σ_b	0.20	0.30	0.33	0.34	0.33	0.26	0.03	0.03	0.19	0.13
	(0.10)	(0.13)	(0.06)	(0.08)	(0.06)	(0.06)	(0.01)	(0.01)	(0.03)	(0.02)
σ_g	0.67	0.67	0.67	0.66	0.62	0.54	0.35	0.22	0.41	0.23
	(0.06)	(0.06)	(0.06)	(0.06)	(0.06)	(0.05)	(0.03)	(0.02)	(0.04)	(0.02)
σ_I	0.55	0.57	0.59	0.58	0.74	0.64	0.14	0.12	0.34	0.37
	(0.11)	(0.11)	(0.10)	(0.11)	(0.11)	(0.11)	(0.03)	(0.02)	(0.06)	(0.06)
σ_r	0.23	0.23	0.23	0.23	0.22	0.20	0.07	0.06	0.17	0.10
	(0.03)	(0.02)	(0.02)	(0.03)	(0.03)	(0.03)	(0.01)	(0.01)	(0.02)	(0.01)
σ_p	0.22	0.22	0.23	0.23	0.29	0.22	0.03	0.03	0.24	0.10
	(0.04)	(0.05)	(0.04)	(0.04)	(0.03)	(0.04)	(0.01)	(0.01)	(0.03)	(0.01)
σ_w	0.27	0.26	0.26	0.26	0.32	0.26	0.03	0.03	0.21	0.08
	(0.04)	(0.04)	(0.03)	(0.04)	(0.04)	(0.04)	(0.01)	(0.01)	(0.03)	(0.01)
ρ_a	0.98	0.98	0.98	0.97	0.68	0.73	0.88	0.91	0.84	0.79
	(0.01)	(0.01)	(0.01)	(0.02)	(0.09)	(0.08)	(0.06)	(0.05)	(0.06)	(0.07)
ρ_b	0.64	0.31	0.18	0.20	0.37	0.50	0.84	0.75	0.64	0.05
	(0.28)	(0.40)	(0.17)	(0.19)	(0.24)	(0.20)	(0.05)	(0.07)	(0.12)	(0.05)
ρ_g	0.97	0.96	0.94	0.92	0.75	0.77	0.89	0.90	0.83	0.83
	(0.02)	(0.03)	(0.03)	(0.05)	(0.08)	(0.08)	(0.04)	(0.04)	(0.08)	(0.07)
ρ_I	0.67	0.63	0.63	0.62	0.37	0.39	0.80	0.75	0.59	0.48
	(0.13)	(0.11)	(0.10)	(0.10)	(0.12)	(0.11)	(0.07)	(0.08)	(0.11)	(0.11)
ρ_r	0.15	0.21	0.16	0.14	0.10	0.12	0.73	0.63	0.10	0.05
	(0.11)	(0.12)	(0.10)	(0.10)	(0.08)	(0.09)	(0.06)	(0.08)	(0.09)	(0.05)
ρ_p	0.95	0.93	0.84	0.85	0.75	0.63	0.77	0.64	0.52	0.17
	(0.04)	(0.05)	(0.08)	(0.08)	(0.15)	(0.18)	(0.07)	(0.11)	(0.24)	(0.16)
ρ_w	0.98	0.97	0.97	0.90	0.80	0.50	0.74	0.70	0.42	0.99
	(0.02)	(0.03)	(0.02)	(0.08)	(0.08)	(0.32)	(0.06)	(0.07)	(0.28)	(0.01)
μ_p	0.78	0.78	0.63	0.65	0.84	0.46	0.04	0.05	0.43	0.79
	(0.11)	(0.13)	(0.19)	(0.16)	(0.10)	(0.24)	(0.04)	(0.05)	(0.27)	(0.07)
μ_w	0.94	0.93	0.92	0.83	0.84	0.48	0.05	0.08	0.42	0.97
	(0.04)	(0.05)	(0.05)	(0.13)	(0.06)	(0.31)	(0.05)	(0.08)	(0.27)	(0.02)
ρ_{ga}	0.60	0.60	0.59	0.58	0.61	0.58	0.40	0.57	0.48	0.33
	(0.17)	(0.17)	(0.16)	(0.18)	(0.17)	(0.18)	(0.20)	(0.19)	(0.19)	(0.17)

Reported are the means of the posterior distributions. Standard errors, calculated as the length of the 90% confidence interval divided by 1.645, are reported in parentheses.
See the note to Table 2.

The parameter estimates under the FD specification (applied to real variables only) are consistent with those in Smets and Wouters (2007). The minor discrepancy comes from the fact that a few steady-state parameters (e.g. β, γ, $\bar{\pi}$, and $\bar{l}$) are calibrated rather than estimated in our paper. When the nominal variables are also first differenced, most of the parameter estimates are robust. There remain a few differences though. For example, the capital adjustment cost function becomes more elastic (φ increases); the response of interest rate to inflation becomes less aggressive (r_π decreases); the risk premium shock becomes more volatile (σ_b increases) but less persistent (ρ_b decreases).

The parameter estimates under the LT and QT specifications are generally in line with the FD results. When the low frequency component is removed from the data by the HP filters, the elasticity of intertemporal substitution ($1/\sigma_c$) is estimated to be much higher and the monetary policy is estimated to be less responsive to inflation. When the high frequency of the data is further removed under the CF2 and CF1 specifications, the parameters μ_p and μ_w are both estimated to be around zero. This is consistent with the purpose of including the MA term in the price and wage markup disturbances. The estimates of the behavioral parameters given by the BNL filter, share some similarity with those obtained under the HP specifications. The BNH filter, instead, returns estimates that are closer to those under the deterministic trend specifications.

While some structural parameters do not differ much among detrending methods, the major difference lies in the persistence and standard error estimates of the structural shocks. In the case of deterministic trends, LT and QT, we need at least some of the structural shocks to be highly persistent to explain the variations in the data. For example, the total factor productivity shock, the government spending shock, and the wage markup shock all have a degree of persistence higher than 0.9. When the low frequency component of the data is removed by the HP filters, all structural shocks become moderately persistent at most. The CF filters keep fluctuations at business cycle frequencies only and remove both the low frequency and the high frequency components from the data. In this case, structural shocks have larger degrees of persistence than those produced by HP filters but they are in general not as persistent as their counterparts under the deterministic trend specifications. We do not observe much difference in the standard error estimates of the structural shocks across the FD filters, the deterministic trends (LT and QT), and the HP filters. However, in the case of CF filters, the standard deviations are estimated to be much lower due to the fact that the variations in the data to be explained are much smaller. The BN filters, especially the one that imposes a relative high signal-to-noise ratio, attribute most of the observed fluctuations to the trend and leave a small fraction to be explained by the model. Thus the estimated standard deviations of the structural shocks are generally small.

4.2 Variance decompositions

Taking the estimated parameters as given, we conduct variance decomposition analysis in this section. As in Canova (2014a), we look at the variance decomposition results at the 5-year horizon; see Table 5. The decomposition results implied by the FD filter applied to real variables only are in general consistent with the findings of Smets and Wouters (2007): (1) Monetary policy shocks do not contribute much to business cycle fluctuations; and (2) The model relies on wage markup (or labor supply) shocks to explain a significant fraction of the variations in employment. The importance of wage markup shocks is also emphasized by the LT and QT specifications. This causes criticisms on the model as tools for both monetary policy and business cycle analysis [see Chari, Kehoe, and McGrattan (2009) and Shimer (2009)].

Table 5: Variance decomposition at the 5-year horizon.

FD filter (real variables)	**TFP shock**	**Risk premium**	**Spending shock**	**Investment shock**	**Monetary shock**	**Price markup**	**Wage markup**
Consumption	10.14	61.69	2.45	0.33	18.07	2.52	4.80
Investment	3.96	8.43	1.26	74.52	6.09	4.32	1.43
Output	19.09	30.43	24.06	9.59	10.95	3.36	2.51
Real wage	8.46	1.30	0.05	0.66	1.19	40.16	48.19
Hours	1.59	14.18	12.59	9.97	11.60	13.81	36.26
Inflation	3.20	2.10	0.56	1.26	8.48	40.17	44.24
Interest rate	3.68	31.28	3.09	12.30	11.48	9.05	29.13
FD filter (all variables)							
Consumption	6.11	70.96	0.67	0.48	16.05	2.00	3.73
Investment	4.15	4.74	1.97	79.34	5.89	2.76	1.15
Output	13.32	28.40	29.39	14.21	9.86	2.50	2.33
Real wage	6.38	0.53	0.08	0.76	1.28	39.95	51.01
Hours	15.60	28.20	29.44	13.83	9.49	1.59	1.86
Inflation	2.89	0.25	0.22	0.38	2.40	79.68	14.19
Interest rate	5.27	22.68	2.27	4.07	53.04	7.19	5.47
Linear trend							

Consumption	38.40	9.36	2.45	3.34	11.05	5.56	29.85
Investment	22.71	0.46	11.43	49.32	3.84	4.61	7.63
Output	46.81	4.07	6.57	12.82	7.48	6.02	16.23
Real wage	46.03	0.31	0.20	3.93	2.70	30.47	16.37
Hours	3.33	8.21	15.05	18.06	13.16	6.83	35.36
Inflation	3.92	0.43	0.99	1.68	8.22	37.42	47.34
Interest rate	6.50	10.38	6.52	17.25	18.41	6.62	34.33
Quadratic trend							
Consumption	40.95	11.85	1.39	3.10	13.21	7.19	22.31
Investment	26.41	0.46	7.77	46.81	3.88	5.20	9.46
Output	47.14	4.71	7.23	12.34	8.17	7.11	13.31
Real wage	42.39	0.28	0.13	3.14	2.35	30.58	21.13
Hours	3.80	9.63	16.33	17.82	14.76	8.54	29.12
Inflation	5.56	0.54	1.05	1.46	9.35	44.79	37.26
Interest rate	9.46	13.35	7.38	15.62	23.19	8.66	22.33
HP filter (2-sided)							
Consumption	3.90	40.00	1.80	2.51	44.19	6.96	0.64
Investment	5.38	5.10	2.71	48.13	33.65	3.52	1.51
Output	14.46	17.61	15.85	9.80	35.95	5.44	0.90
Real wage	6.60	0.38	0.15	2.32	4.46	29.74	56.35
Hours	6.82	20.56	19.15	9.40	37.63	5.10	1.34
Inflation	3.12	0.22	0.16	0.20	6.95	84.73	4.63
Interest rate	11.96	37.12	4.82	2.70	35.68	5.60	2.13
HP filter (1-sided)							
Consumption	5.93	46.83	2.56	2.29	34.32	4.94	3.13
Investment	6.24	8.49	2.16	46.51	25.64	6.38	4.57
Output	17.09	23.63	13.67	8.07	28.10	6.33	3.11
Real wage	5.99	1.21	0.06	1.70	5.02	41.45	44.58
Hours	5.73	28.60	17.39	7.94	31.07	5.35	3.92
Inflation	3.79	0.99	0.21	0.11	9.16	67.88	17.87
Interest rate	10.01	45.47	3.38	1.90	27.67	4.56	7.00
CF filter (2-sided)							
Consumption	6.75	36.51	3.77	1.63	42.25	2.71	6.37
Investment	7.06	13.84	2.65	37.33	25.79	4.83	8.50
Output	12.42	25.64	6.22	7.17	35.70	5.28	7.57
Real wage	13.87	0.82	0.14	1.47	1.67	60.51	21.52
Hours	3.69	30.06	8.44	6.10	39.89	3.44	8.37
Inflation	4.16	4.02	0.14	0.17	11.84	34.67	44.99
Interest rate	4.92	65.10	1.74	1.82	3.86	4.59	17.97
CF filter (1-sided)							
Consumption	12.15	36.85	4.39	2.00	31.01	4.16	9.43
Investment	11.50	9.46	3.03	45.44	15.95	4.23	10.40
Output	22.87	22.08	6.98	8.93	23.86	5.83	9.44
Real wage	22.15	0.80	0.13	1.76	1.43	50.94	22.80
Hours	2.89	29.78	10.67	8.80	30.39	4.95	12.51
Inflation	5.12	3.21	0.23	0.20	8.94	43.80	38.50
Interest rate	6.26	56.48	2.35	3.18	8.63	6.60	16.49
BN filter (low signal-to-noise)							
Consumption	13.27	36.34	4.11	4.62	36.26	3.67	1.72
Investment	13.51	8.89	3.02	47.67	22.26	2.89	1.74
Output	24.16	19.46	10.54	11.31	28.64	4.27	1.62
Real wage	14.55	1.06	0.16	2.64	3.70	37.76	40.14
Hours	4.07	26.65	15.84	11.65	35.72	3.74	2.33
Inflation	4.17	0.84	0.23	0.20	5.60	73.74	15.23
Interest rate	10.72	53.34	3.86	3.39	20.18	2.09	6.42
BNH filter (high signal-to-noise)							
Consumption	1.01	1.99	0.14	2.50	3.03	0.18	91.14
Investment	4.77	0.12	1.73	84.94	2.36	0.28	5.80
Output	5.34	1.46	6.55	19.63	3.68	0.30	63.04
Real wage	1.47	0.01	0.05	6.49	0.67	3.47	87.83
Hours	4.83	1.54	7.11	13.35	3.57	0.26	69.35

Inflation	0.08	0.00	0.01	0.07	0.00	1.91	97.93
Interest rate	1.01	0.28	0.25	0.47	3.75	0.27	93.97

Results are obtained using the mean of the posterior of the parameters.

However, when we remove the low frequency component of the data by using the HP filters, wage markup shocks no longer play such an important role. Instead, risk premium and monetary policy shocks significantly contribute to the cyclical fluctuations. This is consistent with the finding of Justiniano, Primiceri, and Tambalotti (2010) that labor supply shocks are only important for low frequency movements, which might not be relevant to business cycles. It also suggests that monetary policy could be more useful than the FD, LT, and QT specifications suggest. The CF filters further remove the high frequency components of the data but yield similar variance decomposition results as the HP filters at a medium horizon. Similar to the HP filters, the BNL filter also emphasizes the importance of risk premium and monetary policy shocks. However, the BNH filter turns out to be the worst filtering method and attributes most of the observed fluctuations to wage markup shocks only. No matter which detrending method we consider, the markup shocks in the price and wage Phillips curves together explain more than 80% of the fluctuations in inflation.

Given the fact that different detrending methods yield different variance decomposition results, it may not be robust for researchers to claim a particular structural shock playing a major role based on a particular detrending method. Justiniano, Primiceri, and Tambalotti (2010) argue that the expenditures on durable goods should be included in investment instead of consumption and that investment should also include the inventory change. After the data revision, they find that investment shocks drive most of the variability of output and employment. Although the model we use in the current work slightly differs from the one estimated by Justiniano, Primiceri, and Tambalotti (2010), we indeed find that, when using the FD filter on real variables, investment shocks drive about half of output fluctuations. However, this finding is not consistent across alternative detrending methods. For example, under the LT and QT specifications, TFP shocks are still the leading driving force of output fluctuations and the contribution of investment shocks is only half that of TFP shocks; see Table 6.

Table 6: Variance decomposition at the 5-year horizon, durable consumption in investment.

FD filter (real variables)	**TFP shock**	**Risk premium**	**Spending shock**	**Investment shock**	**Monetary shock**	**Price markup**	**Wage markup**
Consumption	6.42	75.38	0.70	0.32	8.89	2.18	6.10
Investment	3.54	3.04	0.28	89.05	1.71	1.36	1.02
Output	9.09	22.29	13.15	45.43	4.66	2.65	2.73
Real wage	6.30	1.32	0.04	1.32	1.23	39.93	49.86
Hours	6.69	8.07	6.97	23.12	7.61	10.18	37.37
Inflation	4.36	0.61	0.16	0.95	5.41	40.36	48.14
Interest rate	9.68	10.41	0.91	10.42	16.94	14.76	36.87
Linear trend							
Consumption	37.88	10.29	0.95	4.78	11.66	5.50	28.94
Investment	28.76	0.42	3.57	57.17	1.83	2.24	6.01
Output	45.29	3.55	3.22	21.84	6.42	5.30	14.37
Real wage	42.04	0.40	0.06	4.81	2.99	27.96	21.75
Hours	8.86	6.38	6.66	32.13	9.96	4.75	31.25
Inflation	4.97	0.40	0.28	1.47	6.22	38.04	48.63
Interest rate	12.04	8.32	1.72	17.62	18.70	7.38	34.22
Quadratic trend							
Consumption	33.20	11.74	0.34	6.61	15.23	7.76	25.12
Investment	29.18	0.26	2.65	59.38	1.72	2.28	4.55
Output	41.51	3.56	3.66	26.47	7.33	6.44	11.02
Real wage	34.38	0.31	0.05	4.58	2.72	30.99	26.97
Hours	9.85	5.99	6.86	37.31	10.79	5.52	23.68
Inflation	6.33	0.34	0.27	1.67	6.66	48.06	36.67
Interest rate	17.02	8.78	1.95	20.76	20.67	8.65	22.17

Results are obtained using the mean of the the posterior of the parameters. The expenditures on durable goods are no longer included in the consumption data, but instead in investment. Investment also includes the change in inventories.

4.3 Optimal monetary policy

Given the differences in parameter estimates, we expect to see different detrending methods producing noticeably different optimal monetary polices as well. It is assumed that the policymaker seeks to minimize an ordinary expected loss criterion as in Giannoni and Woodford (2003):

$$\begin{aligned}\mathfrak{L} =& \lambda_\pi var(\pi_t - \iota_\pi \pi_{t-1}) + \lambda_w var(\pi_{wt} - \iota_w \pi_{t-1}) \\ &+ \lambda_y var\left(y_t - y_t^p\right) + \lambda_r var(r_t - \bar{r}),\end{aligned} \tag{18}$$

where $\bar{r}$ is the steady-state value of the nominal interest rate; $\pi_{wt} = w_t - w_{t-1} + \pi_t$ is the nominal wage inflation. Under the objective $\mathfrak{L}$, the policymaker minimizes a weighted variability of money inflation, wage inflation, output gap, and nominal interest rate, with weights $\lambda_\pi = 0.5$, $\lambda_w = 0.5$, $\lambda_y = 0.048$, and $\lambda_r = 0.236$, as calibrated in Woodford (2003). Given these specifications, we search for the optimal values of the monetary policy parameters r_π, ρ, r_y, and $r_{\Delta y}$ in a bounded space $[1,4] \times [0,4] \times [0,4] \times [0,4]$. Note that the response to inflation needs to be larger than one in order for the model to be determinate.

Table 7 shows the optimal monetary policy parameter estimates and the minimized value of the loss criterion. For a comparison, we also report the estimated monetary policy parameter values and the associated loss criterion value in Table 8. When FD (either applied to real variables only or to all variables), LT, QT, and BNH are specified, the objective function keeps declining as the interest rate rule becomes more persistent and responsive to inflation. In these cases, the optimization problem gives a corner solution at $r_\pi = 4$. The optimal policy parameters look more reasonable under other specifications. When using the HP filters, the optimal parameter values are not too much different from the estimated values, except that the optimal policy responds more aggressively to output gap. The minimized loss criterion is not significantly lower than the estimated value either. This might suggest that the central bank has a fairly good performance in stabilizing economic activities if the volatility of the HP-filtered data is what it cares about. The BNL filter is similar to the HP filters. Under the two-sided CF specification, there is still enough room for improvement by making the interest rate rule more persistent and more responsive to output gap but slightly less responsive to inflation. In the optimal scenario, the weighted variability could be lowered by half.

Table 7: Optimal monetary policy.

Parameter	FDR	FDA	LT	QT	HP2	HP1	CF2	CF1	BNL	BNH
r_π	4.00	4.00	4.00	4.00	1.20	1.34	1.17	1.65	1.19	4.00
ρ	0.90	0.92	0.91	0.92	0.66	0.68	0.95	0.70	0.64	0.94
r_y	0.11	0.18	0.19	0.43	0.53	0.43	3.95	0.56	0.65	0.00
$r_{\Delta y}$	0.88	0.78	0.68	0.57	0.19	0.18	0.19	0.25	0.22	1.67
Minimized $\mathfrak{L}$	1.43	1.20	1.00	0.73	0.30	0.27	0.14	0.06	0.18	0.74

See the note to Table 2.

Table 8: Estimated monetary policy.

Parameter	FDR	FDA	LT	QT	HP2	HP1	CF2	CF1	BNL	BNH
r_π	1.99	1.79	1.83	1.76	1.19	1.32	1.45	1.60	1.18	1.49
ρ	0.88	0.89	0.88	0.88	0.86	0.85	0.88	0.85	0.85	0.76
r_y	0.13	0.14	0.13	0.12	0.24	0.21	0.25	0.20	0.23	0.23
$r_{\Delta y}$	0.18	0.16	0.15	0.15	0.18	0.18	0.20	0.22	0.22	0.11
Estimated $\mathfrak{L}$	1.74	1.51	1.22	0.89	0.38	0.33	0.27	0.09	0.22	1.76

See the note to Table 2.

5 Two possible criteria of choosing a detrending method

Having shown that different detrending procedures produce different business cycle properties and policy implications, how should we make a selection in empirical research? We believe that there is no universally accepted criterion that one can follow because DSGE models are used by researchers for various purposes. Since DSGE models have become an important component of central banks' toolboxes for macroeconomic forecasting, we should pay close attention to the forecasting performance of the model and the effects of alternative detrending methods. We also notice that some properties of the filtered data, the cross correlation structure reported in Table 3 for instance, are largely abstracted from the model. Given a certain model, some detrending methods may outperform others in terms of replicating the correlation properties of the estimation data from multiple random draws of the structural shocks.

5.1 Out-of-sample forecasting

To evaluate the forecasting performance of the model under different trend specifications, we calculate the mean squared error (MSE) of forecasting for each variable-horizon combination. An initial sample using data from 1947:Q2 to 2000:Q4 is used to estimate the model, and 1- to 8-step ahead out-of-sample forecasts are produced. Then, the length of the estimation sample is increased by one recursively, the model is re-estimated, and forecasts are produced again. This yields 16 observations in total for the 1-step ahead forecast, 15 for the 2-step ahead forecast, and down to 9 for the 8-step ahead forecast. For the first difference (FDR and FDA), the deterministic trend (LT and QT), and the one-sided stochastic trend specifications (HP1 and CF1), we not only forecast the cyclical components but also the raw data series. We rely on the cyclical model to forecast the cyclical component of a variable and on the filter itself to forecast the trend. Table 9 reports the MSE of the out-of-sample forecasting for select horizons. The forecasting MSE of the cyclical component is not directly comparable across alternative detrending methods, and thus we divide the MSE by the variance of the relevant variable. We also report an overall MSE by summing up the MSEs across the seven observable variables.

Table 9: Out-of-sample forecasting MSE, cyclical component.

1-step ahead	FDR	FDA	LT	QT	HP2	HP1	CF2	CF1	BNL	BNH	Optimal
Consumption	0.236	0.206	0.024	0.024	0.148	0.127	0.006	0.019	0.108	0.137	CF2
Investment	0.435	0.453	0.049	0.043	0.088	0.092	0.004	0.016	0.046	0.076	CF2
Output	0.400	0.306	0.031	0.015	0.054	0.054	0.007	0.021	0.033	0.046	CF2
Real wage	0.571	0.565	0.008	0.024	0.299	0.257	0.026	0.060	0.218	1.363	LT
Hours	0.045	0.607	0.070	0.067	0.098	0.093	0.013	0.032	0.079	0.106	CF2
Inflation	0.112	0.276	0.107	0.149	0.199	0.235	0.011	0.068	0.206	0.626	CF2
Interest rate	0.013	0.755	0.072	0.059	0.096	0.095	0.010	0.031	0.096	0.161	CF2
Overall	1.812	3.167	0.361	0.381	0.982	0.953	0.078	0.247	0.786	2.515	CF2
2-step ahead											
Consumption	0.191	0.179	0.045	0.046	0.354	0.293	0.039	0.100	0.185	0.227	CF2
Investment	0.664	0.687	0.206	0.169	0.355	0.348	0.033	0.101	0.207	0.273	CF2
Output	0.439	0.289	0.102	0.039	0.173	0.163	0.037	0.099	0.093	0.127	CF2
Real wage	0.631	0.606	0.020	0.070	0.733	0.644	0.159	0.292	0.440	0.522	LT
Hours	0.163	0.647	0.244	0.223	0.293	0.264	0.046	0.128	0.256	0.293	CF2
Inflation	0.091	0.244	0.129	0.196	0.170	0.203	0.041	0.227	0.195	0.671	CF2
Interest rate	0.052	0.844	0.307	0.237	0.316	0.317	0.068	0.169	0.321	0.435	FDR
Overall	2.232	3.496	1.054	0.979	2.394	2.232	0.424	1.116	1.697	2.549	CF2
4-step ahead											
Consumption	0.232	0.271	0.061	0.082	0.485	0.424	0.165	0.328	0.213	0.536	LT
Investment	0.668	0.684	0.745	0.560	1.038	0.974	0.216	0.491	0.721	0.746	CF2
Output	0.373	0.230	0.250	0.077	0.339	0.318	0.168	0.311	0.194	0.230	QT
Real wage	0.521	0.500	0.019	0.160	1.102	0.824	0.479	0.784	0.350	1.202	LT
Hours	0.525	0.367	0.667	0.587	0.468	0.409	0.119	0.320	0.500	0.593	CF2
Inflation	0.077	0.262	0.212	0.228	0.140	0.223	0.051	0.507	0.218	0.695	CF2
Interest rate	0.191	0.443	1.034	0.701	0.574	0.605	0.285	0.630	0.621	0.881	FDR
Overall	2.587	2.757	2.988	2.395	4.145	3.778	1.482	3.370	2.817	4.884	CF2
8-step ahead											
Consumption	0.163	0.226	0.035	0.057	0.254	0.280	0.116	0.241	0.149	1.782	LT
Investment	0.237	0.306	1.147	0.752	1.195	1.163	0.609	0.791	0.796	0.912	FDR
Output	0.217	0.210	0.282	0.023	0.316	0.277	0.303	0.206	0.150	0.389	QT

Real wage	0.570	0.550	0.030	0.730	1.028	0.997	0.454	0.999	0.354	0.520	LT
Hours	1.343	0.268	1.219	1.154	0.239	0.193	0.036	0.089	0.375	0.347	CF2
Inflation	0.047	0.313	0.135	0.080	0.142	0.156	0.082	0.458	0.181	1.252	FDR
Interest rate	0.446	0.094	2.171	0.959	0.276	0.356	0.272	0.639	0.266	1.033	FDA
Overall	3.024	1.967	5.018	3.755	3.448	3.421	1.872	3.422	2.270	6.235	CF2

The reported MSE for the cyclical component has been adjusted by the variance.
See the note to Table 2.

For short horizons such as 1- or 2-step ahead, the Smets and Wouters (2007) model has the best forecasting performance on the cyclical component of the data under the CF2 specification for all variables except real wage, which is slightly better forecasted under the LT specification. The widely used filters, including the FD filter and both the two-sided and one-sided versions of the HP filter, turn out to be the worst specifications from the forecasting perspective. As the forecasting horizon gets longer, we do not find any particular detrending method consistently dominating others. Overall, the best forecasting performance of the model is achieved when the data are prefiltered with the two-sided CF filter.

While real variables typically show long-run drifts, nominal variables just display low frequency fluctuations. Should we filter all the data or only real variables? A comparison between FDR and FDA shows that first differencing nominal variables usually yields larger forecasting errors on those variables.

When it comes to forecasting the raw data series, as Table 10 shows, QT and FD filters are among the best specifications in general, especially for real variables, such as consumption, investment, and output at all horizons, and LT works best for real wage. For nominal variables like inflation and interest rate, FDR performs best. This finding could be meaningful to inflation-targeting central banks. The forecasting performance of the model deteriorates quickly as the horizon increases.

Table 10: Out-of-sample forecasting MSE, raw series.

1-step ahead	**FDR**	**FDA**	**LT**	**QT**	**HP1**	**CF1**	**Optimal**
Consumption	0.168	0.147	0.211	0.186	0.295	0.519	FDA
Investment	3.001	3.122	3.711	2.663	3.225	7.782	QT
Output	0.413	0.317	0.634	0.171	0.251	1.180	QT
Real wage	0.223	0.220	0.196	0.461	0.266	0.966	LT
Hours	0.384	0.309	0.560	0.564	0.243	1.018	HP1
Inflation	0.051	0.062	0.051	0.055	0.056	0.158	FDR
Interest rate	0.010	0.038	0.059	0.024	0.017	0.079	FDR
Overall	4.250	4.216	5.421	4.125	4.353	11.703	QT
2-step ahead							
Consumption	0.235	0.528	0.346	0.281	0.871	0.632	FDR
Investment	10.611	11.735	15.903	10.434	15.584	9.814	CF1
Output	0.642	0.472	2.128	0.418	0.968	1.406	QT
Real wage	0.547	0.588	0.477	1.640	0.844	0.999	LT
Hours	1.394	1.067	1.949	2.013	0.872	1.088	HP1
Inflation	0.042	0.066	0.064	0.069	0.059	0.093	FDR
Interest rate	0.042	0.157	0.252	0.098	0.073	0.082	FDR
Overall	13.511	14.612	21.118	14.952	19.272	14.115	FDR
4-step ahead							
Consumption	2.108	4.282	0.453	0.508	2.347	1.483	LT
Investment	40.295	45.821	58.811	35.562	69.650	45.596	QT
Output	0.945	1.035	5.371	0.809	3.170	3.102	QT
Real wage	1.254	1.599	0.450	6.176	1.819	1.896	LT
Hours	4.507	3.023	5.367	6.139	2.381	1.863	CF1
Inflation	0.035	0.088	0.110	0.074	0.078	0.189	FDR
Interest rate	0.154	0.516	0.877	0.312	0.238	0.272	FDR
Overall	49.298	56.364	71.439	49.581	79.683	54.400	FDR
8-step ahead							
Consumption	12.712	22.239	0.718	0.557	6.403	2.459	QT
Investment	86.110	101.329	98.414	50.093	186.377	132.277	QT
Output	0.333	3.416	6.820	0.570	5.056	2.523	FDR
Real wage	5.842	7.895	0.595	29.003	7.022	5.993	LT
Hours	11.615	7.626	10.000	16.009	4.179	1.573	CF1
Inflation	0.023	0.138	0.090	0.042	0.090	0.365	FDR
Interest rate	0.373	1.108	2.044	0.542	0.520	0.710	FDR
Overall	117.008	143.751	118.681	96.816	209.646	145.900	QT

See the note to Table 2.

5.2 Replication of the correlation properties of the data

How well does each detrending method replicate the correlation properties of the estimation data? We take the cyclical model and the estimated parameters as given, simulate data from faked shocks drawn from normal distributions with zero mean and the estimated standard deviations (see Table 4) or a resampling of the estimated shocks, and examine the performance of alternative detrending methods on reproducing the correlation properties of the data. The correlation properties we focus on include the seven autocorrelation coefficients and the twenty one cross-correlation coefficients of the seven observable variables that we report in Table 3. We calculate the MSE of these correlation coefficients across 10,000 simulation repetitions and report the results in Table 11.

Table 11: MSE of the correlation coefficients.

	Normal distribution			Distribution free		
	Auto	**Cross**	**Overall**	**Auto**	**Cross**	**Overall**
FDR	0.2368	0.4472	0.6840	0.1830	0.9494	1.1324
FDA	0.2883	0.3627	0.6510	0.2710	0.3745	0.6455
LT	0.0095	2.5278	2.5373	0.0093	2.5152	2.5246
QT	0.0092	1.5283	1.5375	0.0086	1.5671	1.5758
HP2	0.0309	0.7406	0.7715	0.0317	0.8320	0.8637
HP1	0.0249	0.6998	0.7247	0.0248	0.7465	0.7713
CF2	0.0193	1.0289	1.0482	0.0193	1.0680	1.0873
CF1	0.0078	1.0213	1.0291	0.0082	1.0496	1.0578
BNL	0.0224	0.7181	0.7406	0.0220	0.7422	0.7643
BNH	2.3473	10.6559	13.0033	2.3529	10.7179	13.0708
Optimal	CF1	FDA	FDA	CF1	FDA	FDA

See the note to Table 2.

The results are in general consistent between the two sampling methods. Overall, the FD filter applied to all variables has the best performance in terms of reproducing the correlation properties of the data, followed by HP filters and the BN filter with a low signal-to-noise ratio, CF filters, and then LT and QT filters. The traditional BN filter with a relatively high signal-to-noise ratio is much worse compared to others. The one-sided CF filter stands out when one mostly cares about the autocorrelation coefficients.

6 Conclusion

In this paper we show how conclusions one gets from a standard DSGE model depend on the detrending method. In particular, we compare nine popular detrending methods and find that model parameter estimates, variance decompositions, optimal monetary polices, and out-of-sample forecasting performances of the model are all sensitive to how the data are detrended.

Short of providing a solution, we propose that one should be clear about the criteria of choosing a certain method. Be it forecasting accuracy or the fit of some features of the data, it is preferred to arbitrarily choosing a method. For a better solution and to stay close to the spirit of micro-founded macroeconomic models, one should work on a DSGE model that simultaneously explains both trends and cycles of aggregate data; the issue of choosing a detrending will then become moot.

Appendix

A: An examination of Canova and Ferroni (2011)

Canova and Ferroni (2011) share Boivin and Giannoni (2006)'s idea that the model variables do not have an exact observable counterpart and treat data filtered with alternative procedures as contaminated proxies of the relevant model variables.

Let x_{it} be a vector of size $n \times 1$ of an observable time series filtered with method $i = 1, 2, \ldots, q$ and let $x_t = [x'_{1t}, x'_{2t}, \ldots, x'_{qt}]'$. Assume that the filtered observables are linked to the true cyclical component, x_t^m, with the structure

$$x_t = v_0 + v_1 x_t^m + u_t, \quad u_t \sim (0, \Sigma_u), \tag{19}$$

where v_0 is a $nq \times 1$ vector of constants, v_1 is an $nq \times n$ matrix of nonstructural parameters, and u_t is an $nq \times 1$ vector of possibly serially correlated errors.

The structural parameters of the model are then estimated from multiple sources of cyclical information. They emphasize that "in the estimation it is important to use filters which overestimate the low frequency contribution of the cyclical components" because standard filters resemble high pass filters and tend to underestimate the low frequency contribution. For example, LT and QT filtered data have a stronger low frequency component and standard filters, e.g. HP2, HP1, CF2, and CF1, have a weaker low frequency component than the actual cyclical data.

We first try LT as the filter that overestimates the low frequency contribution and then replace it by QT while choosing one of the four standard filters to be included at a time in the estimation. The structural and nonstructural parameters are jointly estimated using the Bayesian method.

We plot the implied model variables in Figure 2. The results indicate that the implied model variables are sensitive to the choice of the filter, either LT or QT, that overestimates the low frequency contribution. Given the choice of this filter, model variables are consistent across alternative high pass filters. This method of estimating structural parameters from multiple sources of cyclical information does not resolve the problem of making a choice in detrending.

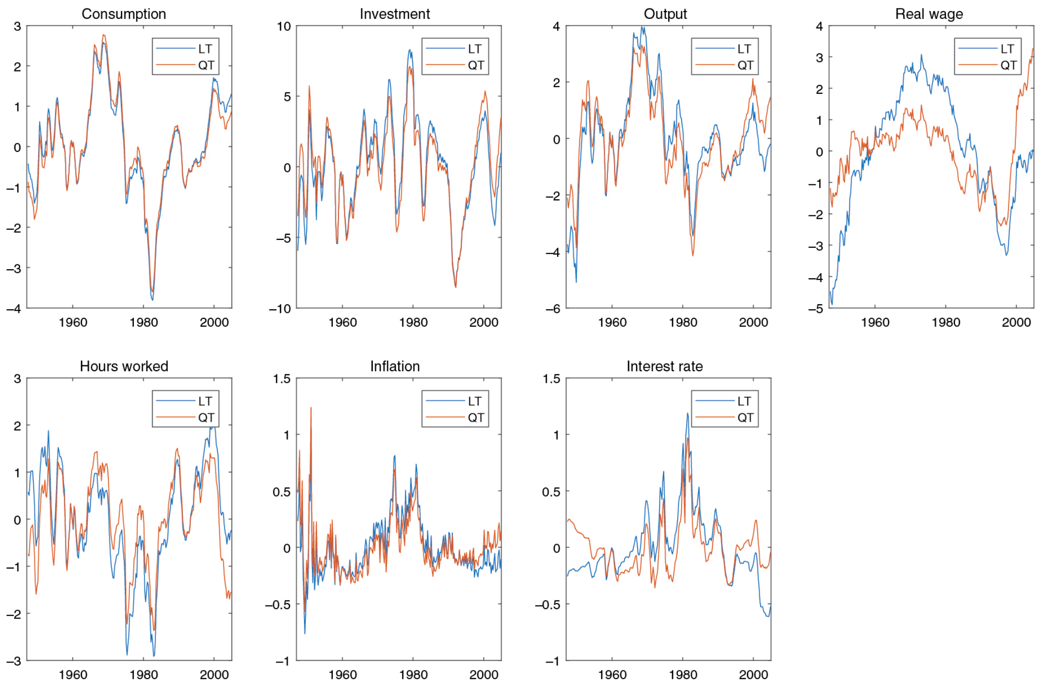


Figure 2: Implied model variables.
Note: LT and QT stand for the implied model variables when the linear trend and quadratic trend filters are combined with any one of the four high pass filters.

B: An extension of the sample

We extend the sample to the last quarter of 2017. All data variables are defined in the same way as in Smets and Wouters (2007). To show that the differences among alternative detrending methods persist with the recent data, we report the variance decomposition results in Table 12; other results are available upon request. Most filters yield consistent variance decomposition results with an extension of the sample except that the risk premium shock explains a much larger fraction of the interest rate fluctuations. This is not surprising given that the model does not take into account the zero lower bound issue. The BN filter with a high signal-to-noise ratio is the only filter that is very sensitive to the data span.

Table 12: Variance decomposition at the 5-year horizon.

FD filter (real variables)	TFP shock	Risk premium	Spending shock	Investment shock	Monetary shock	Price markup	Wage markup
Consumption	11.08	52.60	2.45	0.76	24.80	3.89	4.41
Investment	2.90	15.03	0.57	65.78	8.11	6.21	1.41
Output	20.79	28.82	19.61	10.36	14.11	4.58	1.73
Real wage	4.02	1.90	0.02	0.72	1.11	27.78	64.45
Hours	0.96	22.70	10.21	10.98	13.11	19.36	22.69
Inflation	1.83	10.87	0.63	1.30	8.75	47.50	29.12
Interest rate	1.39	66.45	1.92	11.74	5.71	3.73	9.06
FD filter (all variables)							
Consumption	10.01	51.09	3.03	0.59	28.54	2.94	3.81
Investment	2.74	17.15	0.41	60.35	11.50	6.02	1.83
Output	19.97	29.99	19.11	7.80	17.66	3.88	1.60
Real wage	3.77	1.73	0.02	0.43	1.32	26.63	66.10
Hours	8.36	34.81	22.12	8.68	20.47	3.78	1.79
Inflation	1.20	2.55	0.09	0.19	2.58	84.94	8.45
Interest rate	3.72	50.95	1.70	3.69	32.00	5.84	2.11
Linear trend							
Consumption	31.35	21.72	4.58	2.91	12.00	7.17	20.26
Investment	12.99	13.69	3.43	47.59	8.64	10.43	3.22
Output	39.58	16.61	4.69	11.40	9.70	8.62	9.41
Real wage	31.35	6.03	0.11	4.25	4.22	32.78	21.26
Hours	1.02	28.28	10.62	13.22	16.02	11.24	19.59
Inflation	2.06	8.72	0.65	0.99	7.91	45.68	34.00
Interest rate	1.54	65.01	2.16	10.64	6.01	2.73	11.91
Quadratic trend							
Consumption	25.55	9.85	4.41	3.67	11.99	9.62	34.92
Investment	20.51	1.32	5.71	47.11	6.34	11.27	7.73
Output	39.93	4.84	5.49	12.38	8.74	10.32	18.30
Real wage	30.36	0.62	0.20	4.98	3.63	38.78	21.43
Hours	2.26	9.12	12.40	14.16	14.52	13.35	34.20
Inflation	3.67	1.06	1.19	2.07	11.32	43.61	37.07
Interest rate	6.78	19.28	6.15	20.37	17.16	7.23	23.04
HP filter (2-sided)							
Consumption	6.67	52.90	2.69	2.48	30.64	4.08	0.54
Investment	6.74	18.82	1.14	40.67	26.28	5.20	1.15
Output	17.99	31.76	12.53	6.56	25.84	4.65	0.66
Real wage	6.70	1.25	0.08	1.78	2.51	36.37	51.32
Hours	5.23	38.27	16.00	6.11	28.65	4.70	1.04
Inflation	4.68	1.54	0.15	0.21	5.19	82.80	5.43
Interest rate	7.17	57.83	1.89	0.73	27.70	3.36	1.33
HP filter (1-sided)							
Consumption	8.56	49.31	3.79	2.74	30.02	3.30	2.29
Investment	8.72	16.93	1.83	40.22	24.62	4.52	3.15
Output	21.38	29.79	10.48	6.64	25.30	4.08	2.32
Real wage	10.88	1.67	0.11	2.04	3.34	46.61	35.35
Hours	4.82	38.15	14.48	6.16	29.47	4.11	2.81
Inflation	4.26	1.54	0.19	0.18	5.27	68.88	19.68
Interest rate	7.14	55.19	2.50	1.19	24.64	2.72	6.62

CF filter (2-sided)							
Consumption	7.23	35.75	4.15	1.57	42.76	2.25	6.28
Investment	6.29	14.47	1.60	36.02	27.64	4.94	9.03
Output	12.77	25.83	5.24	7.00	37.19	4.22	7.75
Real wage	14.47	0.92	0.18	2.59	1.95	55.96	23.94
Hours	3.29	30.80	7.47	4.94	41.68	3.39	8.43
Inflation	3.96	4.54	0.15	0.30	13.14	29.55	48.36
Interest rate	3.98	66.65	1.44	1.18	4.18	3.85	18.71
CF filter (1-sided)							
Consumption	16.08	40.09	5.63	2.52	26.32	3.38	5.99
Investment	11.29	11.63	1.71	48.33	14.57	4.61	7.86
Output	25.68	26.22	5.65	8.57	21.87	5.26	6.75
Real wage	19.75	1.89	0.07	2.29	2.46	45.00	28.53
Hours	2.86	37.03	9.15	7.49	29.08	5.16	9.22
Inflation	4.86	5.27	0.24	0.25	9.74	43.06	36.57
Interest rate	5.37	59.75	1.91	2.26	8.63	6.90	15.19
BN filter (low signal-to-noise)							
Consumption	15.69	36.81	5.07	4.96	32.74	3.62	1.11
Investment	15.65	13.31	3.44	41.77	20.77	4.17	0.88
Output	28.76	21.71	8.50	10.71	25.13	4.26	0.95
Real wage	15.06	1.59	0.29	3.12	3.40	33.10	43.44
Hours	3.82	31.79	14.40	10.18	33.51	5.02	1.28
Inflation	4.83	1.08	0.35	0.47	3.42	76.94	12.92
Interest rate	9.21	57.06	3.81	3.00	19.30	2.37	5.25
BN filter (high signal-to-noise)							
Consumption	17.10	25.66	1.00	19.21	31.64	5.12	0.26
Investment	8.50	0.16	2.03	86.32	2.26	0.72	0.01
Output	22.05	4.43	18.89	43.75	8.78	2.05	0.06
Real wage	10.29	0.08	0.16	11.15	1.90	35.82	40.58
Hours	13.37	5.92	26.49	41.20	10.84	2.08	0.09
Inflation	1.99	0.01	0.27	0.84	0.29	94.63	1.97
Interest rate	15.88	5.93	3.73	6.97	57.15	9.34	1.01

Results are obtained using the mean of the the posterior of the parameters.

Notes

1 See also Cogley and Nason (1995) and Murray (2003), among many others.
2 There is also a third but less intensively used approach of connecting DSGE models with data, based on (real or nominal) great ratios [see Cogley (2001) and Whelan (2006)]. As Canova (2014a) points out, taking ratios does not solve the problem because the ratios may still display low frequency movements.
3 Other central bank DSGE models include the Terms-of-Trade Economic Model (ToTEM) at the Bank of Canada [see Murchison and Rennison (2006) and Dorich et al. (2013)], the Bank of England Quarterly Model [BEQM; see Harrison et al. (2005)], the Model for Analysis and Simulations (MAS) at the Central Bank of Chile [see Caputo, Felipe, and Guzman (2006)], the Aggregate General Equilibrium Model with dollarization (MEGA-D) at the Central Reserve Bank of Peru [see Florian and Montoro (2009)], and the Norwegian Economy Model (NEMO) at Norges Bank [see Brubakk and Sveen (2009)].
4 The sensitivity of model parameter estimates to the choice of the detrending method is not a novel finding. Delle Chiaie (2009) also finds that posterior distributions of structural parameters are sensitive to the choice between the HP filter and linear detrending. In this paper, we conduct a more thorough examination of most frequently used detrending methods and show how the difference in model parameter estimates affects other inferences, e.g. variance decompositions, impulse responses, optimal monetary policies, and out-of-sample forecasting performances of the model.
5 See Appendix for a detailed discussion on Canova and Ferroni (2011).
6 As a sensitivity check, we extend the sample to the last quarter of 2017 and find that the differences among alternative detrending methods persist with the recent data. The results are presented in the Appendix. We have to point out that these results do not take in account the zero lower bound (ZLB) effects starting at the last quarter of 2008.
7 Most of the filters we consider here have been used in the literature, for example FD in Smets and Wouters (2007), LT in Ireland (2001) and Bouakez, Cardia, and Ruge-Murcia (2005), QT in Baele et al. (2015), HP2 in Lubik and Schorfheide (2004), HP1 in Jiang (2016), CF2 in Döpke et al. (2008), and the BN filter in Dufourt (2000). Other widely-used filtering methods include the Baxter-King filter (denoted “BK”, see Baxter and King (1999)). The BK filter relies on the use of a symmetric finite odd-order $M = 2K + 1$ moving average, thus it causes a missing of $2K$ observations. We thank one referee for pointing out the possibility of extrapolating the missing observations, but we are concerned that the extrapolation may not be reliable. To keep the amount of observations same across alternative detrending methods for a comparison purpose, we do not consider the BK filter in this work.
8 While real variables usually show long-run drifts, nominal variables such as inflation and interest rate just display low frequency fluctuations. Should we filter all the data or only real variables? While theory suggests nominal variables to be stationary, the stationarity

can hardly be validated empirically. We decide to apply the filters to both real and nominal variables because it is hard to argue that the processes of inflation and interest rate are stationary by looking at the untransformed data as shown in Figure 1 under the "FDR" category. Detrending nominal variables is not something new in the literature. For example, Smets and Wouters (2003) detrend both real and nominal variables by a linear trend and Šustek (2011) detrends all three variables (output, inflation, and interest rate) with the HP filter. Canova and Ferroni (2011) also extract the cyclical component of all variables, including both real and nominal ones, when comparing different detrending procedures.
9 The empirical signal-to-noise ratios obtained from the method proposed by Kamber, Morley, and Wong (2018) are 0.16, 0.29, 0.23, 0.15, 0.28, 0.07, and 0.18 for output, consumption, investment, wage, hours worked, inflation, and interest rate, respectively.

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