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Forecasting exchange rates with elliptically symmetric principal components

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ABSTRACT

We extract elliptically symmetric principal components from a panel of 17 OECD exchange rates and use the deviations from the components to forecast future exchange rate movements, following the method in Engel et al. (2015). Instead of using standard factor models, we apply elliptically symmetric principal component analysis (ESPCA), introduced by Solat and Spanos (2018), which captures both contemporaneous and temporal co-variation among the exchange rates. We find that ESPCA is more accurate than forecasts generated by existing standard methods and the random walk model, with or without including macroeconomic fundamentals.

1. Introduction

The random walk model is hard to beat in forecasting exchange rates, and this finding has more or less survived the numerous studies since Meese and Rogoff (1983b) and Meese and Rogoff (1983a). The model essentially forecasts that the log level of an exchange rate remains the same in the future, and this seemingly simple model beats well-founded, sophisticated models of exchange rates that make use of economic fundamentals like output, interest rates, or inflation rates. It is a well-established finding for horizons from one quarter to three years, while the results are more ambiguous for longer horizons.[1]

Instead of looking for new fundamentals or econometric methods to beat the random walk, some recent papers look for predictability in the exchange rates themselves. In particular, factors are extracted from a panel of exchange rates, and the deviations of the exchange rates from the factors are used to forecast their future changes.[2] Engel et al. (2015) first suggested this new direction and found mixed results. They extracted three factors from a panel of 17 exchange rates (with the US dollar as the base currency), and they found that the factors improved the random walk only for long horizons during the period 1973Q1–2007Q4.

This paper follows the same line of research and extracts factors in a simple and intuitive way using an updated sample from 1973Q1 to 2017Q4. We adopt a more general approach and make use of both temporal (over time) and contemporaneous (synchronous or cross-section) covariations by using elliptically symmetric principal component analysis (ESPCA), proposed in Solat and Spanos (2018).

Though we are agnostic on what the factors represent, we believe that we are better at capturing unobserved fundamentals that make exchange rates persistent and correlated through time. Indeed, we find that relaxing the

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1 Engel, Mark, and West (2015) shows that the random walk dominates when the discount factor is near one and the fundamentals are persistent. For a recent survey on the empirical findings, see Kavtaradze and Mokhtari (2018), Rossi (2013), and Maasoumi and Bulut (2013).

2 For simplicity, we abuse the usage and refer to a principal component as a "factor" in this paper.

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assumptions imposed on classical principal components analysis (PCA) substantially improves the forecasting performance of the factors in Engel et al. (2015) by beating the random walk at all horizons and in all sample periods.

We use ESPCA to extract the elliptically symmetric principal components (ESPCs) and compare our forecasting performance with that in Engel et al. (2015), using the updated data from 1973Q1 to 2017Q4. In addition, in a shorter sample, we compare our results using only ESPCs with those in Engel et al. (2015) using both factors and auxiliary macroeconomics fundamentals.

2. Elliptically symmetric principal component analysis

The idea behind all methods of factorization is to capture the most variations in the data using a few factors with some properties of interest, e.g., independence or orthogonality. Some of the popular factor models are principal component analysis (PCA) and factor analysis (FA), both of which aim at capturing the most variation in the *cross-section* data through decomposition of the *contemporaneous* covariance matrix in a linear regression model.

Solat and Spanos (2018) propose elliptically symmetric principal component analysis (ESPCA) as a twofold extension to PCA. The first extension is to go beyond static (contemporaneous or synchronous) variation and include a certain form of temporal (over time) variation, and the second extension is to go beyond the Gaussian distribution to the elliptically symmetric family of distributions. In other words, if we assume that the list of individual variables remains the same over time, then the ESPCA captures the most variations in the panel of data.

Solat and Spanos (2018) argue that ignoring the nature of the distribution and the temporal dependence in the data results in a biased maximum likelihood estimator (MLE) of the parameters used in PCA. We provide a simple discussion here.

Let $\mathbf{X}_t = (X_{1t}, \ldots, X_{mt})^\top$ be a vector of m random variables. The sampling matrix of order $m \times T$ from the same population is $\mathbf{X} = (\mathbf{X}_1, \ldots, \mathbf{X}_T) \sim \mathrm{ES}_{m,T}(\boldsymbol{\mu}\mathbf{e}_{T\times 1}^\top, \boldsymbol{\Sigma} \otimes \boldsymbol{\Phi}; \psi)$, where $\boldsymbol{\Sigma} \otimes \boldsymbol{\Phi}$ is the Kronecker product between the contemporaneous covariance matrix ($\boldsymbol{\Sigma} : m \times m$) and the temporal covariance matrix ($\boldsymbol{\Phi} : T \times T$), ψ is the characteristic function, $\mathbf{e}_{T\times 1} = (1, \ldots, 1)^\top$, $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_m)^\top$, and $\mu_i, i \in \{1, \ldots, m\}$ is the expected value of the ith row of the sampling matrix $\mathbf{X}$.

Note that when $\psi(.)$ and $\boldsymbol{\Phi}$ are known, the MLE of $\boldsymbol{\Sigma}$ is as follows (see Anderson (2003a) and Gupta, Varga, and Bodnar (2013)):

$$\hat{\boldsymbol{\Sigma}} = \frac{1}{2(T-1)\psi'(0)}\mathbf{X}\Big(\boldsymbol{\Phi}^{-1} - \frac{\boldsymbol{\Phi}^{-1}\mathbf{e}_{T\times 1}\mathbf{e}_{T\times 1}^\top\boldsymbol{\Phi}^{-1}}{\mathbf{e}_{T\times 1}^\top\boldsymbol{\Phi}^{-1}\mathbf{e}_{T\times 1}}\Big)\mathbf{X}^\top \tag{1}$$

where $\Big(\boldsymbol{\Phi}^{-1} - \frac{\boldsymbol{\Phi}^{-1}\mathbf{e}_{T\times 1}\mathbf{e}_{T\times 1}^\top\boldsymbol{\Phi}^{-1}}{\mathbf{e}_{T\times 1}^\top\boldsymbol{\Phi}^{-1}\mathbf{e}_{T\times 1}}\Big)$ is a weighted average matrix presenting the temporal dependence property of the $\mathbf{X}$. Solat and Spanos (2018) argue that assuming temporal independence (i.e., $\boldsymbol{\Phi} = \mathbf{I}_T$), as PCA does, reduces this weighted average matrix to the *deviation from the mean* matrix $(\mathbf{I}_T - \mathbf{e}_{T\times 1}(\mathbf{e}_{T\times 1}^\top\mathbf{e}_{T\times 1})^{-1}\mathbf{e}_{T\times 1}^\top)$, which results in a biased estimation of $\boldsymbol{\Sigma}$.

ESPCA uses the eigenvalue decomposition method to decompose both a contemporaneous covariance matrix ($\boldsymbol{\Sigma}$) and a temporal covariance matrix ($\boldsymbol{\Phi}$) to extract ESPCs as follows:

$$\boldsymbol{\Sigma} = \mathbf{A}^\top\boldsymbol{\Lambda}_m\mathbf{A} \quad , \quad \boldsymbol{\Phi} = \mathbf{B}^\top\boldsymbol{\Gamma}_T\mathbf{B}, \tag{2}$$

where $\boldsymbol{\Lambda}_m = diag(\lambda_1, \ldots, \lambda_m)$ and $\boldsymbol{\Gamma}_T = diag(\gamma_1, \ldots, \gamma_T)$ are diagonal matrices that contain eigenvalues of $\boldsymbol{\Sigma}$ and $\boldsymbol{\Phi}$ in descending order, respectively; also, $\mathbf{A}$ and $\mathbf{B}$ are matrices that consist of the corresponding orthonormal[3] eigenvectors of $\boldsymbol{\Sigma}$ and $\boldsymbol{\Phi}$, respectively.

Define $\mathbf{A}_p$ as an $m \times p$ matrix containing the first p columns of $\mathbf{A}$. The first p ESPCs are

$$\begin{aligned}\mathbf{Y} &= \mathbf{A}_p^\top(\mathbf{X} - \boldsymbol{\mu}\mathbf{e}_{T\times 1}^\top)\mathbf{B} \\ &\sim \mathrm{ES}_{p\times T}(\mathbf{0}_{p\times T}, (\mathbf{A}_p^\top\boldsymbol{\Sigma}\mathbf{A}_p) \otimes (\mathbf{B}^\top\boldsymbol{\Phi}\mathbf{B}); \psi).\end{aligned}$$

Also, because $\mathbf{A}_p$ and $\mathbf{B}$ are orthonormal matrices,

$$\mathbf{Y} = \mathbf{A}_p^\top(\mathbf{X} - \boldsymbol{\mu}\mathbf{e}_{T\times 1}^\top)\mathbf{B} \sim \mathrm{ES}_{p\times T}(\mathbf{0}_{p\times T}, \boldsymbol{\Lambda}_p \otimes \boldsymbol{\Gamma}_T; \psi), \tag{3}$$

which means that the ESPCs are contemporaneously and temporally independent. By assuming normality, Table 1 summarizes the assumptions imposed on the joint distribution of ESPCs together with the statistical generating mechanism.

Under the assumption [3] in Table 1, normal ESPCs are independent contemporaneously and temporally. This property allows for a static factor model when the data exhibit some form of temporal dependence.

3. Empirical results

3.1. Data

We use end-of-quarter data on the log nominal bilateral US dollar exchange rates of 17 OECD countries from 1973Q1 to 2017Q4.[4] The countries are Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Japan, Italy, Korea, Netherlands, Norway, Spain, Sweden, Switzerland, and the United Kingdom. Table 2 presents a descriptive statistical summary of the data.

We construct three sets of out-of-sample forecasts. First, for the nine non-Euro currencies (Australia, Canada, Denmark, Japan, Korea, Norway, Sweden, Switzerland, and the United Kingdom) called the "long sample", the forecasting period is 1987Q1 to 2017Q4. Second, for all 17 currencies (Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Japan, Italy, Korea, Netherlands, Norway, Spain, Sweden, Switzerland, and the United Kingdom) called the "early sample", the forecasting period is 1987Q1 to 1998Q4 (before the Euro). Finally, for 10 currencies (countries included in long sample plus the Euro) called the "late sample", the forecasting period is 1999Q1 to 2017Q4. Table 3 summarizes the out-of-sample forecasting samples for different samples (summary statistics in changes are available in the Online Appendix).[5]

3 Mutually orthogonal and all of unit length.

4 The data source is International Financial Statistics by the IMF.

5 https://www.dropbox.com/s/p25djndnt12gky1/Online%20Appendix.pdf?dl=0.

Table 1
Normal elliptically symmetric principal components analysis model.

General statistical mechanism	$\mathbf{Y}{=}\mathbf{A}_p^\top(\mathbf{X}-\boldsymbol{\mu}\mathbf{e}_{T\times1}^\top)\mathbf{B}+\boldsymbol{\epsilon}$
[1] Normality	$\mathbf{Y}\sim$N(., .),
[2] Linearity	$E(\mathbf{Y}){=}\mathbf{A}_p^\top(\mathbf{X}-\boldsymbol{\mu}\mathbf{e}_{T\times1}^\top)\mathbf{B}$,
[3] Constant covariance	$Cov(\mathbf{Y}){=}\Lambda_p\otimes\Gamma_T$,
[4] Independence	$\{\mathbf{Y}_t, t{\in}\mathbb{N}\}$ is an independent process,
[5] t-invariance	$\boldsymbol{\theta}{:=}(\boldsymbol{\mu}, \mathbf{A}_p, \Lambda_p)$ is not changing with t.

Table 2
Summary statistics for exchange rates.

Country	# Obs.	Mean	Std	Min	Max	Skewness	Kurtosis
Australia	180	0.179	0.246	−0.399	0.715	−0.36	2.63
Canada	180	0.197	0.133	−0.036	0.466	0.01	2.09
Denmark	180	1.867	0.176	1.551	2.421	1.03	3.80
United Kingdom	180	−0.523	0.155	−0.949	−0.145	−0.55	3.20
Japan	180	4.955	0.389	4.339	5.721	0.60	2.05
Korea	180	6.742	0.334	5.985	7.435	−0.60	2.51
Norway	180	1.881	0.164	1.577	2.230	0.18	2.20
Sweden	180	1.893	0.245	1.371	2.384	−0.55	2.49
Switzerland	180	0.392	0.328	−0.181	1.177	0.40	2.43
Austria	180	2.560	0.218	2.164	3.093	0.53	2.32
Belgium	180	3.573	0.182	3.239	4.144	0.93	3.78
France	180	1.709	0.185	1.391	2.261	0.71	3.27
Germany	180	0.605	0.215	0.213	1.147	0.54	2.37
Spain	180	4.762	0.310	4.025	5.279	−0.92	3.04
Italy	180	7.215	0.313	6.335	7.733	−1.06	3.52
Finland	180	1.544	0.164	1.263	1.948	0.40	2.34
Netherlands	180	0.713	0.203	0.332	1.267	0.56	2.61

Quarterly log-exchange rates (in level) based on the US dollar 1973Q1–2017Q4.

Table 3
Out-of-sample forecasting samples.

			Horizon h			
Sample	Prediction period	Number of currencies	$h=1$	$h=4$	$h=8$	$h=12$
Early Sample	1986Q4 + h to 1998Q4 + h	17	49	49	49	49
Late Sample	1999Q1 + h to 2017Q4	10	75	72	68	64
Long Sample	1986Q4 + h to 2017Q4	9	124	121	117	113

- Following the method presented in Engel et al. (2015), we use 17 currencies to estimate the ESPCs for all recursive samples.
- Euro area forecasting is the average of forecasts from eight eurozone countries.

3.2. Models and results

In this section, we show that ESPCA outperforms traditional methods of factor modeling in the context of exchange rate forecasting, following the discussion in Engel et al. (2015). We want to see if there is any improvement in the context of out-of-sample forecasting by replacing their factorization method with ESPCA. Although in some cases the PCA method for British Pound, as a base currency, shows improvement when compared to factor analysis (FA), Engel et al. (2015) conclude that the overall results for the FA method are better than the PCA method. For completeness, we compare the out-of-sample forecasting capacity of ESPCA to FA and PCA (additional results using the British Pound as the base currency are available in the Online Appendix).

To determine the number of ESPCs, we use the eigenvalue test, which gives the percentage of variation explained through the retained ESPCs. It turns out that three components explain 95% of the variation in the data, similar to PCA.[6] Using the method explained in Section 2, we derive ESPCs and estimate the coefficients based on the following model:

$$\begin{aligned} s_{it} &= const. + \delta_{1i} espc_{1t} + \delta_{2i} espc_{2t} + \delta_{3i} espc_{3t} + u_{it}, \\ &= const. + ESPC_{it} + u_{it}, u_{it} \sim \text{NIID}(0, \sigma_u^2), \end{aligned} \tag{4}$$

where s_{it} is the log of nominal exchange rates of currency i based on the US dollar; the derived ESPCs are $espc_{1t}$, $espc_{2t}$ and $espc_{3t}$; and δ_{1i}, δ_{2i}, and δ_{3i} are the corresponding factor loadings, respectively. Furthermore, we have the same unit of measurement for exchange rates based on the dollar, which suggests that there is no need to scale the data. Also, ESPCs aim to capture the most variation in the data in both dimensions, so it would be fair to let the variable with higher variation contribute more.

We use $ESPC_{it} = \delta_{1i} espc_{1t} + \delta_{2i} espc_{2t} + \delta_{3i} espc_{3t}$ to predict s_{it}. The rest of the model is the same as Engel

6 Based on the eigenvalue test, the sufficient number of ESPCs for each round of the recursive out-of-sample forecasting is also three.

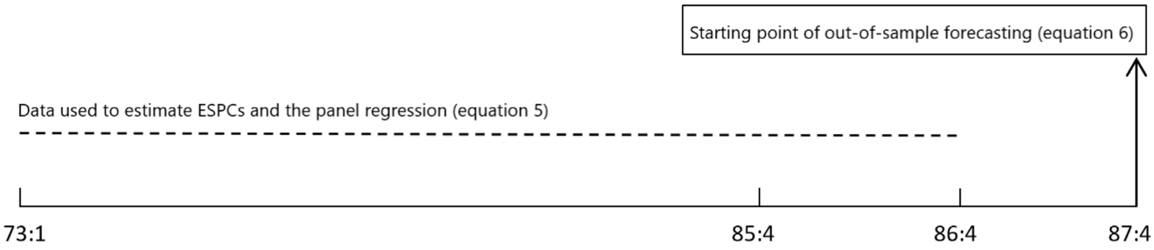

Fig. 1. Forecasting procedure ($h = 4$, long sample).

et al. (2015). First, we assume that $ESPC_{it} - s_{it}$ is stationary and may be useful to capture the stationary regularity of future values of s_{it} through $s_{it+h} - s_{it}$, where $h = 1, 4, 8, 12$ quarterly horizons of forecasting.[7]

Let $\widehat{ESPC}_{it} = \hat{\delta}_{1i}\widehat{espc}_{1t} + \hat{\delta}_{2i}\widehat{espc}_{2t} + \hat{\delta}_{3i}\widehat{espc}_{3t}$ for currency i. We use it as a central tendency to estimate the coefficients of the following regression:

$$s_{it+h} - s_{it} = \alpha_{i,h} + \beta_h(\widehat{ESPC}_{it} - s_{it}) + \epsilon_{it+h}, \ \epsilon_{it+h} \sim \text{NIID}(0, \sigma^2_{\epsilon_i}) \tag{5}$$

where $\alpha_{i,h}$ is the individual effect of currency i for time horizon h. The estimated coefficients $\hat{\alpha}_{i,h}$ and $\hat{\beta}_h$ can be used to predict the future value of the nominal exchange rates.

Fig. 1 illustrates the procedure for quarterly horizon $h = 4$ in forecasting the “long sample".

For this example, we use data from 1973Q1 to 1986Q4 to estimate $\widehat{ESPC}_{it}$ and then estimate the panel regression:

$$s_{it+4} - s_{it} = \alpha_{i,4} + \beta_4(\widehat{ESPC}_{it} - s_{it}) + \epsilon_{it+4}, \quad t \in \{1973Q1, \ldots, 1985Q4\}. \tag{6}$$

Using the estimated coefficients $\hat{\alpha}_{i,4}$ and $\hat{\beta}_4$ from the regression (6), we evaluate the predicted value of $s_{i,1987Q4} - s_{i,1986Q4}$ using the following equation:

$$s_{i,1987Q4} - s_{i,1986Q4} = \hat{\alpha}_{i,4} + \hat{\beta}_4(\widehat{ESPC}_{i,1986Q4} - s_{i,1986Q4}). \tag{7}$$

We repeat this procedure by adding another observation to the end of the sample to produce predictions by a recursive method, re-estimating everything.[8]

Fig. 2 depicts the time plot of three ESPCs derived based on the sample period 1973Q1–2017Q4. For PCA, most of the cross-sectional (contemporaneous) variation is captured by the first few factors. In addition to what has been captured by PCA, ESPCA also captures the temporal (over time) variation across all variables. That is why factors are converging to a similar pattern (except for the volatility), despite some differences in the first few observations.[9] Note that the convergence of ESPCs does not mean they are similar or identical. In fact, they are converging to the same pattern after the very different first few observations, which have extreme values. At some point, the only difference between the components is their scale, but the differences between extreme values in the first few observations will be enough to capture different temporal regularity patterns in the data.

Table 4
Summary statistics for ESPCs.

Factors	# Obs.	Mean	Std	Min	Max	Skew	Kurtosis
First ESPC	175**	0.00	0.08	−0.34	0.33	0.00	5.36
Second ESPC	175**	0.00	0.32	−1.34	1.30	0.01	5.40
Third ESPC	175**	0.00	0.01	−0.03	0.03	0.15	6.83

- Three ESPCs are derived based on the sample period 1973Q1–2017Q4.
** We excluded the first five observations with extreme volatility for each component.

ESPCs are picking up the variation in two dimensions, which make coefficients ambiguous to interpret. Capturing temporal variation along with contemporaneous variation in the data by ESPCs picks up a sort of variation that might be common among individual variables and over time. This makes it hard to decompose the individual effect and time effect of each component in coefficients, unless there is no temporal variation, as assumed in PCA. However, our purpose in this study is to improve the predictive power of the model, so we need not worry much about what each ESPC is picking up individually.

Table 4[10] and Table 5 provide summary statistics of the three ESPCs and their factor loadings derived based on the whole sample (1973Q1 to 2017Q4), respectively. We see from the tables the intuition behind ESPCA. While all factors evolve in a similar manner over time, the third ESPC has the smallest standard deviation and largest skewness and kurtosis (in term of absolute value), and currencies that are more volatile have larger loadings on the factor (e.g., Korea, Japan, Italy, and Spain).

The forecast is evaluated by the root mean squared prediction error (RMSPE). We then compute Theil's

[7] We use the CISP statistic introduced in Pesaran (2007) to test the unit root hypothesis of $ESPC_{it} - s_{it}$. We use the model with intercept and the lag orders of 1 and 4. In both cases, the unit root hypothesis of $ESPC_{it} - s_{it}$ with lag order 1 and lag order 4 are rejected with $p - value = 0.02$ and $p - value = 0.03$, respectively.

[8] We need to centralize the data to extract the factors, and, to make sure that the forecasts are truly out-of-sample, data are centralized using only in-sample data. That is, the data are re-centered recursively to have zero mean.

[9] In order to illustrate the convergence of the patterns of all ESPCs, we refer the reader to the plot inside each box, which is the plot of the same component starting from the sixth observation.

[10] The first few observations in each component capture the most temporal (over time) variation in the data that make them extreme in scale. In order to provide a realistic insight into the regularity pattern of the components, we start from the sixth observation to create Table 4 (summary statistics related to the plots inside the boxes in Fig. 2).

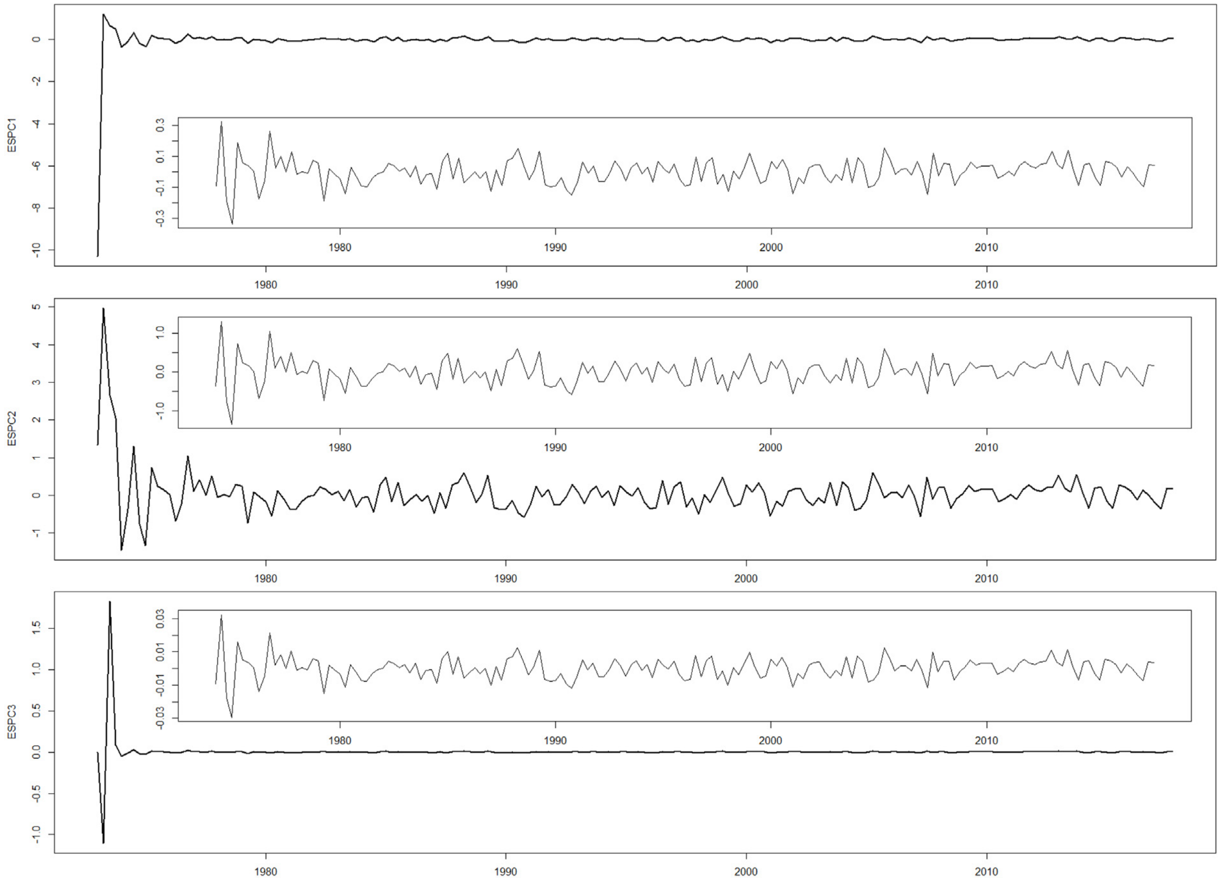

Fig. 2. Elliptically symmetric principal components t-plot (plot without the first five observations inside the box).

Table 5
Factor loadings.

	ESPC1	ESPC2	ESPC3
Australia	0.04	−0.10	−0.11
Canada	0.01	−0.04	−0.03
Denmark	0.00	−0.02	−0.02
United Kingdom	0.03	−0.05	−0.04
Japan	−0.05	0.08	0.10
Korea	0.06	−0.12	−0.13
Norway	0.01	−0.03	−0.02
Sweden	0.03	−0.07	−0.07
Switzerland	−0.06	0.11	0.12
Austria	−0.04	0.06	0.05
Belgium	−0.01	0.01	0.01
France	0.01	−0.05	−0.04
Germany	−0.04	0.05	0.05
Spain	0.05	−0.11	−0.12
Italy	0.07	−0.13	−0.15
Finland	0.01	−0.03	−0.03
Netherlands	−0.03	0.04	0.03

Factor loadings are derived based on the sample period 1973Q1–2017Q4.

U-statistic (Theil, 1971), which is equal to a ratio by dividing the RMSPE of the factor model to the RMSPE of the random walk model. A U-statistic less than one means that the factor model performs better than random walk. Also, to allow for parameter uncertainty, we use the t-test proposed by Clark and West (2006) to test the hypothesis that H_0: $U = 1$ vs. H_1: $U < 1$, based on a 0.10 significance level with a rejection region defined by $(\tau(\mathbf{X}) > 1.282)$.

Table 6 presents the median Theil's U-statistic[11] and Clark–West t-statistics[12] for early, late, and long samples for all models.[13]

The first column indicates the factor model that has been used in the forecasting evaluations. The second column lists the type of sample and the number of currencies in that sample.[14] The last four columns report the median U-statistic used to evaluate the forecastability power of the models for different horizons ($h = 1, 4, 8, 12$) and samples. In addition, we report the number of currencies in the sample that have a U-statistic value less than one and the number of currencies in the sample for which the

11 The U-statistic is defined as the ratio of $RMSPE_{Model}$ to $RMSPE_{RandomWalk}$. Results for individual countries are available upon request.

12 The number of rejections for the t-test proposed by Clark and West (2006) to test the hypothesis that H_0: $U = 1$ vs. H_1: $U < 1$, based on a 0.10 significance level with a rejection region defined by $(\tau(\mathbf{X}) > 1.282)$.

13 We use the codes at http://www.ssc.wisc.edu/~cengel/Data/Factor/FactorData.htm and http://econ.nsysu.edu.tw/files/11-1124-1326.php?Lang=en to reproduce the results in Engel et al. (2015) and Wang and Wu (2015), respectively.

14 We use all the currencies to extract ESPCs.

Table 6
Forecast evaluation: exchange rates only (1973Q1–2017Q4).

		Median U-statistic (#$U<1$) [#$t>1.282$]			
Model	Sample(# Currencies)	horizon=1	horizon=4	horizon=8	horizon=12
$\hat{ESPC}_{it}-s_{it}$	Long sample (9)	**0.993**(9)[2]	**0.963**(8)[1]	**0.922**(7)[1]	**0.917**(7)[0]
$\hat{F}_{it}-s_{it}$	Long sample (9)	**0.998**(6)[0]	**0.991**(6)[2]	1.013(4)[2]	1.027(4)[1]
$\hat{PC}_{it}-s_{it}$	Long sample (9)	1.007(1)[0]	1.048(1)[0]	1.076(1)[0]	1.110(0)[0]
$\hat{ESPC}_{it}-s_{it}$	Early sample (17)	**0.993**(15)[2]	**0.958**(14)[3]	**0.921**(15)[3]	**0.978**(9)[1]
$\hat{F}_{it}-s_{it}$	Early sample (17)	1.000(8)[1]	**0.995**(9)[0]	1.000(8)[0]	1.130(3)[0]
$\hat{PC}_{it}-s_{it}$	Early sample (17)	1.015(3)[1]	1.075(4)[0]	1.101(4)[0]	1.195(2)[0]
$\hat{ESPC}_{it}-s_{it}$	Late sample (10)	**0.992**(9)[2]	**0.965**(9)[1]	**0.916**(9)[1]	**0.870**(9)[1]
$\hat{F}_{it}-s_{it}$	Late sample (10)	**0.997**(7)[1]	**0.987**(6)[2]	**0.963**(6)[1]	**0.925**(6)[3]
$\hat{PC}_{it}-s_{it}$	Late sample (10)	1.015(2)[0]	1.055(2)[1]	1.099(3)[0]	1.147(3)[0]

- $\hat{ESPC}_{it}-s_{it}$, $\hat{F}_{it}-s_{it}$, and $\hat{PC}_{it}-s_{it}$ represent deviations from factors produced by ESPCA, Factor Analysis (FA), and Principal Component Analysis (PCA), respectively.
- The bold numbers represent those median Theil's U-statistics that are less than one, which means the specified factor model is predicting better than the random walk in terms of the root mean squared predict error (RMSPE).
- (#$U<1$) reports the number of currencies for which the Theil's U-statistic is less than one.
- [#$t>1.282$] reports the number of currencies for which the t-test proposed by Clark and West (2006) (H_0: U=1 vs. H_1: $U<1$) has been rejected based on the 0.1 significance level ($t>1.282$).

t-test proposed by Clark and West (2006) (H_0: $U=1$ vs. H_1: $U<1$) has been rejected based on the 0.1 significance level ($t>1.282$).

The results presented in Table 6 show that, in term of the median Theil's U-statistic, ESPCA outperforms the different factor models and the drift-less random walk model in almost all cases.

However, the number of currencies for which the t-test proposed by Clark and West (2006) (H_0: $U=1$ vs. H_1: $U<1$) has been rejected ($t>1.282$) provides mixed results. We believe this discrepancy between the U-statistic and the Clark and West (2006) test is due to the small level of parameter uncertainty in ESPCA. As we can see from Table 4, the three ESPCs stabilize after a few observations. Since ESPCA captures the dependence over time, the forecasts are much less volatile than those from other factor models (plots of forecasts from ESPCA together with those from PCA are available in the Online Appendix). As a result, while ESPCA performs well in terms of having a lower RMSPE, it may not do as well once we take into account the variance of the forecasts.

We also conduct Giacomini–Rossi's fluctuation test. Giacomini and Rossi (2010) propose a method that allows for testing the performance of two competing methods in the context of out-of-sample forecasting in the presence of instabilities. We report all the results in an Online Appendix. The results show time-varying forecasting performance, as expected for exchange rates, but the ESPCs are still more often significantly better than either FA or PCA.

3.3. Auxiliary macro-variables

Engel et al. (2015) use three sets of auxiliary macro-variables as a measure of the central tendency to improve the factor model in a way that captures more regularity patterns in the exchange rates to forecast more accurately. These auxiliary macro-variables are the "monetary model" (Mark, 1995), "Taylor rule" (Molodtsova & Papell, 2009), and PPP (Engel, Mark, West, Rogoff, & Rossi, 2007). Since it is unclear how some variables are calculated, in order to make sure that we are using the same macroeconomic variables, we use the updated data provided by Wang and Wu (2015) for the period 1973Q1 to 2011Q2.

Table 7 compares the results obtained by ESPCA without any auxiliary macro-variables with those of the FA method presented in Engel et al. (2015) using auxiliary macro-variables.

Based on the results presented in Table 7, ESPCA outperforms the FA method with auxiliary macro-variables on forecasting grounds, except for a long horizon ($h=12$) in long and late samples. The forecasting power of ESPCs does not seem to be coming from proxies for the macroeconomic variables.

4. Conclusion

In this paper, we showed that using factors that incorporate both contemporaneous and temporal correlations substantially improves out-of-sample forecasting performance. Exchange rates were found to converge to such factors, while the convergence was not as clear when traditional methods of extracting factors were used (with or without including macroeconomic fundamentals). What do these factors represent? What are the underlying economic forces? Clearly, these questions go beyond the econometric method we have employed, and we leave them for future research.

Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.ijforecast.2020.11.007.

Table 7
Forecast evaluation: ESPCA vs. FA + Macro-variables (1973Q1–2011Q2).

		Median U-statistic (#U < 1) [#t > 1.282]			
Model	Sample(# Currencies)	horizon=1	horizon=4	horizon=8	horizon=12
$\hat{ESPC}_{it} - s_{it}$	Long sample (9)	**0.996**(7)[1]	**0.963**(7)[0]	**0.926**(8)[0]	**0.905**(8)[0]
$\hat{F}_{it} - s_{it} + Taylor$	Long sample (9)	1.008(1)[1]	1.035(0)[0]	1.068(1)[0]	1.052(3)[0]
$\hat{F}_{it} - s_{it} + Monetary$	Long sample (9)	1.008(3)[2]	1.064(3)[2]	1.200(4)[2]	1.456(4)[3]
$\hat{F}_{it} - s_{it} + PPP$	Long sample (9)	1.002(3)[0]	**0.993**(6)[1]	**0.942**(7)[3]	**0.903**(5)[0]
$\hat{ESPC}_{it} - s_{it}$	Early sample (17)	**0.993**(15)[3]	**0.957**(14)[3]	**0.919**(16)[3]	**0.973**(9)[1]
$\hat{F}_{it} - s_{it} + Taylor$	Early sample (17)	1.010(3)[0]	1.041(2)[0]	1.103(4)[0]	1.156(3)[0]
$\hat{F}_{it} - s_{it} + Monetary$	Early sample (17)	**0.995**(10)[3]	**0.997**(9)[4]	1.115(7)[3]	1.190(7)[4]
$\hat{F}_{it} - s_{it} + PPP$	Early sample (17)	**0.998**(7)[0]	**0.972**(14)[1]	1.015(8)[1]	1.098(3)[0]
$\hat{ESPC}_{it} - s_{it}$	Late sample (10)	**0.993**(7)[0]	**0.970**(8)[0]	**0.886**(9)[0]	**0.789**(10)[0]
$\hat{F}_{it} - s_{it} + Taylor$	Late sample (10)	1.009(2)[1]	1.036(2)[0]	1.004(4)[0]	**0.828**(8)[1]
$\hat{F}_{it} - s_{it} + Monetary$	Late sample (10)	1.013(3)[1]	1.033(4)[1]	**0.977**(6)[3]	1.126(5)[3]
$\hat{F}_{it} - s_{it} + PPP$	Late sample (10)	1.005(4)[0]	**0.999**(5)[0]	**0.900**(8)[0]	**0.727**(9)[5]

- ($\hat{ESPC}_{it} - s_{it}$) and ($\hat{F}_{it} - s_{it}$) represent deviations from factors produced by ESPCA and FA, respectively. The auxiliary macro-variables are the "monetary model" (Mark, 1995), "Taylor rule" (Molodtsova & Papell, 2009), and PPP (Engel et al., 2007).
- The bold numbers represent median Theil's U-statistics less than one, meaning that the specified factor model is predicting better than the random walk in terms of the root mean squared predict error (RMSPE).
- (#U < 1) reports the number of currencies for which the Theil's U-statistic is less than one.
- [#t > 1.282] reports the number of currencies for which the t-test proposed by Clark and West (2006) (H_0: U=1 vs. H_1: U < 1) has been rejected based on the 0.1 significance level (t > 1.282).

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