Peak Wireless Power Transfer Using Magnetically Coupled Series Resonators

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***Abstract*—Wireless power transfer can create the illusion of portable devices with infinite power supplies. Power transfer using magnetically coupled series resonators is maximized when the load presented to the sender is matched to the series impedance of the source. This determines an optimal separation distance between the sender and the receiver. However, the maximum power transferred only depends on the losses in the system and is independent of this distance and the resonant frequency. Closed-form expressions describe the power transferred and efficiency for all distances and system values. These expressions are validated with systems operating at 23 kHz, 40 kHz and 59 kHz. The performance of resonant magnetic coupling for wireless power applications is bounded to have efficiencies less than 50% at the maximum power distance and beyond. Higher efficiencies are only possible at shorter distances where the load presented to the sender is larger than the series impedance of the source.**

***Index Terms*—wireless power transfer, magnetic resonance, coupled resonators, ambient power**

I. Introduction

Wireless power transfer is motivated by the desire for devices that are portable at all times, without any interruptions. While wireless communication enables modern devices to offer the illusion of infinite storage and computing power, users are still aware of the energy requirements of their portable devices since they must manage power consumption and periodically recharge or replace batteries. Moreover, energy storage elements are often the largest and heaviest component in portable devices, and their capacities limit the active lifetime of devices.

Ambient power, the technology to obtain power from the surrounding environment at any time without any physical connection, could create the illusion of an infinite power supply. Portable devices could be charged or powered directly without limiting their mobility. They would never have to be connected to a power source, touched or opened to exchange an energy storage element. As a result, they could be permanently sealed, which might be ideal in environments hostile to electronics, such as outer space, under water, or inside living tissue. The application scenarios for ambient power range from powering keyboards, mice and computer monitors to recharging robots or autonomous vehicles. Several functions could become easier, more flexible, and safer, such as powering implanted devices. More importantly, this technology would enable applications that are currently unimaginable because of power constraints, such as high power medical devices that can operate as they move throughout the human body.

Ambient power is a possible scenario that parallels the development of cellular communications or wireless area networks, in a manner conceptually similar to *pervasive* or *ubiquitous computing*. It consists of a system where the receiver is a small part of a portable device and several larger power senders are distributed around it. As long as the device is within some range of distance from a sender it can function indefinitely. As the device moves outside the range of one power sender, either another power sender can take over, or storage elements can sustain its operation until the device is again within range. Such a system may be fundamentally or technologically subject to stringent efficiency and size constraints [1], which may limit the practical feasibility of the system as a universal solution. Nonetheless, there are important cases in which the desire for completely wireless operation of a device outweigh the inefficiencies of the system as a whole, such as in medical applications.

One technique for wireless power transfer that has been extensively used is magnetic induction [2], [3]. However, the capacity to transmit power efficiently and at reasonable levels depends directly on the magnetic coupling between the devices, which decays as the inverse cube of the distance separating the sender and receiver. Therefore, devices that use this technology must transmit power at distances close to contact or increase sender and receiver size [4], [5], [6]. In particular, the voltages induced by external magnetic fields are much smaller than the voltages generated by the self inductance of the device. A solution in this case is to cancel the impedance of the self inductance. This can be achieved by adding or using the device's own structure to create a capacitive impedance and operating the system at resonance. This strategy was first proposed by Tesla, though it is unclear if he managed to use it successfully for power transfer [7]. Inductive resonance has also been used in many applications outside the field of wireless power transfer, such as mitigation of integrated inductor losses in RF IC components [8], [9].

The system described in this paper is composed of two coils, as shown schematically in Figure 1 and physically in Figure 5. The self inductance of each coil is canceled by adding a capacitors in series with each coil and then driving the sending coil with a voltage source and attaching a resistive load to the receiving coil. The capacitors are chosen such that the sender and the receiver share the same resonant frequency, and the source drives the system at that frequency. Several systems have been studied that utilize either resonant senders or receivers [4], [10], but little work has been done on systems that exploit resonance on both ends. The analysis of such a system in [11], [12] uses coupled-mode theory to characterize transmission at mid-range distances with self-resonant coils. However, their results do not show the power peaks that result from the resonant coupling. The present paper extends the work of [1] by characterizing the power transfer and efficiency over all separation distances between the sender and the receiver, including where the magnetic coupling is strong. It focuses on the maximum achievable power transfer, the associated optimal coil separation distance and the efficiency at this point in systems using series resonators as both the sender and receiver.

II. Analysis

The circuit shown in Figure 1 is a model of a wireless power system created using a pair of magnetically coupled resonators as described in the previous section. The power delivered to the load resistance R_o is maximized when the impedances of the inductors L_1 and L_2 are canceled by the impedance of the capacitors C_1 and C_2. Assuming that the system is designed such that these two resonant frequencies coincide, the circuit model reduces to the one shown in Figure 2, where the only remaining resistances in the system are the internal losses of each coil and the load resistance connected to the receiving coil. The dependent voltage sources in each loop represent the electromotive force induced in each coil by the current flowing in the opposite coil, and are related by the impedance of the mutual inductance at resonance. This coupling presents the sender coil with a real impedance such that

$$Z_s = \frac{V_s}{I_s} = R_{L_1}[1 + \kappa^2 Q_1 Q_2 (1 - \eta_x)] \tag{1}$$

where Q_1 and Q_2 are the quality factors of the sender and receiver resonators,

$$Q_1 \stackrel{\text{def}}{=} \frac{\sqrt{L_1/C_1}}{R_{L_1}} \qquad Q_2 \stackrel{\text{def}}{=} \frac{\sqrt{L_2/C_2}}{R_{L_2}}, \tag{2}$$

η_x is the ideal maximum efficiency that the system can achieve at resonance,

$$\eta_x = \frac{R_o}{R_o + R_{L_2}}. \tag{3}$$

and κ is the magnetic coupling coefficient between the coils. The magnetic coupling coefficient κ between two coils is approximately related to the distance between them by

$$\kappa = \frac{1}{(1 + \sqrt[3]{4}\delta^2)^{3/2}} \tag{4}$$

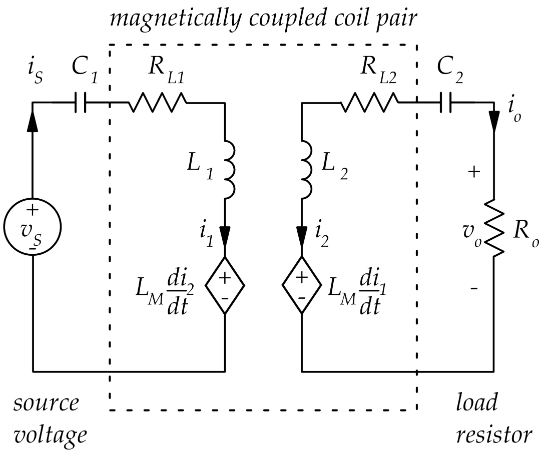


Fig. 1. Circuit model for wireless power transfer using magnetically coupled resonators. The mutual inductance $L_M = \kappa\sqrt{L_1 L_2}$, where κ is the magnetic coupling coefficient, and $0 < \kappa < 1$.

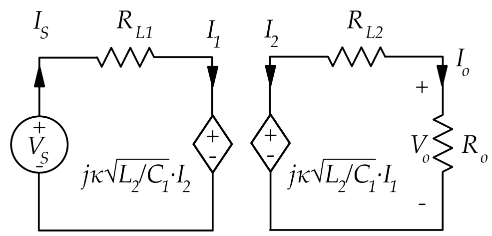


Fig. 2. Equivalent model for the circuit shown in Figure 1 under resonant conditions. I_s and I_o are the complex amplitudes of i_s and i_o, respectively.

where $\delta = d/\sqrt{r_1 r_2}$, d is the distance that separates the coils, and r_1 and r_2 are the effective radius of the sender and receiver coils, respectively [13]. This equation applies to most device geometries since finite magnetic sources behave as dipoles at far enough distances.

The power delivered to the load $P_o = \frac{|V_o|^2}{2R_o}$ is

$$P_o = \frac{V_s^2}{2R_{L_1}} \frac{\kappa^2 Q_1 Q_2 (1 - \eta_x)}{[1 + \kappa^2 Q_1 Q_2 (1 - \eta_x)]^2} \eta_x. \tag{5}$$

The input power $P_s \stackrel{\text{def}}{=} \Re\{V_s I_s^*/2\}$ is

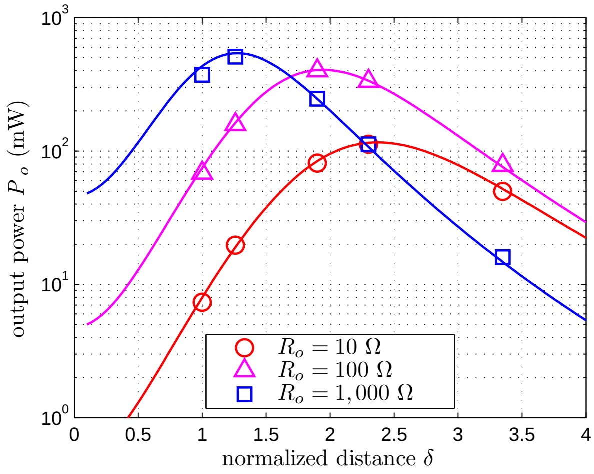
$$P_s = \frac{V_s^2}{2R_{L_1}} \cdot \frac{1}{1 + \kappa^2 Q_1 Q_2 (1 - \eta_x)} \tag{6}$$

and the efficiency $\eta \stackrel{\text{def}}{=} P_o/P_s$ is

$$\eta = \frac{\kappa^2 Q_1 Q_2 (1 - \eta_x)}{1 + \kappa^2 Q_1 Q_2 (1 - \eta_x)} \eta_x. \tag{7}$$

Figure 3 shows the power delivered and efficiency as a function of distance, since the magnetic coupling coefficient κ decreases monotonically with the normalized distance δ.

System performance depends on the separation distance d through the magnetic coupling coefficient κ. Since the coupling coefficient is a function of the separation distance normalized by the geometric mean of the coil radii, the transmission distance must be considered within the context of the sender and



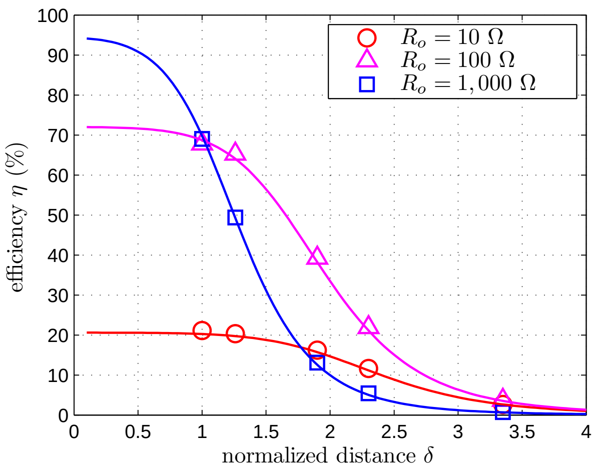


Fig. 3. Power P_o delivered to the load R_o and efficiency η for a 40 kHz system built using the coils described in Appendix A, and three different load resistors R_o. At this frequency, the effective resistance of the sender and receiver coils are $R_{L_1} = 3.6\ \Omega$ and $R_{L_2} = 39\ \Omega$, and the quality factors of the sender and receiver are $Q_1 = 50$ and $Q_2 = 24$, respectively. The maximum efficiencies η_x are 20%, 72% and 96%. The source voltage $V_s = 4$ V, and the distance is normalized to 5 cm, the radii of the coils. Dotted lines represent model predictions.

receiver size. For example, the coupling coefficient between 10 cm devices separated 50 cm is 0.0005, but the coupling coefficient between 1 m devices separated the same distance is 0.24.

Figure 3 also shows that the power P_o delivered to the load R_o is maximized when the impedance seen by the source is equal to the coil resistance R_{L_1}, as expected from impedance matching. The magnetic coupling coefficient under in this condition is constrained such that

$$\kappa^2_{P_{o,max}} = \frac{1}{Q_1 Q_2 (1 - \eta_x)}. \tag{8}$$

This equation indirectly determines the distance at which the maximum power transmission occurs. Since the total load seen by the source under this condition is $2R_{L_1}$, the power delivered by the source is

$$P_{s,max} = \frac{V_s^2}{2(2R_{L_1})}, \tag{9}$$

and the power delivered to the load R_o is

$$\begin{aligned} P_{o,max} &= \frac{V_s^2}{2(2R_{L_1})} \cdot \frac{\eta_x}{2} \\ &= P_{s,max} \cdot \frac{\eta_x}{2}. \end{aligned} \tag{10}$$

The efficiency of the maximum power transfer is therefore

$$\eta_{P_{o,max}} = \frac{\eta_x}{2}. \tag{11}$$

Both the maximum power and the related efficiency are independent of inductances and capacitances in the system, and depend only on the losses in the system. This suggests that any configuration of coils can achieve the same maximum power transfer if the losses are constant, regardless of the operating frequency. Furthermore, the efficiency at this power maximum can only reach 50% for any positive resistances R_{L_2} and R_o.

If the magnetic coupling coefficient κ is increased by moving the resonators closer from the maximum power point, the power delivered to the load will decrease. However, the efficiency η will approach an asymptotic value, as shown in Figure 3. In this case, the magnetic coupling coefficient is larger than the value given by Equation 8. Strong coupling is defined to occur when

$$\kappa^2 \gg \frac{1}{Q_1 Q_2 (1 - \eta_x)}, \tag{12}$$

which implies that the power output can be approximated as

$$P_o \approx \frac{V_s^2}{2R_{L_1}} \cdot \frac{1}{\kappa^2 Q_1 Q_2 (1 - \eta_x)} \eta_x \tag{13}$$

and the efficiency can be approximated as

$$\eta \approx \eta_x. \tag{14}$$

Strongly coupled operation has been used in past work [11] on wireless power transfer in order to achieve relatively constant high levels of efficiency, even at moderately large distances. However, as mentioned earlier, transmission distance is only meaningful in the context of resonator size, particularly since many applications are subject to size limitations.

Note that this power maximum can only exist if

$$Q_1 Q_2 (1 - \eta_x) > 1. \tag{15}$$

such that the magnetic coupling coefficient $\kappa_{P_{o,max}}$ is less than one. This condition places restrictions on R_o such that

$$R_o < (Q_1 Q_2 - 1) R_{L_2} \approx Q_1 Q_2 R_{L_2}, \tag{16}$$

Systems were the resistive loads are so large that this condition is violated do not have a maximum power peak. Furthermore, the efficiency does not keep constant since it does not reach the asymptotic maximum η_x.

Equation 10 can be used as a performance bound since $\eta_x < 1$, such that

$$P_{o,max} < \frac{V_s^2}{8R_{L_1}}. \tag{17}$$

Condition 15 can be used to to give a tighter bound on $P_{o,max}$ in terms of the quality factors of the coils such that

$$P_{o,max} < \frac{V_s^2}{8R_{L_1}}\left(1 - \frac{1}{Q_1 Q_2}\right) < \frac{V_s^2}{8R_{L_1}}. \tag{18}$$

III. Results

Equation 5, the power delivered to the load, Equation 7, the efficiency, and Equation 4, which relates the magnetic coupling coefficient κ and separation distance, predict the behavior of the system model. Comparing these predictions to three systems operating at 23 kHz, 40 kHz and 59 kHz validates the model. These systems are described in Appendix A. Figure 3 shows the power output and efficiency as a function of the normalized distance δ for the 40 kHz system using three different output loads R_o.

Power losses in closely wound coils include skin effect and proximity losses, which increase with frequency [14] thereby decreasing the maximum power transferred, as expected from Equation 10. This effect can be modeled using the internal coil resistances R_{L_1} and R_{L_2}. The losses for each coil were measured empirically and fitted to quadratic functions as shown in Figure 4. The resulting functions are $R_{L_1} = 2.3 + 0.00076 f^2$ for the sender coil, and $R_{L_2} = 3.0 + 0.022 f^2$ for the receiver coil, where f is the frequency in kHz, and R_{L_1} and R_{L_2} are the resistance values in ohms. Figure 4 shows how these losses decrease both the maximum power transfer and efficiency as the resonant frequency of the system increases. The results also indicate that increasing the load R_o increases the efficiency of the system at the maximum power transfer, though it remains bounded at 50% as expected from Equation 11. However, this is only true for the power peak, and higher efficiencies can be attained at shorter separation distances, as shown in Figure 3.

These results highlight the importance of coil losses and the detrimental effect they have on the maximum power transferred. In addition, Equation 8 implies that in order to maximize the distance where the maximum peak occurs, all the losses must be kept to a minimum. It also requires maximizing the inductances and minimizing the capacitances. The maximum inductance might be limited by the size and weight constraints of the sender or the receiver. On the other hand, minimizing the capacitances will increase the operating frequency of the system, which in turn may increase the losses in the coils. Furthermore, at some

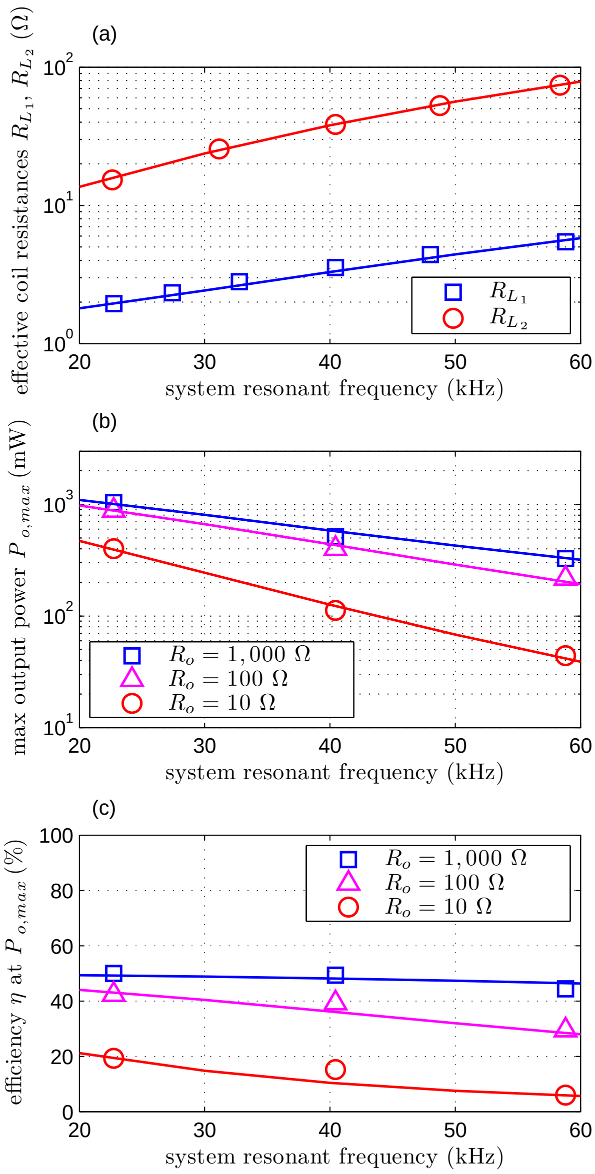


Fig. 4. a) Effective coil resistances R_{L_1} and R_{L_2} derived from measurements of proximity and skin effect losses, shown with corresponding quadratic fits. b) Maximum power $P_{o,max}$ delivered to load R_o as a function of a system's resonant frequency. c) Efficiency $\eta_{P_{o,max}}$ of the maximum power transfer as a function of a system's resonant frequency. The data points in (b) and (c) correspond to measurements made from three systems operating at 23 kHz, 40 kHz and 59 kHz, as described in Appendix A.

point the parasitic capacitances in the system will be comparable to the resonating capacitances, which will limit the maximum resonant frequency of the system. At this point, these parasitic capacitances have to be included in the model in order to find their effect on power transfer and efficiency. Another option is to design the system to use the self-resonance of each coil instead of adding a separate resonating capacitor, which is the approach employed by [11].

IV. Conclusion

Development of ambient power is likely inevitable given the advantages it offers and the applications it can enable. In particular, future portable devices could act as if they had an infinite power source. These devices will probably combine advances in low power consumption, energy storage, energy harvesting, local power generation and power transmission in order to create this illusion.

Resonant magnetic coupling may be an effective way to create wireless power transfer systems. Using resonance on both the receiving and transmitting ends increases achievable power levels and efficiency significantly with respect to nonresonant systems or systems that use resonance only unilaterally [1]. The model presented in this paper describes the power transfer and efficiency of such a system at all distances. The analysis illustrates impedance matching as the underlying reason for the existence of a maximum power peak, and yields compact expressions for the separation, magnitude and efficiency at this maximum. These expressions and the model are useful design tools for developing and for determining the performance bounds of wireless power transfer systems.

The results show the effect of losses that increase with frequency, such as skin effect and proximity losses. These losses decrease the power transferred and limit the range of these systems. Practical systems must minimize these losses and maximize the self resonant frequency of both sender and receiver. These concerns are similar to those of RF transformer and high-frequency switching power converter design.

One of the main concerns presented by this technology and not addressed by this paper is safety. An obvious difference between using electromagnetic fields to send power instead of information is the level of ambient radiation. If the frequency or power level needed to send useful amounts of power at reasonable efficiencies is too high, the use of this technology might be severely limited to niche applications. That being said, most of the power is either sent between or contained in each of the resonating structures, and little energy is sent to nonresonant objects. In fact, this work shows that, at the frequencies and power levels tested, all of the power losses can be accounted by the coil losses, without having to model the power being radiated. In addition, a real impedance implies the source does not have to supply any reactive power to the system.

Another concern is whether the trade off between efficiency, power level and range is sufficient to justify the development of these systems. One of the main objectives of this paper is to provide part of the framework to have this discussion. Even if the amount of power sent to nonresonant objects is unsafe, or the efficiency is too low, the technology may be necessary in scenarios where the need to have wireless power is more important than either the efficiency and range, such as in the case of artificial organs, or the safety, such as in cases where humans are not present long enough to be harmed. Nevertheless, these concerns must be considered in order to determine the usefulness of resonant magnetic coupling as a wireless power transfer technology.

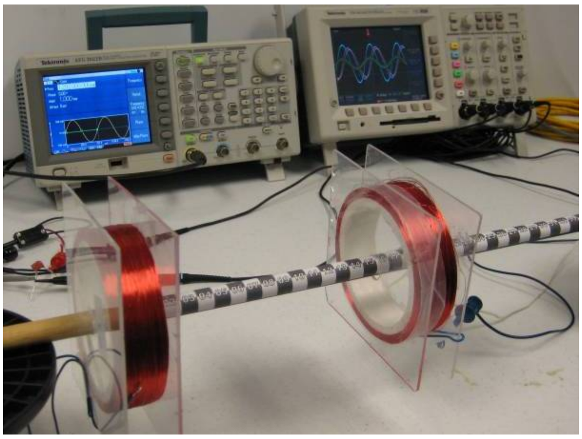


Fig. 5. Physical setup for wireless power transfer tests using magnetically coupled resonators, as described in Appendix A.

TABLE I
PHYSICAL PARAMETER VALUES

f_{res}	C_1	C_2	R_{L_1}	R_{L_2}	Q_1	Q_2
23 kHz	68 nF	13 nF	1.9 Ω	15 Ω	54	35
40 kHz	22 nF	4.0 nF	3.6 Ω	39 Ω	50	24
59 kHz	10 nF	1.7 nF	5.5 Ω	74 Ω	48	20

APPENDIX A
PHYSICAL SYSTEM

The system described in Figure 1 was built using a pair of coils in series with capacitors. The sender was connected to a 4 V amplitude voltage sinusoid which acted as the power source, and the receiver was connected to a resistor which acted as the load. The coils were made using AWG 20 copper wire coils of radii $r_1 = r_2 = 5$ cm, as shown in Figure 5. The coil wire diameter was 0.8 mm with a skin depth of 0.3 mm at 40 kHz. The sender coil had a measured inductance of $L_1 = 0.71$ mH, and the measured inductance of the receiver coil was 3.6 mH. The series capacitors C_1 and C_2 were metal polypropylene film capacitors chosen to resonate at the frequency of the 4 V voltage sinusoid, thereby creating three systems with resonant frequencies at 23 kHz, 40 kHz and 59 kHz. Table I shows the values of the resonant capacitors C_1 and C_2 for each system. Table I also shows the equivalent resistances R_{L_1} and R_{L_2} of the sender and receiver coils for each system and the corresponding quality factors Q_1 and Q_2.

Table II shows the distance d between the coils at the maximum power transfer for each combination of system resonant frequency and load resistance R_0. Distances are rounded to the closest centimeter.

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TABLE II
DISTANCE d BETWEEN COILS AT MAXIMUM POWER TRANSFER

f_{res}	$R_0 = 1\ k\Omega$	$R_0 = 100\ \Omega$	$R_0 = 10\ \Omega$
23 kHz	6 cm	9 cm	12 cm
40 kHz	6 cm	10 cm	12 cm
59 kHz	7 cm	10 cm	12 cm

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