1.

Equation of motion

$$m\ddot{x} + Kx = 0 \quad — \text{①}$$

Boundary Conditions

$$x(t=0) = x_0$$
$$\dot{x}(t=0) = V_0$$

$$m\frac{\partial^2 x}{\partial t^2} + 0\,\frac{\partial x}{\partial t} + Kx = 0$$

$a = m$
$b = 0$
$c = K$

$$m\lambda^2 + 0\cdot\lambda + K = 0$$

$$m\lambda^2 + K = 0$$

$$\lambda = \frac{-0 \pm \sqrt{0^2 - 4mK}}{2m}$$

$$= \frac{0 \pm \sqrt{-4mK}}{2m}$$

$$= 0 \pm \sqrt{\frac{-4mK}{4m^2}} = 0 \pm i\sqrt{\frac{K}{m}}$$

$$\lambda_{1,2} = 0 \pm i\,\omega_n$$

$$x(t) = c_1 e^{0t}\cdot\cos\omega_n t + c_2 e^{0\cdot t}\sin(\omega_n t)$$

$$= c_1\cos(\omega_n t) + c_2\sin(\omega_n t)$$

According to initial conditions

$$x(t=0) = X_0 = C_1 \cos(\omega_n 0) + C_2 \sin(\omega_n \cdot 0)$$

$$\boxed{X_0 = C_1}$$

$$\dot{x} = -C_1 \omega_n \sin(\omega_n t) + C_2 \omega_n \cos(\omega_n t)$$

$$\dot{x}(t=0) = V_0 = -X_0 \omega_n (0) + C_2 \omega_n (1)$$

$$C_2 = \frac{V_0}{\omega_n}$$

$$x(t) = X_0 \cos(\omega_n t) + \frac{V_0}{\omega_n} \sin(\omega_n t)$$

$$X = A_0 \cos(\omega_n t - \phi)$$

$$x(t) = A_0 \cos\omega_n t \cos\phi + A_0 \sin\omega_n t \cdot \sin\phi$$

$$= A_0 \cos\phi \cos\omega_n t + A_0 \sin\phi \sin\omega_n t$$

$$X_0 = A_0 \cos\phi \quad \text{— a}$$

$$\frac{V_0}{\omega_n} = A_0 \sin\phi \quad \text{— b}$$

Squaring the both sides

$$A_0^2 (\cos^2\phi + \sin^2\phi) = X_0^2 + \frac{V_0^2}{\omega_n^2}$$

$$A_0 = \sqrt{X_0^2 + \frac{V_0^2}{\omega_n^2}}$$