Active Learning

CS 203: Software Tools and Techniques for AI

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The Labeling Problem

Scenario: You need to train a classifier

Traditional approach:

1. Collect 10,000 images
2. Label all 10,000 images
3. Train model
4. Hope it works

The cost:

- 10,000 labels × 30 seconds = 83 hours
- At $20/hour = $1,660
- Many labels are redundant

What is Active Learning?

Active Learning: Intelligently select which examples to label to maximize model performance with minimal labeling effort

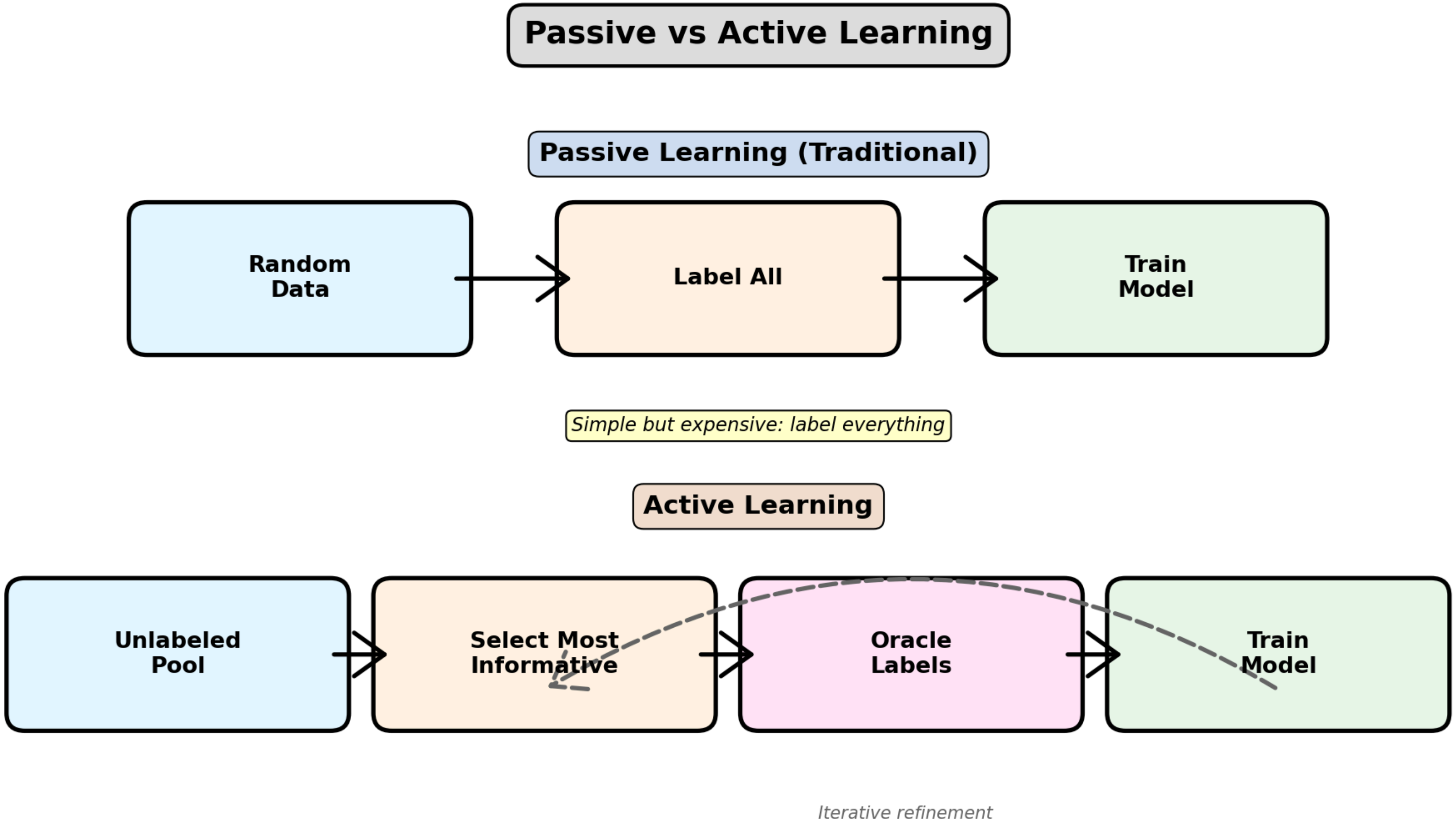
Key Insight: Not all data points are equally valuable for learning!

Example:

- 100 random samples might give 85% accuracy
- 100 carefully chosen samples might give 92% accuracy

Goal: Achieve same performance with 5-10× fewer labels

Passive vs Active Learning



When to Use Active Learning

Use Active Learning when:

- Labeling is expensive (human time, expert knowledge)
- You have large unlabeled dataset
- You need good performance with limited labels
- Labels are imbalanced or rare

Real-world applications:

- Medical imaging (radiologist time is expensive)
- Legal document review (lawyer expertise)
- Rare event detection (fraud, defects)
- Custom domain classification

Active Learning Cycle

The Loop:

1. **Start**: Train initial model on small labeled set
2. **Query**: Select most informative unlabeled examples
3. **Oracle**: Human labels selected examples
4. **Update**: Retrain model with new labels
5. **Repeat**: Until performance target or budget reached

Key components:

- **Learner**: ML model being trained
- **Query Strategy**: How to select examples
- **Oracle**: Human labeler (or simulation)
- **Pool**: Unlabeled data to select from

Query Strategies: Overview

Main strategies:

1. **Uncertainty Sampling**: Pick examples model is most uncertain about
2. **Query-by-Committee**: Pick examples where models disagree
3. **Expected Model Change**: Pick examples that change model most
4. **Expected Error Reduction**: Pick examples that reduce error most
5. **Diversity Sampling**: Pick diverse examples to cover feature space

Most popular: Uncertainty Sampling (simple and effective)

Uncertainty Sampling

Idea: Label examples where the model is most confused

For binary classification:

- Model predicts P(positive) = 0.51
- Model is uncertain! Label this example

For multi-class:

- Model predicts [0.34, 0.33, 0.33]
- Very uncertain! Label this

Intuition: Easy examples don't teach us much. Hard examples are informative.

Uncertainty Measures: Mathematical Foundation

Problem Setup

Given:

- Model f with parameters θ
- Unlabeled example x
- Class predictions $P_\theta(y|x)$ for classes $y \in \{1, \ldots, C\}$

Goal: Define uncertainty $U(x)$ to select most informative examples

Uncertainty Measure 1: Least Confident

Formula

$$U_{LC}(x) = 1 - P_\theta(\hat{y}|x)$$

where $\hat{y} = \arg\max_y P_\theta(y|x)$ is the predicted class

Interpretation

- Measures how uncertain the model is about its top prediction
- Range: [0, 1]
- High value = low confidence = select for labeling

Example

Probabilities: $P(y|x) = [0.6, 0.3, 0.1]$

Uncertainty Measure 2: Margin Sampling

Formula

$$U_M(x) = P_\theta(\hat{y}_1|x) - P_\theta(\hat{y}_2|x)$$

where:

- $\hat{y}_1$ = most probable class
- $\hat{y}_2$ = second most probable class

Then uncertainty is:

$$U_M^{inv}(x) = 1 - U_M(x)$$

Interpretation

- Small margin = model is confused between top 2 classes

Margin Sampling: Example

Scenario

Example A: $P(y|x) = [0.51, 0.49, 0.00]$

$$U_M(A) = 0.51 - 0.49 = 0.02 \quad \text{(very small margin)}$$

Example B: $P(y|x) = [0.99, 0.01, 0.00]$

$$U_M(B) = 0.99 - 0.01 = 0.98 \quad \text{(large margin)}$$

Selection: Example A is more uncertain → select for labeling

Comparison with Least Confident:

- $U_{LC}(A) = 1 - 0.51 = 0.49$
- $U_{LC}(B) = 1 - 0.99 = 0.01$

Both correctly identify A as more uncertain!

Uncertainty Measure 3: Entropy

Formula

$$H(P_\theta(y|x)) = -\sum_{y=1}^{C} P_\theta(y|x) \log P_\theta(y|x)$$

Interpretation

- Measures disorder/uncertainty in probability distribution
- Range: $[0, \log C]$
- Maximum when all classes equally likely
- Most theoretically principled measure

Properties

Entropy: Detailed Example

Binary Classification ($C = 2$)

Example 1 (very confident):

$$P(y|x) = [0.95, 0.05]$$

$$H = -0.95 \log(0.95) - 0.05 \log(0.05) = 0.286$$

Example 2 (uncertain):

$$P(y|x) = [0.5, 0.5]$$

$$H = -0.5 \log(0.5) - 0.5 \log(0.5) = 0.693$$

Maximum possible: $\log(2) = 0.693$

Selection: Example 2 has higher entropy → more uncertain

Entropy: Multi-Class Example

3-Class Classification ($C = 3$)

Example A (confident):

$$P(y|x) = [0.8, 0.15, 0.05]$$

$$H_A = -(0.8 \log 0.8 + 0.15 \log 0.15 + 0.05 \log 0.05) = 0.849$$

Example B (uncertain):

$$P(y|x) = [0.4, 0.35, 0.25]$$

$$H_B = -(0.4 \log 0.4 + 0.35 \log 0.35 + 0.25 \log 0.25) = 1.571$$

Example C (maximum uncertainty):

$$P(y|x) = [0.33, 0.33, 0.33]$$

$$H_C = -3 \times (0.33 \log 0.33) = 1.585 \approx \log(3) = 1.099$$

Comparing Uncertainty Measures

Measure	Formula	Best For	Limitations
Least Confident	$1 - \max_y P(y\|x)$	Simple, fast	Ignores distribution shape
Margin	$P(\hat{y}_1\|x) - P(\hat{y}_2\|x)$	Binary/multiclass	Only considers top 2
Entropy	$-\sum_y P(y\|x) \log P(y\|x)$	Full distribution	More computation

Example Comparison

$$P(y|x) = [0.6, 0.3, 0.05, 0.05]$$

- Least Confident: $U = 0.4$
- Margin: $U = 0.3$
- Entropy: $H = 1.23$

Uncertainty Sampling - Code

Uncertainty Sampling - Code

Basic implementation:

```
from sklearn.linear_model import LogisticRegression
import numpy as np

def uncertainty_sampling(model, X_unlabeled, n_samples=10):
    # Get prediction probabilities
    probs = model.predict_proba(X_unlabeled)

    # Calculate uncertainty (1 - max probability)
    uncertainties = 1 - np.max(probs, axis=1)

    # Select top uncertain samples
    indices = np.argsort(uncertainties)[-n_samples:]

    return indices

# Usage
```

Query-by-Committee: Mathematical Foundation

Setup

Committee: $\mathcal{C} = \{h_1, h_2, \ldots, h_M\}$ of M models

- Each h_i trained on same labeled data $\mathcal{L}$
- Different algorithms or random initializations

For unlabeled example x:

- Get prediction distribution from each model: $P_{h_i}(y|x)$

Goal: Measure disagreement among committee members

QBC Disagreement Measure 1: Vote Entropy

Formula

$$D_{VE}(x) = -\sum_{y=1}^{C} \frac{V(y)}{M} \log \frac{V(y)}{M}$$

where $V(y)$ = number of committee members voting for class y

Example

Committee of 5 models, binary classification:

- 3 models predict class 0
- 2 models predict class 1

$$V(0) = 3, \quad V(1) = 2$$

QBC Disagreement Measure 2: Consensus Entropy

Formula

Average prediction distribution across committee:

$$P_C(y|x) = \frac{1}{M} \sum_{i=1}^{M} P_{h_i}(y|x)$$

Then calculate entropy of consensus:

$$D_{CE}(x) = H(P_C(y|x)) = -\sum_{y=1}^{C} P_C(y|x) \log P_C(y|x)$$

Interpretation

- Uses full probability distributions, not just votes

QBC: Detailed Example

Scenario: 3 Models, Binary Classification

	$P(y=0\|x)$	$P(y=1\|x)$
Model 1	0.9	0.1
Model 2	0.4	0.6
Model 3	0.3	0.7

Calculate Consensus Distribution

$$P_C(y=0|x) = \frac{0.9+0.4+0.3}{3} = 0.533$$

$$P_C(y=1|x) = \frac{0.1+0.6+0.7}{3} = 0.467$$

QBC Disagreement Measure 3: KL Divergence

Formula

Measure divergence of each model from consensus:

$$D_{KL}(x) = \frac{1}{M} \sum_{i=1}^{M} KL(P_{h_i}(y|x) \| P_C(y|x))$$

where KL divergence is:

$$KL(P \| Q) = \sum_{y} P(y) \log \frac{P(y)}{Q(y)}$$

Interpretation

- Measures how much individual predictions differ from average

KL Divergence: Example

Using previous example, consensus is $P_C = [0.533, 0.467]$

Model 1: $P_{h_1} = [0.9, 0.1]$

$$KL_1 = 0.9 \log \frac{0.9}{0.533} + 0.1 \log \frac{0.1}{0.467} = 0.397$$

Model 2: $P_{h_2} = [0.4, 0.6]$

$$KL_2 = 0.4 \log \frac{0.4}{0.533} + 0.6 \log \frac{0.6}{0.467} = 0.056$$

Model 3: $P_{h_3} = [0.3, 0.7]$

$$KL_3 = 0.3 \log \frac{0.3}{0.533} + 0.7 \log \frac{0.7}{0.467} = 0.130$$

Query-by-Committee - Code

Query-by-Committee - Code

```python
from sklearn.ensemble import RandomForestClassifier
from sklearn.linear_model import LogisticRegression
from sklearn.svm import SVC
from scipy.stats import entropy

def query_by_committee(committee, X_unlabeled, n_samples=10):
    # Get predictions from all committee members
    all_probs = []
    for model in committee:
        probs = model.predict_proba(X_unlabeled)
        all_probs.append(probs)

    all_probs = np.array(all_probs)  # Shape: (n_models, n_samples, n_classes)

    # Calculate vote entropy for each sample
    avg_probs = all_probs.mean(axis=0)
    disagreements = entropy(avg_probs.T)

    # Select top disagreement samples
    indices = np.argsort(disagreements)[-n_samples:]
    return indices

# Create committee
committee = [
    RandomForestClassifier(),
    LogisticRegression(),
    SVC(probability=True)
]
```

Expected Model Change

Idea: Select examples that will change the model parameters most if labeled

Approach:

- For each unlabeled example, simulate adding it with each possible label
- Measure how much model parameters change
- Select examples causing largest change

Gradient-based:

```
# For each example x:
gradient = model.compute_gradient(x)
impact = ||gradient||  # Magnitude of gradient
```

Pros: Directly optimizes for model learning

Cons: Computationally expensive (need to retrain or compute gradients)

Diversity Sampling

Problem: Uncertainty sampling can select similar examples

Solution: Also consider diversity

Approaches:

1. **K-means clustering**: Select one example from each cluster
2. **Core-set selection**: Select examples that best represent all data
3. **Hybrid**: Combine uncertainty + diversity

```
from sklearn.cluster import KMeans

def diverse_uncertainty_sampling(model, X_unlabeled, n_samples=10):
    # First, get uncertain examples (2x more than needed)
    probs = model.predict_proba(X_unlabeled)
    uncertainties = 1 - np.max(probs, axis=1)
    uncertain_indices = np.argsort(uncertainties)[-n_samples*2:]
```

Cold Start Problem

Challenge: How to start with no labeled data?

Solutions:

1. **Random Sampling**: Label small random set to bootstrap

```
# Start with 20-50 random examples
initial_indices = np.random.choice(len(X_pool), size=20, replace=False)
X_labeled = X_pool[initial_indices]
```

2. **Cluster-based**: Sample from each cluster

```
kmeans = KMeans(n_clusters=10)
kmeans.fit(X_pool)
# Select one from each cluster
```

3. **Representative Sampling**: Use diversity methods

Active Learning Libraries

1. modAL

```python
from modAL.models import ActiveLearner
from sklearn.ensemble import RandomForestClassifier

learner = ActiveLearner(
    estimator=RandomForestClassifier(),
    query_strategy=uncertainty_sampling,
    X_training=X_initial,
    y_training=y_initial
)

# Query 10 samples
query_idx, query_instance = learner.query(X_pool, n_instances=10)

# Teach with labels
learner.teach(X_pool[query_idx], y_pool[query_idx])
```

Complete Active Learning Example

```
from sklearn.datasets import make_classification
from sklearn.linear_model import LogisticRegression
import numpy as np

# Generate dataset
X, y = make_classification(n_samples=1000, n_features=20, n_classes=2)

# Split into initial labeled set and pool
n_initial = 20
initial_idx = np.random.choice(len(X), size=n_initial, replace=False)
X_labeled = X[initial_idx]
y_labeled = y[initial_idx]

pool_idx = np.setdiff1d(np.arange(len(X)), initial_idx)
X_pool = X[pool_idx]
y_pool = y[pool_idx]

# Active learning loop
model = LogisticRegression()
accuracies = []

for iteration in range(20):  # 20 iterations
    # Train model
    model.fit(X_labeled, y_labeled)

    # Evaluate
    score = model.score(X_test, y_test)
    accuracies.append(score)
    print(f"Iteration {iteration}: Accuracy = {score:.3f}")

    # Query most uncertain samples
    query_idx = uncertainty_sampling(model, X_pool, n_samples=10)

    # Simulate oracle labeling
    X_labeled = np.vstack([X_labeled, X_pool[query_idx]])
    y_labeled = np.hstack([y_labeled, y_pool[query_idx]])

    # Remove from pool
    X_pool = np.delete(X_pool, query_idx, axis=0)
    y_pool = np.delete(y_pool, query_idx, axis=0)
```

Simulating Oracles

For experiments, we need to simulate human labeling

Using existing labels:

```
def oracle(X_query, y_true, query_indices):
    # Return true labels for queried examples
    return y_true[query_indices]
```

With noise:

```
def noisy_oracle(y_true, query_indices, error_rate=0.1):
    labels = y_true[query_indices].copy()
    # Flip some labels randomly
    n_errors = int(len(labels) * error_rate)
    error_idx = np.random.choice(len(labels), size=n_errors, replace=False)
    labels[error_idx] = 1 - labels[error_idx]  # Flip binary labels
    return labels
```

Measuring Active Learning Performance

Learning Curve: Accuracy vs. number of labeled samples

```
import matplotlib.pyplot as plt

def plot_learning_curve(active_accuracies, random_accuracies, n_queries):
    plt.figure(figsize=(10, 6))

    x = np.arange(len(active_accuracies)) * n_queries

    plt.plot(x, active_accuracies, 'o-', label='Active Learning')
    plt.plot(x, random_accuracies, 's-', label='Random Sampling')

    plt.xlabel('Number of Labels')
    plt.ylabel('Accuracy')
    plt.title('Active Learning vs Random Sampling')
    plt.legend()
    plt.grid(True)
    plt.show()
```

Stopping Criteria

When to stop active learning?

1. **Budget exhausted**: Used all labeling budget
2. **Performance plateau**: Accuracy not improving
3. **Uncertainty threshold**: All examples have low uncertainty
4. **Time limit**: Deadline reached

Automatic stopping:

```
def should_stop(accuracies, window=3, threshold=0.01):
    if len(accuracies) < window:
        return False

    recent = accuracies[-window:]
    improvement = max(recent) - min(recent)
```

Active Learning for Deep Learning

Challenges:

- Deep models need more data
- Training is expensive
- Uncertainty estimation harder

Strategies:

1. **MC Dropout**: Use dropout at inference for uncertainty

   ```
   # Enable dropout at test time
   model.train()
   predictions = [model(x) for _ in range(30)]
   uncertainty = np.std(predictions, axis=0)
   ```

2. **Ensemble**: Train multiple models

Batch Mode Active Learning

Problem: Querying one example at a time is inefficient for deep learning

Solution: Query batches of examples

Challenge: Selected examples might be similar

Approaches:

1. **Top-k uncertain**: Simple, but may select similar examples
2. **Diverse batch**: Ensure batch covers feature space
3. **BatchBALD**: Maximize information about model parameters

```python
def batch_uncertainty_sampling(model, X_unlabeled, batch_size=100):
    probs = model.predict_proba(X_unlabeled)
    uncertainties = 1 - np.max(probs, axis=1)

    # Select top-k
```

Active Learning with Label Studio

Label Studio: Open-source annotation tool with active learning

Features:

- Visual interface for labeling
- Built-in active learning
- Custom ML backends
- Export to various formats

Workflow:

1. Upload unlabeled data to Label Studio
2. Connect ML model
3. Model suggests next samples to label

Cost-Effectiveness Analysis

Compare labeling costs:

```
def cost_analysis(active_results, random_results, cost_per_label=1.0):
    target_accuracy = 0.90

    # Find labels needed for target accuracy
    active_labels = np.argmax(active_results >= target_accuracy) * 10
    random_labels = np.argmax(random_results >= target_accuracy) * 10

    active_cost = active_labels * cost_per_label
    random_cost = random_labels * cost_per_label

    savings = random_cost - active_cost
    savings_pct = (savings / random_cost) * 100

    print(f"Target Accuracy: {target_accuracy}")
    print(f"Active Learning: {active_labels} labels (${active_cost})")
    print(f"Random Sampling: {random_labels} labels (${random_cost})")
    print(f"Savings: ${savings} ({savings_pct:.1f}%)")
```

Domain Adaptation with Active Learning

Scenario: Model trained on domain A, deploying to domain B

Problem: Distribution shift causes poor performance

Solution: Use active learning to select examples from domain B

```
# Train initial model on source domain
model.fit(X_source, y_source)

# Active learning on target domain
X_pool = X_target  # Unlabeled target domain data

for iteration in range(n_iterations):
    # Query uncertain examples from target domain
    query_idx = uncertainty_sampling(model, X_pool, n_samples=batch_size)

    # Label (oracle)
    y_new = oracle(X_pool[query_idx])
```

Active Learning for Imbalanced Data

Problem: Rare classes get few queries with standard uncertainty sampling

Solution: Class-balanced active learning

```
def class_balanced_uncertainty_sampling(model, X_unlabeled, n_samples=10):
    probs = model.predict_proba(X_unlabeled)
    predicted_classes = np.argmax(probs, axis=1)
    uncertainties = 1 - np.max(probs, axis=1)

    selected = []
    samples_per_class = n_samples // len(np.unique(predicted_classes))

    for cls in np.unique(predicted_classes):
        cls_mask = predicted_classes == cls
        cls_uncertainties = uncertainties[cls_mask]
        cls_indices = np.where(cls_mask)[0]

        # Select most uncertain from this class
        top_k = min(samples_per_class, len(cls_indices))
```

Common Pitfalls

1. **Not evaluating on separate test set**
   - Always use held-out test data
   - Don't evaluate on the pool
2. **Biased initial sample**
   - Start with diverse/representative sample
   - Not just easiest examples
3. **Ignoring computational cost**
   - Querying and retraining takes time
   - Budget for compute, not just labels
4. **Over-querying similar examples**

Active Learning Best Practices

1. **Start small**: Begin with 5-10 examples per class
2. **Batch wisely**: Query 10-100 examples at once (depends on budget)
3. **Validate strategy**: Compare to random baseline
4. **Monitor convergence**: Track learning curves
5. **Consider human factors**:
    - Annotation fatigue
    - Label quality over time
    - Break large batches into sessions
6. **Save everything**: Log all queries and labels for analysis

Tools and Libraries

Active Learning:

- **modAL**: Python active learning framework
- **alipy**: Comprehensive active learning toolkit
- **libact**: C++ based, Python bindings

Annotation:

- **Label Studio**: Web-based with active learning
- **Prodigy**: Commercial, scriptable
- **CVAT**: Computer vision annotation
- **Labelbox**: Enterprise solution

Experiment tracking:

Real-World Case Studies

1. **Medical Imaging (Chest X-rays)**
    - Random: 5,000 labels for 85% accuracy
    - Active: 1,500 labels for 85% accuracy
    - Savings: 70% reduction in radiologist time
2. **Legal Document Review**
    - Random: 10,000 documents reviewed
    - Active: 3,000 documents for same recall
    - Savings: $140,000 in lawyer fees
3. **Manufacturing Defect Detection**
    - Random: 1% defect rate, need 10,000 labels

Active Learning vs Other Approaches

Active Learning vs Semi-Supervised Learning:

- Active: Choose what to label
- Semi-supervised: Use unlabeled data directly

Active Learning vs Transfer Learning:

- Active: Label task-specific data intelligently
- Transfer: Use pretrained models

Active Learning vs Few-Shot Learning:

- Active: Iteratively grow labeled set
- Few-shot: Learn from very few examples (5-10)

Can combine! Transfer learning + active learning is powerful

Research Directions

Current trends:

1. **Deep active learning**: Better uncertainty for neural nets
2. **Active learning + RL**: Learn query strategy with RL
3. **Human-in-the-loop**: Better human-AI interaction
4. **Active learning at scale**: Billion-sample pools
5. **Weak supervision**: Combine with programmatic labeling

Open problems:

- Theoretical guarantees
- Better uncertainty estimation
- Handling label noise
- Multi-modal active learning

Implementing Your First Active Learning System

Step-by-step:

1. **Load data**: Split into initial labeled set and pool
2. **Train initial model**: Use random sample
3. **Active learning loop**:
    - Predict on pool
    - Calculate uncertainty
    - Select top-k
    - Get labels (oracle or human)
    - Add to training set
    - Retrain model
4. **Evaluate**: Compare to random baseline

Practical Tips for Your Project

1. **Baseline is crucial**: Always compare to random sampling
2. **Start with toy dataset**: Test strategy on iris/digits
3. **Use existing labels**: Simulate oracle with held-out labels
4. **Track everything**:
    - Which samples were queried
    - Model performance at each iteration
    - Time spent
5. **Visualize uncertainty**: Plot samples by uncertainty to understand strategy
6. **Try multiple strategies**: Uncertainty, QBC, diversity

What We've Learned

Core Concepts:

- Active learning reduces labeling costs by 50-80%
- Query strategies: Uncertainty, QBC, diversity
- Oracle simulation for experiments
- Learning curves measure performance

Practical Skills:

- Implementing uncertainty sampling
- Building active learning loop
- Evaluating with learning curves
- Using libraries like modAL

Resources

Papers:

- "Active Learning Literature Survey" by Settles (2009)
- "Deep Active Learning" surveys
- "A Survey of Deep Active Learning" (2020)

Libraries:

- modAL: https://modal-python.readthedocs.io/
- Label Studio: https://labelstud.io/
- alipy: https://github.com/NUAA-AL/alipy

Datasets:

- MNIST, CIFAR-10 for experiments

Questions?

Next: Data Augmentation

Lab: Build active learning system from scratch