Ommited Proof - Lemma 4.1

Proof 1 *With Assumption 4.1, for some constants $\beta > 1$ and c, we have*

$$T(s) = \frac{c}{s^{\beta}}$$

Then, let L be the total affinity of the last $N - O\left(\ln^{1-\epsilon} N\right)$, we have

$$\begin{aligned}
L \leq & \sum_{s=\ln^{1-\epsilon} N}^{N} T(s) \\
= & \sum_{s=\ln^{1-\epsilon} N}^{N} T\left(\ln^{1-\epsilon} N\right) \cdot \frac{\left(\ln^{1-\epsilon} N\right)^{\beta}}{s^{\beta}} \\
= & T\left(\ln^{1-\epsilon} N\right) \cdot \ln^{\beta\cdot(1-\epsilon)} N \cdot \sum_{s=\ln^{1-\epsilon} N}^{N} \frac{1}{s^{\beta}} \\
\leq & O(1) \cdot \int_{\ln^{1-\epsilon} N}^{N} \frac{1}{x^{\beta}} dx \\
\leq & O(1) \cdot \left\{ \left(\ln^{1-\epsilon} N\right)^{1-\beta} - N^{1-\beta} \right\} \\
\leq & O\left(\ln^{(1-\beta)(1-\epsilon)} N\right)
\end{aligned}$$

Let $\gamma = (\beta - 1)(1 - \epsilon)$, then $L = O\left(\frac{1}{\ln^{\gamma} N}\right)$.