Dynamic cash inventory model

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Preliminary and Incomplete, Please do not cite!

Motivation

- Cash usage has been declining over the past decade:
  - lower volume and value of cash transactions at the point-of-sale (POS);
  - higher acceptance of electronic payment instruments at the POS;
- A deeper understanding of cash usage is needed
  - new payment instruments become available;
  - still one of the best ways to store value;
  - lower density of ATM/Bank branches.
- Increases in the cost of withdrawal affect consumer cash holdings and subsequently usage decisions.
- Dynamic cash inventory model (DCIM) to better understand consumer cash withdrawl behavior.

DCIM: intuition

- Consumers keeps cash in wallet & use cash at the POS.
- Consumers withdraw cash from ATMs
- Optimal withdrawal policy should take into account that
  1. Holding cash is costly
     - foregone interest;
     - security concerns;
     - **transaction costs.**??
  2. Have enough cash when needed $\Longrightarrow$ otherwise, unable to make a purchase.
  3. Withdrawal is costly.
- There is a trade-off between more frequent withdrawal trips and larger cash holdings.
- It is conceivable that holding excessive cash is less costly than having insufficient amount of it, i.e., asymmetric loss/reward function.

DCIM: assumptions

- Time is discrete, horizon is infinite, $t = 0, 1, \ldots, \infty$.
- Consumer i's payoff-relevant variables:
  - Cash holding from previous period, $s_{i,t-1} \geq 0$;
  - Current period expected (cash) usage decisions, $u_{it} \geq 0$;
  - Distance from consumer location to ATM, $d_{it} \geq 0$
- Consumer chooses per-period withdrawal/deposit amount $w_t(s_{i,t-1}, d_{it}, u_{it}) \in \mathbb{R}$.
- Per-period reward function is

$$C(s_{i,t-1}, d_{it}, u_{it}, w_{it}) = \begin{cases} \underbrace{-|s_{i,t-1} + w_{it} - u_{it}|^{\gamma_1}}_{\text{cost of holding insufficient cash}} - g(d_{it})\mathbb{1}_{w_{it}>0} & \text{if } s_t < 0 \\ \underbrace{-|s_{i,t-1} + w_{it} - u_{it}|^{\gamma_2}}_{\text{cost of holding excess cash}} - g(d_{it})\mathbb{1}_{w_{it}>0} & \text{if } s_t \geq 0 \end{cases}$$

- Laws of motion for state variables:

$$s_{it} = \max(s_{i,t-1} + w_t - u_t, 0) \quad (1)$$

$$d_{it} = d_i \quad \forall t \quad (2)$$

$$u_{it} = u^*(\mathcal{M}_b, \mathcal{M}_s, \mathcal{J}_b) + \epsilon_t \quad (3)$$

DCIM: dynamic programming problem

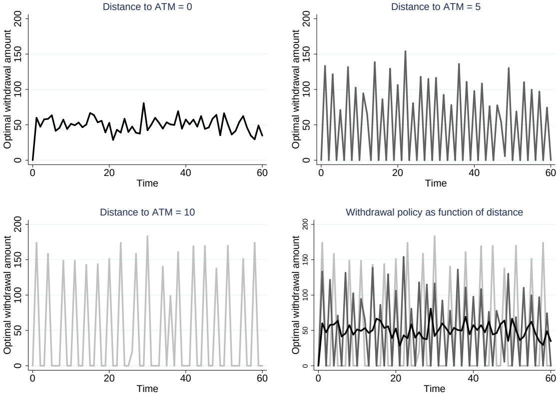
- Assuming consumers maximize present discounted value of infinite sequence of reward flows, $C(s_{i,t-1}, d_{it}, u_{it}, w_{it})$,

$$V(s_{i,t-1}, d_{it}, u_{it}) = \max_{w_{it}} \begin{Bmatrix} C(s_{i,t-1}, d_{it}, u_{it}, w_{it}) + \\ \beta \int V(s_t, d_{it+1}, u_{it+1}) dF_u \end{Bmatrix} \tag{4}$$

- Current period usage is observed.
- Uncertainty comes from unobserved future usage values.
- Solution specifies a dynamical optimal policy $w(s_{i,t-1}, d_{it}, u_{it})$, which we can take to the data:
    - Observable: cash holding, distance to ATM, expected cash usage.
    - Expected cash usage is a function of own adoption decisions, $\mathcal{M}_b$, merchant acceptance decisions, $\mathcal{M}_s$, and consumer demand for transactions, $\mathcal{J}_b$.
    - Note: do not really need a "panel" of observations.

DCIM: policy functions

Figure: Policy functions and distance to ATM



Notes: Optimal policies are computed using the following values of state variables. Holdings from previous period, $s_t \in [0.0, 500.0]$ *with 501 discrete point. Distance,* $d_t \in [0.0, 100.0]$ *with 101 discrete points. Usage,* $u_t \in [0.0, 200.0]$ *with 201 grid point. Decision,* $w_t \in [-500.0, 500.0]$ *with 1001 discrete point. Withdrawal cost is* $5d_{it}$*,* $\gamma_1 = 3$*,* $\gamma_2 = 1.1$ *Discount factor,* $\beta = 0.9$*.*