Dynamic Price Competition with Local Network Effects Online Appendix

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1 Equilibrium Market Structure

Proposition 1. *For any value of $\delta \in [0,1)$, there exist $\psi_1 \leq \psi_2 \leq \psi_3$ such that the equilibrium market structure is*

$$\begin{cases} \text{Unimodal (Region A)} & \text{if } \psi < \psi_1, \\ \text{Bimodal (Global dominance)(RegionB)} & \text{if } \psi_1 < \psi < \psi_2, \\ \text{Four Modal (Region C)} & \text{if } \psi_2 < \psi < \psi_3, \\ \text{Bimodal (Local dominance)(RegionD)} & \text{if } \psi > \psi_3 \end{cases}$$

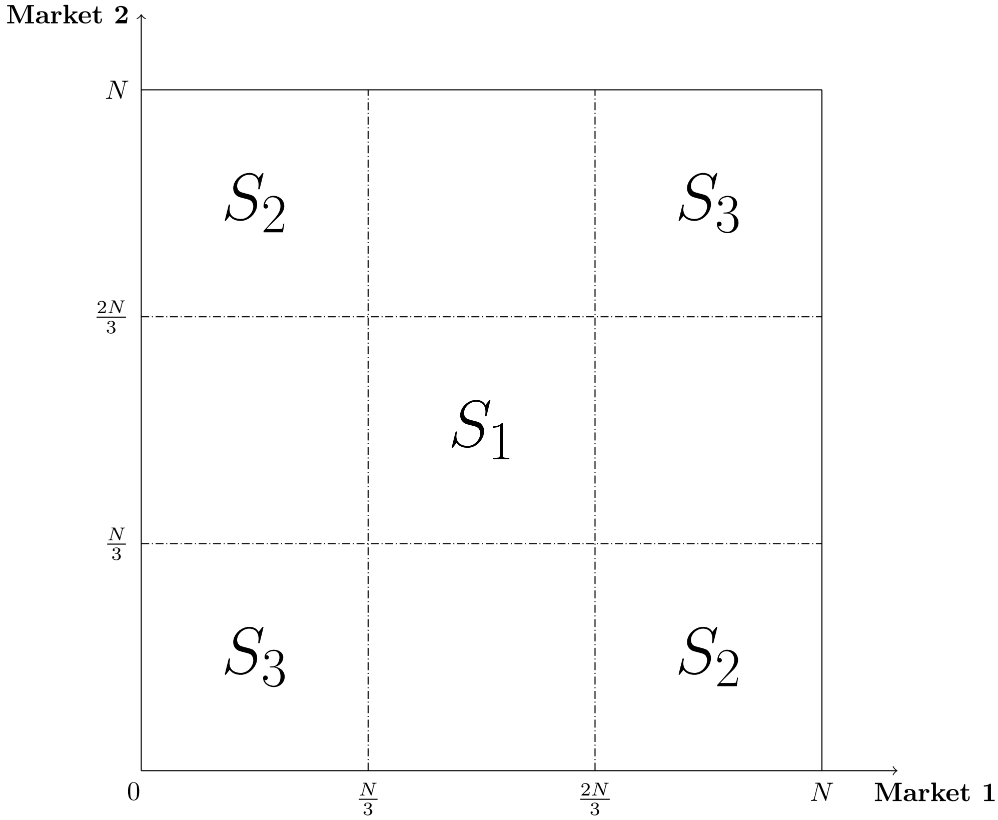

Figure 1: Equilibrium Market Structure

The numerical simulation of the model produces an equilibrium market share matrix for each combination of parameter values δ and psi. The matrix is shown in Figure 1. Then the matrix is divided into 9 regions and I sum up all the market share for each region and focus on three regions S_1, S_2, S_3.[1]

To summarize the result, each equilibrium market structure is categorized into one of the following groups: Unimodal(S_1), Bimodal (Local dominance)(S_2), Bimodal (Global dominance)(S_3), Four Modal(combination of S_2 and (S_3)).[2]

[1] The symmetric model produces symmetric equilibrium market structure, i.e. the two regions of S_2 are equal, and so are the S_3 regions.
[2] See Figure 6 for parameter values that produce each of these four equilibria

	Equilibrium Market Structure
$\mathbf{S_1 > S_2, S_3}$	Unimodal
$\mathbf{S_2 > S_1, S_3}$	
$1 < S_2/S_3 < 1.5$	Four Modal
$S_2/S_3 > 1.5$	Bimodal (Local Dominance)
$\mathbf{S_3 > S_1, S_2}$	
$1 < S_3/S_2 < 1.5$	Four Modal
$S_3/S_2 > 1.5$	Bimodal (Global Dominance)

Table 1: Categorization Rules

The categorization rules used to produce Figure 4 is shown in Table 1.

2 Pricing Function

In the main text of this paper, the firm's optimal pricing strategy is characterized as follows.

$$p_i = h(i_1, i_2) - w(i_1, i_2) \tag{1}$$

where

$$h(i_1, i_2) = -\frac{q(i_1, i_2) + q(i_2, i_1)}{q'(i_1, i_2) + q'(i_2, i_1)} \tag{2}$$

$$w(i_1, i_2) = \frac{q'(i_1, i_2)}{q'(i_1, i_2) + q'(i_2, i_1)} \frac{1}{N} w_1(i_1, i_2) + \frac{q'(i_2, i_1)}{q'(i_1, i_2) + q'(i_2, i_1)} \frac{1}{N} w_2(i_1, i_2) \tag{3}$$

in which

$$\begin{aligned} w_1(i_1, i_2) = & -i_1\theta(i_1 - 1, i_2) + (i_1 - j_1)\theta(i_1, i_2) + j_1\theta(i_1 + 1, i_2) \\ & + \delta[-i_1 v(i_1 - 1, i_2) + (i_1, j_1)v(i_1, i_2) + j_1 v(i_1 + 1, i_2)] \end{aligned} \tag{4}$$

$$\begin{aligned} w_2(i_1, i_2) = & -i_2\theta(i_1, i_2 - 1) + (i_2 - j_2)\theta(i_1, i_2) + j_2\theta(i_1, i_2 + 1) \\ & + \delta[-i_2 v(i_1, i_2 - 1) + (i_2, j_2)v(i_1, i_2) + j_2 v(i_1, i_2 + 1)] \end{aligned} \tag{5}$$

2.1 The Harvesting Effect

As mentioned in the main text, $h(i_1, i_2)$ represents the harvesting effect that motivates the firm to exploit more consumer surplus by raising the price. It is also related to price elasticitiy as explained in the paper. Another simpler intuition can be drawn to show the connection between this harvesting effect and the price elasticity. From Cabral (2011) one can show that

$$h = -\frac{q_i}{q_i'} = \frac{1}{-\frac{q_i'}{q_i}} = \frac{1}{\epsilon} \tag{6}$$

$\epsilon = -\frac{q_i'}{q_i}$ is the price elasticity, in which q_i is the probability that a new consumer choosing network i (Demand) and q_i' is the derivative of the demand with respect to price p_i. So ϵ measures the changes in demand in response to a change in price. In my paper, demand elasticity also determines the harvesting effect:

$$h_{12} = -\frac{q_{12} + q_{21}}{q_{12}' + q_{21}'} = \frac{1}{-\frac{q_{12}' + q_{21}'}{q_{12} + q_{21}}} = \frac{1}{\epsilon} \tag{7}$$

So $\epsilon = -\frac{q_{12}' + q_{21}'}{q_{12} + q_{21}}$ is an weighted average of the harvesting effects in both local market 1 and 2. Here the population N is not in this expression because I assume the same population. In fact, in another version of this model with heterogeneous population N_1 and N_2 at each local market, the price elasticity depends on the relative number of population: $\epsilon = -\frac{N_1 q_{12}' + N_2 q_{21}'}{N_1 q_{12} + N_2 q_{21}}$. When one local market is larger than the other one, the firm puts less weight the smaller market; when one market has no population, the model converges to the model developed by Cabral (2011), where the network effects is global.

2.2 The Investing Effect

$w(i_1, i_2)$ shows the investing effect. It is expressed as the difference in the firm's benefit between attracting and losing a new customer. The total investing effect $w(i_1, i_2)$ is a weighted average of both local investing effects at each local market. w_1 and w_2, and each of the local investing effect includes the change of the firm's current benefit and future benefit by attacting a new customer.

The weights are calculated by the relative price sensitivity. If consumers in local market 1 are significantly more sensitive than those in market 2, the firm focuses more on consumers in market 1.[3]

Again, like the discussion of the harvesting effect above, another version of the model with heterogenous population gives

$$w(i_1, i_2) = \frac{N_1 q'(i_1, i_2)}{N_1 q'(i_1, i_2) + N_2 q'(i_2, i_1)} \frac{1}{N_1} w_1(i_1, i_2) + \frac{N_2 q'(i_2, i_1)}{N_2 q'(i_1, i_2) + N_1 q'(i_2, i_1)} \frac{1}{N_2} w_2(i_1, i_2) \tag{8}$$

In this case, firms adjust their pricing strategy by taking the market sizes into consideration.

[3]For example, $w_1 = 100$, $w_2 = 5$ means the additional value brought by an additional consumer from market 1 and market 2 are 100 and 5 respectively. By this measure alone, it seems that market 1 is a more profitable investment. But if $q'(i_1, i_2) = 0.01$ and $q'(i_2, i_1) = 1$, meaning that consumers in local market 2 are much more price sensitive to those in market 1, then lowering prices in market 2 becomes a better choice.

References