Platform Competition with Local Network Effects

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Abstract

This paper presents a dynamic model of price competition between two platforms in which consumers value *local* network effects and choose one of the platforms (single homing). Specifically, each consumer's utility level depends on the number of her *neighbors* on the same platform. Each platform competes for new customers in different *neighborhoods* with a homogeneous entry price. I characterize equilibrium market structure with a combination of analytical and numerical solutions, and compare them to results from models with network effects that are *global*, in which a consumer benefits from *all* other consumers on the same platform. I provide sufficient conditions such that one platform dominates all market *segments*, as well as sufficient conditions that one platform only dominate one market *segment*.

Keywords: platform competition, price competition, network effects, local network externalities, two-sided markets

JEL Classification Numbers: D43, D62, L11, L13, L14

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1 Introduction

In many markets (e.g., video games) consumer utility is a funciton of the number of other consumers of the same product (e.g., game platform), either because compatibility allows for consumers to play each other (direct network effects) or because a bigger installed base leads to the development of third party products and services (indirect network effects, e.g., new games).

An important distinction regarding network effects dinstinguishes *global* from *local* effects. With *global* network effects, the value of a network to consumers depends on the total number of consumers in the same network,[1] regardless of their identity.[2] With *local* network effects, a consumer only values other customers in her *neighborhood*. Here *neighborhood* can refer to social group, geographical segment, profession, or generally speaking, a group of people who have certain relationships. This paper models price competition of firms with consumers who enjoy *local* network effects.

For example, a hardcore gamer enjoys playing games with other hardcore gamers, but not with casual gamers. Her utility depends on the number of hardcore gamers playing on the same game console, but not the number of casual gamers. However, models with *global* network effects assume consumer utility depends on the total number of gamers owning the same game console, which might lead to a very different analysis of firms' and consumers' optimal strategy. In this context, *local* network effects arise when the installed base of a given platform includes many users with similar tastes (in the same *neighborhood*), so that the games developed for the platform have greater appeal to users in that *neighborhood*.

Another important aspect of network effects is the distinction between platform size and strength of network effects. The size of a network refers to the number of users (direct network effects) or the availability of complementary products (indirect network effects) in that network. The strength of network effects refers to how strongly users value others on the same platform. Consumers' valuation of a platform depends on both the size and the strength of network effects.

This paper investigates the relative role of local and global network effects on platform competition. In my model, each platform competes for new customers in two market segments by setting a single platform adoption price,[3] and new consumers choose a platform given the adoption prices and current platform size in both market segments. Both firms and consumers benefit from network effects, but consumers' *local* network benefits are greater than the *global* network benefits, in the sense that a consumer benefits more from additional users on the same platform *and* in the same market segment. Both firms and consumers make their decisions sequentially and consumers "switch" platforms through a birth-death process (that is, each consumer reassesses its platform membership at random moments in time).

I solve for the consumers' optimal platform choices and the firms' optimal pricing decisions, both of which depend on current market size in all market segments. I also characterize the equilibrium using both analytical and numerical results. Moreover, I provide sufficient conditions for different equilibrium market structures — mainly by varying the discount factor and the strength of local network effects — and investigate the rationale behind these results.[4]

If the strength of network effects is weak, i.e. consumers do not value highly the size of existing customer base, then consumers' idiosyncratic brand preferences and platform adoption prices dominate their platform choice. In equilibrium, the stationary distribution of market shares is uni-modal, that is, both platforms retain a significant market share in each market segment. In this case, consumers' platform decisions are highly elastic to prices, so accordingly, firms compete fiercely for new customers through their platform adoption prices.

If strength of network effects is strong, i.e. the size of existing customers is highly valued, then a larger platform is more likely to dominate the market. Models with *global* network effects show that in equilibrium one platform dominates the *whole* market. However, my model with *local* network effects predicts that in equilibrium each platform dominates only one of the market segments, with the local dominant firm charging a high adoption price.

Roadmap

The rest of the paper is organized as follows. In Section 2, I discuss the similarities and differences between (global and local) network effects and social networks by going through related literature in both fields. In Section 3, I present the dynamic competition model with local network effects and solve the dynamic optimization problems for consumers and firms. In Section 4, I provide analytical implications characterizing equilibrium price and market structure. In Section 5, I give a summary of numerical results and provide parameter values for different equilibrium market structures. I also discuss the intuition and rationales behind the numerical results. Section 6 concludes with how this framework can be improved with certain features and how it can be applied to various industries for dynamic demand estimation.

[1] I will use the terms "platform", "firm" and "network" interchangeably. Consumers are on the same platform/network when they all use products or services from the same firm.

[2] Please note that the global network effect is not related to *global games* in game theory, which is an incomplete information game where the payoff structure is determined from a class of games and where each player makes a noisy observation of the game. (See Carlsson and Van Damme (1993))

[3] I will use "market segment" to denote the idea of "neighborhood" mentioned above.

[4] In this dynamic model, the market structure includes stationary distribution of market share, probability of a sale to new customers, and equilibrium prices charged by platforms.

2 Literature Review

Network Effects in IO Literature

Early literature in the study of network effects focused on static models.[5] The rapid development of dynamic models provides deeper understanding through the analysis of strategic consumer and firm behavior.

Early works in dynamic models include Farrell and Saloner (1985). They build a dynamic model with strategic consumers who enter into the market at different times. Fudenberg and Tirole (2000) develop a model of pricing to deter entry by a sole supplier of a network good. They show that with network effects, the threat of entry can lead the incumbent to set low prices.

Markovich and Moenius (2009) model the hardware purchase decisions by consumers as well as hardware adoption choices by software developers. Every period software developers decide whether to enter or exit the market, and if they are in, whether or not to invest to improve the quality of the software. Consumers choose a hardware platform given the entry price as well as the quality of the software available on that platform. They find that a successful software developer increases the hardware's market share, which in turn results in more quality investment by software developers, thus speeding up the market dominance process. In a sense, my paper is simpler than theirs since I do not consider quality investment nor entry/exit decisions in order to focus the effect of strength of local network effects on equilibrium market structure.

Jenkins et al. (2004) develop a dynamic competition model to analyze the browser war between Netscape and Microsoft. They analyze the barriers to entry effect caused by network effects, and investigate a firm's optimal strategy in order to gain market share faster than competitors. They show that an entrant firm has an incentive to fight with the incumbent firm. My paper provides conditions such that the fringe firm has a strong incentive to start a price war against the dominant firm in order to gain market share, as well as conditions such that the incentive is not enough for the fringe firm to fight.[6]

My paper looks at similar questions raised by Zhu and Iansiti (2011). They analyze the effect of the quality of a platform and consumers' discount factor on equilibrium market structure using a theoretical dynamic model in a two-sided market setup, with both consumers and developers. They also provide an empirical study on the gaming industry and explain the successful entry of Xbox into the market. A serious limitation of their model is that it assumes homogeneous prices set by different platforms, i.e. there are no pricing decisions. In fact, firms do not make any decisions in their model. Both pricing decisions and network effects are essential for firms to attract new customers and keep existing customers.

Lee (2013) investigates the impact of vertical integration and exclusivity in the gaming industry using a structural dynamic model in which consumers choose both hardware and software and developers adopt hardware. He estimates parameter values of the model using sales of PlayStation and Xbox as well as game titles over time. Then he conducts welfare analysis by simulating counterfactuals where exclusive vertical arrangements were prohibited. He concludes that exclusivity favored the entrant platform and is key to the successful entry of Xbox. His model is much more complicated than mine because his focus is on the structural demand estimation. My paper looks at a different set of research questions and focuses on the effect of local network effects on equilibrium market structure by providing both analytical and numerical implications.

The underlying model of my paper is based on Cabral (2011). He models dynamic competition with consumers making network choices and firms making price decisions. He then provides results regarding market equilibria, firms' pricing strategy, and network size dynamics. My paper differs from his in several aspects. One, different types of network effects are addressed: instead of *global* network effects, I analyze dynamic competition for consumers who value *local* network effects. Second, the timeline of the model is different: I models a new customer's decision with simultaneous birth and death process, so instead of having full information of the current state of the market share, the new customer has an expectation of future payoff. Third, I model market segmentation and the equilibrium market structure in different market segments.

Social Networks

Strategic decisions and dynamic interaction are prevalent in the literature of network effects in IO, as mentioned above. However, the models of network effects in IO have very simple network structure, as the standard utility is a linear function of the user base in the case of direct network effects. At the same time, current literature on social networks adopts very complex structures to explain the interaction among individuals. (See Jackson and Zenou (2012) for a detailed literature review in social networks). However, current literature in social networks has little dynamic strategic interaction compared to that in the IO literature. Both types of networks have thus respectively provided deep insights in their own fields.

Candogan, Bimpikis and Ozdaglar (2010) study the optimal pricing strategies of a monopolist selling a divisible good to customers who experience local network effects. They analyze a broader set of problems, including price

[5]See, e.g. Katz and Shapiro (1985). Moreover, see Shy (2004) and Farrell and Klemperer (2007) for a thorough literature review for early literature on static models

[6]See Doraszelski and Pakes (2007), for other early research on dynamic models with network effects.

discrimination of customers depending on the popularity of that person and algorithms to achieve the optimal set of customers to serve. There are two main differences from my paper. First, I consider a duopolistic competition framework and analyze strategic behavior from both the firms' and the consumers' sides. Second, I develop a dynamic model where consumers are forward-looking. Fundamentally, Candogan, Bimpikis and Ozdaglar (2010) is a paper in social networks and mine is in IO with certain social networks features.

Campbell (2012) tries to bridge these two areas of research from one end. He constructs a framework where a monopoly sells a good to consumers who are in social networks. He then shows that the optimal price of the monopoly can be higher or lower given different structures of the network. Campbell (2012) starts from models in social networks literature and adds some elements of network effects in IO.

Local Network Effects

Local network effect is an extension of network effects towards the literature of social networks. Current research in social networks focuses on the formation and behavioral implications of social networks, using graph theory and cooperative game theory. However, research of network effects in IO focuses on the effect of network externalities on the decisions of consumers and firms, therefore providing welfare implications. The topics of network effects in IO include compatibility, standardization, platforms, etc. Local network effects add certain structures of social networks into network effects in IO.

Very little research has been done on local network effects in an IO setting, i.e. analyzing firms and consumers behavior due to the benefits from local network effects. Fjeldstad, Moen and Riis (2010) explore the competition and welfare effect using a local network effect framework where two firms compete by offering differentiated products. In their static model, each consumer has a social location on a Salop circle and each has a technical preference. Their model is more complex since each consumer values others' platform decisions depending on their distance. However, their equilibrium analysis depends on the assumption that social and technical preferences are correlated. In my model, I focus on the differences of products caused by network effects, not by the preference of different social group. Moreover, with a dynamic setup, my model does not require coordination among all consumers.

Existing literature of symmetric models with (global) network effects all show that whenever the network is very strong, i.e. consumers value highly of existing user base on the same platform, in equilibrium, the market always consists of a dominant firm and a competitive fringe, and each firm can become the dominant firm with equal probability. So what if there are two market segments where consumers only value local network effects. If firms can charge different prices in different market segments, then in each market segment the competition is exactly the same as predicted by models with global network effects, thus in each segment, both firms are equally likely to become a dominant firm. However, in reality, firms are usually unable to price discriminate by market segments. My model will show that when the network is strong enough, the equilibrium market structure will always be local dominance where each firm dominants one market segment.

3 Model

I create an infinite-period model where two platforms compete for consumers and consumers choose a platform by paying the entry price. The timeline of the model is shown in Figure 1.[7]

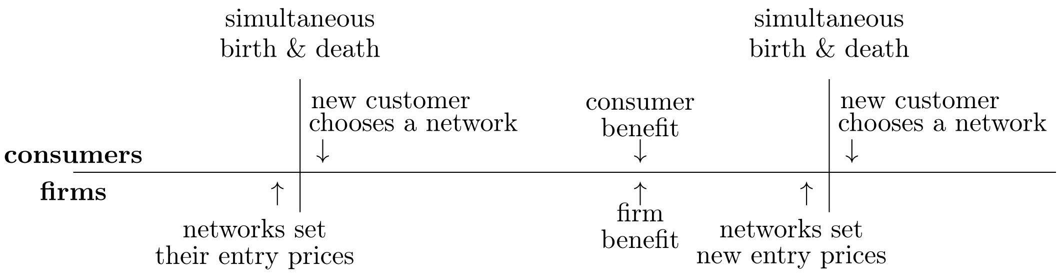

Figure 1: Timeline of Model

To model local network effects, I assume two local market segments, denoted by A and B, and two platforms, represented by i and j. (i_A, i_B) and (j_A, j_B) are the customer bases of platforms i and j in each market segment. I assume that two local market segments have the same population size N.[8]

[7] My model has many similar features to Cabral (2011), who models *global* network effects.
[8] $i_A, i_B, j_A, j_B \in [0, N]$, and $i_A = N - j_A$, $i_B = N - j_B$

The network switching decision is modeled as a simultaneous birth-death process.[9] Before the birth-death process, both networks set entry prices for new customers, taking into consideration of their current user bases and expected future period payoff and firm value. A higher price increases the firm's current profit when it's chosen by the consumer, but reduces the chance of being chosen. If the future payoff is valued highly (i.e. high discount factor), a network will fight fiercely for more user base and trade short-term losses for long-term gains. Then firms solve for the dynamic optimization problem by offering entry prices $p_i(i_A, i_B)$ and $p_j(j_A, j_B)$.

At the beginning of each period, exactly one randomly chosen customer dies (leaves a network) and one new customer is born (re-evaluates her network choice) in the same segment.[10][11] Then the new born customer chooses a network to join by paying the network entry price p_i or p_j, taking into consideration of her own network preference, network entry prices, and current user bases.

After the birth-death process, new user bases are known to both firms and consumers. All consumers and firms receive their period payoffs. For example, consumers in network i in market segment A receive $\lambda(i_A)$, and network i receives a payoff of $\theta(i_A, i_B)$. Firms' period payoff are increasing function of both i_A and i_B, but consumers' period benefit only depend on the number of other consumers in the same network and same segment. I assume the aftermarket consumer and firm payoffs satisfy the following properties:

Property 1. *$\lambda(i_A)$ and $\lambda(i_B)$ are increasing in i_A and i_B, respectively.*

Property 2. *$\theta(i_A, i_B)$ is an additively separable, and is increasing in both i_A and i_B. Moreover, $\theta(i_A+1, i_B)-\theta(i_A, i_B)$ is increasing in i_A only, and $\theta(i_A, i_B+1)-\theta(i_A, i_B)$ is increasing in i_B only.*

Property 1 says that consumers benefit from more existing customer base on the same platform.[12] The first part of property 2 shows that firms benefit from larger network size in both market segments. The second part of property 2 says that marginal aftermarket benefit from an additional customer is increasing in the network size, i.e. increasing return to scale.[13]

The "localness" of *local* network effect comes from two sources. First, consumers only value others on the same platform *and* in same market segment, not those on the same platform but different segment, nor those in the same segment but on a different platform. For example, hardcore Xbox gamers do not want to play against amateur Xbox gamers and cannot play against any Wii gamers. However, the number of consumers in other segments still matters through firms' pricing strategy. Second, firms can only set one price for all consumers, i.e. firms cannot price discriminate against consumers based on their market segmentation.[14] So each firm takes both market segments into consideration when maximizing its firm value.

In the subsequent analysis, I will solve for firms' optimal pricing strategy and consumers' optimal network adoption strategy. Then I will provide both analytical and numerical analysis.

3.1 Consumers' Network Choices

When a consumer is born, she has a relative platform (brand) preference for platform i over platform j, denoted by γ.[15] I assume the distribution of γ satisfies the following assumption.

Assumption 1. *(i) $\Phi(\gamma)$ is continuously differentiable; (ii) $\phi(\gamma) = \phi(-\gamma)$; (iii) $\phi(\gamma) > 0$, $\forall\gamma$; (iv) $\frac{\Phi(\gamma)}{\phi(\gamma)}$ are strictly increasing; (v) $\frac{\phi'(\gamma)}{\phi(\gamma)}$ is strictly decreasing.*

This assumption has been widely used, especially in the literature of auction theory, and many distributions, including the normal distribution, satisfy Assumption 1.

When a new consumer chooses which platform to join, three factors are taken into consideration: 1. relative platform (brand) preference; 2. platform entry prices; 3. current user bases and future expected user bases. When a consumer is born, she is assigned a random platform preference γ which follows certain distribution. In each period, both platforms chooses the platform entry prices p_i and p_j, given their current market share. The customer observes the existing user bases of both platforms and makes prediction on the future evolution of the market structure. In

[9]This discrete birth-death process is essentially the same as the expected value of a continuous case such as Poisson process. For a Poisson process with arrival rate λ, the mean time interval between any two consecutive events is $\frac{1}{\lambda}$. So the discount factor in the discrete case is $\delta = exp(-r/\lambda)$.

[10]By assuming a consumer dies and is born in the same segment, I can hold the population in both segments fixed in order to reduce the state space and allow the model to be analytically and computationally tractable.

[11]The proportion of consumers who re-evaluate their network decision is $\frac{1}{N}$. A larger N means that a smaller proportion of consumers go through the birth-death process.

[12]Here I model positive network effects, i.e. more users make a product more valuable. In reality, negative network effects, or "congestion effect", can also occur, where where more users make a product less valuable.

[13]Many industries satisfy these two properties, especially those with large investments in R&D or infrastructure, e.g. game consoles, telecommunications, etc.

[14]The inability to price discriminate may result from logistical or legal reasons.

[15]In other words, if $\gamma = 0$ then the consumer is indifferent between two platforms; if $\gamma > 0$, then she prefers platform i; and if $\gamma < 0$, then she prefers platform j.

each period, consumers benefit from existing customer base on the same platform, denoted by $\lambda(.)$. $u(i_A, i_B)$ is the consumer value function for a consumer on platform i in market segment A, which is the expected discounted value of all future period payoffs.[16]

The demand for one platform is measured by the probability of a new consumer choosing that platform. Given the current user base (i_A, i_B), I first calculate the position of the indifferent customer in market segment A: $x(i_A, i_B)$

$$\begin{aligned} x(i_A,i_B) - p_i + \frac{i_A}{N}u(i_A,i_B) + \frac{j_A}{N}u(i_A+1,i_B) &= -p_j + \frac{j_A}{N}u(j_A,j_B) + \frac{i_A}{N}u(j_A+1,j_B) \\ x(i_A,i_B) = p_i - p_j + \frac{j_A}{N}u(j_A,j_B) &+ \frac{i_A}{N}u(j_A+1,j_B) - \frac{i_A}{N}u(i_A,i_B) - \frac{j_A}{N}u(i_A+1,i_B) \\ \equiv p_i - p_j + Eu(j_A,j_B) &- Eu(i_A,i_B) \end{aligned} \tag{1}$$

Similarly, the position of the indifferent customer in segment B is $x(i_B, i_A)$.

$$\begin{aligned} x(i_B,i_A) - p_i + \frac{i_B}{N}u(i_B,i_A) + \frac{j_B}{N}u(i_B+1,i_A) &= -p_j + \frac{j_B}{N}u(j_B,j_A) + \frac{i_B}{N}u(j_B+1,j_A) \\ x(i_B,i_A) = p_i - p_j + \frac{j_B}{N}u(j_B,j_A) &+ \frac{i_B}{N}u(j_B+1,j_A) - \frac{i_B}{N}u(i_B,i_A) - \frac{j_B}{N}u(i_B+1,i_A) \\ \equiv p_i - p_j + Eu(j_B,j_A) &- Eu(i_B,i_A) \end{aligned} \tag{2}$$

The probability that a new consumer in segment A choosing network i is given by

$$\begin{aligned} q(i_A,i_B) &= 1 - \Phi(x(i_A,i_B)) \\ &= 1 - \Phi(p_i - p_j + Eu(j_A,j_B) - Eu(i_A,i_B)) \end{aligned} \tag{3}$$

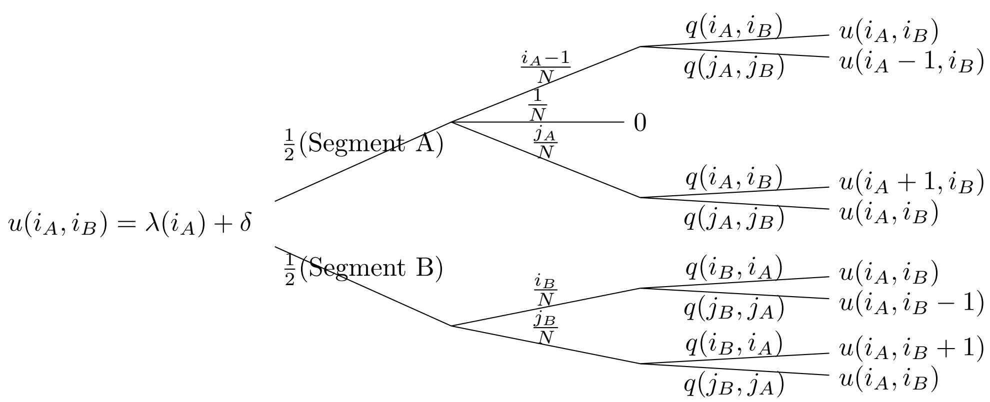

Figure 2: Consumer value function

The consumer value function and the birth-death process are shown in Figure 2. For example, if I were a consumer on platform i in market segment A, I have a value function $u(i_A, i_B)$, which is a expected discounted present value of all future period payoffs $\lambda(.)$, when there are i_A and i_B users on platform i in segments A and B. In the current period, I enjoy $\lambda(i_A)$ from other consumers on the same platform in the same market segment.

In each of the future periods, only one consumer is randomly chosen to die among all consumers. The probability of death in each segment is $\frac{1}{2}$, since the population in each segment is assumed to be the same. The probability of death of one person is thus $\frac{1}{2N}$.

So in next period, given that the death is in segment A (with probability $\frac{1}{2}$, the probability that I were to die is $\frac{1}{N}$ (I get value 0), the probability that someone else dies on platform i is $\frac{i_A-1}{N}$, and the probability that someone dies on platform j is $\frac{j_A}{N}$. The birth-death process is assumed to be simultaneous, so the new born consumer cannot identify who died in the current period until she joins one platform. So the probability of a new consumer from segment A joining platform i and j is $q(i_A, i_B)$ and $q(j_A, j_B)$, respectively.[17] After the new consumer joins a platform, the new user base for each platform is realized, and I have a new consumer value function $u(i_A \pm 1, i_B)$.

[16] Similarly, $u(i_A + 1, i_B)$ is consumer value function for a consumer on platform i in segment A when there are $i_A + 1$ and i_B in each segment, $u(i_B, i_A)$ is consumer value function for a consumer on platform i in segment B, $u(j_A, j_B)$ is consumer value function for a consumer on platform j in segment A, etc.

[17] Note that I assume that the outside option is always dominated, so here $q(i_A, i_B) = 1 - q(j_A, j_B)$.

Thus the consumer value function $u(i_A, i_B)$ is calculated as the expected present value of all future period payoffs:

$$\begin{aligned} u(i_A, i_B) = \lambda(i_A) + \delta\{ &\frac{i_A - 1}{2N}[q(i_A, i_B)u(i_A, i_B) + q(j_A, j_B)u(i_A - 1, i_B)] \\ &+ \frac{j_A}{2N}[q(i_A, i_B)u(i_A + 1, i_B) + q(j_A, j_B)u(i_A, i_B)] \\ &+ \frac{i_B}{2N}[q(i_B, i_A)u(i_A, i_B) + q(j_B, j_A)u(i_A, i_B - 1)] \\ &+ \frac{j_B}{2N}[q(i_B, i_A)u(i_A, i_B + 1) + q(j_B, j_A)u(i_A, i_B)]\} \end{aligned} \tag{4}$$

Similarly the consumer value for those in segment B is written as $u(i_B, i_A)$, and consumer values in platform j are written as $u(j_A, j_B)$ and $u(j_B, j_A)$, for market segment A and B respectively.[18]

Equation (4) can be simplified into a linear system of equations (5) and then u can be solved easily.

$$c_1 u(i_A - 1, i_B) + c_2 u(i_A, i_B - 1) + c_3 u(i_A, i_B) + c_4 u(i_A, i_B + 1) + c_5 u(i_A + 1, i_B) = \lambda(i_A) \tag{5}$$

where c_1, c_2, c_3, c_4, c_5 are functions of δ, q, N, i_A, i_B, j_A, j_B

3.2 Firms' Pricing Decisions

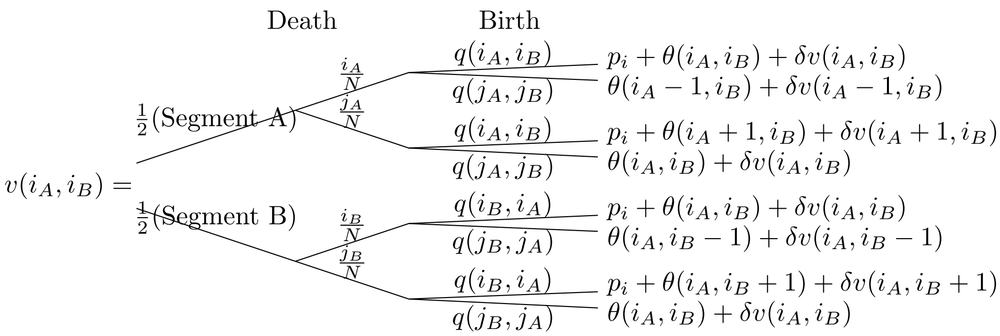

Figure 3: Firm value function

The firm value function is illustrated in Figure 3. Similar to consumer value function, the probability of death in each segment is $\frac{1}{2}$. $v(i_A, i_B)$ is the value function for firm i. $v(i_A, i_B)$ and $v(i_B, i_A)$ are always equal, since both market segments are assumed to be identical.[19]

The corresponding formula for the firms' value function is given by:[20]

$$\begin{aligned} v(i_A, i_B) = &\frac{i_A}{2N}[q(i_A, i_B)(p_i + \theta(i_A, i_B) + \delta v(i_A, i_B)) + q(j_A, j_B)(\theta(i_A - 1, i_B) + \delta v(i_A - 1, i_B))] \\ &+ \frac{j_A}{N}[q(i_A, i_B)(p_i + \theta(i_A + 1, i_B) + \delta v(i_A + 1, i_B)) + q(j_A, j_B)(\theta(i_A, i_B) + \delta v(i_A, i_B))] \\ &+ \frac{i_B}{N}[q(i_B, i_A)(p_i + \theta(i_A, i_B) + \delta v(i_A, i_B)) + q(j_B, j_A)(\theta(i_A, i_B - 1) + \delta v(i_A, i_B - 1))] \\ &+ \frac{j_B}{N}[q(i_B, i_A)(p_i + \theta(i_A, i_B + 1) + \delta v(i_A, i_B + 1)) + q(j_B, j_A)(\theta(i_A, i_B) + \delta v(i_A, i_B))] \end{aligned} \tag{6}$$

As we see from equation (6), $q(i_A, i_B)$ depends on p_i through equation (3). So we can solve for the optimal pricing decision by equating the first-order condition with respect to p_i to zero:

$$p_i = h(i_A, i_B) - w(i_A, i_B) \tag{7}$$

where

$$h(i_A, i_B) = -\frac{q(i_A, i_B) + q(i_B, i_A)}{q'(i_A, i_B) + q'(i_B, i_A)}{}^{21} \tag{8}$$

$$w(i_A, i_B) = \frac{q'(i_A, i_B)}{q'(i_A, i_B) + q'(i_B, i_A)} w_1(i_A, i_B) + \frac{q'(i_B, i_A)}{q'(i_A, i_B) + q'(i_B, i_A)} w_2(i_A, i_B) \tag{9}$$

[18] Note that I do not distinguish between u_i and u_j because the model is a symmetric one. In equilibrium, $u_i(z_1, z_2) = u_j(z_1, z_2)$, $\forall z_1, z_2 \in [0, N]$

[19] The only state variables are market shares (customer bases) in both each market segments. So in equilibrium, firm value depends on market share only, not on the identity of the firm, i.e. $v_i(z_1, z_2) = v_j(z_1, z_2)$, $\forall z_1, z_2 \in [0, N]$

[20] Throughout the paper, I write the network entry price for firm i as p_i instead of $p_i(i_A, i_B)$ for simplicity.

in which

$$w_1(i_A,i_B) = -\frac{i_A}{N}\theta(i_A-1,i_B) + \frac{i_A-j_A}{N}\theta(i_A,i_B) + \frac{j_A}{N}\theta(i_A+1,i_B) \tag{10}$$
$$+\delta[-\frac{i_A}{N}v(i_A-1,i_B) + \frac{i_A-j_A}{N}v(i_A,i_B) + \frac{j_A}{N}v(i_A+1,i_B)]$$
$$w_2(i_A,i_B) = -\frac{i_B}{N}\theta(i_A,i_B-1) + \frac{i_B-j_B}{\theta}(i_A,i_B) + \frac{j_B}{N}\theta(i_A,i_B+1) \tag{11}$$
$$+\delta[-\frac{i_B}{N}v(i_A,i_B-1) + \frac{i_B-j_B}{v}(i_A,i_B) + \frac{j_B}{N}v(i_A,i_B+1)]$$

Firms' optimal pricing decisions (equation (7)) show a tradeoff between harvesting and investing effects, denoted as h and w. Harvesting and investing effects are terminologies commonly adopted in the literature of switching cost.[22] Through harvesting effect, firms can charge a high price for existing customers because they are unwilling to switch due to switching costs. At the same time, investing effect shows the incentive for firms to lower its entry price in order to lock more customers into their product so that more surplus value can exploited in the future. In the model of network effects, however, switching cost is modeled through a birth-death process, so instead of exploiting existing customers, firms can exploit new customers by charging a high entry price due to the network effects valued by the new customer. At the same time, the firm finds incentive to lowers its entry price in order to enlarge its network in order to exploit more surplus value from new customers later.

In the harvesting effect shown by equation (8)[23], $q(i_A,i_B)$ and $q(i_B,i_A)$ are the probability that network i is chosen by a new consumer if death occurred in market segment A and B, respectively.[24] $q'(i_A,i_B)$ and $q'(i_B,i_A)$ are the first order conditions of consumer demand with respect to the entry price p_i. They measure the sensitivity of consumers' platform choice to the firm's pricing decision. The more sensitive consumers are to prices (higher elasticity of demand), the weaker the harvesting effect is for the firm.

The investing effect, denoted by $w(i_A,i_B)$ in equation (9), equals to a weighted average of the investing effects in both market segments. Relative elasticities between two market segments are used as weights. If consumers in market segment A are very sensitive to price changes compared to those in segment B, then firms would focus more on segment A, since a small change in price would cause a larger response from consumer demand.[25] The investing effects in each market segment are denoted by $w_1(i_A,i_B)$ and $w_2(i_A,i_B)$ in equations (10) and (11)[26]. Technically speaking, the investing effect is a measure of how much discounted present value the firm can gain by winning over a new customer. Each investing effect has two parts: current gain in period payoff, and future gain in firm value, which is a discounted value of all future payoffs.

Firms' optimal pricing decision is essentially measured by the average elasticity of consumer demand, which is very similar to the optimization rule in the standard profit maximization model. This can be shown by transforming equation (7) into:

$$\frac{p_i-(-w(i_A,i_B))}{p_i} = \frac{q(i_A,i_B)+q(i_B,i_A)}{-(q(i_A,i_B)+q(i_B,i_A))'p_i} = \frac{1}{\epsilon} \tag{12}$$

If we substitute equation (7) back into firm's value function (6), we can get a simplified version of firm's optimization problem:

$$c_1v(i_A-1,i_B) + c_2v(i_A,i_B-1) + c_3v(i_A,i_B) + c_4v(i_A,i_B+1) + c_5v(i_A+1,i_B) = B(i_A,i_B) \tag{13}$$
where c_1,c_2,c_3,c_4,c_5 and B are functions of δ, q, q', θ, N, i_A, i_B, j_A, j_B

Then the linear system of equations (13) can be solved easily to get the value of v.

---

[21] $q' = \frac{dq}{dp}$

[22] Please see Farrell and Klemperer (2007) for a literature review related to these concepts.

[23] Note that all calculation and explanation shown here are done from the perspective of platform i.

[24] The average probability is $\frac{q(i_A,i_B)+q(i_B,i_A)}{2}$, assuming homogeneous and constant population in both market segments. Please refer to Online Appendix for a version with heterogeneous population.

[25] The weights between two market segments are also shown in the harvesting effect. Please see the Online Appendix for more detailed discussion.

[26] Here the subscript of w refers to the market segment noted by the argument of w, e.g. subscript 2 in $w_2(i_A,i_B)$ refers to the second argument of w, which is segment B.

3.3 Transition Matrix

Given the equilibrium values of q, I calculate the transition matrix of the Markov process $M = m((z_1, z_2), (z_3, z_4)), \forall z_1, z_2, z_3, z_4 \in [0, N]$ where $m((z_1, z_2), (z_3, z_4))$ is the probability of moving from state (z_1, z_2) to state (z_3, z_4). Therefore we have:

$$
\begin{aligned}
m[(i_A, i_B), (i_A - 1, i_B)] &= \frac{i_A}{2N} q(j_A, j_B) \\
m[(i_A, i_B), (i_A, i_B - 1)] &= \frac{i_B}{2N} q(j_B, j_A) \\
m[(i_A, i_B), (i_A, i_B)] &= \frac{i_A}{2N} q(i_A, i_B) + \frac{i_B}{2N} q(i_B, i_A) + \frac{j_A}{2N} q(j_A, j_B) + \frac{j_B}{2N} q(j_B, j_A) \\
m[(i_A, i_B), (i_A, i_B + 1)] &= \frac{j_B}{2N} q(i_B, i_A) \\
m[(i_A, i_B), (i_A + 1, i_B)] &= \frac{j_A}{2N} q(i_A, i_B)
\end{aligned}
$$

With the transition matrix M, the equilibrium stationary market share μ can be retrieved by solving $\mu M = \mu$.

4 Analytical Results

Proposition 1. *Suppose Property 1 and 2 hold. There exists a δ' such that, if $\delta < \delta'$, there exists an unique equilibrium.*

The proof of Proposition 1 can be found in the appendix. The uniqueness of the equilibrium can only be shown with low values of discount factor only. The logic of the proof is then used for numerical simulation, which confirms the existence and uniquess of the equilibrium.

Proposition 2. *Suppose Property 1 and 2 hold. There exists a δ' such that, if $\delta < \delta'$, then there exists ψ' such that when $\psi > \psi'$, the stationary distribution of market shares is bimodal with local dominance.*

Proposition 2 states that if the strength of network effects is strong enough, i.e. consumers value highly the number of other users on the same platform, then market equilibrium shows local dominance where each platform dominates one market segment, and both platforms charge a high entry price to new customers.

As mentioned before, new consumers take into consideration of three factors when choosing a platform: brand preferences, entry prices, and total consumer value. Consumer value is calculated as the discounted present value of all future payoffs from a platform, and each period's payoff is a function of size and strength of network effects. If the strength of network effects is weak, then consumer value is less important compared to brand preferences and prices. Both platforms compete fiercely for consumers through prices. In equilibrium, both charge a relative low price and retain significant market shares in both segments. However, if strength of network effects is strong, consumer value outweighs the other two factors. In each market segment, the larger platform enjoys greater advantage since consumers value highly of existing user base. In equilibrium, each platform dominates one market segment.

Proposition 2 characterizes the equilibrium with low discount factors and strong network effects, and contrasts with equilibrium from models with *global* network effects. In those models, there is no market segmentation, and consumers value *all* other consumers on the same platform. If network effects are strong, equilibrium predicts one-firm dominance of the whole market. When extending the model to *local* network effects with two market segments, it seems that both global dominance and local dominance are possible with strong network effects. However, my model predicts that with *local* network effects, only local dominance can be observed. The main reason that only local dominance is achieved is due to firms' inability to price discriminate among market segments. In this case, neither firms have the incentive to fight through lowering their prices.

In the following section, numerical simulation complements theoretical propositions by providing a complete picture behind model equilibria.

5 Numerical Results

In this section, I summarize and plot the model equilibria with different values of discount factors (δ) and strength of network effects (ψ), and provide detailed explanation of the rationales behind different patterns of results.

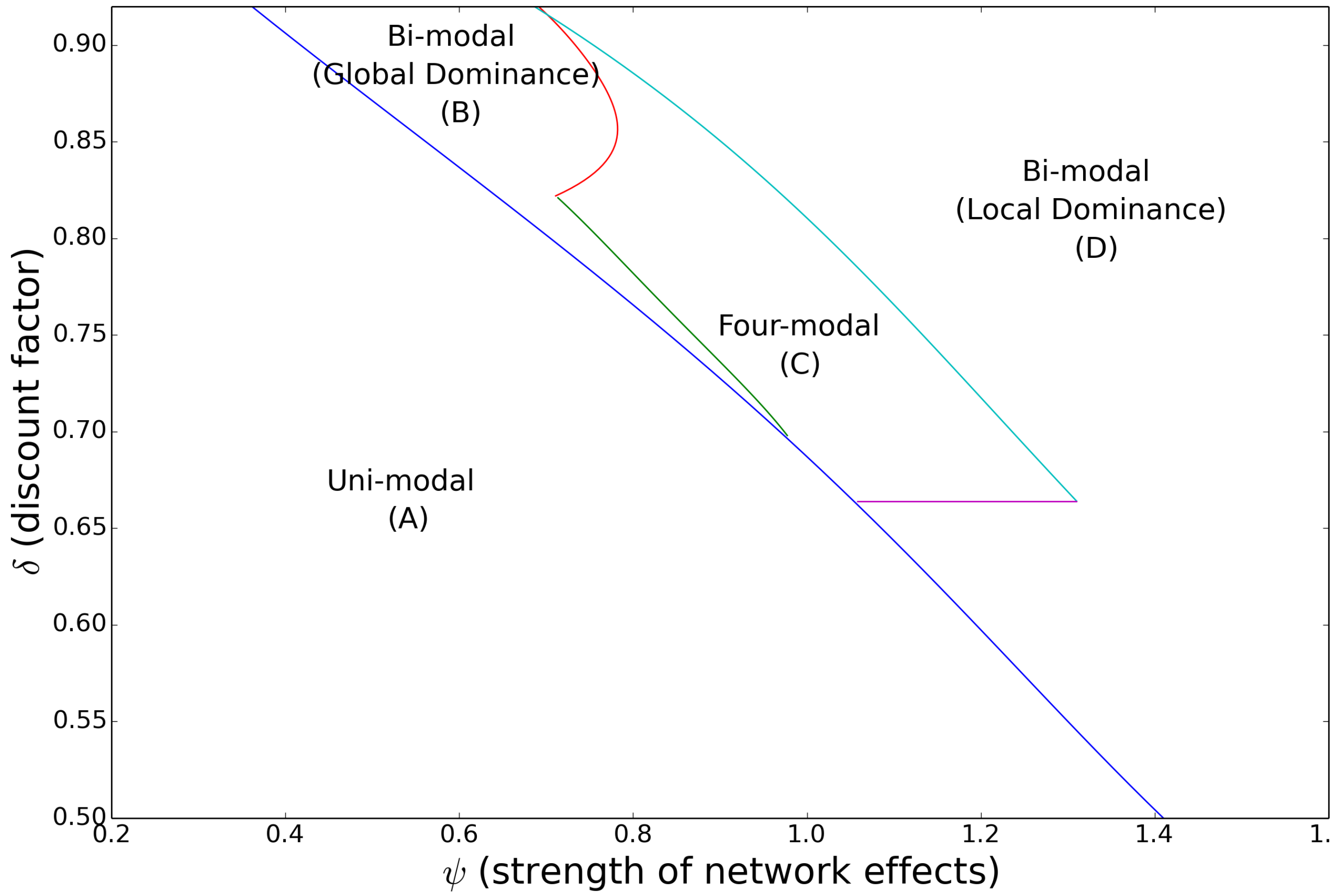

Figure 4: Stationary Distribution of Market Shares

I solve for the stationary distribution of market shares of the model with all values of δ and ψ.[27] I group different shapes of distribution into four groups:

$$\begin{cases} A\text{: Uni-modal (both platforms have similar market shares in both segments)} \\ B\text{: Bi-modal with Global Dominance (one platform dominates both segments)} \\ C\text{: Four-modal (mixture between Global and Local Dominance)} \\ D\text{: Bi-modal with Local Dominance (each platform dominates one segment)} \end{cases}$$

In the following subsection, I will explain each of the four types of equilibrium market structure in detail for low and high values of discount factor δ.

5.1 Low Discount Factor ($\delta = 0.60$)

Figure 5 lists equilibrium results (distribution of market shares and prices) with different strengths of network effects when the discount factor is small ($\delta = 0.6$).

Graphs on the left column show the stationary distribution of market shares for any *one* platform. Points on the horizontal plane represent market shares in segment A and B, and the vertical axis shows the probability of observing each point.[28] Graphs on the right show the equilibrium prices charged by a platform given its market shares.

In industries without network effects ($\psi = 0$), the distribution of market shares concentrates in the center of the domain, i.e. platforms split the market in each segment. Platforms always charge the same price regardless of their market shares since a larger market share does not bring extra utility to new consumers. Then new consumers choose a platform based purely on their own brand preferences. The distribution of market shares corresponds to region A in Figure 4.

In industries where the strength of network effects is strong (e.g. $\psi = 0.6$), the stationary distribution of market shares is bimodal with local dominance, i.e. one platform dominates either segment A or B, but not both. Numerical

[27] The numerical simulation of Figure 4 is done using $N = 5$, which can be interpreted as 20% of the consumers re-evaluate their network decisions every period. The main reason of using a small value of N is because of computational power limit. The time required to compute the equilibrium is exponential in N. Models with higher N produces similar qualitative results. Later numerical results with specific values of δ and ψ are computed using $N = 20$.

[28] When the market shares of one platform are (x_A, x_B), market shares of the other platform are $(1 - x_A, 1 - x_B)$.

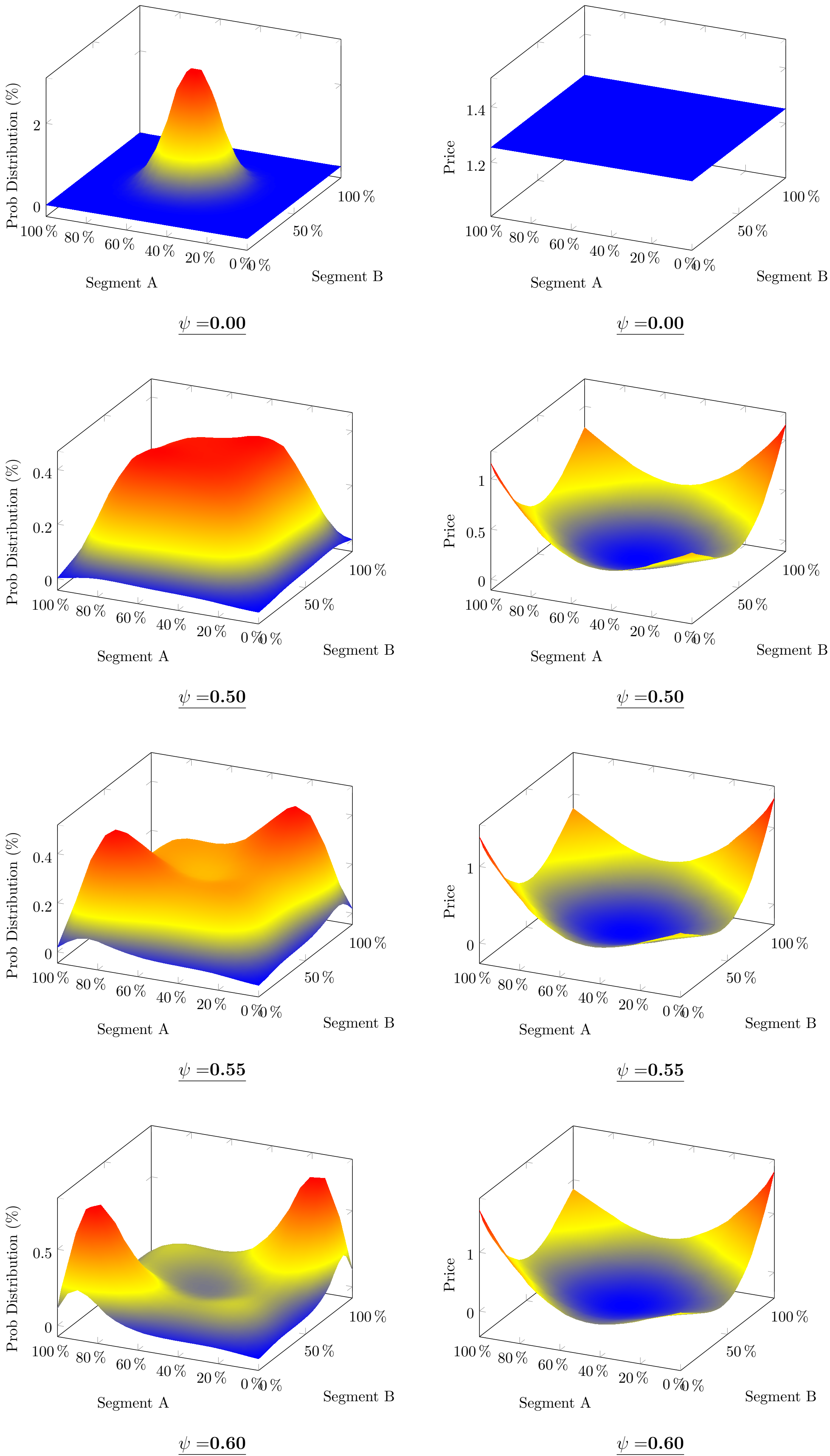

Figure 5: Stationary Distribution of Market Shares and Equilibrium Prices ($\delta = 0.60$)

simulation here not only confirms the analytical result, but also expands the result to all values of δ. It corresponds to region D in Figure 4.

Local dominance equilibrium can be explained using the harvesting and investing effects. When each platform dominates one segment, harvesting effect is large because because a new consumer in that segment has a high reservation price for a larger network. Both platforms exploit new customers in their respective dominant market segment by charging a high price. Since price discrimination is not allowed, neither platforms have the incentive to fight for customers in the competitor's segment by lowering its price and forgo the profit from its own dominant segment.

5.2 High Discount Factor ($\delta = 0.92$)

Figure 6 lists equilibrium results when the discount factor is large ($\delta = 0.92$).

In the model with small discount factor ($\delta = 0.60$), when increasing the strength of network effects, the stationary distribution of market shares changes from uni-modal to bi-modal with local dominance. However, in the model with large discount factor ($\delta = 0.92$), the stationary distribution of market shares changes from uni-modal to bi-modal with global dominance, then to four-modal, and lastly to bi-modal with local dominance.

In this case, global dominance is achievable where one platform dominates both segments ($\psi = 0.12$). In this equilibrium, both small and large platforms charge a high price. This is possible only when ψ is not too large, so that certain new customers still choose the smaller platform over the larger platform due to their own platform preference. However, as the strength of network effects becomes higher ($\psi = 0.18$), small platform cannot attract customers and charge a high price at the same time any more.[29]

6 Concluding Remarks

In this paper, I model local network effects based on an extension of the platform competition model proposed by Cabral (2011). I derive several analytical results and numerical results that characterizes equilibrium, including the consumers' platform decision, market structure and the platforms' optimal pricing strategy. I show that, if network effects are weak, then the stationary distribution of market shares is unimodal. If network effects are strong, however, then equilibrium market structure feature local dominance, that is, each firm dominates one local market. Finally, in the intermediate case when the discount factor is high and network effects relatively weak, the equilibrium features global dominance where one firm dominates both local markets; or four-modal equilibrium, a mixture between global and local dominance. I provide a rationale for why the different distributions are observed.

There are many other interesting issues that one can analyze with this local network effects model. One is to include firm entry and exit decisions: I would expect the optimal platform pricing strategy to changes with the potential threat of entry (see, for example, Fudenberg and Tirole (2000)). A second area of research is the role of heterogeneous network effects. Another promising extension would allow firms to increase the strength of network effects as a way to attract new users and keep existing ones. Finally, adding social networking elements (e.g., allowing communication across different market segments) would extend the model in a realistic model. All in all, I believe my model provides a first step in the effort to connect the IO and social networks literatures.

7 Appendix

Proof of Proposition 1. Combining firms' optimal pricing strategy equation (7) and consumer choice decision equation (3), we get:

$$p_i = \frac{2 - \Phi(x(i_A, i_B)) - \Phi(x(i_B, i_A))}{\phi(x(i_A, i_B)) + \phi(x(i_B, i_A))} - \frac{\phi(x(i_A, i_B))}{\phi(x(i_A, i_B)) + \phi(x(i_B, i_A))} w_1(i_A, i_B) - \frac{\Phi(x(i_B, i_A))}{\phi(x(i_A, i_B)) + \phi(x(i_B, i_A))} w_2(j_A, j_B) \tag{14}$$

$$\text{where } w_1(i_A, i_B) = -\frac{i_A}{N}\theta(i_A - 1, i_B) + \frac{(i_A - j_A)}{N}\theta(i_A, i_B) + \frac{j_A}{N}\theta(i_A + 1, i_B)$$
$$w_2(i_A, i_B) = -\frac{i_B}{N}\theta(i_A, i_B - 1) + \frac{(i_B - j_B)}{N}\theta(i_A, i_B) + \frac{j_B}{N}\theta(i_A, i_B + 1)$$

[29]More detailed analysis and insights are provided in the Online Appendix.

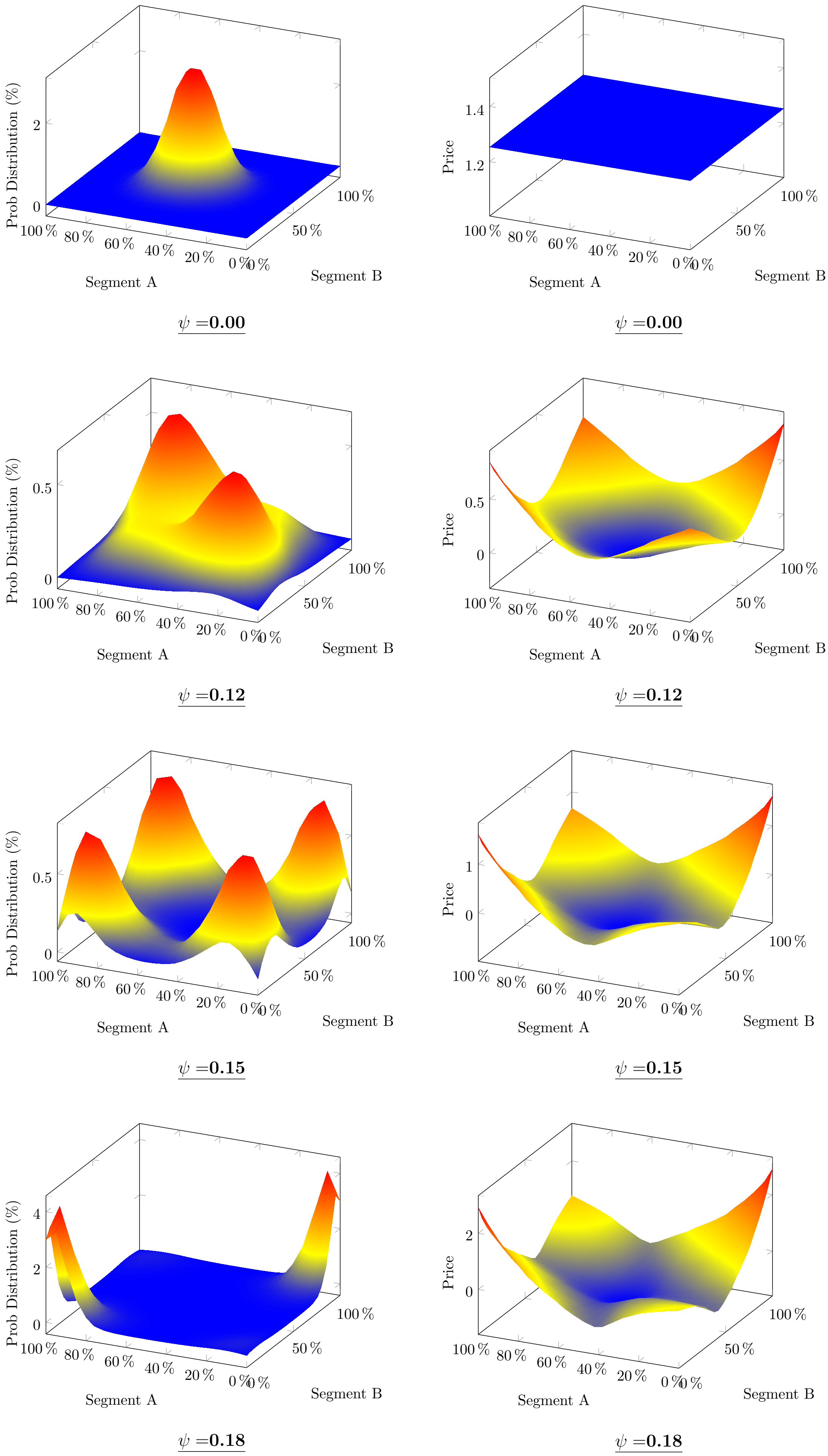

Figure 6: Stationary Distribution of Market Shares and Equilibrium Prices ($\delta = 0.92$)

Similarly,

$$p_j = \frac{2-\Phi(x(j_A,j_B))-\Phi(x(j_B,j_A))}{\Phi(x(j_A,j_B))+\Phi(x(j_B,j_A))} - \frac{\Phi(x(j_A,j_B))}{\Phi(x(j_A,j_B))+\Phi(x(j_B,j_A))} w_1(j_A,j_B) - \frac{\Phi(x(j_B,j_A))}{\Phi(x(j_A,j_B))+\Phi(x(j_B,j_A))} w_2(j_A,j_B) \tag{15}$$

$$\text{where } w_1(j_A,j_B) = -\frac{j_A}{N}\theta(j_A-1,j_B) + \frac{(j_A-i_A)}{N}\theta(j_A,j_B) + \frac{j_A}{N}\theta(j_A+1,j_B)$$
$$w_2(j_A,j_B) = -\frac{j_B}{N}\theta(j_A,j_B-1) + \frac{(j_B-i_B)}{N}\theta(j_A,j_B) + \frac{j_B}{N}\theta(j_A,j_B+1)$$

From the location of the indifference consumer in equation (1) and (2) and $i_A = N - j_A$, $i_B = N - j_B$, one can get $x(i_A, i_B) = -x(j_A, j_B)$, and $x(i_B, i_A) = -x(j_B, j_A)$. Given Assumption 1 (ii), $p_i - p_j$ can be written as:

$$\begin{aligned}
p_i - p_j = & \frac{2-2\Phi(x(i_A,i_B))-2\Phi(x(i_B,i_A))}{\phi(x(i_A,i_B))+\phi(x(i_B,i_A))} + \frac{\phi(x(i_A,i_B))}{\phi(x(i_A,i_B))+\phi(x(i_B,i_A))}(w_1(j_A,j_B)-w_1(i_A,i_B)) \\
& + \frac{\Phi(x(i_B,i_A))}{\phi(x(i_A,i_B))+\phi(x(i_B,i_A))}(w_2(i_A,i_B)-w_2(j_A,j_B)) \\
= & \frac{\Phi(-x(i_A,i_B))+\Phi(-x(i_B,i_A))}{\phi(-x(i_A,i_B))+\phi(-x(i_B,i_A))} - \frac{\Phi(x(i_A,i_B))+\Phi(x(i_B,i_A))}{\phi(x(i_A,i_B))+\phi(x(i_B,i_A))} + \frac{\phi(x(i_A,i_B))}{\phi(x(i_A,i_B))+\phi(x(i_B,i_A))} \\
& (w_1(j_A,j_B)-w_1(i_A,i_B)) + \frac{\Phi(x(i_B,i_A))}{\phi(x(i_A,i_B))+\phi(x(i_B,i_A))}(w_2(i_A,i_B)-w_2(j_A,j_B))
\end{aligned} \tag{16}$$

In order to show the steps of the proof in a clearer way, I simplify the expressions by using the following notations: $\Delta p_i = p_i - p_j, \Phi_{AB} = \Phi(x(i_A,i_B)), \Phi_{BA} = \Phi(x(i_B,i_A)), \Phi_{-AB} = \Phi(-x(i_A,i_B)), \Phi_{-BA} = \Phi(-x(i_B,i_A)), \phi_{AB} = \phi(x(i_A,i_B)), \phi_{BA} = \phi(x(i_B,i_A)), \phi_{-AB} = \phi(-x(i_A,i_B)), \phi_{-BA} = \phi(-x(i_B,i_A)), w_1^i = w_1(i_A,i_B), w_2^i = w_2(i_A,i_B), w_1^j = w_1(j_A,j_B), w_2^j = w_2(j_A,j_B)$. Thus equation (16) can be re-written as:

$$\begin{aligned}
\Delta p_i + \frac{\Phi_{AB}+\Phi_{BA}}{\phi_{AB}+\phi_{BA}} - \frac{\Phi_{-AB}+\Phi_{-BA}}{\phi_{-AB}+\phi_{-BA}} &= \frac{\phi_{AB}}{\phi_{AB}+\phi_{BA}}(w_1^j - w_1^i) + \frac{\phi_{BA}}{\phi_{AB}+\phi_{BA}}(w_2^j - w_2^i) \\
&= \frac{\phi_{AB}}{\phi_{AB}+\phi_{BA}}(w_1^j - w_1^i - w_2^j + w_2^i) + w_2^j - w_2^i
\end{aligned} \tag{17}$$

From Assumption 1 (iv) $\frac{\Phi(x)}{\phi(x)}$ is strictly increasing in x, it can be shown that $\frac{\Phi_{AB}+\Phi_{BA}}{\phi_{AB}+\phi_{BA}} - \frac{\Phi_{-AB}+\Phi_{-BA}}{\phi_{-AB}+\phi_{-BA}}$ increases when both $x(i_A,i_B)$ and $x(i_B,i_A)$ increase. From equation (1) and (2), both $x(i_A,i_B)$ and $x(i_B,i_A)$ increase in Δp_i. Therefore, the left-hand side of equation (17) increases in Δp_i.

In order to show the uniqueness of the equilibrium, we need to show that the value of the right-hand side of the equation (17) is decreasing in Δp_i.

$$\begin{aligned}
\frac{dRHS}{d\Delta p_i} &= \frac{\phi'_{AB}(\phi_{AB}+\phi_{BA}) - \phi_{AB}(\phi'_{AB}+\phi'_{BA})}{[\phi_{AB}+\phi_{BA}]^2}(w_1^j - w_1^i - w_2^j + w_2^i) \\
&= \frac{1}{[\phi_{AB}+\phi_{BA}]^2}(\phi'_{AB}\phi_{BA} - \phi_{AB}\phi'_{BA})(w_1^j - w_1^i - w_2^j + w_2^i)
\end{aligned} \tag{18}$$

To determine the sign of equation (18), we need to know the signs for each one of the three parts. The first part $\frac{1}{[\phi_{AB}+\phi_{BA}]^2}$ is obviously positive. However, the sign of the second and third parts of equation (18) $\phi'_{AB}\phi_{BA} - \phi_{AB}\phi'_{BA}$ and $w_1^j - w_1^i - w_2^j + w_2^i$ are unclear, and we need to rely on the following lemmas:

Lemma 1. *For small values of δ, $x_{BA} < x_{AB}$ if an only if $i_A < i_B$.*

Proof. Suppose $\delta = 0$, then from equation (4)), $u(i_A, i_B) = \lambda_{i_A}$,[30] then we can get:

$$x_{AB} = \Delta p_i + \frac{1}{N}[j_A\lambda_{j_A} + i_A\lambda_{j_A+1} - i_A\lambda_{i_A} - j_A\lambda_{i_A+1}]$$
$$x_{BA} = \Delta p_i + \frac{1}{N}[j_B\lambda_{j_B} + i_B\lambda_{j_B+1} - i_B\lambda_{i_B} - j_B\lambda_{i_B+1}]$$

Subtracting equation x_{AB} from x_{BA}, we get:

$$\begin{aligned}
N(x_{BA} - x_{AB}) &= N(\lambda_{j_B} - \lambda_{j_A} + \lambda_{i_A+1} - \lambda_{i_B+1}) + i_A(\lambda_{j_A} - \lambda_{j_A+1} + \lambda_{i_A} - \lambda_{i_A+1}) + i_B(\lambda_{i_B+1} - \lambda_{i_B} + \lambda_{j_B+1} - \lambda_{j_B}) \\
&= j_B(\lambda_{j_B} - \lambda_{j_A} + \lambda_{i_A+1} - \lambda_{i_B+1}) + i_A(\lambda_{j_A} - \lambda_{j_A+1} + \lambda_{i_A} - \lambda_{i_A+1}) + i_B(\lambda_{i_A+1} - \lambda_{i_B} + \lambda_{j_B+1} - \lambda_{j_A})
\end{aligned}$$

[30] Here I write $\lambda_{i_A} \equiv \lambda(i_A)$ for better legibility.

From $i_A < i_B$, which also implies $j_A > j_B$, together with Property 1, we conclude that $x_{BA} < x_{AB}$ if an only if $i_A < i_B$. $\square$

Lemma 2. *$\phi'_{AB}\phi_{BA} - \phi_{AB}\phi'_{BA} > 0$ if and only if $x_{BA} < x_{AB}$.*

From Assumption 1 (v), $x_{BA} < x_{AB}$ means $\frac{\phi'(x_{AB})}{\phi(x_{AB})} < \frac{\phi'(x_{BA})}{\phi(x_{BA})}$, and the result follows.

Lemma 3.

$$\begin{aligned} w_1(i_A, i_B) &= w_2(i_B, i_A) \\ w_1(j_A, j_B) &= w_2(j_B, j_A) \end{aligned}$$

Note that w is called the "investing" effect, and it measures firm's additional aftermarket benefit from making a sale compared to losing it. From equation (9), we know that $w_1(i_A, i_B)$ means firm i's investing effect for market segment A when the sizes are i_A and i_B, and $w_2(i_B, i_A)$ also means the investing effect for market segment B when sizes are i_B and i_A. By definition $w_1(i_A, i_B)$ and $w_2(i_B, i_A)$ refer to the same value, so do $w_1(j_A, j_B)$ and $w_2(j_B, j_A)$.

Lemma 4. *For small values of δ, $w_1(i_B, i_A) - w_1(i_A, i_B) > 0$ if $i_B > i_A$.*

Proof. Suppose $\delta = 0$, then from equation (9), we get:

$$\begin{aligned} w_1(i_B, i_A) &= \frac{i_B}{N}(\theta_{i_A,i_B} - \theta_{i_A,i_B-1}) + \frac{i_B}{N}(\theta_{i_A,i_B+1} - \theta_{i_A,i_B}) \\ w_1(i_A, i_B) &= \frac{i_A}{N}(\theta_{i_A,i_B} - \theta_{i_A-1,i_B}) + \frac{j_A}{N}(\theta_{i_A+1,i_B} - \theta_{i_A,i_B}) \end{aligned}$$

Subtracting $w_1(i_A, i_B)$ from $w_1(i_B, i_A)$ we have:

$$\begin{aligned} & w_1(i_B, i_A) - w_1(i_A, i_B) \\ =& [\theta_{i_A,i_B+1} - \theta_{i_A+1,i_B}] + \frac{i_B}{N}[\theta_{i_A,i_B} - \theta_{i_A,i_B-1} - \theta_{i_A,i_B+1} + \theta_{i_A,i_B}] + \frac{i_A}{N}[\theta_{i_A+1,i_B} - \theta_{i_A,i_B} - (\theta_{i_A,i_B} - \theta_{i_A-1,i_B})] \\ =& \frac{j_B}{N}[\theta_{i_A,i_B+1} - \theta_{i_A+1,i_B}] + \frac{i_B}{N}[\theta_{i_A,i_B} - \theta_{i_A,i_B-1} - (\theta_{i_A+1,i_B} - \theta_{i_A,i_B})] + \frac{i_A}{N}[\theta_{i_A+1,i_B} - \theta_{i_A,i_B} - (\theta_{i_A,i_B} - \theta_{i_A-1,i_B})] \end{aligned}$$

According to Property 2, each component of the equation above gives a positive value when $i_B > i_A$. So $w_1(i_B, i_A) - w_1(i_A, i_B) > 0$. $\square$

Comining Lemma 1 and Lemma 2, we know that the second part of equation (18) $\phi'_{AB}\phi_{BA} - \phi_{AB}\phi'_{BA} > 0$ if $i_A < i_B$.

Using Lemma 3, we can transform the third part of equation (18) $w_1^j - w_1^i - w_2^j + w_2^i$ into:

$$\begin{aligned} w_1^j - w_1^i - w_2^j + w_2^i &= w_1(j_A, j_B) - w_1(i_A, i_B) - w_2(j_A, j_B) + w_2(i_A, i_B) \\ &= w_1(j_A, j_B) - w_1(i_A, i_B) - w_1(j_B, j_A) + w_1(i_B, i_A) \\ &= w_1(j_A, j_B) - w_1(j_B, j_A) + w_1(i_B, i_A) - w_1(i_A, i_B) \end{aligned} \tag{19}$$

From Lemma 4, we know that $w_1^j - w_1^i - w_2^j + w_2^i > 0$ when $i_A < i_B$.

Therefore, when $i_A < i_B$ (or $j_A > j_B$), the second term of equation (18) is negative and the third term is positive; on the other hand, when $i_A > i_B$ (or $j_A < j_B$), the second term of equation (18) is positive and the third term is negative. At the same time, the first term is always positive. Thus, the value of equation (18) is always negative, i.e. the right-hand side of equation (16) is decreasing in Δp_i.

To conclude, there exists a unique value of Δp_i that satisfies equation (16). Together with the value of $u(i_A, i_B)$, $w_1(i_A, i_B)$, $w_2(i_A, i_B)$, we can characterize the full equilibrium including optimal prices $p(i_A, i_B)$ and probability of sales $q(i_A, i_B)$ $\forall i_A, i_B$.

$\square$

Proof of Proposition 2. Suppose $\delta = 0$, from equation (4)), $u(i_A, i_B) = \lambda_{i_A}$. From equation (1), I simiplify the equation by denoting

$$\begin{aligned} x(i_A, i_B) &= p_i - p_j + Eu(j_A, j_B) - Eu(i_A, i_B) \\ &\equiv \Delta p + L_1 \\ x(i_B, i_A) &= p_i - p_j + Eu(j_B, j_A) - Eu(i_B, i_A) \\ &\equiv \Delta p + L_2 \end{aligned}$$

If Property (1) and (2) hold, then I can get:

$$L_1 > 0 \text{ if } j_A > i_A$$
$$L_2 > 0 \text{ if } j_B > i_B$$

From equations (14) and (15), I denote:

$$w_1^j - w_1^i \equiv L_3 > 0 \text{ if } j_A > i_A$$
$$w_2^j - w_2^i \equiv L_4 > 0 \text{ if } j_B > i_B$$

I can write equation (16) as:

$$\Delta p = \frac{2 - 2\Phi(\Delta p + L_1) - 2\Phi(\Delta p + L_2)}{\phi(\Delta p + L_1) + \phi(\Delta p + L_2)} + \frac{\phi(\Delta p + L_1)}{\phi(\Delta p + L_1) + \phi(\Delta p + L_2)} L_3 + \frac{\Phi(\Delta p + L_2)}{\phi(\Delta p + L_1) + \phi(\Delta p + L_2)} L_4 \tag{20}$$

In order to show that the probability distribution of market share is bi-modal with local dominance, it is sufficient to shwo that if $i_A + i_B = N$, $i_A > j_A$, and $i_B < j_B$, then both $q(i_A, i_B)$ and $q(i_B, i_A) \to 1$ as $\psi \to \infty$.

When $i_A + i_B = N$, $i_A > j_A$, and $i_B < j_B$, we can show the following properties regarding L_1, L_2, L_3, L_4:

$$L_1 < 0$$
$$L_2 > 0$$
$$L_3 < 0$$
$$L_4 > 0$$
$$L_1 = -L_2$$
$$L_3 = -L_4$$

Using these properties, equation (20) can be simplified as:

$$\Delta p = \frac{2 - 2\Phi(\Delta p - L_2) - 2\Phi(\Delta p + L_2)}{\phi(\Delta p - L_2) + \phi(\Delta p + L_2)} + \frac{L_4}{\phi(\Delta p - L_2) + \phi(\Delta p + L_2)} [\phi(\Delta p + L_2) - \phi(\Delta p - L_2)] \tag{21}$$

Lemma 5. *Under the setup mentioned above in Proposition (2), the only possible value of Δp that satisfies equation (21) is zero.*

Proof. In order to show the proof with clarity, I denote parts of equation (21) using the following notations:

$$RHS_1 = \frac{2 - 2\Phi(\Delta p - L_2) - 2\Phi(\Delta p + L_2)}{\phi(\Delta p - L_2) + \phi(\Delta p + L_2)}$$
$$RHS_2 = \frac{L_4}{\phi(\Delta p - L_2) + \phi(\Delta p + L_2)} [\phi(\Delta p + L_2) - \phi(\Delta p - L_2)]$$

There are two possible cases regarding the values of of $\Delta p - L_2$:

Case 1: $\Delta p - L_2 \geq 0 \Rightarrow \Delta p > L_2 \geq 0 \Rightarrow RHS_1 < 0, RHS_2 < 0 \Rightarrow \Delta p < 0$ (contradiction)

Case 2: $\Delta p - L_2 < 0 \Rightarrow \Delta p < L_2$. Then $\Delta p + L_2$ has two possible values:

Case 2.1: $\Delta p + L_2 \leq 0 \Rightarrow \Delta p \leq -L_2 < 0 \Rightarrow RHS_1 > 0, RHS_2 > 0 \Rightarrow \Delta p > 0$ (contradiction)

Case 2.2: $\Delta p + L_2 > 0 \Rightarrow \Delta p > -L_2 \Rightarrow -L_2 < \Delta p < L_2$. Then again Δp can fall into one of the following three cases:

Case 2.2.1: $0 < \Delta p < L_2 \Rightarrow RHS_1 < 0, RHS_2 < 0 \Rightarrow \Delta p < 0$ (contradiction)

Case 2.2.2: $-L_2 < \Delta p < 0 \Rightarrow RHS_1 > 0, RHS_2 > 0 \Rightarrow \Delta p > 0$ (contradiction)

Case 2.2.3: $\Delta p = 0 \Rightarrow RHS_1 = 0, RHS_2 = 0$, and $\Delta p = RHS_1 + RHS_2 = 0$.

Therefore, $\Delta p = 0$ is the only case that satisfies equation (21) □

Thus, the position of the indifferent consumer $x(i_A, i_B)$ becomes:

$$\begin{aligned} x(i_A, i_B) &= p_i - p_j + Eu(j_A, j_B) - Eu(i_A, i_B) \\ &\equiv \Delta p + L_1 \\ &= L_1 \\ &= \frac{1}{N}[j_A \lambda_{j_A} + i_A \lambda_{j_A+1} - i_A \lambda_{i_A} - j_B \lambda_{i_A+1}] \\ &= \frac{1}{N}[j_A(\lambda_{j_A} - \lambda_{i_A+1}) + i_A(\lambda_{j_A+1} - \lambda_{i_A})] \\ &< 0 \end{aligned}$$

The same steps can also show that $x(i_A, i_B) < 0$

Accordingt to Property (1), consumer period payofff $\lambda(i)$ is a function of strength of network effects ψ and is increasing in user base i. So $\frac{\partial x(i_A,i_B)}{\partial \psi} < 0$ and $\frac{\partial x(i_B,i_A)}{\partial \psi} < 0$. As $\psi \to \infty$, $x(i_A, i_B)$ and $x(i_B, i_A) \to -\infty$. From equation (3), it can be shown that both $q(i_A, i_B)$ and $q(i_B, i_A) \to 1$.

$\square$

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