3.1 Example. *Calculate the stiffness* $[A]$, $[B]$, $[D]$ *and the compliance* $[\alpha]$, $[\beta]$, $[\delta]$ *matrices of a* $[0_{10}/45_{10}]$ *laminate made of graphite epoxy unidirectional plies. The ply properties are given in Table 3.6.*

Solution. The stiffness matrix of a unidirectional ply with the fibers in the 0-degree direction is $[\overline{Q}]^0 = [Q]$. The stiffness matrix $[Q]$ is given by Eq. (2.147),

Table 3.6. Properties of the material used in the examples

		[0]	±45[f]
Longitudinal Young's modulus (GPa)	E_1	148	16.39
Transverse Young's modulus (GPa)	E_2	9.65	16.39
Longitudinal shear modulus (GPa)	G_{12}	4.55	38.19
Longitudinal Poission's ratio	ν_{12}	0.3	0.801
Thickness (mm)	h_0	0.1	0.2

and thus $[\overline{Q}]^0$ is

$$[\overline{Q}]^0 = [Q] = \begin{bmatrix} 148.87 & 2.91 & 0 \\ 2.91 & 9.71 & 0 \\ 0 & 0 & 4.55 \end{bmatrix} 10^9 \, \frac{\text{N}}{\text{m}^2}. \tag{3.49}$$

The stiffness matrix $[\overline{Q}]$ of a ply not in the 0-degree direction is (Eq. 3.14)

$$\begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} = [T_\sigma]^{-1} [Q] [T_\epsilon], \tag{3.50}$$

where $[T_\sigma]$ and $[T_\epsilon]$ are given by Eq. (3.15). For the 45-degree ply $c = \cos 45° = 0.707$ and $s = \sin 45° = 0.707$, and we have

$$[T_\sigma] = \begin{bmatrix} 0.5 & 0.5 & 1.0 \\ 0.5 & 0.5 & -1.0 \\ -0.5 & 0.5 & 0 \end{bmatrix} \qquad [T_\epsilon] = \begin{bmatrix} 0.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & -0.5 \\ -1.0 & 1.0 & 0 \end{bmatrix}. \tag{3.51}$$

By substituting Eqs. (3.49) and (3.51) into Eq. (3.50), we obtain the stiffness matrix of the 45-degree ply as follows:

$$[\overline{Q}]^{45} = \begin{bmatrix} 45.65 & 36.55 & 34.79 \\ 36.55 & 45.65 & 34.79 \\ 34.79 & 34.79 & 38.19 \end{bmatrix} 10^9 \, \frac{\text{N}}{\text{m}^2}. \tag{3.52}$$

The layup is shown in Fig 3.15. In calculating the $[A], [B], [D]$ matrices we treat the ten 0-degree plies as one layer and the ten 45-degree plies as another layer. The $[A], [B], [D]$ matrices are (Eq. 3.20)

$$\begin{aligned} [A] &= [\overline{Q}]^0 (z_1 - z_0) + [\overline{Q}]^{45} (z_2 - z_1) \\ [B] &= [\overline{Q}]^0 \frac{z_1^2 - z_0^2}{2} + [\overline{Q}]^{45} \frac{z_2^2 - z_1^2}{2} \\ [D] &= [\overline{Q}]^0 \frac{z_1^3 - z_0^3}{3} + [\overline{Q}]^{45} \frac{z_2^3 - z_1^3}{3}. \end{aligned} \tag{3.53}$$

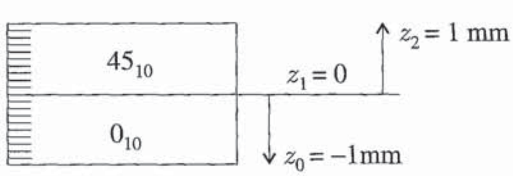


Figure 3.15: The $[0_{10}/45_{10}]$ laminate in Example 3.1.

The $[\overline{Q}]$ matrices are given by Eqs. (3.49) and (3.52). The distances (in meters) are $z_0 = -0.001$, $z_1 = 0$, $z_2 = 0.001$ (Fig. 3.15). With these values Eq. (3.53) yields

$$[A] = [\overline{Q}]^0[0 - (-0.001)] + [\overline{Q}]^{45}(0.001 - 0) = \begin{bmatrix} 194.52 & 39.46 & 34.79 \\ 39.46 & 55.36 & 34.79 \\ 34.79 & 34.79 & 42.74 \end{bmatrix} 10^6 \frac{\text{N}}{\text{m}}$$

$$[B] = [\overline{Q}]^0 \frac{0^2 - (-0.001)^2}{2} + [\overline{Q}]^{45} \frac{0.001^2 - 0^2}{2} = \begin{bmatrix} -51.61 & 16.82 & 17.40 \\ 16.82 & 17.97 & 17.40 \\ 17.40 & 17.40 & 16.82 \end{bmatrix} 10^3\,\text{N}$$

$$[D] = [\overline{Q}]^0 \frac{0^3 - (-0.001)^3}{3} + [\overline{Q}]^{45} \frac{0.001^3 - 0^3}{3} = \begin{bmatrix} 64.84 & 13.15 & 11.60 \\ 13.15 & 18.45 & 11.60 \\ 11.60 & 11.60 & 14.25 \end{bmatrix} \text{N} \cdot \text{m}. \tag{3.54}$$

The compliance matrices are (Eq. 3.23)

$$\begin{bmatrix} \alpha & \beta \\ \beta^T & \delta \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix}^{-1}. \tag{3.55}$$

Hence, we have

$$[\alpha] = \begin{bmatrix} 13.44 & -4.85 & -7.14 \\ -4.85 & 41.81 & -21.23 \\ -7.14 & -21.23 & 64.95 \end{bmatrix} 10^{-9} \frac{\text{m}}{\text{N}}$$

$$[\beta] = \begin{bmatrix} 17.07 & -6.01 & -11.06 \\ -6.01 & -5.04 & -11.06 \\ -11.06 & -11.06 & -24.05 \end{bmatrix} 10^{-6} \frac{1}{\text{N}} \tag{3.56}$$

$$[\delta] = \begin{bmatrix} 40.32 & -14.56 & -21.41 \\ -14.56 & 125.42 & -63.68 \\ -21.41 & -63.68 & 194.86 \end{bmatrix} 10^{-3} \frac{1}{\text{N} \cdot \text{m}}.$$

The compliance and stiffness matrices of $[45_2/-45_2/0_{12}/-45_2/45_2]$, $[-30_4/15_4/0_2]_s$, $[0_2/45_2/90_2/-45_2]_s$, $[45_6/0_4]_s$, and $[0_2/45_2/0_2/45_2]$ laminates are calculated similarly. The results are in Tables 3.7, 3.8, and 3.9 (pages 84–86). Note that, for symmetrical laminates, the following simplifications apply: $[B] = [\beta] = 0$ and $[\alpha] = [a]$, $[\delta] = [d]$.

3.2 Example. *Calculate the stiffness $[A]$, $[B]$, $[D]$ and the compliance $[\alpha]$, $[\beta]$, $[\delta]$ matrices of a $[0_{20}]$ laminate made of graphite epoxy unidirectional plies. The ply properties are given in Table 3.6 (page 81).*

Solution. The unidirectional laminate is symmetrical, and the $[B]$ matrix is zero:

$$[B] = 0. \tag{3.57}$$

Table 3.7. **The [A] and [D] matrices for symmetrical laminates. The unit of [A] is $10^6\,\frac{\text{N}}{\text{m}}$ and the unit of [D] is N · m. The material properties are given in Table 3.6 (page 81).**

[A]	[D]
$[0_{20}]$ (orthotropic, symmetrical)	
$\begin{bmatrix} 297.75 & 5.82 & 0 \\ 5.82 & 19.41 & 0 \\ 0 & 0 & 9.10 \end{bmatrix}$	$\begin{bmatrix} 99.25 & 1.94 & 0 \\ 1.94 & 6.47 & 0 \\ 0 & 0 & 3.03 \end{bmatrix}$
$[\pm 45^{\text{f}}_2/0_{12}/\pm 45^{\text{f}}_2]$ (orthotropic, symmetrical)	
$\begin{bmatrix} 215.17 & 32.74 & 0 \\ 32.74 & 48.17 & 0 \\ 0 & 0 & 36.01 \end{bmatrix}$	$\begin{bmatrix} 45.30 & 19.52 & 0 \\ 19.52 & 25.26 & 0 \\ 0 & 0 & 20.62 \end{bmatrix}$
$[45_2/-45_2/0_{12}/-45_2/45_2]$ (balanced, symmetrical)	
$\begin{bmatrix} 215.17 & 32.74 & 0 \\ 32.74 & 48.17 & 0 \\ 0 & 0 & 36.01 \end{bmatrix}$	$\begin{bmatrix} 45.30 & 19.52 & 4.45 \\ 19.52 & 25.26 & 4.45 \\ 4.45 & 4.45 & 20.62 \end{bmatrix}$
$[-30_4/15_4/0_2]_{\text{s}}$ (symmetrical)	
$\begin{bmatrix} 235.54 & 32.74 & -10.19 \\ 32.74 & 27.79 & -10.19 \\ -10.19 & -10.19 & 36.01 \end{bmatrix}$	$\begin{bmatrix} 65.42 & 16.29 & -18.93 \\ 16.29 & 11.60 & -7.74 \\ -18.93 & -7.74 & 17.39 \end{bmatrix}$
$[0_2/45_2/90_2/-45_2]_{\text{s}}$ (quasi-isotropic, symmetrical)	
$\begin{bmatrix} 99.95 & 31.57 & 0 \\ 31.57 & 99.95 & 0 \\ 0 & 0 & 34.19 \end{bmatrix}$	$\begin{bmatrix} 34.61 & 4.58 & 3.34 \\ 4.58 & 12.34 & 3.34 \\ 3.34 & 3.34 & 5.14 \end{bmatrix}$
$[45_6/0_4]_{\text{s}}$ (symmetrical)	
$\begin{bmatrix} 173.88 & 46.19 & 41.75 \\ 46.19 & 62.55 & 41.75 \\ 41.75 & 41.75 & 49.47 \end{bmatrix}$	$\begin{bmatrix} 34.84 & 22.93 & 21.71 \\ 22.93 & 28.90 & 21.71 \\ 21.71 & 21.71 & 24.02 \end{bmatrix}$

By treating the 20 plies as a single layer, the [A] and [D] matrices are (Eq. 3.20)

$$[A] = h[\overline{Q}] \qquad [D] = \frac{h^3}{12}[\overline{Q}], \tag{3.58}$$

where $h = 0.002$ m is the thickness of the laminate.

By substituting Eqs. (3.20) and (2.139) into Eq. (3.58) and by using the engineering constants in Table 3.6 (page 81) ($E_1 = 148 \times 10^9\,\text{N/m}^2$, $E_2 = 9.65 \times$

Table 3.8. The $[a]$ and $[d]$ matrices for symmetrical laminates. The unit of $[a]$ is $10^{-9}\,\frac{\mathrm{m}}{\mathrm{N}}$ and the unit of $[d]$ is $10^{-3}\,\frac{1}{\mathrm{N \cdot m}}$. The material properties are given in Table 3.6 (page 81).

$[a]$	$[d]$
$[0_{20}]$ (orthotropic, symmetrical)	
$\begin{bmatrix} 3.38 & -1.01 & 0 \\ -1.01 & 51.81 & 0 \\ 0 & 0 & 109.89 \end{bmatrix}$	$\begin{bmatrix} 10.14 & -3.04 & 0 \\ -3.04 & 155.44 & 0 \\ 0 & 0 & 329.67 \end{bmatrix}$
$[\pm 45^{\mathrm{f}}_{2}/0_{12}/\pm 45^{\mathrm{f}}_{2}]$ (orthotropic, symmetrical)	
$\begin{bmatrix} 5.18 & -3.52 & 0 \\ -3.52 & 23.15 & 0 \\ 0 & 0 & 27.77 \end{bmatrix}$	$\begin{bmatrix} 33.10 & -25.59 & 0 \\ -25.59 & 59.37 & 0 \\ 0 & 0 & 48.51 \end{bmatrix}$
$[45_2/-45_2/0_{12}/-45_2/45_2]$ (balanced, symmetrical)	
$\begin{bmatrix} 5.18 & -3.52 & 0 \\ -3.52 & 23.15 & 0 \\ 0 & 0 & 27.77 \end{bmatrix}$	$\begin{bmatrix} 33.16 & -25.33 & -1.69 \\ -25.33 & 60.51 & -7.60 \\ -1.69 & -7.60 & 50.51 \end{bmatrix}$
$[-30_4/15_4/0_2]_{\mathrm{s}}$ (symmetrical)	
$\begin{bmatrix} 5.08 & -6.09 & -0.29 \\ -6.09 & 47.44 & 11.70 \\ -0.29 & 11.70 & 31.00 \end{bmatrix}$	$\begin{bmatrix} 26.87 & -25.93 & 17.70 \\ -25.93 & 147.76 & 37.57 \\ 17.70 & 37.57 & 93.52 \end{bmatrix}$
$[0_2/45_2/90_2/-45_2]_{\mathrm{s}}$ (quasi-isotropic, symmetrical)	
$\begin{bmatrix} 11.11 & -3.51 & 0 \\ -3.51 & 11.11 & 0 \\ 0 & 0 & 29.25 \end{bmatrix}$	$\begin{bmatrix} 31.38 & -7.44 & -15.55 \\ -7.44 & 100.06 & -60.17 \\ -15.55 & -60.17 & 243.70 \end{bmatrix}$
$[45_6/0_4]_{\mathrm{s}}$ (symmetrical)	
$\begin{bmatrix} 7.45 & -2.99 & -3.77 \\ -2.99 & 37.81 & -29.39 \\ -3.77 & -29.39 & 48.20 \end{bmatrix}$	$\begin{bmatrix} 71.24 & -25.43 & -41.40 \\ -25.43 & 116.82 & -82.58 \\ -41.40 & -82.58 & 153.66 \end{bmatrix}$

10^9 N/m^2, $G_{12} = 4.55 \times 10^9$ N/m^2, $\nu_{12} = 0.3$), we obtain

$$A_{11} = \frac{E_1 h}{1 - \nu_{12}^2 \frac{E_2}{E_1}} = 297.75 \times 10^6\,\frac{\mathrm{N}}{\mathrm{m}} \qquad A_{22} = \frac{E_2 h}{1 - \nu_{12}^2 \frac{E_2}{E_1}} = 19.41 \times 10^6\,\frac{\mathrm{N}}{\mathrm{m}}$$
$$A_{12} = \nu_{12} A_{22} = 5.82 \times 10^6\,\frac{\mathrm{N}}{\mathrm{m}} \qquad A_{66} = G_{12} h = 9.10 \times 10^6\,\frac{\mathrm{N}}{\mathrm{m}} \tag{3.59}$$

$$D_{11} = \frac{E_1 h^3}{12\left(1 - \nu_{12}^2 \frac{E_2}{E_1}\right)} = 99.25\ \mathrm{N \cdot m} \qquad D_{22} = \frac{E_2 h^3}{12\left(1 - \nu_{12}^2 \frac{E_2}{E_1}\right)} = 6.47\ \mathrm{N \cdot m}$$
$$D_{12} = \nu_{12} D_{22} = 1.94\ \mathrm{N \cdot m} \qquad D_{66} = \frac{G_{12} h^3}{12} = 3.03\ \mathrm{N \cdot m}. \tag{3.60}$$

Table 3.9. The $[A]$, $[B]$, $[D]$ and the $[\alpha]$, $[\beta]$, and $[\delta]$ matrices for unsymmetrical laminates. $[A]$ is in $10^6\,\frac{\text{N}}{\text{m}}$, $[B]$ is in 10^3N, $[D]$ is in N · m, $[\alpha]$ is in $10^{-9}\,\frac{\text{m}}{\text{N}}$, $[\beta]$ is in $10^{-6}\,\frac{1}{\text{N}}$, and $[\delta]$ is in $10^{-3}\,\frac{1}{\text{N}\cdot\text{m}}$. The material properties are given in Table 3.6 (page 81).

$[A]$	$[B]$	$[D]$
$[0_{10}/45_{10}]$		
$\begin{bmatrix} 194.52 & 39.46 & 34.79 \\ 39.46 & 55.36 & 34.79 \\ 34.79 & 34.79 & 42.74 \end{bmatrix}$	$\begin{bmatrix} -51.61 & 16.82 & 17.40 \\ 16.82 & 17.97 & 17.40 \\ 17.40 & 17.40 & 16.82 \end{bmatrix}$	$\begin{bmatrix} 64.84 & 13.15 & 11.60 \\ 13.15 & 18.45 & 11.60 \\ 11.60 & 11.60 & 14.25 \end{bmatrix}$
$[0_2/45_2/0_2/45_2]$		
$\begin{bmatrix} 77.81 & 15.79 & 13.92 \\ 15.79 & 22.14 & 13.92 \\ 13.92 & 13.92 & 17.10 \end{bmatrix}$	$\begin{bmatrix} -4.129 & 1.346 & 1.392 \\ 1.346 & 1.438 & 1.392 \\ 1.392 & 1.392 & 1.346 \end{bmatrix}$	$\begin{bmatrix} 4.150 & 0.842 & 0.742 \\ 0.842 & 1.181 & 0.742 \\ 0.742 & 0.742 & 0.912 \end{bmatrix}$
$[\pm 45^{f}_{5}/0_{10}]$		
$\begin{bmatrix} 194.52 & 39.46 & 0 \\ 39.46 & 55.36 & 0 \\ 0 & 0 & 42.74 \end{bmatrix}$	$\begin{bmatrix} 51.61 & -16.82 & 0 \\ -16.82 & -17.97 & 0 \\ 0 & 0 & -16.82 \end{bmatrix}$	$\begin{bmatrix} 64.84 & 13.15 & 0 \\ 13.15 & 18.45 & 0 \\ 0 & 0 & 14.25 \end{bmatrix}$

$[\alpha]$	$[\beta]$	$[\delta]$
$[0_{10}/45_{10}]$		
$\begin{bmatrix} 13.44 & -4.85 & -7.14 \\ -4.85 & 41.81 & -21.23 \\ -7.14 & -21.23 & 64.95 \end{bmatrix}$	$\begin{bmatrix} 17.07 & -6.01 & -11.06 \\ -6.01 & -5.04 & -11.06 \\ -11.06 & -11.06 & -24.05 \end{bmatrix}$	$\begin{bmatrix} 40.32 & -14.56 & -21.41 \\ -14.56 & 125.42 & -63.68 \\ -21.41 & -63.68 & 194.86 \end{bmatrix}$
$[0_2/45_2/0_2/45_2]$		
$\begin{bmatrix} 17.88 & -7.14 & -8.79 \\ -7.14 & 96.35 & -69.68 \\ -8.79 & -69.68 & 128.51 \end{bmatrix}$	$\begin{bmatrix} 28.37 & -9.99 & -18.38 \\ -9.99 & -8.38 & -18.38 \\ -18.38 & -18.38 & -39.96 \end{bmatrix}$	$\begin{bmatrix} 335 & -134 & -165 \\ -134 & 1\,807 & -1\,306 \\ -165 & -1\,306 & 2\,410 \end{bmatrix}$
$[\pm 45^{f}_{5}/0_{10}]$		
$\begin{bmatrix} 11.65 & -8.58 & 0 \\ -8.58 & 32.94 & 0 \\ 0 & 0 & 43.70 \end{bmatrix}$	$\begin{bmatrix} -13.97 & 12.22 & 0 \\ 12.22 & 15.55 & 0 \\ 0 & 0 & 51.60 \end{bmatrix}$	$\begin{bmatrix} 34.94 & -25.74 & 0 \\ -25.74 & 98.83 & 0 \\ 0 & 0 & 131.11 \end{bmatrix}$

The compliance matrices $[a]$ and $[d]$ are obtained by inverting the $[A]$ and $[D]$ stiffness matrices as follows:

$$[a] = [A]^{-1} \qquad [d] = [D]^{-1}. \tag{3.61}$$

Equations (3.59)–(3.61) give

$$a_{11} = \frac{1}{E_1 h} = 3.38 \times 10^{-9} \frac{\text{m}}{\text{N}} \qquad a_{22} = \frac{1}{E_2 h} = 51.81 \times 10^{-9} \frac{\text{m}}{\text{N}}$$
$$a_{12} = -\nu_{12} a_{11} = -1.01 \times 10^{-9} \frac{\text{m}}{\text{N}} \qquad a_{66} = \frac{1}{G_{12} h} = 109.89 \times 10^{-9} \frac{\text{m}}{\text{N}} \tag{3.62}$$

$$d_{11} = \frac{12}{E_1 h^3} = 10.14 \times 10^{-3} \frac{1}{\text{N} \cdot \text{m}} \qquad d_{22} = \frac{12}{E_2 h^3} = 155.44 \times 10^{-3} \frac{1}{\text{N} \cdot \text{m}}$$
$$d_{12} = -\nu_{12} d_{11} = -3.04 \times 10^{-3} \frac{1}{\text{N} \cdot \text{m}} \qquad d_{66} = \frac{12}{G_{12} h^3} = 329.67 \times 10^{-3} \frac{1}{\text{N} \cdot \text{m}}. \tag{3.63}$$

3.3 Example. *Calculate the stiffness and the compliance matrices of (i) a laminated composite consisting of two layers of ±45-degree woven fabric, twelve layers of 0-degree unidirectional plies, and two layers of ±45-degree woven fabric* ($[\pm 45^{\mathrm{f}}_2/0_{12}/\pm 45^{\mathrm{f}}_2]$)*; and (ii) a laminate consisting five layers of ±45-degree woven fabric and ten layers of 0-degree unidirectional plies* ($[\pm 45^{\mathrm{f}}_5/0_{10}]$)*. The material properties are given in Table 3.6 (page 81).*

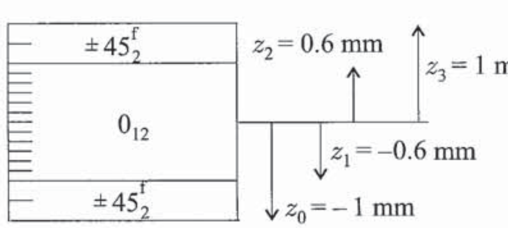
Solution. First we consider the laminate with ($[\pm 45^{\mathrm{f}}_2/0_{12}/\pm 45^{\mathrm{f}}_2]$) layup (Fig. 3.16). The laminate is symmetrical, and the $[B]$ matrix is zero:

$$[B] = 0. \tag{3.64}$$

The compliance matrix of a unidirectional ply with the fibers in the 0-degree direction is $[\overline{Q}]^0 = [Q]$. The stiffness matrix $[Q]$ is given by Eq. (2.147), and thus $[\overline{Q}]^0$ is

$$[\overline{Q}]^0 = [Q] = \begin{bmatrix} 148.87 & 2.91 & 0 \\ 2.91 & 9.71 & 0 \\ 0 & 0 & 4.55 \end{bmatrix} 10^9 \frac{\text{N}}{\text{m}^2}. \tag{3.65}$$

Figure 3.16: The layup of the laminate in Example 3.3. The supercript f denotes fabric.



For a ±45-degree woven fabric the stiffness matrix $[\overline{Q}]$ is (Eq. 2.150)

$$[\overline{Q}]^{\pm 45} = \begin{bmatrix} 45.65 & 36.55 & 0 \\ 36.55 & 45.65 & 0 \\ 0 & 0 & 38.19 \end{bmatrix} 10^9 \, \frac{\text{N}}{\text{m}^2}. \tag{3.66}$$

In calculating the $[A]$, $[B]$, $[D]$ matrices we treat the twelve 0-degree plies as one layer and each adjacent woven fabric as one layer. The $[A]$ and $[D]$ matrices are

$$\begin{aligned} [A] &= [\overline{Q}]^{\pm 45}(z_1 - z_0) + [\overline{Q}]^0 (z_2 - z_1) + [\overline{Q}]^{\pm 45}(z_3 - z_2) \\ [D] &= [\overline{Q}]^{\pm 45} \frac{z_1^3 - z_0^3}{3} + [\overline{Q}]^0 \frac{z_2^3 - z_1^3}{3} + [\overline{Q}]^{\pm 45} \frac{z_3^3 - z_2^3}{3}. \end{aligned} \tag{3.67}$$

The $[\overline{Q}]$ matrices are given by Eqs. (3.65) and (3.66). The distances (in meters) are $z_0 = -0.001$, $z_1 = -0.0006$, $z_2 = 0.0006$, and $z_3 = 0.001$ (Fig. 3.16). With these values Eq. (3.67) yields

$$[A] = \begin{bmatrix} 215.17 & 32.74 & 0 \\ 32.74 & 48.17 & 0 \\ 0 & 0 & 36.01 \end{bmatrix} 10^6 \, \frac{\text{N}}{\text{m}} \tag{3.68}$$

$$[D] = \begin{bmatrix} 45.30 & 19.52 & 0 \\ 19.52 & 25.26 & 0 \\ 0 & 0 & 20.62 \end{bmatrix} \text{N} \cdot \text{m}. \tag{3.69}$$

The compliance matrices $[a]$ and $[d]$ are (Eqs. 3.29 and 3.30)

$$[a] = [A]^{-1} = \begin{bmatrix} 5.18 & -3.52 & 0 \\ -3.52 & 23.15 & 0 \\ 0 & 0 & 27.77 \end{bmatrix} 10^{-9} \, \frac{\text{m}}{\text{N}} \tag{3.70}$$

$$[d] = [D]^{-1} = \begin{bmatrix} 33.10 & -25.59 & 0 \\ -25.59 & 59.37 & 0 \\ 0 & 0 & 48.51 \end{bmatrix} 10^{-3} \, \frac{1}{\text{N} \cdot \text{m}}. \tag{3.71}$$

The compliance and stiffness matrices of the $[\pm 45_5^{\text{f}}/0_{10}]$ laminate are calculated similarly. The results are given in Table 3.9.