FLOWFoil Theory Document

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1 Geometry

1.1 Panels

For any geometric input, we assume that the airfoil geometry begins at the trailing egde and proceeds clockwise around the leading edge and back to the trailing edge. It is likely convenient to input geometry data that positions the airfoil leading edge at the origin and has a chord length of one.

Linear panels are defines using the input coordinates as the panel end points. Figure 1 shows the geometry convention uses in FLOWFoil as well as the linear paneling of an airfoil.

Figure 1: Linear Paneling

2 Inviscid Solution Details

2.1 Vortex Influence Coefficients

Following the formulation in [1], the inviscid system is assembled as follows.

$$\mathbf{A}\gamma = \mathbf{\Psi}^{\infty} \tag{1}$$

where $\mathbf{A}$ is comprized of the coefficients of influence between the panels and evaluation points.

For a given evaluation point, $i \in \mathbb{N}^N$, and panel, $k \in \mathbb{N}^{N-1}$, (comprised of nodes $j \in \mathbb{N}^N$ and $j+1$) the streamfunction at the evaluation point due to a linear vortex distribution across the panel (with vortex strengths at the nodes of γ_j and γ_{j+1}, respectively) is

$$\Psi_{ik} = \mathbf{\Psi}^{\gamma}_{ik}[\gamma_j, \gamma_{j+1}]^{\top}, \tag{2}$$

where

$$\mathbf{\Psi}^{\gamma}_{ik} = [\overline{\psi}^{\gamma}_{ik} - \widetilde{\psi}^{\gamma}_{ik}, \widetilde{\psi}^{\gamma}_{ik}] \tag{3}$$

Therefore, the influence coefficient of the jth node on the ith node (the ijth element of $\mathbf{A}$) is comprized of portions of the influence seen from each panel of which it is part:

$$A_{ij} = \begin{cases} \overline{\psi}^{\gamma}_{ij} - \widetilde{\psi}^{\gamma}_{ij} & j = 1 \\ \widetilde{\psi}^{\gamma}_{i,j-1} & j = N \\ \widetilde{\psi}^{\gamma}_{i,j-1} + \overline{\psi}^{\gamma}_{ij} - \widetilde{\psi}^{\gamma}_{ij} & \text{otherwise.} \end{cases} \tag{4}$$

2.1.1 Influence Geometry

The components of $\mathbf{\Psi}^{\gamma}_{ik}$ are defined as

$$\overline{\psi}^{\gamma}_{ik} = \frac{1}{2\pi}\left(h(\theta_{j+1} - \theta j) - d + a\ln(r_j) - (a-d)\ln(r_j + 1)\right) \tag{5}$$

$$\widetilde{\psi}^{\gamma}_{ik} = \frac{a}{d}\overline{\psi}^{\gamma}_{ij} + \frac{1}{4\pi d}\left(r^2_{j+1}\ln(r_{j+1}) - r^2_j\ln(r_j) - \frac{1}{2}r^2_{j+1} + \frac{1}{2}r^2_j\right). \tag{6}$$

In order to evaluate the the componenets of $\mathbf{\Psi}^{\gamma}_{ik}$ we will need to understand the geometry of the problem.

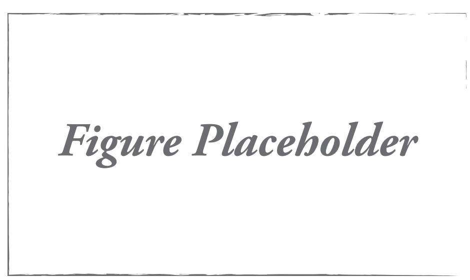

Figure 2: Influence Geometry

Figure 2 shows the relative geometry for the problem, and we calculate each of the geometric values as follows.

- The kth panel length, from the jth to $j+1$th node:

$$\mathbf{d}_k = \mathbf{n}_{j+1} - \mathbf{n}_j \tag{7}$$

  where n is the node position.

- The vector from the jth node to the evaluation point:

$$\mathbf{r}_j = \mathbf{p} - \mathbf{n}_j \tag{8}$$

  where p is the evaluation point position.

- The angle between the kth panel and the evaluation point centered at the jth node:

$$\theta_j = \cos^{-1}\left(\frac{\mathbf{r}_j \cdot \mathbf{d}_k}{r_j d_k}\right) \tag{9}$$

  where $r = ||\mathbf{r}||$ and $d = ||\mathbf{d}||$.

- The height of the triangle made from the evaluation point and jth and $j+1$th nodes:

$$h_k = 2A/d_k \tag{10}$$

  where $A = [s(s-d_k)(s-r_j)(s-r_{j+1})]^{1/2}$, where $s = (d_k + r_j + r_{j+1})/2$ (this is Heron's formula for the area of a triangle).

- The length of the right triangle with base aligned with the panel, and height, h_k:

$$a_k = \begin{cases} r_j \cos(\pi - \theta_j) + d_k & \text{if} \quad \theta_j > \frac{\pi}{2} \\ d_k & \text{if} \quad \theta_j = \frac{\pi}{2} \\ r_j \cos(\theta_j) & \text{otherwise} \end{cases} \tag{11}$$

- The natural log of the distance between node and evaluation point:

$$\ln(r) = \begin{cases} 0 & \text{if} \quad r = 0 \\ \ln(r) & \text{otherwise} \end{cases} \tag{12}$$

References

[1] Fidkowski, K. J., "A Coupled Inviscid–Viscous Airfoil Analysis Solver, Revisited," *AIAA Journal*, Vol. 60, No. 5, May 2022, pp. 2961–2971.

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