FLOWFoil Theory Document

Judd Mehr,

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1 Geometry

1.1 Panels

For any geometric input, we assume that the airfoil geometry begins at the trailing egde and proceeds clockwise around the leading edge and back to the trailing edge. It is likely convenient to input geometry data that positions the airfoil leading edge at the origin and has a chord length of one.

Linear panels are defined using the input coordinates as the panel end points. Figure 1 shows the geometry convention uses in FLOWFoil as well as the linear paneling of an airfoil.

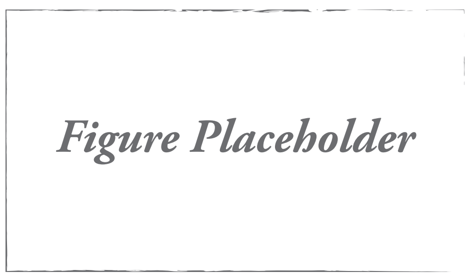

Figure 1: Linear Paneling (see fig1 in mfoil paper)

2 Inviscid Solution Details

2.1 Vortex Influence Coefficients

Following the formulation in [1], the inviscid system is assembled as follows.

$$\begin{gathered} \sum_{j=1}^{N}(a_{ij}\gamma_j) - \Psi_0 = V_\infty(z_i\cos\alpha - x_i\sin\alpha) \\ \gamma_1 + \gamma_N = 0 \qquad \text{(Kutta Condition)} \end{gathered} \tag{1}$$

or

$$\mathbf{A}\gamma = \mathbf{\Psi}^\infty \tag{2}$$

where $\mathbf{\Psi}^\infty = V_\infty(\mathbf{z}\cos\alpha - \mathbf{x}\sin\alpha)$ with α being the airfoil angle of attack, Ψ_0 is an unknown constant streamfunction at each node, and $\mathbf{A}$ is comprized of the coefficients of influence between the panels and evaluation points (a_{ij}), the Kutta condition, and the influence of Ψ_0.

For a given evaluation point, $i \in \mathbb{N}^N$, and panel, $k \in \mathbb{N}^{N-1}$, (comprised of nodes $j \in \mathbb{N}^{N-1}$ and $j+1$) the streamfunction at the evaluation point due to a linear vortex distribution across the panel (with vortex strengths at the nodes of γ_j and γ_{j+1}, respectively) is

$$\Psi_{ik}^{\gamma} = \mathbf{\Psi}_{ik}^{\gamma}[\gamma_j, \gamma_{j+1}]^\top, \tag{3}$$

where

$$\mathbf{\Psi}_{ik}^{\gamma} = \left[\overline{\psi}_{ik}^{\gamma} - \widetilde{\psi}_{ik}^{\gamma}, \widetilde{\psi}_{ik}^{\gamma}\right] \tag{4}$$

Therefore, the influence coefficient of the jth node on the ith node (the ijth element of $\mathbf{A}$) is comprized of portions of the influence seen from each panel of which it is part:

$$a_{ij} = \begin{cases} \overline{\psi}_{ij}^{\gamma} - \widetilde{\psi}_{ij}^{\gamma} & j = 1 \\ \widetilde{\psi}_{i,j-1}^{\gamma} & j = N \\ \widetilde{\psi}_{i,j-1}^{\gamma} + \left(\overline{\psi}_{ij}^{\gamma} - \widetilde{\psi}_{ij}^{\gamma}\right) & \text{otherwise.} \end{cases} \tag{5}$$

Note that panel 1 and panel N-1 do not share nodes 1 and N. That is, node 1 is associated only with panel 1, and node N only with panel N-1.

IS EQN 5 CORRECT? AM I UNDERSTANDING HOW THE MATRIX IS ASSEMBLED??

2.1.1 Influence Geometry

The components of $\mathbf{\Psi}_{ik}^{\gamma}$ are defined as

$$\overline{\psi}_{ik}^{\gamma} = \frac{1}{2\pi}\left(h_{ik}(\theta_{i,j+1} - \theta_{ij}) - d_k + a_{ik}\ln(r_{ij}) - (a_{ik} - d_k)\ln(r_{i,j+1})\right) \tag{6}$$

$$\widetilde{\psi}_{ik}^{\gamma} = \frac{a_{ik}}{d_k}\overline{\psi}_{ik}^{\gamma} + \frac{1}{4\pi d_k}\left(r_{i,j+1}^2\ln(r_{i,j+1}) - r_{ij}^2\ln(r_{ij}) - \frac{1}{2}r_{i,j+1}^2 + \frac{1}{2}r_{ij}^2\right). \tag{7}$$

In order to evaluate the the componenets of $\mathbf{\Psi}_{ik}^{\gamma}$ we will need to understand the geometry of the problem.

Figure 2 shows the relative geometry for the problem, and we calculate each of the geometric values as follows.

- The kth panel vector and length, from the jth to $j+1$th node:

$$\mathbf{d}_k = \mathbf{n}_{j+1} - \mathbf{n}_j \tag{8}$$

$$d_k = ||\mathbf{n}_{j+1} - \mathbf{n}_j||, \tag{9}$$

  where $\mathbf{n}$ is the node position.

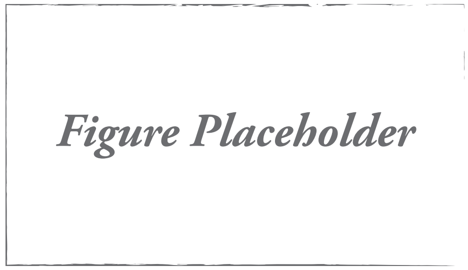

Figure 2: Influence Geometry (see fig10 in mfoil paper)

- The vector and distance from the jth node to the evaluation point:

$$\mathbf{r}_{ij} = \mathbf{p}_i - \mathbf{n}_j \tag{10}$$
$$r_{ij} = ||\mathbf{p}_i - \mathbf{n}_j||, \tag{11}$$

  where $\mathbf{p}$ is the evaluation point position.

- The angles between the kth panel and the evaluation point centered at the jth node:

$$\theta_{ij} = \cos^{-1}\left(\frac{\mathbf{r}_{ij} \cdot \mathbf{d}_k}{r_{ij} d_k}\right) \tag{12}$$

- The height of the triangle made from the evaluation point and jth and $j+1$th nodes:

$$h_{ik} = 2A_{ik}/d_k, \tag{13}$$

  where $A_{ik} = [s_{ik}(s_{ik} - d_k)(s_{ik} - r_j)(s_{ik} - r_{j+1})]^{1/2}$, with $s_{ik} = (d_k + r_{ij} + r_{i,j+1})/2$ (this is Heron's formula for the area of a triangle).

- The length of the base (colinear with the panel) of the right triangle of height, h_{ik}:

$$a_k = \begin{cases} r_{ij}\cos(\pi - \theta_{ij}) + d_k & \text{if } \theta_{ij} > \frac{\pi}{2} \\ d_k & \text{if } \theta_{ij} = \frac{\pi}{2} \\ r_{ij}\cos(\theta_{ij}) & \text{otherwise.} \end{cases} \tag{14}$$

- The natural log of the distance between node and evaluation point:

$$\ln(r_{ij}) = \begin{cases} 0 & \text{if } r_{ij} = 0 \\ \ln(r_{ij}) & \text{otherwise.} \end{cases} \tag{15}$$

2.1.2 Completing the Coefficient Matrix

To complete the coefficient matrix, $\mathbf{A}$ after putting together the vortex influence coefficients, we need to first add in the contribution of the unknown constant, Ψ_0. We do this by adding a column of -1's to the current $N \times N$ matrix of coefficients

$$\begin{pmatrix} & & & & -1 \\ & & & & -1 \\ & a_{ij} & & & -1 \\ & & & & -1 \\ & & & & -1 \end{pmatrix}. \tag{16}$$

We then add in the kutta condition, $\gamma_1 + \gamma_N = 0$, as an additional row to our matrix

$$\begin{pmatrix} & & & & & & -1 \\ & & & & & & -1 \\ & & a_{ij} & & & & -1 \\ & & & & & & -1 \\ & & & & & & -1 \\ 1 & 0 & \cdots & & 0 & 1 & 0 \end{pmatrix}. \tag{17}$$

We now have an $N+1 \times N+1$ system of equations for γ and Ψ_0

$$\begin{pmatrix} & & & & & & -1 \\ & & & & & & -1 \\ & & a_{ij} & & & & -1 \\ & & & & & & -1 \\ & & & & & & -1 \\ 1 & 0 & \cdots & & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_N \\ \Psi_0 \end{pmatrix} = \begin{pmatrix} V_\infty(z_1 \cos\alpha - x_1 \sin\alpha) \\ V_\infty(z_2 \cos\alpha - x_2 \sin\alpha) \\ \vdots \\ V_\infty(z_N \cos\alpha - x_N \sin\alpha) \\ 0 \end{pmatrix} \tag{18}$$

2.1.3 Trailing Edge Treatment

We have a few modifications to make to our system depending on whether the airfoil has a blunt or sharp trailing edge.

Sharp Trailing Edge In the case of the sharp trailing edge, the first and last nodes are coincident, which leads them to have identical equations, causing the matrix to be singular. In this case, we discard the Nth row and substitute it for an extrapolation of the mean (between upper and lower sides) to the trailing edge as

$$\gamma_1 + 2\gamma_2 - \gamma_3 + \gamma_{N-2} - 2\gamma_{N-1} - \gamma_N = 0. \tag{19}$$

This gives us the Nth row of the coefficient matrix to be

$$\begin{bmatrix} 1 & 2 & -1 & 0 & \cdots & 0 & 1 & -2 & -1 & 0 \end{bmatrix}. \tag{20}$$

(Where the $N+1$th entry is set to zero, since $-\Psi_0$ was already applied at node 1.) We also set the Nth boundary condition value to be 0 (rather than $V_\infty[z_N \cos\alpha - x_N \sin\alpha]$). This yields the following system of equations for the inviscid solution of an airfoil with a sharp trailing edge:

$$
\begin{pmatrix}
a_{11} & a_{12} & & & \cdots & & & a_{1,N-1} & a_{1N} & -1 \\
a_{21} & a_{22} & & & \cdots & & & a_{2,N-1} & a_{2N} & -1 \\
\vdots & \vdots & & & \ddots & & & \vdots & \vdots & -1 \\
a_{N-1,1} & a_{N-1,2} & & & \cdots & & & a_{N-1,N-1} & a_{N-1,N} & -1 \\
1 & 2 & -1 & 0 & \cdots & 0 & 1 & -2 & -1 & 0 \\
1 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
\gamma_1 \\ \gamma_2 \\ \vdots \\ \gamma_{N-1} \\ \gamma_N \\ \Psi_0
\end{pmatrix}
=
\begin{pmatrix}
V_\infty(z_1 \cos\alpha - x_1 \sin\alpha) \\
V_\infty(z_2 \cos\alpha - x_2 \sin\alpha) \\
\vdots \\
V_\infty(z_{N-1} \cos\alpha - x_{N-1} \sin\alpha) \\
0 \\
0
\end{pmatrix}
\tag{21}
$$

Blunt Trailing Edge

References

[1] Fidkowski, K. J., "A Coupled Inviscid–Viscous Airfoil Analysis Solver, Revisited," *AIAA Journal*, Vol. 60, No. 5, May 2022, pp. 2961–2971.

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