Chapter 5

Validation of Ohm's Law

What physicists do is to try to understand how the universe works. To do this we use the Scientific Method. And what makes the Scientific Method different from philosophy is the use of experimentation to verify our ideas. So in a physics lab class, we need to test ideas about how the universe works. We call these ideas "mental models" or just "models." We have been using one of these models in making voltage measuring devices already. It was called Ohm's law. Let's start out by testing Ohm's law to see if it really works.

5.1 Ohm's Law Revisited

We learned several labs ago that voltage and current are linearly related to each other. This is what we would call a *model*, a mental understanding of how part of the universe works. Usually in physics we distil the model into an equation . We call this equation a law. In this case, Ohm's law.

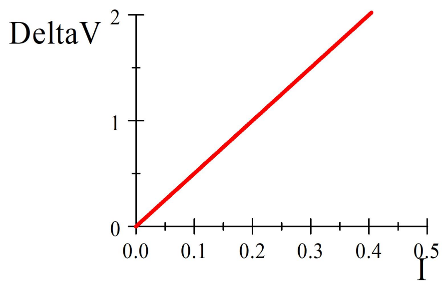
$$\Delta V = IR$$

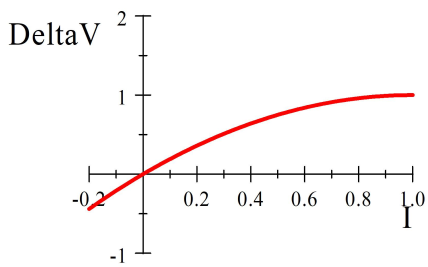
where R is the slope of the ΔV vs. I curve. We can see the model relationship between ΔV and I reflected in the equation. Note that being a "law" doesn't mean the equation is always true. The word "law" generally implies that the equation is true at least some of the time, but really it is telling us we have distilled our model into math.

We might plot our equation to show the ΔV vs. I relationship.

But as scientists, we should ask, does our model work for all materials? What if we graphed ΔV vs I for some device and found a graph that looks like this?Such a device would *not* follow Ohm's law. We would say that such a device is *nonohmic*.

Today we will test our model by taking ΔV and I measurements and seeing if the equation $\Delta V = IR$ describes the data well. Of course, this means we need to measure two things at once with our Arduino. We need both ΔV and I. But this isn't a problem because our Arduninos have five analog inputs. So



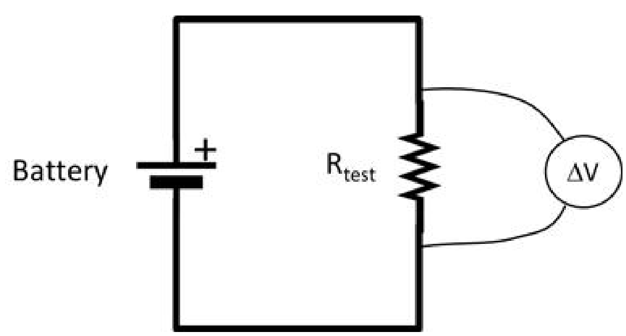


we just need to have one measurement attached to, say, pin A0 and another to, say, pin A1. Of course both will need to be connected to GND as the second measurement because ΔV measurements take two leads.

But wait if we are testing Ohm's law, we don't want two ΔV measurements, we want ΔV and I. How do we get I measured by an Arduino?

5.1.1 Measuring current with our Arduino

Arduinos and other DAQs only measure voltages. Let's review how we measure the voltage across a resistor, and then review how to turn that voltage measurement into a current measurement.



We put the two leads of a voltmeter (shown as a circle with a ΔV in it) on either side of the resistor that we are testing. If the voltmeter is our Arduino, the leads on the side of the resister connected to the positive side of the battery should go to A0 and the lead connected to the negative side of the battery should be connected to GND. This is all what an Arduino (or any other DAQ) can do.

To measure a current with our Arduino we have to somehow turn that current into a voltage. This is true of most measurements we will do. We need to turn temperature, or humidity, or magnetic field, or light intensity into a voltage. Turning magnetic field into a voltage is a little tricky, but we already know all we need to know to handle current.

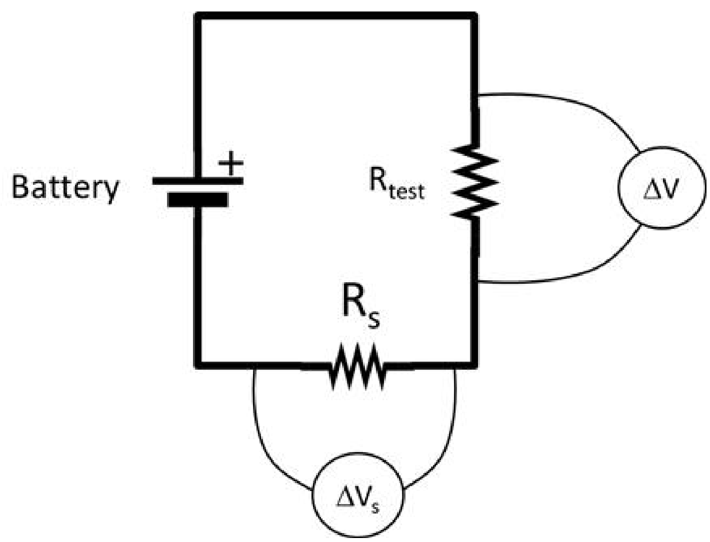
To turn a current into a voltage, think of Ohm's law again.

$$\Delta V_s = I R_s$$

we can solve for I

$$I = \frac{\Delta V_s}{R_s}$$

so if we add a new resistor, R_s to the circuit, and measure the voltage across



that circuit, we will be able to calculate the current.

Of course, if R_s is very large, then R_s, itself, will slow down the current. So we want to choose a R_s that is much less than R_{test}.

$$R_s \ll R_{test}$$

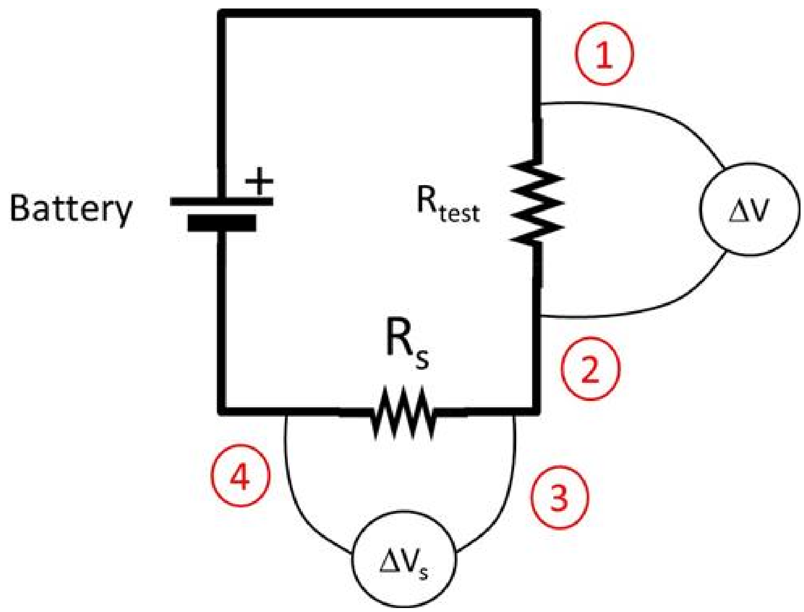
But so long as this is true, our R_s won't change the current much, and since we know R_s we know the current

$$I = \frac{\Delta V_s}{R_s}$$

We have turned our current measurement into a voltage measurement!

5.1.2 Actually making an Arduino measure current

This idea is great, but let's talk a little bit about how to wire this dual measurement. Think again about our two voltage measurements, ΔV and ΔV_s. Each Δ implies two measurements. That means we need a total of four measurements to make this work! Let's see where these measurements would be on our circuit diagram.



By drawing the diagram, we realize that we can create both ΔV measurements with only three individual voltage measurements because the voltage at point 2 and the voltage at point 3 should be the same. So we could wire our circuit like this:and of course we need to wire the negative pole of the battery or power supply to the GND pin. Then our two voltage difference measurements will be formed from

$$\begin{aligned}\Delta V &= V_{A2} - V_{A1} \\ \Delta V_s &= V_{A1} - V_{A0}\end{aligned}$$

If we keep our voltage from our battery or power supply in the 0V to +5V range, then we can use our simple voltmeter sketch. We do need to modify it to take three different voltage measurements. And we need to add the math to make ΔV_s into I. We could even modify this so that our code would report out

$$R = \frac{\Delta V}{I}$$

and we might as well. Here is an example sketch.

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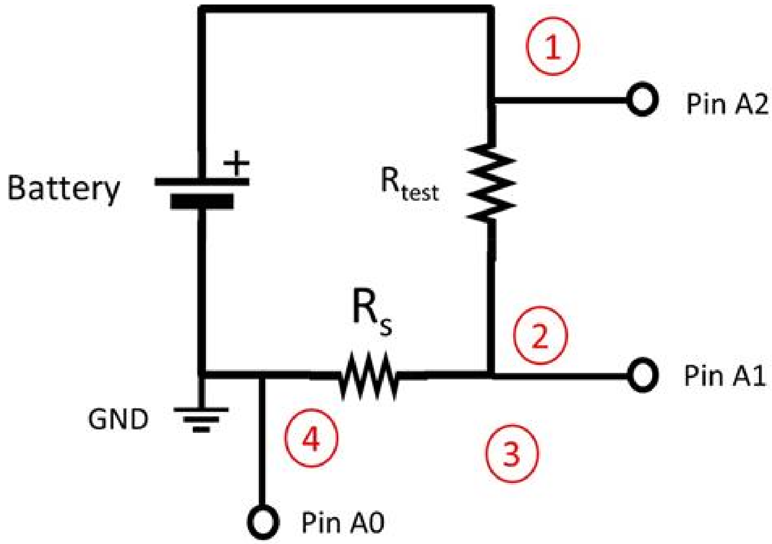


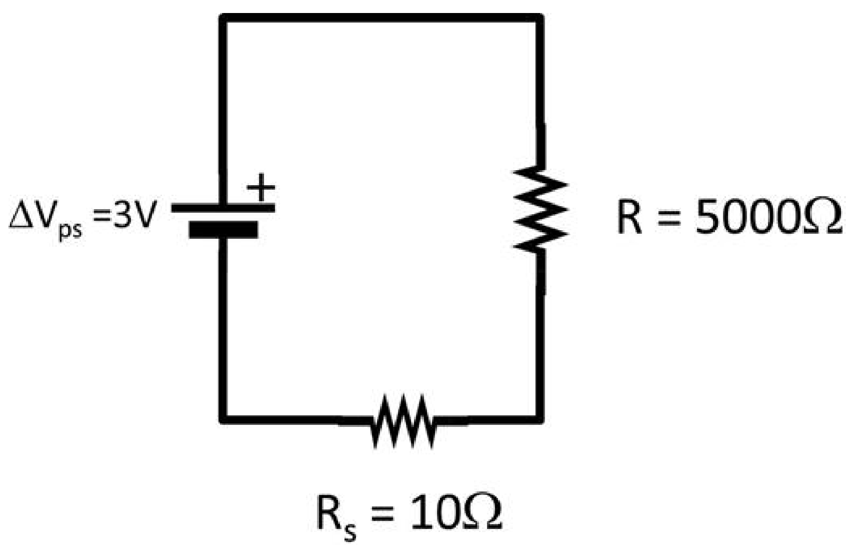

```
//////////////////////////////////////////////////////////
// very simple voltmeter and equally simple ammeter
// will measure 0 to 5V only!
// Voltages outside 0 to 5V will destroy your Arduino!!!
// Delta_V_shunt must therefore be much much less than 5V
// The shunt resistor should be much less than the
// resistance of the rest of the circuit.
/////////////////////////////////////////////////////////
// Shunt resistor value goes here:
  float R_shunt= 220;
    //ohms  -  remember you have to replace this with your
    // actual shunt resistor value

// make some integer variables that identify
//the analog input pins we will use:
   int AI0 = 0;
   int AI1 = 1;
   int AI2 = 2;

// you also need a place to put the analog to
//digital converter values from the Arduino
   int ADC0 = 0;
   int ADC1 = 0;
   int ADC2 = 0;

// Remember we will have to convert from Analog to
```

```
// digital converter(ADC) units to volts. We need
// our delta_V_min just like we did in lab 3
   float delta_v_min=0.0049;    // volts per A2D unit

// We need a place to put the
// calculated voltage and current.
float voltage = 0.0;
float amperage = 0.0;

//////////////////////////////////////////////////////////
 void setup() {
  // put your setup code here, to run once:
  //Initiate Serial Communication
  Serial.begin(9600);     //9600 baud rate
}
//////////////////////////////////////////////////////////
 void loop() {
  // Read in the voltages in A2D units form the
  // Arduino Analog pins
  ADC0 = analogRead(AI0);
  ADC1 = analogRead(AI1);
  ADC2 = analogRead(AI2);

  // Convert the voltage across the
  // test resistor to voltage
  //   units using delta_v_min
  voltage = (ADC2-ADC1) * delta_v_min;

  // Convert the voltage across R_shunt to voltage units
  // using delt_v_min, then convert to
  // current using R_shunt
  amperage = (ADC1-ADC0)* delta_v_min / R_shunt;

  // output the voltage, amperage, and resistance
  Serial.print(" voltage ");
  Serial.print(voltage, 6);
  Serial.print(" amperage ");
  Serial.print(amperage,6);
  Serial.print( " resistance " );
  Serial.println(voltage/amperage, 4);
}
```

Choosing shunt resistors

It's harder than you might think to choose a good shunt resistor. The shunt resistor resistance shouldn't be big enough to cause too much error in our ΔV measurement for the resistor we want to measure. Nor should it be so large that it effects the current much. But suppose we find the smallest resistor that we can, say, 10Ω. Surely that will not affect the actual ΔV or I measurements. But still, we may have a problem. Let's consider an actual circuit to see why.

Suppose we have a circuit where the input voltage from the power supply is $\Delta V_{ps} = 2\text{V}$ and our test resistor is $R = 5000\Omega$. And suppose we try to use $R_s = 10\Omega$.



The two resistors together are a voltage divider. We recognize this from our previous labs. So we expect the voltage drop across each resistor to sum to the voltage given by the power supply

$$\Delta V_{ps} = \Delta V_R + \Delta V_s$$

and we know the current will be the same in the entire circuit. We can use Ohm's law to find the current.

$$\Delta V_{ps} = IR_{total}$$

so that

$$\begin{aligned} I &= \frac{\Delta V_{ps}}{R_{total}} \\ &= \frac{\Delta V_{ps}}{R + R_s} \end{aligned}$$

Now we can find the voltage drop across just R_s

$$\begin{aligned}\Delta V_s &= IR_s \\ &= \left(\frac{\Delta V_{ps}}{R+R_s}\right)R_s\end{aligned}$$

and let's put in numbers

$$\begin{aligned}\Delta V_s &= \left(\frac{2\mathrm{V}}{5000\Omega+10\Omega}\right)(10\Omega) \\ &= 3.992\times10^{-3}\mathrm{V} \\ &= 3.992\mathrm{mV}\end{aligned}$$

Remember that for our simple voltmeter,

$$\Delta V_{\min} = \frac{5\mathrm{V}}{1024} = 4.88\mathrm{mV}$$

and this is larger than ΔV_s so once we use our Arduino analog to digital converter (ADC), ΔV_s will appear to be zero! Our current meter that we built will say our current measurement will be zero even thought there is a current flowing. That is a 100% error!

We might try to improve things by increasing the power supply voltage. Even if we increased the voltage from the power supply to, say, 5V (our maximum) we would only have

$$\begin{aligned}\Delta V_s &= \left(\frac{5\mathrm{V}}{5000\Omega+10\Omega}\right)(10\Omega) \\ &= 9.98\mathrm{mV}\end{aligned}$$

We should compare this value to our ADC minimum detectable value

$$\frac{\Delta V_s}{\Delta V_{\min}} = N$$

the number of ADC units that will be used. We can see that for $R_s = 10\Omega$

$$N = \frac{9.98\mathrm{mV}}{4.88\mathrm{mV}} = 2$$

ADC units. With our entire value of ΔV_s split into only two numbers our uncertainty in our ΔV_s would be something like 50%. That won't make a very good current measurement

Suppose instead, we use $R_s = 170\Omega$. This is much bigger, so it will affect the voltage measurement of the test resistor a little. But in the end will work better. If we set our power supply back to $\Delta V_{ps} = 2\mathrm{V}$ the $R_s = 170\Omega$ gives.

$$\begin{aligned}\Delta V_s &= \left(\frac{2\mathrm{V}}{5000\Omega+170\Omega}\right)(170\Omega) \\ &= 65.764\mathrm{mV}\end{aligned}$$

This would give

$$N=\frac{65.764\text{mV}}{4.88\text{mV}}=13.476$$

or about 13 ADC units spread across our 65.764mV. Then each of our ADC units would be worth

$$\delta\Delta V_s=\frac{65.764\text{mV}}{13}=5.1\text{mV}$$

This is very near the $\Delta V_{\min}=4.88\,\text{mV}$ value, but a little bit higher. If $\delta\Delta V_s$ from our calculation is larger than $\delta V_{\min,}$ then we have to use the larger value as our uncertainty in ΔV_s. So we would say $\delta\Delta V_s=5.1$mV. But still this is not a terrible error.

$$100\times\frac{5.0588\text{mV}}{65.764\text{mV}}=7.7\%$$

This is much better than 50% or 100% error. You might guess that we can do a little better by trying other resistance values. And you would be right. But if you only need an 8% error, this value would be fine.

Our stand-alone meters have lots of shunt resistors inside of them. You are changing shunt resistors when you change the dial setting, trying to balance these errors. By changing shunt resistors in our circuit we are doing the same thing as turning the dial on the current settings of a multimeter.

5.1.3 Finding Uncertainty in a calculated value

In the last section I gave errors in ΔV_s, but didn't finish the error in the current, I. Of course, since we had to calculate the current, we also need to find the uncertainty in our current using error propagation. Fortunately we "remember" how to do this from PH150. We use our basic form for standard error propagation. If we have a function $f(x,y,z)$ then the uncertainty in f would be

$$\delta f=\sqrt{\left(\left(\frac{\partial f}{\partial x}\right)(\delta x)\right)^2+\left(\left(\frac{\partial f}{\partial y}\right)(\delta y)\right)^2+\left(\left(\frac{\partial f}{\partial z}\right)(\delta z)\right)^2}$$

In our current case, our function f is the current I and it is a function of ΔV_s and R_s

$$f=I=\frac{\Delta V_s}{R_s}$$

so we will have an uncertainty like this

$$\delta I=\sqrt{\left(\left(\frac{\partial I}{\partial\Delta V_s}\right)(\delta\Delta V_s)\right)^2+\left(\left(\frac{\partial I}{\partial R_s}\right)(\delta R_s)\right)^2}$$

and we can find the partial derivatives

$$\frac{\partial I}{\partial\Delta V_s}=\frac{1}{R_s}$$

$$\frac{\partial I}{\partial R_s} = -\frac{\Delta V_s}{R_s^2}$$

so we have

$$\delta I = \sqrt{\left(\left(\frac{1}{R_s}\right)(\delta \Delta V_s)\right)^2 + \left(\left(-\frac{\Delta V_s}{R_s^2}\right)(\delta R_s)\right)^2}$$

Let's try this for our example in the last section. We have $\Delta V_s = 9\,.98\text{mV}$ and $R_s = 170\Omega$. We know that $\delta \Delta V_s = 5.058\,8\text{mV}$ and our resistors are only good to 1% so that would be $\delta R_S = 1.7\Omega$

$$\begin{aligned} \delta I &= \sqrt{\left(\left(\frac{1}{170\Omega}\right)(5.058\,8\text{mV})\right)^2 + \left(\left(-\frac{65.764\text{mV}}{(170\Omega)^2}\right)(1.7\Omega)\right)^2} \\ &= 3.000\,8 \times 10^{-5}\text{A} \end{aligned}$$

This looks small. Is it a good uncertainty? We can't tell until we compare it to our expected current. We expect for our example

$$\begin{aligned} I &= \frac{\Delta V_{ps}}{R} \\ &= \frac{2\text{V}}{5000\Omega} \\ &= 0.000\,4\text{A} \\ &= 4 \times 10^4\text{A} \end{aligned}$$

so the fractional uncertainty in the current will be

$$100\frac{3.000\,8 \times 10^{-5}\text{A}}{4 \times 10^4\text{A}} = 7.502\%$$

This still isn't great, it's about what we got for the error in $\delta \Delta V_s$ (but it's better than 50%). We might be able to do better. But if 8% is OK for our application, then we stop here!

Let's try to figure out what the biggest contributor to our uncertainty might be. To do this we look at the terms in our uncertainty calculation separately

$$\begin{aligned} \left(\left(\frac{1}{R_s}\right)(\delta \Delta V_s)\right)^2 &= \left(\left(\frac{1}{170\Omega}\right)(5.058\,8\text{mV})\right)^2 = 8.855\,2 \times 10^{-10}\text{A}^2 \\ \left(\left(-\frac{\Delta V_s}{R_s^2}\right)(\delta R_s)\right)^2 &= \left(\left(-\frac{65.764\text{mV}}{(170\Omega)^2}\right)(1.7\Omega.)\right)^2 = 1.496\,5 \times 10^{-11}\text{A}^2 \end{aligned}$$

The first term is about sixty times the second. So to make an improvement we would want to first concentrate on the first term. We could change our $\delta \Delta V_s$ or change our R_s value. Changing ΔV_s is harder than changing R_s. Maybe we could even make R_s a little bigger to improve our current measurement. *Notice that this was not the obvious solution!* At first it seemed that smaller R_s values would give better uncertainties. But after doing the uncertainty calculations, we find that there is an optimal range for R_s. Big R_s is still bad, but very small R_s is also bad. You have to do the math to find this out.

Iterate to find an optimal value

Since there is an R_s in the bottom of both terms in our current uncertainty, let's try changing the R_s value and see if the uncertainty gets better. We have to start all the way back at the top with ΔV_s. We will have to go through all our calculations again! A spreadsheet or symbolic math processor might be a good way to go so you aren't putting the same things in your calculator over and over.

We start by finding the current in the circuit

$$I = \left(\frac{\Delta V_{ps}}{R + R_s}\right)$$

Then an estimate for ΔV_s across the shunt resistor would be

$$\begin{aligned} \Delta V_s &= I R_s \\ &= \left(\frac{\Delta V_{ps}}{R + R_s}\right) R_s \end{aligned}$$

and then the number of ADC units we used will be

$$N_{ADC} = \frac{\Delta V_s}{\Delta V_{\min}}$$

rounded to the smallest integer, which gives a new estimate of our uncertainty in ΔV_s

$$\delta \Delta V_s = \frac{\Delta V_s}{N_{ADC}}$$

and now we need can find the uncertainty in I

$$\delta I = \sqrt{\left(\left(\frac{1}{R_s}\right)(\delta \Delta V_s)\right)^2 + \left(\left(-\frac{\Delta V_s}{R_s^2}\right)(\delta R_s)\right)^2}$$

and its fractional uncertainty

$$f_I = \frac{dI}{I}$$

As you can see, it is probably best to put all this in a symbolic package (like Mathmatica or Maple, or Sage, or whatever your favorite symbolic math processor might be). That way, you can change values of, say, R_s and ΔV_s without redoing everything. At least consider using a spreadsheet program or even in Python!.

Let's try this once more with $R_s = 500\Omega$ just to see what would happen.

$$\begin{aligned} I &= \left(\frac{2\text{V}}{5000\Omega + 500\Omega}\right) \\ &= 3.6364 \times 10^{-4}\text{A} \end{aligned}$$

so

$$\begin{aligned}\Delta V_s &= IR_s \\ &= \left(\frac{2\text{V}}{5000\Omega + 500\Omega}\right)(500\Omega) \\ &= 0.181\,82\text{V}\end{aligned}$$

and then the number of ADC units we used will be

$$N_{ADC} = \frac{0.181\,82\text{V}}{4.\,88\text{mV}} = 37.\,258$$

which gives a new estimate of our uncertainty in ΔV_s

$$\delta\Delta V_s = \frac{0.181\,82\text{V}}{37} = 4.\,914\,1 \times 10^{-3}\text{V}$$

and now we need can find the uncertainty in I. We will need $\delta R_s = 500\Omega \times 0.01 = 5.0\Omega$

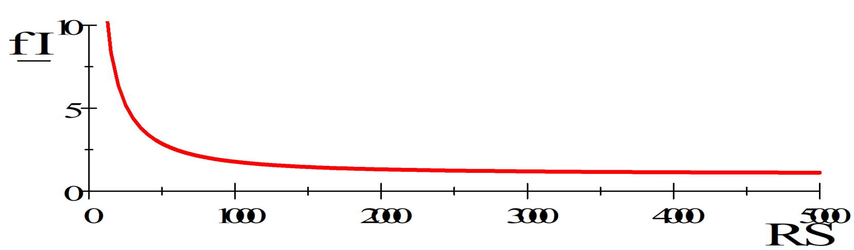
$$\delta I = \sqrt{\left(\left(\frac{1}{500\Omega}\right)(\delta\Delta V_s)\right)^2 + \left(\left(-\frac{\Delta V_s}{(500\Omega)^2}\right)(\delta R_s)\right)^2}$$

$$\begin{aligned}\delta I &= \sqrt{\left(\left(\frac{1}{500\Omega}\right)(4.\,914\,1 \times 10^{-3}\text{V})\right)^2 + \left(\left(-\frac{0.181\,82\text{V}}{(500\Omega)^2}\right)(5.0\Omega)\right)^2} \\ &= 1.\,047\,9 \times 10^{-5}\text{A}\end{aligned}$$

and its fractional uncertainty

$$f_I = 100 \times \frac{1.\,047\,9 \times 10^{-5}\text{A}}{3.\,636\,4 \times 10^{-4}\text{A}} = 2.\,881\,7\%$$

This was a bit of an improvement! We could continue to iterate. I had my computer do this for $R = 5000\Omega$ and $\Delta V_{ps} = 2\text{V}$ I asked it to plot f_I as a function of R_s Notice that after about 500Ω we are not going to get much of an



improvement. So our choice of $R_s = 500\Omega$ seems good for this situation. Once you have a symbolic package or spreadsheet version of this calculation, picking different shunt resistors becomes fairly easy.

We should check, though. What did our 500Ω resistor do to our ΔV measurement? We found our current in the circuit to be

$$I = 3.6364 \times 10^{-4}\text{A}$$

and our test resistor is 5000Ω so

$$\Delta V_{test} = \left(3.6364 \times 10^{-4}\text{A}\right)(5000\Omega) = 1.8182\text{V}$$

We know the power supply was providing $\Delta V_{ps} = 2\text{V}$. So we have introduced an error. We can find the percent difference

$$(100)\,\frac{1.8182\text{V} - 2\text{V}}{1.8182\text{V}} = -9.9989\%$$

which means the ΔV measurement will be 10% low due to our inserting the shunt resistance. If we can live with a 10% error, then we are fine. If not, it is back to iteration to find a better shunt resistance.

Of course, so far we have just found uncertainty in ΔV and I. These are the uncertainties in our measuring devices that we built. Since you are the manufacturer of these devices, you have had to calculate what their uncertainties will be. When we design our own measuring devices, we always have to do this. Of course you could have built the devices and then watched the output to see where the digits fluctuate like we did with our stand-alone multimeters. But the risk is that it might take a long time to find a value for each part of our device that works together with the other parts, and in the mean time we might burn up our equipment if we don't plan for what we want first. You can check your uncertainty calculations by looking at the fluctuation of the digits to see if we are right (or if some other uncertainty factor has crept in that we haven't handled yet).

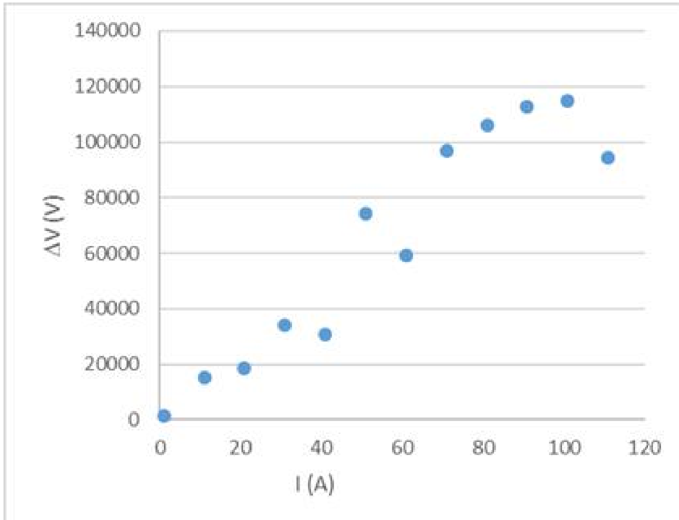
When we started this lab we said we were testing Ohm's law, and we wanted to find R_{test} uncertainty δR_{test} to see if Ohm's law really works. In finding the uncertainty in our measuring devices we haven't found δR_{test}. We will review a different way to do that analysis.

5.1.4 Using statistics to calculate uncertainty

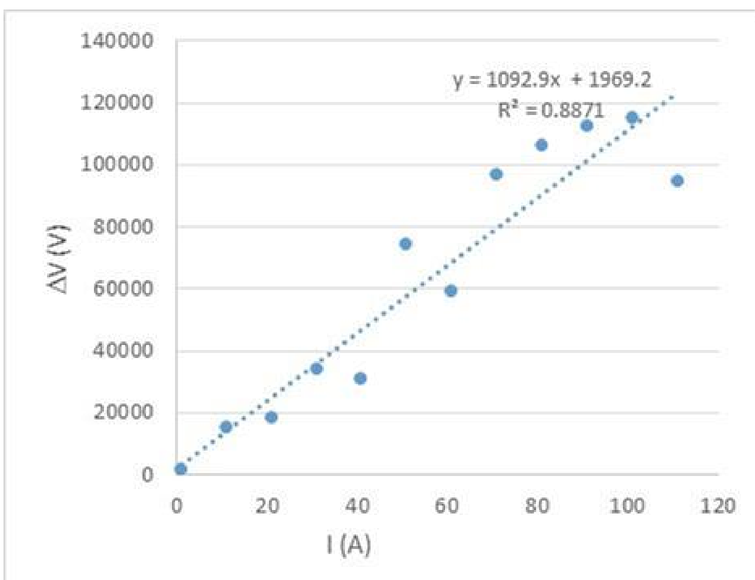
By now in a experimental design, I am usually at my tolerance limit for calculating uncertainties. You might ask, can't we get our powerful computers to help us out a little bit with finding the uncertainty? After all, we went to all the trouble to get the data on the computer. The answer is, yes!

Let's suppose we have done our experiment and we have some data that look like this:

This looks pretty good. It seems to be sort of linear. We might guess from this that Ohm's law is begin obeyed. But we want R_{test} and R_{test} is the slope of this line. We could calculate R_{test} from each pair of ΔV and I points and find



it's uncertainty using standard error propagation. But it seems that it would be better to take all the points into our analysis to find R_{test}. More data should give us a better estimate for R_{test}. Back in PH150 you may have done such a thing using a *curve fit*. In the next figure, a curve fit is shown for the data from the last figure.



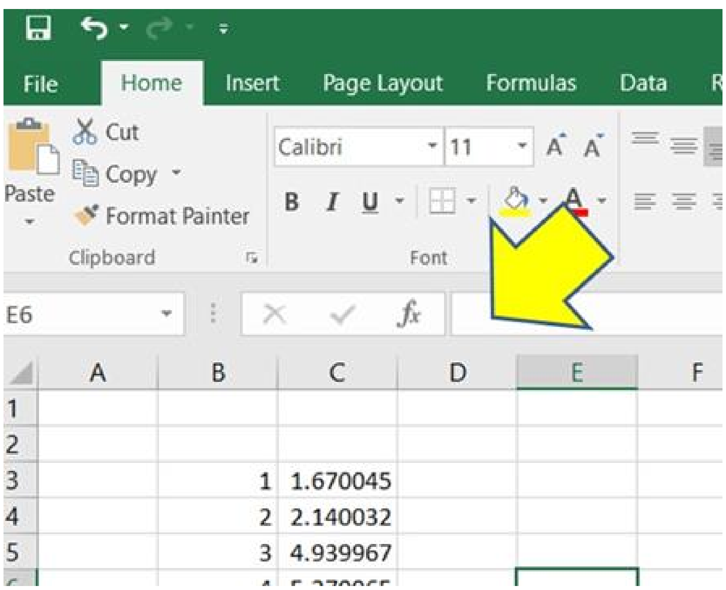
Notice that I performed this curve fit in MicroSoft Excel, but if you are a great Python programer you could do this directly in Python. You could also do this in LoggerPro or many other data analysis programs. From the curve fit equation we can see that we have a slope of 1092.9. Since our graph has ΔV on the vertical axis and I on the horizontal axis we recognize

$$\begin{aligned} \Delta V &= R_{test} I + 0 \\ y &= mx + b \end{aligned}$$

that R_{test} must be the slope. In the data above the we can see that the resistance is a little more than 1092Ω because that is the slope of our fit line. But we know we need an uncertainty along with this nominal value. Can we get the computer to do this as well?

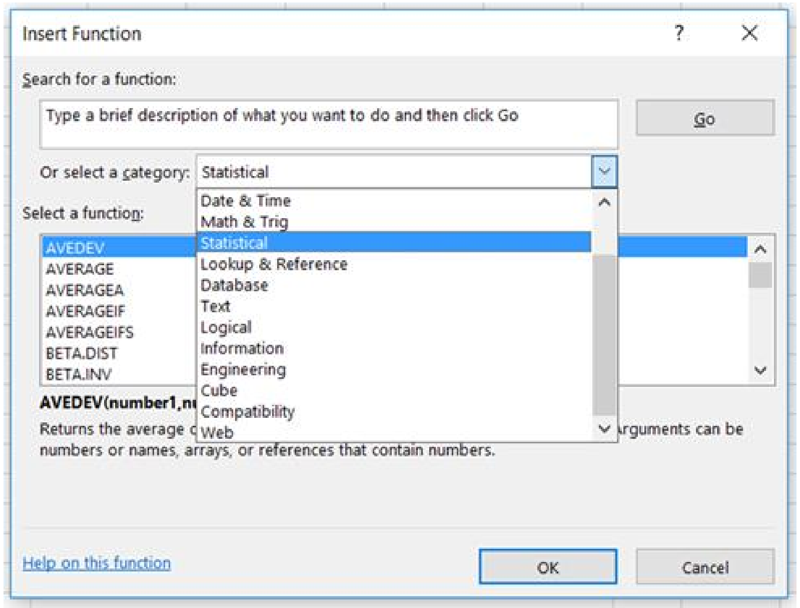
The answer is, of course, Yes! And that will save us a bunch of math! In some data analysis programs this is easy. LoggerPro, for example, just gives you the uncertainty in m and b. Excel does not. If you want to use LoggerPro, that is fine. If you know how to do this in Python, go ahead. But, if you want to use Excel, let's see how to find the uncertainty.

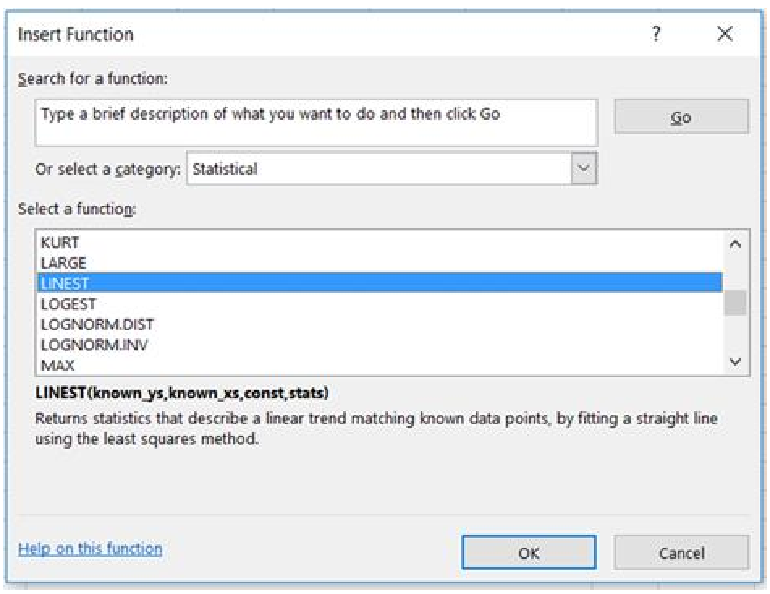
We will use the `linst` function in Excel. To find this, select the *insert function tool.*



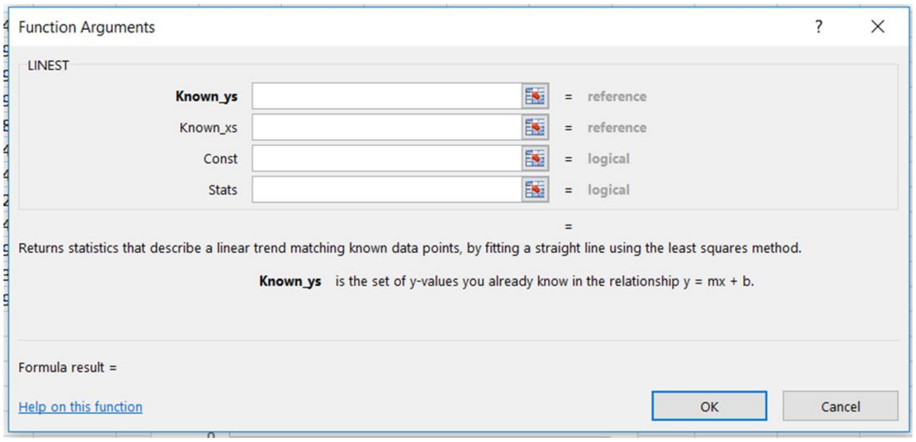
This will bring up a dialog box that allows you to select functions. We want the linst function and it is categorized under *statistical*, so in the category drop down box Then in the *Select a function:* box you should find `LINST` in the list.When you choose `LINST`, another dialog box will open. It will ask you for your y-values and your x-values. It asks for a Constant, but leave that input blank. It also asks if we want statistics. We do, so fill this in with the word "TRUE" in all caps.

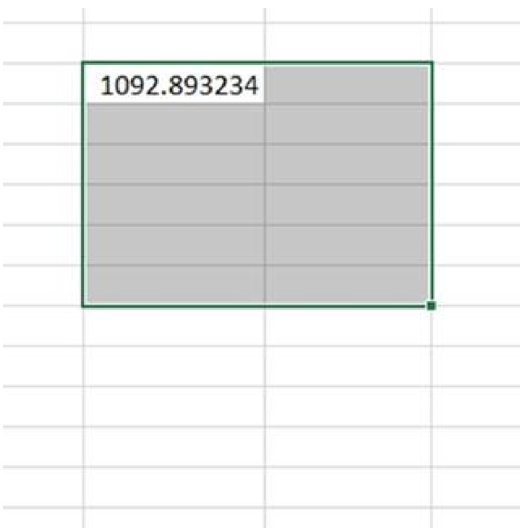
When you click on OK, you will get a number in a spreadsheet cell. That is good, It should be our slope. But we want the uncertainty in that slope.





To do this highlight the cells near the slope value. The region needs to be two columns by five rows. With the region highlighted, click the formula in the formula bar with your mouse. It should light up parts of your spreadsheet. Type CTRL+SHIFT+ENTER all at the same time. This will fill in the region with statistics on our data. The top left cell in our region is still the slope, but now right under this slope is the uncertainty in the slope. We also have in the





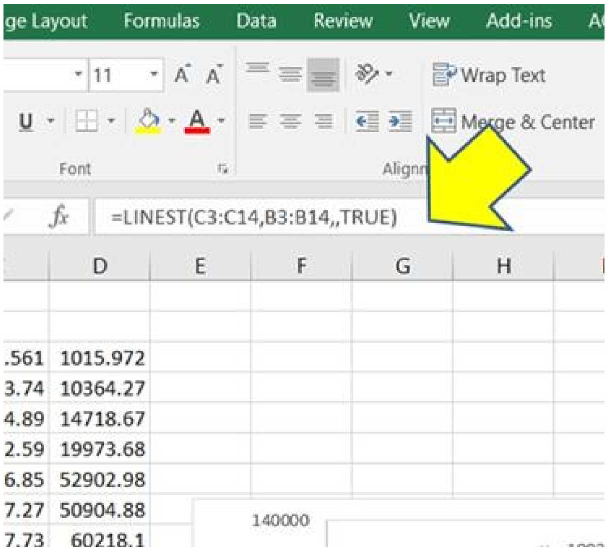
top, right cell the y-intercept and below it the y-intercept uncertainty.

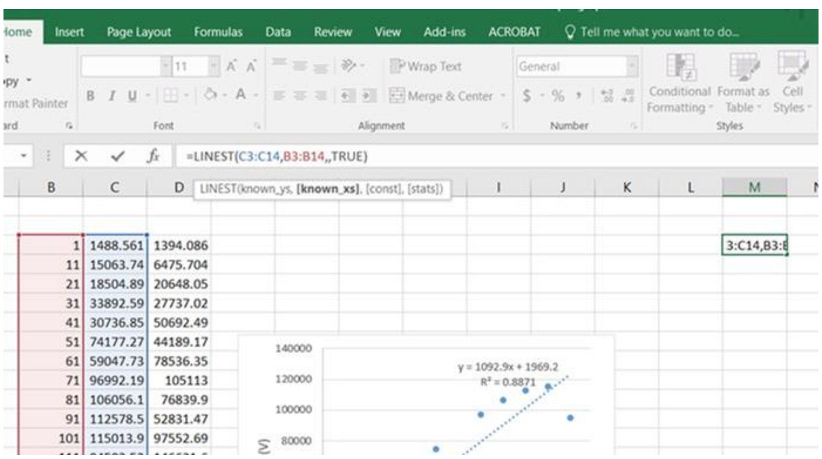
There are other numbers that are useful, but for now the two rows have given us what we needed. We have the slope, m, the y-intercept, b, and their uncertainties. For the data in the figures, we would have

$$R_{test} = 1100\Omega \pm 100\Omega$$

This was a little tricky, but was far less mathematical work than doing the uncertainty for every $(I, \Delta V)$ pair. We will often use this technique to find uncertainty.

Also notice that in this analysis technique, we can afford some error in our ΔV and I values. So maybe a 9% error (like we found in one of our instrument desigs) is not so bad. We may not have to work too hard to get a wonderful



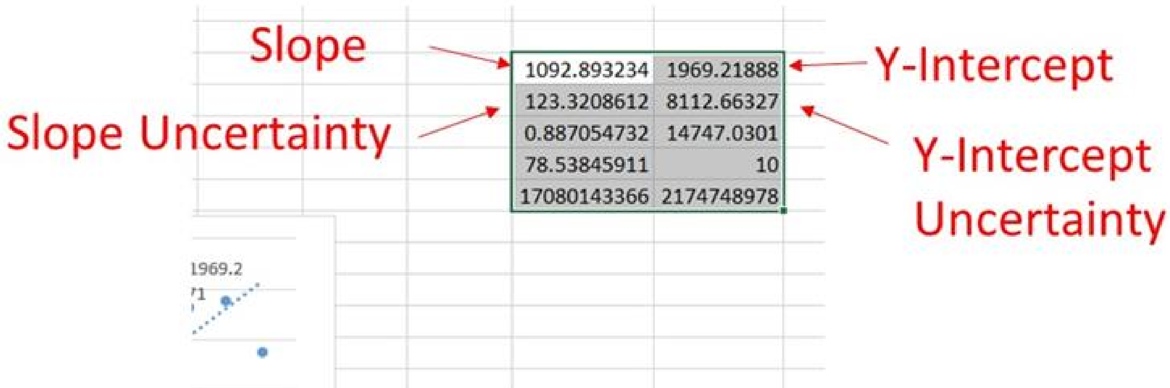


choice of shunt resistor if we are going to use many data points and the power of statistics to analyze the data in the end!

5.1.5 Philosophical warning

There was a lot to this reading. We talked about designing and building an instrument. We used some physics, Ohm's law, in our design process. Then we tested a physical model, Ohm's law, with our instrument. All these were good things, but maybe you wondered along the way if it is acceptable to test Ohm's law with an instrument that depends on Ohm's law. And the answer is a big NO!

This lab is practice, but it is imperfect practice. We do know that Ohm's



law works, so we are going to use it in designing instruments. But you really need a different instrument, one that does not depend on Ohm's law, to test Ohm's law in a credible way. In next week's lab, we will test a different physical model with the same basic instrument that we build today. The instrument will depend on Ohm's law, but the new physical model must not if the experiment is to be valid.

5.2 Proposals

It's time to start thinking of what experiment you and your group will design. You are required to write a proposal for this experiment. This is a document that is intended to persuade someone (your professor, funding agency, yourself, etc.) that you should be given the resources and support to perform the experiment. The proposal consists of the following parts:

1. Statement of the experimental problem
2. Procedures and anticipated difficulties
3. Proposed analysis and expected results
4. Preliminary List of equipment needed

Each of these are discussed below.

5.2.1 Statement of the experimental problem

This is a physics class, so our experiment should be a physics experiment. The job of an experimental physicist is to test physics theory. So your statement of the experimental problem should include what theory you are testing and a brief, high level, overview of what you plan to do to test this theory.

5.2.2 Procedures and anticipated difficulties

Hopefully, your reader will be so excited by the thought of you solving your experimental problem that he or she will want to know the details of what you plan to do. You should describe in some detail what you are planning. If there are hard parts of the procedure, tell how you plan to get through them.

5.2.3 Proposed analysis and expected results

You might think this is unfair, how are you supposed to know what analysis will be needed and what the results should be until you take the data? But really you both can, and should, make a good plan for your data analysis and figure out what your expected results should be. After all, you have a theory your are testing! You can encapsulate that theory into a predictive equation for your experiment. You can design your experimental apparatus, and put in the numbers from your experimental design. From this you can calculate what should be the outcome.

If you don't do this, you don't know what equipment you will need or how sensitive that equipment needs to be. If you are trying to measure the size of your text book, an odometer that only measures in whole miles may not be the best choice of equipment. To know what you need, do the calculations in advance.

You should also do the error analysis. You will want to predict the uncertainty. A measurement of your text book length that is good to ± 3m is not very satisfying in most cases. Uncertainty in your result is governed by the uncertainty inherent in the measurements you will take. The uncertainty calculation tells you what sensitivity you will need in your measurement devices. Since you are choosing those measurement devices as part of your proposal, and you are choosing the inputs to your model equaiton (like the resistance and the capacitance in today's lab) you will know how much uncertatinty they have, so you can do the calculation in advance.

You should do all of this symbolically if you can, numerically if you must, but almost never by hand (meaning not using your calculator) giving single value results. Some measurements will come back poorer than you anticipated, or some piece of equipment will be unavailable. You don't want to have to redo all your calculations from scratch each time this happens. For example, in the event of an equipment problem, your analysis tells you if another piece of equipment is sufficiently sensitive, or if you need to find an exact replacement. When I perform an analysis like this, try for a symbolic equation for uncertainty. I like to program these equations into Scientific Workplace, or Maple, or SAGE, or MathCAD, or whatever symbolic math processor I have. Alternatevly, you could code it into Python. Then, as actual measurements change, I instantly get new predictions. In the absence of a symbolic package, a spreadsheet program will do fine. A numerical program also is quick and easy to re-run with new numbers when no symbolic answer is found.

5.2.4 Preliminary List of equipment needed

Once you have done your analysis, you are ready to list the equipment you need and the sensitivity of the measurement equipment you need. Final approval of the project and the ultimate success of your experiment depend on the equipment you choose or are granted. You want to do a good job analyzing so you know what you need, and a good job describing the experiment so you are likely to have the equipment granted.

5.2.5 Designing the Experiment

Of course, as part of your proposal, you will have to design your experiment. In PH150 we learned that to design an experiment we needed the following steps. Some evidence of these steps should be found in your lab notebook:

1. Identify the system to be examined. Identify the inputs and outputs. Describe your system in your lab notebook.
2. Identify the model to be tested. Express the model in terms of an equation representing a prediction of the measurement you will make. Record this in your lab notebook.
3. Plan how you will know if you are successful in your experiment. Plan graphs or other reporting devices. Record this in your lab notebook. This usually requires you to calculate the predicted uncertainty and to evaluate the relative size of the terms in the uncertainty equation (see below).
4. Rectify your equation if needed. Record this in your lab notebook.
5. Choose ranges of the variables. Record this in your lab notebook.
6. Plan the experimental procedure. Record this in your lab notebook.
7. Perform the experiment . Record this in your lab notebook (see next section).

5.2.6 Using Uncertainty to refine experimental design.

Suppose you plan to test our model for resistance from your PH220 text book. The equation for resistance is

$$R = \rho \frac{\ell}{A}$$

where ρ is the resistivity, the material properties of the material that makes wire or resistor have friction. The length of the wire or resistor is ℓ, and A is the cross sectional area. We could find the uncertainty in R

$$\delta R = \sqrt{\left(\frac{\partial R}{\partial \rho}\delta\rho\right)^2 + \left(\frac{\partial R}{\partial \ell}\delta\ell\right)^2 + \left(\frac{\partial R}{\partial A}\delta A\right)^2}$$

The first term in the square root is

$$\left(\frac{\partial R}{\partial \rho}\delta\rho\right)^2=\left(\frac{\ell}{A}\delta\rho\right)^2$$

and the other two terms are

$$\left(\frac{\partial R}{\partial \ell}\delta\ell\right)^2=\left(\frac{\rho}{A}\delta\ell\right)^2$$

$$\left(\frac{\partial R}{\partial A}\delta A\right)^2=\left(-\rho\frac{\ell}{A^2}\delta A\right)^2$$

And suppose that our design is to have a copper wire with

$$\begin{aligned}\rho &= 1.68\pm 0.03\times 10^{-8}\Omega\text{m}\\ \ell &= 5.0\pm 0.1\text{m}\\ A &= 5.0\times 10^{-10}\text{m}^2\text{m}^2\end{aligned}$$

This would give a resistance of

$$\begin{aligned}R_{new} &= 1.68\times 10^{-8}\Omega\text{m}\frac{5\text{m}}{5.0\times 10^{-10}\text{m}^2}\\ &= 168.0\Omega\end{aligned}$$

We can calculate each of our terms from the δR equation.

$$\left(\frac{\ell}{A}\delta\rho\right)^2=\left(\frac{5\text{m}}{5.0\times 10^{-10}\text{m}^2}\left(0.03\times 10^{-8}\Omega\text{m}\right)\right)^2=9.0\Omega^2$$

$$\left(\frac{\rho}{A}\delta\ell\right)^2=\left(\frac{1.68\times 10^{-8}\Omega\text{m}}{5.0\times 10^{-10}\text{m}^2}\left(0.1\text{m}\right)\right)^2=11.290\Omega^2$$

$$\left(-\rho\frac{\ell}{A^2}\delta A\right)^2=\left(-\left(1.68\times 10^{-8}\Omega\text{m}\right)\frac{(5\text{m})}{\left(5.0\times 10^{-10}\text{m}^2\right)^2}\left(0.1\times 10^{-9}\text{m}^2\right)\right)^2=1129.0\Omega^2$$

The overall uncertainty then would be

$$\delta R=\sqrt{9.0\Omega^2+11.290\Omega^2+1129.0\Omega^2}=33.901\Omega$$

So with this design we predict a fractional uncertainty of

$$\frac{33.901\Omega}{168.0\Omega}=0.20179$$

or a little over 20%. This is not a great design. We would like a much lower uncertainty, something that gives a fractional uncertainty more like 1%. It is clear that the last term has the highest contribution to the uncertainty, so this is the term that needs fixing. One method of fixing the problem would be to increase δA. We could try $1.0 \pm 0.1 \times 10^{-9}\text{m}^2$. In order to have the same resistance we will also have to change the length of the wire from 10m to 5m.

$$\begin{aligned}
\rho &= 1.68 \pm 0.03 \times 10^{-8}\Omega\text{m} \\
\ell &= 10.0 \pm 0.1\text{m} \\
A &= 1.0 \pm 0.1 \times 10^{-9}\text{m}^2
\end{aligned}$$

Checking we see we do get the same resistance

$$\begin{aligned}
R &= 1.68 \times 10^{-8}\Omega\text{m}\frac{10\text{m}}{1.0 \times 10^{-9}\text{m}^2} \\
&= 168\Omega
\end{aligned}$$

But now for the last term we would get

$$\left(-\rho\frac{\ell}{A^2}\delta A\right)^2 = \left(-\left(1.68 \times 10^{-8}\Omega\text{m}\right)\frac{(10\text{m})}{\left(1.0 \times 10^{-9}\text{m}^2\right)^2}\left(0.1 \times 10^{-9}\text{m}^2\right)\right)^2 = 282.24\Omega^2$$

which is better. But we have to check to make sure our design change didn't cause a large rise in the other two terms.

$$\left(\frac{\ell}{A}\delta\rho\right)^2 = \left(\frac{10\text{m}}{1.0 \times 10^{-9}\text{m}^2}\left(0.03 \times 10^{-8}\Omega\text{m}\right)\right)^2 = 9.0\Omega^2$$

$$\left(\frac{\rho}{A}\delta\ell\right)^2 = \left(\frac{1.68 \times 10^{-8}\Omega\text{m}}{1.0 \times 10^{-9}\text{m}^2}(0.1\text{m})\right)^2 = 2.8224\Omega^2$$

The first term was hurt by our new design change, but not badly. So with the new design the overall uncertainty would be

$$\delta R = \sqrt{9.0\Omega^2 + 2.8224\Omega^2 + 282.24\Omega^2} = 17.148\Omega$$

So with this new design we predict a fractional uncertainty of

$$\frac{17.148\Omega}{168.0\Omega} = 0.10207$$

which is about 10.%. This is much better. From our uncertainty terms, we can see that to do better we need to improve both the δA term and the $\delta\ell$ terms because they are now about the same size. The terms in our uncertainty calculation tell us how to modify our experimental design.

There is a refinement we could make to our process. really there are no area measurement devices available, so what we would do is measure the diameter of the wire and calculate the area.

$$A = \frac{1}{4}\pi D^2$$

we could find δA by using our propagation of uncertainty equation again, or we could modify our resistance equation so that it is in terms of what we actually measure.

$$R = \rho \frac{4\ell}{\pi D^2}$$

and calculate our uncertainty in terms of ρ, ℓ, and D. That is preferred and usually less work. The general rule is to express your model equation in terms of what you will actually measure before you calculate the uncertainty terms.

The moral of this long story is that we must calculate the uncertainty *as part of the design process.* It is probably best to use a symbolic math processor or at lease a spreadsheet so that as the design changes your uncertainty estimate will change too without having to manually recalculate it.

5.3 Lab Assignment

1. Build the instrument

   (a) Choose a test resistor in the 1kΩ to 10kΩ range and a shunt resistor. You will have to check your values using the math we discussed above to make sure they will work. If your first shunt resistor choice works, use it. If not, iterate until you have a shunt resistance that will work.

   (b) Modify your voltmeter sketch to measure both the voltage and the current. (Check the voltage, currents, and their uncertainties with the serial monitor to make sure things seem good).

   (c) Build your voltmeter and ammeter so your Arduino is taking $(I, \Delta V)$ pares and reporting them. Reporting to the serial monitor is fine for a start. if you based your sketch on the simple voltmeter, make

   sure you don't use voltages outside the 0V to +5V range! Include expected uncertainties for your ΔV and I measurements.

   (d) Check your lab group's instruments to see if they work, and have your lab group members check yours.

2. Test Ohm's law

   (a) Take 10-15 measurements of ΔV and I. For each ΔV measurement change the ΔV setting on the power supply a small amount (don't go over 5V if you are using the simple voltmeter!).

   (b) Plot voltage vs. current and fit a curve to the data.

   (c) Determine the resistance from this curve fit and its uncertainty.

   (d) Finally, determine if your results support the Ohm model for how potential and current are related?

   (e) Compare your data and conclusions to the data and conclusions of your lab group members. Have them look at your results as well.

3. If you still have time, repeat part 2 for a diode. Do your results support the Ohm model for how potential and current are related?

4. Could you have your Arduino sketch report the calculated uncertainties for ΔV, I, and R? If you have time (you probably won't) give this a try.