

User Guide

A concise description of all the routines and applications: what they do, calling syntax, output format, and return values or messages.

Routines:

The bis function is a modification of the algorithm for the bisection method. It approximates the solution to a root finding problem by checking smaller intervals to apply the intermediate value theorem.

`>bis(a,b,tolb,func,pars)`

a and b are the endpoints of the initial interval

tolb is the error tolerance for the bisection method

func is the root solving problem expressed as a function set equal to 0

`%func=@(x)(x-e*sin(x)-Mt); %Kepler's equation to find root E(t)`

pars stores all constants in the Kepler's equation, Mt and e.

output format: [root a b]

return values:

the bis function returns root, which is the midpoint of the final computed interval [a,b]

The nwt function is a modification of the algorithm for Newton's method. This method provides faster convergence to the actual root when $f'(p)$ does not equal 0.

`>nwt(seed,toln,nmax,func,dfunc,pars)`

seed is the initial approximation p0

toln is the error tolerance set for Newton's method

nmax is the maximum number of iterations

func is the root solving problem $f(x)=0$

dfunc is the first derivative of func

`%func=@(x)(x-e*sin(x)-Mt); %Kepler's equation to find root E(t)`

`%dfunc=@(x)1-e*cos(x); %derivative of f`

pars stores all constants in the Kepler's equation, Mt and e

output format: [root found iter]

return values:

the nwt function returns root, the approximation for the zeros of the equation

found is a boolean logical indicating whether the root is found

iter is the number of iterations actually used

The bisnwt function uses algorithm 1 of the project description to find a root of a function. It combines the bisection method and Newton's method.

>bisnwt(a,b,tolb,toln,nmax,func,dfunc,pars)

a and b are the endpoints of the initial interval

tolb is the error tolerance for the bisection method

toln is the error tolerance set for Newton's method

nmax is the maximum number of iterations

func is the root solving problem $f(x)=0$

dfunc is the first derivative of func

%func=@(x)(x-e*sin(x)-Mt); %Kepler's equation to find root E(t)

%dfunc=@(x)1-e*cos(x); %derivative of f

pars stores all constants in the Kepler's equation, Mt and e

output format: [root iter itern]

return values:

iter is the number of iterations used

itern is the number of iterations used in Newton's method

The application, tanom, solves the Kepler equation and gives the position of the satellite in its orbital plane.

>tanom(T,,e,n,varargin)

T : the period of the orbit in hours

e: the eccentricity of the orbit

n: the number of points where we want to localize the satellite on its orbit

the following arguments can optionally be passed through varargin

tolb : the error for the bisection method

toln ; the error for Newton's method

nmax: the maximum iterations that we allow to Newton's method

output format: [orbit]

orbit is a matrix of the format where the columns in order are:

ti, ei, vi, ri, xi, yi

Implementation:

The argument `toln` in the routine `tanom.m` represents the error that we allow in the approximation of the Kepler equation. If we want the error in the coordinates of the orbit to be smaller than 10^{-6} , we use `toln=1e-16`.

We could also change the computations to work with a vector x in R^n to find all the points of the orbit at once. It would be a faster way to compute a function for many values, because it avoids setting up each function evaluation.

Test:

First I verified that there were no issues with `bis` function, by testing a simple function, and setting `pars=0`. Then I tested `nwt` function similarly and found no errors. I debugged for an infinite loop that occurred in `bisnwt`.

In the console window type:

```
>> tanom(4,.25,100)
```

```
ans =
```

```
1.0e+04 *  
      0      0      0      0.9595 0.9595      0  
0.0000 0.0000 0.0000 0.9606 0.9550 0.1036  
0.0000 0.0000 0.0000 0.9639 0.9416 0.2063  
0.0000 0.0000 0.0000 0.9694 0.9196 0.3070  
0.0000 0.0000 0.0000 0.9770 0.8892 0.4050
```

```
...
```

Solutions:

Plot the orbits that you have obtained with your selected initial data and parameters.

```
>> orbit=tanom(4,.25,100)
```

```
>> plot(orbit(:,5),orbit(:,6))
```

