#3. Let $I = \int e^{2x}\cos 3x\,dx$.

Apply integration by parts:

keep $\quad$ d/dx

$$\int e^{2x}\cos 3x\,dx = e^{2x}\,\frac{1}{3}\sin 3x - \int 2e^{2x}\,\frac{1}{3}\sin 3x\,dx = \frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\underbrace{\int e^{2x}\sin 3x\,dx}_{\text{we have to know this.}}$$

$\int$ $\quad$ keep

Apply integration by parts again:

keep $\quad$ d/dx

$$\int e^{2x}\sin 3x\,dx = e^{2x}\left(-\frac{1}{3}\cos 3x\right) - \int 2e^{2x}\left(-\frac{1}{3}\right)\cos 3x\,dx = -\frac{1}{3}e^{2x}\cos 3x + \frac{2}{3}\underbrace{\int e^{2x}\cos 3x\,dx}_{=\,I}$$

$\int$ $\quad$ keep

Therefore,

$$I = \frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\left(-\frac{1}{3}e^{2x}\cos 3x + \frac{2}{3}I\right)$$

$$= \frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x - \frac{4}{9}I$$

$$\Leftrightarrow \left(1+\frac{4}{9}\right)I = \frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x$$

$$I = \frac{9}{13}\left(\frac{1}{3}e^{2x}\sin 3x + \frac{2}{9}e^{2x}\cos 3x\right) = \underline{\frac{3}{13}e^{2x}\sin 3x + \frac{2}{13}e^{2x}\cos 3x + C}$$

Answer.