Michele Kholodova

2/2 Excellent!

$$\frac{4 \cdot 3}{3}$$

$$\frac{7}{7} f(x,y) = \left(\frac{x}{\sqrt{x^2 + y^2}}, \frac{y}{\sqrt{x^2 + y^2}}\right)$$

$$F(1,1) = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$$

$$F(1,-1) = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$$

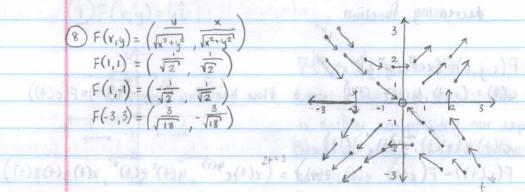
$$F(2,2) = \left(\frac{2}{\sqrt{8}}, \frac{2}{\sqrt{8}}\right)$$

$$F(2,1) = \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$$

$$F(x_1, y) = \begin{pmatrix} \frac{y}{\sqrt{x^2 + y^2}} & \frac{x}{\sqrt{x^2 + y^2}} \end{pmatrix}$$

$$F(1, 1) = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$F(3, 3) = \begin{pmatrix} \frac{3}{\sqrt{18}} & -\frac{3}{\sqrt{18}} \end{pmatrix}$$



(6)
$$c(t) = (t^2, 2t-1, \sqrt{t}), t>0$$
 \Rightarrow $c'(t) = (2t, 2, \frac{1}{2}t^{-1/2}) = (2t, 2, \frac{1}{2\sqrt{t}})$
 $F(x,y,z) = (y+1, 2, \frac{1}{2z})$ \Rightarrow $F(c(t)) = F(t^2, 2t-1, \sqrt{2}) = (2t, 2, \frac{1}{2\sqrt{t}})$
* Thus, $F(c(t)) = c'(t)$ $y+1 = 2t$

$$\begin{array}{ccc}
(1) c(t) = (sint, cost, e^t) &\Rightarrow c'(t) = (cost, -sint, e^t) &\Rightarrow \\
F(x,y,z) = (y,-x,z) &F(c(t)) = F(sint, cost, e^t) = (cost, -sint, e^t) \\
*Thus, F(c(t)) = c'(t) &\Rightarrow \\
\end{array}$$

21) a)
$$F(x,y,z) = (yz, xz, xy)$$
 $f: \mathbb{R}^3 \to \mathbb{R}$ such that $F = \nabla f$

$$F = \nabla f = \frac{\partial f}{\partial x} \hat{c} + \frac{\partial f}{\partial y} \hat{j} + \frac{\partial f}{\partial z} \hat{k} \qquad \frac{\partial f}{\partial x} = yz \qquad \frac{\partial f}{\partial y} = xz \qquad \frac{\partial f}{\partial z} = xy$$

$$f(x,y,z) = xyz \qquad \qquad \int yz \, dx \qquad \int xz \, dy \qquad \int xy \, dz$$

b)
$$F(x,y,z) = (x,y,z)$$
 f: $\mathbb{R}^3 \to \mathbb{R}$ such that $F = \mathbb{P}f$

$$\frac{\partial f}{\partial x} = x \quad \frac{\partial f}{\partial y} = y \quad \frac{\partial f}{\partial z} = Z$$

$$f(x) = x \, dx \quad f(y) = y \, dy \quad f(z) = z \, dz$$

$$f(x) = \frac{x^2}{2} \quad f(y) = \frac{y^2}{2} \quad f(z) = \frac{z^2}{2}$$

	The Comment
	(24) Let c(t) be flow line of F=-DV . Prove V(c(t)) is decreasing function of t.
	(24) let c(t) be flow line of F=-VV - From (CC)
	f(t) = V(c(t)) $f(t) = V(c(t)) $ $f(t) = F(c(t))$
	$f'(t) = \Delta \Lambda(c(t)) \cdot c_1(t) = \Delta \Lambda(c(t)) \cdot E(c(t))$
	Since F=- DV, then f'(t) = DV - DV - (+V) = (1-1) =
	Then $f'(t) = - \nabla V ^2 < 0$
	Thus, f'(t) = av(c(t))/at < 0 and v(c(t)) is a
	decreasing function
	5 (mm) = (m) = (m) = (m)
	27 $F(x_1y_1z) = (xe^y, y^2z^2, xyz)$
	(27) $F(x_1y_1z) = (xe^{-1}, ye^{-1}, xye^{-1})$ $c(+) = (xc+), y(+), z(+))$ is a flow line for $F \sim c'(+) = F(c(+))$
	(1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1 - 1
	$C'(+) = (x'(+), y'(+), z'(+))$ $(x(+) e^{y(+)} y(+)^{2} z(+)^{2} x(+) y(+) z(+))$
	$c'(+) = (x'(+), y'(+), z'(+))$ $F(c(+)) = F(x(+), y(+), z(+)) = (x(+)e^{y(+)}, y(+)^{2}z(+)^{2}, x(+)y(+)z(+))$
	The state of the s
	(c'(+) = F(c(+))) = (+)
	(x, y) = (
	$y'(t) = y(t)^{2} \neq (t)^{2}$
	$a'(1) = x(1)u(1) \neq (1)$
	(15 trans tend) (10 m2 (15 trans trans) (10 16 (1)
	(2, 5 - 4, 1) = (4, 5) + (5, x - p) = (5, y - p) = (5, y - p)
Ž	mis: (+) '= (+)
	(21) a) F(x, 9, 3) = (92, x2, x4) F. R R such that F- VF
	1 x x x x x x x x x x x x x x x x x x x
	spirit press report 26x = (2'6'x) 4
	$10 = 7 + 1000 \text{M} \leftarrow 2 \text{M} \cdot 1 \qquad (x, y, x) = (x, y, x) = (d)$
	$4q = 7 + nnt \text{ slow } 21 \leftarrow 21 + 1 $ $(x, y, x) = (x, y, x) = (x$
	+(x) = (x,y,x) + (x) = (x) (
	The state of the s

4.4

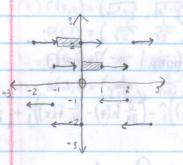
$$(4) V(x,y,z) = x^{2} \hat{i} + (x+y)^{2} \hat{j} + (x+y+z)^{2} \hat{k}$$

$$div V = \nabla \cdot V = \frac{\partial x^{2}}{x} + \frac{\partial (x+y)^{2}}{y} + \frac{\partial (x+y+z)^{2}}{z}$$

$$= 2x + 2(x+y) + 2(x+y+z) = 2x + 2x + 2y + 2x + 2y + 2z$$

$$= 6x + 4y + 2z = 2(3x + 2y + z)$$

2((ex) 200 p) (7) F(x,y) = yîx) + 2(== e2x8) =



$$\nabla \cdot F = \frac{\partial}{\partial x} (y) = O(M_{\text{obs}} + O(M_{\text{obs}}))$$

Since it is 0, it means the fluid

is neither compressing nor expanding.

Based on image, shaded regions

have some area.

(14) $F(x,y,z) = yz\hat{i} + xz\hat{j} + xy\hat{k}$

$$\nabla_{x} F = \begin{vmatrix} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & xz & xy \end{vmatrix} = \left(\frac{\partial}{\partial y} xy - \frac{\partial}{\partial z} xz \right) \hat{i} - \left(\frac{\partial}{\partial x} xy - \frac{\partial}{\partial z} yz \right) \hat{j} + \left(\frac{\partial}{\partial x} xz - \frac{\partial}{\partial y} yz \right) \hat{k}$$

$$= (x-x)\hat{i} + (y-y)\hat{j} + (z-z)\hat{k} = (0,0,0) = 0$$

17 F(x,y) = sinxî + cosxĵ

$$\nabla_{x} F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial t} \end{vmatrix} = (\hat{i} - \hat{i} - \hat{i}) + (-\sin x - \hat{i})$$

(21) $F(x,y,t) = (x^2, x^2y, z+zx)$

a)
$$\nabla \cdot (\nabla x \vec{z}) = \nabla \cdot \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & x^2 y & 2+2x \end{vmatrix} = \nabla \cdot \left[(0-0)\hat{i} - (z-0)\hat{j} + (2xy-0)\hat{k} \right]$$

$$\begin{vmatrix} x^2 & x^2 y & 2+2x \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix} = O + O + O = O \checkmark$$

b) No, because let there be a f such that $\vec{F} = P\vec{f}$. The curl of a gradient is \vec{O} (ex: $\nabla \times Pf = O$) which implies that $\nabla \times \vec{F} = O$. But as we saw from (a) $\nabla \times \vec{F} = -z\hat{j} + 2xy\hat{k}$ which does not equal to O.

23) $F(x_1y_1^2) = (e^{x^2}, \sin(xy), x^5y^3 + 2)$ a) $\nabla \cdot F = \frac{\partial}{\partial x} e^{xz} + \frac{\partial}{\partial y} \sin(xy) + \frac{\partial}{\partial z} x^{5} y^{3} z^{2}$ $= 2 e^{x^2} + x \cos(xy) + 2 x^5 y^3 2$ b) $\nabla x = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \end{vmatrix} = (3y^2x^5z^2 - 0)\hat{i} - (5x^4y^3z^2 - xe^{xz})\hat{j} + (y\cos(xy) - 0)\hat{k}$ ext sin(xy) x 5 y 3 22 = $(3x^{5}y^{2}z^{2})\hat{c} + (xe^{x^{2}} - 5x^{4}y^{3}z^{2})\hat{j} + (y\cos(xy))\hat{k}$ 26) f,g,h:R→R F(x,y,z)= (f(x), g(y), h(z)) is irrotational ~ 7x = 0 $\nabla_{x} F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ f(x) & g(y) & h(z) \end{vmatrix} = \left(\frac{\partial}{\partial y} & h(z) - \frac{\partial}{\partial z} & g(y) \right) \hat{i} - \left(\frac{\partial}{\partial x} & h(z) - \frac{\partial}{\partial z} & f(x) \right) \hat{j} + \left(\frac{\partial}{\partial x} & g(y) - \frac{\partial}{\partial y} & f(x) \right) \hat{k}$ = Oî - Oĵ + Oĥ = O ANX + 7 3 x 1 1 5 p = (5, p, x) = (P) (27) f, q, h: R3 → R F(x,y,z) = (f(y,z),g(x,z),h(x,y)) has zero divergence ~ $\nabla \cdot F = 0$ $\nabla \cdot F = \frac{\partial}{\partial x} f(y,z) + \frac{\partial}{\partial y} g(x,z) + \frac{\partial}{\partial z} h(x,y) = 0$ 1(0-px) - 1(0-5) - 5(0-0) -V 1. (0, 2, 2, 3) + (0, 0) · (0, 0) · (0, 0) · (0, 0) and of agradient is O (ex 7x 7x = 0) which inchies that VaF = O. But as us as from (a) VXF = -27+ 2xyk which