# Midterm III Solution

**#1.** $6 - 2x \geq 18$.

$-2x \geq 12$

$2x \leq -12$

$\underline{x \leq -6.}$

**#2.** $2x - y = -3 \quad : \ell$

$\Leftrightarrow y = 2x + 3. \quad : \ell$

So slope $= 2$

The straightline perpendicular to $\ell$ has its slope $-\frac{1}{2}$.

Now Given slope : $-\frac{1}{2}$

point : $(3, -4)$

The Straightline we are looking for is

$y = -\frac{1}{2}(x - 3) - 4$

$= -\frac{1}{2}x + \frac{3}{2} - 4$

Hence $\underline{y = -\frac{1}{2}x - \frac{5}{2}}$.

**#3.** $(x^2 - 4x + 4)(x + 2)$

$= (x - 2)^2 (x + 2)$    or you can multiply directly.

$= (x - 2)(x^2 - 4)$

$= \underline{x^3 - 2x^2 - 4x + 8.}$

**#4.** $\left(-3 a^{-3} b^2\right)^{-2} (8ab)^2$

$= \dfrac{(8ab)^2}{(-3a^{-3}b^2)^2} = \dfrac{8a^2 b^2}{9a^{-6}b^4} = \underline{\dfrac{8a^8}{b^2}.}$

**#5.** $40500000000 = 405 \times 10^8 = \underline{4.05 \times 10^{10}.}$

**#6.** $8x^4 - 18x^8 = 2(4x^4 - 9x^8) = 2x^4(4 - 9x^4) = \underline{2x^4(2 + 3x^2)(2 - 3x^2)}$

If you can, factor in $\mathbb{C}$.

**#7.** $x^2 - 6x = 1$. $\Leftrightarrow x^2 - 6x + 9 = 10$.

$\Leftrightarrow (x - 3)^2 = 10$.

$\Leftrightarrow \underline{x = 3 \pm \sqrt{10}}$,

**#8.** $\dfrac{x-2}{x+1} - \dfrac{3 - 12x}{2x^2 - x - 3} = \dfrac{x-2}{x+1} + \dfrac{3(4x-1)}{(2x-3)(x+1)} = \dfrac{(2x-3)(x-2)}{(2x-3)(x+1)} + \dfrac{3(4x-1)}{(2x-3)(x+1)}$

$= \dfrac{2x^2 - 7x + 6 + 12x - 3}{(2x-3)(x+1)} = \dfrac{2x^2 + 5x + 3}{(2x-3)(x+1)} = \dfrac{(2x+3)(x+1)}{(2x-3)(x+1)}$

$= \underline{\dfrac{2x+3}{2x-3}.}$

**#9.** $\dfrac{x^2 - x - 2}{9x^3} \div \dfrac{x^2 - 4}{3x^6} = \dfrac{(x-2)(x+1)}{3 \cdot 9x^3} \times \dfrac{3x^6}{(x-2)(x+2)} = \underline{\dfrac{(x+1) \cdot x^3}{3(x+2)}}$

#10. $\dfrac{\dfrac{15}{x^2} - \dfrac{2}{x} - 1}{\dfrac{4}{x^2} - \dfrac{5}{x} + 4} \times \dfrac{x^2}{x^2} = \dfrac{15 - 2x - x^2}{4 - 5x + 4x^2}$

$= -\dfrac{(x-3)(x+5)}{4x^2 - 5x + 4}$ .

#11. $f(1) = 4 \cdot 1 - 1^2 = \underline{3}$.

#12. $y = x^2 + 8x + 16 - 16$

$= (x+4)^2 - 16$.

Vertex: $(-4, -16)$.

#13. $x^2 - x + 1 = 0$.

$\Leftrightarrow x^2 - x + \tfrac{1}{4} = -\tfrac{3}{4}$

$\Leftrightarrow (x - \tfrac{1}{2})^2 = -\tfrac{3}{4}$

$x = \tfrac{1}{2} \pm \dfrac{\sqrt{3}i}{2}$

$x = \dfrac{1 \pm \sqrt{3}i}{2}$

#14. $y = x^2 + 3x = x^2 + 3x + (\tfrac{3}{2})^2 - (\tfrac{3}{2})^2$

$= (x + \tfrac{3}{2})^2 - \tfrac{9}{4}$.

Axis of Symmetry: $x = -\tfrac{3}{2}$

#15. $y = x^2 + x + 1 = x^2 + x + \tfrac{1}{4} - \tfrac{1}{4} + 1 = (x + \tfrac{1}{2})^2 + \tfrac{3}{4}$.

Vertex: $(-\tfrac{1}{2}, \tfrac{3}{4})$.