Your name:

Instructions: Please answer the following and be sure to show your work or support your answer. You are not allowed to use the textbook, workbook, or notes. You cannot talk to other sudents. You can use your calculator. You also can refer a 1-page formula sheet of your own, upon approval.

1. Draw the graph of $y = 2\sin x$ where $0 \le x \le 2\pi$. Indicate both x- and y-coordinates of the points where the y value is in its maximum and minimum.
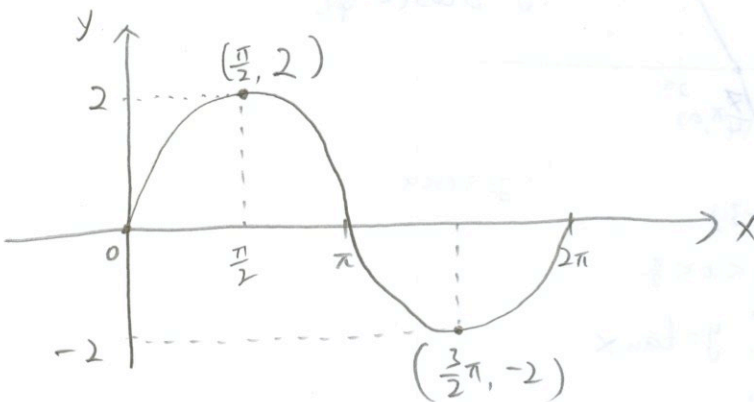
2. What is the period and the amplitude of the function $y = \frac{1}{2}\sin\frac{\pi}{3}x$?

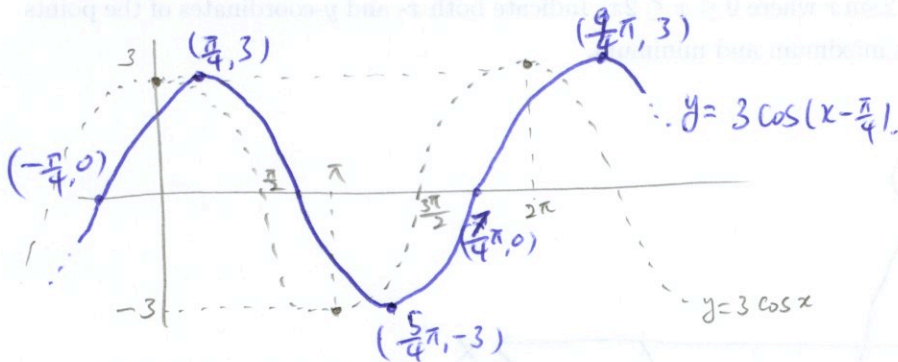$$\text{Period} = \frac{2\pi}{\left|\frac{\pi}{3}\right|} = \frac{2\pi \cdot 3}{\frac{\pi}{3} \cdot 3} = 6.$$

$$\text{Amplitude} = \frac{1}{2}$$

3. Determine the amplitude, the period, and the phase-shift of the function $y = 3\cos(x - \frac{\pi}{4})$ and graph it.
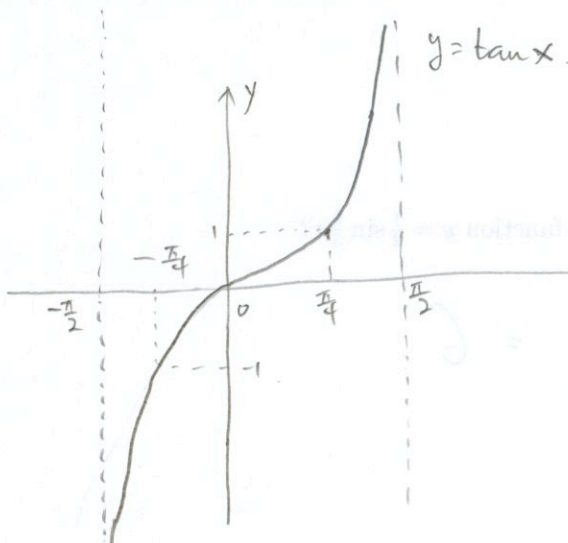
Amplitude $= 3$

Period $= 2\pi$

phase shift $= \frac{\pi}{4}$

$(\frac{\pi}{4}, 3)$

$(\frac{9}{4}\pi, 3)$

$\therefore y = 3\cos(x - \frac{\pi}{4})$

$(-\frac{\pi}{4}, 0)$

$(\frac{7}{4}\pi, 0)$

$y = 3\cos x$

$(\frac{5}{4}\pi, -3)$

4. Graph one cycle of $y = \tan x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

$y = \tan x$

5. Prove the following identity:
$$\frac{1 - \sin x}{\sin x \cot x} = \frac{\cos x}{1 + \sin x}.$$

$$LHS = \frac{1 - \sin x}{\sin x \frac{\cos x}{\sin x}} \cdot \frac{1 + \sin x}{1 + \sin x} = \frac{1 - \sin^2 x}{\cos x (1 + \sin x)} = \frac{\cos^2 x}{\cos x (1 + \sin x)} = \frac{\cos x}{1 + \sin x} \checkmark$$

6. Recall that $\sin 45° = \frac{\sqrt{2}}{2} = \cos 45°$, $\sin 30° = \frac{1}{2}$, and $\cos 30° = \frac{\sqrt{3}}{2}$. What is the exact value of $\sin 15°$?

$$\sin 15° = \sin(45°-30°) = \sin 45° \cos 30° - \cos 45° \sin 30°$$

$$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2}$$

$$= \frac{\sqrt{6} - \sqrt{2}}{4}.$$

7. Simplify the following expression:
$$\frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x}.$$

$$\frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x} = \frac{\sin 3x \cos x - \cos 3x \sin x}{\sin x \cos x}$$

$$= \frac{\sin(3x-x)}{\sin x \cos x} = \frac{2 \sin x \cos x}{\sin x \cos x} = 2.$$

8. Simplify the following expression:
$$\sqrt{\frac{9 + 9\cos 6x}{2}}.$$

$\sqrt{\frac{9 + 9\cos 6x}{2}} = 3\sqrt{\frac{1 + \cos 6x}{2}} = 3 \cdot \cos 3x$ ✓

9. Solve the equation: $\sin x - \sin \frac{x}{2} = 0$, $(0 \le x < 2\pi)$.

$\sin x - \sin \frac{x}{2} = 0$

$\Leftrightarrow$ $\sin^2 x = \sin^2 \frac{x}{2} = \frac{1 - \cos x}{2}$

Since $\sin^2 x = 1 - \cos^2 x$,

$1 - \cos^2 x = \frac{1 - \cos x}{2}$

$2 - 2\cos^2 x = 1 - \cos x$

So $2\cos^2 x - \cos x - 1 = 0$.

$(2\cos x + 1)(\cos x - 1) = 0$

$2\cos x + 1 = 0$ or $\cos x = 1$

$\Leftrightarrow \cos x = -\frac{1}{2}$ $\Leftrightarrow x = 0.$
$\Leftrightarrow x = \frac{2\pi}{3}, \frac{4\pi}{3}$

Check:

$x = \frac{2\pi}{3}$

$\sin \frac{2\pi}{3} - \sin \frac{\pi}{3} = 0.$ ✓

$x = \frac{4\pi}{3}$

$\sin \frac{4\pi}{3} - \sin \frac{2\pi}{3} = -\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} = 0$ ✓

$x = 0$ ✓

Solutions $= \left\{ 0, \frac{2\pi}{3}, \frac{4\pi}{3} \right\}$.

10. What is the value of $\sin^{-1}(\sin 3\pi)$?

$$\sin^{-1}\left(\sin 3\pi\right) = \sin^{-1}\left(0\right) = 0 \checkmark$$