**Foundation of Mathematics I**
**Course Outline**
**Course 8996051 Section 01, Spring 2020**
**Tuesdays 19:00–21:50, Room: E1-1 #116**
**Chungbuk National University**
Updated on 24 March 2020

**Instructor:** Dr. Byungdo Park

**Email:** byungdo@chungbuk.ac.kr

**Office hours:** Tuesdays 16:00–17:00 at E1-1 #110 or by appointment.

**Class webpage:** Announcements, homework, exam schedules and other relevant information will be posted on the following webpage: `https://byungdo.github.io/teaching/s2020_fom1.html` which is also accessible via instructor's webpage: `https://byungdo.github.io/`

**Textbook:**

- Patrick J. Ryan, *Euclidean and Non-Euclidean Geometry: An Analytic Approach*, 1st Edition (1989), Cambridge University Press, ISBN-13: 9780521276351.

**References:**

- I. M. Yaglom, *Felix Klein and Sophus Lie* 1st Edition (1990), Birkhäuser, ISBN-13: 9780817633165
- Lizhen Ji and Athanase Papadopoulos (Editors), *Sophus Lie and Felix Klein: The Erlangen Program and Its Impact in Mathematics and Physics* IRMA Lectures in Mathematics and Theoretical Physics (Book 23), 1st Edition (2015), European Mathematical Society, ISBN-13: 9783037191484

**References on geometry general:**

- Claire F. Adler, *Modern Geometry : an integrated first course*, 2nd Edition (1967), McGraw-Hill, ISBN-13: 9780070004214
- Marvin J. Greenberg, *Euclidean and Non-Euclidean Geometries: Development and History*, 4th st Edition (2007), W. H. Freeman, ISBN-13: 9780716799481
- Robin Hartshone, *Geometry: Euclid and Beyond* (Undergraduate Texts in Mathematics), 1st Edition (2005), Springer New York, ISBN-13: 9780387986500
- Shoshichi Kobayashi, *From Euclid geometry to modern geometry*, translated in Korean by D. Won (1999), Cheongmoongak, ISBN-10: 8970881816

**References on differential geometry:**

- Martin M. Lipschutz, *Schaum's Outline of Differential Geometry*, 1st Edition (1969), McGraw-Hill Education, ISBN-13: 9780070379855
- Barrett O'Neill, *Elementary Differential Geometry*, Revised 2nd Edition (2006), Academic Press, ISBN-13: 9780120887354
- Manfredo P. do Carmo, *Differential Geometry of Curves and Surfaces: Revised and Updated Second Edition* (Dover Books on Mathematics) Updated, Revised Edition (2016), Dover Publications, ISBN-13: 9780486806990

**Prerequisites:** Undergraduate linear algebra, for example Linear Algebra and Mathematics Education Ⅰ, II (7412068, 7412069). Those who also have taken Algebra and School Mathematics Ⅰ (7412038) will find it easier to follow. Undergraduate courses Differential Geometry I and II (7412005, 7412005) would definitely help understanding, but those are not required. A coursework in Geometry for teachers II (7412075) with a use of C. F. Adler's book would be nicely paired with the contents of this course. In particular, the instructor highly recommends those who has gotten A or higher from Fall 2019 Geometry for teachers II taking this course. The instructor does not dissuade students without meeting the prerequisite criteria registering for this course at his/her own risk.

**Course description:** This course aims to thoroughly study foundations of Euclidean, projective, and non-Euclidean geometries which provide the underpinnings of the secondary school mathematics curricula. The treatment of the material will be highly in algebraic flavor as opposed to the geometry of straightedge and compass. We shall put a special emphasis on transformation groups which will lead to an understanding of F. Klein's vision in his "Erlanger Programm."

**Course objectives:** At the end of the course students should be able to:

- Understand geometries in Euclidean 2- and 3-spaces as well as the projective 2-space and transformation groups therein.
- Understand various models of non-Euclidean geometries.
- Understand history of non-Euclidean geometry especially about its birth.
- Envision secondary school mathematics curriula from the perspective of "Erlanger Programm" of Felix Klein.

**Details on problem solving:** Problems arising in this course will be requiring proofs and calculations based on the mathematical discourse in class. Through dialogues and discussions during each lecture as well as the instructor's office hours, the instructor will guide students approaching to problems that they will have to address.

**Details on class proceeding:** The instructor will give lectures on the material following the weekly lesson plan and assign weekly homework problems. There is a term paper every student in this class has to write with the following title: **An implementation of F. Klein's "Erlanger Programm" in secondary school mathematics curricula.** It is allowed to work on this term

paper as a teamwork, co-authored paper, and the instructor encourages collaboratively discussing and working on this task. Everyone has to give an in-class presentation on term paper.

**Grading policies:** 40% from final exam (No midterm exam), 30% from term paper, 10% from term paper presentation, and 20% homework.

**Homework policies:** A list of homework problems will be posted on the class webpage roughly in weekly basis. Late homework will be accepted. The instructor will assign as many homework problems as it is needed to master the subject. The instructor will scan through each submitted homework and assign a score 2, 1, or 0 depending on quality of work. The homework score for the total grade will be calculated based on the following formula: $(\sum_{i=1}^{h} h_i \cdot n_i)/(\sum_{i=1}^{h} 2 \cdot n_i)$, where $h$ is total number of homework assignment, $h_i$ is the score for the $i^{\text{th}}$ homework score, $n_i$ is the number of problems in the $i^{\text{th}}$ homework.

**Attendance policies:** Attendance data will be collected in every class meeting and will be used for determining your final grade. In a class meeting consisting of three-consecutive class hours, there will be only one attendance call, but if you miss it, it will be recorded as a 3-hour absence. You will get a grade F if you have missed more than 25% of class meeting hours. If you have permissible reasons for your absence in accordance with the Regulation on Academic Management of the CBNU Article 52(1) (충북대학교 학사운영규정 제52조(공결승인) 제1항), you will need to contact the Department Assistant to follow the procedure for getting an approval on your absence bringing proper documentation as proof. That said, you have to fill out a form and submit it along with appropriate proofs before the absence or after seven days of the date of absence.

**Assessment of term paper/in-class presentation:** For both term paper and in-class presentation, the assessment will be done as follows: 3/3 all in all good work. 2/3 lacking important examples, theorem, proofs or there are significant mathematical errors. 1/3 overall poor contents of the material. 0/3 no hand-in.

**Assessment of learning:** The assessment will be primarily done by the abovementioned grading policy. Nonetheless, the instructor will also take into account students' devotions and efforts for this course as well as their enthusiasm as a future educator so that those qualitative elements are not going to be neglected.

**Weekly lesson plan:**

Week 1: Plane Euclidean geometry: Lines, perpendicularity, parallelism (Chapter 1). – *Assignment-based classes*

Week 2: Plane Euclidean geometry: Reflections, congruence, isometries, symmetry groups, translations, rotations (Chapter 1). – *Assignment-based classes*

Week 3: Plane Euclidean geometry: Structure of the isometry group. Affine transformations in the Euclidean plane: Affine transformations, The affine group AF(2) (Chapters 1, 2). – *Assignment-based classes*

Week 4: Affine transformations in the Euclidean plane: Fundamental theorem of affine geometry, affine reflections, shears, dilations, similarities, affine symmetries, etc (Chapter 2).

Week 5: Affine transformations in the Euclidean plane: Symmetries of a segment and an angle, barycentric coordinates, addition of angles, triangles, symmetries of a triangle (Chapter 2).

Week 6: Affine transformations in the Euclidean plane: Congruence theorem for triangles, angle sums for triangles. Finite group of isometries of Euclidean 2-space: Cyclic and dihedral groups, conjugate subgroups (Chapters 2, 3).

Week 7: Finite group of isometries of a Euclidean 2-space: Orbits and stabilizers, Leonardo's theorem, regular polygons and their similarity and symmetry, figures with no vertices (Chapter 3).

Week 8: Geometry of the sphere: Euclidean 3-space, incidence geometry of the sphere, distance and the triangle inequality, perpendicular lines, motions of the 2-sphere (Chapter 4).

Week 9: Geometry of the sphere: Orthogonal transformations of Euclidean 3-space, Euler's theorem, isometries, representation theorems, etc. (Chapter 4).

Week 10: Geometry of the sphere: Spherical trigonometry, rectilinear figures, congruence theorems, concurrence theorems (Chapter 4).

Week 11: Geometry of the sphere: Concurrence theorems for triangles, finite rotation groups, finite group of isometries of the 2-sphere. The projective 2-plane: Incidence properties of the projective 2-plane, homogeneous coordinates, two famous theorems (Chapter 4, 5).

Week 12: The projective 2-plane: Desargue's theorem in Euclidean 2-plane, the projective group, the fundamental theorem of projective geometry, projective collineation, polarities (Chapter 5).

Week 13: Distance geometry on the projective 2-space (Chapter 6).

Week 14: The hyperbolic plane: Incidence geometry in the hyperbolic plane, perpendicular lines, pencils, distance, isometries, and other topics as time permits. (Chapter 7).

Week 15: Final exam.

**Accommodating disabilities in learning and assessment:** The instructor is committed to providing access to all students. If you need accommodation in classroom or in assessment, you are encouraged to set up an appointment with the instructor at your soonest availability so that we can figure out the best way to acoommodate you. Possible accommodations include, but not limited to, provision of materials from lectures, permission to hire an assistant for taking notes, audio-recording lectures, and aid/assistant devices, extension of due dates for assignments, alternative assessment for in-class presentations, extension of exam hours, and provision of an accommodating exam locations and exam sheets.

**Academic integrity:** It is expected that you will complete all exams without giving or receiving help from anyone. The minimum penalty for giving or receiving help on an exam is a grade of 0 on

that test. Electronic devices are not allowed in any in-class exam. You may talk to other students about the homework but you must then complete the homework yourself. If your homework is identical to someone else's in the class, you will be summoned to explain your solution in front of the instructor. A failure in justifying your solution would lead score 0 to that homework. The abovementioned violation of academic integrity can be a subject of filing a report in accordance with the university policy.

**Disclaimer:** Lectures in this course will be given in English as well as all other course materials. This will include, for example, the course syllabus, most of boardwork, exam problems, homework, solutions to exams, course webpage, announcements, but not limited to those. The instructor will try to maintain a helping attitude and will accept questions which the questioner deems it more appropriate to be posed in Korean, but please remember that the most important objective of a course running in English is to make the class atmosphere as international as possible to the extent that it is not distinguishable compared to classes in English-speaking countries.

**Updates regarding the new Corona Virus (COVID-19) outbreak:** Following the decision of the university center (교무과-2459, 2020.02.12.), the first day of class has been deferred to March 16th. Also following another decision of the university (학사지원과-2288, 2020.03.05.), we shall not meet during the period March 16th to 28th and make-up those 6-contact hours by an "Assignment-based Class (과제물 활용수업)." Details below. Please stay tuned for any further updates which will be posted on the course webpage.

- **Update as of 2020-03-24:** The university center has decided not to meet in classroom for an extra week (학사지원과-2983, 2020.03.23). Accordingly, we shall not meet during the period March 29th to April 4th and continue our "Assignment-based Class" to make up an additional 3 contact hours. In addition, there will be skype meetings on Thursdays March 26th and April 2nd at 19:00–20:00. Please check your email for a web link to the skype chatroom for this class.

**Plan for Assignment-based Class (과제물 활용수업):** Here is how it works.

- Our **platform** will be the class webpage
  `https://byungdo.github.io/teaching/s2020_fom1.html`
  and you must visit this online platform (webpage) to regularly check any update.

- On the platform, there will be a section "Assignment-based Class" wherein you will be able to find weekly **assignment**. It will be of the form of downloading reading materials (if you do not have a textbook yet) from CBNU e-Campus (blackboard), learn the material while getting necessary help from the lecture note (will be available on the platform) and completing assigned homework problems in the "Homework" section on the platform.

- You must **submit** your work on each weekly assignment by (1) making it into a PDF and attaching to an email to `byungdo@g.cbnu.ac.kr` (2) dropping it off to the instructor's mailbox, or (3) bringing it in-person when we meet at the classroom.

- You will receive **feedback** on your submitted assignment.

- **Important:** Your attendance will be counted based on your submitted assignment. Not handing-in an assignment will not only affect your homework score but also your attendance record.
- If you completed the assignment but do not want to submit your homework for any reason, you must still submit an assignment on a sheet with your name and CBNU ID written on it for each class hour. For example, if you do not want to submit your homework for first two weeks, you must still submit six sheets of papers each of which contains your name, CBNU ID, and a claim of your attendance to a class hour.