Midterm Exam

Fall 2019, Geometry for teachers II

Mathematics Education, Chungbuk National University

28.10.2019 10:00–11:50

Instructions: On each page of your answer sheet, please write your name, page number, and total pages for example "홍길순 2/4면." Be sure to use your answer sheets as single-page. If you want some portion of your writings on your answer sheet not to be graded, just cross it out. You are not allowed to use your textbook or notes. You cannot use any electronic device in this exam. You are not allowed to talk to other students. Please write all details explicitly. Answers without justifications and/or calculation steps may receive no score.

1. Answer whether each of the following statements is true or false. No need to give reasons or details. **Just say true or false**. 2 points for each correct answer, 0 point for no answer, and -2 points for each incorrect answer.

(1) The axiomatic system of Euclid geometry is a perfect axiomatic system that is flawless.

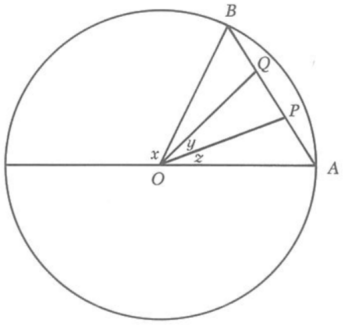
(2) If A, B, C, D are four points of the Euclidean plane, and if lines AB and CD intersect at O, the condition that $OA \cdot OB = OC \cdot OD$ is a necessary, but not a sufficient condition that the four points lie on a circle.

(3) There is no square in elliptic geometry.

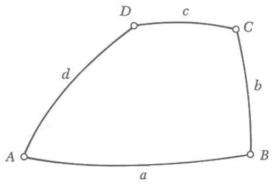
(4) The stereographic projection maps all points on the 2-sphere to a 2-dimensional plane under a one-to-one correspondence.

(5) In hyperbolic geometry, the sum of internal angles of a quadrilateral is less than 2π.

2. If P and Q are trisection points of chord AB of a circle with center O, do lines OP and OQ trisect angle $\angle AOB$? Hint: Prove $\angle AOP = \angle QOB = x$; then let $\angle POQ = y$ and show that $x = y$ leads to a contradiction. [10 points]



3. Does the figure consisting of the four points shown below satisfy the following axiomatic system? [10 points]



Axiomatic system: (1) Each pair of lines of S is on one and only one point. (2) Each point of S is on two and only two lines. (3) The total number of lines of S is four.

4. Precisely state what the parallel postulate (the fifth postulate) is. [10 points]

5. Prove by using Pasch's axiom that, if a line through a vertex A of a triangle ABC enters the triangle, it intersects the opposite side BC at a point between B and C. [10 points]

6. Show that a line *not* through O inverts into a circle through O. [10 points]

7. Prove that, in hyperbolic geometry, the summit angles of a Saccheri quadrilateral are equal and acute. (**Hint:** You may use the following theorems without proof. A triangle in the Poincaré's disk model is called an **Ω-triangle** if a triangle $AB\Omega$ has one of its vertex Ω on the boundary of the disk while two other vertices A, B are in the interior of the disk. (1) *Exterior angle theorem:* If $\triangle AB\Omega$ is an Ω-triangle, and C is an interior point of the disk such that B is between A and C, then $\angle CA\Omega > \angle AB\Omega$. (2) *$\Omega$-triangle congruency theorem:* If the finite sides and one pair of angles of two Ω-triangles are congruent, then the Ω-triangles are congruent.) 5 points for equal and 15 points for acute.

8. Complete the following table. 1 point each.

Questions	Euclidean	Hyperbolic	Single elliptic	Double elliptic
Number of parallels?	(1)	(2)	(3)	(4)
Saccheri quadrilateral summit angles?	(5)	(6)	(7)	(8)
The sum of internal angles of a $\triangle$?	(9)	(10)	(11)	(12)
Number of lines determined by two distict points?	(13)	(14)	(15)	(16)
Can a line have an infinite length (Y/N)?	(17)	(18)	(19)	(20)