

Score:

Name:

Sample Final Exam Calculus 1 - MAT 175

Instructions: This exam should be taken without text or notes or electronic devices. Show your work, and indicate answers clearly. Cross out all work that you do not want to be graded.

1. (5pts.) Compute the derivative $\frac{dy}{dx}$ for $y = \sqrt{5} + \frac{x^5}{5} - 5 \ln x$.
2. (5pts.) Compute the derivative $p'(y)$ of the function $p(y) = \frac{2y+3}{7y+5}$.
3. (5pts.) Write down an equation of the tangent line to the graph of $y = 1 + 2e^x$ at the point where $x = 0$.
4. (5pts.) Determine the slope of the tangent line to the graph of the equation $5x^2 + 3y^2 + xy = 15$ at the point $(-1, 2)$.
5. (5pts.) Compute the derivative $\frac{dw}{dz}$ of the function $w = \sqrt{1 + ze^z}$.
6. (5pts.) Find the limit:

$$\lim_{\theta \rightarrow 0} \frac{3 \sin 4\theta}{5\theta} =$$

7. (5pts.) Find the limit:

$$\lim_{x \rightarrow 1} \frac{x^2 - 4x + 3}{x^2 - 1} =$$

8. (5pts.) Find the limit:

$$\lim_{x \rightarrow \infty} \frac{6x^2 + 100}{7x^2 - 100} =$$

9. (5pts.) For which constant k is the following function $Q(x)$ continuous for all x ? Justify your answer.

$$Q(x) = \begin{cases} x^2 + k & \text{if } x \leq 0 \\ \cos x & \text{if } 0 < x \end{cases}$$

10. (10pts.) Show that the derivative of $f(x) = \frac{1}{x}$ is $f'(x) = \frac{-1}{x^2}$ by using the definition of the derivative as the limit of a difference quotient.
11. (5pts.) If the volume $V = s^3$ of an expanding cube is increasing at the constant rate of 120 cubic inches per second, how fast is the length s of the sides increasing when the volume is 8 cubic inches?
12. (5pts.) Find where the graph of $y = x^3 - 6x^2$ is concave up and concave down, and find all inflection points.
13. (5pts.) Find and classify all the relative extrema of $F(x) = x^4 - 4x^2 + 2$.
14. (8pts.) Find the absolute maximum and minimum values of $f(x) = 4x^3 - 3x^2$ on the closed interval $[-1, 1]$.
15. (8 pts.) A particle moves along the x-axis with an acceleration given by where t is measured in seconds and s (position) is measured in meters. If the initial position is given by $s(0) = 3$ and the initial velocity is given by $v(0) = 4$ then find the position of the particle at t seconds.
16. (4pts.) Find the area under the curve $y = 5 - 2x^2$ from $x = 0$ to $x = 1$.
17. (4pts.) Evaluate the derivative $F'(x)$ of the function $F(x)$ defined by:

$$F(x) = \int_0^x \frac{1}{\sqrt{1+t^3}} dt$$

at $x = 2$.

18. (6pts.) SOLVE **ONLY ONE** OF THE FOLLOWING TWO PROBLEMS:

- (a) A rectangular poster is to have an area of 100 square inches with a 1-inch margin on the right and left sides and a 2-inch margin at the top and bottom. Find the dimensions of the poster with the largest printed area.
- (b) Evaluate $\int x \sin(x^2) dx$