교사를 위한 기하학 II 중간시험 모범답안 Fall 2019

#1. (1) F Hilbert's axiomatic system was developed as an attempt of making Euclid geometry perfect

(2) F Suppose $OA \cdot OB = OC \cdot OD$, assume that A, B, C, D are not on the same circle. Since A, B, C determines a circle passing through these points, we may assume that either CD or its extension meets at D' the aforementioned circle. Since $OA \cdot OB = OC \cdot OD'$, we get $OC \cdot OD' = OC \cdot OD$, and hence $OD = OD'$.

(3) T It follows from the fact that the sum of internal angles of an elliptic quadrilateral is $> 2\pi$.

(4) F There is no point in a 2-dimensional plane corresponding to the north pole of a 2-sphere.

(5) T It follows from the fact that the sum of internal angles of a hyperbolic triangle is less than π.

#2. No.

Proof: Suppose $\angle BOQ = \angle QOP = \angle POA$.

Since $BQ = QP = PA$, consider the following proposition:

Prop: Given $\triangle ABC$ with a bisector of $\angle A$ meeting BC at D. Then $AB : AC = BD : DC$.

Accordingly in the given triangle $\triangle OPB$, $OP : OB = PQ : QB = 1 : 1$

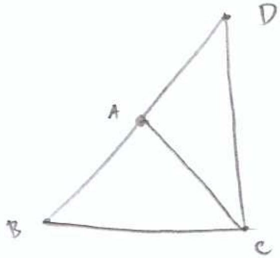
Hence $OP = OB$, whereas OP is strictly less than OB (the radius)

Hence a contradiction.

#3. No, lines c and a are not on one and only one point. The same for d and b.

#4. If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.

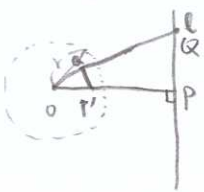
#5. Consider the following triangle:



Suppose that the line through A enters $\triangle ABC$ but does not intersect BC. By pasch's axiom, the line passing through A between B and D of $\triangle DBC$ has to pass through CD at a point between C and D, if it does not pass through C.

But this contradicts to the assumption that the line enters $\triangle ABC$. $\square$.

#6. Let $(O)_r$ be the circle of inversion and ℓ a line not through O.



$$OP' \cdot OP = r^2 = OQ' \cdot OQ$$

$$\Leftrightarrow \quad OP : OQ = OQ' : OP'$$

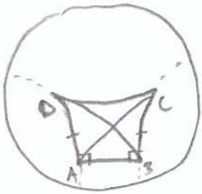
$$\Leftrightarrow \quad \triangle OP'Q' \cong \triangle OQP$$

Hence $\angle Q' = \angle P = \frac{\pi}{2}$.

by the circumference angle theorem, the inversion of ℓ is a circle whose diameter is OP'. $\square$

#7. We first show that the two summit angles are equal.

Consider the following quadrilateral and draw diagonals

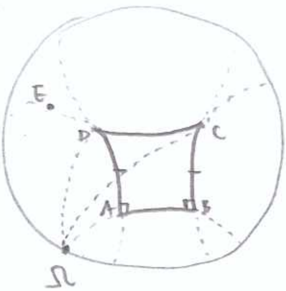


DB and AC. Since $\Delta DAB \cong \Delta CBA$, $\angle ADB = \angle BCA$.

Also from $\Delta ACD \cong \Delta BDC$, $\angle BDC = \angle ACD$. $\therefore \angle D = \angle C$.

Now we show that both $\angle C$, $\angle D$ are acute.

Consider the following quadrilateral, and take E on the extension of DC so that D lies between E and C. Take Ω so that it is on the boundary of the disc meeting the extension of AB so that A is between Ω and B.



By the exterior angle theorem, $\angle ED\Omega > \angle EC\Omega$

By the Ω-triangle congruency theorem, $\angle AD\Omega = \angle BC\Omega$

Adding these two, we get $\angle EDA > \angle BCD$

From $\pi = \angle EDA + \angle ADC > \angle BCD + \angle ADC$

$= 2\angle ADC$

$\uparrow$ from equality earlier in the proof

we see that $\angle ADC < \frac{\pi}{2}$. $\square$

#8

	Euclidean	Hyperbolic	Single elliptic	Double elliptic
# of parallels	(1) 1	(2) infinitely many	(3) 0	(4) 0
Saccheri Quadrilateral Summit angles	(5) $\frac{\pi}{2}$	(6) acute	(7) obtuse	(8) obtuse
Sum of internal angles of a Δ	(9) π	(10) $< \pi$	(11) $> \pi$	(12) $> \pi$
Number of lines determined by two distinct points.	(13) 1	(14) 1	(15) More than 1	(16) More than 1
Can a line have an infinite length?	(17) Yes	(18) Yes	(19) No	(20) NO