Final Exam

Fall 2019, Geometry for teachers II

Mathematics Education, Chungbuk National University

16.12.2019 10:00–11:50

Instructions: On each page of your answer sheet, please write your name, page number, and total pages for example “홍길순 2/4면.” Be sure to use your answer sheets as single-page. If you want some portion of your writings on your answer sheet not to be graded, just cross it out. You are not allowed to use your textbook or notes. You cannot use any electronic device in this exam. You are not allowed to talk to other students. Please write all details explicitly. Answers without justifications and/or calculation steps may receive no score. You may use the following set of axioms of projective geometry.

◎ Incidence axioms

Axiom 1. If A and B are distinct points, there is at least one line on both A and B.

Axiom 2. If A and B are distinct points, there is not more than one line on both A and B.

Axiom 3. If A, B, C are points that are not all on the same line and if D, E are distinct points such that B, C, D are on a line and C, A, E are also on a line, there is a point F such that points A, B, F are on a line and points D, E, F are also on a line.

◎ Existence axioms

Axiom 4. There exists at least one line.

Axiom 5. There are at least three distinct points on every line.

Axiom 6. Not all points are on the same line.

Axiom 7. Not all points are on the same plane.

Axiom 8. If S is a 3-space (i.e. the totality of points on lines joining a point P not in the plane ABC to points of this plane), every point is on S.

◎ The quadrangle axiom

Axiom 9. The diagonal points of a quadrangle are not collinear.

◎ Separation axioms

Axiom 10. The pairs A, B and C, D of a harmonic set of points $H(AB, CD)$ separate each other.

Axiom 11. If the pairs A, B and D_1, C separate each other and if also the pairs A, D_1 and D_2, B separate each other, then the pairs A, B and C, D_2 separate each other.

Axiom 12. If the pairs A, B and C, D separate each other, then A, B, C, D are distinct points.

◎ A continuity axiom

Axiom 13. There exists a projective line L containing a set of points isomorphic with the set of numbers of the extended real number system.

1. Answer whether each of the following statements is true or false. No need to give reasons or details. **Just say true or false**. 2 points for each correct answer, 0 point for no answer, and -2 points for each incorrect answer.

(1) Under an inversion with respect to a given circle, any circle gets inverted to a circle.

(2) The single elliptic geometry abandons the separation principle and two straight lines meet at one point.

(3) Straightedge and compass constructions in projective geometry have no need of compass because projective geometry is solely based on metric properties.

(4) A sufficient condition for a projective line to contain at least four points is the **Axiom 9.**

(5) One can prove the Desargues' theorem (See Problem 3 below) in projective geometry by using **Axiom 1** to **Axiom 6** only.

2. Prove that the medians of a triangle are concurrent. [10 points]

3. Prove the Desargues' theorem: If two triangles $A_1B_1C_1$ and $A_2B_2C_2$ are so situated that the three lines A_1A_2, B_1B_2, C_1C_2 joining corresponding vertices are concurrent, the pairs of corresponding sides intersect in three collinear points. [20 points]

4. Show that, in the harmonic set of points $H(AB, CD)$ the lengths of the line segments AC, AB, AD form a harmonic progression. [10 points]

5. Show that the absolute value of the cross ratio of four points in the Euclidean plane is invariant on projection. [10 points]

6. Prove that the following statements in projective geometry formulated using **Axioms 1** to **13** in the above:

(1) A line L in the plane π always intersects its generating line m. [10 points]

(2) Two lines a, b of plane π intersect if either one of them passes through the generating point C of the plane. [10 points]

7. If points C, D are harmonic conjugates with respect to points A, B, then points A, B are harmonic conjugates with respect to points C, D.[10 points]

8. When three points A, B, C of a harmonic set of points are given, do the straightedge construction of the fourth point D, the harmonic conjugate of C with respect to A and B. [10 points]