Final Exam
Spring 2014, MAT155 Section 04LB[51293]
May 22nd, 2014. 11:00AM--12:40PM.

Your Name:

Instructions: You can use only MAPLE program and a web browser(only for the purpose of submitting your exam solution to the instructor). You may not use any other programs other than these two. You can look up your MAPLE source files, but otherwise this exam is closed-book, closed-note, and you may not use any electronic device in this exam except your PC. You are not allowed to talk to other students. Type all details explicitly. All solutions should be obtained by using MAPLE codes.

Problem 1. (10 points) This problem is about using loops in MAPLE. Define a function f(x)=(sin(x))/x and evaluate it(use evalf command) when x=10^k where k is an integer among -1,...,-10. Compare this result with the result when the limit of f(x) as x goes to 0 is calculated using MAPLE.

```
> f:=x->(sin(x))/x;
```

$$f := x \rightarrow \frac{\sin(x)}{x} \tag{1}$$

```
> for k from 1 to 10 do evalf(f(10^(-k))) od;
```

$$\begin{gathered} 0.9983341665 \\ 0.9999833334 \\ 0.9999998333 \\ 0.9999999983 \\ 1.000000000 \\ 1.000000000 \\ 1.000000000 \\ 1.000000000 \\ 1.000000000 \\ 1.000000000 \end{gathered} \tag{2}$$

```
> limit(f(x),x=0);
```

$$1 \tag{3}$$

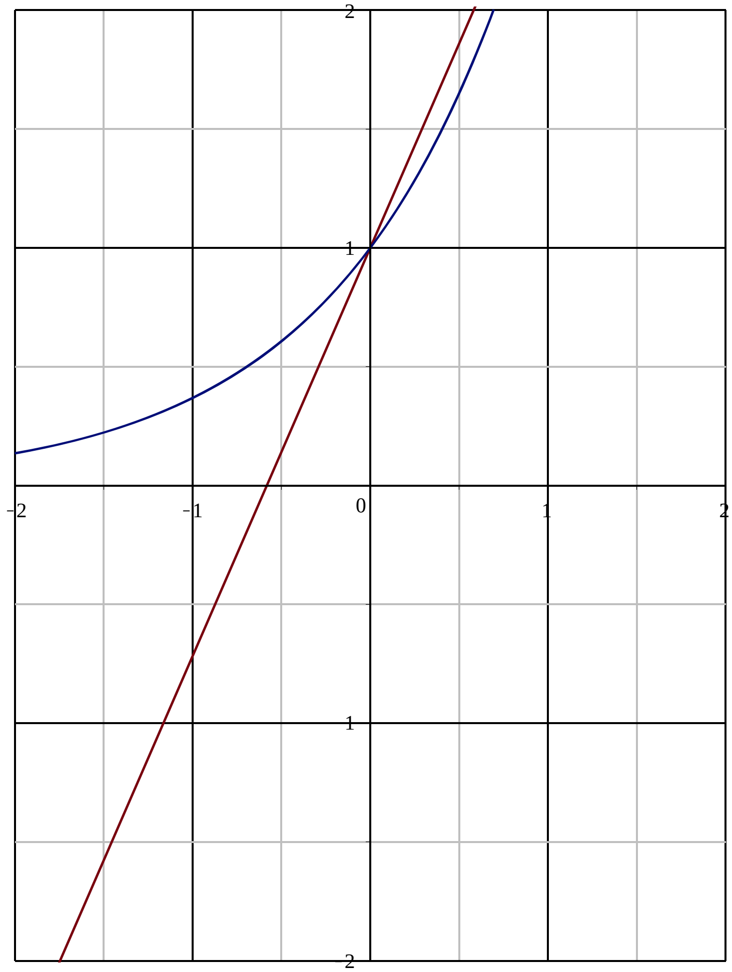
Problem 2. (10 points) Using loop, plot five graphs of secant line to f(x)=exp(x) passing through (0,1) and (1/10^j,exp(1/10^j)), where j=1,2,3,4,5. In the same plot, include the graph of exp(x) as well. On the top of the graph, show the slope of the secant line.

```
> f:=x->exp(x);
```

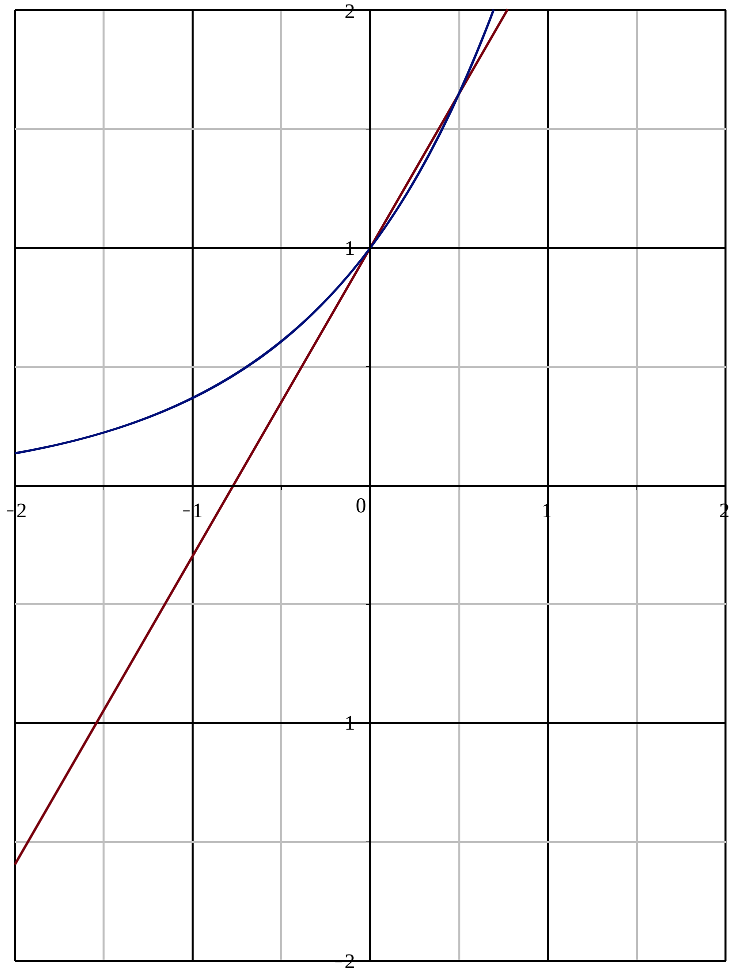
$$f := x \rightarrow \mathrm{e}^{x} \tag{4}$$

```
> for j to 5 do m := evalf((f(1/j)-f(0))/(1/j+0)); plot({m*x+1, f
  (x)}, x = -2 .. 2, y = -2 .. 2) od;
```

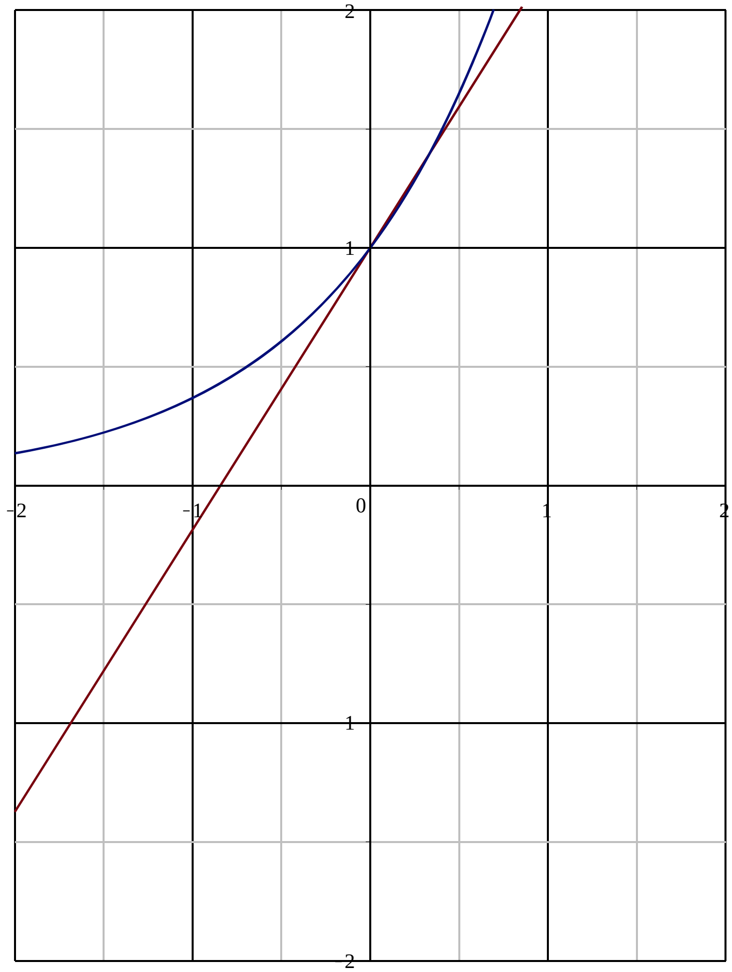
$$m := 1.718281828$$



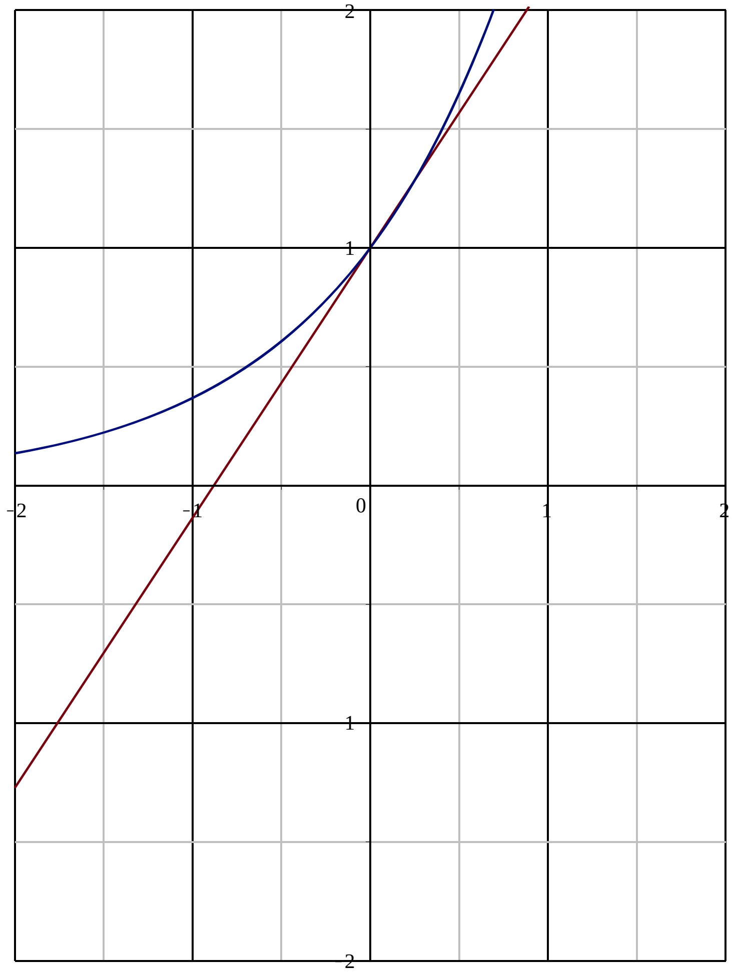
$m := 1.297442542$



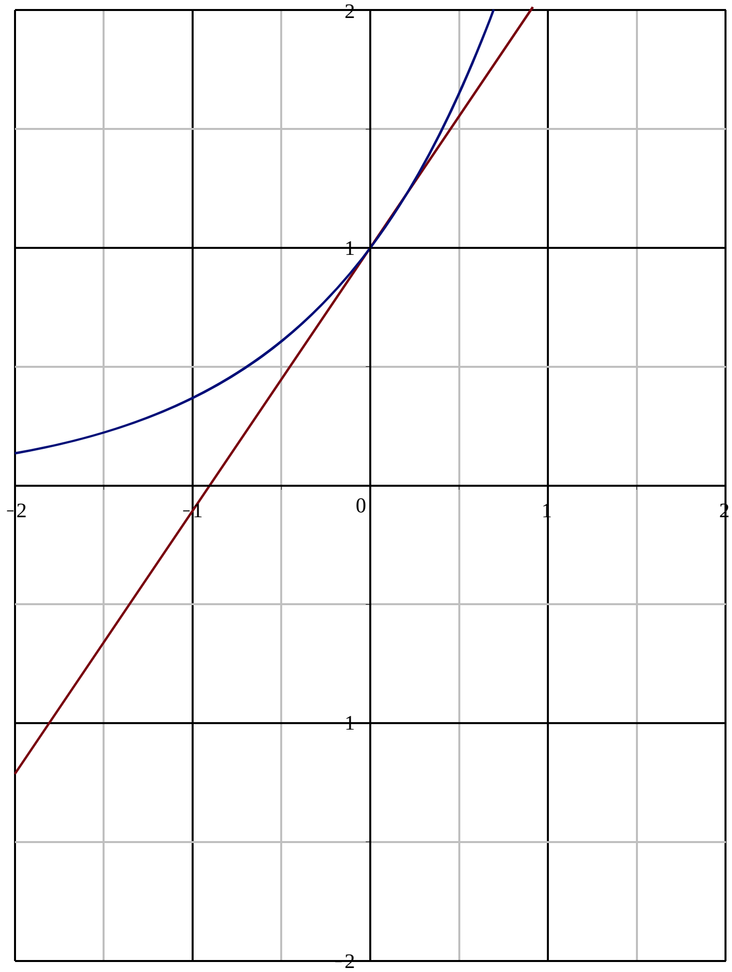
$m := 1.186837275$



$m := 1.136101668$



$m := 1.107013790$



Problem 3. (20 points, 5 points each) Suppose a particle is moving in a stright line satisfying a position function p(t)=-t^2-9t+3.

(1) Find the velocity and acceleration.

```
> p:=t->-t^2+9*t+3;
```

$$p := t \rightarrow -t^2 + 9\,t + 3 \tag{5}$$

```
> D(p)(t);
```

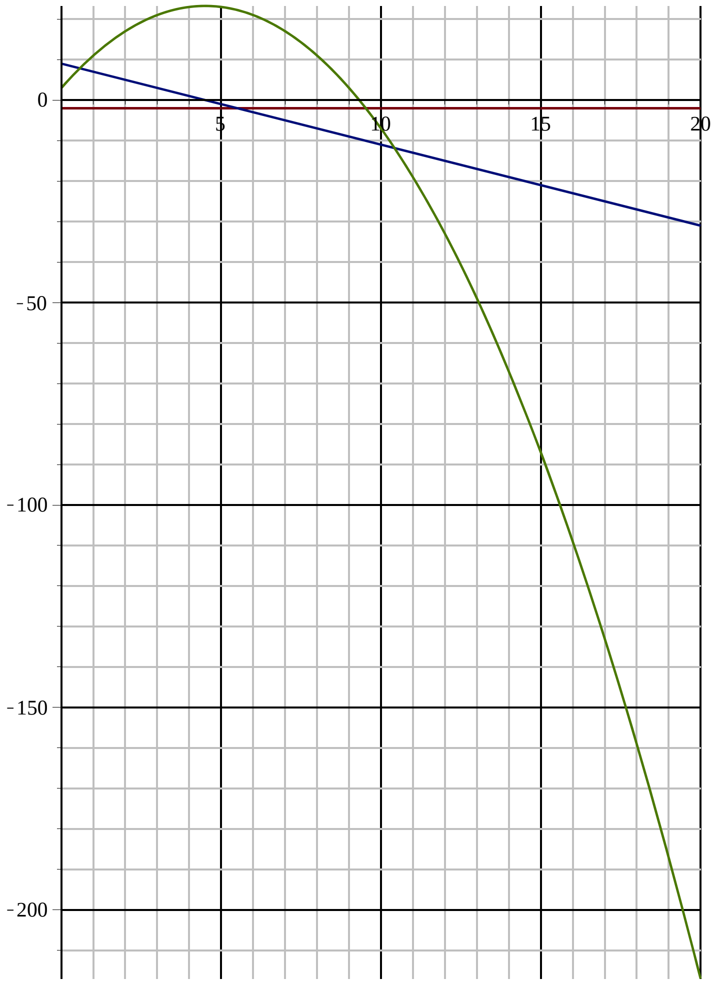
$$-2\,t - 9 \tag{6}$$

```
> D(-2*t-9)(t);
```

$$-2\,\mathrm{D}(t)\,(t) \tag{7}$$

(2) Draw the position-function graph, velocity-function graph, and acceleration-function graph.

```
> plot({p(t),D(p)(t),-2},t=0..20);
```



(3) When does the particle stops?

```
> solve(D(p)(t)=0,t);
```

$$\frac{9}{2} \qquad (8)$$

(4) When does the particle comes back to its original position?

```
> fsolve(p(t)=0,t=8..10);
```

$$9.321825380 \tag{9}$$

Problem 4. (10 points) Write a brief description about what the idea of the Newton's method is.

```
> The Newton's method is an algorithm to find a root of given real-
  valued differentiable function on reals. One first take a point
  x=x_0 and consider the tangent line to the graph of the given
  function. That tangent line crosses x-axis which we call x_1. Now
  consider the tangent line to the graph of the same function, and
  take the new x-intercept of this tangent line. One can repeat
  this process ad infinitum to approximate a zero of the given
  function.
```

Problem 5. (10 points) For f(x)=sin(x) starting at x=4, by using Newton's method to find the value of Pi up to 8 decimal places.

```
> a:=4; err:=1; f:=x->sin(x);
  for j from 1 by 1 to 30 while (err>0.000000001) do j;crossa:=a-f
  (a)/(D(f)(a)); err:=evalf(abs(crossa-a)); a:=evalf(crossa); od;
```

$$4 \tag{10}$$

$$1$$

$$x \rightarrow \sin(x)$$

$$1$$

$$4 - \frac{\sin(4)}{\cos(4)}$$

$$1.157821282$$

$$2.842178718$$

$$2$$

$$3.150872940$$

$$0.308694222$$

$$3.150872940$$

$$3$$

$$3.141592387$$

$$0.009280553$$

$$3.141592387$$

$$4$$

$$3.141592654$$

$$2.67\ 10^{-7}$$

$$3.141592654$$

$$5$$

$$3.141592654$$

$$0.$$

$$3.141592654$$

```
> The approximated value of Pi is 3.14159265.
```

Problem 6. (15 points, 5 points each) Consider the function f(x)=x^3-50*x + 1

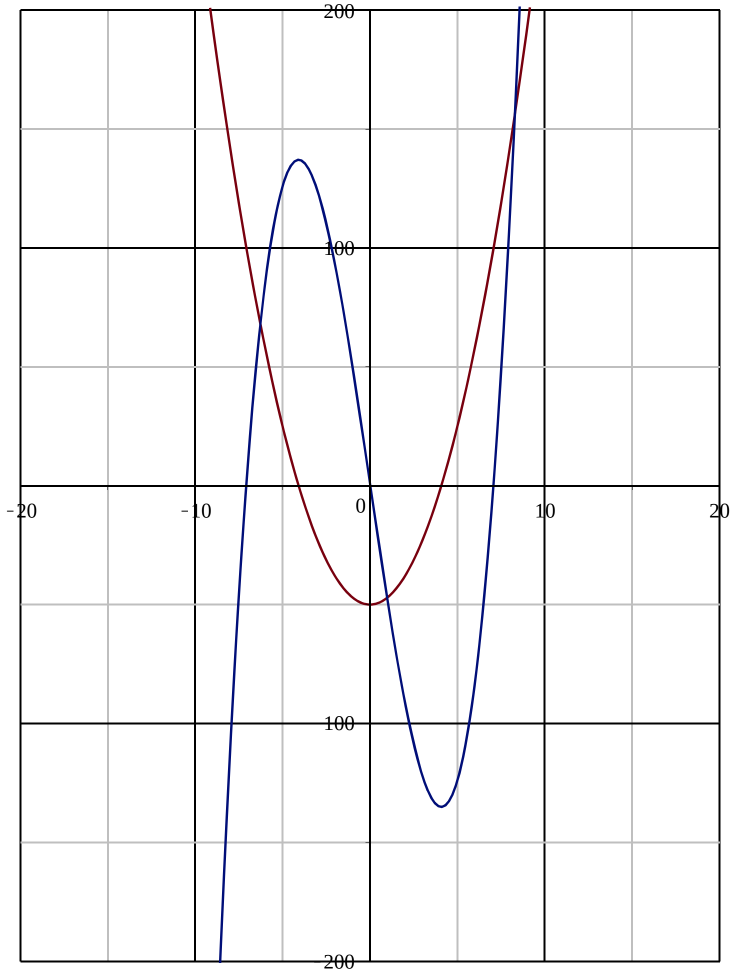
(1) Plot the graph of f(x) and its derivative in the same graph. Make sure that the graph shows all intersections with x-axis.

(2) Using MAPLE, find all critical numbers of f(x). Find all relative extrema.
(3) Find the second derivative of f(x). Using MAPLE, find all inflection points. (Note that an inflection point is a pair of x- and y-coordinate.)

```
> f:=x -> x^3-50*x + 1; D(f); plot({f(x),D(f)(x)}, x=-20..20, y=
  -200..200);
```

$$f := x \rightarrow x^3 - 50\,x + 1$$

$$x \rightarrow 3\,x^2 - 50$$




```
> fsolve(D(f)(x)=0,x);
```

$$-4.082482905, 4.082482905 \tag{11}$$

```
> f(-4.082482905); f(4.082482905);
```

$$137.0827634 \tag{12}$$

$$-135.0827634$$

```
> D(D(f));
```

$$x \rightarrow 6\,x \tag{13}$$

```
> fsolve(6*x=0);
```

$$0. \tag{14}$$

```
> f(0);
```

$$1 \tag{15}$$

Problem 7. (25 points. 5 points each) Let g(x)=1/x^2. Consider the interval [1,2]. Run the student package using "with(student):"

(1) Show graphically the left hand sum, right hand sum for 10 subintervals. Give it a name.

(2) Using "with(plots):", display the above three pictures in a single graph.

(3) Give a numerical value(in decimal expression) for left hand sum and right hand sum for 10 subintervals.

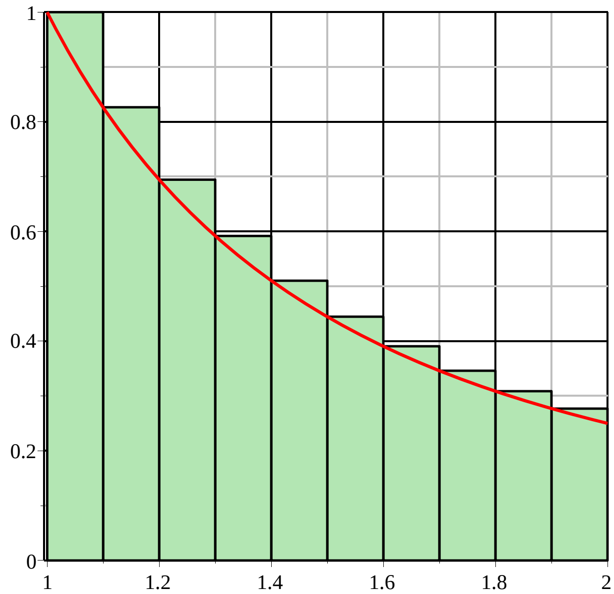
(4) Using MAPLE, calculate the integral of g(x) from 1 to 2. Explain, for the result from (3), which one is overestimating and underestimating.

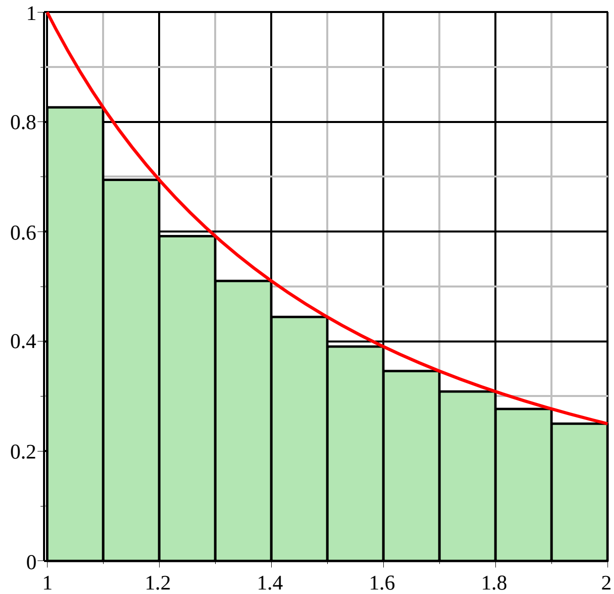
(5) Using a loop, write commands that calculate the left-hand sums and right-hand sums with 5, 10, 20, 40, 80, 160, 320, 640, 1280, 2560 subintervals.

```
> with(student): g:=x->1/x^2;
```

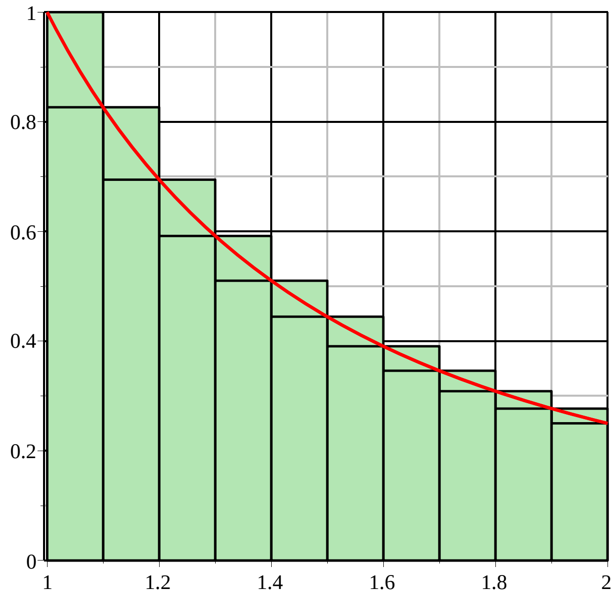
$$x \rightarrow \frac{1}{x^2} \tag{16}$$

```
> leftbox(g(x),x=1..2,10); rightbox(g(x),x=1..2,10);
```


1
0.8
0.6
0.4
0.2
0
1
1.2
1.4
1.6
1.8
2




```
> lhs10:=leftbox(g(x),x=1..2,10): rhs10:=rightbox(g(x),x=1..2,10):
> with(plots): display(rhs10,lhs10);
```




```
> evalf(leftsum(g(x),x=1..2,10)); evalf(rightsum(g(x),x=1..2,10));
```

0.5389551275 **(17)**
0.4639551275

```
> evalf(Int(g(x),x=1..2));
```

0.5000000000 **(18)**

> Hence the left hand sum is overestimating, and the rightsum is underestimating.

```
> for k from 0 to 9 do N := 5*2^k; evalf(leftsum(g(x),x=1..2,N));
  evalf(rightsum(g(x),x=1..2,N)) od;
```

5 **(19)**
0.5807831002
0.4307831002
10
0.5389551275
0.4639551275
20
0.5191143820
0.4816143819
40

0.5094661332
0.4907161332
80
0.5047102856
0.4953352856
160
0.5023494466
0.4976619466
320
0.5011732991
0.4988295491
640
0.5005862936
0.4994144186
1280
0.5002930577
0.4997071202
2560
0.5001465066
0.4998535379