IE 303 Modelling and Methods in Optimization
Fall 2022

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1 December 2022

CD Collection Sell Out

An internet site allows you to sell your classical music CD collection online. Your collection has some CDs. The selling procedure works as follows: every potential buyer sends online his/her valuations. Having collected the valuations matrix, the site decides which customer gets which CD, and what price he/she has to pay. The rule is that the CDs being rare collection items, every customer can only get one CD. The site is trying to maximize its total revenue.

1 LP Modeling

In first stage of the modeling, decision variables and parameters are decided. To define the model's main purpose and approach, some simple remarks should be made. For the working principle, LP model assigns a price to each CD with respect to some restrictions. The model is using one decision variable for that price assignment. In addition to that, to control every assignment's compatibility with the restrictions, another binary decision variable which matches CD's and assigned customers should be made. In notation form these decision variables can be shown as;

$$P_j = \text{ The price assigned to j'th CD} \tag{1}$$

$$X_{ij} = \begin{cases} 1, & \text{if the i'th customer purchases j'th CD.} \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

For the parameters, the model should have a matrix shaped parameter to store the information about the evaluations made by people for the CD's. By using these data sets, the model will define which customer to buy which CD at what price. In notation form these parameters can be shown as;

$$V_{ij} = \text{ The evaluation made by i'th customer for the j'th CD} \tag{3}$$

Secondly, objective function should be defined. By assuming that every CD is going to be sold and we have m customer, it can be demonstrated as;

$$\text{Maximize: } \sum_{j=1}^{m} X_{ij} \tag{4}$$

For the last part, restrictions should be decided. To set the balance between customer and CD there are two matching restrictions should be made;

$$\sum_{i=1}^{n} X_{ij} = 1 \quad \text{for all j = 1,2...m (every CD must be assigned)} \tag{5}$$

$$\sum_{j=1}^{m} X_{ij} \leq 1 \quad \text{for all i = 1,2...n (every customer can buy 1 CD at most)} \tag{6}$$

In addition, 2 more restrictions should be included to assign customer and product fairly and avoid infeasible prices to customers;

$$V_{ij} - P_j - 40X_{ij} \geq -40 + V_{il} - P_l \quad \text{for all i = 1,2...n j,l = 1,2...m , } l \neq j \tag{7}$$

$$V_{ij} - P_j - 20X_{ij} \geq -20 \quad \text{for all i = 1,2...n j = 1,2...m} \tag{8}$$

By defining small m, I used the interval -[1,20]- for the customer evaluations and subsequently the prices. Therefore, I arbitrarily choose -40 as a lower bound for the fairness constraint and -20 for the feasibility constraint.

2 Demonstrations

Remark: In all of these demonstrations, V_{ij} values are distributed uniformly $U = (1, 20)$ and randomly. While constructing the V matrix, a random matrix generator is used to provide randomness. Different V matrices are used for every demonstrations. V matrices can be shown as;

V Matrix for N=20 M=10

16	4	5	17	11	15	19	15	5	8
20	7	7	11	18	19	8	12	12	18
12	19	15	18	2	20	9	9	13	3
6	4	2	20	7	7	5	12	20	18
14	18	6	3	2	10	1	13	6	19
16	13	16	16	2	6	7	16	3	10
20	5	4	6	19	10	13	19	7	9
19	14	4	20	1	9	14	19	15	20
16	3	14	2	11	15	12	5	16	10
2	12	7	15	8	18	20	11	8	6
14	4	12	1	19	19	18	15	4	14
15	16	3	2	6	19	9	2	5	9
3	2	18	14	12	10	8	9	3	11
11	10	6	15	12	9	3	4	4	11
5	8	6	12	10	12	20	7	8	9
18	11	20	10	11	2	6	6	12	3
10	9	13	10	4	8	2	7	11	12
9	10	17	4	12	15	2	14	16	8
19	3	3	20	10	10	11	5	20	18
15	4	18	20	17	11	6	19	14	20

V Matrix for N=20 M=15

20	19	7	19	6	8	1	11	6	13	19	13	10	8	20
10	8	18	19	19	1	20	13	19	5	10	16	5	12	1
10	9	19	19	8	8	11	17	19	7	11	20	6	8	7
15	10	11	12	20	12	13	8	15	5	3	7	9	17	18
6	1	13	9	2	20	20	8	7	11	1	5	15	17	8
16	1	10	3	5	2	11	10	2	12	16	17	5	2	19
7	14	6	12	20	13	18	10	6	15	20	14	4	2	18
18	11	11	10	12	20	10	8	9	16	9	10	8	17	17
6	5	11	1	20	20	3	7	7	11	4	19	16	1	20
1	4	1	14	6	3	15	5	8	10	12	8	8	16	7
20	1	11	6	8	3	20	19	19	7	14	14	14	5	18
8	9	14	5	16	10	17	12	2	20	1	9	19	12	13
15	3	9	19	17	9	10	3	18	2	18	15	2	4	18
13	18	12	4	13	16	12	9	17	1	15	11	4	9	8
12	4	20	5	8	18	20	20	19	18	5	13	8	16	7
1	6	10	20	18	4	11	7	18	11	3	16	12	11	9
15	7	14	20	14	14	16	7	12	13	13	16	20	12	4
10	16	14	1	11	16	7	5	5	1	5	20	20	10	19
5	14	16	20	20	12	8	2	16	17	4	1	19	17	13
12	6	13	13	13	12	19	4	13	15	11	16	6	5	14

V Matrix for N=20 M=19

2	18	4	4	20	20	14	12	9	16	7	9	11	13	15	5	10	14	8
7	2	8	11	20	14	18	20	1	9	18	11	9	13	7	15	4	4	18
17	7	17	16	15	2	18	6	5	7	6	5	17	19	11	11	5	16	16
14	9	18	12	5	3	12	4	10	2	4	15	2	6	3	10	15	2	19
7	11	3	3	2	10	20	19	2	18	5	11	7	4	8	19	8	8	8
10	13	7	14	10	2	7	13	5	4	9	19	10	9	13	9	11	6	8
2	8	19	20	20	16	15	20	5	1	3	4	17	7	8	19	5	15	12
9	17	7	6	19	14	1	1	2	9	3	13	11	14	7	14	10	14	18
3	16	12	10	15	5	9	11	15	18	13	15	18	8	10	4	19	14	1
10	17	19	8	1	17	12	17	8	12	10	18	20	12	7	2	18	10	9
11	10	13	12	11	5	16	2	19	18	19	15	1	3	18	11	19	16	16
9	15	11	18	6	15	11	9	16	11	8	13	8	2	14	10	15	6	9
10	14	10	18	11	5	12	18	6	1	11	13	8	9	17	2	18	18	11
3	20	20	2	1	5	11	10	19	12	16	4	3	9	1	15	6	6	4
12	3	3	17	8	2	1	8	17	5	6	10	6	4	13	6	19	17	14
7	14	8	12	1	2	19	3	4	5	13	16	12	16	10	5	16	19	1
11	9	13	1	8	2	8	5	17	8	6	20	15	6	17	10	18	5	6
12	5	3	13	4	15	20	9	2	14	20	17	17	8	17	2	2	11	20
9	16	12	8	5	17	11	9	7	19	15	20	16	11	10	1	1	18	11
15	15	12	13	17	8	4	18	6	3	20	8	18	13	11	9	18	3	15

```
model ModelName
  uses "mmxprs", "mmnl"
  declarations
    numcustomerset = 1..20
    numcdset = 1..10
    V: array(numcustomerset,numcdset) of integer
    P: array(numcdset) of mpvar
    X: array(numcustomerset,numcdset) of mpvar
  end-declarations
  forall(j in numcdset) create(P(j))
  forall(j in numcdset) P(j) is_integer
  forall(i in numcustomerset,j in numcdset) create(X(i,j))
  forall(i in numcustomerset,j in numcdset) X(i,j) is_binary
  !objective (max)
  Revenue:= sum(j in numcdset) P(j)
  !constraints
  !every cd being assigned to somebody
  forall(j in numcdset)sum(i in numcustomerset) X(i,j) = 1
  !every customer can buy at most 1 cd
  forall(i in numcustomerset)sum(j in numcdset) X(i,j) <= 1
  !assignment constraint with respect to cd's advantage to customer
  forall(i in numcustomerset,j in numcdset, l in numcdset|l<>j) V(i,j)-P(j)-40*X(i,j) >= -40 + V(i,l)-P(l)
  !customers do not pay more than their evaluation
  forall(i in numcustomerset,j in numcdset) V(i,j)-P(j)-20*X(i,j) >= -20
  maximize(Revenue)
  writeln("Objective value is ", getobjval)
  forall(i in numcustomerset,j in numcdset)writeln(" X(",i,j,") = ",getsol(X(i,j)))
  forall(j in numcdset)writeln(" P(",j,") = ",getsol(P(j)))
end-model
```

Figure 1: General XPress-MP model

```
model ModelName
  uses "mmxprs", "mmnl"
  declarations
    numcustomerset = 1..20
    numcdset = 1..10
    V: array(numcustomerset,numcdset) of integer
    P: array(numcdset) of mpvar
    X: array(numcustomerset,numcdset) of mpvar
  end-declarations
  forall(j in numcdset) create(P(j))
  forall(j in numcdset) P(j) is_integer
  forall(i in numcustomerset,j in numcdset) create(X(i,j))
  forall(i in numcustomerset,j in numcdset) X(i,j) is_binary
    V:: [16,4,5,17,11,15,19,15,5,8,    20,7,7,11,18,19,8,12,12,18,    12,19,15,18,2,20,9,9,13,3,    6,4,2,20,7,7,5,12,20,18,    14,18,6,3,2,10,1,13,6,19,
    16,13,16,16,2,6,7,16,3,10,    20,5,4,6,19,10,13,19,7,9,    19,14,4,20,1,9,14,19,15,20,    16,3,14,2,11,15,12,5,16,10,    2,12,7,15,8,18,20,11,8,6,
    14,4,12,1,19,19,18,15,4,14,    15,16,3,2,6,19,9,2,5,9,    3,2,18,14,12,10,8,9,3,11,    11,10,6,15,12,9,3,4,4,11,    5,8,6,12,10,12,20,7,8,9,
    18,11,20,10,11,2,6,6,12,3,    10,9,13,10,4,8,2,7,11,12,    9,10,17,4,12,15,2,14,16,8,    19,3,3,20,10,10,11,5,20,18,    15,4,18,20,17,11,6,19,14,20]
  !objective (max)
  Revenue:= sum(j in numcdset) P(j)
  !constraints
  !every cd being assigned to somebody
  forall(j in numcdset)sum(i in numcustomerset) X(i,j) = 1
  !every customer can buy at most 1 cd
  forall(i in numcustomerset)sum(j in numcdset) X(i,j) <= 1
  !assignment constraint with respect to cd's advantage to customer
  forall(i in numcustomerset,j in numcdset, l in numcdset|l<>j) V(i,j)-P(j)-40*X(i,j) >= -40 + V(i,l)-P(l)
  !customers do not pay more than their evaluation
  forall(i in numcustomerset,j in numcdset) V(i,j)-P(j)-20*X(i,j) >= -20
  maximize(Revenue)
  writeln("Objective value is ", getobjval)
  forall(i in numcustomerset,j in numcdset)writeln(" X(",i,j,") = ",getsol(X(i,j)))
  forall(j in numcdset)writeln(" P(",j,") = ",getsol(P(j)))
end-model
```

Figure 2: XPress-MP model with V matrix

2.1 Scenario 1

In this scenario, n = 20, m = 10. In the model, variables n and m are adjusted accordingly. The results are;

```
FICO Xpress Mosel 64-bit v6.0.1, FICO Xpress v8.14.2
(c) Copyright Fair Isaac Corporation 2001-2022. All rights reserved
Compiling IE-303PROJECT2.mos to out\IE-303PROJECT2.bim with -g
Running model
Objective value is 195
 X(36) = 1
 X(44) = 1
 X(52) = 1
 X(71) = 1
 X(810) = 1
 X(115) = 1
 X(157) = 1
 X(163) = 1
 X(199) = 1
 X(208) = 1
 P(1) = 20
 P(2) = 18
 P(3) = 20
 P(4) = 20
 P(5) = 19
 P(6) = 19
 P(7) = 20
 P(8) = 19
 P(9) = 20
 P(10) = 20

Process exited with code: 0
```

Figure 3: Optimal Solution for n=20 m=10

2.2 Scenario 2

In this scenario, n = 20, m = 15. In the model, variables n and m are adjusted accordingly and solution set is gathered via Xpress-mosel. The results are;

```
Fri Dec 02 2022 11:28:58 GMT+0300 (GMT+03:00)
FICO Xpress Mosel 64-bit v6.0.1, FICO Xpress v8.14.2
(c) Copyright Fair Isaac Corporation 2001-2022. All rights reserved
Compiling IE-303PROJECT2.mos to out\IE-303PROJECT2.bim with -g
Running model
Objective value is 293
 X(12) = 1
 X(29) = 1
 X(33) = 1
 X(414) = 1
 X(57) = 1
 X(711) = 1
 X(86) = 1
 X(915) = 1
 X(111) = 1
 X(1210) = 1
 X(158) = 1
 X(164) = 1
 X(1713) = 1
 X(1812) = 1
 X(195) = 1
 P(1) = 20
 P(2) = 19
 P(3) = 19
 P(4) = 20
 P(5) = 20
 P(6) = 20
 P(7) = 20
 P(8) = 19
 P(9) = 19
 P(10) = 20
 P(11) = 20
 P(12) = 20
 P(13) = 20
 P(14) = 17
 P(15) = 20

Process exited with code: 0
```

Figure 4: Optimal Solution for n=20 m=15

2.3 Scenario 3

In this scenario, n = 20, m = 19. In the model, variables n and m are adjusted accordingly and solution set is gathered via Neos Solver because, Xpress-mosel had a constraint limit. The results are;

```
Objective value is 337
 X(16) = 1          P(1) = 13
 X(28) = 1          P(2) = 17
 X(314) = 1         P(3) = 17
 X(419) = 1         P(4) = 18
 X(57) = 1          P(5) = 19
 X(716) = 1         P(6) = 19
 X(85) = 1          P(7) = 18
 X(913) = 1         P(8) = 19
 X(103) = 1         P(9) = 18
 X(119) = 1         P(10) = 19
 X(124) = 1         P(11) = 18
 X(1315) = 1        P(12) = 20
 X(142) = 1         P(13) = 18
 X(1517) = 1        P(14) = 15
 X(1618) = 1        P(15) = 17
 X(1712) = 1        P(16) = 17
 X(1811) = 1        P(17) = 19
 X(1910) = 1        P(18) = 18
 X(201) = 1         P(19) = 18
```

Figure 5: Optimal Solution for n=20 m=19

2.4 Result

To understand the revenue change with respect to the costumer number, 3 different matrices were generated independently from each other for parameter V. In order to make meaningful conclusions, one should look for marginal revenue gathered from one CD because, sold CD number is different for every scenario. Parallel with the increase of CD number sold, it is expected that revenue will increase in magnitude however the situation will be different in margin. Looking up for marginal revenues;

	Scenario 1	19.5
Marginal Revenues	Scenario 2	19.53
	Scenario 3	17.74

As it can be seen from the table, marginal revenue is decreasing. With the increase in CD number, CD is more likely to be sold to lower prices than 20, because, it is a compulsory to sell every CD for every scenario. Therefore, it would be more accurate to set a limit price for every CD.