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1 计算几何

1.1 二维计算几何基本操作

```
1 const double PI = 3.14159265358979323846264338327950288;
2 double arcSin(const double &a) {
3     return a <= -1.0 ? -PI / 2 : (a >= 1.0 ? PI / 2 : asin(a));
4 }
5 double arcCos(const double &a) {
6     return a <= -1.0 ? PI : (a >= 1.0 ? 0 : acos(a));
7 }
8 struct point {
9     double x, y; // something omitted
10 point rot(const double &a) const { // counter-clockwise
11     return point(x * cos(a) - y * sin(a), x * sin(a) + y * cos(a));
12 }
13 point rot90() const { // counter-clockwise
14     return point(-y, x);
15 }
16 point project(const point &p1, const point &p2) const {
17     const point &q = *this;
18     return p1 + (p2 - p1) * (dot(p2 - p1, q - p1) / (p2 - p1).norm());
19 }
20 bool onSeg(const point &a, const point &b) const { // a, b inclusive
21     const point &c = *this;
22     return sign(dot(a - c, b - c)) <= 0 && sign(det(b - a, c - a)) == 0;
23 }
24 double distLP(const point &p1, const point &p2) const { // dist from *this to line p1->p2
25     const point &q = *this;
26     return fabs(det(p2 - p1, q - p1)) / (p2 - p1).len();
27 }
28 double distSP(const point &p1, const point &p2) const { // dist from *this to segment [p1,
29     p2]
30     const point &q = *this;
31     if (dot(p2 - p1, q - p1) < EPS) return (q - p1).len();
32     if (dot(p1 - p2, q - p2) < EPS) return (q - p2).len();
33     return distLP(p1, p2);
34 }
35 bool inAngle(const point &p1, const point &p2) const { // det(p1, p2) ≥ 0
36     const point &q = *this; return det(p1, q) > -EPS && det(p2, q) < EPS;
37 }
38 bool lineIntersect(const point &a, const point &b, const point &c, const point &d, point
39     &e) {
40     double s1 = det(c - a, d - a);
41     double s2 = det(d - b, c - b);
42     if (!sign(s1 + s2)) return false;
43     e = (b - a) * (s1 / (s1 + s2)) + a;
44     return true;
```

```

44 }
45 int segIntersectCheck(const point &a, const point &b, const point &c, const point &d,
    point &o) {
46     static double s1, s2, s3, s4;
47     static int iCnt;
48     int d1 = sign(s1 = det(b - a, c - a));
49     int d2 = sign(s2 = det(b - a, d - a));
50     int d3 = sign(s3 = det(d - c, a - c));
51     int d4 = sign(s4 = det(d - c, b - c));
52     if ((d1 ^ d2) == -2 && (d3 ^ d4) == -2) {
53         o = (c * s2 - d * s1) / (s2 - s1);
54         return true;
55     }
56     iCnt = 0;
57     if (d1 == 0 && c.onSeg(a, b)) o = c, ++iCnt;
58     if (d2 == 0 && d.onSeg(a, b)) o = d, ++iCnt;
59     if (d3 == 0 && a.onSeg(c, d)) o = a, ++iCnt;
60     if (d4 == 0 && b.onSeg(c, d)) o = b, ++iCnt;
61     return iCnt ? 2 : 0; // 不相交返回 0, 严格相交返回 1, 非严格相交返回 2
62 }
63 struct circle {
64     point o;
65     double r, rSqure;
66     bool inside(const point &a) { // 非严格
67         return (a - o).len() < r + EPS;
68     }
69     bool contain(const circle &b) const { // 非严格
70         return sign(b.r + (o - b.o).len() - r) <= 0;
71     }
72     bool disjunct(const circle &b) const { // 非严格
73         return sign(b.r + r - (o - b.o).len()) <= 0;
74     }
75     int isCL(const point &p1, const point &p2, point &a, point &b) const {
76         double x = dot(p1 - o, p2 - p1), y = (p2 - p1).norm();
77         double d = x * x - y * ((p1 - o).norm() - rSqure);
78         if (d < -EPS) return 0;
79         if (d < 0) d = 0;
80         point q1 = p1 - (p2 - p1) * (x / y);
81         point q2 = (p2 - p1) * (sqrt(d) / y);
82         a = q1 - q2; b = q1 + q2;
83         return q2.len() < EPS ? 1 : 2;
84     }
85     int tanCP(const point &p, point &a, point &b) const { // 返回切点, 注意可能与 p 重合
86         double x = (p - o).norm(), d = x - rSqure;
87         if (d < -EPS) return 0;
88         if (d < 0) d = 0;
89         point q1 = (p - o) * (rSqure / x);
90         point q2 = ((p - o) * (-r * sqrt(d) / x)).rot90();
91         a = o + (q1 - q2); b = o + (q1 + q2);

```

```

92         return q2.len() < EPS ? 1 : 2;
93     }
94 };
95 bool checkCrossCS(const circle &cir, const point &p1, const point &p2) { // 非严格
96     const point &c = cir.o;
97     const double &r = cir.r;
98     return c.distSP(p1, p2) < r + EPS
99         && (r < (c - p1).len() + EPS || r < (c - p2).len() + EPS);
100 }
101 bool checkCrossCC(const circle &cir1, const circle &cir2) { // 非严格
102     const double &r1 = cir1.r, &r2 = cir2.r, d = (cir1.o - cir2.o).len();
103     return d < r1 + r2 + EPS && fabs(r1 - r2) < d + EPS;
104 }
105 int isCC(const circle &cir1, const circle &cir2, point &a, point &b) {
106     const point &c1 = cir1.o, &c2 = cir2.o;
107     double x = (c1 - c2).norm(), y = ((cir1.rSqure - cir2.rSqure) / x + 1) / 2;
108     double d = cir1.rSqure / x - y * y;
109     if (d < -EPS) return 0;
110     if (d < 0) d = 0;
111     point q1 = c1 + (c2 - c1) * y;
112     point q2 = ((c2 - c1) * sqrt(d)).rot90();
113     a = q1 - q2; b = q1 + q2;
114     return q2.len() < EPS ? 1 : 2;
115 }
116 vector<pair<point, point>> tanCC(const circle &cir1, const circle &cir2) {
117     // 注意: 如果只有三条切线, 即 s1 = 1, s2 = 1, 返回的切线可能重复, 切点没有问题
118     vector<pair<point, point>> list;
119     if (cir1.contain(cir2) || cir2.contain(cir1)) return list;
120     const point &c1 = cir1.o, &c2 = cir2.o;
121     double r1 = cir1.r, r2 = cir2.r;
122     point p, a1, b1, a2, b2;
123     int s1, s2;
124     if (sign(r1 - r2) == 0) {
125         p = c2 - c1;
126         p = (p * (r1 / p.len())).rot90();
127         list.push_back(make_pair(c1 + p, c2 + p));
128         list.push_back(make_pair(c1 - p, c2 - p));
129     } else {
130         p = (c2 * r1 - c1 * r2) / (r1 - r2);
131         s1 = cir1.tanCP(p, a1, b1);
132         s2 = cir2.tanCP(p, a2, b2);
133         if (s1 >= 1 && s2 >= 1) {
134             list.push_back(make_pair(a1, a2));
135             list.push_back(make_pair(b1, b2));
136         }
137     }
138     p = (c1 * r2 + c2 * r1) / (r1 + r2);
139     s1 = cir1.tanCP(p, a1, b1);
140     s2 = cir2.tanCP(p, a2, b2);

```

```

141     if (s1 >= 1 && s2 >= 1) {
142         list.push_back(make_pair(a1, a2));
143         list.push_back(make_pair(b1, b2));
144     }
145     return list;
146 }
147 bool distConvexPIn(const point &p1, const point &p2, const point &p3, const point &p4,
148     const point &q) {
149     point o12 = (p1 - p2).rot90(), o23 = (p2 - p3).rot90(), o34 = (p3 - p4).rot90();
150     return (q - p1).inAngle(o12, o23) || (q - p3).inAngle(o23, o34)
151         || ((q - p2).inAngle(o23, p3 - p2) && (q - p3).inAngle(p2 - p3, o23));
152 }
153 double distConvexP(int n, point ps[], const point &q) { // 外部点到多边形的距离
154     int left = 0, right = n;
155     while (right - left > 1) {
156         int mid = (left + right) / 2;
157         if (distConvexPIn(ps[(left + n - 1) % n], ps[left], ps[mid], ps[(mid + 1) % n], q))
158             right = mid;
159         else left = mid;
160     }
161     return q.distSP(ps[left], ps[right % n]);
162 }
163 double areaCT(const circle &cir, point pa, point pb) {
164     pa = pa - cir.o; pb = pb - cir.o;
165     double R = cir.r;
166     if (pa.len() < pb.len()) swap(pa, pb);
167     if (pb.len() < EPS) return 0;
168     point pc = pb - pa;
169     double a = pa.len(), b = pb.len(), c = pc.len(), S, h, theta;
170     double cosB = dot(pb, pc) / b / c, B = acos(cosB);
171     double cosC = dot(pa, pb) / a / b, C = acos(cosC);
172     if (b > R) {
173         S = C * 0.5 * R * R;
174         h = b * a * sin(C) / c;
175         if (h < R && B < PI * 0.5)
176             S -= acos(h / R) * R * R - h * sqrt(R * R - h * h);
177     } else if (a > R) {
178         theta = PI - B - asin(sin(B) / R * b);
179         S = 0.5 * b * R * sin(theta) + (C - theta) * 0.5 * R * R;
180     } else S = 0.5 * sin(C) * b * a;
181     return S;
182 }
183 circle minCircle(const point &a, const point &b) {
184     return circle((a + b) * 0.5, (b - a).len() * 0.5);
185 }
186 circle minCircle(const point &a, const point &b, const point &c) { // 钝角三角形没有被考虑
187     double a2((b - c).norm()), b2((a - c).norm()), c2((a - b).norm());
188     if (b2 + c2 <= a2 + EPS) return minCircle(b, c);
189     if (a2 + c2 <= b2 + EPS) return minCircle(a, c);

```

```

189     if (a2 + b2 <= c2 + EPS) return minCircle(a, b);
190     double A = 2.0 * (a.x - b.x), B = 2.0 * (a.y - b.y);
191     double D = 2.0 * (a.x - c.x), E = 2.0 * (a.y - c.y);
192     double C = a.norm() - b.norm(), F = a.norm() - c.norm();
193     point p((C * E - B * F) / (A * E - B * D), (A * F - C * D) / (A * E - B * D));
194     return circle(p, (p - a).len());
195 }
196 circle minCircle(point P[], int N) { // 1-based
197     if (N == 1) return circle(P[1], 0.0);
198     random_shuffle(P + 1, P + N + 1); circle O = minCircle(P[1], P[2]);
199     Rep(i, 1, N) if(!O.inside(P[i])) { O = minCircle(P[1], P[i]);
200         Foru(j, 1, i) if(!O.inside(P[j])) { O = minCircle(P[i], P[j]);
201             Foru(k, 1, j) if(!O.inside(P[k])) O = minCircle(P[i], P[j], P[k]); }
202     } return O;
203 }

```

1.2 圆的面积模板

```

1 struct Event { point p; double alpha; int add; // 构造函数省略
2     bool operator < (const Event &other) const { return alpha < other.alpha; } };
3 void circleKCover(circle *c, int N, double *area) { // area[k] : 至少被覆盖 k 次
4     static bool overlap[MAXN][MAXN], g[MAXN][MAXN];
5     Rep(i, 0, N + 1) area[i] = 0.0; Rep(i, 1, N) Rep(j, 1, N) overlap[i][j] = c[i].contain(
6         c[j]);
7     Rep(i, 1, N) Rep(j, 1, N) g[i][j] = !(overlap[i][j] || overlap[j][i] || c[i].disjunct(c
8         [j]));
9     Rep(i, 1, N) { static Event events[MAXN * 2 + 1]; int totE = 0, cnt = 1;
10         Rep(j, 1, N) if (j != i && overlap[j][i]) ++cnt;
11         Rep(j, 1, N) if (j != i && g[i][j]) {
12             circle &a = c[i], &b = c[j]; double l = (a.o - b.o).norm();
13             double s = ((a.r - b.r) * (a.r + b.r) / l + 1) * 0.5;
14             double t = sqrt(-(l - sqrt(a.r - b.r)) * (l - sqrt(a.r + b.r)) / (l * l * 4.0));
15             point dir = b.o - a.o, nDir = point(-dir.y, dir.x);
16             point aa = a.o + dir * s + nDir * t;
17             point bb = a.o + dir * s - nDir * t;
18             double A = atan2(aa.y - a.o.y, aa.x - a.o.x);
19             double B = atan2(bb.y - a.o.y, bb.x - a.o.x);
20             events[totE++] = Event(bb, B, 1); events[totE++] = Event(aa, A, -1); if (B > A) ++
21                 cnt;
22         } if (totE == 0) { area[cnt] += PI * c[i].rSquare; continue; }
23         sort(events, events + totE); events[totE] = events[0];
24         Foru(j, 0, totE) {
25             cnt += events[j].add; area[cnt] += 0.5 * det(events[j].p, events[j + 1].p);
26             double theta = events[j + 1].alpha - events[j].alpha; if (theta < 0) theta += 2.0 *
27                 PI;
28             area[cnt] += 0.5 * c[i].rSquare * (theta - sin(theta));
29         }
30     }
31 }

```

1.3 多边形相关

```

1 struct Polygon { // stored in [0, n)
2     int n; point ps[MAXN];
3     Polygon cut(const point &a, const point &b) {
4         static Polygon res; static point o; res.n = 0;
5         for (int i = 0; i < n; ++i) {
6             int s1 = sign(det(ps[i] - a, b - a));
7             int s2 = sign(det(ps[(i + 1) % n] - a, b - a));
8             if (s1 <= 0) res.ps[res.n++] = ps[i];
9             if (s1 * s2 < 0) {
10                 lineIntersect(a, b, ps[i], ps[(i + 1) % n], o);
11                 res.ps[res.n++] = o;
12             }
13         } return res;
14     }
15     bool contain(const point &p) const { // 1 if on border or inner, 0 if outer
16         static point A, B; int res = 0;
17         for (int i = 0; i < n; ++i) {
18             A = ps[i]; B = ps[(i + 1) % n];
19             if (p.onSeg(A, B)) return 1;
20             if (sign(A.y - B.y) <= 0) swap(A, B);
21             if (sign(p.y - A.y) > 0) continue;
22             if (sign(p.y - B.y) <= 0) continue;
23             res += (int)(sign(det(B - p, A - p)) > 0);
24         } return res & 1;
25     }
26     #define qs(x) (ps[x] - ps[0])
27     bool convexContain(point p) const { // counter-clockwise
28         point q = qs(n - 1); p = p - ps[0];
29         if (!p.inAngle(qs(1), q)) return false;
30         int L = 0, R = n - 1;
31         while (L + 1 < R) { int M((L + R) >> 1);
32             if (p.inAngle(qs(M), q)) L = M; else R = M;
33         } if (L == 0) return false; point l(qs(L)), r(qs(R));
34         return sign(fabs(det(l, p)) + fabs(det(p, r)) + fabs(det(r - l, p - l)) - det(l, r)
35             ) == 0;
36     }
37     #undef qs
38     double isPLAtan2(const point &a, const point &b) {
39         double k = (b - a).alpha(); if (k < 0) k += 2 * PI;
40         return k;
41     }
42     point isPL_Get(const point &a, const point &b, const point &s1, const point &s2) {
43         double k1 = det(b - a, s1 - a), k2 = det(b - a, s2 - a);
44         if (sign(k1) == 0) return s1;
45         if (sign(k2) == 0) return s2;
46         return (s1 * k2 - s2 * k1) / (k2 - k1);
47     }

```

```

47 int isPL_Dic(const point &a, const point &b, int l, int r) {
48     int s = (det(b - a, ps[l] - a) < 0) ? -1 : 1;
49     while (l <= r) {
50         int mid = (l + r) / 2;
51         if (det(b - a, ps[mid] - a) * s <= 0) r = mid - 1;
52         else l = mid + 1;
53     }
54     return r + 1;
55 }
56 int isPL_Find(double k, double w[]) {
57     if (k <= w[0] || k > w[n - 1]) return 0;
58     int l = 0, r = n - 1, mid;
59     while (l <= r) {
60         mid = (l + r) / 2;
61         if (w[mid] >= k) r = mid - 1;
62         else l = mid + 1;
63     } return r + 1;
64 }
65 bool isPL(const point &a, const point &b, point &cp1, point &cp2) { // O(logN)
66     static double w[MAXN * 2]; // pay attention to the array size
67     for (int i = 0; i <= n; ++i) ps[i + n] = ps[i];
68     for (int i = 0; i < n; ++i) w[i] = w[i + n] = isPLAtan2(ps[i], ps[i + 1]);
69     int i = isPL_Find(isPLAtan2(a, b), w);
70     int j = isPL_Find(isPLAtan2(b, a), w);
71     double k1 = det(b - a, ps[i] - a), k2 = det(b - a, ps[j] - a);
72     if (sign(k1) * sign(k2) > 0) return false; // no intersection
73     if (sign(k1) == 0 || sign(k2) == 0) { // intersect with a point or a line in the convex
74         if (sign(k1) == 0) {
75             if (sign(det(b - a, ps[i + 1] - a)) == 0) cp1 = ps[i], cp2 = ps[i + 1];
76             else cp1 = cp2 = ps[i];
77             return true;
78         }
79         if (sign(k2) == 0) {
80             if (sign(det(b - a, ps[j + 1] - a)) == 0) cp1 = ps[j], cp2 = ps[j + 1];
81             else cp1 = cp2 = ps[j];
82         }
83         return true;
84     }
85     if (i > j) swap(i, j);
86     int x = isPL_Dic(a, b, i, j), y = isPL_Dic(a, b, j, i + n);
87     cp1 = isPL_Get(a, b, ps[x - 1], ps[x]);
88     cp2 = isPL_Get(a, b, ps[y - 1], ps[y]);
89     return true;
90 }
91 double getI(const point &o) const {
92     if (n <= 2) return 0;
93     point G(0.0, 0.0);
94     double S = 0.0, I = 0.0;
95     for (int i = 0; i < n; ++i) {

```

```

96     const point &x = ps[i], &y = ps[(i + 1) % n];
97     double d = det(x, y);
98     G = G + (x + y) * d / 3.0;
99     S += d;
100 } G = G / S;
101 for (int i = 0; i < n; ++i) {
102     point x = ps[i] - G, y = ps[(i + 1) % n] - G;
103     I += fabs(det(x, y)) * (x.norm() + dot(x, y) + y.norm());
104 }
105 return I = I / 12.0 + fabs(S * 0.5) * (O - G).norm();
106 }
107 };

```

1.4 直线与凸包求交点

```

1 int isPL(point a, point b, vector<point> &res) { // 点逆时针给出, 无三点共线
2     static double theta[MAXN];
3     for (int i = 0; i < n; ++i) theta[i] = (list[(i + 1) % n] - list[i]).atan2();
4     double delta = theta[0];
5     for (int i = 0; i < n; ++i) theta[i] = normalize(theta[i] - delta);
6     int x = lower_bound(theta, theta + n, normalize((b - a).atan2() - delta)) - theta;
7     int y = lower_bound(theta, theta + n, normalize((a - b).atan2() - delta)) - theta;
8     for (int k = 0; k <= 1; ++k, swap(a, b), swap(x, y)) {
9         if (y < x) y += n;
10        int l = x, r = y, m;
11        while (l + 1 < r) {
12            if (sign(det(b - a, list[(m = (l + r) / 2) % n] - a)) < 0) l = m;
13            else r = m;
14        }
15        l %= n, r %= n;
16        if (sign(det(b - a, list[r] - list[l])) == 0) {
17            if (sign(det(b - a, list[l] - a)) == 0)
18                return -1; // 直线与 (list[l], list[r]) 重合
19        }
20        else {
21            point p; lineIntersect(list[l], list[r], a, b, p);
22            if (p.onSeg(list[l], list[r]))
23                res.push_back(p);
24        }
25    }
26    return res.size();
27 }

```

1.5 半平面交

```

1 struct Border {

```

```

2     point p1, p2; double alpha;
3     Border() : p1(), p2(), alpha(0.0) {}
4     Border(const point &a, const point &b): p1(a), p2(b), alpha( atan2(p2.y - p1.y, p2.x -
        p1.x) ) {}
5     bool operator == (const Border &b) const { return sign(alpha - b.alpha) == 0; }
6     bool operator < (const Border &b) const {
7         int c = sign(alpha - b.alpha); if (c != 0) return c > 0;
8         return sign(det(b.p2 - b.p1, p1 - b.p1)) >= 0;
9     }
10 };
11 point isBorder(const Border &a, const Border &b) { // a and b should not be parallel
12     point is; lineIntersect(a.p1, a.p2, b.p1, b.p2, is); return is;
13 }
14 bool checkBorder(const Border &a, const Border &b, const Border &me) {
15     point is; lineIntersect(a.p1, a.p2, b.p1, b.p2, is);
16     return sign(det(me.p2 - me.p1, is - me.p1)) > 0;
17 }
18 double HPI(int N, Border border[]) {
19     static Border que[MAXN * 2 + 1]; static point ps[MAXN];
20     int head = 0, tail = 0, cnt = 0; // [head, tail)
21     sort(border, border + N); N = unique(border, border + N) - border;
22     for (int i = 0; i < N; ++i) {
23         Border &cur = border[i];
24         while (head + 1 < tail && !checkBorder(que[tail - 2], que[tail - 1], cur)) --tail;
25         while (head + 1 < tail && !checkBorder(que[head], que[head + 1], cur)) ++head;
26         que[tail++] = cur;
27     } while (head + 1 < tail && !checkBorder(que[tail - 2], que[tail - 1], que[head])) --
        tail;
28     while (head + 1 < tail && !checkBorder(que[head], que[head + 1], que[tail - 1])) ++head
        ;
29     if (tail - head <= 2) return 0.0;
30     Foru(i, head, tail) ps[cnt++] = isBorder(que[i], que[(i + 1 == tail) ? (head) : (i + 1)
        ]);
31     double area = 0; Foru(i, 0, cnt) area += det(ps[i], ps[(i + 1) % cnt]);
32     return fabs(area * 0.5); // or (-area * 0.5)
33 }

```

1.6 最大面积空凸包

```

1 inline bool toUpRight(const point &a, const point &b) {
2     int c = sign(b.y - a.y); if (c > 0) return true;
3     return c == 0 && sign(b.x - a.x) > 0;
4 }
5 inline bool cmpByPolarAngle(const point &a, const point &b) { // counter-clockwise, shorter
        first if they share the same polar angle
6     int c = sign(det(a, b)); if (c != 0) return c > 0;
7     return sign(b.len() - a.len()) > 0;
8 }

```

```

9 double maxEmptyConvexHull(int N, point p[]) {
10     static double dp[MAXN][MAXN];
11     static point vec[MAXN];
12     static int seq[MAXN]; // empty triangles formed with (0,0),vec[o],vec[seq[i]]
13     double ans = 0.0;
14     Rep(o, 1, N) {
15         int totVec = 0;
16         Rep(i, 1, N) if (toUpRight(p[o], p[i])) vec[++totVec] = p[i] - p[o];
17         sort(vec + 1, vec + totVec + 1, cmpByPolarAngle);
18         Rep(i, 1, totVec) Rep(j, 1, totVec) dp[i][j] = 0.0;
19         Rep(k, 2, totVec) {
20             int i = k - 1;
21             while (i > 0 && sign( det(vec[k], vec[i]) ) == 0) --i;
22             int totSeq = 0;
23             for (int j; i > 0; i = j) {
24                 seq[++totSeq] = i;
25                 for (j = i - 1; j > 0 && sign(det(vec[i] - vec[k], vec[j] - vec[k])) > 0; --j);
26                 double v = det(vec[i], vec[k]) * 0.5;
27                 if (j > 0) v += dp[i][j];
28                 dp[k][i] = v;
29                 cMax(ans, v);
30             } for (int i = totSeq - 1; i >= 1; --i) cMax( dp[k][ seq[i] ], dp[k][seq[i + 1]] );
31         }
32     } return ans;
33 }

```

1.7 最近点对

```

1 int N; point p[maxn];
2 bool cmpByX(const point &a, const point &b) { return sign(a.x - b.x) < 0; }
3 bool cmpByY(const int &a, const int &b) { return p[a].y < p[b].y; }
4 double minimalDistance(point *c, int n, int *ys) {
5     double ret = 1e+20;
6     if (n < 20) {
7         Foru(i, 0, n) Foru(j, i + 1, n) cMin(ret, (c[i] - c[j]).len() );
8         sort(ys, ys + n, cmpByY); return ret;
9     } static int mergeTo[maxn];
10     int mid = n / 2; double xmid = c[mid].x;
11     ret = min(minimalDistance(c, mid, ys), minimalDistance(c + mid, n - mid, ys + mid));
12     merge(ys, ys + mid, ys + mid, ys + n, mergeTo, cmpByY);
13     copy(mergeTo, mergeTo + n, ys);
14     Foru(i, 0, n) {
15         while (i < n && sign(fabs(p[ys[i]].x - xmid) - ret) > 0) ++i;
16         int cnt = 0;
17         Foru(j, i + 1, n)
18             if (sign(p[ys[j]].y - p[ys[i]].y - ret) > 0) break;
19             else if (sign(fabs(p[ys[j]].x - xmid) - ret) <= 0) {
20                 ret = min(ret, (p[ys[i]] - p[ys[j]]).len());

```

```

21         if (++cnt >= 10) break;
22     }
23     } return ret;
24 }
25 double work() {
26     sort(p, p + n, cmpByX); Foru(i, 0, n) ys[i] = i; return minimalDistance(p, n, ys);
27 }

```

1.8 凸包与点集直径

```

1 vector<point> convexHull(int n, point ps[]) { // counter-clockwise, strict
2     static point qs[MAXN * 2];
3     sort(ps, ps + n, cmpByXY);
4     if (n <= 2) return vector(ps, ps + n);
5     int k = 0;
6     for (int i = 0; i < n; qs[k++] = ps[i++])
7         while (k > 1 && det(qs[k - 1] - qs[k - 2], ps[i] - qs[k - 1]) < EPS) --k;
8     for (int i = n - 2, t = k; i >= 0; qs[k++] = ps[i--])
9         while (k > t && det(qs[k - 1] - qs[k - 2], ps[i] - qs[k - 1]) < EPS) --k;
10    return vector<point>(qs, qs + k);
11 }
12 double convexDiameter(int n, point ps[]) {
13     if (n < 2) return 0; if (n == 2) return (ps[1] - ps[0]).len();
14     double k, ans = 0;
15     for (int x = 0, y = 1, nx, ny; x < n; ++x) {
16         for(nx = (x == n - 1) ? (0) : (x + 1); ; y = ny) {
17             ny = (y == n - 1) ? (0) : (y + 1);
18             if ( sign(k = det(ps[nx] - ps[x], ps[ny] - ps[y])) <= 0) break;
19             } ans = max(ans, (ps[x] - ps[y]).len());
20             if (sign(k) == 0) ans = max(ans, (ps[x] - ps[ny]).len());
21         } return ans;
22 }

```

1.9 Farmland

```

1 struct node { int begin[MAXN], *end; } a[MAXN]; // 按对 p[i] 的极角的 atan2 值排序
2 bool check(int n, point p[], int b1, int b2, bool vis[MAXN][MAXN]) {
3     static pii l[MAXN * 2 + 1]; static bool used[MAXN];
4     int tp(0), *k, p, p1, p2; double area(0.0);
5     for (l[0] = pii(b1, b2); ; ) {
6         vis[p1 = l[tp].first][p2 = l[tp].second] = true;
7         area += det(p[p1], p[p2]);
8         for (k = a[p2].begin; k != a[p2].end; ++k) if (*k == p1) break;
9         k = (k == a[p2].begin) ? (a[p2].end - 1) : (k - 1);
10        if ((l[++tp] = pii(p2, *k)) == l[0]) break;
11    } if (sign(area) < 0 || tp < 3) return false;

```

```

12 Rep(i, 1, n) used[i] = false;
13 for (int i = 0; i < tp; ++i) if (used[p = l[i].first]) return false; else used[p] =
    true;
14 return true; // a face with tp vertices
15 }
16 int countFaces(int n, point p[]) {
17     static bool vis[MAXN][MAXN]; int ans = 0;
18     Rep(x, 1, n) Rep(y, 1, n) vis[x][y] = false;
19     Rep(x, 1, n) for (int *itr = a[x].begin; itr != a[x].end; ++itr) if (!vis[x][*itr])
20         if (check(n, p, x, *itr, vis)) ++ans;
21     return ans;
22 }

```

1.10 Voronoi 图

不能有重点, 点数应当不小于 2

```

1 #define Oi(e) ((e)->oi)
2 #define Dt(e) ((e)->dt)
3 #define On(e) ((e)->on)
4 #define Op(e) ((e)->op)
5 #define Dn(e) ((e)->dn)
6 #define Dp(e) ((e)->dp)
7 #define Other(e, p) ((e)->oi == p ? (e)->dt : (e)->oi)
8 #define Next(e, p) ((e)->oi == p ? (e)->on : (e)->dn)
9 #define Prev(e, p) ((e)->oi == p ? (e)->op : (e)->dp)
10 #define V(p1, p2, u, v) (u = p2->x - p1->x, v = p2->y - p1->y)
11 #define C2(u1, v1, u2, v2) (u1 * v2 - v1 * u2)
12 #define C3(p1, p2, p3) ((p2->x - p1->x) * (p3->y - p1->y) - (p2->y - p1->y) * (p3->x - p1->x))
13 #define Dot(u1, v1, u2, v2) (u1 * u2 + v1 * v2)
14 #define dis(a,b) (sqrt( (a->x - b->x) * (a->x - b->x) + (a->y - b->y) * (a->y - b->y) ))
15 const int maxn = 110024;
16 const int aix = 4;
17 const double eps = 1e-7;
18 int n, M, k;
19 struct gEdge {
20     int u, v; double w;
21     bool operator <(const gEdge &e1) const { return w < e1.w - eps; }
22 } E[aix * maxn], MST[maxn];
23 struct point {
24     double x, y; int index; edge *in;
25     bool operator <(const point &p1) const { return x < p1.x - eps || (abs(x - p1.x) <= eps
        && y < p1.y - eps); }
26 };
27 struct edge { point *oi, *dt; edge *on, *op, *dn, *dp; };
28
29 point p[maxn], *Q[maxn];

```

```

30 edge mem[aix * maxn], *elist[aix * maxn];
31 int nfree;
32 void Alloc_memory() { nfree = aix * n; edge *e = mem; for (int i = 0; i < nfree; i++)
    elist[i] = e++; }
33 void Splice(edge *a, edge *b, point *v) {
34     edge *next;
35     if (Oi(a) == v) next = On(a), On(a) = b; else next = Dn(a), Dn(a) = b;
36     if (Oi(next) == v) Op(next) = b; else Dp(next) = b;
37     if (Oi(b) == v) On(b) = next, Op(b) = a; else Dn(b) = next, Dp(b) = a;
38 }
39 edge *Make_edge(point *u, point *v) {
40     edge *e = elist[--nfree];
41     e->on = e->op = e->dn = e->dp = e; e->oi = u; e->dt = v;
42     if (!u->in) u->in = e;
43     if (!v->in) v->in = e;
44     return e;
45 }
46 edge *Join(edge *a, point *u, edge *b, point *v, int side) {
47     edge *e = Make_edge(u, v);
48     if (side == 1) {
49         if (Oi(a) == u) Splice(Op(a), e, u);
50         else Splice(Dp(a), e, u);
51         Splice(b, e, v);
52     } else {
53         Splice(a, e, u);
54         if (Oi(b) == v) Splice(Op(b), e, v);
55         else Splice(Dp(b), e, v);
56     } return e;
57 }
58 void Remove(edge *e) {
59     point *u = Oi(e), *v = Dt(e);
60     if (u->in == e) u->in = e->on;
61     if (v->in == e) v->in = e->dn;
62     if (Oi(e->on) == u) e->on->op = e->op; else e->on->dp = e->op;
63     if (Oi(e->op) == u) e->op->on = e->on; else e->op->dn = e->on;
64     if (Oi(e->dn) == v) e->dn->op = e->dp; else e->dn->dp = e->dp;
65     if (Oi(e->dp) == v) e->dp->on = e->dn; else e->dp->dn = e->dn;
66     elist[nfree++] = e;
67 }
68 void Low_tangent(edge *e_l, point *o_l, edge *e_r, point *o_r, edge **l_low, point **OL,
    edge **r_low, point **OR) {
69     for (point *d_l = Other(e_l, o_l), *d_r = Other(e_r, o_r); ; )
70         if (C3(o_l, o_r, d_l) < -eps) e_l = Prev(e_l, d_l), o_l = d_l, d_l = Other(e_l,
            o_l);
71         else if (C3(o_l, o_r, d_r) < -eps) e_r = Next(e_r, d_r), o_r = d_r, d_r = Other(e_r,
            o_r);
72         else break;
73     *OL = o_l, *OR = o_r; *l_low = e_l, *r_low = e_r;
74 }

```



```

75 void Merge(edge *lr, point *s, edge *rl, point *u, edge **tangent) {
76     double l1, l2, l3, l4, r1, r2, r3, r4, cot_L, cot_R, u1, v1, u2, v2, n1, cot_n, P1,
        cot_P;
77     point *O, *D, *OR, *OL; edge *B, *L, *R;
78     Low_tangent(lr, s, rl, u, &L, &OL, &R, &OR);
79     for (*tangent = B = Join(L, OL, R, OR, O), O = OL, D = OR; ; ) {
80         edge *El = Next(B, O), *Er = Prev(B, D), *next, *prev;
81         point *l = Other(El, O), *r = Other(Er, D);
82         V(l, O, l1, l2); V(l, D, l3, l4); V(r, O, r1, r2); V(r, D, r3, r4);
83         double c1 = C2(l1, l2, l3, l4), cr = C2(r1, r2, r3, r4);
84         bool BL = c1 > eps, BR = cr > eps;
85         if (!BL && !BR) break;
86         if (BL) {
87             double d1 = Dot(l1, l2, l3, l4);
88             for (cot_L = d1 / c1; ; Remove(El), El = next, cot_L = cot_n) {
89                 next = Next(El, O); V(Other(next, O), O, u1, v1); V(Other(next, O), D, u2, v2);
90                 n1 = C2(u1, v1, u2, v2); if (!(n1 > eps)) break;
91                 cot_n = Dot(u1, v1, u2, v2) / n1;
92                 if (cot_n > cot_L) break;
93             }
94         } if (BR) {
95             double dr = Dot(r1, r2, r3, r4);
96             for (cot_R = dr / cr; ; Remove(Er), Er = prev, cot_R = cot_P) {
97                 prev = Prev(Er, D); V(Other(prev, D), O, u1, v1); V(Other(prev, D), D, u2, v2);
98                 P1 = C2(u1, v1, u2, v2); if (!(P1 > eps)) break;
99                 cot_P = Dot(u1, v1, u2, v2) / P1;
100                if (cot_P > cot_R) break;
101            }
102        } l = Other(El, O); r = Other(Er, D);
103        if (!BL || (BL && BR && cot_R < cot_L)) B = Join(B, O, Er, r, O), D = r;
104        else B = Join(El, l, B, D, O), O = l;
105    }
106 }
107 void Divide(int s, int t, edge **L, edge **R) {
108     edge *a, *b, *c, *l1, *lr, *rl, *rr, *tangent;
109     int n = t - s + 1;
110     if (n == 2) *L = *R = Make_edge(Q[s], Q[t]);
111     else if (n == 3) {
112         a = Make_edge(Q[s], Q[s + 1]), b = Make_edge(Q[s + 1], Q[t]);
113         Splice(a, b, Q[s + 1]);
114         double v = C3(Q[s], Q[s + 1], Q[t]);
115         if (v > eps) c = Join(a, Q[s], b, Q[t], O), *L = a, *R = b;
116         else if (v < -eps) c = Join(a, Q[s], b, Q[t], 1), *L = c, *R = c;
117         else *L = a, *R = b;
118     } else if (n > 3) {
119         int split = (s + t) / 2;
120         Divide(s, split, &l1, &lr); Divide(split + 1, t, &rl, &rr);
121         Merge(lr, Q[split], rl, Q[split + 1], &tangent);
122         if (Oi(tangent) == Q[s]) l1 = tangent;

```

```

123         if (Dt(tangent) == Q[t]) rr = tangent;
124         *L = l1; *R = rr;
125     }
126 }
127 void Make_Graph() {
128     edge *start, *e; point *u, *v;
129     for (int i = 0; i < n; i++) {
130         start = e = (u = &p[i])->in;
131         do{ v = Other(e, u);
132             if (u < v) E[M++].u = (u - p, v - p, dis(u, v)); // M < aix * maxn
133         } while ((e = Next(e, u)) != start);
134     }
135 }
136 int b[maxn];
137 int Find(int x) { while (x != b[x]) { b[x] = b[b[x]]; x = b[x]; } return x; }
138 void Kruskal() {
139     memset(b, 0, sizeof(b)); sort(E, E + M);
140     for (int i = 0; i < n; i++) b[i] = i;
141     for (int i = 0, kk = 0; i < M && kk < n - 1; i++) {
142         int m1 = Find(E[i].u), m2 = Find(E[i].v);
143         if (m1 != m2) b[m1] = m2, MST[kk++] = E[i];
144     }
145 }
146 void solve() {
147     scanf("%d", &n);
148     for (int i = 0; i < n; i++) scanf("%lf%lf", &p[i].x, &p[i].y), p[i].index = i, p[i].in
        = NULL;
149     Alloc_memory(); sort(p, p + n);
150     for (int i = 0; i < n; i++) Q[i] = p + i;
151     edge *L, *R; Divide(0, n - 1, &L, &R);
152     M = 0; Make_Graph(); Kruskal();
153 }
154 int main() { solve(); return 0; }

```

1.11 四边形双费马点

```

1  typedef complex<double> Tpoint;
2  const double eps = 1e-8;
3  const double sqrt3 = sqrt(3.0);
4  bool cmp(const Tpoint &a, const Tpoint &b) {
5      return a.real() < b.real() - eps || (a.real() < b.real() + eps && a.imag() < b.imag());
6  }
7  Tpoint rotate(const Tpoint &a, const Tpoint &b, const Tpoint &c) {
8      Tpoint d = b - a; d = Tpoint(-d.imag(), d.real());
9      if (Sign(cross(a, b, c)) == Sign(cross(a, b, a + d))) d *= -1.0;
10     return unit(d);
11 }
12 Tpoint p[10], a[10], b[10];

```

```

13 int N, T;
14 double totlen(const Tpoint &p, const Tpoint &a, const Tpoint &b, const Tpoint &c) {
15     return abs(p - a) + abs(p - b) + abs(p - c);
16 }
17 double fermat(const Tpoint &x, const Tpoint &y, const Tpoint &z, Tpoint &cp) {
18     a[0] = a[3] = x; a[1] = a[4] = y; a[2] = a[5] = z;
19     double len = 1e100, len2;
20     for (int i = 0; i < 3; i++) {
21         len2 = totlen(a[i], x, y, z);
22         if (len2 < len) len = len2, cp = a[i];
23     }
24     for (int i = 0; i < 3; i++) {
25         b[i] = rotate(a[i + 1], a[i], a[i + 2]);
26         b[i] = (a[i + 1] + a[i]) / 2.0 + b[i] * (abs(a[i + 1] - a[i]) * sqrt3 / 2.0);
27     }
28     b[3] = b[0];
29     Tpoint cp2 = intersect(b[0], a[2], b[1], a[3]);
30     len2 = totlen(cp2, x, y, z);
31     if (len2 < len) len = len2, cp = cp2;
32     return len;
33 }
34 double getans(const Tpoint &a) {
35     double len = 0; for (int i = 0; i < N; i++) len += abs(a - p[i]);
36     return len;
37 }
38 double mindist(const Tpoint &p, const Tpoint &a, const Tpoint &b, const Tpoint &c, const
    Tpoint &d) {
39     return min( min(abs(p - a), abs(p - b)), min(abs(p - c), abs(p - d)));
40 }
41 int main() {
42     N = 4;
43     for (cin >> T; T; T--) {
44         double ret = 1e100, len_cur, len_before, len1, len2, len;
45         Tpoint cp, cp1, cp2;
46         Foru(i, 0, N) cin >> p[i];
47         Foru(i, 0, N) ret = min(ret, getans(p[i]));
48         Foru(i, 1, N) Foru(j, 1, N) if (j != i) Foru(k, 1, N) if (k != i && k != j) {
49             cMin(ret, abs(p[0] - p[i]) + abs(p[j] - p[k])
50                 + min( min(abs(p[0] - p[j]), abs(p[0] - p[k])),
51                     min(abs(p[i] - p[j]), abs(p[i] - p[k]))
52                 ));
53             ret = min(ret, getans(intersect(p[0], p[i], p[j], p[k])));
54         }
55         Foru(i, 0, N) Foru(j, i + 1, N) Foru(k, j + 1, N) {
56             double len = fermat(p[i], p[j], p[k], cp);
57             ret = min(ret, len + mindist(p[6 - i - j - k], p[i], p[j], p[k], cp));
58         }
59         sort(p, p + N, cmp);
60         for (int i = 1; i < N; i++) {

```

```

61         cp1 = (p[0] + p[i]) / 2.0;
62         int j, k;
63         for (j = 1; j < N && j == i; j++);
64         for (k = 6 - i - j, len_before = 1e100; ; ) {
65             len1 = fermat(cp1, p[j], p[k], cp2);
66             len1 = fermat(cp2, p[0], p[i], cp1);
67             len = len1 + abs(cp2 - p[j]) + abs(cp2 - p[k]);
68             if (len < len_before - (1e-6)) len_before = len;
69             else break;
70         } ret = min(ret, len_before);
71         } printf("%.4f\n", ret);
72     }
73     return 0;
74 }

```

1.12 三角形和四边形的费马点

- 费马点: 距几个顶点距离之和最小的点

- 三角形:

- 若每个角都小于 120° : 以每条边向外作正三角形, 得到 $\triangle ABF$, $\triangle BCD$, $\triangle CAE$, 连接 AD , BE , CF , 三线必共点于费马点. 该点对三边的张角必然是 120° , 也必然是三个三角形外接圆的交点
- 否则费马点一定是那个大于等于 120° 的顶角

- 四边形:

- 在凸四边形中, 费马点为对角线的交点
- 在凹四边形中, 费马点位凹顶点

1.13 三维计算几何基本操作

```

1 struct point { double x, y, z; // something omitted
2     friend point det(const point &a, const point &b) {
3         return point(a.y * b.z - a.z * b.y, a.z * b.x - a.x * b.z, a.x * b.y - a.y * b.x);
4     }
5     friend double mix(const point &a, const point &b, const point &c) {
6         return a.x * b.y * c.z + a.y * b.z * c.x + a.z * b.x * c.y - a.z * b.y * c.x - a.x *
            b.z * c.y - a.y * b.x * c.z;
7     }
8     double distLP(const point &p1, const point &p2) const {
9         return det(p2 - p1, *this - p1).len() / (p2 - p1).len();
10    }
11    double distFP(const point &p1, const point &p2, const point &p3) const {
12        point n = det(p2 - p1, p3 - p1); return fabs( dot(n, *this - p1) / n.len() );
13    }

```

```

14 };
15 double distLL(const point &p1, const point &p2, const point &q1, const point &q2) {
16     point p = q1 - p1, u = p2 - p1, v = q2 - q1;
17     double d = u.norm() * v.norm() - dot(u, v) * dot(u, v);
18     if (sign(d) == 0) return p1.distLP(q1, q2);
19     double s = (dot(p, u) * v.norm() - dot(p, v) * dot(u, v)) / d;
20     return (p1 + u * s).distLP(q1, q2);
21 }
22 double distSS(const point &p1, const point &p2, const point &q1, const point &q2) {
23     point p = q1 - p1, u = p2 - p1, v = q2 - q1;
24     double d = u.norm() * v.norm() - dot(u, v) * dot(u, v);
25     if (sign(d) == 0) return min( min((p1 - q1).len(), (p1 - q2).len()),
26                                 min((p2 - q1).len(), (p2 - q2).len()));
27     double s1 = (dot(p, u) * v.norm() - dot(p, v) * dot(u, v)) / d;
28     double s2 = (dot(p, v) * u.norm() - dot(p, u) * dot(u, v)) / d;
29     if (s1 < 0.0) s1 = 0.0; if (s1 > 1.0) s1 = 1.0;
30     if (s2 < 0.0) s2 = 0.0; if (s2 > 1.0) s2 = 1.0;
31     point r1 = p1 + u * s1; point r2 = q1 + v * s2;
32     return (r1 - r2).len();
33 }
34 bool isFL(const point &p, const point &o, const point &q1, const point &q2, point &res) {
35     double a = dot(o, q2 - p), b = dot(o, q1 - p), d = a - b;
36     if (sign(d) == 0) return false;
37     res = (q1 * a - q2 * b) / d;
38     return true;
39 }
40 bool isFF(const point &p1, const point &o1, const point &p2, const point &o2, point &a,
41           point &b) {
42     point e = det(o1, o2), v = det(o1, e);
43     double d = dot(o2, v); if (sign(d) == 0) return false;
44     point q = p1 + v * (dot(o2, p2 - p1) / d);
45     a = q; b = q + e;
46     return true;
47 }

```

1.14 凸多面体切割

```

1 vector<vector<point>> > convexCut(const vector<vector<point>> > &pss, const point &p, const
2   point &o) {
3     vector<vector<point>> > res;
4     vector<point> sec;
5     for (unsigned itr = 0, size = pss.size(); itr < size; ++itr) {
6         const vector<point> &ps = pss[itr];
7         int n = ps.size();
8         vector<point> qs;
9         bool dif = false;
10        for (int i = 0; i < n; ++i) {

```

```

11        int d2 = sign( dot(o, ps[(i + 1) % n] - p) );
12        if (d1 <= 0) qs.push_back(ps[i]);
13        if (d1 * d2 < 0) {
14            point q;
15            isFL(p, o, ps[i], ps[(i + 1) % n], q); // must return true
16            qs.push_back(q);
17            sec.push_back(q);
18        }
19        if (d1 == 0) sec.push_back(ps[i]);
20        else dif = true;
21        dif |= dot(o, det(ps[(i + 1) % n] - ps[i], ps[(i + 2) % n] - ps[i])) < -EPS;
22    }
23    if (!qs.empty() && dif)
24        res.insert(res.end(), qs.begin(), qs.end());
25 }
26 if (!sec.empty()) {
27     vector<point> tmp( convexHull2D(sec, o) );
28     res.insert(res.end(), tmp.begin(), tmp.end());
29 }
30 return res;
31 }
32
33 vector<vector<point>> > initConvex() {
34     vector<vector<point>> > pss(6, vector<point>(4));
35     pss[0][0] = pss[1][0] = pss[2][0] = point(-INF, -INF, -INF);
36     pss[0][3] = pss[1][1] = pss[5][2] = point(-INF, -INF, INF);
37     pss[0][1] = pss[2][3] = pss[4][2] = point(-INF, INF, -INF);
38     pss[0][2] = pss[5][3] = pss[4][1] = point(-INF, INF, INF);
39     pss[1][3] = pss[2][1] = pss[3][2] = point( INF, -INF, -INF);
40     pss[1][2] = pss[5][1] = pss[3][3] = point( INF, -INF, INF);
41     pss[2][2] = pss[4][3] = pss[3][1] = point( INF, INF, -INF);
42     pss[5][0] = pss[4][0] = pss[3][0] = point( INF, INF, INF);
43     return pss;
44 }

```

1.15 三维凸包

不能有重点

```

1 namespace ConvexHull3D {
2     #define volume(a, b, c, d) (mix(ps[b] - ps[a], ps[c] - ps[a], ps[d] - ps[a]))
3     vector<Facet> getHull(int n, point ps[]) {
4         static int mark[MAXN][MAXN], a, b, c;
5         int stamp = 0;
6         bool exist = false;
7         vector<Facet> facet;
8         random_shuffle(ps, ps + n);
9         for (int i = 2; i < n && !exist; i++) {

```

```

10     point ndir = det(ps[0] - ps[i], ps[1] - ps[i]);
11     if (ndir.len() < EPS) continue;
12     swap(ps[i], ps[2]);
13     for (int j = i + 1; j < n && !exist; j++)
14         if (sign(volume(0, 1, 2, j)) != 0) {
15             exist = true;
16             swap(ps[j], ps[3]);
17             facet.push_back(Facet(0, 1, 2));
18             facet.push_back(Facet(0, 2, 1));
19         }
20     }
21     if (!exist) return ConvexHull2D(n, ps);
22     for (int i = 0; i < n; ++i)
23         for (int j = 0; j < n; ++j)
24             mark[i][j] = 0;
25     stamp = 0;
26     for (int v = 3; v < n; ++v) {
27         vector<Facet> tmp;
28         ++stamp;
29         for (unsigned i = 0; i < facet.size(); i++) {
30             a = facet[i].a;
31             b = facet[i].b;
32             c = facet[i].c;
33             if (sign(volume(v, a, b, c)) < 0)
34                 mark[a][b] = mark[a][c] =
35                 mark[b][a] = mark[b][c] =
36                 mark[c][a] = mark[c][b] = stamp;
37             else tmp.push_back(facet[i]);
38         } facet = tmp;
39         for (unsigned i = 0; i < tmp.size(); i++) {
40             a = facet[i].a; b = facet[i].b; c = facet[i].c;
41             if (mark[a][b] == stamp) facet.push_back(Facet(b, a, v));
42             if (mark[b][c] == stamp) facet.push_back(Facet(c, b, v));
43             if (mark[c][a] == stamp) facet.push_back(Facet(a, c, v));
44         }
45     } return facet;
46 }
47 #undef volume
48 }
49 namespace Gravity {
50     using ConvexHull3D::Facet;
51     point findG(point ps[], const vector<Facet> &facet) {
52         double ws = 0; point res(0.0, 0.0, 0.0), o = ps[ facet[0].a ];
53         for (int i = 0, size = facet.size(); i < size; ++i) {
54             const point &a = ps[ facet[i].a ], &b = ps[ facet[i].b ], &c = ps[ facet[i].c ];
55             point p = (a + b + c + o) * 0.25;
56             double w = mix(a - o, b - o, c - o);
57             ws += w;
58             res = res + p * w;

```

```

59         } res = res / ws;
60         return res;
61     }
62 }

```

1.16 球面点表面点距离

```

1 double distOnBall(double lati1, double longi1, double lati2, double longi2, double R) {
2     lati1 *= PI / 180; longi1 *= PI / 180;
3     lati2 *= PI / 180; longi2 *= PI / 180;
4     double x1 = cos(lati1) * sin(longi1);
5     double y1 = cos(lati1) * cos(longi1);
6     double z1 = sin(lati1);
7     double x2 = cos(lati2) * sin(longi2);
8     double y2 = cos(lati2) * cos(longi2);
9     double z2 = sin(lati2);
10    double theta = acos(x1 * x2 + y1 * y2 + z1 * z2);
11    return R * theta;
12 }

```

1.17 长方体表面点距离

```

1 int r;
2 void turn(int i, int j, int x, int y, int z, int x0, int y0, int L, int W, int H) {
3     if (z == 0) r = min(r, x * x + y * y);
4     else {
5         if (i >= 0 && i < 2) turn(i + 1, j, x0 + L + z, y, x0 + L - x, x0 + L, y0, H, W, L);
6         if (j >= 0 && j < 2) turn(i, j + 1, x, y0 + W + z, y0 + W - y, x0, y0 + W, L, H, W);
7         if (i <= 0 && i > -2) turn(i - 1, j, x0 - z, y, x - x0, x0 - H, y0, H, W, L);
8         if (j <= 0 && j > -2) turn(i, j - 1, x, y0 - z, y - y0, x0, y0 - H, L, H, W);
9     }
10 }
11 int calc(int L, int H, int W, int x1, int y1, int z1, int x2, int y2, int z2) {
12     if (z1 != 0 && z1 != H)
13         if (y1 == 0 || y1 == W) swap(y1, z1), swap(y2, z2), swap(W, H);
14     else swap(x1, z1), swap(x2, z2), swap(L, H);
15     if (z1 == H) z1 = 0, z2 = H - z2;
16     r = INF; turn(0, 0, x2 - x1, y2 - y1, z2, -x1, -y1, L, W, H);
17     return r;
18 }

```

1.18 最小覆盖球

```

1 int outCnt; point out[4], res; double radius;
2 void ball() {

```

```

3 static point q[3];
4 static double m[3][3], sol[3], L[3], det;
5 int i, j; res = point(0.0, 0.0, 0.0); radius = 0.0;
6 switch (outCnt) {
7 case 1: res = out[0]; break;
8 case 2: res = (out[0] + out[1]) * 0.5; radius = (res - out[0]).norm();
9         break;
10 case 3:
11     q[0] = out[1] - out[0]; q[1] = out[2] - out[0];
12     for (i = 0; i < 2; ++i) for (j = 0; j < 2; ++j)
13         m[i][j] = dot(q[i], q[j]) * 2.0;
14     for (i = 0; i < 2; ++i) sol[i] = dot(q[i], q[i]);
15     det = m[0][0] * m[1][1] - m[0][1] * m[1][0];
16     if (sign(det) == 0) return;
17     L[0] = (sol[0] * m[1][1] - sol[1] * m[0][1]) / det;
18     L[1] = (sol[1] * m[0][0] - sol[0] * m[1][0]) / det;
19     res = out[0] + q[0] * L[0] + q[1] * L[1];
20     radius = (res - out[0]).norm();
21     break;
22 case 4:
23     q[0] = out[1] - out[0]; q[1] = out[2] - out[0]; q[2] = out[3] - out[0];
24     for (i = 0; i < 3; ++i) for (j = 0; j < 3; ++j) m[i][j] = dot(q[i], q[j]) * 2;
25     for (i = 0; i < 3; ++i) sol[i] = dot(q[i], q[i]);
26     det = m[0][0] * m[1][1] * m[2][2] + m[0][1] * m[1][2] * m[2][0]
27           + m[0][2] * m[2][1] * m[1][0] - m[0][2] * m[1][1] * m[2][0]
28           - m[0][1] * m[1][0] * m[2][2] - m[0][0] * m[1][2] * m[2][1];
29     if (sign(det) == 0) return;
30     for (j = 0; j < 3; ++j) { for (i = 0; i < 3; ++i) m[i][j] = sol[i];
31         L[j] = (m[0][0] * m[1][1] * m[2][2] + m[0][1] * m[1][2] * m[2][0]
32               + m[0][2] * m[2][1] * m[1][0] - m[0][2] * m[1][1] * m[2][0]
33               - m[0][1] * m[1][0] * m[2][2] - m[0][0] * m[1][2] * m[2][1]) / det;
34         for (i = 0; i < 3; ++i) m[i][j] = dot(q[i], q[j]) * 2;
35     } res = out[0];
36     for (i = 0; i < 3; ++i) res += q[i] * L[i]; radius = (res - out[0]).norm();
37 }
38 }
39 void minball(int n, point pt[]) {
40     ball();
41     if (outCnt < 4) for (int i = 0; i < n; ++i)
42         if ((res - pt[i]).norm() > +radius + EPS) {
43             out[outCnt] = pt[i]; ++outCnt; minball(i, pt); --outCnt;
44             if (i > 0) {
45                 point Tt = pt[i];
46                 memmove(&pt[1], &pt[0], sizeof(point) * i);
47                 pt[0] = Tt;
48             }
49         }
50 }
51 pair<point, double> main(int npoint, point pt[]) { // 0-based

```

```

52     random_shuffle(pt, pt + npoint); radius = -1;
53     for (int i = 0; i < npoint; i++) { if ((res - pt[i]).norm() > EPS + radius) {
54         outCnt = 1; out[0] = pt[i]; minball(i, pt); } }
55     return make_pair(res, sqrt(radius));
56 }

```

1.19 三维向量操作矩阵

- 绕单位向量 $u = (u_x, u_y, u_z)$ 右手方向旋转 θ 度的矩阵:

$$\begin{bmatrix} \cos\theta + u_x^2(1-\cos\theta) & u_x u_y(1-\cos\theta) - u_z \sin\theta & u_x u_z(1-\cos\theta) + u_y \sin\theta \\ u_y u_x(1-\cos\theta) + u_z \sin\theta & \cos\theta + u_y^2(1-\cos\theta) & u_y u_z(1-\cos\theta) - u_x \sin\theta \\ u_z u_x(1-\cos\theta) - u_y \sin\theta & u_z u_y(1-\cos\theta) + u_x \sin\theta & \cos\theta + u_z^2(1-\cos\theta) \end{bmatrix}$$

$$= \cos\theta I + \sin\theta \begin{bmatrix} 0 & -u_z & u_y \\ u_z & 0 & -u_x \\ -u_y & u_x & 0 \end{bmatrix} + (1-\cos\theta) \begin{bmatrix} u_x^2 & u_x u_y & u_x u_z \\ u_y u_x & u_y^2 & u_y u_z \\ u_z u_x & u_z u_y & u_z^2 \end{bmatrix}$$

- 点 a 绕单位向量 $u = (u_x, u_y, u_z)$ 右手方向旋转 θ 度的对应点为 $a' = a \cos\theta + (u \times a) \sin\theta + (u \otimes u)a(1-\cos\theta)$

- 关于向量 v 作对称变换的矩阵 $H = I - 2 \frac{vv^T}{v^T v}$,

- 点 a 对称点: $a' = a - 2 \frac{v^T a}{v^T v} \cdot v$

1.20 立体角

对于任意一个四面体 $OABC$, 从 O 点观察 $\triangle ABC$ 的立体角 $\tan \frac{\Omega}{2} = \frac{\text{mix}(\vec{a}, \vec{b}, \vec{c})}{|a||b||c| + (\vec{a} \cdot \vec{b})|c| + (\vec{a} \cdot \vec{c})|b| + (\vec{b} \cdot \vec{c})|a|}$.

2 数据结构

2.1 动态凸包 (只支持插入)

```

1 #define x first // upperHull ← (x,y)
2 #define y second // lowerHull ← (x,-y)
3 typedef map<int, int> mii;
4 typedef map<int, int>::iterator mit;
5 struct point { point(const mit &p): x(p->first), y(p->second) {} };
6 inline bool checkInside(mii &a, const point &p) { // border inclusive
7     int x = p.x, y = p.y; mit p1 = a.lower_bound(x);
8     if (p1 == a.end()) return false; if (p1->x == x) return y <= p1->y;
9     if (p1 == a.begin()) return false; mit p2(p1--);
10    return sign(det(p - point(p1), point(p2) - p)) >= 0;
11 } inline void addPoint(mii &a, const point &p) { // no collinear points
12     int x = p.x, y = p.y; mit pnt = a.insert(make_pair(x, y)).first, p1, p2;
13     for (pnt->y = y; ; a.erase(p2)) {
14         p1 = pnt; if (++p1 == a.end()) break;
15         p2 = p1; if (++p1 == a.end()) break;
16         if (det(point(p2) - p, point(p1) - p) < 0) break;

```

```

17     } for ( ; ; a.erase(p2)) {
18         if ((p1 = pnt) == a.begin()) break;
19         if (--p1 == a.begin()) break; p2 = p1--;
20         if (det(point(p2) - p, point(p1) - p) > 0) break;
21     }
22 }

```

2.2 Rope 用法

```

1 #include <ext/rope>
2 using __gnu_cxx::crope; using __gnu_cxx::rope;
3 a = b.substr(from, len);          // [from, from + len)
4 a = b.substr(from);              // [from, from]
5 b.c_str();                       // might lead to memory leaks
6 b.delete_c_str();               // delete the c_str that created before
7 a.insert(p, str);                // insert str before position p
8 a.erase(i, n);                  // erase [i, i + n)

```

2.3 Treap

```

1 struct node { int key, prio, size; node *ch[2]; } base[MAXN], *top, *root, *null, nil;
2 typedef node *tree;
3 tree newNode(int key) {
4     static int seed = 3312;
5     top->key = key; top->prio = seed = int(seed * 48271LL % 2147483647);
6     top->size = 1; top->ch[0] = top->ch[1] = null; return top++;
7 }
8 void Rotate(tree &x, int d) {
9     tree y = x->ch[!d]; x->ch[!d] = y->ch[d]; y->ch[d] = x; y->size = x->size;
10    x->size = x->ch[0]->size + 1 + x->ch[1]->size; x = y;
11 }
12 void Insert(tree &t, int key) {
13     if (t == null) t = newNode(key);
14     else { int d = t->key < key; Insert(t->ch[d], key); ++t->size;
15         if (t->ch[d]->prio < t->prio) Rotate(t, !d);
16     }
17 }
18 void Delete(tree &t, int key) {
19     if (t->key != key) { Delete(t->ch[t->key < key], key); --t->size; }
20     else if (t->ch[0] == null) t = t->ch[1];
21     else if (t->ch[1] == null) t = t->ch[0];
22     else { int d = t->ch[0]->prio < t->ch[1]->prio;
23         Rotate(t, d); Delete(t->ch[d], key); --t->size;
24     }
25 }

```

2.4 可持久化 Treap

```

1 inline bool randomBySize(int a, int b) {
2     static long long seed = 1;
3     return (seed = seed * 48271 % 2147483647) * (a + b) < 2147483647LL * a;
4 }
5 tree merge(tree x, tree y) {
6     if (x == null) return y; if (y == null) return x;
7     tree t = NULL;
8     if (randomBySize(x->size, y->size)) t = newNode(x), t->r = merge(x->r, y);
9     else t = newNode(y), t->l = merge(x, y->l);
10    update(t); return t;
11 }
12 void splitByKey(tree t, int k, tree &l, tree &r) { //  $[-\infty, k)[k, +\infty)$ 
13     if (t == null) l = r = null;
14     else if (t->key < k) l = newNode(t), splitByKey(t->r, k, l->r, r), update(l);
15     else r = newNode(t), splitByKey(t->l, k, l, r->l), update(r);
16 }
17 void splitBySize(tree t, int k, tree &l, tree &r) { //  $[1, k)[k, +\infty)$ 
18     static int s; if (t == null) l = r = null;
19     else if ((s = t->l->size + 1) < k) l = newNode(t), splitBySize(t->r, k - s, l->r, r),
20         update(l);
21     else r = newNode(t), splitBySize(t->l, k, l, r->l),
22         update(r);
23 }

```

2.5 左偏树

```

1 tree merge(tree a, tree b) {
2     if (a == null) return b;
3     if (b == null) return a;
4     if (a->key > b->key) swap(a, b);
5     a->rc = merge(a->rc, b);
6     a->rc->fa = a;
7     if (a->lc->dist < a->rc->dist) swap(a->lc, a->rc);
8     a->dist = a->rc->dist + 1;
9     return a;
10 }
11 void erase(tree t) {
12     tree x = t->fa, y = merge(t->lc, t->rc);
13     if (y != null) y->fa = x;
14     if (x == null) root = y;
15     else
16     for ((x->lc == t ? x->lc : x->rc) = y; x != null; y = x, x = x->fa) {
17         if (x->lc->dist < x->rc->dist) swap(x->lc, x->rc);
18         if (x->rc->dist + 1 == x->dist) return;
19         x->dist = x->rc->dist + 1;
20     }

```

21 }

2.6 Link-Cut Tree

```

1 struct node { int rev; node *pre, *ch[2]; } base[MAXN], nil, *null;
2 typedef node *tree;
3 #define isRoot(x) (x->pre->ch[0] != x && x->pre->ch[1] != x)
4 #define isRight(x) (x->pre->ch[1] == x)
5 inline void MakeRev(tree t) { if (t != null) { t->rev ^= 1; swap(t->ch[0], t->ch[1]); } }
6 inline void PushDown(tree t) { if (t->rev) { MakeRev(t->ch[0]); MakeRev(t->ch[1]); t->rev
    = 0; } }
7 inline void Rotate(tree x) {
8     tree y = x->pre; PushDown(y); PushDown(x);
9     int d = isRight(x);
10    if (!isRoot(y)) y->pre->ch[isRight(y)] = x; x->pre = y->pre;
11    if ((y->ch[d] = x->ch[!d]) != null) y->ch[d]->pre = y;
12    x->ch[!d] = y; y->pre = x; Update(y);
13 }
14 inline void Splay(tree x) {
15     PushDown(x); for (tree y; !isRoot(x); Rotate(x)) {
16         y = x->pre; if (!isRoot(y)) Rotate(isRight(x) != isRight(y) ? x : y);
17     } Update(x);
18 }
19 inline void Splay(tree x, tree to) {
20     PushDown(x); for (tree y; (y = x->pre) != to; Rotate(x)) if (y->pre != to)
21         Rotate(isRight(x) != isRight(y) ? x : y);
22     Update(x);
23 }
24 inline tree Access(tree t) {
25     tree last = null; for (; t != null; last = t, t = t->pre) Splay(t), t->ch[1] = last,
        Update(t);
26     return last;
27 }
28 inline void MakeRoot(tree t) { Access(t); Splay(t); MakeRev(t); }
29 inline tree FindRoot(tree t) { Access(t); Splay(t); tree last = null;
30     for (; t != null; last = t, t = t->ch[0]) PushDown(t); Splay(last); return last;
31 }
32 inline void Join(tree x, tree y) { MakeRoot(y); y->pre = x; }
33 inline void Cut(tree t) { Access(t); Splay(t); t->ch[0]->pre = null; t->ch[0] = null;
    Update(t); }
34 inline void Cut(tree x, tree y) {
35     tree upper = (Access(x), Access(y));
36     if (upper == x) { Splay(x); y->pre = null; x->ch[1] = null; Update(x); }
37     else if (upper == y) { Access(x); Splay(y); x->pre = null; y->ch[1] = null; Update(y);
        }
38     else assert(0); // impossible to happen
39 }
40 inline int Query(tree a, tree b) { // query the cost in path a <-> b, lca inclusive

```

```

41     Access(a); tree c = Access(b); // c is lca
42     int v1 = c->ch[1]->maxCost; Access(a);
43     int v2 = c->ch[1]->maxCost;
44     return max(max(v1, v2), c->cost);
45 }
46 void Init() {
47     null = &nil; null->ch[0] = null->ch[1] = null->pre = null; null->rev = 0;
48     Rep(i, 1, N) { node &n = base[i]; n.rev = 0; n.pre = n.ch[0] = n.ch[1] = null; }
49 }

```

2.7 K-D Tree Nearest

```

1 struct Point { int x, y; };
2 struct Rectangle {
3     int lx, rx, ly, ry;
4     void set(const Point &p) { lx = rx = p.x; ly = ry = p.y; }
5     void merge(const Point &o) {
6         lx = min(lx, o.x); rx = max(rx, o.x); ly = min(ly, o.y); ry = max(ry, o.y);
7     } void merge(const Rectangle &o) {
8         lx = min(lx, o.lx); rx = max(rx, o.rx); ly = min(ly, o.ly); ry = max(ry, o.ry);
9     } LL dist(const Point &p) {
10         LL res = 0;
11         if (p.x < lx) res += sqr(lx - p.x); else if (p.x > rx) res += sqr(p.x - rx);
12         if (p.y < ly) res += sqr(ly - p.y); else if (p.y > ry) res += sqr(p.y - ry);
13         return res;
14     }
15 };
16 struct Node { int child[2]; Point p; Rectangle rect; };
17 const int MAX_N = 1111111;
18 const LL INF = 100000000;
19 int n, m, tot, root; LL result;
20 Point a[MAX_N], p; Node tree[MAX_N];
21 int build(int s, int t, bool d) {
22     int k = ++tot, mid = (s + t) >> 1;
23     nth_element(a + s, a + mid, a + t, d ? cmpXY : cmpYX);
24     tree[k].p = a[mid]; tree[k].rect.set(a[mid]); tree[k].child[0] = tree[k].child[1] = 0;
25     if (s < mid)
26         tree[k].child[0] = build(s, mid, d ^ 1), tree[k].rect.merge(tree[tree[k].child[0]].
            rect);
27     if (mid + 1 < t)
28         tree[k].child[1] = build(mid + 1, t, d ^ 1), tree[k].rect.merge(tree[tree[k].child
            [1]].rect);
29     return k;
30 }
31 int insert(int root, bool d) {
32     if (root == 0) {
33         tree[++tot].p = p; tree[tot].rect.set(p); tree[tot].child[0] = tree[tot].child[1] =
            0;

```

```

34     return tot;
35 } tree[root].rect.merge(p);
36 if ((d && cmpXY(p, tree[root].p)) || (!d && cmpYX(p, tree[root].p)))
37     tree[root].child[0] = insert(tree[root].child[0], d ^ 1);
38 else tree[root].child[1] = insert(tree[root].child[1], d ^ 1);
39 return root;
40 }
41 void query(int k, bool d) {
42     if (tree[k].rect.dist(p) >= result) return;
43     cMin(result, dist(tree[k].p, p));
44     if ((d && cmpXY(p, tree[k].p)) || (!d && cmpYX(p, tree[k].p))) {
45         if (tree[k].child[0]) query(tree[k].child[0], d ^ 1);
46         if (tree[k].child[1]) query(tree[k].child[1], d ^ 1);
47     } else {
48         if (tree[k].child[1]) query(tree[k].child[1], d ^ 1);
49         if (tree[k].child[0]) query(tree[k].child[0], d ^ 1);
50     }
51 }
52 void example(int n) {
53     root = tot = 0; scan(a); root = build(0, n, 0); // init, a[0...n-1]
54     scan(p); root = insert(root, 0); // insert
55     scan(p); result = INF; ans = query(root, 0); // query
56 }

```

2.8 K-D Tree Farthest

输入 n 个点, 对每个询问 px, py, k , 输出 k 远点的编号

```

1 struct Point { int x, y, id; };
2 struct Rectangle {
3     int lx, rx, ly, ry;
4     void set(const Point &p) { lx = rx = p.x; ly = ry = p.y; }
5     void merge(const Rectangle &o) {
6         lx = min(lx, o.lx); rx = max(rx, o.rx); ly = min(ly, o.ly); ry = max(ry, o.ry);
7     }
8     LL dist(const Point &p) { LL res = 0;
9         res += max(sqr(rx - p.x), sqr(lx - p.x));
10        res += max(sqr(ry - p.y), sqr(ly - p.y));
11        return res;
12    }
13 }; struct Node { Point p; Rectangle rect; };
14 const int MAX_N = 111111;
15 const LL INF = 1LL << 60;
16 int n, m;
17 Point a[MAX_N], b[MAX_N];
18 Node tree[MAX_N * 3];
19 Point p; // p is the query point
20 pair<LL, int> result[22];

```

```

21 void build(int k, int s, int t, bool d) {
22     int mid = (s + t) >> 1;
23     nth_element(a + s, a + mid, a + t, d ? cmpX : cmpY);
24     tree[k].p = a[mid];
25     tree[k].rect.set(a[mid]);
26     if (s < mid)
27         build(k << 1, s, mid, d ^ 1), tree[k].rect.merge(tree[k << 1].rect);
28     if (mid + 1 < t)
29         build(k << 1 | 1, mid + 1, t, d ^ 1), tree[k].rect.merge(tree[k << 1 | 1].rect);
30 }
31 void query(int k, int s, int t, bool d, int kth) {
32     if (tree[k].rect.dist(p) < result[kth].first) return;
33     pair<LL, int> tmp(dist(tree[k].p, p), -tree[k].p.id);
34     for (int i = 1; i <= kth; i++) if (tmp > result[i]) {
35         for (int j = kth + 1; j > i; j--) result[j] = result[j - 1]; result[i] = tmp;
36         break;
37     }
38     int mid = (s + t) >> 1;
39     if ((d && cmpX(p, tree[k].p)) || (!d && cmpY(p, tree[k].p))) {
40         if (mid + 1 < t) query(k << 1 | 1, mid + 1, t, d ^ 1, kth);
41         if (s < mid) query(k << 1, s, mid, d ^ 1, kth);
42     } else {
43         if (s < mid) query(k << 1, s, mid, d ^ 1, kth);
44         if (mid + 1 < t) query(k << 1 | 1, mid + 1, t, d ^ 1, kth);
45     }
46 }
47 void example(int n) {
48     scan(a); build(1, 0, n, 0); // init, a[0...n-1]
49     scan(p, k); // query
50     Rep(j, 1, k) result[j].first = -1;
51     query(1, 0, n, 0, k); ans = -result[k].second + 1;
52 }

```

2.9 树链剖分

```

1 int N, fa[MAXN], dep[MAXN], que[MAXN], size[MAXN], own[MAXN];
2 int LCA(int x, int y) { if (x == y) return x;
3     for (; x = fa[own[x]]; ) {
4         if (dep[x] < dep[y]) swap(x, y); if (own[x] == own[y]) return y;
5         if (dep[own[x]] < dep[own[y]]) swap(x, y);
6     } return -1;
7 }
8 void Decomposition() {
9     static int path[MAXN]; int x, y, a, next, head = 0, tail = 0, cnt; // BFS omitted
10    for (int i = 1; i <= N; ++i) if (own[a = que[i]] == -1)
11        for (x = a, cnt = 0; ; x = next) { next = -1; own[x] = a; path[++cnt] = x;
12            for (edge e(fir[x]); e; e = e->next) if ( (y = e->to) != fa[x] )
13                if (next == -1 || size[y] > size[next]) next = y;

```



```

14     if (next == -1) { tree[a].init(cnt, path); break; }
15     }
16 }

```

3 字符串相关

3.1 Manacher

```

1 // len[i] : the max length of palindrome whose mid point is (i / 2)
2 void Manacher(int n, char cs[], int len[]) { // 0-based, len[] must be double sized
3     for (int i = 0; i < n + n; ++i) len[i] = 0;
4     for (int i = 0, j = 0, k; i < n * 2; i += k, j = max(j - k, 0)) {
5         while (i - j >= 0 && i + j + 1 < n * 2 && cs[(i - j) / 2] == cs[(i + j + 1) / 2]) j
            ++;
6         len[i] = j; for (k = 1; i - k >= 0 && j - k >= 0 && len[i - k] != j - k; k++)
7             len[i + k] = min(len[i - k], j - k);
8     }
9 }

```

3.2 KMP

$next[i] = \max\{len|A[0 \dots len - 1] = A \text{ 的第 } i \text{ 位向前或后的长度为 } len \text{ 的串}\}$

$ext[i] = \max\{len|A[0 \dots len - 1] = B \text{ 的第 } i \text{ 位向前或后的长度为 } len \text{ 的串}\}$

```

1 void KMP(char *a, int la, char *b, int lb, int *next, int *ext) {
2     --a; --b; --next; --ext;
3     for (int i = 2, j = next[1] = 0; i <= la; i++) {
4         while (j && a[j + 1] != a[i]) j = next[j]; if (a[j + 1] == a[i]) ++j; next[i] = j;
5     } for (int i = 1, j = 0; i <= lb; ++i) {
6         while (j && a[j + 1] != b[i]) j = next[j]; if (a[j + 1] == b[i]) ++j; ext[i] = j;
7         if (j == la) j = next[j];
8     }
9 } void ExKMP(char *a, int la, char *b, int lb, int *next, int *ext) {
10     next[0] = la; for (int &j = next[1] = 0; j + 1 < la && a[j] == a[j + 1]; ++j);
11     for (int i = 2, k = 1; i < la; ++i) {
12         int p = k + next[k], l = next[i - k]; if (l < p - i) next[i] = l;
13         else for (int &j = next[k = i] = max(0, p - i); i + j < la && a[j] == a[i + j]; ++j);
14     } for (int &j = ext[0] = 0; j < la && j < lb && a[j] == b[j]; ++j);
15     for (int i = 1, k = 0; i < lb; ++i) {
16         int p = k + ext[k], l = next[i - k]; if (l < p - i) ext[i] = l;
17         else for (int &j = ext[k = i] = max(0, p - i); j < la && i + j < lb && a[j] == b[i +
            j]; ++j);
18     }
19 }

```

3.3 Aho-Corasick 自动机

```

1 void construct() {
2     static tree Q[MAX_NODE]; int head = 0, tail = 0;
3     for (root->fail = root, Q[++tail] = root; head < tail; ) {
4         tree x = Q[++head];
5         // if (x->fail->danger) x->danger = true;
6         Rep(d, 0, sigma - 1) if (!x->next[d])
7             x->next[d] = (x == root) ? (root) : (x->fail->next[d]);
8         else {
9             x->next[d]->fail = (x == root) ? (root) : (x->fail->next[d]);
10            Q[++tail] = x->next[d];
11        }
12    }
13 }

```

3.4 后缀自动机

```

1 struct node { int len; node *fa, *go[26]; } base[MAXNODE], *top = base, *root, *que[
    MAXNODE];
2 typedef node *tree;
3 inline tree newNode(int len) {
4     top->len = len; top->fa = NULL; memset(top->go, 0, sizeof(top->go)); return top++;
5 } inline tree newNode(int len, tree fa, tree *go) {
6     top->len = len; top->fa = fa; memcpy(top->go, go, sizeof(top->go)); return top++;
7 } void construct(char *A, int N) {
8     tree p = root = newNode(0), q, up, fa;
9     for (int i = 0; i < N; ++i) {
10        int w = A[i] - 'a'; up = p; p = newNode(i + 1);
11        for ( ; up && !up->go[w]; up = up->fa) up->go[w] = p;
12        if (!up) p->fa = root;
13        else { q = up->go[w];
14            if (up->len + 1 == q->len) p->fa = q;
15            else { fa = newNode(up->len + 1, q->fa, q->go);
16                for (p->fa = q->fa = fa; up && up->go[w] == q; up = up->fa) up->go[w] = fa;
17            }
18        }
19    } static int cnt[MAXLEN]; memset(cnt, 0, sizeof(int) * (N + 1));
20    for (tree i(base); i != top; ++i) ++cnt[i->len];
21    Rep(i, 1, N) cnt[i] += cnt[i - 1];
22    for (tree i(base); i != top; ++i) Q[ cnt[i->len]-- ] = i;
23 }

```

3.5 后缀数组

待排序的字符串放在 $r[0 \dots n - 1]$ 中, 最大值小于 m .

$r[0 \dots n - 2] > 0, r[n - 1] = 0$.

结果放在 $sa[0 \dots n - 1]$.

```

1 namespace SuffixArrayDoubling {
2     int wa[MAXN], wb[MAXN], wv[MAXN], ws[MAXN];
3     int cmp(int *r, int a, int b, int l) {
4         return r[a] == r[b] && r[a + 1] == r[b + 1];
5     }
6     void da(int *r, int *sa, int n, int m) {
7         int i, j, p, *x = wa, *y = wb, *t;
8         for (i = 0; i < m; i++) ws[i] = 0;
9         for (i = 0; i < n; i++) ws[x[i]] = r[i]++;
10        for (i = 1; i < m; i++) ws[i] += ws[i - 1];
11        for (i = n - 1; i >= 0; i--) sa[--ws[x[i]]] = i;
12        for (j = 1, p = 1; p < n; j *= 2, m = p) {
13            for (p = 0, i = n - j; i < n; i++) y[p++] = i;
14            for (i = 0; i < n; i++) if (sa[i] >= j) y[p++] = sa[i] - j;
15            for (i = 0; i < n; i++) wv[i] = x[y[i]];
16            for (i = 0; i < m; i++) ws[i] = 0;
17            for (i = 0; i < n; i++) ws[wv[i]]++;
18            for (i = 1; i < m; i++) ws[i] += ws[i - 1];
19            for (i = n - 1; i >= 0; i--) sa[--ws[wv[i]]] = y[i];
20            for (t = x, x = y, y = t, p = 1, x[sa[0]] = 0, i = 1; i < n; i++)
21                x[sa[i]] = cmp(y, sa[i - 1], sa[i], j) ? p - 1 : p++;
22        }
23    }
24 }
25 namespace SuffixArrayDC3 { // r 与 sa 大小需 3 倍
26     #define F(x) ((x) / 3 + ((x) % 3 == 1 ? 0 : tb))
27     #define G(x) ((x) < tb ? (x) * 3 + 1 : ((x) - tb) * 3 + 2)
28     int wa[MAXN], wb[MAXN], wv[MAXN], ws[MAXN];
29     int c0(int *r, int a, int b) {
30         return r[a] == r[b] && r[a + 1] == r[b + 1] && r[a + 2] == r[b + 2];
31     }
32     int c12(int k, int *r, int a, int b) {
33         if (k == 2) return r[a] < r[b] || (r[a] == r[b] && c12(1, r, a + 1, b + 1));
34         else return r[a] < r[b] || (r[a] == r[b] && wv[a + 1] < wv[b + 1]);
35     }
36     void sort(int *r, int *a, int *b, int n, int m) {
37         for (int i = 0; i < n; i++) wv[i] = r[a[i]];
38         for (int i = 0; i < m; i++) ws[i] = 0;
39         for (int i = 0; i < n; i++) ws[wv[i]]++;
40         for (int i = 1; i < m; i++) ws[i] += ws[i - 1];
41         for (int i = n - 1; i >= 0; i--) b[--ws[wv[i]]] = a[i];
42     }
43     void dc3(int *r, int *sa, int n, int m) {
44         int i, j, *rn = r + n, *san = sa + n, ta = 0, tb = (n + 1) / 3, tbc = 0, p;
45         r[n] = r[n + 1] = 0;
46         for (i = 0; i < n; i++) if (i % 3 != 0) wa[tbc++] = i;

```

```

47         sort(r + 2, wa, wb, tbc, m);
48         sort(r + 1, wb, wa, tbc, m);
49         sort(r, wa, wb, tbc, m);
50         for (p = 1, rn[F(wb[0])] = 0, i = 1; i < tbc; i++)
51             rn[F(wb[i])] = c0(r, wb[i - 1], wb[i]) ? p - 1 : p++;
52         if (p < tbc) dc3(rn, san, tbc, p);
53         else for (i = 0; i < tbc; i++) san[rn[i]] = i;
54         for (i = 0; i < tbc; i++) if (san[i] < tb) wb[ta++] = san[i] * 3;
55         if (n % 3 == 1) wb[ta++] = n - 1;
56         sort(r, wb, wa, ta, m);
57         for (i = 0; i < tbc; i++) wv[wb[i] = G(san[i])] = i;
58         for (i = 0, j = 0, p = 0; i < ta && j < tbc; p++)
59             sa[p] = c12(wb[j] % 3, r, wa[i], wb[j]) ? wa[i++] : wb[j++];
60         for (; i < ta; p++) sa[p] = wa[i++];
61         for (; j < tbc; p++) sa[p] = wb[j++];
62     }
63     #undef F
64     #undef G
65 }
66 namespace CalcHeight {
67     int rank[MAXN], height[MAXN];
68     void calheight(int *r, int *sa, int n) {
69         int i, j, k = 0;
70         for (i = 1; i <= n; i++) rank[sa[i]] = i;
71         for (i = 0; i < n; height[rank[i++]] = k)
72             for (k ? k-- : 0, j = sa[rank[i] - 1]; r[i + k] == r[j + k]; k++);
73     }
74 }

```

3.6 环串最小表示

```

1 int minimalRepresentation(int N, char *s) { // s must be double-sized and 0-based
2     int i, j, k, l; for (i = 0; i < N; ++i) s[i + N] = s[i]; s[N + N] = 0;
3     for (i = 0, j = 1; j < N; ) {
4         for (k = 0; k < N && s[i + k] == s[j + k]; ++k);
5         if (k >= N) break; if (s[i + k] < s[j + k]) j += k + 1;
6         else l = i + k, i = j, j = max(l, j) + 1;
7     } return i; // [i, i + N) is the minimal representation
8 }

```

3.7 回文自动机

```

1 #include <cstdlib>
2 #include <cstdio>
3 #include <cstring>
4 #include <algorithm>

```

```

5
6 const int C = 26;
7 const int N = 100000;
8 const int S = N + 2 + C;
9
10 char string[N + 2];
11 int s, length[S], suffix[S], go[S][C];
12
13 int extend(int p, int i) {
14     while (string[i - 1 - length[p]] != string[i]) {
15         p = suffix[p];
16     }
17     int q = suffix[p];
18     while (string[i - 1 - length[q]] != string[i]) {
19         q = suffix[q];
20     }
21     int c = string[i] - 'a';
22     int pp = go[p][c];
23     int qq = go[q][c];
24     if (pp == -1) {
25         length[pp] = go[p][c] = s ++ = length[p] + 2;
26         suffix[pp] = qq;
27         memset(go[pp], -1, sizeof(go[pp]));
28     }
29     return pp;
30 }
31
32 int main() {
33     int tests;
34     scanf("%d", &tests);
35     for (int t = 1; t <= tests; ++t) {
36         printf("Case_%d: ", t);
37         for (int i = 0; i < C + 2; ++i) {
38             suffix[i] = 1;
39             length[i] = std::min(i - 1, 1);
40             memset(go[i], -1, sizeof(go[i]));
41         }
42         suffix[0] = suffix[1] = 0;
43         for (int i = 0; i < C; ++i) {
44             go[0][i] = 2 + i;
45         }
46         s = C + 2;
47         string[0] = '#';
48         scanf("%s", string + 1);
49         int n = strlen(string + 1);
50         int p = 0;
51         for (int i = 1; i <= n; ++i) {
52             p = extend(p, i);
53         }

```

```

54     int result = s - (C + 2);
55     std::sort(string + 1, string + n + 1);
56     result += std::unique(string + 1, string + n + 1) - string - 1;
57     printf("%d\n", result);
58 }
59 return 0;
60 }

```

4 图论

4.1 带花树

```

1 namespace Blossom {
2     int n, head, tail, S, T, lca;
3     int match[MAXN], Q[MAXN], pred[MAXN], label[MAXN], inq[MAXN], inb[MAXN];
4     vector<int> link[MAXN];
5     inline void push(int x) { Q[tail++] = x; inq[x] = true; }
6     int findCommonAncestor(int x, int y) {
7         static bool inPath[MAXN]; for (int i = 0; i < n; ++i) inPath[i] = 0;
8         for ( ; ; x = pred[ match[x] ]) { x = label[x]; inPath[x] = true; if (x == S) break;
9             }
10         for ( ; ; y = pred[ match[y] ]) { y = label[y]; if (inPath[y]) break; } return y;
11     }
12     void resetTrace(int x, int lca) {
13         while (label[x] != lca) { int y = match[x]; inb[ label[x] ] = inb[ label[y] ] = true;
14             x = pred[y]; if (label[x] != lca) pred[x] = y; }
15     void blossomContract(int x, int y) {
16         lca = findCommonAncestor(x, y);
17         Foru(i, 0, n) inb[i] = 0; resetTrace(x, lca); resetTrace(y, lca);
18         if (label[x] != lca) pred[x] = y; if (label[y] != lca) pred[y] = x;
19         Foru(i, 0, n) if (inb[ label[i] ]) { label[i] = lca; if (!inq[i]) push(i); }
20     }
21     bool findAugmentingPath() {
22         Foru(i, 0, n) pred[i] = -1, label[i] = i, inq[i] = 0;
23         int x, y, z; head = tail = 0;
24         for (push(S); head < tail; ) for (int i = (int)link[x = Q[head++]].size() - 1; i >=
25             0; --i) {
26             y = link[x][i]; if (label[x] == label[y] || x == match[y]) continue;
27             if (y == S || (match[y] >= 0 && pred[ match[y] ] >= 0)) blossomContract(x, y);
28             else if (pred[y] == -1) {
29                 pred[y] = x; if (match[y] >= 0) push(match[y]);
30                 else {
31                     for (x = y; x >= 0; x = z) {
32                         y = pred[x], z = match[y]; match[x] = y, match[y] = x;
33                     } return true; } } } return false;
34     }
35     int findMaxMatching() {

```

```

34     int ans = 0; Foru(i, 0, n) match[i] = -1;
35     for (S = 0; S < n; ++S) if (match[S] == -1) if (findAugmentingPath()) ++ans;
36     return ans;
37 }
38 }

```

4.2 最大流

```

1 namespace Maxflow {
2     int h[MAXNODE], vh[MAXNODE], S, T, Ncnt; edge cur[MAXNODE], pe[MAXNODE];
3     void init(int _S, int _T, int _Ncnt) { S = _S; T = _T; Ncnt = _Ncnt; }
4     int maxflow() {
5         static int Q[MAXNODE]; int x, y, augc, flow = 0, head = 0, tail = 0; edge e;
6         Rep(i, 0, Ncnt) cur[i] = fir[i]; Rep(i, 0, Ncnt) h[i] = INF; Rep(i, 0, Ncnt) vh[i] =
            0;
7         for (Q[++tail] = T, h[T] = 0; head < tail; ) {
8             x = Q[++head]; ++vh[ h[x] ];
9             for (e = fir[x]; e; e = e->next) if (e->op->c)
10                 if (h[y = e->to] >= INF) h[y] = h[x] + 1, Q[++tail] = y;
11         } for (x = S; h[S] < Ncnt; ) {
12             for (e = cur[x]; e; e = e->next) if (e->c)
13                 if (h[y = e->to] + 1 == h[x]) { cur[x] = pe[y] = e; x = y; break; }
14             if (!e) {
15                 if (--vh[ h[x] ] == 0) break; h[x] = Ncnt; cur[x] = NULL;
16                 for (e = fir[x]; e; e = e->next) if (e->c)
17                     if ( cMin( h[x], h[e->to] + 1 ) ) cur[x] = e;
18                 ++vh[ h[x] ];
19                 if (x != S) x = pe[x]->op->to;
20             } else if (x == T) { augc = INF;
21                 for (x = T; x != S; x = pe[x]->op->to) cMin(augc, pe[x]->c);
22                 for (x = T; x != S; x = pe[x]->op->to) {
23                     pe[x]->c -= augc; pe[x]->op->c += augc;
24                 } flow += augc;
25             }
26         } return flow;
27     }
28 }

```

4.3 最高标号预流推进

```

1 namespace Network {
2     int S, T, Ncnt, hsize, heap[MAXN], h[MAXN], inq[MAXN], Q[MAXN], vh[MAXN * 2 + 1];
3     LL E[MAXN]; edge cur[MAXN];
4     inline void pushFlow(int x, int y, edge e) {
5         int d = (int)min(E[x], (LL)e->c);
6         E[x] -= d; e->c -= d; E[y] += d; e->op->c += d;

```

```

7     } inline bool heapCmp(int x, int y) { return h[x] < h[y]; }
8     inline void hpush(int x) {
9         inq[x] = true; heap[++hsize] = x; push_heap(heap + 1, heap + hsize + 1, heapCmp);
10    } inline void hpop(int x) {
11        inq[x] = false; pop_heap(heap + 1, heap + hsize + 1, heapCmp); --hsize;
12    } LL maxFlow() {
13        int head = 0, tail = 0, x, y, h0;
14        memset(h, 63, sizeof(int) * (Ncnt + 1));
15        memset(vh, 0, sizeof(int) * (2 * Ncnt + 2));
16        memset(E, 0, sizeof(LL) * (Ncnt + 1));
17        memset(inq, 0, sizeof(int) * (Ncnt + 1));
18        memcpy(cur, fir, sizeof(edge) * (Ncnt + 1));
19        for (Q[++tail] = T, h[T] = 0; head < tail; )
20            for (edge e(fir[x = Q[++head]]); e; e = e->next) if (e->op->c)
21                if (h[y = e->to] >= INF) h[y] = h[x] + 1, Q[++tail] = y;
22        if (h[S] >= Ncnt) return 0;
23        h[S] = Ncnt; E[S] = LL_INF;
24        for (int i = 1; i <= Ncnt; ++i) if (h[i] <= Ncnt) ++vh[ h[i] ];
25        hsize = 0;
26        for (edge e(fir[S]); e; e = e->next) if (e->c && h[y = e->to] < Ncnt) {
27            pushFlow(S, y, e); if (!inq[y] && y != S && y != T) hpush(y);
28        } while (hsize) {
29            bool good = false;
30            for (edge &e(cur[x = heap[1]]); e; e = e->next) if (e->c)
31                if (h[x] == h[y = e->to] + 1) {
32                    good = true; pushFlow(x, y, e); if (E[x] == 0) hpop(x);
33                    if (inq[y] == false && y != S && y != T) hpush(y);
34                    break;
35                }
36            if (!good) { // relabel
37                hpop(x); --vh[ h0 = h[x] ];
38                int &minH = h[x] = INF; cur[x] = NULL;
39                for (edge e(fir[x]); e; e = e->next) if (e->c)
40                    if ( cMin(minH, h[e->to] + 1) ) cur[x] = fir[x];
41                hpush(x); ++vh[ h[x] ];
42                if (vh[h0] == 0 && h0 < Ncnt) {
43                    hsize = 0;
44                    for (int i = 1; i <= Ncnt; ++i) {
45                        if (h[i] > h0 && h[i] < Ncnt) --vh[ h[i] ], ++vh[ h[i] = Ncnt + 1 ];
46                        if (i != S && i != T && E[i]) heap[++hsize] = i;
47                    } make_heap(heap + 1, heap + hsize + 1, heapCmp);
48                }
49            }
50        } return E[T];
51    }
52 }

```

4.4 KM

```

1 int N, Tcnt, w[MAXN][MAXN], slack[MAXN];
2 int lx[MAXN], linkx[MAXN], visy[MAXN], ly[MAXN], linky[MAXN], visx[MAXN]; // 初值全为 0
3 bool DFS(int x) { visx[x] = Tcnt;
4     Rep(y, 1, N) if(visy[y] != Tcnt) { int t = lx[x] + ly[y] - w[x][y];
5         if (t == 0) { visy[y] = Tcnt;
6             if (!linky[y] || DFS(linky[y])) { linkx[x] = y; linky[y] = x; return true; }
7         } else cMin(slack[y], t);
8     } return false;
9 } void KM() {
10     Tcnt = 0; Rep(x, 1, N) Rep(y, 1, N) cMax(lx[x], w[x][y]);
11     Rep(S, 1, N) { Rep(i, 1, N) slack[i] = INF;
12         for (++Tcnt; !DFS(S); ++Tcnt) { int d = INF;
13             Rep(y, 1, N) if(visy[y] != Tcnt) cMin(d, slack[y]);
14             Rep(x, 1, N) if(visx[x] == Tcnt) lx[x] -= d;
15             Rep(y, 1, N) if(visy[y] == Tcnt) ly[y] += d; else slack[y] -= d;
16         }
17     }
18 }

```

4.5 2-SAT 与 Kosaraju

注意 Kosaraju 需要建反图

```

1 namespace SCC {
2     int code[MAXN * 2], seq[MAXN * 2], sCnt;
3     void DFS_1(int x) { code[x] = 1;
4         for (edge e(fir[x]); e; e = e->next) if (code[e->to] == -1) DFS_1(e->to);
5         seq[++sCnt] = x;
6     } void DFS_2(int x) { code[x] = sCnt;
7         for (edge e(fir2[x]); e; e = e->next) if (code[e->to] == -1) DFS_2(e->to); }
8     void SCC(int N) {
9         sCnt = 0; for (int i = 1; i <= N; ++i) code[i] = -1;
10        for (int i = 1; i <= N; ++i) if (code[i] == -1) DFS_1(i);
11        sCnt = 0; for (int i = 1; i <= N; ++i) code[i] = -1;
12        for (int i = N; i >= 1; --i) if (code[seq[i]] == -1) {
13            ++sCnt; DFS_2(seq[i]); }
14    }
15 } // true - 2i - 1
16 // false - 2i
17 bool TwoSat() { SCC::SCC(N + N);
18     // if code[2i - 1] = code[2i]: no solution
19     // if code[2i - 1] > code[2i]: i selected. else i not selected
20 }

```

4.6 全局最小割 Stoer-Wagner

```

1 int minCut(int N, int G[MAXN][MAXN]) { // 0-based
2     static int weight[MAXN], used[MAXN]; int ans = INT_MAX;
3     while (N > 1) {
4         for (int i = 0; i < N; ++i) used[i] = false; used[0] = true;
5         for (int i = 0; i < N; ++i) weight[i] = G[i][0];
6         int S = -1, T = 0;
7         for (int _r = 2; _r <= N; ++_r) { // N - 1 selections
8             int x = -1;
9             for (int i = 0; i < N; ++i) if (!used[i])
10                 if (x == -1 || weight[i] > weight[x]) x = i;
11             for (int i = 0; i < N; ++i) weight[i] += G[x][i];
12             S = T; T = x; used[x] = true;
13         } ans = min(ans, weight[T]);
14         for (int i = 0; i < N; ++i) G[i][S] += G[i][T], G[S][i] += G[i][T];
15         G[S][S] = 0; --N;
16         for (int i = 0; i <= N; ++i) swap(G[i][T], G[i][N]);
17         for (int i = 0; i < N; ++i) swap(G[T][i], G[N][i]);
18     } return ans;
19 }

```

4.7 Hopcroft-Karp

```

1 int N, M, level[MAXN], matchX[MAXN], matchY[MAXN];
2 bool used[MAXN];
3 bool DFS(int x) {
4     used[x] = true; for (edge e(fir[x]); e; e = e->next) {
5         int y = e->to, z = matchY[y];
6         if (z == -1 || (!used[z] && level[x] < level[z] && DFS(z))) {
7             matchX[x] = y; matchY[y] = x; return true;
8         }
9     } return false;
10 }
11 int maxMatch() {
12     for (int i = 0; i < N; ++i) used[i] = false;
13     for (int i = 0; i < N; ++i) matchX[i] = -1;
14     for (int i = 0; i < M; ++i) matchY[i] = -1;
15     for (int i = 0; i < N; ++i) level[i] = -1;
16     int match = 0, d;
17     for ( ; ; match += d) {
18         static int Q[MAXN * 2 + 1];
19         int head = 0, tail = d = 0;
20         for (int x = 0; x < N; ++x) level[x] = -1;
21         for (int x = 0; x < N; ++x) if (matchX[x] == -1)
22             level[x] = 0, Q[++tail] = x;
23         while (head < tail)
24             for (edge e(fir[x = Q[++head]]); e; e = e->next) {
25                 int y = e->to, z = matchY[y];

```

```

26         if (z != -1 && level[z] < 0) level[z] = level[x] + 1, Q[++tail] = z;
27     }
28     for (int x = 0; x < N; ++x) used[x] = false;
29     for (int x = 0; x < N; ++x) if (matchX[x] == -1) if (DFS(x)) ++d;
30     if (d == 0) break;
31 } return match;
32 }

```

4.8 欧拉路

```

1 vector<int> eulerianWalk(int N, int S) {
2     static int res[MAXN], stack[MAXN]; static edge cur[MAXN];
3     int rcnt = 0, top = 0, x; for (int i = 1; i <= N; ++i) cur[i] = fir[i];
4     for (stack[top++] = S; top; ) {
5         for (x = stack[--top]; ; ) {
6             edge &e = cur[x]; if (e == NULL) break;
7             stack[top++] = x; x = e->to; e = e->next;
8             // 对于无向图需要删掉反向边
9         } res[rcnt++] = x;
10    } reverse(res, res + rcnt); return vector<int>(res, res + rcnt);
11 }

```

4.9 稳定婚姻

```

1 namespace StableMatching {
2     int pairM[MAXN], pairW[MAXN], p[MAXN];
3     // init: pairM[0...n - 1] = pairW[0...n - 1] = -1, p[0...n - 1] = 0
4     void stableMatching(int n, int orderM[MAXN][MAXN], int preferW[MAXN][MAXN]) {
5         for (int i = 0; i < n; i++) while (pairM[i] < 0) {
6             int w = orderM[i][p[i]++], m = pairW[w];
7             if (m == -1) pairM[i] = w, pairW[w] = i;
8             else if (preferW[w][i] < preferW[w][m])
9                 pairM[m] = -1, pairM[i] = w, pairW[w] = i, i = m;
10        }
11    }
12 }

```

4.10 最大团搜索

```

1 namespace MaxClique { // 1-based
2     int g[MAXN][MAXN], len[MAXN], list[MAXN][MAXN], mc[MAXN], ans, found;
3     void DFS(int size) {
4         if (len[size] == 0) { if (size > ans) ans = size, found = true; return; }
5         for (int k = 0; k < len[size] && !found; ++k) {
6             if (size + len[size] - k <= ans) break;

```

```

7             int i = list[size][k]; if (size + mc[i] <= ans) break;
8             for (int j = k + 1, len[size + 1] = 0; j < len[size]; ++j) if (g[i][list[size][j]])
9                 list[size + 1][len[size + 1]++] = list[size][j];
10            DFS(size + 1);
11        }
12    }
13    int work(int n) {
14        mc[n] = ans = 1; for (int i = n - 1; i; --i) { found = false; len[i] = 0;
15            for (int j = i + 1; j <= n; ++j) if (g[i][j]) list[i][len[i]++] = j;
16            DFS(i); mc[i] = ans;
17        } return ans;
18    }
19 }

```

4.11 极大团计数

```

1 namespace MaxCliqueCounting {
2     int n, ans;
3     int ne[MAXN], ce[MAXN];
4     int g[MAXN][MAXN], list[MAXN][MAXN];
5     void dfs(int size) {
6         int i, j, k, t, cnt, best = 0;
7         bool bb;
8         if (ne[size] == ce[size]) {
9             if (ce[size] == 0)
10                 ++ans;
11             return;
12        }
13        for (t = 0, i = 1; i <= ne[size]; ++i) {
14            for (cnt = 0, j = ne[size] + 1; j <= ce[size]; ++j)
15                if (!g[list[size][i]][list[size][j]])
16                    ++cnt;
17            if (t == 0 || cnt < best)
18                t = i, best = cnt;
19        }
20        if (t && best <= 0)
21            return;
22        for (k = ne[size] + 1; k <= ce[size]; ++k) {
23            if (t > 0) {
24                for (i = k; i <= ce[size]; ++i)
25                    if (!g[list[size][t]][list[size][i]])
26                        break;
27                swap(list[size][k], list[size][i]);
28            }
29            i = list[size][k];
30            ne[size + 1] = ce[size + 1] = 0;
31            for (j = 1; j < k; ++j)
32                if (g[i][list[size][j]])

```

```

33     list[size + 1][++ne[size + 1]] = list[size][j];
34     for (ce[size + 1] = ne[size + 1], j = k + 1; j <= ce[size]; ++j)
35         if (g[i][list[size][j]])
36             list[size + 1][++ce[size + 1]] = list[size][j];
37     dfs(size + 1);
38     ++ne[size];
39     --best;
40     for (j = k + 1, cnt = 0; j <= ce[size]; ++j)
41         if (!g[i][list[size][j]])
42             ++cnt;
43     if (t == 0 || cnt < best)
44         t = k, best = cnt;
45     if (t && best <= 0)
46         break;
47 }
48 }
49 void work() {
50     int i;
51     ne[0] = 0;
52     ce[0] = 0;
53     for (i = 1; i <= n; ++i)
54         list[0][++ce[0]] = i;
55     ans = 0;
56     dfs(0);
57 }
58 }

```

4.12 最小树形图

```

1 namespace EdmondsAlgorithm { //  $O(E \log E + V^2)$  !!! 0-based !!!
2     struct enode { int from, c, key, delta, dep; enode *ch[2], *next;
3     } ebase[maxm], *etop, *fir[maxn], nil, *null, *inEdge[maxn], *chs[maxn];
4     typedef enode *edge; typedef enode *tree;
5     int n, m, setFa[maxn], deg[maxn], que[maxn];
6     inline void pushDown(tree x) { if (x->delta) {
7         x->ch[0]->key += x->delta; x->ch[0]->delta += x->delta;
8         x->ch[1]->key += x->delta; x->ch[1]->delta += x->delta; x->delta = 0;
9     }}
10    tree merge(tree x, tree y) {
11        if (x == null) return y; if (y == null) return x;
12        if (x->key > y->key) swap(x, y); pushDown(x); x->ch[1] = merge(x->ch[1], y);
13        if (x->ch[0]->dep < x->ch[1]->dep) swap(x->ch[0], x->ch[1]);
14        x->dep = x->ch[1]->dep + 1; return x;
15    }
16    void addEdge(int u, int v, int w) {
17        etop->from = u; etop->c = etop->key = w; etop->delta = etop->dep = 0;
18        etop->next = fir[v]; etop->ch[0] = etop->ch[1] = null;
19        fir[v] = etop; inEdge[v] = merge(inEdge[v], etop++);

```

```

20    }
21    void deleteMin(tree &r) { pushDown(r); r = merge(r->ch[0], r->ch[1]); }
22    int findSet(int x) { return setFa[x] == x ? x : setFa[x] = findSet(setFa[x]); }
23    void clear(int V, int E) {
24        null = &nil; null->ch[0] = null->ch[1] = null; null->dep = -1;
25        n = V; m = E; etop = ebase; Foru(i, 0, V) fir[i] = NULL; Foru(i, 0, V) inEdge[i] =
26            null;
27    }
28    int solve(int root) { int res = 0, head, tail;
29        for (int i = 0; i < n; ++i) setFa[i] = i;
30        for ( ; ; ) { memset(deg, 0, sizeof(int) * n); chs[root] = inEdge[root];
31            for (int i = 0; i < n; ++i) if (i != root && setFa[i] == i) {
32                while (findSet(inEdge[i]->from) == findSet(i)) deleteMin(inEdge[i]);
33                ++deg[ findSet((chs[i] = inEdge[i])->from) ];
34            }
35            for (int i = head = tail = 0; i < n; ++i)
36                if (i != root && setFa[i] == i && deg[i] == 0) que[tail++] = i;
37            while (head < tail) {
38                int x = findSet(chs[que[head++]]->from);
39                if (--deg[x] == 0) que[tail++] = x;
40            } bool found = false;
41            for (int i = 0; i < n; ++i) if (i != root && setFa[i] == i && deg[i] > 0) {
42                int j = i; tree temp = null; found = true;
43                do {setFa[j] = findSet(chs[j]->from)} = i;
44                deleteMin(inEdge[j]); res += chs[j]->key;
45                inEdge[j]->key -= chs[j]->key; inEdge[j]->delta -= chs[j]->key;
46                temp = merge(temp, inEdge[j]);
47                } while (j != i); inEdge[i] = temp;
48            } if (!found) break;
49        } for (int i = 0; i < n; ++ i) if (i != root && setFa[i] == i) res += chs[i]->key;
50        return res;
51    }
52 }
53 namespace ChuLiu { //  $O(V^3)$  !!! 1-based !!!
54     int n, used[maxn], pass[maxn], eg[maxn], more, que[maxn], g[maxn][maxn];
55     void combine(int id, int &sum) { int tot = 0, from, i, j, k;
56         for ( ; id != 0 && !pass[id]; id = eg[id]) que[tot++] = id, pass[id] = 1;
57         for (from = 0; from < tot && que[from] != id; from++);
58         if (from == tot) return; more = 1;
59         for (i = from; i < tot; i++) {
60             sum += g[eg[que[i]]][que[i]]; if (i == from) continue;
61             for (j = used[que[i]] = 1; j <= n; j++) if (!used[j])
62                 if (g[que[i]][j] < g[id][j]) g[id][j] = g[que[i]][j];
63         }
64         for (i = 1; i <= n; i++) if (!used[i] && i != id)
65             for (j = from; j < tot; j++) {
66                 k = que[j]; if (g[i][id] > g[i][k] - g[eg[k]][k])
67                 g[i][id] = g[i][k] - g[eg[k]][k];

```

```

68 }
69 void clear(int V) { n = V; Rep(i, 1, V) Rep(j, 1, V) g[i][j] = inf; }
70 int solve(int root) {
71     int i, j, k, sum = 0; memset(used, 0, sizeof(int) * (n + 1));
72     for (more = 1; more; ) {
73         more = 0; memset(eg, 0, sizeof(int) * (n + 1));
74         for (i = 1; i <= n; i++) if (!used[i] && i != root) {
75             for (j = 1, k = 0; j <= n; j++) if (!used[j] && i != j)
76                 if (k == 0 || g[j][i] < g[k][i]) k = j;
77             eg[i] = k;
78         } memset(pass, 0, sizeof(int) * (n + 1));
79         for (i = 1; i <= n; i++) if (!used[i] && !pass[i] && i != root)
80             combine(i, sum);
81     } for (i = 1; i <= n; i++) if (!used[i] && i != root) sum += g[eg[i]][i];
82     return sum;
83 }
84 }

```

4.13 离线动态最小生成树

$O(Q \log^2 Q)$. $(qx[i], qy[i])$ 表示将编号为 $qx[i]$ 的边的权值改为 $qy[i]$, 删除一条边相当于将其权值改为 ∞ , 加入一条边相当于将其权值从 ∞ 变成某个值.

```

1  const int maxn = 100000 + 5;
2  const int maxm = 1000000 + 5;
3  const int maxq = 1000000 + 5;
4  const int qsize = maxm + 3 * maxq;
5  int n, m, Q, x[qsize], y[qsize], z[qsize], qx[maxq], qy[maxq], a[maxn], *tz;
6  int kx[maxn], ky[maxn], kt, vd[maxn], id[maxm], app[maxm];
7  bool extra[maxm];
8  void init() {
9      scanf("%d%d", &n, &m); for (int i = 0; i < m; i++) scanf("%d%d%d", x + i, y + i, z + i)
10         ;
11     scanf("%d", &Q); for (int i = 0; i < Q; i++) { scanf("%d%d", qx + i, qy + i); qx[i]--;
12         }
13 }
14 int find(int x) {
15     int root = x, next; while (a[root]) root = a[root];
16     while ((next = a[x]) != 0) a[x] = root, x = next; return root;
17 }
18 inline bool cmp(const int &a, const int &b) { return tz[a] < tz[b]; }
19 void solve(int *qx, int *qy, int Q, int n, int *x, int *y, int *z, int m, long long ans)
20 {
21     int ri, rj;
22     if (Q == 1) {
23         for (int i = 1; i <= n; i++) a[i] = 0; z[qx[0]] = qy[0];
24         for (int i = 0; i < m; i++) id[i] = i;
25         tz = z; sort(id, id + m, cmp);

```

```

23     for (int i = 0; i < m; i++) {
24         ri = find(x[id[i]]); rj = find(y[id[i]]);
25         if (ri != rj) ans += z[id[i]], a[ri] = rj;
26     } printf("%I64d\n", ans);
27     return;
28 } int tm = kt = 0, n2 = 0, m2 = 0;
29 for (int i = 1; i <= n; i++) a[i] = 0;
30 for (int i = 0; i < Q; i++) {
31     ri = find(x[qx[i]]); rj = find(y[qx[i]]); if (ri != rj) a[ri] = rj;
32 }
33 for (int i = 0; i < m; i++) extra[i] = true;
34 for (int i = 0; i < Q; i++) extra[qx[i]] = false;
35 for (int i = 0; i < m; i++) if (extra[i]) id[tm++] = i;
36 tz = z; sort(id, id + tm, cmp);
37 for (int i = 0; i < tm; i++) {
38     ri = find(x[id[i]]); rj = find(y[id[i]]);
39     if (ri != rj)
40         a[ri] = rj, ans += z[id[i]], kx[kt] = x[id[i]], ky[kt] = y[id[i]], kt++;
41 }
42 for (int i = 1; i <= n; i++) a[i] = 0;
43 for (int i = 0; i < kt; i++) a[find(kx[i])] = find(ky[i]);
44 for (int i = 1; i <= n; i++) if (a[i] == 0) vd[i] = ++n2;
45 for (int i = 1; i <= n; i++) if (a[i] != 0) vd[i] = vd[find(i)];
46 int *Nx = x + m, *Ny = y + m, *Nz = z + m;
47 for (int i = 0; i < m; i++) app[i] = -1;
48 for (int i = 0; i < Q; i++)
49     if (app[qx[i]] == -1)
50         Nx[m2] = vd[x[qx[i]]], Ny[m2] = vd[y[qx[i]]], Nz[m2] = z[qx[i]], app[qx[i]] = m2,
51         m2++;
52 for (int i = 0; i < Q; i++) {
53     z[qx[i]] = qy[i];
54     qx[i] = app[qx[i]];
55 }
56 for (int i = 1; i <= n2; i++) a[i] = 0;
57 for (int i = 0; i < tm; i++) {
58     ri = find(vd[x[id[i]]]); rj = find(vd[y[id[i]]]);
59     if (ri != rj)
60         a[ri] = rj, Nx[m2] = vd[x[id[i]]], Ny[m2] = vd[y[id[i]]], Nz[m2] = z[id[i]], m2++;
61 }
62 int mid = Q / 2;
63 solve(qx, qy, mid, n2, Nx, Ny, Nz, m2, ans);
64 solve(qx + mid, qy + mid, Q - mid, n2, Nx, Ny, Nz, m2, ans);
65 }
66 void work() { if (Q) solve(qx, qy, Q, n, x, y, z, m, 0); }
67 int main() { init(); work(); return 0; }

```


4.14 弦图

- 任何一个弦图都至少有一个单纯点, 不是完全图的弦图至少有两个不相邻的单纯点.
- 设第 i 个点在弦图的完美消除序列第 $p(i)$ 个. 令 $N(v) = \{w | w \text{ 与 } v \text{ 相邻且 } p(w) > p(v)\}$ 弦图的极大团一定是 $v \cup N(v)$ 的形式.
- 弦图最多有 n 个极大团.
- 设 $next(v)$ 表示 $N(v)$ 中最前的点. 令 w^* 表示所有满足 $A \in B$ 的 w 中最后的一个点. 判断 $v \cup N(v)$ 是否为极大团, 只需判断是否存在一个 w , 满足 $Next(w) = v$ 且 $|N(v)| + 1 \leq |N(w)|$ 即可.
- 最小染色: 完美消除序列从后往前依次给每个点染色, 给每个点染上可以染的最小的颜色. (团数 = 色数)
- 最大独立集: 完美消除序列从前往后能选就选.
- 最小团覆盖: 设最大独立集为 $\{p_1, p_2, \dots, p_t\}$, 则 $\{p_1 \cup N(p_1), \dots, p_t \cup N(p_t)\}$ 为最小团覆盖. (最大独立数 = 最小团覆盖数)

```

1 class Chordal { // 1-Based, G is the Graph, must be sorted before call Check_Chordal
2 public: // Construct will sort it automatically
3     int v[Maxn], id[Maxn]; bool inseq[Maxn]; priority_queue<pair<int, int> > pq;
4     vector<int> Construct_Perfect_Elimination_Sequence(vector<int> *G, int n) { // O(m +
        n log n)
5         vector<int> seq(n + 1, 0);
6         for (int i = 0; i <= n; ++i) inseq[i] = false, sort(G[i].begin(), G[i].end()), v[i] =
            0;
7         int cur = n; pair<int, int> Mx; while(!pq.empty()) pq.pop(); pq.push(make_pair(0, 1))
            ;
8         for (int i = n; i >= 1; --i) {
9             while (!pq.empty() && (Mx = pq.top(), inseq[Mx.second] || Mx.first != v[Mx.second])
                ) pq.pop();
10            id[Mx.second] = cur;
11            int x = seq[cur--] = Mx.second, sz = (int)G[Mx.second].size(); inseq[x] = true;
12            for (int j = 0; j < sz; ++j) {
13                int y = G[x][j]; if(!inseq[y]) pq.push(make_pair(++v[y], y));
14            }
15        } return seq;
16    }
17    bool Check_Chordal(vector<int> *G, vector<int> &seq, int n) { // O(n + m log n), plz gen
        seq first
18        bool isChordal = true;
19        for (int i = n - 1; i >= 1 && isChordal; --i) {
20            int x = seq[i], sz, y = -1;
21            if ((sz = (int)G[x].size()) == 0) continue;
22            for(int j = 0; j < sz; ++j) {
23                if (id[G[x][j]] < i) continue;
24                if (y == -1 || id[y] > id[G[x][j]]) y = G[x][j];
25            } if (y == -1) continue;

```

```

26        for (int j = 0; j < sz; ++j) {
27            int y1 = G[x][j]; if (id[y1] < i) continue;
28            if (y1 == y || binary_search(G[y].begin(), G[y].end(), y1)) continue;
29            isChordal = false; break;
30        }
31    } return isChordal;
32    }
33 };

```

4.15 K 短路 (允许重复)

```

1 #define for_each(it, v) for (vector<Edge*>::iterator it = (v).begin(); it != (v).end();
    ++it)
2 const int MAX_N = 10000, MAX_M = 50000, MAX_K = 10000, INF = 1000000000;
3 struct Edge { int from, to, weight; };
4 struct HeapNode { Edge* edge; int depth; HeapNode* child[4]; }; // child[0..1] for heap G
    , child[2..3] for heap out edge
5
6 int n, m, k, s, t; Edge* edge[MAX_M];
7 int dist[MAX_N]; Edge* prev[MAX_N];
8 vector<Edge*> graph[MAX_N]; vector<Edge*> graphR[MAX_N];
9 HeapNode* nullNode; HeapNode* heapTop[MAX_N];
10
11 HeapNode* createHeap(HeapNode* curNode, HeapNode* newNode) {
12     if (curNode == nullNode) return newNode; HeapNode* rootNode = new HeapNode;
13     memcpy(rootNode, curNode, sizeof(HeapNode));
14     if (newNode->edge->weight < curNode->edge->weight) {
15         rootNode->edge = newNode->edge; rootNode->child[2] = newNode->child[2]; rootNode->
            child[3] = newNode->child[3];
16         newNode->edge = curNode->edge; newNode->child[2] = curNode->child[2]; newNode->child
            [3] = curNode->child[3];
17     } if (rootNode->child[0]->depth < rootNode->child[1]->depth) rootNode->child[0] =
        createHeap(rootNode->child[0], newNode);
18     else rootNode->child[1] = createHeap(rootNode->child[1], newNode);
19     rootNode->depth = max(rootNode->child[0]->depth, rootNode->child[1]->depth) + 1;
20     return rootNode;
21 }
22 bool heapNodeMoreThan(HeapNode* node1, HeapNode* node2) { return node1->edge->weight >
    node2->edge->weight; }
23
24 int main() {
25     scanf("%d%d%d", &n, &m, &k); scanf("%d%d", &s, &t); s--, t--;
26     while (m--) { Edge* newEdge = new Edge;
27         int i, j, w; scanf("%d%d%d", &i, &j, &w);
28         i--, j--; newEdge->from = i; newEdge->to = j; newEdge->weight = w;
29         graph[i].push_back(newEdge); graphR[j].push_back(newEdge);
30     }
31     //Dijkstra

```

```

32 queue<int> dfsOrder; memset(dist, -1, sizeof(dist));
33 typedef pair<int, pair<int, Edge*>> > DijkstraQueueItem;
34 priority_queue<DijkstraQueueItem, vector<DijkstraQueueItem>, greater<DijkstraQueueItem>
    > dq;
35 dq.push(make_pair(0, make_pair(t, (Edge*) NULL)));
36 while (!dq.empty()) {
37     int d = dq.top().first; int i = dq.top().second.first;
38     Edge* edge = dq.top().second.second; dq.pop();
39     if (dist[i] != -1) continue;
40     dist[i] = d; prev[i] = edge; dfsOrder.push(i);
41     for_each(it, graphR[i]) dq.push(make_pair(d + (*it)->weight, make_pair((*it)->from, *
        it)));
42 }
43 //Create edge heap
44 nullNode = new HeapNode; nullNode->depth = 0; nullNode->edge = new Edge; nullNode->edge
    ->weight = INF;
45 fill(nullNode->child, nullNode->child + 4, nullNode);
46 while (!dfsOrder.empty()) {
47     int i = dfsOrder.front(); dfsOrder.pop();
48     if (prev[i] == NULL) heapTop[i] = nullNode;
49     else heapTop[i] = heapTop[prev[i]->to];
50     vector<HeapNode*> heapNodeList;
51     for_each(it, graph[i]) { int j = (*it)->to; if (dist[j] == -1) continue;
52         (*it)->weight += dist[j] - dist[i]; if (prev[i] != *it) {
53             HeapNode* curNode = new HeapNode;
54             fill(curNode->child, curNode->child + 4, nullNode);
55             curNode->depth = 1; curNode->edge = *it;
56             heapNodeList.push_back(curNode);
57         }
58     } if (!heapNodeList.empty()) { //Create heap out
59         make_heap(heapNodeList.begin(), heapNodeList.end(), heapNodeMoreThan);
60         int size = heapNodeList.size();
61         for (int p = 0; p < size; p++) {
62             heapNodeList[p]->child[2] = 2 * p + 1 < size ? heapNodeList[2 * p + 1] : nullNode
                ;
63             heapNodeList[p]->child[3] = 2 * p + 2 < size ? heapNodeList[2 * p + 2] : nullNode
                ;
64         } heapTop[i] = createHeap(heapTop[i], heapNodeList.front());
65     }
66 } //Walk on DAG
67 typedef pair<long long, HeapNode*> DAGQueueItem;
68 priority_queue<DAGQueueItem, vector<DAGQueueItem>, greater<DAGQueueItem> > aq;
69 if (dist[s] == -1) printf("NO\n");
70 else { printf("%d\n", dist[s]);
71     if (heapTop[s] != nullNode) aq.push(make_pair(dist[s] + heapTop[s]->edge->weight,
        heapTop[s]));
72 } k--; while (k--) {
73     if (aq.empty()) { printf("NO\n"); continue; }
74     long long d = aq.top().first; HeapNode* curNode = aq.top().second; aq.pop();

```

```

75     printf("%I64d\n", d);
76     if (heapTop[curNode->edge->to] != nullNode)
77         aq.push(make_pair(d + heapTop[curNode->edge->to]->edge->weight, heapTop[curNode->
            edge->to]));
78     for (int i = 0; i < 4; i++) if (curNode->child[i] != nullNode)
79         aq.push(make_pair(d - curNode->edge->weight + curNode->child[i]->edge->weight,
            curNode->child[i]));
80 } return 0;
81 }

```

4.16 K 短路 (不允许重复)

```

1 int Num[10005][205], Path[10005][205], dev[10005], from[10005], value[10005], dist[205],
    Next[205], Graph[205][205];
2 int N, M, K, s, t, tot, cnt; bool forbid[205], hasNext[10005][205];
3 struct cmp {
4     bool operator()(const int &a, const int &b) {
5         int *i, *j; if (value[a] != value[b]) return value[a] > value[b];
6         for (i = Path[a], j = Path[b]; (*i) == (*j); i++, j++);
7         return (*i) > (*j);
8     }
9 };
10 void Check(int idx, int st, int *path, int &res) {
11     int i, j; for (i = 0; i < N; i++) dist[i] = 1000000000, Next[i] = t;
12     dist[t] = 0; forbid[t] = true; j = t;
13     for ( ; ) {
14         for (i = 0; i < N; i++) if (!forbid[i] && (i != st || !hasNext[idx][j]) && (dist[j] +
            Graph[i][j] < dist[i] || (dist[j] + Graph[i][j] == dist[i] && j < Next[i])))
15             Next[i] = j, dist[i] = dist[j] + Graph[i][j];
16         j = -1; for (i = 0; i < N; i++) if (!forbid[i] && (j == -1 || dist[i] < dist[j])) j =
            i;
17         if (j == -1) break; forbid[j] = 1; if (j == st) break;
18     } res += dist[st]; for (i = st; i != t; i = Next[i], path++) (*path) = i; (*path) = i;
19 }
20 int main() {
21     int i, j, k, l;
22     while (scanf("%d%d%d%d", &N, &M, &K, &s, &t) && N) {
23         priority_queue<int, vector<int>, cmp> Q;
24         for (i = 0; i < N; i++) for (j = 0; j < N; j++) Graph[i][j] = 1000000000;
25         for (i = 0; i < M; i++) { scanf("%d%d", &j, &k, &l); Graph[j - 1][k - 1] = 1; }
26         s--; t--;
27         memset(forbid, false, sizeof(forbid)); memset(hasNext[0], false, sizeof(hasNext[0]));
28         Check(0, s, Path[0], value[0]); dev[0] = 0; from[0] = 0; Num[0][0] = 0; Q.push(0);
29         cnt = 1; tot = 1;
30         for (i = 0; i < K; i++) {
31             if (Q.empty()) break; l = Q.top(); Q.pop();
32             for (j = 0; j <= dev[l]; j++) Num[l][j] = Num[from[l]][j];
33             for (; Path[l][j] != t; j++) {

```

```

34     memset(hasNext[tot], false, sizeof(hasNext[tot])); Num[l][j] = tot++;
35 } for (j = 0; Path[l][j] != t; j++) hasNext[Num[l][j]][Path[l][j + 1]] = true;
36 for (j = dev[l]; Path[l][j] != t; j++) {
37     memset(forbid, false, sizeof(forbid)); value[cnt] = 0;
38     for (k = 0; k < j; k++) {
39         forbid[Path[l][k]] = true;
40         Path[cnt][k] = Path[l][k];
41         value[cnt] += Graph[Path[l][k]][Path[l][k + 1]];
42     } Check(Num[l][j], Path[l][j], &Path[cnt][j], value[cnt]);
43     if (value[cnt] > 2000000) continue;
44     dev[cnt] = j; from[cnt] = 1; Q.push(cnt); cnt++;
45 }
46 }
47 if (i < K || value[l] > 2000000) printf("None\n");
48 else {
49     for (i = 0; Path[l][i] != t; i++) printf("%d-", Path[l][i] + 1);
50     printf("%d\n", t + 1);
51 }
52 } return 0;
53 }

```

4.17 小知识

- 平面图: 一定存在一个度小于等于 5 的点. $E \leq 3V - 6$. 欧拉公式: $V + F - E = 1$ + 连通块数

- 图连通度:

- k - 连通 (k -connected): 对于任意一对结点都至少存在结点各不相同的 k 条路
- 点连通度 ($vertex\ connectivity$): 把图变成非连通图所需删除的最少点数
- Whitney 定理: 一个图是 k - 连通的当且仅当它的点连通度至少为 k

- Lindstroem-Gessel-Viennot Lemma: 给定一个图的 n 个起点和 n 个终点, 令 A_{ij} = 第 i 个起点到第 j 个终点的路径条数, 则从起点到终点的不相交路径条数为 $\det(A)$

- 欧拉回路与树形图的联系: 对于出度等于入度的连通图 $s(G) = t_i(G) \prod_{j=1}^n (d^+(v_j) - 1)!$

- 密度子图: 给定无向图, 选取点集及其导出子图, 最大化 $W_e + P_v$ (点权可负).

- $(S, u) = U, (u, T) = U - 2P_u - D_u, (u, v) = (v, u) = W_e$
- $\text{ans} = \frac{Un - C[S, T]}{2}$, 解集为 $S - \{s\}$

- 最大权闭合图: 选 a 则 a 的后继必须被选

- $P_u > 0, (S, u) = P_u, P_u < 0, (u, T) = -P_u$
- $\text{ans} = \sum_{P_u > 0} P_u - C[S, T]$, 解集为 $S - \{s\}$

- 判定边是否属于最小割:

- 可能属于最小割: (u, v) 不属于同一 SCC
- 一定在所有最小割中: (u, v) 不属于同一 SCC, 且 S, u 在同一 SCC, u, T 在同一 SCC

- 图同构 Hash: $F_t(i) = (F_{t-1}(i) \times A + \sum_{i \rightarrow j} F_{t-1}(j) \times B + \sum_{j \leftarrow i} F_{t-1}(j) \times C + D \times (i = a)) \pmod{P}$, 枚举点 a , 迭代 K 次后求得的 $F_k(a)$ 就是 a 点所对应的 Hash 值.

5 数学

5.1 单纯形 Cpp

$$\max \{cx | Ax \leq b, x \geq 0\}$$

```

1  const int MAXN = 11000, MAXM = 1100;
2  // here MAXN is the MAX number of conditions, MAXM is the MAX number of vars
3
4  int aveli[MAXN], avacnt;
5  double A[MAXN][MAXM];
6  double b[MAXN], c[MAXN];
7  double* simplex(int n, int m) {
8  // here n is the number of conditions, m is the number of vars
9      m++;
10     int r = n, s = m - 1;
11     static double D[MAXN + 2][MAXM + 1];
12     static int ix[MAXN + MAXM];
13     for (int i = 0; i < n + m; i++) ix[i] = i;
14     for (int i = 0; i < n; i++) {
15         for (int j = 0; j < m - 1; j++) D[i][j] = -A[i][j];
16         D[i][m - 1] = 1;
17         D[i][m] = b[i];
18         if (D[r][m] > D[i][m]) r = i;
19     }
20     for (int j = 0; j < m - 1; j++) D[n][j] = c[j];
21     D[n + 1][m - 1] = -1;
22     for (double d; ; ) {
23         if (r < n) {
24             int t = ix[s]; ix[s] = ix[r + m]; ix[r + m] = t;
25             D[r][s] = 1.0 / D[r][s];
26             for (int j = 0; j <= m; j++) if (j != s) D[r][j] *= -D[r][s];
27             avacnt = 0;
28             for (int i = 0; i <= m; ++i)
29                 if (fabs(D[r][i]) > EPS)
30                     aveli[avacnt++] = i;
31             for (int i = 0; i <= n + 1; i++) if (i != r) {
32                 if (fabs(D[i][s]) < EPS) continue;
33                 double *cur1 = D[i], *cur2 = D[r], tmp = D[i][s];
34                 //for (int j = 0; j <= m; j++) if (j != s) cur1[j] += cur2[j] * tmp;

```

```

35         for(int j = 0; j < avacnt; ++j) if(aval[i][j] != s) cur1[aval[i][j]] += cur2[aval[i][j]] * tmp;
36         D[i][s] *= D[r][s];
37     }
38 }
39 r = -1; s = -1;
40 for (int j = 0; j < m; j++) if (s < 0 || ix[s] > ix[j]) {
41     if (D[n + 1][j] > EPS || D[n + 1][j] > -EPS && D[n][j] > EPS) s = j;
42 }
43 if (s < 0) break;
44 for (int i = 0; i < n; i++) if (D[i][s] < -EPS) {
45     if (r < 0 || (d = D[r][m] / D[r][s] - D[i][m] / D[i][s]) < -EPS
46         || d < EPS && ix[r + m] > ix[i + m])
47         r = i;
48 }
49 if (r < 0) return null; // 非有界
50 }
51 if (D[n + 1][m] < -EPS) return null; // 无法执行
52 static double x[MAXM - 1];
53 for (int i = m; i < n + m; i++) if (ix[i] < m - 1) x[ix[i]] = D[i - m][m];
54 return x; // 值为  $D[n][m]$ 
55 }

```

5.2 单纯形 Java

```

1 double[] simplex(double[][] A, double[] b, double[] c) {
2     int n = A.length, m = A[0].length + 1, r = n, s = m - 1;
3     double[][] D = new double[n + 2][m + 1];
4     int[] ix = new int[n + m];
5     for (int i = 0; i < n + m; i++) ix[i] = i;
6     for (int i = 0; i < n; i++) {
7         for (int j = 0; j < m - 1; j++) D[i][j] = -A[i][j];
8         D[i][m - 1] = 1; D[i][m] = b[i]; if (D[r][m] > D[i][m]) r = i;
9     }
10    for (int j = 0; j < m - 1; j++) D[n][j] = c[j];
11    D[n + 1][m - 1] = -1;
12    for (double d; ; ) {
13        if (r < n) {
14            int t = ix[s]; ix[s] = ix[r + m]; ix[r + m] = t; D[r][s] = 1.0 / D[r][s];
15            for (int j = 0; j <= m; j++) if (j != s) D[r][j] *= -D[r][s];
16            for (int i = 0; i <= n + 1; i++) if (i != r) {
17                for (int j = 0; j <= m; j++) if (j != s) D[i][j] += D[r][j] * D[i][s];
18                D[i][s] *= D[r][s];
19            }
20        } r = -1; s = -1;
21        for (int j = 0; j < m; j++) if (s < 0 || ix[s] > ix[j]) {
22            if (D[n + 1][j] > EPS || D[n + 1][j] > -EPS && D[n][j] > EPS) s = j;
23        }

```

```

24        if (s < 0) break;
25        for (int i = 0; i < n; i++) if (D[i][s] < -EPS) {
26            if (r < 0 || (d = D[r][m] / D[r][s] - D[i][m] / D[i][s]) < -EPS
27                || d < EPS && ix[r + m] > ix[i + m])
28                r = i;
29        }
30        if (r < 0) return null; // 非有界
31    } if (D[n + 1][m] < -EPS) return null; // 无法执行
32    double[] x = new double[m - 1];
33    for (int i = m; i < n + m; i++) if (ix[i] < m - 1) x[ix[i]] = D[i - m][m];
34    return x; // 值为  $D[n][m]$ 
35 }

```

5.3 高斯消元

```

1 #define Zero(x) (fabs(x) <= EPS)
2 bool GaussElimination(double G[MAXN][MAXM], int N, int M) {
3     int rb = 1; memset(res, 0, sizeof(res));
4     Rep(i_th, 1, N) { int maxRow = 0;
5         Rep(row, rb, N) if (!Zero(G[row][i_th]))
6             if (!maxRow || fabs(G[row][i_th]) > fabs(G[maxRow][i_th]))
7                 maxRow = row;
8         if (!maxRow) continue;
9         swapRow(G[rb], G[maxRow]);
10        maxRow = rb++;
11        Rep(row, 1, N) if (row != maxRow && !Zero(G[row][i_th])) {
12            double coef = G[row][i_th] / G[maxRow][i_th];
13            Rep(col, 0, M) G[row][col] -= coef * G[maxRow][col];
14        }
15    }
16    Rep(row, 1, N) if (!Zero(G[row][0])) {
17        int i_th = 1;
18        for ( ; i_th <= M; ++i_th) if (!Zero(G[row][i_th])) break;
19        if (i_th > N) return false;
20        res[i_th] = G[row][0] / G[row][i_th];
21    }
22    return true;
23 }

```

5.4 FFT

```

1 namespace FFT {
2     #define mul(a, b) (Complex(a.x * b.x - a.y * b.y, a.x * b.y + a.y * b.x))
3     struct Complex {}; // something omitted
4     void FFT(Complex P[], int n, int oper) {
5         for (int i = 1, j = 0; i < n - 1; i++) {

```

```

6     for (int s = n; j ^= s >= 1, ~j & s; );
7     if (i < j) swap(P[i], P[j]);
8 }
9 for (int d = 0; (1 << d) < n; d++) {
10     int m = 1 << d, m2 = m * 2;
11     double p0 = PI / m * oper;
12     Complex unit_p0(cos(p0), sin(p0));
13     for (int i = 0; i < n; i += m2) {
14         Complex unit(1.0, 0.0);
15         for (int j = 0; j < m; j++) {
16             Complex &P1 = P[i + j + m], &P2 = P[i + j];
17             Complex t = mul(unit, P1);
18             P1 = Complex(P2.x - t.x, P2.y - t.y);
19             P2 = Complex(P2.x + t.x, P2.y - t.y);
20             unit = mul(unit, unit_p0);
21         }}}
22 vector<int> doFFT(const vector<int> &a, const vector<int> &b) {
23     vector<int> ret(max(0, (int) a.size() + (int) b.size() - 1), 0);
24     static Complex A[MAXB], B[MAXB], C[MAXB];
25     int len = 1; while (len < (int)ret.size()) len *= 2;
26     for (int i = 0; i < len; i++) A[i] = i < (int)a.size() ? a[i] : 0;
27     for (int i = 0; i < len; i++) B[i] = i < (int)b.size() ? b[i] : 0;
28     FFT(A, len, 1); FFT(B, len, 1);
29     for (int i = 0; i < len; i++) C[i] = mul(A[i], B[i]);
30     FFT(C, len, -1);
31     for (int i = 0; i < (int)ret.size(); i++)
32         ret[i] = (int) (C[i].x / len + 0.5);
33     return ret;
34 }
35 }

```

5.5 整数 FFT

```

1 namespace FFT {
2 // 替代方案: 23068673(= 11 * 221 + 1), 原根为 3
3 const int MOD = 786433, PRIMITIVE_ROOT = 10; // 3 * 218 + 1
4 const int MAXB = 1 << 20;
5 int getMod(int downLimit) { // 或者现场自己找一个 MOD
6     for (int c = 3; ; ++c) { int t = (c << 21) | 1;
7         if (t >= downLimit && isPrime(t)) return t;
8     }}
9 int modInv(int a) { return a <= 1 ? a : (long long) (MOD - MOD / a) * modInv(MOD % a) %
    MOD; }
10 void NTT(int P[], int n, int oper) {
11     for (int i = 1, j = 0; i < n - 1; i++) {
12         for (int s = n; j ^= s >= 1, ~j & s; );
13         if (i < j) swap(P[i], P[j]);
14     }

```

```

15     for (int d = 0; (1 << d) < n; d++) {
16         int m = 1 << d, m2 = m * 2;
17         long long unit_p0 = powMod(PRIMITIVE_ROOT, (MOD - 1) / m2);
18         if (oper < 0) unit_p0 = modInv(unit_p0);
19         for (int i = 0; i < n; i += m2) {
20             long long unit = 1;
21             for (int j = 0; j < m; j++) {
22                 int &P1 = P[i + j + m], &P2 = P[i + j];
23                 int t = unit * P1 % MOD;
24                 P1 = (P2 - t + MOD) % MOD; P2 = (P2 + t) % MOD;
25                 unit = unit * unit_p0 % MOD;
26             }}}
27 vector<int> mul(const vector<int> &a, const vector<int> &b) {
28     vector<int> ret(max(0, (int) a.size() + (int) b.size() - 1), 0);
29     static int A[MAXB], B[MAXB], C[MAXB];
30     int len = 1; while (len < (int)ret.size()) len <= 1;
31     for (int i = 0; i < len; i++) A[i] = i < (int)a.size() ? a[i] : 0;
32     for (int i = 0; i < len; i++) B[i] = i < (int)b.size() ? b[i] : 0;
33     NTT(A, len, 1); NTT(B, len, 1);
34     for (int i = 0; i < len; i++) C[i] = (long long) A[i] * B[i] % MOD;
35     NTT(C, len, -1); for (int i = 0, inv = modInv(len); i < (int)ret.size(); i++) ret[i]
        = (long long) C[i] * inv % MOD;
36     return ret;
37 }
38 }

```

5.6 扩展欧几里得

$$ax + by = g = \gcd(x, y)$$

```

1 void exgcd(LL x, LL y, LL &a0, LL &b0, LL &g) {
2     LL a1 = b0 = 0, b1 = a0 = 1, t;
3     while (y != 0) {
4         t = a0 - x / y * a1, a0 = a1, a1 = t;
5         t = b0 - x / y * b1, b0 = b1, b1 = t;
6         t = x % y, x = y, y = t;
7     } if (x < 0) a0 = -a0, b0 = -b0, x = -x;
8     g = x;
9 }

```

5.7 线性同余方程

- 中国剩余定理: 设 m_1, m_2, \dots, m_k 两两互素, 则同余方程组 $x \equiv a_i \pmod{m_i}$ for $i = 1, 2, \dots, k$ 在 $[0, M = m_1 m_2 \dots m_k]$ 内有唯一解. 记 $M_i = M/m_i$, 找出 p_i 使得 $M_i p_i \equiv 1 \pmod{m_i}$, 记 $e_i = M_i p_i$, 则 $x \equiv e_1 a_1 + e_2 a_2 + \dots + e_k a_k \pmod{M}$
- 多变元线性同余方程组: 方程的形式为 $a_1 x_1 + a_2 x_2 + \dots + a_n x_n + b \equiv 0 \pmod{m}$, 令 $d = (a_1, a_2, \dots, a_n, m)$, 有解的充要条件是 $d|b$, 解的个数为 $m^{n-1}d$

5.8 Miller-Rabin 素性测试

```

1 bool test(LL n, int base) {
2     LL m = n - 1, ret = 0; int s = 0;
3     for ( ; m % 2 == 0; ++s) m >>= 1; ret = pow_mod(base, m, n);
4     if (ret == 1 || ret == n - 1) return true;
5     for (--s; s >= 0; --s) {
6         ret = multiply_mod(ret, ret, n); if (ret == n - 1) return true;
7     } return false;
8 }
9 LL special[7] = {
10     1373653LL,      25326001LL,
11     3215031751LL,   25000000000LL,
12     2152302898747LL, 3474749660383LL, 341550071728321LL};
13 /*
14  * n < 2047          test[] = {2}
15  * n < 1,373,653     test[] = {2, 3}
16  * n < 9,080,191     test[] = {31, 73}
17  * n < 25,326,001    test[] = {2, 3, 5}
18  * n < 4,759,123,141 test[] = {2, 7, 61}
19  * n < 1,122,004,669,633 test[] = {2, 13, 23, 1662803}
20  * n < 2,152,302,898,747 test[] = {2, 3, 5, 7, 11}
21  * n < 3,474,749,660,383 test[] = {2, 3, 5, 7, 11, 13}
22  * n < 341,550,071,728,321 test[] = {2, 3, 5, 7, 11, 13, 17}
23  * n < 3,825,123,056,546,413,051 test[] = {2, 3, 5, 7, 11, 13, 17, 19, 23}
24  */
25 bool is_prime(LL n) {
26     if (n < 2) return false;
27     if (n < 4) return true;
28     if (!test(n, 2) || !test(n, 3)) return false;
29     if (n < special[0]) return true;
30     if (!test(n, 5)) return false;
31     if (n < special[1]) return true;
32     if (!test(n, 7)) return false;
33     if (n == special[2]) return false;
34     if (n < special[3]) return true;
35     if (!test(n, 11)) return false;
36     if (n < special[4]) return true;
37     if (!test(n, 13)) return false;
38     if (n < special[5]) return true;
39     if (!test(n, 17)) return false;
40     if (n < special[6]) return true;
41     return test(n, 19) && test(n, 23) && test(n, 29) && test(n, 31) && test(n, 37);
42 }

```

5.9 PollardRho

```

1 LL pollardRho(LL n, LL seed) {
2     LL x, y, head = 1, tail = 2; x = y = random() % (n - 1) + 1;
3     for ( ; ; ) {
4         x = addMod(multiplyMod(x, x, n), seed, n);
5         if (x == y) return n; LL d = gcd(myAbs(x - y), n);
6         if (1 < d && d < n) return d;
7         if (++head == tail) y = x, tail <= 1;
8     } vector<LL> divisors;
9     void factorize(LL n) { // 需要保证 n > 1
10         if (isPrime(n)) divisors.push_back(n);
11         else { LL d = n;
12             while (d >= n) d = pollardRho(n, random() % (n - 1) + 1);
13             factorize(n / d); factorize(d);
14         }

```

5.10 多项式求根

```

1 const double error = 1e-12;
2 const double infi = 1e+12;
3 int n; double a[10], x[10];
4 double f(double a[], int n, double x) {
5     double tmp = 1, sum = 0;
6     for (int i = 0; i <= n; i++) sum = sum + a[i] * tmp, tmp = tmp * x;
7     return sum;
8 }
9 double binary(double l, double r, double a[], int n) {
10     int sl = sign(f(a, n, l)), sr = sign(f(a, n, r));
11     if (sl == 0) return l; if (sr == 0) return r;
12     if (sl * sr > 0) return infi;
13     while (r - l > error) {
14         double mid = (l + r) / 2;
15         int ss = sign(f(a, n, mid));
16         if (ss == 0) return mid;
17         if (ss * sl > 0) l = mid; else r = mid;
18     } return l;
19 }
20 void solve(int n, double a[], double x[], int &nx) {
21     if (n == 1) { x[1] = -a[0] / a[1]; nx = 1; return; }
22     double da[10], dx[10]; int ndx;
23     for (int i = n; i >= 1; i--) da[i - 1] = a[i] * i;
24     solve(n - 1, da, dx, ndx); nx = 0;
25     if (ndx == 0) {
26         double tmp = binary(-infi, infi, a, n);
27         if (tmp < infi) x[++nx] = tmp; return;
28     } double tmp = binary(-infi, dx[1], a, n);
29     if (tmp < infi) x[++nx] = tmp;
30     for (int i = 1; i <= ndx - 1; i++) {
31         tmp = binary(dx[i], dx[i + 1], a, n);

```

```

32     if (tmp < infi) x[++nx] = tmp;
33 } tmp = binary(dx[ndx], infi, a, n);
34 if (tmp < infi) x[++nx] = tmp;
35 }
36 int main() {
37     scanf("%d", &n);
38     for (int i = n; i >= 0; i--) scanf("%lf", &a[i]);
39     int nx; solve(n, a, x, nx);
40     for (int i = 1; i <= nx; i++) printf("%.6f\n", x[i]);
41     return 0;
42 }

```

5.11 线性递推

$$\text{for } a_{i+n} = (\sum_{j=0}^{n-1} k_j a_{i+j}) + d, a_m = (\sum_{i=0}^{n-1} c_i a_i) + c_n d$$

```

1 vector<int> recFormula(int n, int k[], int m) {
2     vector<int> c(n + 1, 0);
3     if (m < n) c[m] = 1;
4     else {
5         static int a[MAX_K * 2 + 1];
6         vector<int> b = recFormula(n, k, m >> 1);
7         for (int i = 0; i < n + n; ++i) a[i] = 0;
8         int s = m & 1;
9         for (int i = 0; i < n; i++) {
10             for (int j = 0; j < n; j++) a[i + j + s] += b[i] * b[j];
11             c[n] += b[i];
12         } c[n] = (c[n] + 1) * b[n];
13         for (int i = n * 2 - 1; i >= n; i--) {
14             int add = a[i]; if (add == 0) continue;
15             for (int j = 0; j < n; j++) a[i - n + j] += k[j] * add;
16             c[n] += add;
17         } for (int i = 0; i < n; ++i) c[i] = a[i];
18     } return c;
19 }

```

5.12 原根

原根 g : g 是模 n 简化剩余系构成的乘法群的生成元. 模 n 有原根的充要条件是 $n = 2, 4, p^n, 2p^n$, 其中 p 是奇质数, n 是正整数

```

1 vector<int> findPrimitiveRoot(int N) {
2     if (N <= 4) return vector<int>(1, max(1, N - 1));
3     static int factor[100];
4     int phi = N, totF = 0;
5     { // check no solution and calculate phi
6         int M = N, k = 0;
7         if (~M & 1) M >>= 1, phi >>= 1;

```

```

8         if (~M & 1) return vector<int>(0);
9         for (int d = 3; d * d <= M; ++d) if (M % d == 0) {
10             if (++k > 1) return vector<int>(0);
11             for (phi -= phi / d; M % d == 0; M /= d);
12         } if (M > 1) {
13             if (++k > 1) return vector<int>(0); phi -= phi / M;
14         }
15     } { // factorize phi
16         int M = phi;
17         for (int d = 2; d * d <= M; ++d) if (M % d == 0) {
18             for (; M % d == 0; M /= d); factor[++totF] = d;
19         } if (M > 1) factor[++totF] = M;
20     } vector<int> ans;
21     for (int g = 2; g <= N; ++g) if (Gcd(g, N) == 1) {
22         bool good = true;
23         for (int i = 1; i <= totF && good; ++i)
24             if (powMod(g, phi / factor[i], N) == 1) good = false;
25         if (!good) continue;
26         for (int i = 1, gp = g; i <= phi; ++i, gp = (LL)gp * g % N)
27             if (Gcd(i, phi) == 1) ans.push_back(gp);
28         break;
29     } sort(ans.begin(), ans.end());
30     return ans;
31 }

```

5.13 离散对数

$A^x \equiv B \pmod{C}$, 对非质数 C 也适用.

```

1 int modLog(int A, int B, int C) {
2     static pii baby[MAX_SQRT_C + 11];
3     int d = 0; LL k = 1, D = 1; B %= C;
4     for (int i = 0; i < 100; ++i, k = k * A % C) // [0, log C]
5         if (k == B) return i;
6     for (int g; ; ++d) {
7         g = gcd(A, C); if (g == 1) break;
8         if (B % g != 0) return -1;
9         B /= g; C /= g; D = (A / g * D) % C;
10    } int m = (int) ceil(sqrt((double) C)); k = 1;
11    for (int i = 0; i <= m; ++i, k = k * A % C) baby[i] = pii(k, i);
12    sort(baby, baby + m + 1); // [0, m]
13    int n = unique(baby, baby + m + 1, equalFirst) - baby, am = powMod(A, m, C);
14    for (int i = 0; i <= m; ++i) {
15        LL e, x, y; exgcd(D, C, x, y, e); e = x * B % C;
16        if (e < 0) e += C;
17        if (e >= 0) {
18            int k = lower_bound(baby, baby + n, pii(e, -1)) - baby;
19            if (baby[k].first == e) return i * m + baby[k].second + d;

```

```
20     } D = D * am % C;
21   } return -1;
22 }
```

5.14 平方剩余

- Legendre Symbol: 对奇质数 p , $(\frac{a}{p}) = \begin{cases} 1 & \text{是平方剩余} \\ -1 & \text{是非平方剩余} = a^{\frac{p-1}{2}} \bmod p \\ 0 & a \equiv 0 \pmod{p} \end{cases}$
- 若 p 是奇质数, $(\frac{-1}{p}) = 1$ 当且仅当 $p \equiv 1 \pmod{4}$
- 若 p 是奇质数, $(\frac{2}{p}) = 1$ 当且仅当 $p \equiv \pm 1 \pmod{8}$
- 若 p, q 是奇素数且互质, $(\frac{p}{q})(\frac{q}{p}) = (-1)^{\frac{p-1}{2} \times \frac{q-1}{2}}$
- Jacobi Symbol: 对奇数 $n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$, $(\frac{a}{n}) = (\frac{a}{p_1})^{\alpha_1} (\frac{a}{p_2})^{\alpha_2} \cdots (\frac{a}{p_k})^{\alpha_k}$
- Jacobi Symbol 为 -1 则一定不是平方剩余, 所有平方剩余的 Jacobi Symbol 都是 1, 但 1 不一定是平方剩余

$ax^2 + bx + c \equiv 0 \pmod{p}$, 其中 $a \not\equiv 0 \pmod{p}$, 且 p 是质数

```
1 inline int normalize(LL a, int P) { a %= P; return a < 0 ? a + P : a; }
2 vector<int> QuadraticResidue(LL a, LL b, LL c, int P) {
3     int h, t; LL r1, r2, delta, pb = 0;
4     a = normalize(a, P); b = normalize(b, P); c = normalize(c, P);
5     if (P == 2) { vector<int> res;
6         if (c % P == 0) res.push_back(0);
7         if ((a + b + c) % P == 0) res.push_back(1);
8         return res;
9     } delta = b * rev(a + a, P) % P;
10    a = normalize(-c * rev(a, P) + delta * delta, P);
11    if (powMod(a, P / 2, P) + 1 == P) return vector<int>(0);
12    for (t = 0, h = P / 2; h % 2 == 0; ++t, h /= 2);
13    r1 = powMod(a, h / 2, P);
14    if (t > 0) { do b = random() % (P - 2) + 2;
15        while (powMod(b, P / 2, P) + 1 != P); }
16    for (int i = 1; i <= t; ++i) {
17        LL d = r1 * r1 % P * a % P;
18        for (int j = 1; j <= t - i; ++j) d = d * d % P;
19        if (d + 1 == P) r1 = r1 * pb % P; pb = pb * pb % P;
20    } r1 = a * r1 % P; r2 = P - r1;
21    r1 = normalize(r1 - delta, P); r2 = normalize(r2 - delta, P);
22    if (r1 > r2) swap(r1, r2); vector<int> res(1, r1);
23    if (r1 != r2) res.push_back(r2);
24    return res;
25 }
```

5.15 N 次剩余

- 若 p 为奇质数, a 为 p 的 n 次剩余的充要条件是 $a^{\frac{p-1}{n}} \equiv 1 \pmod{p}$.

$x^N \equiv a \pmod{p}$, 其中 p 是质数

```
1 vector<int> solve(int p, int N, int a) {
2     if ((a %= p) == 0) return vector<int>(1, 0);
3     int g = findPrimitiveRoot(p), m = modLog(g, a, p); // g ^ m = a (mod p)
4     if (m == -1) return vector<int>(0);
5     LL B = p - 1, x, y, d; exgcd(N, B, x, y, d);
6     if (m % d != 0) return vector<int>(0);
7     vector<int> ret; x = (x * (m / d) % B + B) % B; // g ^ B mod p = g ^ (p - 1) mod p = 1
8     for (int i = 0, delta = B / d; i < d; ++i) {
9         x = (x + delta) % B; ret.push_back((int)powMod(g, x, p));
10    } sort(ret.begin(), ret.end());
11    ret.resize(unique(ret.begin(), ret.end()) - ret.begin());
12    return ret;
13 }
```

5.16 Pell 方程

$$\begin{pmatrix} x_k \\ y_k \end{pmatrix} = \begin{pmatrix} x_1 & dy_1 \\ y_1 & x_1 \end{pmatrix}^{k-1} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$$

```
1 pair<ULL, ULL> Pell(int n) {
2     static ULL p[50] = {0, 1}, q[50] = {1, 0}, g[50] = {0, 0}, h[50] = {0, 1}, a[50];
3     ULL t = a[2] = Sqrt(n);
4     for (int i = 2; ; ++i) {
5         g[i] = -g[i - 1] + a[i] * h[i - 1];
6         h[i] = (n - g[i] * g[i]) / h[i - 1];
7         a[i + 1] = (g[i] + t) / h[i];
8         p[i] = a[i] * p[i - 1] + p[i - 2];
9         q[i] = a[i] * q[i - 1] + q[i - 2];
10        if (p[i] * p[i] - n * q[i] * q[i] == 1) return make_pair(p[i], q[i]);
11    } return make_pair(-1, -1);
12 }
```

5.17 Romberg 积分

```
1 template <class T> double Romberg(const T&f, double a, double b, double eps = 1e-8) {
2     vector<double> t; double h = b - a, last, now; int k = 1, i = 1;
3     t.push_back(h * (f(a) + f(b)) / 2); // 梯形
4     do {
5         last = t.back(); now = 0; double x = a + h / 2;
6         for (int j = 0; j < k; ++j, x += h) now += f(x);
7         now = (t[0] + h * now) / 2; double k1 = 4.0 / 3.0, k2 = 1.0 / 3.0;
8         for (int j = 0; j < i; ++j, k1 = k2 + 1) {
```



```
9      double tmp = k1 * now - k2 * t[j];
10      t[j] = now; now = tmp; k2 /= 4 * k1 - k2; // 防止溢出
11      } t.push_back(now); k *= 2; h /= 2; ++i;
12      } while (fabs(last - now) > eps);
13      return t.back();
14 }
```

5.18 公式

5.18.1 级数与三角

- $\sum_{k=1}^n k^3 = (\frac{n(n+1)}{2})^2$
- $\sum_{k=1}^n k^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$
- $\sum_{k=1}^n k^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$
- $\sum_{k=1}^n k(k+1) = \frac{n(n+1)(n+2)}{3}$
- $\sum_{k=1}^n k(k+1)(k+2) = \frac{n(n+1)(n+2)(n+3)}{4}$
- $\sum_{k=1}^n k(k+1)(k+2)(k+3) = \frac{n(n+1)(n+2)(n+3)(n+4)}{5}$
- 错排: $D_n = n!(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + \frac{(-1)^n}{n!}) = (n-1)(D_{n-2} - D_{n-1})$
- $\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$
- $\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$
- $\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$
- $\tan \alpha \pm \tan \beta = \frac{\sin(\alpha \pm \beta)}{\cos \alpha \cos \beta}$
- $\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$
- $\sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$
- $\cos \alpha + \cos \alpha = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$
- $\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$
- $\cos n\alpha = \binom{n}{0} \cos^n \alpha - \binom{n}{2} \cos^{n-2} \alpha \sin^2 \alpha + \binom{n}{4} \cos^{n-4} \alpha \sin^4 \alpha \dots$
- $\sin n\alpha = \binom{n}{1} \cos^{n-1} \alpha \sin \alpha - \binom{n}{3} \cos^{n-3} \alpha \sin^3 \alpha + \binom{n}{5} \cos^{n-5} \alpha \sin^5 \alpha \dots$
- $\sum_{n=1}^N \cos nx = \frac{\sin(N+\frac{1}{2})x - \sin \frac{x}{2}}{2 \sin \frac{x}{2}}$
- $\sum_{n=1}^N \sin nx = \frac{-\cos(N+\frac{1}{2})x + \cos \frac{x}{2}}{2 \sin \frac{x}{2}}$

- $\int_0^{\frac{\pi}{2}} \sin^n x dx = \begin{cases} \frac{(n-1)!!}{n!!} \times \frac{\pi}{2} & n \text{ 是偶数} \\ \frac{(n-1)!!}{n!!} & n \text{ 是奇数} \end{cases}$
- $\int_0^{+\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$
- $\int_0^{+\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$
- 傅里叶级数: 设周期为 $2T$. 函数分段连续. 在不连续点的值为左右极限的平均数.
 - $a_n = \frac{1}{T} \int_{-T}^T f(x) \cos \frac{n\pi}{T} x dx$
 - $b_n = \frac{1}{T} \int_{-T}^T f(x) \sin \frac{n\pi}{T} x dx$
 - $f(x) = \frac{a_0}{2} + \sum_{n=1}^{+\infty} (a_n \cos \frac{n\pi}{T} x + b_n \sin \frac{n\pi}{T} x)$
- Beta 函数: $B(p, q) = \int_0^1 x^{p-1} (1-x)^{q-1} dx$
 - 定义域 $(0, +\infty) \times (0, +\infty)$, 在定义域上连续
 - $B(p, q) = B(q, p) = \frac{q-1}{p+q-1} B(p, q-1) = 2 \int_0^{\frac{\pi}{2}} \cos^{2p-1} \phi \sin^{2p-1} \phi d\phi = \int_0^{+\infty} \frac{t^{q-1}}{(1+t)^{p+q}} dt = \int_0^1 \frac{t^{p-1} + t^{q-1}}{(1+t)^{(p+q)}} dt$
 - $B(\frac{1}{2}, \frac{1}{2}) = \pi$
- Gamma 函数: $\Gamma = \int_0^{+\infty} x^{s-1} e^{-x} dx$
 - 定义域 $(0, +\infty)$, 在定义域上连续
 - $\Gamma(1) = 1, \Gamma(\frac{1}{2}) = \sqrt{\pi}$
 - $\Gamma(s) = (s-1)\Gamma(s-1)$
 - $B(p, q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$
 - $\Gamma(s)\Gamma(1-s) = \frac{\pi}{\sin \pi s}$ for $s > 0$
 - $\Gamma(s)\Gamma(s+\frac{1}{2}) = 2\sqrt{\pi} \frac{\Gamma(s)}{2^{2s-1}}$ for $0 < s < 1$
- 积分: 平面图形面积、曲线弧长、旋转体体积、旋转曲面面积 $y = f(x), \int_a^b f(x) dx, \int_a^b \sqrt{1+f'^2(x)} dx,$
 $\pi \int_a^b f^2(x) dx, 2\pi \int_a^b |f(x)| \sqrt{1+f'^2(x)} dx$
- $x = x(t), y = y(t), t \in [T_1, T_2], \int_{T_1}^{T_2} |y(t)x'(t)| dt, \int_{T_1}^{T_2} \sqrt{x'^2(t) + y'^2(t)} dt, \pi \int_{T_1}^{T_2} |x'(t)| y^2(t) dt,$
 $2\pi \int_{T_1}^{T_2} |y(t)| \sqrt{x'^2(t) + y'^2(t)} dt,$

$$\begin{aligned}r &= r(\theta), \theta \in [\alpha, \beta], & \frac{1}{2} \int_{\alpha}^{\beta} r^2(\theta) \mathrm{d}\theta, & \int_{\alpha}^{\beta} \sqrt{r^2(\theta) + r'^2(\theta)} \mathrm{d}\theta, & \frac{2}{3} \pi \int_{\alpha}^{\beta} r^3(\theta) \sin \theta \mathrm{d}\theta, \\ & 2\pi \int_{\alpha}^{\beta} r(\theta) \sin \theta \sqrt{r^2(\theta) + r'^2(\theta)} \mathrm{d}\theta\end{aligned}$$

5.18.2 三次方程求根公式

对一元三次方程 $x^3 + px + q = 0$, 令

$$\begin{aligned}A &= \sqrt[3]{-\frac{q}{2} + \sqrt{(\frac{q}{2})^2 + (\frac{p}{3})^3}} \\ B &= \sqrt[3]{-\frac{q}{2} - \sqrt{(\frac{q}{2})^2 + (\frac{p}{3})^3}} \\ \omega &= \frac{(-1 + \mathrm{i}\sqrt{3})}{2}\end{aligned}$$

则 $x_j = A\omega^j + B\omega^{2j}$ ($j = 0, 1, 2$).
当求解 $ax^3 + bx^2 + cx + d = 0$ 时, 令 $x = y - \frac{b}{3a}$, 再求解 y , 即转化为 $y^3 + py + q = 0$ 的形式. 其中,

$$\begin{aligned}p &= \frac{b^2 - 3ac}{3a^2} \\ q &= \frac{2b^3 - 9abc + 27a^2d}{27a^3}\end{aligned}$$

卡尔丹判别法: 令 $\Delta = (\frac{q}{2})^2 + (\frac{p}{3})^3$. 当 $\Delta > 0$ 时, 有一个实根和一对共轭虚根; 当 $\Delta = 0$ 时, 有三个实根, 其中两个相等; 当 $\Delta < 0$ 时, 有三个不相等的实根.

5.18.3 椭圆

- 椭圆 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, 其中离心率 $e = \frac{c}{a}, c = \sqrt{a^2 - b^2}$; 焦点参数 $p = \frac{b^2}{a}$
- 椭圆上 (x, y) 点处的曲率半径为 $R = a^2b^2(\frac{x^2}{a^4} + \frac{y^2}{b^4})^{\frac{3}{2}} = \frac{(r_1r_2)^{\frac{3}{2}}}{ab}$, 其中 r_1 和 r_2 分别为 (x, y) 与两焦点 F_1 和 F_2 的距离.

$$L_{AM} = a \int_0^{\arccos \frac{x}{a}} \sqrt{1 - e^2 \cos^2 t} \mathrm{d}t = a \int_{\arccos \frac{x}{a}}^{\frac{\pi}{2}} \sqrt{1 - e^2 \sin^2 t} \mathrm{d}t$$

- 椭圆的周长 $L = 4a \int_0^{\frac{\pi}{2}} \sqrt{1 - e^2 \sin^2 t} \mathrm{d}t = 4aE(e, \frac{\pi}{2})$, 其中

$$E(e, \frac{\pi}{2}) = \frac{\pi}{2} [1 - (\frac{1}{2})^2 e^2 - (\frac{1 \times 3}{2 \times 4})^2 \frac{e^4}{3} - (\frac{1 \times 3 \times 5}{2 \times 4 \times 6})^2 \frac{e^6}{5} - \dots]$$

- 设椭圆上点 $M(x, y), N(x, -y), x, y > 0, A(a, 0)$, 原点 $O(0, 0)$, 扇形 OAM 的面积 $S_{OAM} = \frac{1}{2} ab \arccos \frac{x}{a}$, 弓形 MAN 的面积 $S_{MAN} = ab \arccos \frac{x}{a} - xy$.

- 需要 5 个点才能确定一个圆锥曲线.

- 设 θ 为 (x, y) 点关于椭圆中心的极角, r 为 (x, y) 到椭圆中心的距离, 椭圆极坐标方程:

$$x = r \cos \theta, y = r \sin \theta, r^2 = \frac{b^2 a^2}{b^2 \cos^2 \theta + a^2 \sin^2 \theta}$$

5.18.4 抛物线

- 标准方程 $y^2 = 2px$, 曲率半径 $R = \frac{(p + 2x)^{\frac{3}{2}}}{\sqrt{p}}$
- 弧长: 设 $M(x, y)$ 是抛物线上一点, 则 $L_{OM} = \frac{p}{2} [\sqrt{\frac{2x}{p}(1 + \frac{2x}{p})} + \ln(\sqrt{\frac{2x}{p}} + \sqrt{1 + \frac{2x}{p}})]$
- 弓形面积: 设 M, D 是抛物线上两点, 且分居一, 四象限. 做一条平行于 MD 且与抛物线相切的直线 L . 若 M 到 L 的距离为 h . 则有 $S_{MOD} = \frac{2}{3} MD \cdot h$.

5.18.5 重心

- 半径 r , 圆心角为 θ 的扇形的重心与圆心的距离为 $\frac{4r \sin \frac{\theta}{2}}{3\theta}$
- 半径 r , 圆心角为 θ 的圆弧的重心与圆心的距离为 $\frac{4r \sin^3 \frac{\theta}{2}}{3(\theta - \sin \theta)}$
- 椭圆上半部分的重心与圆心的距离为 $\frac{4b}{3\pi}$
- 抛物线中弓形 MOD 的重心满足 $CQ = \frac{2}{5} PQ$, P 是直线 L 与抛物线的切点, Q 在 MD 上且 PQ 平行 x 轴, C 是重心

5.18.6 向量恒等式

- $\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} \times \vec{a}) = \vec{c} \cdot (\vec{a} \times \vec{b})$
- $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{c} \times \vec{b}) \times \vec{a} = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$

5.18.7 常用几何公式

- 三角形的五心
 - 重心 $\vec{G} = \frac{\vec{A} + \vec{B} + \vec{C}}{3}$
 - 内心 $\vec{I} = \frac{a\vec{A} + b\vec{B} + c\vec{C}}{a + b + c}$, $R = \frac{2S}{a + b + c}$
 - 外心 $x = \frac{\vec{A} + \vec{B} - \frac{\vec{B}\vec{C} \cdot \vec{A}\vec{C}}{\vec{A}\vec{B} \times \vec{B}\vec{C}} \vec{A}\vec{B}^T}{2}$, $y = \frac{\vec{A} + \vec{B} + \frac{\vec{B}\vec{C} \cdot \vec{A}\vec{C}}{\vec{A}\vec{B} \times \vec{B}\vec{C}} \vec{A}\vec{B}^T}{2}$, $R = \frac{abc}{4S}$
 - 垂心 $\vec{H} = 3\vec{G} - 2\vec{O}$
 - 旁心 (三个) $\frac{-a\vec{A} + b\vec{B} + c\vec{C}}{-a + b + c}$
- 四边形: 设 D_1, D_2 为对角线, M 为对角线中点连线, A 为对角线夹角
 - $a^2 + b^2 + c^2 + d^2 = D_1^2 + D_2^2 + 4M^2$
 - $S = \frac{1}{2} D_1 D_2 \sin A$
 - $ac + bd = D_1 D_2$ (内接四边形适用)
 - Bretschneider 公式: $S = \sqrt{(p - a)(p - b)(p - c)(p - d) - abcd \cos^2(\frac{\theta}{2})}$, 其中 θ 为对角和

- 棱锥:
 - 体积 $V = \frac{1}{3}Ah$, A 为底面积, h 为高
 - (对正棱锥) 侧面积 $S = \frac{1}{2}lp$, l 为斜高, p 为底面周长

- 棱台:
 - 体积 $V = \frac{(A_1 + A_2 + \sqrt{A_1A_2}) \cdot h}{3}$, A_1, A_2 分别为上下底面面积, h 为高
 - (对正棱台) 侧面积 $S = \frac{1}{2}(p_1 + p_2) \cdot l$, p_1, p_2 为上下底面周长, l 为斜高.

5.18.8 树的计数

- 有根数计数: 令 $S_{n,j} = \sum_{1 \leq i \leq n/j} a_{n+1-ij} = S_{n-j,j} + a_{n+1-j}$
于是, $n+1$ 个结点的有根数的总数为 $a_{n+1} = \frac{\sum_{1 \leq j \leq n} j \cdot a_j \cdot S_{n,j}}{n}$
附: $a_1 = 1, a_2 = 1, a_3 = 2, a_4 = 4, a_5 = 9, a_6 = 20, a_9 = 286, a_{11} = 1842$
- 无根树计数: 当 n 是奇数时, 则有 $a_n - \sum_{1 \leq i \leq \frac{n}{2}} a_i a_{n-i}$ 种不同的无根树
当 n 是偶数时, 则有 $a_n - \sum_{1 \leq i \leq \frac{n}{2}} a_i a_{n-i} + \frac{1}{2} a_{\frac{n}{2}} (a_{\frac{n}{2}} + 1)$ 种不同的无根树
- Matrix-Tree 定理: 对任意图 G , 设 $\text{mat}[i][i] = i$ 的度数, $\text{mat}[i][j] = i$ 与 j 之间边数的相反数, 则 $\text{mat}[i][j]$ 的任意余子式的行列式就是该图的生成树个数

5.19 小知识

- 勾股数: 设正整数 n 的质因数分解为 $n = \prod p_i^{a_i}$, 则 $x^2 + y^2 = n$ 有整数解的充要条件是 n 中不存在形如 $p_i \equiv 3 \pmod{4}$ 且指数 a_i 为奇数的质因数 p_i . $(\frac{a-b}{2})^2 + ab = (\frac{a+b}{2})^2$.
- 素勾股数: 若 m 和 n 互质, 而且 m 和 n 中有一个是偶数, 则 $a = m^2 - n^2, b = 2mn, c = m^2 + n^2$, 则 a, b, c 是素勾股数.
- Stirling 公式: $n! \approx \sqrt{2\pi n} (\frac{n}{e})^n$
- Pick 定理: 简单多边形, 不自交, 顶点如果全是整点. 则: 严格在多边形内部的整点数 + $\frac{1}{2}$ 在边上的整点数 - 1 = 面积
- Mersenne 素数: p 是素数且 $2^p - 1$ 的数是素数. (10000 以内的 p 有: 2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127, 521, 607, 1279, 2203, 2281, 3217, 4253, 4423, 9689, 9941)
- 序列差分表: 差分表的第 0 条对角线确定原序列. 设原序列为 h_i , 第 0 条对角线为 $c_0, c_1, \dots, c_p, 0, 0, \dots$ 有这样两个公式: $h_n = \binom{n}{0}c_0 + \binom{n}{1}c_1 + \dots + \binom{n}{p}c_p, \sum_{k=0}^n h_k = \binom{n+1}{1}c_0 + \binom{n+1}{2}c_2 + \dots + \binom{n+1}{p+1}c_p$
- GCD: $\text{gcd}(2^a - 1, 2^b - 1) = 2^{\text{gcd}(a,b)} - 1$

- Fermat 分解算法: 从 $t = \sqrt{n}$ 开始, 依次检查 $t^2 - n, (t+1)^2 - n, (t+2)^2 - n, \dots$, 直到出现一个平方数 y , 由于 $t^2 - y^2 = n$, 因此分解得 $n = (t-y)(t+y)$. 显然, 当两个因数很接近时这个方法能很快找到结果, 但如果遇到一个素数, 则需要检查 $\frac{n+1}{2} - \sqrt{n}$ 个整数
- 牛顿迭代: $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$
- 球与盒子的动人故事: (n 个球, m 个盒子, S 为第二类斯特林数)

- 球同, 盒同, 无空: dp
- 球同, 盒同, 可空: dp
- 球同, 盒不同, 无空: $\binom{n-1}{m-1}$
- 球同, 盒不同, 可空: $\binom{n+m-1}{n-1}$
- 球不同, 盒同, 无空: $S(n, m)$
- 球不同, 盒同, 可空: $\sum_{k=1}^m S(n, k)$
- 球不同, 盒不同, 无空: $m!S(n, m)$
- 球不同, 盒不同, 可空: m^n

- 组合数奇偶性: 若 $(n \& m) = m$, 则 $\binom{n}{m}$ 为奇数, 否则为偶数

- 格雷码 $G(x) = x \otimes (x >> 1)$

- Fibonacci 数:

- $F_0 = F_1 = 1, F_i = F_{i-1} + F_{i-2}, F_{-i} = (-1)^{i-1} F_i$
- $F_i = \frac{1}{\sqrt{5}}((\frac{1+\sqrt{5}}{2})^n - (\frac{1-\sqrt{5}}{2})^n)$
- $\text{gcd}(F_n, F_m) = F_{\text{gcd}(n,m)}$
- $F_{i+1}F_i - F_i^2 = (-1)^i$
- $F_{n+k} = F_kF_{n+1} + F_{k-1}F_n$

- 第一类 Stirling 数: $[n_k]$ 代表第一类无符号 Stirling 数, 代表将 n 阶置换群中有 k 个环的置换个数; $s(n, k)$ 代表有符号型, $s(n, k) = (-1)^{n-k} [n_k]$.

- $(x)^{(n)} = \sum_{k=0}^n [n_k] x^k, (x)_n = \sum_{k=0}^n s(n, k) x^k$
- $[n_k] = n [n_{k-1}^{1}] + [n_{k-1}^{1-1}], [0] = 1, [n_0] = [n_1^0] = 0$
- $[n_{-2}] = \frac{1}{4}(3n-1)\binom{n}{3}, [n_{-3}] = \binom{n}{2}\binom{n}{4}$
- $\sum_{k=0}^a [n_k] = n! - \sum_{k=0}^n [k+a+1]$
- $\sum_{p=k}^n [p] \binom{p}{k} = [k+1^{n+1}]$

- 第二类 Stirling 数: $\{n_k\} = S(n, k)$ 代表 n 个不同的球, 放到 k 个相同的盒子里, 盒子非空.

- $\{n_k\} = \frac{1}{k!} \sum_{j=0}^k (-1)^j \binom{k}{j} (k-j)^n$
 - $\{n+1_k\} = k\{n_k\} + \{n_{k-1}\}, \{0_0\} = 1, \{n_0\} = \{0_n\} = 0$
 - 奇偶性: $(n-k) \& \frac{k-1}{2} == 0$
- Bell 数: B_n 代表将 n 个元素划分成若干个非空集合的方案数
 - $B_0 = B_1 = 1, B_n = \sum_{k=0}^{n-1} \binom{n-1}{k} B_k$
 - $B_n = \sum_{k=0}^n \{n_k\}$
 - Bell 三角形: $a_{1,1} = 1, a_{n,1} = a_{n-1,n-1}, a_{n,m} = a_{n,m-1} + a_{n-1,m-1}, B_n = a_{n,1}$
 - 对质数 $p, B_{n+p} \equiv B_n + B_{n+1} \pmod{p}$
 - 对质数 $p, B_{n+p^m} \equiv mB_n + B_{n+1} \pmod{p}$
 - 对质数 p , 模的周期一定是 $\frac{p^p-1}{p-1}$ 的约数, $p \leq 101$ 时就是这个值
 - 从 B_0 开始, 前几项是 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975...

- Bernoulli 数
- $B_0 = 1, B_1 = \frac{1}{2}, B_2 = \frac{1}{6}, B_4 = -\frac{1}{30}, B_6 = \frac{1}{42}, B_8 = B_4, B_{10} = \frac{5}{66}$
- $\sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k n^{m+1-k}$
- $B_m = 1 - \sum_{k=0}^{m-1} \binom{m}{k} \frac{B_k}{m-k+1}$

- 完全数: x 是偶完全数等价于 $x = 2^{n-1}(2^n - 1)$, 且 $2^n - 1$ 是质数.

6 其他

6.1 Extended LIS

```
1 int G[MAXN][MAXN];
2 void insertYoung(int v) {
3     for (int x = 1, y = INT_MAX; ; ++x) {
4         Down(y, *G[x]); while (y > 0 && G[x][y] >= v) --y;
5         if (++y > *G[x]) { ++*G[x]; G[x][y] = v; break; }
6         else swap(G[x][y], v);
7     }
8 }
9 int solve(int N, int seq[]) {
10     Rep(i, 1, N) *G[i] = 0;
11     Rep(i, 1, N) insertYoung(seq[i]);
```

```
12     printf("%d\n", *G[1] + *G[2]);
13     return 0;
14 }
```

6.2 生成 nCk

```
1 void nCk(int n, int k) {
2     for (int comb = (1 << k) - 1; comb < (1 << n); ) {
3         int x = comb & -comb, y = comb + x;
4         comb = (((comb & ~y) / x) >> 1) | y;
5     }
6 }
```

6.3 nextPermutation

```
1 boolean nextPermutation(int[] is) {
2     int n = is.length;
3     for (int i = n - 1; i > 0; i--) {
4         if (is[i - 1] < is[i]) {
5             int j = n; while (is[i - 1] >= is[--j]);
6             swap(is, i - 1, j); // swap is[i - 1], is[j]
7             rev(is, i, n); // reverse is[i, n)
8             return true;
9         }
10    } rev(is, 0, n);
11    return false;
12 }
```

6.4 Josephus 数与逆 Josephus 数

```
1 int josephus(int n, int m, int k) { int x = -1;
2     for (int i = n - k + 1; i <= n; i++) x = (x + m) % i; return x;
3 }
4 int invJosephus(int n, int m, int x) {
5     for (int i = n; ; i--) { if (x == i) return n - i; x = (x - m % i + i) % i; }
6 }
```

6.5 表达式求值

```
1 inline int getLevel(char ch) {
2     switch (ch) { case '+': case '-': return 0; case '*': return 1; } return -1;
3 }
4 int evaluate(char *&p, int level) {
```

```

5   int res;
6   if (level == 2) {
7       if (*p == '(') ++p, res = evaluate(p, 0);
8       else res = isdigit(*p) ? *p - '0' : value[*p - 'a'];
9       ++p; return res;
10  } res = evaluate(p, level + 1);
11  for (int next; *p && getLevel(*p) == level; ) {
12      char op = *p++; next = evaluate(p, level + 1);
13      switch (op) {
14          case '+': res += next; break;
15          case '-': res -= next; break;
16          case '*': res *= next; break;
17      }
18  } return res;
19 }
20 int makeEvaluation(char *str) { char *p = str; return evaluate(p, 0); }

```

6.6 曼哈顿最小生成树

```

1  const int INF = 1000000005;
2  struct TreeEdge {
3      int x, y, z; void make(int _x, int _y, int _z) { x = _x; y = _y; z = _z; }
4  } data[maxn * 4];
5  int n, x[maxn], y[maxn], px[maxn], py[maxn], id[maxn], tree[maxn], node[maxn], val[maxn],
    fa[maxn];
6  bool operator < (const TreeEdge& x, const TreeEdge& y) { return x.z < y.z; }
7  bool cmp1(int a, int b) { return x[a] < x[b]; }
8  bool cmp2(int a, int b) { return y[a] < y[b]; }
9  bool cmp3(int a, int b) { return (y[a] - x[a] < y[b] - x[b] || (y[a] - x[a] == y[b] - x[b]
10     ] && y[a] > y[b])); }
11 bool cmp4(int a, int b) { return (y[a] - x[a] > y[b] - x[b] || (y[a] - x[a] == y[b] - x[b]
12     ] && x[a] > x[b])); }
13 bool cmp5(int a, int b) { return (x[a] + y[a] > x[b] + y[b] || (x[a] + y[a] == x[b] + y[b]
14     ] && x[a] < x[b])); }
15 bool cmp6(int a, int b) { return (x[a] + y[a] < x[b] + y[b] || (x[a] + y[a] == x[b] + y[b]
16     ] && y[a] > y[b])); }
17 void Change_X() {
18     for (int i = 0; i < n; ++i) val[i] = x[i];
19     for (int i = 0; i < n; ++i) id[i] = i;
20     sort(id, id + n, cmp1);
21     int cntM = 1, last = val[id[0]]; px[id[0]] = 1;
22     for (int i = 1; i < n; ++i) {
23         if (val[id[i]] > last) ++cntM, last = val[id[i]];
24         px[id[i]] = cntM;
25     }
26 }
27 void Change_Y() {
28     for (int i = 0; i < n; ++i) val[i] = y[i];
29 }

```

```

25 for (int i = 0; i < n; ++i) id[i] = i;
26 sort(id, id + n, cmp2);
27 int cntM = 1, last = val[id[0]]; py[id[0]] = 1;
28 for (int i = 1; i < n; ++i) {
29     if (val[id[i]] > last)
30         ++cntM, last = val[id[i]];
31     py[id[i]] = cntM;
32 }
33 }
34 inline int Cost(int a, int b) { return abs(x[a] - x[b]) + abs(y[a] - y[b]); }
35 int find(int x) { return (fa[x] == x) ? x : (fa[x] = find(fa[x])); }
36 int main() {
37     for (int i = 0; i < n; ++i) scanf("%d%d", x + i, y + i);
38     Change_X(); Change_Y();
39     int cntE = 0; for (int i = 0; i < n; ++i) id[i] = i;
40     sort(id, id + n, cmp3);
41     for (int i = 1; i <= n; ++i) tree[i] = INF, node[i] = -1;
42     for (int i = 0; i < n; ++i) {
43         int Min = INF, Tnode = -1;
44         for (int k = py[id[i]]; k <= n; k += k & (-k))
45             if (tree[k] < Min) Min = tree[k], Tnode = node[k];
46         if (Tnode >= 0) data[cntE++].make(id[i], Tnode, Cost(id[i], Tnode));
47         int tmp = x[id[i]] + y[id[i]];
48         for (int k = py[id[i]]; k; k -= k & (-k))
49             if (tmp < tree[k]) tree[k] = tmp, node[k] = id[i];
50     } sort(id, id + n, cmp4);
51     for (int i = 1; i <= n; ++i) tree[i] = INF, node[i] = -1;
52     for (int i = 0; i < n; ++i) {
53         int Min = INF, Tnode = -1;
54         for (int k = px[id[i]]; k <= n; k += k & (-k))
55             if (tree[k] < Min) Min = tree[k], Tnode = node[k];
56         if (Tnode >= 0) data[cntE++].make(id[i], Tnode, Cost(id[i], Tnode));
57         int tmp = x[id[i]] + y[id[i]];
58         for (int k = px[id[i]]; k; k -= k & (-k))
59             if (tmp < tree[k]) tree[k] = tmp, node[k] = id[i];
60     }
61     sort(id, id + n, cmp5);
62     for (int i = 1; i <= n; ++i) tree[i] = INF, node[i] = -1;
63     for (int i = 0; i < n; ++i) {
64         int Min = INF, Tnode = -1;
65         for (int k = px[id[i]]; k; k -= k & (-k))
66             if (tree[k] < Min) Min = tree[k], Tnode = node[k];
67         if (Tnode >= 0) data[cntE++].make(id[i], Tnode, Cost(id[i], Tnode));
68         int tmp = -x[id[i]] + y[id[i]];
69         for (int k = px[id[i]]; k <= n; k += k & (-k))
70             if (tmp < tree[k]) tree[k] = tmp, node[k] = id[i];
71     } sort(id, id + n, cmp6);
72     for (int i = 1; i <= n; ++i) tree[i] = INF, node[i] = -1;
73     for (int i = 0; i < n; ++i) {

```

```

74     int Min = INF, Tnode = -1;
75     for (int k = py[id[i]]; k <= n; k += k & (-k))
76         if (tree[k] < Min) Min = tree[k], Tnode = node[k];
77     if (Tnode >= 0) data[cntE++].make(id[i], Tnode, Cost(id[i], Tnode));
78     int tmp = -x[id[i]] + y[id[i]];
79     for (int k = py[id[i]]; k; k -= k & (-k))
80         if (tmp < tree[k]) tree[k] = tmp, node[k] = id[i];
81 }
82 long long Ans = 0; sort(data, data + cntE);
83 for (int i = 0; i < n; ++i) fa[i] = i;
84 for (int i = 0; i < cntE; ++i) if (find(data[i].x) != find(data[i].y)) {
85     Ans += data[i].z;
86     fa[fa[data[i].x]] = fa[data[i].y];
87 } cout << Ans << endl;
88 }

```

6.7 直线下的整点个数

求 $\sum_{i=0}^{n-1} \lfloor \frac{a+bi}{m} \rfloor$

```

1 LL count(LL n, LL a, LL b, LL m) {
2     if (b == 0) return n * (a / m);
3     if (a >= m) return n * (a / m) + count(n, a % m, b, m);
4     if (b >= m) return (n - 1) * n / 2 * (b / m) + count(n, a, b % m, m);
5     return count((a + b * n) / m, (a + b * n) % m, m, b);
6 }

```

6.8 Java 多项式

```

1 class Polynomial {
2     final static Polynomial ZERO = new Polynomial(new int[] { 0 });
3     final static Polynomial ONE = new Polynomial(new int[] { 1 });
4     final static Polynomial X = new Polynomial(new int[] { 0, 1 });
5     int[] coef;
6     static Polynomial valueOf(int val) { return new Polynomial(new int[] { val }); }
7     Polynomial(int[] coef) { this.coef = Arrays.copyOf(coef, coef.length); }
8     Polynomial add(Polynomial o, int mod); // omitted
9     Polynomial subtract(Polynomial o, int mod); // omitted
10    Polynomial multiply(Polynomial o, int mod); // omitted
11    Polynomial scale(int o, int mod); // omitted
12    public String toString() {
13        int n = coef.length; String ret = "";
14        for (int i = n - 1; i > 0; --i) if (coef[i] != 0)
15            ret += coef[i] + "x^" + i + "+";
16        return ret + coef[0];
17    }
18    static Polynomial lagrangeInterpolation(int[] x, int[] y, int mod) {

```

```

19        int n = x.length; Polynomial ret = Polynomial.ZERO;
20        for (int i = 0; i < n; ++i) {
21            Polynomial poly = Polynomial.valueOf(y[i]);
22            for (int j = 0; j < n; ++j) if (i != j) {
23                poly = poly.multiply(
24                    Polynomial.X.subtract(Polynomial.valueOf(x[j]), mod), mod);
25                poly = poly.scale(powMod(x[i] - x[j] + mod, mod - 2, mod), mod);
26            } ret = ret.add(poly, mod);
27        } return ret;
28    }
29 }

```

6.9 long long 乘法取模

```

1 LL multiplyMod(LL a, LL b, LL P) { // 需要保证 a 和 b 非负
2     LL t = (a * b - LL((long double)a / P * b + 1e-3) * P) % P;
3     return t < 0 : t + P : t;
4 }

```

6.10 重复覆盖

```

1 namespace DLX {
2     struct node { int x, y; node *l, *r, *u, *d; } base[MAX * MAX], *top, *head;
3     typedef node *link;
4     int row, col, nGE, ans, stamp, cntc[MAX], vis[MAX];
5     vector<link> eachRow[MAX], eachCol[MAX];
6     inline void addElement(int x, int y) {
7         top->x = x; top->y = y; top->l = top->r = top->u = top->d = NULL;
8         eachRow[x].push_back(top); eachCol[y].push_back(top++);
9     }
10    void init(int _row, int _col, int _nGE) {
11        row = _row; col = _col; nGE = _nGE; top = base; stamp = 0;
12        for (int i = 0; i <= col; ++i) vis[i] = 0;
13        for (int i = 0; i <= row; ++i) eachRow[i].clear();
14        for (int i = 0; i <= col; ++i) eachCol[i].clear();
15        for (int i = 0; i <= col; ++i) addElement(0, i);
16        head = eachCol[0].front();
17    }
18    void build() {
19        for (int i = 0; i <= row; ++i) {
20            vector<link> &v = eachRow[i];
21            sort(v.begin(), v.end(), cmpByY);
22            int s = v.size();
23            for (int j = 0; j < s; ++j) {
24                link l = v[j], r = v[(j + 1) % s]; l->r = r; r->l = l;
25            }

```

```

26     }
27     for (int i = 0; i <= col; ++i) {
28         vector<link> &v = eachCol[i];
29         sort(v.begin(), v.end(), cmpByX);
30         int s = v.size();
31         for (int j = 0; j < s; ++j) {
32             link u = v[j], d = v[(j + 1) % s]; u->d = d; d->u = u;
33         }
34     } for (int i = 0; i <= col; ++i) cntc[i] = (int) eachCol[i].size() - 1;
35 }
36 void removeExact(link c) {
37     c->l->r = c->r; c->r->l = c->l;
38     for (link i = c->d; i != c; i = i->d)
39         for (link j = i->r; j != i; j = j->r) {
40             j->d->u = j->u; j->u->d = j->d; --cntc[j->y];
41         }
42 }
43 void resumeExact(link c) {
44     for (link i = c->u; i != c; i = i->u)
45         for (link j = i->l; j != i; j = j->l) {
46             j->d->u = j; j->u->d = j; ++cntc[j->y];
47         }
48     c->l->r = c; c->r->l = c;
49 }
50 void removeRepeat(link c) {
51     for (link i = c->d; i != c; i = i->d) {
52         i->l->r = i->r; i->r->l = i->l;
53     }
54 }
55 void resumeRepeat(link c) {
56     for (link i = c->u; i != c; i = i->u) {
57         i->l->r = i; i->r->l = i;
58     }
59 }
60 int calcH() {
61     int y, res = 0; ++stamp;
62     for (link c = head->r; (y = c->y) <= row && c != head; c = c->r) {
63         if (vis[y] != stamp) {
64             vis[y] = stamp; ++res;
65             for (link i = c->d; i != c; i = i->d)
66                 for (link j = i->r; j != i; j = j->r) vis[j->y] = stamp;
67         }
68     } return res;
69 }
70 void DFS(int dep) { if (dep + calcH() >= ans) return;
71     if (head->r->y > nGE || head->r == head) {
72         if (ans > dep) ans = dep; return;
73     } link c = NULL;
74     for (link i = head->r; i->y <= nGE && i != head; i = i->r)

```

```

75         if (!c || cntc[i->y] < cntc[c->y]) c = i;
76     for (link i = c->d; i != c; i = i->d) {
77         removeRepeat(i);
78         for (link j = i->r; j != i; j = j->r) if (j->y <= nGE) removeRepeat(j);
79         for (link j = i->r; j != i; j = j->r) if (j->y > nGE) removeExact(base + j->y);
80         DFS(dep + 1);
81         for (link j = i->l; j != i; j = j->l) if (j->y > nGE) resumeExact(base + j->y);
82         for (link j = i->l; j != i; j = j->l) if (j->y <= nGE) resumeRepeat(j);
83         resumeRepeat(i);
84     }
85 }
86 int solve() { build(); ans = INF; DFS(0); return ans; }
87 }

```

6.11 星期几判定

```

1 int getDay(int y, int m, int d) {
2     if (m <= 2) m += 12, y--;
3     if (y < 1752 || (y == 1752 && m < 9) || (y == 1752 && m == 9 && d < 3))
4         return (d + 2 * m + 3 * (m + 1) / 5 + y + y / 4 + 5) % 7 + 1;
5     return (d + 2 * m + 3 * (m + 1) / 5 + y + y / 4 - y / 100 + y / 400) % 7 + 1;
6 }

```

6.12 LCSequence Fast

```

1 ULL *a, *b, *s, c, d;
2 for (i = 0, a = appear[(int)B[k]], b = row[max(k - 1, 0)], s = X; i < bitSetLen; ++i)
3     *s++ = *a++ | *b++; // X = row[i - 1] or appear[ B[i] ]
4 for (i = 0, a = dp, c = d = 0; i < bitSetLen; ++a, c = d, ++i)
5     d = *a >> 63, *a = ~((*a << 1) + c); // row[i] = -((row[i] << 1) + 1)
6 for (i = 0, a = dp, b = X, c = 0; i < bitSetLen; ++a, ++b, ++i)
7     d = *b + c, c = (*a >= -d), *a += d; // row[i] += X
8 for (i = 0, a = dp, b = X; i < bitSetLen; ++a, ++b, ++i)
9     *a = (*a ^ *b) & *b; // row[i] = X and (row[i] xor X)

```

6.13 C Split

```

1 for (char *tok = strtok(ins, delimiters); tok; tok = strtok(NULL, delimiters))
2     puts(tok); // '会破坏原字符串ins'

```

6.14 builtin 系列

- int __builtin_ffs (unsigned int x) 返回 x 的最后一位 1 的是从后向前第几位, 比如 7368(1110011001000) 返回 4.

- `int __builtin_clz` (unsigned int x) 返回前导的 0 的个数.
- `int __builtin_ctz` (unsigned int x) 返回后面的 0 个数, 和 `__builtin_clz` 相对.
- `int __builtin_popcount` (unsigned int x) 返回二进制表示中 1 的个数.
- `int __builtin_parity` (unsigned int x) 返回 x 的奇偶校验位, 也就是 x 的 1 的个数模 2 的结果.

7 Templates

7.1 泰勒级数

$$\begin{aligned}
 \frac{1}{1-x} &= 1 + x + x^2 + x^3 + x^4 + \cdots &= \sum_{i=0}^{\infty} x^i \\
 \frac{1}{1-cx} &= 1 + cx + c^2x^2 + c^3x^3 + \cdots &= \sum_{i=0}^{\infty} c^i x^i \\
 \frac{1}{1-x^n} &= 1 + x^n + x^{2n} + x^{3n} + \cdots &= \sum_{i=0}^{\infty} x^{ni} \\
 \frac{x}{(1-x)^2} &= x + 2x^2 + 3x^3 + 4x^4 + \cdots &= \sum_{i=0}^{\infty} ix^i \\
 \sum_{k=0}^n \begin{Bmatrix} n \\ k \end{Bmatrix} \frac{k!z^k}{(1-z)^{k+1}} &= x + 2^n x^2 + 3^n x^3 + 4^n x^4 + \cdots &= \sum_{i=0}^{\infty} i^n x^i \\
 e^x &= 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \cdots &= \sum_{i=0}^{\infty} \frac{x^i}{i!} \\
 \ln(1+x) &= x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots &= \sum_{i=1}^{\infty} (-1)^{i+1} \frac{x^i}{i} \\
 \ln \frac{1}{1-x} &= x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \cdots &= \sum_{i=1}^{\infty} \frac{x^i}{i} \\
 \sin x &= x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \cdots &= \sum_{i=0}^{\infty} (-1)^i \frac{x^{2i+1}}{(2i+1)!} \\
 \cos x &= 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \cdots &= \sum_{i=0}^{\infty} (-1)^i \frac{x^{2i}}{(2i)!} \\
 \tan^{-1} x &= x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \cdots &= \sum_{i=0}^{\infty} (-1)^i \frac{x^{2i+1}}{(2i+1)} \\
 (1+x)^n &= 1 + nx + \frac{n(n-1)}{2}x^2 + \cdots &= \sum_{i=0}^{\infty} \binom{n}{i} x^i
 \end{aligned}$$

$$\begin{aligned}
 \frac{1}{(1-x)^{n+1}} &= 1 + (n+1)x + \binom{n+2}{2}x^2 + \cdots &= \sum_{i=0}^{\infty} \binom{i+n}{i} x^i \\
 \frac{x}{e^x - 1} &= 1 - \frac{1}{2}x + \frac{1}{12}x^2 - \frac{1}{720}x^4 + \cdots &= \sum_{i=0}^{\infty} \frac{B_i x^i}{i!} \\
 \frac{1}{2x}(1 - \sqrt{1-4x}) &= 1 + x + 2x^2 + 5x^3 + \cdots &= \sum_{i=0}^{\infty} \frac{1}{i+1} \binom{2i}{i} x^i \\
 \frac{1}{\sqrt{1-4x}} &= 1 + 2x + 6x^2 + 20x^3 + \cdots &= \sum_{i=0}^{\infty} \binom{2i}{i} x^i \\
 \frac{1}{\sqrt{1-4x}} \left(\frac{1 - \sqrt{1-4x}}{2x} \right)^n &= 1 + (2+n)x + \binom{4+n}{2}x^2 + \cdots &= \sum_{i=0}^{\infty} \binom{2i+n}{i} x^i \\
 \frac{1}{1-x} \ln \frac{1}{1-x} &= x + \frac{3}{2}x^2 + \frac{11}{6}x^3 + \frac{25}{12}x^4 + \cdots &= \sum_{i=1}^{\infty} H_i x^i \\
 \frac{1}{2} \left(\ln \frac{1}{1-x} \right)^2 &= \frac{1}{2}x^2 + \frac{3}{4}x^3 + \frac{11}{24}x^4 + \cdots &= \sum_{i=2}^{\infty} \frac{H_{i-1} x^i}{i} \\
 \frac{x}{1-x-x^2} &= x + x^2 + 2x^3 + 3x^4 + \cdots &= \sum_{i=0}^{\infty} F_i x^i \\
 \frac{F_n x}{1 - (F_{n-1} + F_{n+1})x - (-1)^n x^2} &= F_n x + F_{2n} x^2 + F_{3n} x^3 + \cdots &= \sum_{i=0}^{\infty} F_{ni} x^i
 \end{aligned}$$

7.2 积分表

- $d(\tan x) = \sec^2 x dx$
- $d(\cot x) = \csc^2 x dx$
- $d(\sec x) = \tan x \sec x dx$
- $d(\csc x) = -\cot x \csc x dx$
- $d(\arcsin x) = \frac{1}{\sqrt{1-x^2}} dx$
- $d(\arccos x) = \frac{-1}{\sqrt{1-x^2}} dx$
- $d(\arctan x) = \frac{1}{1+x^2} dx$
- $d(\operatorname{arccot} x) = \frac{-1}{1+x^2} dx$

- $d(\operatorname{arcsec} x) = \frac{1}{x\sqrt{1-x^2}}dx$
- $d(\operatorname{arccsc} x) = \frac{-1}{u\sqrt{1-x^2}}dx$
- $\int cu \, dx = c \int u \, dx$
- $\int (u+v) \, dx = \int u \, dx + \int v \, dx$
- $\int x^n \, dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1$
- $\int \frac{1}{x} dx = \ln x$
- $\int e^x \, dx = e^x$
- $\int \frac{dx}{1+x^2} = \arctan x$
- $\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$
- $\int \sin x \, dx = -\cos x$
- $\int \cos x \, dx = \sin x$
- $\int \tan x \, dx = -\ln |\cos x|$
- $\int \cot x \, dx = \ln |\cos x|$
- $\int \sec x \, dx = \ln |\sec x + \tan x|$
- $\int \csc x \, dx = \ln |\csc x + \cot x|$
- $\int \arcsin \frac{x}{a} dx = \arcsin \frac{x}{a} + \sqrt{a^2 - x^2}, \quad a > 0$
- $\int \arccos \frac{x}{a} dx = \arccos \frac{x}{a} - \sqrt{a^2 - x^2}, \quad a > 0$
- $\int \arctan \frac{x}{a} dx = x \arctan \frac{x}{a} - \frac{a}{2} \ln(a^2 + x^2), \quad a > 0$

- $\int \sin^2(ax) dx = \frac{1}{2a}(ax - \sin(ax) \cos(ax))$
- $\int \cos^2(ax) dx = \frac{1}{2a}(ax + \sin(ax) \cos(ax))$
- $\int \sec^2 x \, dx = \tan x$
- $\int \csc^2 x \, dx = -\cot x$
- $\int \sin^n x \, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x \, dx$
- $\int \cos^n x \, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx$
- $\int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx, \quad n \neq 1$
- $\int \cot^n x \, dx = -\frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x \, dx, \quad n \neq 1$
- $\int \sec^n x \, dx = \frac{\tan x \sec^{n-1} x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx, \quad n \neq 1$
- $\int \csc^n x \, dx = -\frac{\cot x \csc^{n-1} x}{n-1} + \frac{n-2}{n-1} \int \csc^{n-2} x \, dx, \quad n \neq 1$
- $\int \sinh x \, dx = \cosh x$
- $\int \cosh x \, dx = \sinh x$
- $\int \tanh x \, dx = \ln |\cosh x|$
- $\int \coth x \, dx = \ln |\sinh x|$
- $\int \operatorname{sech} x \, dx = \arctan \sinh x$
- $\int \operatorname{csch} x \, dx = \ln \left| \tanh \frac{x}{2} \right|$
- $\int \sinh^2 x \, dx = \frac{1}{4} \sinh(2x) - \frac{1}{2} x$

$$\bullet \int \cosh^2 x \, dx = \frac{1}{4} \sinh(2x) + \frac{1}{2}x$$

$$\bullet \int \operatorname{sech}^2 x \, dx = \tanh x$$

$$\bullet \int \operatorname{arcsinh} \frac{x}{a} dx = x \operatorname{arcsinh} \frac{x}{a} - \sqrt{x^2 + a^2}, \quad a > 0$$

$$\bullet \int \operatorname{arctanh} \frac{x}{a} dx = x \operatorname{arctanh} \frac{x}{a} + \frac{a}{2} \ln |a^2 - x^2|$$

$$\bullet \int \operatorname{arccosh} \frac{x}{a} dx = \begin{cases} x \operatorname{arccosh} \frac{x}{a} - \sqrt{x^2 + a^2}, & \text{if } \operatorname{arccosh} \frac{x}{a} > 0 \text{ and } a > 0 \\ x \operatorname{arccosh} \frac{x}{a} + \sqrt{x^2 + a^2}, & \text{if } \operatorname{arccosh} \frac{x}{a} < 0 \text{ and } a > 0 \end{cases}$$

$$\bullet \int \frac{dx}{\sqrt{a^2 + x^2}} = \ln \left(x + \sqrt{a^2 + x^2} \right), \quad a > 0$$

$$\bullet \int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a}, \quad a > 0$$

$$\bullet \int \sqrt{a^2 - x^2} \, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin \frac{x}{a}, \quad a > 0$$

$$\bullet \int (a^2 - x^2)^{3/2} dx = \frac{x}{8} (5a^2 - 2x^2) \sqrt{a^2 - x^2} + \frac{3a^4}{8} \arcsin \frac{x}{a}, \quad a > 0$$

$$\bullet \int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a}, \quad a > 0$$

$$\bullet \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right|$$

$$\bullet \int \frac{dx}{(a^2 - x^2)^{3/2}} = \frac{x}{a^2 \sqrt{a^2 - x^2}}$$

$$\bullet \int \sqrt{a^2 \pm x^2} \, dx = \frac{x}{2} \sqrt{a^2 \pm x^2} \pm \frac{a^2}{2} \ln \left| x + \sqrt{a^2 \pm x^2} \right|$$

$$\bullet \int \frac{dx}{\sqrt{x^2 - a^2}} = \ln \left| x + \sqrt{x^2 - a^2} \right|, \quad a > 0$$

$$\bullet \int \frac{dx}{ax^2 + bx} = \frac{1}{a} \ln \left| \frac{x}{a + bx} \right|$$

$$\bullet \int x \sqrt{a + bx} \, dx = \frac{2(3bx - 2a)(a + bx)^{3/2}}{15b^2}$$

$$\bullet \int \frac{\sqrt{a + bx}}{x} dx = 2\sqrt{a + bx} + a \int \frac{1}{x\sqrt{a + bx}} dx$$

$$\bullet \int \frac{x}{\sqrt{a + bx}} dx = \frac{1}{\sqrt{2}} \ln \left| \frac{\sqrt{a + bx} - \sqrt{a}}{\sqrt{a + bx} + \sqrt{a}} \right|, \quad a > 0$$

$$\bullet \int \frac{\sqrt{a^2 - x^2}}{x} dx = \sqrt{a^2 - x^2} - a \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right|$$

$$\bullet \int x \sqrt{a^2 - x^2} \, dx = -\frac{1}{3} (a^2 - x^2)^{3/2}$$

$$\bullet \int x^2 \sqrt{a^2 - x^2} \, dx = \frac{x}{8} (2x^2 - a^2) \sqrt{a^2 - x^2} + \frac{a^4}{8} \arcsin \frac{x}{a}, \quad a > 0$$

$$\bullet \int \frac{dx}{\sqrt{a^2 - x^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right|$$

$$\bullet \int \frac{x \, dx}{\sqrt{a^2 - x^2}} = -\sqrt{a^2 - x^2}$$

$$\bullet \int \frac{x^2 \, dx}{\sqrt{a^2 - x^2}} = -\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin \frac{x}{a}, \quad a > 0$$

$$\bullet \int \frac{\sqrt{a^2 + x^2}}{x} dx = \sqrt{a^2 + x^2} - a \ln \left| \frac{a + \sqrt{a^2 + x^2}}{x} \right|$$

$$\bullet \int \frac{\sqrt{x^2 - a^2}}{x} dx = \sqrt{x^2 - a^2} - a \arccos \frac{a}{|x|}, \quad a > 0$$

$$\bullet \int x \sqrt{x^2 \pm a^2} \, dx = \frac{1}{3} (x^2 \pm a^2)^{3/2}$$

$$\bullet \int \frac{dx}{x \sqrt{x^2 + a^2}} = \frac{1}{a} \ln \left| \frac{x}{a + \sqrt{a^2 + x^2}} \right|$$

$$\bullet \int \frac{dx}{x \sqrt{x^2 - a^2}} = \frac{1}{a} \arccos \frac{a}{|x|}, \quad a > 0$$

$$\bullet \int \frac{dx}{x^2 \sqrt{x^2 \pm a^2}} = \mp \frac{\sqrt{x^2 \pm a^2}}{a^2 x}$$

$$\bullet \int \frac{x \, dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}$$

- $\int \frac{\sqrt{x^2 \pm a^2}}{x^4} dx = \mp \frac{(x^2 + a^2)^{3/2}}{3a^2 x^3}$
- $\int \frac{dx}{ax^2 + bx + c} = \begin{cases} \frac{1}{\sqrt{b^2 - 4ac}} \ln \left| \frac{2ax + b - \sqrt{b^2 - 4ac}}{2ax + b + \sqrt{b^2 - 4ac}} \right|, & \text{if } b^2 > 4ac \\ \frac{2}{\sqrt{4ac - b^2}} \arctan \frac{2ax + b}{\sqrt{4ac - b^2}}, & \text{if } b^2 < 4ac \end{cases}$
- $\int \frac{dx}{\sqrt{ax^2 + bx + c}} = \begin{cases} \frac{1}{\sqrt{a}} \ln \left| 2ax + b + 2\sqrt{a}\sqrt{ax^2 + bx + c} \right|, & \text{if } a > 0 \\ \frac{1}{\sqrt{-a}} \arcsin \frac{-2ax - b}{\sqrt{b^2 - 4ac}}, & \text{if } a < 0 \end{cases}$
- $\int \sqrt{ax^2 + bx + c} dx = \frac{2ax + b}{4a} \sqrt{ax^2 + bx + c} + \frac{4ax - b^2}{8a} \int \frac{dx}{\sqrt{ax^2 + bx + c}}$
- $\int \frac{x dx}{\sqrt{ax^2 + bx + c}} = \frac{\sqrt{ax^2 + bx + c}}{a} - \frac{b}{2a} \int \frac{dx}{\sqrt{ax^2 + bx + c}}$
- $\int \frac{dx}{x\sqrt{ax^2 + bx + c}} = \begin{cases} \frac{-1}{\sqrt{c}} \ln \left| \frac{2\sqrt{c}\sqrt{ax^2 + bx + c} + bx + 2c}{x} \right|, & \text{if } c > 0 \\ \frac{1}{\sqrt{-c}} \arcsin \frac{bx + 2c}{|x|\sqrt{b^2 - 4ac}}, & \text{if } c < 0 \end{cases}$
- $\int x^3 \sqrt{x^2 + a^2} dx = \left(\frac{1}{3}x^2 - \frac{2}{15}a^2\right)(x^2 + a^2)^{3/2}$
- $\int x^n \sin(ax) dx = -\frac{1}{a}x^n \cos(ax) + \frac{n}{a} \int x^{n-1} \cos(ax) dx$
- $\int x^n \cos(ax) dx = \frac{1}{a}x^n \sin(ax) - \frac{n}{a} \int x^{n-1} \sin(ax) dx$
- $\int x^n e^{ax} dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} dx$
- $\int x^n \ln(ax) dx = x^{n+1} \left(\frac{\ln(ax)}{n+1} - \frac{1}{(n+1)^2} \right)$
- $\int x^n (\ln ax)^m dx = \frac{x^{n+1}}{n+1} (\ln ax)^m - \frac{m}{n+1} \int x^n (\ln ax)^{m-1} dx$

7.3 Eclipse 配置

Exec=env UBUNTU_MENUPROXY= /opt/eclipse/eclipse

preference general keys 把 word completion 设置成 alt+c, 把 content assistant 设置成 alt + /

7.4 C++

```

1  #pragma comment(linker, "/STACK:10240000")
2  #include <cstdio>
3  #include <cstdlib>
4  #include <cstring>
5  #include <iostream>
6  #include <algorithm>
7  #define Rep(i, a, b) for(int i = (a); i <= (b); ++i)
8  #define Foru(i, a, b) for(int i = (a); i < (b); ++i)
9  using namespace std;
10 typedef long long LL;
11 typedef pair<int, int> pii;
12 namespace BufferedReader {
13     char buff[MAX_BUFFER + 5], *ptr = buff, c; bool flag;
14     bool nextChar(char &c) {
15         if ( (c = *ptr++) == 0 ) {
16             int tmp = fread(buff, 1, MAX_BUFFER, stdin);
17             buff[tmp] = 0; if (tmp == 0) return false;
18             ptr = buff; c = *ptr++;
19         } return true;
20     }
21     bool nextUnsignedInt(unsigned int &x) {
22         for (;;) {if (!nextChar(c)) return false; if ('0'<=c && c<='9') break;}
23         for (x=c-'0'; nextChar(c); x = x * 10 + c - '0') if (c < '0' || c > '9') break;
24         return true;
25     }
26     bool nextInt(int &x) {
27         for (;;) { if (!nextChar(c)) return false; if (c=='-' || ('0'<=c && c<='9')) break; }
28         for ((c=='-') ? (x=0,flag=true) : (x=c-'0',flag=false); nextChar(c); x=x*10+c-'0')
29             if (c<'0' || c>'9') break;
30         if (flag) x=-x; return true;
31     }
32 };
33 #endif

```

7.5 Java

```

1  import java.io.*;
2  import java.util.*;
3  import java.math.*;
4
5  public class Main {
6      public void solve() {}
7      public void run() {
8          tokenizer = null; out = new PrintWriter(System.out);
9          in = new BufferedReader(new InputStreamReader(System.in));
10         solve();

```

```
11     out.close();
12 }
13 public static void main(String[] args) {
14     new Main().run();
15 }
16 public StringTokenizer tokenizer;
17 public BufferedReader in;
18 public PrintWriter out;
19 public String next() {
20     while (tokenizer == null || !tokenizer.hasMoreTokens()) {
21         try { tokenizer = new StringTokenizer(in.readLine()); }
22         catch (IOException e) { throw new RuntimeException(e); }
23     } return tokenizer.nextToken();
24 }
25 }
```

7.6 vim 配置

在.bashrc 中加入 export CXXFLAGS="-Wall -Wconversion -Wextra -g3"

```
1 set nu ru nobk cindent si
2 set mouse=a sw=4 sts=4 ts=4
3 set hlsearch incsearch
4 set whichwrap=b,s,<,>,[,]
5 syntax on
6
7 nmap <C-A> ggVG
8 vmap <C-C> "+y
9
10 autocmd BufNewFile *.cpp Or ~/Templates/cpp.cpp
11 map<F9>:!g++ -o %<-Wall -Wconversion -Wextra -g3<CR>
12 map<F5>:!. /%<CR>
13 map<F8>:!. /%<%<.in<CR>
14
15 map<F3>:vnew %<.in<CR>
16 map<F4>:!(gedit %&)<CR>
```