

Contents

1	计算几何	2	4	图论	19
1.1	二维计算几何基本操作	2	4.1	带花树	19
1.2	圆的面积模板	4	4.2	最大流	20
1.3	多边形相关	5	4.3	最高标号预流推进	20
1.4	直线与凸包求交点	6	4.4	KM	21
1.5	半平面交	6	4.5	2-SAT 与 Kosaraju	21
1.6	最大面积空凸包	6	4.6	全局最小割 Stoer-Wagner	21
1.7	最近点对	7	4.7	Hopcroft-Karp	21
1.8	凸包与点集直径	7	4.8	欧拉路	22
1.9	Farmland	7	4.9	稳定婚姻	22
1.10	Voronoi 图	8	4.10	最大团搜索	22
1.11	四边形双费马点	9	4.11	极大团计数	22
1.12	三角形和四边形的费马点	10	4.12	最小树形图	23
1.13	三维计算几何基本操作	10	4.13	离线动态最小生成树	24
1.14	凸多面体切割	11	4.14	弦图	25
1.15	三维凸包	11	4.15	K 短路 (允许重复)	25
1.16	球面点表面点距离	12	4.16	K 短路 (不允许重复)	26
1.17	长方体表面点距离	12	4.17	小知识	27
1.18	最小覆盖球	12	5	数学	27
1.19	三维向量操作矩阵	13	5.1	单纯形 Cpp	27
1.20	立体角	13	5.2	单纯形 Java	28
2	数据结构	13	5.3	高斯消元	28
2.1	动态凸包 (只支持插入)	13	5.4	FFT	28
2.2	Rope 用法	14	5.5	整数 FFT	29
2.3	Treap	14	5.6	扩展欧几里得	29
2.4	可持久化 Treap	14	5.7	线性同余方程	29
2.5	左偏树	14	5.8	Miller-Rabin 素性测试	30
2.6	Link-Cut Tree	15	5.9	PollardRho	30
2.7	K-D Tree Nearest	15	5.10	多项式求根	30
2.8	K-D Tree Farthest	16	5.11	线性递推	31
2.9	树链剖分	16	5.12	原根	31
3	字符串相关	17	5.13	离散对数	31
3.1	Manacher	17	5.14	平方剩余	32
3.2	KMP	17	5.15	N 次剩余	32
3.3	Aho-Corasick 自动机	17	5.16	Pell 方程	32
3.4	后缀自动机	17	5.17	Romberg 积分	32
3.5	后缀数组	17	5.18	公式	33
3.6	环串最小表示	18	5.18.1	级数与三角	33
			5.18.2	三次方程求根公式	34
			3.7	回文自动机	18

5.18.3 椭圆	34
5.18.4 抛物线	34
5.18.5 重心	34
5.18.6 向量恒等式	34
5.18.7 常用几何公式	34
5.18.8 树的计数	35
5.19 小知识	35
6 其他	36
6.1 Extended LIS	36
6.2 生成 nCk	36
6.3 nextPermutation	36
6.4 Josephus 数与逆 Josephus 数	36
6.5 表达式求值	36
6.6 曼哈顿最小生成树	37
6.7 直线下的整点个数	38
6.8 Java 多项式	38
6.9 long long 乘法取模	38
6.10 重复覆盖	38
6.11 星期几判定	39
6.12 LCSequence Fast	39
7 Templates	39
7.1 builtin 系列	39
7.2 泰勒级数	40
7.3 积分表	40
7.4 Eclipse 配置	43
7.5 C++	43
7.6 Java	43
7.7 vim 配置	44

1 计算几何

1.1 二维计算几何基本操作

```
1 const double PI = 3.14159265358979323846264338327950288;
2 double arcSin(const double &a) {
3     return a <= -1.0 ? -PI / 2 : (a >= 1.0 ? PI / 2 : asin(a));
4 }
5 double arcCos(const double &a) {
6     return a <= -1.0 ? PI : (a >= 1.0 ? 0 : acos(a));
7 }
8 struct point {
9     double x, y; // something omitted
10 point rot(const double &a) const { // counter-clockwise
11     return point(x * cos(a) - y * sin(a), x * sin(a) + y * cos(a));
12 }
13 point rot90() const { // counter-clockwise
14     return point(-y, x);
15 }
16 point project(const point &p1, const point &p2) const {
17     const point &q = *this;
18     return p1 + (p2 - p1) * (dot(p2 - p1, q - p1) / (p2 - p1).norm());
19 }
20 bool onSeg(const point &a, const point &b) const { // a, b inclusive
21     const point &c = *this;
22     return sign(dot(a - c, b - c)) <= 0 && sign(det(b - a, c - a)) == 0;
23 }
24 double distLP(const point &p1, const point &p2) const { // dist from *this to line p1->p2
25     const point &q = *this;
26     return fabs(det(p2 - p1, q - p1)) / (p2 - p1).len();
27 }
28 double distSP(const point &p1, const point &p2) const { // dist from *this to segment [p1,
29     p2]
30     const point &q = *this;
31     if (dot(p2 - p1, q - p1) < EPS) return (q - p1).len();
32     if (dot(p1 - p2, q - p2) < EPS) return (q - p2).len();
33     return distLP(p1, p2);
34 }
35 bool inAngle(const point &p1, const point &p2) const { // det(p1, p2) ≥ 0
36     const point &q = *this; return det(p1, q) > -EPS && det(p2, q) < EPS;
37 }
38 bool lineIntersect(const point &a, const point &b, const point &c, const point &d, point
39     &e) {
40     double s1 = det(c - a, d - a);
41     double s2 = det(d - b, c - b);
42     if (!sign(s1 + s2)) return false;
43     e = (b - a) * (s1 / (s1 + s2)) + a;
44     return true;
45 }
```

```

44 }
45 int segIntersectCheck(const point &a, const point &b, const point &c, const point &d,
    point &o) {
46     static double s1, s2, s3, s4;
47     static int iCnt;
48     int d1 = sign(s1 = det(b - a, c - a));
49     int d2 = sign(s2 = det(b - a, d - a));
50     int d3 = sign(s3 = det(d - c, a - c));
51     int d4 = sign(s4 = det(d - c, b - c));
52     if ((d1 ^ d2) == -2 && (d3 ^ d4) == -2) {
53         o = (c * s2 - d * s1) / (s2 - s1);
54         return true;
55     }
56     iCnt = 0;
57     if (d1 == 0 && c.onSeg(a, b)) o = c, ++iCnt;
58     if (d2 == 0 && d.onSeg(a, b)) o = d, ++iCnt;
59     if (d3 == 0 && a.onSeg(c, d)) o = a, ++iCnt;
60     if (d4 == 0 && b.onSeg(c, d)) o = b, ++iCnt;
61     return iCnt ? 2 : 0; // 不相交返回 0, 严格相交返回 1, 非严格相交返回 2
62 }
63 struct circle {
64     point o;
65     double r, rSquire;
66     bool inside(const point &a) { // 非严格
67         return (a - o).len() < r + EPS;
68     }
69     bool contain(const circle &b) const { // 非严格
70         return sign(b.r + (o - b.o).len() - r) <= 0;
71     }
72     bool disjunct(const circle &b) const { // 非严格
73         return sign(b.r + r - (o - b.o).len()) <= 0;
74     }
75     int isCL(const point &p1, const point &p2, point &a, point &b) const {
76         double x = dot(p1 - o, p2 - p1), y = (p2 - p1).norm();
77         double d = x * x - y * ((p1 - o).norm() - rSquire);
78         if (d < -EPS) return 0;
79         if (d < 0) d = 0;
80         point q1 = p1 - (p2 - p1) * (x / y);
81         point q2 = (p2 - p1) * (sqrt(d) / y);
82         a = q1 - q2; b = q1 + q2;
83         return q2.len() < EPS ? 1 : 2;
84     }
85     int tanCP(const point &p, point &a, point &b) const { // 返回切点, 注意可能与 p 重合
86         double x = (p - o).norm(), d = x - rSquire;
87         if (d < -EPS) return 0;
88         if (d < 0) d = 0;
89         point q1 = (p - o) * (rSquire / x);
90         point q2 = ((p - o) * (-r * sqrt(d) / x)).rot90();
91         a = o + (q1 - q2); b = o + (q1 + q2);

```

```

92         return q2.len() < EPS ? 1 : 2;
93     }
94 };
95 bool checkCrossCS(const circle &cir, const point &p1, const point &p2) { // 非严格
96     const point &c = cir.o;
97     const double &r = cir.r;
98     return c.distSP(p1, p2) < r + EPS
99         && (r < (c - p1).len() + EPS || r < (c - p2).len() + EPS);
100 }
101 bool checkCrossCC(const circle &cir1, const circle &cir2) { // 非严格
102     const double &r1 = cir1.r, &r2 = cir2.r, d = (cir1.o - cir2.o).len();
103     return d < r1 + r2 + EPS && fabs(r1 - r2) < d + EPS;
104 }
105 int isCC(const circle &cir1, const circle &cir2, point &a, point &b) {
106     const point &c1 = cir1.o, &c2 = cir2.o;
107     double x = (c1 - c2).norm(), y = ((cir1.rSquire - cir2.rSquire) / x + 1) / 2;
108     double d = cir1.rSquire / x - y * y;
109     if (d < -EPS) return 0;
110     if (d < 0) d = 0;
111     point q1 = c1 + (c2 - c1) * y;
112     point q2 = ((c2 - c1) * sqrt(d)).rot90();
113     a = q1 - q2; b = q1 + q2;
114     return q2.len() < EPS ? 1 : 2;
115 }
116 vector<pair<point, point>> tanCC(const circle &cir1, const circle &cir2) {
117     // 注意: 如果只有三条切线, 即 s1 = 1, s2 = 1, 返回的切线可能重复, 切点没有问题
118     vector<pair<point, point>> list;
119     if (cir1.contain(cir2) || cir2.contain(cir1)) return list;
120     const point &c1 = cir1.o, &c2 = cir2.o;
121     double r1 = cir1.r, r2 = cir2.r;
122     point p, a1, b1, a2, b2;
123     int s1, s2;
124     if (sign(r1 - r2) == 0) {
125         p = c2 - c1;
126         p = (p * (r1 / p.len())).rot90();
127         list.push_back(make_pair(c1 + p, c2 + p));
128         list.push_back(make_pair(c1 - p, c2 - p));
129     } else {
130         p = (c2 * r1 - c1 * r2) / (r1 - r2);
131         s1 = cir1.tanCP(p, a1, b1);
132         s2 = cir2.tanCP(p, a2, b2);
133         if (s1 >= 1 && s2 >= 1) {
134             list.push_back(make_pair(a1, a2));
135             list.push_back(make_pair(b1, b2));
136         }
137     }
138     p = (c1 * r2 + c2 * r1) / (r1 + r2);
139     s1 = cir1.tanCP(p, a1, b1);
140     s2 = cir2.tanCP(p, a2, b2);

```

```

141     if (s1 >= 1 && s2 >= 1) {
142         list.push_back(make_pair(a1, a2));
143         list.push_back(make_pair(b1, b2));
144     }
145     return list;
146 }
147 bool distConvexPIn(const point &p1, const point &p2, const point &p3, const point &p4,
148     const point &q) {
149     point o12 = (p1 - p2).rot90(), o23 = (p2 - p3).rot90(), o34 = (p3 - p4).rot90();
150     return (q - p1).inAngle(o12, o23) || (q - p3).inAngle(o23, o34)
151         || ((q - p2).inAngle(o23, p3 - p2) && (q - p3).inAngle(p2 - p3, o23));
152 }
153 double distConvexP(int n, point ps[], const point &q) { // 外部点到多边形的距离
154     int left = 0, right = n;
155     while (right - left > 1) {
156         int mid = (left + right) / 2;
157         if (distConvexPIn(ps[(left + n - 1) % n], ps[left], ps[mid], ps[(mid + 1) % n], q))
158             right = mid;
159         else left = mid;
160     }
161     return q.distSP(ps[left], ps[right % n]);
162 }
163 double areaCT(const circle &cir, point pa, point pb) {
164     pa = pa - cir.o; pb = pb - cir.o;
165     double R = cir.r;
166     if (pa.len() < pb.len()) swap(pa, pb);
167     if (pb.len() < EPS) return 0;
168     point pc = pb - pa;
169     double a = pa.len(), b = pb.len(), c = pc.len(), S, h, theta;
170     double cosB = dot(pb, pc) / b / c, B = acos(cosB);
171     double cosC = dot(pa, pb) / a / b, C = acos(cosC);
172     if (b > R) {
173         S = C * 0.5 * R * R;
174         h = b * a * sin(C) / c;
175         if (h < R && B < PI * 0.5)
176             S -= acos(h / R) * R * R - h * sqrt(R * R - h * h);
177     } else if (a > R) {
178         theta = PI - B - asin(sin(B) / R * b);
179         S = 0.5 * b * R * sin(theta) + (C - theta) * 0.5 * R * R;
180     } else S = 0.5 * sin(C) * b * a;
181     return S;
182 }
183 circle minCircle(const point &a, const point &b) {
184     return circle((a + b) * 0.5, (b - a).len() * 0.5);
185 }
186 circle minCircle(const point &a, const point &b, const point &c) { // 钝角三角形没有被考虑
187     double a2((b - c).norm()), b2((a - c).norm()), c2((a - b).norm());
188     if (b2 + c2 <= a2 + EPS) return minCircle(b, c);
189     if (a2 + c2 <= b2 + EPS) return minCircle(a, c);

```

```

189     if (a2 + b2 <= c2 + EPS) return minCircle(a, b);
190     double A = 2.0 * (a.x - b.x), B = 2.0 * (a.y - b.y);
191     double D = 2.0 * (a.x - c.x), E = 2.0 * (a.y - c.y);
192     double C = a.norm() - b.norm(), F = a.norm() - c.norm();
193     point p((C * E - B * F) / (A * E - B * D), (A * F - C * D) / (A * E - B * D));
194     return circle(p, (p - a).len());
195 }
196 circle minCircle(point P[], int N) { // 1-based
197     if (N == 1) return circle(P[1], 0.0);
198     random_shuffle(P + 1, P + N + 1); circle O = minCircle(P[1], P[2]);
199     Rep(i, 1, N) if(!O.inside(P[i])) { O = minCircle(P[1], P[i]);
200         Foru(j, 1, i) if(!O.inside(P[j])) { O = minCircle(P[i], P[j]);
201             Foru(k, 1, j) if(!O.inside(P[k])) O = minCircle(P[i], P[j], P[k]); }
202     } return O;
203 }

```

1.2 圆的面积模板

```

1 struct Event { point p; double alpha; int add; // 构造函数省略
2     bool operator < (const Event &other) const { return alpha < other.alpha; } };
3 void circleKCover(circle *c, int N, double *area) { // area[k] : 至少被覆盖 k 次
4     static bool overlap[MAXN][MAXN], g[MAXN][MAXN];
5     Rep(i, 0, N + 1) area[i] = 0.0; Rep(i, 1, N) Rep(j, 1, N) overlap[i][j] = c[i].contain(
6         c[j]);
7     Rep(i, 1, N) Rep(j, 1, N) g[i][j] = !(overlap[i][j] || overlap[j][i] || c[i].disjunct(c
8         [j]));
9     Rep(i, 1, N) { static Event events[MAXN * 2 + 1]; int totE = 0, cnt = 1;
10         Rep(j, 1, N) if (j != i && overlap[j][i]) ++cnt;
11         Rep(j, 1, N) if (j != i && g[i][j]) {
12             circle &a = c[i], &b = c[j]; double l = (a.o - b.o).norm();
13             double s = ((a.r - b.r) * (a.r + b.r) / l + 1) * 0.5;
14             double t = sqrt(-(l - sqrt(a.r - b.r)) * (l - sqrt(a.r + b.r)) / (l * l * 4.0));
15             point dir = b.o - a.o, nDir = point(-dir.y, dir.x);
16             point aa = a.o + dir * s + nDir * t;
17             point bb = a.o + dir * s - nDir * t;
18             double A = atan2(aa.y - a.o.y, aa.x - a.o.x);
19             double B = atan2(bb.y - a.o.y, bb.x - a.o.x);
20             events[totE++] = Event(bb, B, 1); events[totE++] = Event(aa, A, -1); if (B > A) ++
21                 cnt;
22         } if (totE == 0) { area[cnt] += PI * c[i].rSquare; continue; }
23         sort(events, events + totE); events[totE] = events[0];
24         Foru(j, 0, totE) {
25             cnt += events[j].add; area[cnt] += 0.5 * det(events[j].p, events[j + 1].p);
26             double theta = events[j + 1].alpha - events[j].alpha; if (theta < 0) theta += 2.0 *
27                 PI;
28             area[cnt] += 0.5 * c[i].rSquare * (theta - sin(theta));
29         }
30     }
31 }

```

1.3 多边形相关

```

1 struct Polygon { // stored in [0, n)
2     int n; point ps[MAXN];
3     Polygon cut(const point &a, const point &b) {
4         static Polygon res; static point o; res.n = 0;
5         for (int i = 0; i < n; ++i) {
6             int s1 = sign(det(ps[i] - a, b - a));
7             int s2 = sign(det(ps[(i + 1) % n] - a, b - a));
8             if (s1 <= 0) res.ps[res.n++] = ps[i];
9             if (s1 * s2 < 0) {
10                 lineIntersect(a, b, ps[i], ps[(i + 1) % n], o);
11                 res.ps[res.n++] = o;
12             }
13         } return res;
14     }
15     bool contain(const point &p) const { // 1 if on border or inner, 0 if outer
16         static point A, B; int res = 0;
17         for (int i = 0; i < n; ++i) {
18             A = ps[i]; B = ps[(i + 1) % n];
19             if (p.onSeg(A, B)) return 1;
20             if (sign(A.y - B.y) <= 0) swap(A, B);
21             if (sign(p.y - A.y) > 0) continue;
22             if (sign(p.y - B.y) <= 0) continue;
23             res += (int)(sign(det(B - p, A - p)) > 0);
24         } return res & 1;
25     }
26     #define qs(x) (ps[x] - ps[0])
27     bool convexContain(point p) const { // counter-clockwise
28         point q = qs(n - 1); p = p - ps[0];
29         if (!p.inAngle(qs(1), q)) return false;
30         int L = 0, R = n - 1;
31         while (L + 1 < R) { int M((L + R) >> 1);
32             if (p.inAngle(qs(M), q)) L = M; else R = M;
33         } if (L == 0) return false; point l(qs(L)), r(qs(R));
34         return sign( fabs(det(l, p)) + fabs(det(p, r)) + fabs(det(r - l, p - l)) - det(l, r)
35                     ) == 0;
36     }
37     #undef qs
38     double isPLAtan2(const point &a, const point &b) {
39         double k = (b - a).alpha(); if (k < 0) k += 2 * PI;
40         return k;
41     }
42     point isPL_Get(const point &a, const point &b, const point &s1, const point &s2) {
43         double k1 = det(b - a, s1 - a), k2 = det(b - a, s2 - a);
44         if (sign(k1) == 0) return s1;
45         if (sign(k2) == 0) return s2;
46         return (s1 * k2 - s2 * k1) / (k2 - k1);
47     }

```

```

47 int isPL_Dic(const point &a, const point &b, int l, int r) {
48     int s = (det(b - a, ps[l] - a) < 0) ? -1 : 1;
49     while (l <= r) {
50         int mid = (l + r) / 2;
51         if (det(b - a, ps[mid] - a) * s <= 0) r = mid - 1;
52         else l = mid + 1;
53     }
54     return r + 1;
55 }
56 int isPL_Find(double k, double w[]) {
57     if (k <= w[0] || k > w[n - 1]) return 0;
58     int l = 0, r = n - 1, mid;
59     while (l <= r) {
60         mid = (l + r) / 2;
61         if (w[mid] >= k) r = mid - 1;
62         else l = mid + 1;
63     } return r + 1;
64 }
65 bool isPL(const point &a, const point &b, point &cp1, point &cp2) { // O(logN)
66     static double w[MAXN * 2]; // pay attention to the array size
67     for (int i = 0; i <= n; ++i) ps[i + n] = ps[i];
68     for (int i = 0; i < n; ++i) w[i] = w[i + n] = isPLAtan2(ps[i], ps[i + 1]);
69     int i = isPL_Find(isPLAtan2(a, b), w);
70     int j = isPL_Find(isPLAtan2(b, a), w);
71     double k1 = det(b - a, ps[i] - a), k2 = det(b - a, ps[j] - a);
72     if (sign(k1) * sign(k2) > 0) return false; // no intersection
73     if (sign(k1) == 0 || sign(k2) == 0) { // intersect with a point or a line in the convex
74         if (sign(k1) == 0) {
75             if (sign(det(b - a, ps[i + 1] - a)) == 0) cp1 = ps[i], cp2 = ps[i + 1];
76             else cp1 = cp2 = ps[i];
77             return true;
78         }
79         if (sign(k2) == 0) {
80             if (sign(det(b - a, ps[j + 1] - a)) == 0) cp1 = ps[j], cp2 = ps[j + 1];
81             else cp1 = cp2 = ps[j];
82         }
83         return true;
84     }
85     if (i > j) swap(i, j);
86     int x = isPL_Dic(a, b, i, j), y = isPL_Dic(a, b, j, i + n);
87     cp1 = isPL_Get(a, b, ps[x - 1], ps[x]);
88     cp2 = isPL_Get(a, b, ps[y - 1], ps[y]);
89     return true;
90 }
91 double getI(const point &o) const {
92     if (n <= 2) return 0;
93     point G(0.0, 0.0);
94     double S = 0.0, I = 0.0;
95     for (int i = 0; i < n; ++i) {

```

```

96     const point &x = ps[i], &y = ps[(i + 1) % n];
97     double d = det(x, y);
98     G = G + (x + y) * d / 3.0;
99     S += d;
100 } G = G / S;
101 for (int i = 0; i < n; ++i) {
102     point x = ps[i] - G, y = ps[(i + 1) % n] - G;
103     I += fabs(det(x, y)) * (x.norm() + dot(x, y) + y.norm());
104 }
105 return I = I / 12.0 + fabs(S * 0.5) * (O - G).norm();
106 }
107 };

```

1.4 直线与凸包求交点

```

1 int isPL(point a, point b, vector<point> &res) { // 点逆时针给出, 无三点共线
2     static double theta[MAXN];
3     for (int i = 0; i < n; ++i) theta[i] = (list[(i + 1) % n] - list[i]).atan2();
4     double delta = theta[0];
5     for (int i = 0; i < n; ++i) theta[i] = normalize(theta[i] - delta);
6     int x = lower_bound(theta, theta + n, normalize((b - a).atan2() - delta)) - theta;
7     int y = lower_bound(theta, theta + n, normalize((a - b).atan2() - delta)) - theta;
8     for (int k = 0; k <= 1; ++k, swap(a, b), swap(x, y)) {
9         if (y < x) y += n;
10        int l = x, r = y, m;
11        while (l + 1 < r) {
12            if (sign(det(b - a, list[(m = (l + r) / 2) % n] - a)) < 0) l = m;
13            else r = m;
14        }
15        l %= n, r %= n;
16        if (sign(det(b - a, list[r] - list[l])) == 0) {
17            if (sign(det(b - a, list[l] - a)) == 0)
18                return -1; // 直线与 (list[l], list[r]) 重合
19        }
20        else {
21            point p; lineIntersect(list[l], list[r], a, b, p);
22            if (p.onSeg(list[l], list[r]))
23                res.push_back(p);
24        }
25    }
26    return res.size();
27 }

```

1.5 半平面交

```

1 struct Border {

```

```

2     point p1, p2; double alpha;
3     Border() : p1(), p2(), alpha(0.0) {}
4     Border(const point &a, const point &b): p1(a), p2(b), alpha( atan2(p2.y - p1.y, p2.x -
5         p1.x) ) {}
6     bool operator == (const Border &b) const { return sign(alpha - b.alpha) == 0; }
7     bool operator < (const Border &b) const {
8         int c = sign(alpha - b.alpha); if (c != 0) return c > 0;
9         return sign(det(b.p2 - b.p1, p1 - b.p1)) >= 0;
10    }
11    point isBorder(const Border &a, const Border &b) { // a and b should not be parallel
12        point is; lineIntersect(a.p1, a.p2, b.p1, b.p2, is); return is;
13    }
14    bool checkBorder(const Border &a, const Border &b, const Border &me) {
15        point is; lineIntersect(a.p1, a.p2, b.p1, b.p2, is);
16        return sign(det(me.p2 - me.p1, is - me.p1)) > 0;
17    }
18    double HPI(int N, Border border[]) {
19        static Border que[MAXN * 2 + 1]; static point ps[MAXN];
20        int head = 0, tail = 0, cnt = 0; // [head, tail)
21        sort(border, border + N); N = unique(border, border + N) - border;
22        for (int i = 0; i < N; ++i) {
23            Border &cur = border[i];
24            while (head + 1 < tail && !checkBorder(que[tail - 2], que[tail - 1], cur)) --tail;
25            while (head + 1 < tail && !checkBorder(que[head], que[head + 1], cur)) ++head;
26            que[tail++] = cur;
27        } while (head + 1 < tail && !checkBorder(que[tail - 2], que[tail - 1], que[head])) --
            tail;
28        while (head + 1 < tail && !checkBorder(que[head], que[head + 1], que[tail - 1])) ++head
            ;
29        if (tail - head <= 2) return 0.0;
30        Foru(i, head, tail) ps[cnt++] = isBorder(que[i], que[(i + 1 == tail) ? (head) : (i + 1)
            ]);
31        double area = 0; Foru(i, 0, cnt) area += det(ps[i], ps[(i + 1) % cnt]);
32        return fabs(area * 0.5); // or (-area * 0.5)
33    }

```

1.6 最大面积空凸包

```

1 inline bool toUpRight(const point &a, const point &b) {
2     int c = sign(b.y - a.y); if (c > 0) return true;
3     return c == 0 && sign(b.x - a.x) > 0;
4 }
5 inline bool cmpByPolarAngle(const point &a, const point &b) { // counter-clockwise, shorter
6     first if they share the same polar angle
7     int c = sign(det(a, b)); if (c != 0) return c > 0;
8     return sign(b.len() - a.len()) > 0;
9 }

```

```

9 double maxEmptyConvexHull(int N, point p[]) {
10     static double dp[MAXN][MAXN];
11     static point vec[MAXN];
12     static int seq[MAXN]; // empty triangles formed with (0,0),vec[o],vec[seq[i]]
13     double ans = 0.0;
14     Rep(o, 1, N) {
15         int totVec = 0;
16         Rep(i, 1, N) if (toUpRight(p[o], p[i])) vec[++totVec] = p[i] - p[o];
17         sort(vec + 1, vec + totVec + 1, cmpByPolarAngle);
18         Rep(i, 1, totVec) Rep(j, 1, totVec) dp[i][j] = 0.0;
19         Rep(k, 2, totVec) {
20             int i = k - 1;
21             while (i > 0 && sign( det(vec[k], vec[i]) ) == 0) --i;
22             int totSeq = 0;
23             for (int j; i > 0; i = j) {
24                 seq[++totSeq] = i;
25                 for (j = i - 1; j > 0 && sign(det(vec[i] - vec[k], vec[j] - vec[k])) > 0; --j);
26                 double v = det(vec[i], vec[k]) * 0.5;
27                 if (j > 0) v += dp[i][j];
28                 dp[k][i] = v;
29                 cMax(ans, v);
30             } for (int i = totSeq - 1; i >= 1; --i) cMax( dp[k][ seq[i] ], dp[k][seq[i + 1]] );
31         }
32     } return ans;
33 }

```

1.7 最近点对

```

1 int N; point p[maxn];
2 bool cmpByX(const point &a, const point &b) { return sign(a.x - b.x) < 0; }
3 bool cmpByY(const int &a, const int &b) { return p[a].y < p[b].y; }
4 double minimalDistance(point *c, int n, int *ys) {
5     double ret = 1e+20;
6     if (n < 20) {
7         Foru(i, 0, n) Foru(j, i + 1, n) cMin(ret, (c[i] - c[j]).len() );
8         sort(ys, ys + n, cmpByY); return ret;
9     } static int mergeTo[maxn];
10    int mid = n / 2; double xmid = c[mid].x;
11    ret = min(minimalDistance(c, mid, ys), minimalDistance(c + mid, n - mid, ys + mid));
12    merge(ys, ys + mid, ys + mid, ys + n, mergeTo, cmpByY);
13    copy(mergeTo, mergeTo + n, ys);
14    Foru(i, 0, n) {
15        while (i < n && sign(fabs(p[ys[i]].x - xmid) - ret) > 0) ++i;
16        int cnt = 0;
17        Foru(j, i + 1, n)
18            if (sign(p[ys[j]].y - p[ys[i]].y - ret) > 0) break;
19            else if (sign(fabs(p[ys[j]].x - xmid) - ret) <= 0) {
20                ret = min(ret, (p[ys[i]] - p[ys[j]]).len());

```

```

21         if (++cnt >= 10) break;
22     }
23     } return ret;
24 }
25 double work() {
26     sort(p, p + n, cmpByX); Foru(i, 0, n) ys[i] = i; return minimalDistance(p, n, ys);
27 }

```

1.8 凸包与点集直径

```

1 vector<point> convexHull(int n, point ps[]) { // counter-clockwise, strict
2     static point qs[MAXN * 2];
3     sort(ps, ps + n, cmpByXY);
4     if (n <= 2) return vector(ps, ps + n);
5     int k = 0;
6     for (int i = 0; i < n; qs[k++] = ps[i++])
7         while (k > 1 && det(qs[k - 1] - qs[k - 2], ps[i] - qs[k - 1]) < EPS) --k;
8     for (int i = n - 2, t = k; i >= 0; qs[k++] = ps[i--])
9         while (k > t && det(qs[k - 1] - qs[k - 2], ps[i] - qs[k - 1]) < EPS) --k;
10    return vector<point>(qs, qs + k);
11 }
12 double convexDiameter(int n, point ps[]) {
13     if (n < 2) return 0; if (n == 2) return (ps[1] - ps[0]).len();
14     double k, ans = 0;
15     for (int x = 0, y = 1, nx, ny; x < n; ++x) {
16         for(nx = (x == n - 1) ? (0) : (x + 1); ; y = ny) {
17             ny = (y == n - 1) ? (0) : (y + 1);
18             if ( sign(k = det(ps[nx] - ps[x], ps[ny] - ps[y])) <= 0) break;
19             } ans = max(ans, (ps[x] - ps[y]).len());
20             if (sign(k) == 0) ans = max(ans, (ps[x] - ps[ny]).len());
21         } return ans;
22 }

```

1.9 Farmland

```

1 struct node { int begin[MAXN], *end; } a[MAXN]; // 按对 p[i] 的极角的 atan2 值排序
2 bool check(int n, point p[], int b1, int b2, bool vis[MAXN][MAXN]) {
3     static pii l[MAXN * 2 + 1]; static bool used[MAXN];
4     int tp(0), *k, p, p1, p2; double area(0.0);
5     for (l[0] = pii(b1, b2); ; ) {
6         vis[p1 = l[tp].first][p2 = l[tp].second] = true;
7         area += det(p[p1], p[p2]);
8         for (k = a[p2].begin; k != a[p2].end; ++k) if (*k == p1) break;
9         k = (k == a[p2].begin) ? (a[p2].end - 1) : (k - 1);
10        if ((l[++tp] = pii(p2, *k)) == l[0]) break;
11    } if (sign(area) < 0 || tp < 3) return false;

```

```

12 Rep(i, 1, n) used[i] = false;
13 for (int i = 0; i < tp; ++i) if (used[p = l[i].first]) return false; else used[p] =
    true;
14 return true; // a face with tp vertices
15 }
16 int countFaces(int n, point p[]) {
17     static bool vis[MAXN][MAXN]; int ans = 0;
18     Rep(x, 1, n) Rep(y, 1, n) vis[x][y] = false;
19     Rep(x, 1, n) for (int *itr = a[x].begin; itr != a[x].end; ++itr) if (!vis[x][*itr])
20         if (check(n, p, x, *itr, vis)) ++ans;
21     return ans;
22 }

```

1.10 Voronoi 图

不能有重点, 点数应当不小于 2

```

1 #define Oi(e) ((e)->oi)
2 #define Dt(e) ((e)->dt)
3 #define On(e) ((e)->on)
4 #define Op(e) ((e)->op)
5 #define Dn(e) ((e)->dn)
6 #define Dp(e) ((e)->dp)
7 #define Other(e, p) ((e)->oi == p ? (e)->dt : (e)->oi)
8 #define Next(e, p) ((e)->oi == p ? (e)->on : (e)->dn)
9 #define Prev(e, p) ((e)->oi == p ? (e)->op : (e)->dp)
10 #define V(p1, p2, u, v) (u = p2->x - p1->x, v = p2->y - p1->y)
11 #define C2(u1, v1, u2, v2) (u1 * v2 - v1 * u2)
12 #define C3(p1, p2, p3) ((p2->x - p1->x) * (p3->y - p1->y) - (p2->y - p1->y) * (p3->x - p1->x))
13 #define Dot(u1, v1, u2, v2) (u1 * u2 + v1 * v2)
14 #define dis(a,b) (sqrt( (a->x - b->x) * (a->x - b->x) + (a->y - b->y) * (a->y - b->y) ))
15 const int maxn = 110024;
16 const int aix = 4;
17 const double eps = 1e-7;
18 int n, M, k;
19 struct gEdge {
20     int u, v; double w;
21     bool operator <(const gEdge &e1) const { return w < e1.w - eps; }
22 } E[aix * maxn], MST[maxn];
23 struct point {
24     double x, y; int index; edge *in;
25     bool operator <(const point &p1) const { return x < p1.x - eps || (abs(x - p1.x) <= eps
        && y < p1.y - eps); }
26 };
27 struct edge { point *oi, *dt; edge *on, *op, *dn, *dp; };
28
29 point p[maxn], *Q[maxn];

```

```

30 edge mem[aix * maxn], *elist[aix * maxn];
31 int nfree;
32 void Alloc_memory() { nfree = aix * n; edge *e = mem; for (int i = 0; i < nfree; i++)
    elist[i] = e++; }
33 void Splice(edge *a, edge *b, point *v) {
34     edge *next;
35     if (Oi(a) == v) next = On(a), On(a) = b; else next = Dn(a), Dn(a) = b;
36     if (Oi(next) == v) Op(next) = b; else Dp(next) = b;
37     if (Oi(b) == v) On(b) = next, Op(b) = a; else Dn(b) = next, Dp(b) = a;
38 }
39 edge *Make_edge(point *u, point *v) {
40     edge *e = elist[--nfree];
41     e->on = e->op = e->dn = e->dp = e; e->oi = u; e->dt = v;
42     if (!u->in) u->in = e;
43     if (!v->in) v->in = e;
44     return e;
45 }
46 edge *Join(edge *a, point *u, edge *b, point *v, int side) {
47     edge *e = Make_edge(u, v);
48     if (side == 1) {
49         if (Oi(a) == u) Splice(Op(a), e, u);
50         else Splice(Dp(a), e, u);
51         Splice(b, e, v);
52     } else {
53         Splice(a, e, u);
54         if (Oi(b) == v) Splice(Op(b), e, v);
55         else Splice(Dp(b), e, v);
56     } return e;
57 }
58 void Remove(edge *e) {
59     point *u = Oi(e), *v = Dt(e);
60     if (u->in == e) u->in = e->on;
61     if (v->in == e) v->in = e->dn;
62     if (Oi(e->on) == u) e->on->op = e->op; else e->on->dp = e->op;
63     if (Oi(e->op) == u) e->op->on = e->on; else e->op->dn = e->on;
64     if (Oi(e->dn) == v) e->dn->op = e->dp; else e->dn->dp = e->dp;
65     if (Oi(e->dp) == v) e->dp->on = e->dn; else e->dp->dn = e->dn;
66     elist[nfree++] = e;
67 }
68 void Low_tangent(edge *e_l, point *o_l, edge *e_r, point *o_r, edge **l_low, point **OL,
    edge **r_low, point **OR) {
69     for (point *d_l = Other(e_l, o_l), *d_r = Other(e_r, o_r); ; )
70         if (C3(o_l, o_r, d_l) < -eps) e_l = Prev(e_l, d_l), o_l = d_l, d_l = Other(e_l,
            o_l);
71         else if (C3(o_l, o_r, d_r) < -eps) e_r = Next(e_r, d_r), o_r = d_r, d_r = Other(e_r,
            o_r);
72         else break;
73     *OL = o_l, *OR = o_r; *l_low = e_l, *r_low = e_r;
74 }

```



```

75 void Merge(edge *lr, point *s, edge *rl, point *u, edge **tangent) {
76     double l1, l2, l3, l4, r1, r2, r3, r4, cot_L, cot_R, u1, v1, u2, v2, n1, cot_n, P1,
        cot_P;
77     point *O, *D, *OR, *OL; edge *B, *L, *R;
78     Low_tangent(lr, s, rl, u, &L, &OL, &R, &OR);
79     for (*tangent = B = Join(L, OL, R, OR, O), O = OL, D = OR; ; ) {
80         edge *El = Next(B, O), *Er = Prev(B, D), *next, *prev;
81         point *l = Other(El, O), *r = Other(Er, D);
82         V(l, O, l1, l2); V(l, D, l3, l4); V(r, O, r1, r2); V(r, D, r3, r4);
83         double c1 = C2(l1, l2, l3, l4), cr = C2(r1, r2, r3, r4);
84         bool BL = c1 > eps, BR = cr > eps;
85         if (!BL && !BR) break;
86         if (BL) {
87             double d1 = Dot(l1, l2, l3, l4);
88             for (cot_L = d1 / c1; ; Remove(El), El = next, cot_L = cot_n) {
89                 next = Next(El, O); V(Other(next, O), O, u1, v1); V(Other(next, O), D, u2, v2);
90                 n1 = C2(u1, v1, u2, v2); if (!(n1 > eps)) break;
91                 cot_n = Dot(u1, v1, u2, v2) / n1;
92                 if (cot_n > cot_L) break;
93             }
94         } if (BR) {
95             double dr = Dot(r1, r2, r3, r4);
96             for (cot_R = dr / cr; ; Remove(Er), Er = prev, cot_R = cot_P) {
97                 prev = Prev(Er, D); V(Other(prev, D), O, u1, v1); V(Other(prev, D), D, u2, v2);
98                 P1 = C2(u1, v1, u2, v2); if (!(P1 > eps)) break;
99                 cot_P = Dot(u1, v1, u2, v2) / P1;
100                if (cot_P > cot_R) break;
101            }
102        } l = Other(El, O); r = Other(Er, D);
103        if (!BL || (BL && BR && cot_R < cot_L)) B = Join(B, O, Er, r, O), D = r;
104        else B = Join(El, l, B, D, O), O = l;
105    }
106 }
107 void Divide(int s, int t, edge **L, edge **R) {
108     edge *a, *b, *c, *l1, *lr, *rl, *rr, *tangent;
109     int n = t - s + 1;
110     if (n == 2) *L = *R = Make_edge(Q[s], Q[t]);
111     else if (n == 3) {
112         a = Make_edge(Q[s], Q[s + 1]), b = Make_edge(Q[s + 1], Q[t]);
113         Splice(a, b, Q[s + 1]);
114         double v = C3(Q[s], Q[s + 1], Q[t]);
115         if (v > eps) c = Join(a, Q[s], b, Q[t], O), *L = a, *R = b;
116         else if (v < -eps) c = Join(a, Q[s], b, Q[t], 1), *L = c, *R = c;
117         else *L = a, *R = b;
118     } else if (n > 3) {
119         int split = (s + t) / 2;
120         Divide(s, split, &l1, &lr); Divide(split + 1, t, &rl, &rr);
121         Merge(lr, Q[split], rl, Q[split + 1], &tangent);
122         if (Oi(tangent) == Q[s]) l1 = tangent;

```

```

123         if (Dt(tangent) == Q[t]) rr = tangent;
124         *L = l1; *R = rr;
125     }
126 }
127 void Make_Graph() {
128     edge *start, *e; point *u, *v;
129     for (int i = 0; i < n; i++) {
130         start = e = (u = &p[i])->in;
131         do{ v = Other(e, u);
132             if (u < v) E[M++].u = (u - p, v - p, dis(u, v)); // M < aix * maxn
133         } while ((e = Next(e, u)) != start);
134     }
135 }
136 int b[maxn];
137 int Find(int x) { while (x != b[x]) { b[x] = b[b[x]]; x = b[x]; } return x; }
138 void Kruskal() {
139     memset(b, 0, sizeof(b)); sort(E, E + M);
140     for (int i = 0; i < n; i++) b[i] = i;
141     for (int i = 0, kk = 0; i < M && kk < n - 1; i++) {
142         int m1 = Find(E[i].u), m2 = Find(E[i].v);
143         if (m1 != m2) b[m1] = m2, MST[kk++] = E[i];
144     }
145 }
146 void solve() {
147     scanf("%d", &n);
148     for (int i = 0; i < n; i++) scanf("%lf%lf", &p[i].x, &p[i].y), p[i].index = i, p[i].in
        = NULL;
149     Alloc_memory(); sort(p, p + n);
150     for (int i = 0; i < n; i++) Q[i] = p + i;
151     edge *L, *R; Divide(0, n - 1, &L, &R);
152     M = 0; Make_Graph(); Kruskal();
153 }
154 int main() { solve(); return 0; }

```

1.11 四边形双费马点

```

1  typedef complex<double> Tpoint;
2  const double eps = 1e-8;
3  const double sqrt3 = sqrt(3.0);
4  bool cmp(const Tpoint &a, const Tpoint &b) {
5      return a.real() < b.real() - eps || (a.real() < b.real() + eps && a.imag() < b.imag());
6  }
7  Tpoint rotate(const Tpoint &a, const Tpoint &b, const Tpoint &c) {
8      Tpoint d = b - a; d = Tpoint(-d.imag(), d.real());
9      if (Sign(cross(a, b, c)) == Sign(cross(a, b, a + d))) d *= -1.0;
10     return unit(d);
11 }
12 Tpoint p[10], a[10], b[10];

```

```

13 int N, T;
14 double totlen(const Tpoint &p, const Tpoint &a, const Tpoint &b, const Tpoint &c) {
15     return abs(p - a) + abs(p - b) + abs(p - c);
16 }
17 double fermat(const Tpoint &x, const Tpoint &y, const Tpoint &z, Tpoint &cp) {
18     a[0] = a[3] = x; a[1] = a[4] = y; a[2] = a[5] = z;
19     double len = 1e100, len2;
20     for (int i = 0; i < 3; i++) {
21         len2 = totlen(a[i], x, y, z);
22         if (len2 < len) len = len2, cp = a[i];
23     }
24     for (int i = 0; i < 3; i++) {
25         b[i] = rotate(a[i + 1], a[i], a[i + 2]);
26         b[i] = (a[i + 1] + a[i]) / 2.0 + b[i] * (abs(a[i + 1] - a[i]) * sqrt3 / 2.0);
27     }
28     b[3] = b[0];
29     Tpoint cp2 = intersect(b[0], a[2], b[1], a[3]);
30     len2 = totlen(cp2, x, y, z);
31     if (len2 < len) len = len2, cp = cp2;
32     return len;
33 }
34 double getans(const Tpoint &a) {
35     double len = 0; for (int i = 0; i < N; i++) len += abs(a - p[i]);
36     return len;
37 }
38 double mindist(const Tpoint &p, const Tpoint &a, const Tpoint &b, const Tpoint &c, const
    Tpoint &d) {
39     return min( min(abs(p - a), abs(p - b)), min(abs(p - c), abs(p - d)));
40 }
41 int main() {
42     N = 4;
43     for (cin >> T; T; T--) {
44         double ret = 1e100, len_cur, len_before, len1, len2, len;
45         Tpoint cp, cp1, cp2;
46         Foru(i, 0, N) cin >> p[i];
47         Foru(i, 0, N) ret = min(ret, getans(p[i]));
48         Foru(i, 1, N) Foru(j, 1, N) if (j != i) Foru(k, 1, N) if (k != i && k != j) {
49             cMin(ret, abs(p[0] - p[i]) + abs(p[j] - p[k])
50                 + min( min(abs(p[0] - p[j]), abs(p[0] - p[k])),
51                     min(abs(p[i] - p[j]), abs(p[i] - p[k]))
52                 ));
53             ret = min(ret, getans(intersect(p[0], p[i], p[j], p[k])));
54         }
55         Foru(i, 0, N) Foru(j, i + 1, N) Foru(k, j + 1, N) {
56             double len = fermat(p[i], p[j], p[k], cp);
57             ret = min(ret, len + mindist(p[6 - i - j - k], p[i], p[j], p[k], cp));
58         }
59         sort(p, p + N, cmp);
60         for (int i = 1; i < N; i++) {

```

```

61         cp1 = (p[0] + p[i]) / 2.0;
62         int j, k;
63         for (j = 1; j < N && j == i; j++);
64         for (k = 6 - i - j, len_before = 1e100; ; ) {
65             len1 = fermat(cp1, p[j], p[k], cp2);
66             len1 = fermat(cp2, p[0], p[i], cp1);
67             len = len1 + abs(cp2 - p[j]) + abs(cp2 - p[k]);
68             if (len < len_before - (1e-6)) len_before = len;
69             else break;
70         } ret = min(ret, len_before);
71     } printf("%.4f\n", ret);
72 }
73 return 0;
74 }

```

1.12 三角形和四边形的费马点

- 费马点: 距几个顶点距离之和最小的点

- 三角形:

- 若每个角都小于 120° : 以每条边向外作正三角形, 得到 $\triangle ABF$, $\triangle BCD$, $\triangle CAE$, 连接 AD , BE , CF , 三线必共点于费马点. 该点对三边的张角必然是 120° , 也必然是三个三角形外接圆的交点
- 否则费马点一定是那个大于等于 120° 的顶角

- 四边形:

- 在凸四边形中, 费马点为对角线的交点
- 在凹四边形中, 费马点位凹顶点

1.13 三维计算几何基本操作

```

1 struct point { double x, y, z; // something omitted
2     friend point det(const point &a, const point &b) {
3         return point(a.y * b.z - a.z * b.y, a.z * b.x - a.x * b.z, a.x * b.y - a.y * b.x);
4     }
5     friend double mix(const point &a, const point &b, const point &c) {
6         return a.x * b.y * c.z + a.y * b.z * c.x + a.z * b.x * c.y - a.z * b.y * c.x - a.x *
            b.z * c.y - a.y * b.x * c.z;
7     }
8     double distLP(const point &p1, const point &p2) const {
9         return det(p2 - p1, *this - p1).len() / (p2 - p1).len();
10    }
11    double distFP(const point &p1, const point &p2, const point &p3) const {
12        point n = det(p2 - p1, p3 - p1); return fabs( dot(n, *this - p1) / n.len() );
13    }

```

```

14 };
15 double distLL(const point &p1, const point &p2, const point &q1, const point &q2) {
16     point p = q1 - p1, u = p2 - p1, v = q2 - q1;
17     double d = u.norm() * v.norm() - dot(u, v) * dot(u, v);
18     if (sign(d) == 0) return p1.distLP(q1, q2);
19     double s = (dot(p, u) * v.norm() - dot(p, v) * dot(u, v)) / d;
20     return (p1 + u * s).distLP(q1, q2);
21 }
22 double distSS(const point &p1, const point &p2, const point &q1, const point &q2) {
23     point p = q1 - p1, u = p2 - p1, v = q2 - q1;
24     double d = u.norm() * v.norm() - dot(u, v) * dot(u, v);
25     if (sign(d) == 0) return min( min((p1 - q1).len(), (p1 - q2).len()),
26                                 min((p2 - q1).len(), (p2 - q2).len()));
27     double s1 = (dot(p, u) * v.norm() - dot(p, v) * dot(u, v)) / d;
28     double s2 = (dot(p, v) * u.norm() - dot(p, u) * dot(u, v)) / d;
29     if (s1 < 0.0) s1 = 0.0; if (s1 > 1.0) s1 = 1.0;
30     if (s2 < 0.0) s2 = 0.0; if (s2 > 1.0) s2 = 1.0;
31     point r1 = p1 + u * s1; point r2 = q1 + v * s2;
32     return (r1 - r2).len();
33 }
34 bool isFL(const point &p, const point &o, const point &q1, const point &q2, point &res) {
35     double a = dot(o, q2 - p), b = dot(o, q1 - p), d = a - b;
36     if (sign(d) == 0) return false;
37     res = (q1 * a - q2 * b) / d;
38     return true;
39 }
40 bool isFF(const point &p1, const point &o1, const point &p2, const point &o2, point &a,
41           point &b) {
42     point e = det(o1, o2), v = det(o1, e);
43     double d = dot(o2, v); if (sign(d) == 0) return false;
44     point q = p1 + v * (dot(o2, p2 - p1) / d);
45     a = q; b = q + e;
46     return true;
47 }

```

1.14 凸多面体切割

```

1 vector<vector<point>> > convexCut(const vector<vector<point>> > &pss, const point &p, const
2   point &o) {
3     vector<vector<point>> > res;
4     vector<point> sec;
5     for (unsigned itr = 0, size = pss.size(); itr < size; ++itr) {
6         const vector<point> &ps = pss[itr];
7         int n = ps.size();
8         vector<point> qs;
9         bool dif = false;
10        for (int i = 0; i < n; ++i) {

```

```

11        int d2 = sign( dot(o, ps[(i + 1) % n] - p) );
12        if (d1 <= 0) qs.push_back(ps[i]);
13        if (d1 * d2 < 0) {
14            point q;
15            isFL(p, o, ps[i], ps[(i + 1) % n], q); // must return true
16            qs.push_back(q);
17            sec.push_back(q);
18        }
19        if (d1 == 0) sec.push_back(ps[i]);
20        else dif = true;
21        dif |= dot(o, det(ps[(i + 1) % n] - ps[i], ps[(i + 2) % n] - ps[i])) < -EPS;
22    }
23    if (!qs.empty() && dif)
24        res.insert(res.end(), qs.begin(), qs.end());
25 }
26 if (!sec.empty()) {
27     vector<point> tmp( convexHull2D(sec, o) );
28     res.insert(res.end(), tmp.begin(), tmp.end());
29 }
30 return res;
31 }
32
33 vector<vector<point>> > initConvex() {
34     vector<vector<point>> > pss(6, vector<point>(4));
35     pss[0][0] = pss[1][0] = pss[2][0] = point(-INF, -INF, -INF);
36     pss[0][3] = pss[1][1] = pss[5][2] = point(-INF, -INF, INF);
37     pss[0][1] = pss[2][3] = pss[4][2] = point(-INF, INF, -INF);
38     pss[0][2] = pss[5][3] = pss[4][1] = point(-INF, INF, INF);
39     pss[1][3] = pss[2][1] = pss[3][2] = point( INF, -INF, -INF);
40     pss[1][2] = pss[5][1] = pss[3][3] = point( INF, -INF, INF);
41     pss[2][2] = pss[4][3] = pss[3][1] = point( INF, INF, -INF);
42     pss[5][0] = pss[4][0] = pss[3][0] = point( INF, INF, INF);
43     return pss;
44 }

```

1.15 三维凸包

不能有重点

```

1 namespace ConvexHull3D {
2     #define volume(a, b, c, d) (mix(ps[b] - ps[a], ps[c] - ps[a], ps[d] - ps[a]))
3     vector<Facet> getHull(int n, point ps[]) {
4         static int mark[MAXN][MAXN], a, b, c;
5         int stamp = 0;
6         bool exist = false;
7         vector<Facet> facet;
8         random_shuffle(ps, ps + n);
9         for (int i = 2; i < n && !exist; i++) {

```

```

10     point ndir = det(ps[0] - ps[i], ps[1] - ps[i]);
11     if (ndir.len() < EPS) continue;
12     swap(ps[i], ps[2]);
13     for (int j = i + 1; j < n && !exist; j++)
14         if (sign(volume(0, 1, 2, j)) != 0) {
15             exist = true;
16             swap(ps[j], ps[3]);
17             facet.push_back(Facet(0, 1, 2));
18             facet.push_back(Facet(0, 2, 1));
19         }
20     }
21     if (!exist) return ConvexHull2D(n, ps);
22     for (int i = 0; i < n; ++i)
23         for (int j = 0; j < n; ++j)
24             mark[i][j] = 0;
25     stamp = 0;
26     for (int v = 3; v < n; ++v) {
27         vector<Facet> tmp;
28         ++stamp;
29         for (unsigned i = 0; i < facet.size(); i++) {
30             a = facet[i].a;
31             b = facet[i].b;
32             c = facet[i].c;
33             if (sign(volume(v, a, b, c)) < 0)
34                 mark[a][b] = mark[a][c] =
35                 mark[b][a] = mark[b][c] =
36                 mark[c][a] = mark[c][b] = stamp;
37             else tmp.push_back(facet[i]);
38         } facet = tmp;
39         for (unsigned i = 0; i < tmp.size(); i++) {
40             a = facet[i].a; b = facet[i].b; c = facet[i].c;
41             if (mark[a][b] == stamp) facet.push_back(Facet(b, a, v));
42             if (mark[b][c] == stamp) facet.push_back(Facet(c, b, v));
43             if (mark[c][a] == stamp) facet.push_back(Facet(a, c, v));
44         }
45     } return facet;
46 }
47 #undef volume
48 }
49 namespace Gravity {
50     using ConvexHull3D::Facet;
51     point findG(point ps[], const vector<Facet> &facet) {
52         double ws = 0; point res(0.0, 0.0, 0.0), o = ps[ facet[0].a ];
53         for (int i = 0, size = facet.size(); i < size; ++i) {
54             const point &a = ps[ facet[i].a ], &b = ps[ facet[i].b ], &c = ps[ facet[i].c ];
55             point p = (a + b + c + o) * 0.25;
56             double w = mix(a - o, b - o, c - o);
57             ws += w;
58             res = res + p * w;

```

```

59         } res = res / ws;
60         return res;
61     }
62 }

```

1.16 球面点表面点距离

```

1 double distOnBall(double lati1, double longi1, double lati2, double longi2, double R) {
2     lati1 *= PI / 180; longi1 *= PI / 180;
3     lati2 *= PI / 180; longi2 *= PI / 180;
4     double x1 = cos(lati1) * sin(longi1);
5     double y1 = cos(lati1) * cos(longi1);
6     double z1 = sin(lati1);
7     double x2 = cos(lati2) * sin(longi2);
8     double y2 = cos(lati2) * cos(longi2);
9     double z2 = sin(lati2);
10    double theta = acos(x1 * x2 + y1 * y2 + z1 * z2);
11    return R * theta;
12 }

```

1.17 长方体表面点距离

```

1 int r;
2 void turn(int i, int j, int x, int y, int z, int x0, int y0, int L, int W, int H) {
3     if (z == 0) r = min(r, x * x + y * y);
4     else {
5         if (i >= 0 && i < 2) turn(i + 1, j, x0 + L + z, y, x0 + L - x, x0 + L, y0, H, W, L);
6         if (j >= 0 && j < 2) turn(i, j + 1, x, y0 + W + z, y0 + W - y, x0, y0 + W, L, H, W);
7         if (i <= 0 && i > -2) turn(i - 1, j, x0 - z, y, x - x0, x0 - H, y0, H, W, L);
8         if (j <= 0 && j > -2) turn(i, j - 1, x, y0 - z, y - y0, x0, y0 - H, L, H, W);
9     }
10 }
11 int calc(int L, int H, int W, int x1, int y1, int z1, int x2, int y2, int z2) {
12     if (z1 != 0 && z1 != H)
13         if (y1 == 0 || y1 == W) swap(y1, z1), swap(y2, z2), swap(W, H);
14     else swap(x1, z1), swap(x2, z2), swap(L, H);
15     if (z1 == H) z1 = 0, z2 = H - z2;
16     r = INF; turn(0, 0, x2 - x1, y2 - y1, z2, -x1, -y1, L, W, H);
17     return r;
18 }

```

1.18 最小覆盖球

```

1 int outCnt; point out[4], res; double radius;
2 void ball() {

```

```

3 static point q[3];
4 static double m[3][3], sol[3], L[3], det;
5 int i, j; res = point(0.0, 0.0, 0.0); radius = 0.0;
6 switch (outCnt) {
7 case 1: res = out[0]; break;
8 case 2: res = (out[0] + out[1]) * 0.5; radius = (res - out[0]).norm();
9         break;
10 case 3:
11     q[0] = out[1] - out[0]; q[1] = out[2] - out[0];
12     for (i = 0; i < 2; ++i) for (j = 0; j < 2; ++j)
13         m[i][j] = dot(q[i], q[j]) * 2.0;
14     for (i = 0; i < 2; ++i) sol[i] = dot(q[i], q[i]);
15     det = m[0][0] * m[1][1] - m[0][1] * m[1][0];
16     if (sign(det) == 0) return;
17     L[0] = (sol[0] * m[1][1] - sol[1] * m[0][1]) / det;
18     L[1] = (sol[1] * m[0][0] - sol[0] * m[1][0]) / det;
19     res = out[0] + q[0] * L[0] + q[1] * L[1];
20     radius = (res - out[0]).norm();
21     break;
22 case 4:
23     q[0] = out[1] - out[0]; q[1] = out[2] - out[0]; q[2] = out[3] - out[0];
24     for (i = 0; i < 3; ++i) for (j = 0; j < 3; ++j) m[i][j] = dot(q[i], q[j]) * 2;
25     for (i = 0; i < 3; ++i) sol[i] = dot(q[i], q[i]);
26     det = m[0][0] * m[1][1] * m[2][2] + m[0][1] * m[1][2] * m[2][0]
27           + m[0][2] * m[2][1] * m[1][0] - m[0][2] * m[1][1] * m[2][0]
28           - m[0][1] * m[1][0] * m[2][2] - m[0][0] * m[1][2] * m[2][1];
29     if (sign(det) == 0) return;
30     for (j = 0; j < 3; ++j) { for (i = 0; i < 3; ++i) m[i][j] = sol[i];
31         L[j] = (m[0][0] * m[1][1] * m[2][2] + m[0][1] * m[1][2] * m[2][0]
32               + m[0][2] * m[2][1] * m[1][0] - m[0][2] * m[1][1] * m[2][0]
33               - m[0][1] * m[1][0] * m[2][2] - m[0][0] * m[1][2] * m[2][1]) / det;
34         for (i = 0; i < 3; ++i) m[i][j] = dot(q[i], q[j]) * 2;
35     } res = out[0];
36     for (i = 0; i < 3; ++i) res += q[i] * L[i]; radius = (res - out[0]).norm();
37 }
38 }
39 void minball(int n, point pt[]) {
40     ball();
41     if (outCnt < 4) for (int i = 0; i < n; ++i)
42         if ((res - pt[i]).norm() > +radius + EPS) {
43             out[outCnt] = pt[i]; ++outCnt; minball(i, pt); --outCnt;
44             if (i > 0) {
45                 point Tt = pt[i];
46                 memmove(&pt[1], &pt[0], sizeof(point) * i);
47                 pt[0] = Tt;
48             }
49         }
50 }
51 pair<point, double> main(int npoint, point pt[]) { // 0-based

```

```

52     random_shuffle(pt, pt + npoint); radius = -1;
53     for (int i = 0; i < npoint; i++) { if ((res - pt[i]).norm() > EPS + radius) {
54         outCnt = 1; out[0] = pt[i]; minball(i, pt); } }
55     return make_pair(res, sqrt(radius));
56 }

```

1.19 三维向量操作矩阵

- 绕单位向量 $u = (u_x, u_y, u_z)$ 右手方向旋转 θ 度的矩阵:

$$\begin{bmatrix} \cos\theta + u_x^2(1-\cos\theta) & u_x u_y(1-\cos\theta) - u_z \sin\theta & u_x u_z(1-\cos\theta) + u_y \sin\theta \\ u_y u_x(1-\cos\theta) + u_z \sin\theta & \cos\theta + u_y^2(1-\cos\theta) & u_y u_z(1-\cos\theta) - u_x \sin\theta \\ u_z u_x(1-\cos\theta) - u_y \sin\theta & u_z u_y(1-\cos\theta) + u_x \sin\theta & \cos\theta + u_z^2(1-\cos\theta) \end{bmatrix}$$

$$= \cos\theta I + \sin\theta \begin{bmatrix} 0 & -u_z & u_y \\ u_z & 0 & -u_x \\ -u_y & u_x & 0 \end{bmatrix} + (1-\cos\theta) \begin{bmatrix} u_x^2 & u_x u_y & u_x u_z \\ u_y u_x & u_y^2 & u_y u_z \\ u_z u_x & u_z u_y & u_z^2 \end{bmatrix}$$

- 点 a 绕单位向量 $u = (u_x, u_y, u_z)$ 右手方向旋转 θ 度的对应点为 $a' = a \cos\theta + (u \times a) \sin\theta + (u \otimes u)a(1-\cos\theta)$

- 关于向量 v 作对称变换的矩阵 $H = I - 2 \frac{vv^T}{v^T v}$,

- 点 a 对称点: $a' = a - 2 \frac{v^T a}{v^T v} \cdot v$

1.20 立体角

对于任意一个四面体 $OABC$, 从 O 点观察 $\triangle ABC$ 的立体角 $\tan \frac{\Omega}{2} = \frac{\text{mix}(\vec{a}, \vec{b}, \vec{c})}{|a||b||c| + (\vec{a} \cdot \vec{b})|c| + (\vec{a} \cdot \vec{c})|b| + (\vec{b} \cdot \vec{c})|a|}$.

2 数据结构

2.1 动态凸包 (只支持插入)

```

1 #define x first // upperHull ← (x,y)
2 #define y second // lowerHull ← (x,-y)
3 typedef map<int, int> mii;
4 typedef map<int, int>::iterator mit;
5 struct point { point(const mit &p): x(p->first), y(p->second) {} };
6 inline bool checkInside(mii &a, const point &p) { // border inclusive
7     int x = p.x, y = p.y; mit p1 = a.lower_bound(x);
8     if (p1 == a.end()) return false; if (p1->x == x) return y <= p1->y;
9     if (p1 == a.begin()) return false; mit p2(p1--);
10    return sign(det(p - point(p1), point(p2) - p)) >= 0;
11 } inline void addPoint(mii &a, const point &p) { // no collinear points
12     int x = p.x, y = p.y; mit pnt = a.insert(make_pair(x, y)).first, p1, p2;
13     for (pnt->y = y; ; a.erase(p2)) {
14         p1 = pnt; if (++p1 == a.end()) break;
15         p2 = p1; if (++p1 == a.end()) break;
16         if (det(point(p2) - p, point(p1) - p) < 0) break;

```

```

17     } for ( ; ; a.erase(p2)) {
18         if ((p1 = pnt) == a.begin()) break;
19         if (--p1 == a.begin()) break; p2 = p1--;
20         if (det(point(p2) - p, point(p1) - p) > 0) break;
21     }
22 }

```

2.2 Rope 用法

```

1 #include <ext/rope>
2 using __gnu_cxx::crope; using __gnu_cxx::rope;
3 a = b.substr(from, len);          // [from, from + len)
4 a = b.substr(from);              // [from, from]
5 b.c_str();                       // might lead to memory leaks
6 b.delete_c_str();               // delete the c_str that created before
7 a.insert(p, str);                // insert str before position p
8 a.erase(i, n);                  // erase [i, i + n)

```

2.3 Treap

```

1 struct node { int key, prio, size; node *ch[2]; } base[MAXN], *top, *root, *null, nil;
2 typedef node *tree;
3 tree newNode(int key) {
4     static int seed = 3312;
5     top->key = key; top->prio = seed = int(seed * 48271LL % 2147483647);
6     top->size = 1; top->ch[0] = top->ch[1] = null; return top++;
7 }
8 void Rotate(tree &x, int d) {
9     tree y = x->ch[!d]; x->ch[!d] = y->ch[d]; y->ch[d] = x; y->size = x->size;
10    x->size = x->ch[0]->size + 1 + x->ch[1]->size; x = y;
11 }
12 void Insert(tree &t, int key) {
13     if (t == null) t = newNode(key);
14     else { int d = t->key < key; Insert(t->ch[d], key); ++t->size;
15         if (t->ch[d]->prio < t->prio) Rotate(t, !d);
16     }
17 }
18 void Delete(tree &t, int key) {
19     if (t->key != key) { Delete(t->ch[t->key < key], key); --t->size; }
20     else if (t->ch[0] == null) t = t->ch[1];
21     else if (t->ch[1] == null) t = t->ch[0];
22     else { int d = t->ch[0]->prio < t->ch[1]->prio;
23         Rotate(t, d); Delete(t->ch[d], key); --t->size;
24     }
25 }

```

2.4 可持久化 Treap

```

1 inline bool randomBySize(int a, int b) {
2     static long long seed = 1;
3     return (seed = seed * 48271 % 2147483647) * (a + b) < 2147483647LL * a;
4 }
5 tree merge(tree x, tree y) {
6     if (x == null) return y; if (y == null) return x;
7     tree t = NULL;
8     if (randomBySize(x->size, y->size)) t = newNode(x), t->r = merge(x->r, y);
9     else t = newNode(y), t->l = merge(x, y->l);
10    update(t); return t;
11 }
12 void splitByKey(tree t, int k, tree &l, tree &r) { //  $[-\infty, k)[k, +\infty)$ 
13     if (t == null) l = r = null;
14     else if (t->key < k) l = newNode(t), splitByKey(t->r, k, l->r, r), update(l);
15     else r = newNode(t), splitByKey(t->l, k, l, r->l), update(r);
16 }
17 void splitBySize(tree t, int k, tree &l, tree &r) { //  $[1, k)[k, +\infty)$ 
18     static int s; if (t == null) l = r = null;
19     else if ((s = t->l->size + 1) < k) l = newNode(t), splitBySize(t->r, k - s, l->r, r),
20         update(l);
21     else r = newNode(t), splitBySize(t->l, k, l, r->l),
22         update(r);
23 }

```

2.5 左偏树

```

1 tree merge(tree a, tree b) {
2     if (a == null) return b;
3     if (b == null) return a;
4     if (a->key > b->key) swap(a, b);
5     a->rc = merge(a->rc, b);
6     a->rc->fa = a;
7     if (a->lc->dist < a->rc->dist) swap(a->lc, a->rc);
8     a->dist = a->rc->dist + 1;
9     return a;
10 }
11 void erase(tree t) {
12     tree x = t->fa, y = merge(t->lc, t->rc);
13     if (y != null) y->fa = x;
14     if (x == null) root = y;
15     else
16     for ((x->lc == t ? x->lc : x->rc) = y; x != null; y = x, x = x->fa) {
17         if (x->lc->dist < x->rc->dist) swap(x->lc, x->rc);
18         if (x->rc->dist + 1 == x->dist) return;
19         x->dist = x->rc->dist + 1;
20     }

```

21 }

2.6 Link-Cut Tree

```

1 struct node { int rev; node *pre, *ch[2]; } base[MAXN], nil, *null;
2 typedef node *tree;
3 #define isRoot(x) (x->pre->ch[0] != x && x->pre->ch[1] != x)
4 #define isRight(x) (x->pre->ch[1] == x)
5 inline void MakeRev(tree t) { if (t != null) { t->rev ^= 1; swap(t->ch[0], t->ch[1]); } }
6 inline void PushDown(tree t) { if (t->rev) { MakeRev(t->ch[0]); MakeRev(t->ch[1]); t->rev
    = 0; } }
7 inline void Rotate(tree x) {
8     tree y = x->pre; PushDown(y); PushDown(x);
9     int d = isRight(x);
10    if (!isRoot(y)) y->pre->ch[isRight(y)] = x; x->pre = y->pre;
11    if ((y->ch[d] = x->ch[!d]) != null) y->ch[d]->pre = y;
12    x->ch[!d] = y; y->pre = x; Update(y);
13 }
14 inline void Splay(tree x) {
15     PushDown(x); for (tree y; !isRoot(x); Rotate(x)) {
16         y = x->pre; if (!isRoot(y)) Rotate(isRight(x) != isRight(y) ? x : y);
17     } Update(x);
18 }
19 inline void Splay(tree x, tree to) {
20     PushDown(x); for (tree y; (y = x->pre) != to; Rotate(x)) if (y->pre != to)
21         Rotate(isRight(x) != isRight(y) ? x : y);
22     Update(x);
23 }
24 inline tree Access(tree t) {
25     tree last = null; for (; t != null; last = t, t = t->pre) Splay(t), t->ch[1] = last,
        Update(t);
26     return last;
27 }
28 inline void MakeRoot(tree t) { Access(t); Splay(t); MakeRev(t); }
29 inline tree FindRoot(tree t) { Access(t); Splay(t); tree last = null;
30     for (; t != null; last = t, t = t->ch[0]) PushDown(t); Splay(last); return last;
31 }
32 inline void Join(tree x, tree y) { MakeRoot(y); y->pre = x; }
33 inline void Cut(tree t) { Access(t); Splay(t); t->ch[0]->pre = null; t->ch[0] = null;
    Update(t); }
34 inline void Cut(tree x, tree y) {
35     tree upper = (Access(x), Access(y));
36     if (upper == x) { Splay(x); y->pre = null; x->ch[1] = null; Update(x); }
37     else if (upper == y) { Access(x); Splay(y); x->pre = null; y->ch[1] = null; Update(y);
        }
38     else assert(0); // impossible to happen
39 }
40 inline int Query(tree a, tree b) { // query the cost in path a <-> b, lca inclusive

```

```

41     Access(a); tree c = Access(b); // c is lca
42     int v1 = c->ch[1]->maxCost; Access(a);
43     int v2 = c->ch[1]->maxCost;
44     return max(max(v1, v2), c->cost);
45 }
46 void Init() {
47     null = &nil; null->ch[0] = null->ch[1] = null->pre = null; null->rev = 0;
48     Rep(i, 1, N) { node &n = base[i]; n.rev = 0; n.pre = n.ch[0] = n.ch[1] = null; }
49 }

```

2.7 K-D Tree Nearest

```

1 struct Point { int x, y; };
2 struct Rectangle {
3     int lx, rx, ly, ry;
4     void set(const Point &p) { lx = rx = p.x; ly = ry = p.y; }
5     void merge(const Point &o) {
6         lx = min(lx, o.x); rx = max(rx, o.x); ly = min(ly, o.y); ry = max(ry, o.y);
7     } void merge(const Rectangle &o) {
8         lx = min(lx, o.lx); rx = max(rx, o.rx); ly = min(ly, o.ly); ry = max(ry, o.ry);
9     } LL dist(const Point &p) {
10         LL res = 0;
11         if (p.x < lx) res += sqr(lx - p.x); else if (p.x > rx) res += sqr(p.x - rx);
12         if (p.y < ly) res += sqr(ly - p.y); else if (p.y > ry) res += sqr(p.y - ry);
13         return res;
14     }
15 };
16 struct Node { int child[2]; Point p; Rectangle rect; };
17 const int MAX_N = 1111111;
18 const LL INF = 100000000;
19 int n, m, tot, root; LL result;
20 Point a[MAX_N], p; Node tree[MAX_N];
21 int build(int s, int t, bool d) {
22     int k = ++tot, mid = (s + t) >> 1;
23     nth_element(a + s, a + mid, a + t, d ? cmpXY : cmpYX);
24     tree[k].p = a[mid]; tree[k].rect.set(a[mid]); tree[k].child[0] = tree[k].child[1] = 0;
25     if (s < mid)
26         tree[k].child[0] = build(s, mid, d ^ 1), tree[k].rect.merge(tree[tree[k].child[0]].
            rect);
27     if (mid + 1 < t)
28         tree[k].child[1] = build(mid + 1, t, d ^ 1), tree[k].rect.merge(tree[tree[k].child
            [1]].rect);
29     return k;
30 }
31 int insert(int root, bool d) {
32     if (root == 0) {
33         tree[++tot].p = p; tree[tot].rect.set(p); tree[tot].child[0] = tree[tot].child[1] =
            0;

```

```

34     return tot;
35 } tree[root].rect.merge(p);
36 if ((d && cmpXY(p, tree[root].p)) || (!d && cmpYX(p, tree[root].p)))
37     tree[root].child[0] = insert(tree[root].child[0], d ^ 1);
38 else tree[root].child[1] = insert(tree[root].child[1], d ^ 1);
39 return root;
40 }
41 void query(int k, bool d) {
42     if (tree[k].rect.dist(p) >= result) return;
43     cMin(result, dist(tree[k].p, p));
44     if ((d && cmpXY(p, tree[k].p)) || (!d && cmpYX(p, tree[k].p))) {
45         if (tree[k].child[0]) query(tree[k].child[0], d ^ 1);
46         if (tree[k].child[1]) query(tree[k].child[1], d ^ 1);
47     } else {
48         if (tree[k].child[1]) query(tree[k].child[1], d ^ 1);
49         if (tree[k].child[0]) query(tree[k].child[0], d ^ 1);
50     }
51 }
52 void example(int n) {
53     root = tot = 0; scan(a); root = build(0, n, 0); // init, a[0...n-1]
54     scan(p); root = insert(root, 0); // insert
55     scan(p); result = INF; ans = query(root, 0); // query
56 }

```

2.8 K-D Tree Farthest

输入 n 个点, 对每个询问 px, py, k , 输出 k 远点的编号

```

1 struct Point { int x, y, id; };
2 struct Rectangle {
3     int lx, rx, ly, ry;
4     void set(const Point &p) { lx = rx = p.x; ly = ry = p.y; }
5     void merge(const Rectangle &o) {
6         lx = min(lx, o.lx); rx = max(rx, o.rx); ly = min(ly, o.ly); ry = max(ry, o.ry);
7     }
8     LL dist(const Point &p) { LL res = 0;
9         res += max(sqr(rx - p.x), sqr(lx - p.x));
10        res += max(sqr(ry - p.y), sqr(ly - p.y));
11        return res;
12    }
13 }; struct Node { Point p; Rectangle rect; };
14 const int MAX_N = 111111;
15 const LL INF = 1LL << 60;
16 int n, m;
17 Point a[MAX_N], b[MAX_N];
18 Node tree[MAX_N * 3];
19 Point p; // p is the query point
20 pair<LL, int> result[22];

```

```

21 void build(int k, int s, int t, bool d) {
22     int mid = (s + t) >> 1;
23     nth_element(a + s, a + mid, a + t, d ? cmpX : cmpY);
24     tree[k].p = a[mid];
25     tree[k].rect.set(a[mid]);
26     if (s < mid)
27         build(k << 1, s, mid, d ^ 1), tree[k].rect.merge(tree[k << 1].rect);
28     if (mid + 1 < t)
29         build(k << 1 | 1, mid + 1, t, d ^ 1), tree[k].rect.merge(tree[k << 1 | 1].rect);
30 }
31 void query(int k, int s, int t, bool d, int kth) {
32     if (tree[k].rect.dist(p) < result[kth].first) return;
33     pair<LL, int> tmp(dist(tree[k].p, p), -tree[k].p.id);
34     for (int i = 1; i <= kth; i++) if (tmp > result[i]) {
35         for (int j = kth + 1; j > i; j--) result[j] = result[j - 1]; result[i] = tmp;
36         break;
37     }
38     int mid = (s + t) >> 1;
39     if ((d && cmpX(p, tree[k].p)) || (!d && cmpY(p, tree[k].p))) {
40         if (mid + 1 < t) query(k << 1 | 1, mid + 1, t, d ^ 1, kth);
41         if (s < mid) query(k << 1, s, mid, d ^ 1, kth);
42     } else {
43         if (s < mid) query(k << 1, s, mid, d ^ 1, kth);
44         if (mid + 1 < t) query(k << 1 | 1, mid + 1, t, d ^ 1, kth);
45     }
46 }
47 void example(int n) {
48     scan(a); build(1, 0, n, 0); // init, a[0...n-1]
49     scan(p, k); // query
50     Rep(j, 1, k) result[j].first = -1;
51     query(1, 0, n, 0, k); ans = -result[k].second + 1;
52 }

```

2.9 树链剖分

```

1 int N, fa[MAXN], dep[MAXN], que[MAXN], size[MAXN], own[MAXN];
2 int LCA(int x, int y) { if (x == y) return x;
3     for (; x = fa[own[x]]; ) {
4         if (dep[x] < dep[y]) swap(x, y); if (own[x] == own[y]) return y;
5         if (dep[own[x]] < dep[own[y]]) swap(x, y);
6     } return -1;
7 }
8 void Decomposition() {
9     static int path[MAXN]; int x, y, a, next, head = 0, tail = 0, cnt; // BFS omitted
10    for (int i = 1; i <= N; ++i) if (own[a = que[i]] == -1)
11        for (x = a, cnt = 0; ; x = next) { next = -1; own[x] = a; path[++cnt] = x;
12            for (edge e(fir[x]); e; e = e->next) if ( (y = e->to) != fa[x] )
13                if (next == -1 || size[y] > size[next]) next = y;

```



```

14     if (next == -1) { tree[a].init(cnt, path); break; }
15     }
16 }

```

3 字符串相关

3.1 Manacher

```

1 // len[i] : the max length of palindrome whose mid point is (i / 2)
2 void Manacher(int n, char cs[], int len[]) { // 0-based, len[] must be double sized
3     for (int i = 0; i < n + n; ++i) len[i] = 0;
4     for (int i = 0, j = 0, k; i < n * 2; i += k, j = max(j - k, 0)) {
5         while (i - j >= 0 && i + j + 1 < n * 2 && cs[(i - j) / 2] == cs[(i + j + 1) / 2]) j
            ++;
6         len[i] = j; for (k = 1; i - k >= 0 && j - k >= 0 && len[i - k] != j - k; k++)
7             len[i + k] = min(len[i - k], j - k);
8     }
9 }

```

3.2 KMP

$next[i] = \max\{len|A[0 \dots len - 1] = A \text{ 的第 } i \text{ 位向前或后的长度为 } len \text{ 的串}\}$

$ext[i] = \max\{len|A[0 \dots len - 1] = B \text{ 的第 } i \text{ 位向前或后的长度为 } len \text{ 的串}\}$

```

1 void KMP(char *a, int la, char *b, int lb, int *next, int *ext) {
2     --a; --b; --next; --ext;
3     for (int i = 2, j = next[1] = 0; i <= la; i++) {
4         while (j && a[j + 1] != a[i]) j = next[j]; if (a[j + 1] == a[i]) ++j; next[i] = j;
5     } for (int i = 1, j = 0; i <= lb; ++i) {
6         while (j && a[j + 1] != b[i]) j = next[j]; if (a[j + 1] == b[i]) ++j; ext[i] = j;
7         if (j == la) j = next[j];
8     }
9 } void ExKMP(char *a, int la, char *b, int lb, int *next, int *ext) {
10     next[0] = la; for (int &j = next[1] = 0; j + 1 < la && a[j] == a[j + 1]; ++j);
11     for (int i = 2, k = 1; i < la; ++i) {
12         int p = k + next[k], l = next[i - k]; if (1 < p - i) next[i] = 1;
13         else for (int &j = next[k = i] = max(0, p - i); i + j < la && a[j] == a[i + j]; ++j);
14     } for (int &j = ext[0] = 0; j < la && j < lb && a[j] == b[j]; ++j);
15     for (int i = 1, k = 0; i < lb; ++i) {
16         int p = k + ext[k], l = next[i - k]; if (1 < p - i) ext[i] = 1;
17         else for (int &j = ext[k = i] = max(0, p - i); j < la && i + j < lb && a[j] == b[i +
            j]; ++j);
18     }
19 }

```

3.3 Aho-Corasick 自动机

```

1 void construct() {
2     static tree Q[MAX_NODE]; int head = 0, tail = 0;
3     for (root->fail = root, Q[++tail] = root; head < tail; ) {
4         tree x = Q[++head];
5         // if (x->fail->danger) x->danger = true;
6         Rep(d, 0, sigma - 1) if (!x->next[d])
7             x->next[d] = (x == root) ? (root) : (x->fail->next[d]);
8         else {
9             x->next[d]->fail = (x == root) ? (root) : (x->fail->next[d]);
10            Q[++tail] = x->next[d];
11        }
12    }
13 }

```

3.4 后缀自动机

```

1 struct node { int len; node *fa, *go[26]; } base[MAXNODE], *top = base, *root, *que[
    MAXNODE];
2 typedef node *tree;
3 inline tree newNode(int len) {
4     top->len = len; top->fa = NULL; memset(top->go, 0, sizeof(top->go)); return top++;
5 } inline tree newNode(int len, tree fa, tree *go) {
6     top->len = len; top->fa = fa; memcpy(top->go, go, sizeof(top->go)); return top++;
7 } void construct(char *A, int N) {
8     tree p = root = newNode(0), q, up, fa;
9     for (int i = 0; i < N; ++i) {
10        int w = A[i] - 'a'; up = p; p = newNode(i + 1);
11        for ( ; up && !up->go[w]; up = up->fa) up->go[w] = p;
12        if (!up) p->fa = root;
13        else { q = up->go[w];
14            if (up->len + 1 == q->len) p->fa = q;
15            else { fa = newNode(up->len + 1, q->fa, q->go);
16                for (p->fa = q->fa = fa; up && up->go[w] == q; up = up->fa) up->go[w] = fa;
17            }
18        }
19    } static int cnt[MAXLEN]; memset(cnt, 0, sizeof(int) * (N + 1));
20    for (tree i(base); i != top; ++i) ++cnt[i->len];
21    Rep(i, 1, N) cnt[i] += cnt[i - 1];
22    for (tree i(base); i != top; ++i) Q[ cnt[i->len]-- ] = i;
23 }

```

3.5 后缀数组

待排序的字符串放在 $r[0 \dots n - 1]$ 中, 最大值小于 m .

$r[0 \dots n - 2] > 0, r[n - 1] = 0$.

结果放在 $sa[0 \dots n - 1]$.

```

1 namespace SuffixArrayDoubling {
2     int wa[MAXN], wb[MAXN], wv[MAXN], ws[MAXN];
3     int cmp(int *r, int a, int b, int l) {
4         return r[a] == r[b] && r[a + 1] == r[b + 1];
5     }
6     void da(int *r, int *sa, int n, int m) {
7         int i, j, p, *x = wa, *y = wb, *t;
8         for (i = 0; i < m; i++) ws[i] = 0;
9         for (i = 0; i < n; i++) ws[x[i]]++;
10        for (i = 1; i < m; i++) ws[i] += ws[i - 1];
11        for (i = n - 1; i >= 0; i--) sa[--ws[x[i]]] = i;
12        for (j = 1, p = 1; p < n; j *= 2, m = p) {
13            for (p = 0, i = n - j; i < n; i++) y[p++] = i;
14            for (i = 0; i < n; i++) if (sa[i] >= j) y[p++] = sa[i] - j;
15            for (i = 0; i < n; i++) wv[i] = x[y[i]];
16            for (i = 0; i < m; i++) ws[i] = 0;
17            for (i = 0; i < n; i++) ws[wv[i]]++;
18            for (i = 1; i < m; i++) ws[i] += ws[i - 1];
19            for (i = n - 1; i >= 0; i--) sa[--ws[wv[i]]] = y[i];
20            for (t = x, x = y, y = t, p = 1, x[sa[0]] = 0, i = 1; i < n; i++)
21                x[sa[i]] = cmp(y, sa[i - 1], sa[i], j) ? p - 1 : p++;
22        }
23    }
24 }
25 namespace SuffixArrayDC3 { // r 与 sa 大小需 3 倍
26     #define F(x) ((x) / 3 + ((x) % 3 == 1 ? 0 : tb))
27     #define G(x) ((x) < tb ? (x) * 3 + 1 : ((x) - tb) * 3 + 2)
28     int wa[MAXN], wb[MAXN], wv[MAXN], ws[MAXN];
29     int c0(int *r, int a, int b) {
30         return r[a] == r[b] && r[a + 1] == r[b + 1] && r[a + 2] == r[b + 2];
31     }
32     int c12(int k, int *r, int a, int b) {
33         if (k == 2) return r[a] < r[b] || (r[a] == r[b] && c12(1, r, a + 1, b + 1));
34         else return r[a] < r[b] || (r[a] == r[b] && wv[a + 1] < wv[b + 1]);
35     }
36     void sort(int *r, int *a, int *b, int n, int m) {
37         for (int i = 0; i < n; i++) wv[i] = r[a[i]];
38         for (int i = 0; i < m; i++) ws[i] = 0;
39         for (int i = 0; i < n; i++) ws[wv[i]]++;
40         for (int i = 1; i < m; i++) ws[i] += ws[i - 1];
41         for (int i = n - 1; i >= 0; i--) b[--ws[wv[i]]] = a[i];
42     }
43     void dc3(int *r, int *sa, int n, int m) {
44         int i, j, *rn = r + n, *san = sa + n, ta = 0, tb = (n + 1) / 3, tbc = 0, p;
45         r[n] = r[n + 1] = 0;
46         for (i = 0; i < n; i++) if (i % 3 != 0) wa[tbc++] = i;

```

```

47         sort(r + 2, wa, wb, tbc, m);
48         sort(r + 1, wb, wa, tbc, m);
49         sort(r, wa, wb, tbc, m);
50         for (p = 1, rn[F(wb[0])] = 0, i = 1; i < tbc; i++)
51             rn[F(wb[i])] = c0(r, wb[i - 1], wb[i]) ? p - 1 : p++;
52         if (p < tbc) dc3(rn, san, tbc, p);
53         else for (i = 0; i < tbc; i++) san[rn[i]] = i;
54         for (i = 0; i < tbc; i++) if (san[i] < tb) wb[ta++] = san[i] * 3;
55         if (n % 3 == 1) wb[ta++] = n - 1;
56         sort(r, wb, wa, ta, m);
57         for (i = 0; i < tbc; i++) wv[wb[i] = G(san[i])] = i;
58         for (i = 0, j = 0, p = 0; i < ta && j < tbc; p++)
59             sa[p] = c12(wb[j] % 3, r, wa[i], wb[j]) ? wa[i++] : wb[j++];
60         for (; i < ta; p++) sa[p] = wa[i++];
61         for (; j < tbc; p++) sa[p] = wb[j++];
62     }
63     #undef F
64     #undef G
65 }
66 namespace CalcHeight {
67     int rank[MAXN], height[MAXN];
68     void calheight(int *r, int *sa, int n) {
69         int i, j, k = 0;
70         for (i = 1; i <= n; i++) rank[sa[i]] = i;
71         for (i = 0; i < n; height[rank[i++]] = k)
72             for (k ? k-- : 0, j = sa[rank[i] - 1]; r[i + k] == r[j + k]; k++);
73     }
74 }

```

3.6 环串最小表示

```

1 int minimalRepresentation(int N, char *s) { // s must be double-sized and 0-based
2     int i, j, k, l; for (i = 0; i < N; ++i) s[i + N] = s[i]; s[N + N] = 0;
3     for (i = 0, j = 1; j < N; ) {
4         for (k = 0; k < N && s[i + k] == s[j + k]; ++k);
5         if (k >= N) break; if (s[i + k] < s[j + k]) j += k + 1;
6         else l = i + k, i = j, j = max(l, j) + 1;
7     } return i; // [i, i + N) is the minimal representation
8 }

```

3.7 回文自动机

```

1 #include <cstdlib>
2 #include <cstdio>
3 #include <cstring>
4 #include <algorithm>

```

```

5
6 const int C = 26;
7 const int N = 100000;
8 const int S = N + 2 + C;
9
10 char string[N + 2];
11 int s, length[S], suffix[S], go[S][C];
12
13 int extend(int p, int i) {
14     while (string[i - 1 - length[p]] != string[i]) {
15         p = suffix[p];
16     }
17     int q = suffix[p];
18     while (string[i - 1 - length[q]] != string[i]) {
19         q = suffix[q];
20     }
21     int c = string[i] - 'a';
22     int pp = go[p][c];
23     int qq = go[q][c];
24     if (pp == -1) {
25         length[pp = go[p][c] = s++] = length[p] + 2;
26         suffix[pp] = qq;
27         memset(go[pp], -1, sizeof(go[pp]));
28     }
29     return pp;
30 }
31
32 int main() {
33     int tests;
34     scanf("%d", &tests);
35     for (int t = 1; t <= tests; ++t) {
36         printf("Case_%d: ", t);
37         for (int i = 0; i < C + 2; ++i) {
38             suffix[i] = 1;
39             length[i] = std::min(i - 1, 1);
40             memset(go[i], -1, sizeof(go[i]));
41         }
42         suffix[0] = suffix[1] = 0;
43         for (int i = 0; i < C; ++i) {
44             go[0][i] = 2 + i;
45         }
46         s = C + 2;
47         string[0] = '#';
48         scanf("%s", string + 1);
49         int n = strlen(string + 1);
50         int p = 0;
51         for (int i = 1; i <= n; ++i) {
52             p = extend(p, i);
53         }

```

```

54     int result = s - (C + 2);
55     std::sort(string + 1, string + n + 1);
56     result += std::unique(string + 1, string + n + 1) - string - 1;
57     printf("%d\n", result);
58 }
59 return 0;
60 }

```

4 图论

4.1 带花树

```

1 namespace Blossom {
2     int n, head, tail, S, T, lca;
3     int match[MAXN], Q[MAXN], pred[MAXN], label[MAXN], inq[MAXN], inb[MAXN];
4     vector<int> link[MAXN];
5     inline void push(int x) { Q[tail++] = x; inq[x] = true; }
6     int findCommonAncestor(int x, int y) {
7         static bool inPath[MAXN]; for (int i = 0; i < n; ++i) inPath[i] = 0;
8         for ( ; ; x = pred[ match[x] ]) { x = label[x]; inPath[x] = true; if (x == S) break;
9             }
10         for ( ; ; y = pred[ match[y] ]) { y = label[y]; if (inPath[y]) break; } return y;
11     }
12     void resetTrace(int x, int lca) {
13         while (label[x] != lca) { int y = match[x]; inb[ label[x] ] = inb[ label[y] ] = true;
14             x = pred[y]; if (label[x] != lca) pred[x] = y; }
15     void blossomContract(int x, int y) {
16         lca = findCommonAncestor(x, y);
17         Foru(i, 0, n) inb[i] = 0; resetTrace(x, lca); resetTrace(y, lca);
18         if (label[x] != lca) pred[x] = y; if (label[y] != lca) pred[y] = x;
19         Foru(i, 0, n) if (inb[ label[i] ]) { label[i] = lca; if (!inq[i]) push(i); }
20     }
21     bool findAugmentingPath() {
22         Foru(i, 0, n) pred[i] = -1, label[i] = i, inq[i] = 0;
23         int x, y, z; head = tail = 0;
24         for (push(S); head < tail; ) for (int i = (int)link[x = Q[head++]].size() - 1; i >=
25             0; --i) {
26             y = link[x][i]; if (label[x] == label[y] || x == match[y]) continue;
27             if (y == S || (match[y] >= 0 && pred[ match[y] ] >= 0)) blossomContract(x, y);
28             else if (pred[y] == -1) {
29                 pred[y] = x; if (match[y] >= 0) push(match[y]);
30                 else {
31                     for (x = y; x >= 0; x = z) {
32                         y = pred[x], z = match[y]; match[x] = y, match[y] = x;
33                     } return true; }
34             } return false;
35         }
36     }
37     int findMaxMatching() {

```

```

34     int ans = 0; Foru(i, 0, n) match[i] = -1;
35     for (S = 0; S < n; ++S) if (match[S] == -1) if (findAugmentingPath()) ++ans;
36     return ans;
37 }
38 }

```

4.2 最大流

```

1 namespace Maxflow {
2     int h[MAXNODE], vh[MAXNODE], S, T, Ncnt; edge cur[MAXNODE], pe[MAXNODE];
3     void init(int _S, int _T, int _Ncnt) { S = _S; T = _T; Ncnt = _Ncnt; }
4     int maxflow() {
5         static int Q[MAXNODE]; int x, y, augc, flow = 0, head = 0, tail = 0; edge e;
6         Rep(i, 0, Ncnt) cur[i] = fir[i]; Rep(i, 0, Ncnt) h[i] = INF; Rep(i, 0, Ncnt) vh[i] =
            0;
7         for (Q[++tail] = T, h[T] = 0; head < tail; ) {
8             x = Q[++head]; ++vh[ h[x] ];
9             for (e = fir[x]; e; e = e->next) if (e->op->c)
10                 if (h[y = e->to] >= INF) h[y] = h[x] + 1, Q[++tail] = y;
11         } for (x = S; h[S] < Ncnt; ) {
12             for (e = cur[x]; e; e = e->next) if (e->c)
13                 if (h[y = e->to] + 1 == h[x]) { cur[x] = pe[y] = e; x = y; break; }
14             if (!e) {
15                 if (--vh[ h[x] ] == 0) break; h[x] = Ncnt; cur[x] = NULL;
16                 for (e = fir[x]; e; e = e->next) if (e->c)
17                     if ( cMin( h[x], h[e->to] + 1 ) ) cur[x] = e;
18                 ++vh[ h[x] ];
19                 if (x != S) x = pe[x]->op->to;
20             } else if (x == T) { augc = INF;
21                 for (x = T; x != S; x = pe[x]->op->to) cMin(augc, pe[x]->c);
22                 for (x = T; x != S; x = pe[x]->op->to) {
23                     pe[x]->c -= augc; pe[x]->op->c += augc;
24                 } flow += augc;
25             }
26         } return flow;
27     }
28 }

```

4.3 最高标号预流推进

```

1 namespace Network {
2     int S, T, Ncnt, hsize, heap[MAXN], h[MAXN], inq[MAXN], Q[MAXN], vh[MAXN * 2 + 1];
3     LL E[MAXN]; edge cur[MAXN];
4     inline void pushFlow(int x, int y, edge e) {
5         int d = (int)min(E[x], (LL)e->c);
6         E[x] -= d; e->c -= d; E[y] += d; e->op->c += d;

```

```

7     } inline bool heapCmp(int x, int y) { return h[x] < h[y]; }
8     inline void hpush(int x) {
9         inq[x] = true; heap[++hsize] = x; push_heap(heap + 1, heap + hsize + 1, heapCmp);
10    } inline void hpop(int x) {
11        inq[x] = false; pop_heap(heap + 1, heap + hsize + 1, heapCmp); --hsize;
12    } LL maxFlow() {
13        int head = 0, tail = 0, x, y, h0;
14        memset(h, 63, sizeof(int) * (Ncnt + 1));
15        memset(vh, 0, sizeof(int) * (2 * Ncnt + 2));
16        memset(E, 0, sizeof(LL) * (Ncnt + 1));
17        memset(inq, 0, sizeof(int) * (Ncnt + 1));
18        memcpy(cur, fir, sizeof(edge) * (Ncnt + 1));
19        for (Q[++tail] = T, h[T] = 0; head < tail; )
20            for (edge e(fir[x = Q[++head]]); e; e = e->next) if (e->op->c)
21                if (h[y = e->to] >= INF) h[y] = h[x] + 1, Q[++tail] = y;
22        if (h[S] >= Ncnt) return 0;
23        h[S] = Ncnt; E[S] = LL_INF;
24        for (int i = 1; i <= Ncnt; ++i) if (h[i] <= Ncnt) ++vh[ h[i] ];
25        hsize = 0;
26        for (edge e(fir[S]); e; e = e->next) if (e->c && h[y = e->to] < Ncnt) {
27            pushFlow(S, y, e); if (!inq[y] && y != S && y != T) hpush(y);
28        } while (hsize) {
29            bool good = false;
30            for (edge &e(cur[x = heap[1]]); e; e = e->next) if (e->c)
31                if (h[x] == h[y = e->to] + 1) {
32                    good = true; pushFlow(x, y, e); if (E[x] == 0) hpop(x);
33                    if (inq[y] == false && y != S && y != T) hpush(y);
34                    break;
35                }
36            if (!good) { // relabel
37                hpop(x); --vh[ h0 = h[x] ];
38                int &minH = h[x] = INF; cur[x] = NULL;
39                for (edge e(fir[x]); e; e = e->next) if (e->c)
40                    if ( cMin(minH, h[e->to] + 1) ) cur[x] = fir[x];
41                hpush(x); ++vh[ h[x] ];
42                if (vh[h0] == 0 && h0 < Ncnt) {
43                    hsize = 0;
44                    for (int i = 1; i <= Ncnt; ++i) {
45                        if (h[i] > h0 && h[i] < Ncnt) --vh[ h[i] ], ++vh[ h[i] = Ncnt + 1 ];
46                        if (i != S && i != T && E[i]) heap[++hsize] = i;
47                    } make_heap(heap + 1, heap + hsize + 1, heapCmp);
48                }
49            }
50        } return E[T];
51    }
52 }

```

4.4 KM

```

1 int N, Tcnt, w[MAXN][MAXN], slack[MAXN];
2 int lx[MAXN], linkx[MAXN], visy[MAXN], ly[MAXN], linky[MAXN], visx[MAXN]; // 初值全为 0
3 bool DFS(int x) { visx[x] = Tcnt;
4     Rep(y, 1, N) if(visy[y] != Tcnt) { int t = lx[x] + ly[y] - w[x][y];
5         if (t == 0) { visy[y] = Tcnt;
6             if (!linky[y] || DFS(linky[y])) { linkx[x] = y; linky[y] = x; return true; }
7         } else cMin(slack[y], t);
8     } return false;
9 } void KM() {
10     Tcnt = 0; Rep(x, 1, N) Rep(y, 1, N) cMax(lx[x], w[x][y]);
11     Rep(S, 1, N) { Rep(i, 1, N) slack[i] = INF;
12         for (++Tcnt; !DFS(S); ++Tcnt) { int d = INF;
13             Rep(y, 1, N) if(visy[y] != Tcnt) cMin(d, slack[y]);
14             Rep(x, 1, N) if(visx[x] == Tcnt) lx[x] -= d;
15             Rep(y, 1, N) if(visy[y] == Tcnt) ly[y] += d; else slack[y] -= d;
16         }
17     }
18 }

```

4.5 2-SAT 与 Kosaraju

注意 Kosaraju 需要建反图

```

1 namespace SCC {
2     int code[MAXN * 2], seq[MAXN * 2], sCnt;
3     void DFS_1(int x) { code[x] = 1;
4         for (edge e(fir[x]); e; e = e->next) if (code[e->to] == -1) DFS_1(e->to);
5         seq[++sCnt] = x;
6     } void DFS_2(int x) { code[x] = sCnt;
7         for (edge e(fir2[x]); e; e = e->next) if (code[e->to] == -1) DFS_2(e->to); }
8     void SCC(int N) {
9         sCnt = 0; for (int i = 1; i <= N; ++i) code[i] = -1;
10        for (int i = 1; i <= N; ++i) if (code[i] == -1) DFS_1(i);
11        sCnt = 0; for (int i = 1; i <= N; ++i) code[i] = -1;
12        for (int i = N; i >= 1; --i) if (code[seq[i]] == -1) {
13            ++sCnt; DFS_2(seq[i]); }
14    }
15 } // true - 2i - 1
16 // false - 2i
17 bool TwoSat() { SCC::SCC(N + N);
18     // if code[2i - 1] = code[2i]: no solution
19     // if code[2i - 1] > code[2i]: i selected. else i not selected
20 }

```

4.6 全局最小割 Stoer-Wagner

```

1 int minCut(int N, int G[MAXN][MAXN]) { // 0-based
2     static int weight[MAXN], used[MAXN]; int ans = INT_MAX;
3     while (N > 1) {
4         for (int i = 0; i < N; ++i) used[i] = false; used[0] = true;
5         for (int i = 0; i < N; ++i) weight[i] = G[i][0];
6         int S = -1, T = 0;
7         for (int _r = 2; _r <= N; ++_r) { // N - 1 selections
8             int x = -1;
9             for (int i = 0; i < N; ++i) if (!used[i])
10                 if (x == -1 || weight[i] > weight[x]) x = i;
11             for (int i = 0; i < N; ++i) weight[i] += G[x][i];
12             S = T; T = x; used[x] = true;
13         } ans = min(ans, weight[T]);
14         for (int i = 0; i < N; ++i) G[i][S] += G[i][T], G[S][i] += G[i][T];
15         G[S][S] = 0; --N;
16         for (int i = 0; i <= N; ++i) swap(G[i][T], G[i][N]);
17         for (int i = 0; i < N; ++i) swap(G[T][i], G[N][i]);
18     } return ans;
19 }

```

4.7 Hopcroft-Karp

```

1 int N, M, level[MAXN], matchX[MAXN], matchY[MAXN];
2 bool used[MAXN];
3 bool DFS(int x) {
4     used[x] = true; for (edge e(fir[x]); e; e = e->next) {
5         int y = e->to, z = matchY[y];
6         if (z == -1 || (!used[z] && level[x] < level[z] && DFS(z))) {
7             matchX[x] = y; matchY[y] = x; return true;
8         }
9     } return false;
10 }
11 int maxMatch() {
12     for (int i = 0; i < N; ++i) used[i] = false;
13     for (int i = 0; i < N; ++i) matchX[i] = -1;
14     for (int i = 0; i < M; ++i) matchY[i] = -1;
15     for (int i = 0; i < N; ++i) level[i] = -1;
16     int match = 0, d;
17     for ( ; ; match += d) {
18         static int Q[MAXN * 2 + 1];
19         int head = 0, tail = d = 0;
20         for (int x = 0; x < N; ++x) level[x] = -1;
21         for (int x = 0; x < N; ++x) if (matchX[x] == -1)
22             level[x] = 0, Q[++tail] = x;
23         while (head < tail)
24             for (edge e(fir[x = Q[++head]]); e; e = e->next) {
25                 int y = e->to, z = matchY[y];

```

```

26         if (z != -1 && level[z] < 0) level[z] = level[x] + 1, Q[++tail] = z;
27     }
28     for (int x = 0; x < N; ++x) used[x] = false;
29     for (int x = 0; x < N; ++x) if (matchX[x] == -1) if (DFS(x)) ++d;
30     if (d == 0) break;
31 } return match;
32 }

```

4.8 欧拉路

```

1 vector<int> eulerianWalk(int N, int S) {
2     static int res[MAXN], stack[MAXN]; static edge cur[MAXN];
3     int rcnt = 0, top = 0, x; for (int i = 1; i <= N; ++i) cur[i] = fir[i];
4     for (stack[top++] = S; top; ) {
5         for (x = stack[--top]; ; ) {
6             edge &e = cur[x]; if (e == NULL) break;
7             stack[top++] = x; x = e->to; e = e->next;
8             // 对于无向图需要删掉反向边
9         } res[rcnt++] = x;
10    } reverse(res, res + rcnt); return vector<int>(res, res + rcnt);
11 }

```

4.9 稳定婚姻

```

1 namespace StableMatching {
2     int pairM[MAXN], pairW[MAXN], p[MAXN];
3     // init: pairM[0...n - 1] = pairW[0...n - 1] = -1, p[0...n - 1] = 0
4     void stableMatching(int n, int orderM[MAXN][MAXN], int preferW[MAXN][MAXN]) {
5         for (int i = 0; i < n; i++) while (pairM[i] < 0) {
6             int w = orderM[i][p[i]++], m = pairW[w];
7             if (m == -1) pairM[i] = w, pairW[w] = i;
8             else if (preferW[w][i] < preferW[w][m])
9                 pairM[m] = -1, pairM[i] = w, pairW[w] = i, i = m;
10        }
11    }
12 }

```

4.10 最大团搜索

```

1 namespace MaxClique { // 1-based
2     int g[MAXN][MAXN], len[MAXN], list[MAXN][MAXN], mc[MAXN], ans, found;
3     void DFS(int size) {
4         if (len[size] == 0) { if (size > ans) ans = size, found = true; return; }
5         for (int k = 0; k < len[size] && !found; ++k) {
6             if (size + len[size] - k <= ans) break;

```

```

7             int i = list[size][k]; if (size + mc[i] <= ans) break;
8             for (int j = k + 1, len[size + 1] = 0; j < len[size]; ++j) if (g[i][list[size][j]])
9                 list[size + 1][len[size + 1]++] = list[size][j];
10            DFS(size + 1);
11        }
12    }
13    int work(int n) {
14        mc[n] = ans = 1; for (int i = n - 1; i; --i) { found = false; len[i] = 0;
15            for (int j = i + 1; j <= n; ++j) if (g[i][j]) list[i][len[i]++] = j;
16            DFS(i); mc[i] = ans;
17        } return ans;
18    }
19 }

```

4.11 极大团计数

```

1 namespace MaxCliqueCounting {
2     int n, ans;
3     int ne[MAXN], ce[MAXN];
4     int g[MAXN][MAXN], list[MAXN][MAXN];
5     void dfs(int size) {
6         int i, j, k, t, cnt, best = 0;
7         bool bb;
8         if (ne[size] == ce[size]) {
9             if (ce[size] == 0)
10                 ++ans;
11             return;
12        }
13        for (t = 0, i = 1; i <= ne[size]; ++i) {
14            for (cnt = 0, j = ne[size] + 1; j <= ce[size]; ++j)
15                if (!g[list[size][i]][list[size][j]])
16                    ++cnt;
17            if (t == 0 || cnt < best)
18                t = i, best = cnt;
19        }
20        if (t && best <= 0)
21            return;
22        for (k = ne[size] + 1; k <= ce[size]; ++k) {
23            if (t > 0) {
24                for (i = k; i <= ce[size]; ++i)
25                    if (!g[list[size][t]][list[size][i]])
26                        break;
27                swap(list[size][k], list[size][i]);
28            }
29            i = list[size][k];
30            ne[size + 1] = ce[size + 1] = 0;
31            for (j = 1; j < k; ++j)
32                if (g[i][list[size][j]])

```

```

20 }
21 void deleteMin(tree &r) { pushDown(r); r = merge(r->ch[0], r->ch[1]); }
22 int findSet(int x) { return setFa[x] == x ? x : setFa[x] = findSet(setFa[x]); }
23 void clear(int V, int E) {
24     null = &nil; null->ch[0] = null->ch[1] = null; null->dep = -1;
25     n = V; m = E; etop = ebase; Foru(i, 0, V) fir[i] = NULL; Foru(i, 0, V) inEdge[i] =
        null;
26 }
27 int solve(int root) { int res = 0, head, tail;
28     for (int i = 0; i < n; ++i) setFa[i] = i;
29     for ( ; ) { memset(deg, 0, sizeof(int) * n); chs[root] = inEdge[root];
30         for (int i = 0; i < n; ++i) if (i != root && setFa[i] == i) {
31             while (findSet(inEdge[i]->from) == findSet(i)) deleteMin(inEdge[i]);
32             ++deg[ findSet((chs[i] = inEdge[i])->from) ];
33         }
34         for (int i = head = tail = 0; i < n; ++i)
35             if (i != root && setFa[i] == i && deg[i] == 0) que[tail++] = i;
36         while (head < tail) {
37             int x = findSet(chs[que[head++]]->from);
38             if (--deg[x] == 0) que[tail++] = x;
39         } bool found = false;
40         for (int i = 0; i < n; ++i) if (i != root && setFa[i] == i && deg[i] > 0) {
41             int j = i; tree temp = null; found = true;
42             do {setFa[j] = findSet(chs[j]->from)} = i;
43             deleteMin(inEdge[j]); res += chs[j]->key;
44             inEdge[j]->key -= chs[j]->key; inEdge[j]->delta -= chs[j]->key;
45             temp = merge(temp, inEdge[j]);
46             } while (j != i); inEdge[i] = temp;
47         } if (!found) break;
48     } for (int i = 0; i < n; ++ i) if (i != root && setFa[i] == i) res += chs[i]->key;
49     return res;
50 }
51 }

```

```

52 namespace Chuliu { // O(V ^ 3) !!! 1-based !!!
53     int n, used[maxn], pass[maxn], eg[maxn], more, que[maxn], g[maxn][maxn];
54     void combine(int id, int &sum) { int tot = 0, from, i, j, k;
55         for ( ; id != 0 && !pass[id]; id = eg[id]) que[tot++] = id, pass[id] = 1;
56         for (from = 0; from < tot && que[from] != id; from++);
57         if (from == tot) return; more = 1;
58         for (i = from; i < tot; i++) {
59             sum += g[eg[que[i]]][que[i]]; if (i == from) continue;
60             for (j = used[que[i]] = 1; j <= n; j++) if (!used[j])
61                 if (g[que[i]][j] < g[id][j]) g[id][j] = g[que[i]][j];
62         }
63         for (i = 1; i <= n; i++) if (!used[i] && i != id)
64             for (j = from; j < tot; j++) {
65                 k = que[j]; if (g[i][id] > g[i][k] - g[eg[k]][k])
66                     g[i][id] = g[i][k] - g[eg[k]][k];
67             }

```

```

68 }
69 void clear(int V) { n = V; Rep(i, 1, V) Rep(j, 1, V) g[i][j] = inf; }
70 int solve(int root) {
71     int i, j, k, sum = 0; memset(used, 0, sizeof(int) * (n + 1));
72     for (more = 1; more; ) {
73         more = 0; memset(eg, 0, sizeof(int) * (n + 1));
74         for (i = 1; i <= n; i++) if (!used[i] && i != root) {
75             for (j = 1, k = 0; j <= n; j++) if (!used[j] && i != j)
76                 if (k == 0 || g[j][i] < g[k][i]) k = j;
77             eg[i] = k;
78         } memset(pass, 0, sizeof(int) * (n + 1));
79         for (i = 1; i <= n; i++) if (!used[i] && !pass[i] && i != root)
80             combine(i, sum);
81     } for (i = 1; i <= n; i++) if (!used[i] && i != root) sum += g[eg[i]][i];
82     return sum;
83 }
84 }

```

4.13 离线动态最小生成树

$O(Q \log^2 Q)$. $(qx[i], qy[i])$ 表示将编号为 $qx[i]$ 的边的权值改为 $qy[i]$, 删除一条边相当于将其权值改为 ∞ , 加入一条边相当于将其权值从 ∞ 变成某个值.

```

1  const int maxn = 100000 + 5;
2  const int maxm = 1000000 + 5;
3  const int maxq = 1000000 + 5;
4  const int qsize = maxm + 3 * maxq;
5  int n, m, Q, x[qsize], y[qsize], z[qsize], qx[maxq], qy[maxq], a[maxn], *tz;
6  int kx[maxn], ky[maxn], kt, vd[maxn], id[maxm], app[maxm];
7  bool extra[maxm];
8  void init() {
9      scanf("%d%d", &n, &m); for (int i = 0; i < m; i++) scanf("%d%d%d", x + i, y + i, z + i)
10         ;
11     scanf("%d", &Q); for (int i = 0; i < Q; i++) { scanf("%d%d", qx + i, qy + i); qx[i]--;
12     }
13 int find(int x) {
14     int root = x, next; while (a[root]) root = a[root];
15     while ((next = a[x]) != 0) a[x] = root, x = next; return root;
16 }
17 inline bool cmp(const int &a, const int &b) { return tz[a] < tz[b]; }
18 void solve(int *qx, int *qy, int Q, int n, int *x, int *y, int *z, int m, long long ans)
19 {
20     int ri, rj;
21     if (Q == 1) {
22         for (int i = 1; i <= n; i++) a[i] = 0; z[qx[0]] = qy[0];
23         for (int i = 0; i < m; i++) id[i] = i;
24         tz = z; sort(id, id + m, cmp);

```

```

23         for (int i = 0; i < m; i++) {
24             ri = find(x[id[i]]); rj = find(y[id[i]]);
25             if (ri != rj) ans += z[id[i]], a[ri] = rj;
26         } printf("%I64d\n", ans);
27         return;
28     } int tm = kt = 0, n2 = 0, m2 = 0;
29     for (int i = 1; i <= n; i++) a[i] = 0;
30     for (int i = 0; i < Q; i++) {
31         ri = find(x[qx[i]]); rj = find(y[qx[i]]); if (ri != rj) a[ri] = rj;
32     }
33     for (int i = 0; i < m; i++) extra[i] = true;
34     for (int i = 0; i < Q; i++) extra[qx[i]] = false;
35     for (int i = 0; i < m; i++) if (extra[i]) id[tm++] = i;
36     tz = z; sort(id, id + tm, cmp);
37     for (int i = 0; i < tm; i++) {
38         ri = find(x[id[i]]); rj = find(y[id[i]]);
39         if (ri != rj)
40             a[ri] = rj, ans += z[id[i]], kx[kt] = x[id[i]], ky[kt] = y[id[i]], kt++;
41     }
42     for (int i = 1; i <= n; i++) a[i] = 0;
43     for (int i = 0; i < kt; i++) a[find(kx[i])] = find(ky[i]);
44     for (int i = 1; i <= n; i++) if (a[i] == 0) vd[i] = ++n2;
45     for (int i = 1; i <= n; i++) if (a[i] != 0) vd[i] = vd[find(i)];
46     int *Nx = x + m, *Ny = y + m, *Nz = z + m;
47     for (int i = 0; i < m; i++) app[i] = -1;
48     for (int i = 0; i < Q; i++)
49         if (app[qx[i]] == -1)
50             Nx[m2] = vd[x[qx[i]]], Ny[m2] = vd[y[qx[i]]], Nz[m2] = z[qx[i]], app[qx[i]] = m2,
51             m2++;
52     for (int i = 0; i < Q; i++) {
53         z[qx[i]] = qy[i];
54         qx[i] = app[qx[i]];
55     }
56     for (int i = 1; i <= n2; i++) a[i] = 0;
57     for (int i = 0; i < tm; i++) {
58         ri = find(vd[x[id[i]]]); rj = find(vd[y[id[i]]]);
59         if (ri != rj)
60             a[ri] = rj, Nx[m2] = vd[x[id[i]]], Ny[m2] = vd[y[id[i]]], Nz[m2] = z[id[i]], m2++;
61     }
62     int mid = Q / 2;
63     solve(qx, qy, mid, n2, Nx, Ny, Nz, m2, ans);
64     solve(qx + mid, qy + mid, Q - mid, n2, Nx, Ny, Nz, m2, ans);
65 }
66 void work() { if (Q) solve(qx, qy, Q, n, x, y, z, m, 0); }
67 int main() { init(); work(); return 0; }

```


4.14 弦图

- 任何一个弦图都至少有一个单纯点, 不是完全图的弦图至少有两个不相邻的单纯点.
- 设第 i 个点在弦图的完美消除序列第 $p(i)$ 个. 令 $N(v) = \{w | w \text{ 与 } v \text{ 相邻且 } p(w) > p(v)\}$ 弦图的极大团一定是 $v \cup N(v)$ 的形式.
- 弦图最多有 n 个极大团.
- 设 $next(v)$ 表示 $N(v)$ 中最前的点. 令 w^* 表示所有满足 $A \in B$ 的 w 中最后的一个点. 判断 $v \cup N(v)$ 是否为极大团, 只需判断是否存在一个 w , 满足 $Next(w) = v$ 且 $|N(v)| + 1 \leq |N(w)|$ 即可.
- 最小染色: 完美消除序列从后往前依次给每个点染色, 给每个点染上可以染的最小的颜色. (团数 = 色数)
- 最大独立集: 完美消除序列从前往后能选就选.
- 最小团覆盖: 设最大独立集为 $\{p_1, p_2, \dots, p_t\}$, 则 $\{p_1 \cup N(p_1), \dots, p_t \cup N(p_t)\}$ 为最小团覆盖. (最大独立数 = 最小团覆盖数)

```

1 class Chordal { // 1-Based, G is the Graph, must be sorted before call Check_Chordal
2 public: // Construct will sort it automatically
3     int v[Maxn], id[Maxn]; bool inseq[Maxn]; priority_queue<pair<int, int> > pq;
4     vector<int> Construct_Perfect_Elimination_Sequence(vector<int> *G, int n) { // O(m +
        n log n)
5         vector<int> seq(n + 1, 0);
6         for (int i = 0; i <= n; ++i) inseq[i] = false, sort(G[i].begin(), G[i].end()), v[i] =
            0;
7         int cur = n; pair<int, int> Mx; while(!pq.empty()) pq.pop(); pq.push(make_pair(0, 1))
            ;
8         for (int i = n; i >= 1; --i) {
9             while (!pq.empty() && (Mx = pq.top(), inseq[Mx.second] || Mx.first != v[Mx.second])
                ) pq.pop();
10            id[Mx.second] = cur;
11            int x = seq[cur--] = Mx.second, sz = (int)G[Mx.second].size(); inseq[x] = true;
12            for (int j = 0; j < sz; ++j) {
13                int y = G[x][j]; if(!inseq[y]) pq.push(make_pair(++v[y], y));
14            }
15        } return seq;
16    }
17    bool Check_Chordal(vector<int> *G, vector<int> &seq, int n) { // O(n + m log n), plz gen
        seq first
18        bool isChordal = true;
19        for (int i = n - 1; i >= 1 && isChordal; --i) {
20            int x = seq[i], sz, y = -1;
21            if ((sz = (int)G[x].size()) == 0) continue;
22            for(int j = 0; j < sz; ++j) {
23                if (id[G[x][j]] < i) continue;
24                if (y == -1 || id[y] > id[G[x][j]]) y = G[x][j];
25            } if (y == -1) continue;

```

```

26        for (int j = 0; j < sz; ++j) {
27            int y1 = G[x][j]; if (id[y1] < i) continue;
28            if (y1 == y || binary_search(G[y].begin(), G[y].end(), y1)) continue;
29            isChordal = false; break;
30        }
31    } return isChordal;
32    }
33 };

```

4.15 K 短路 (允许重复)

```

1 #define for_each(it, v) for (vector<Edge*>::iterator it = (v).begin(); it != (v).end();
    ++it)
2 const int MAX_N = 10000, MAX_M = 50000, MAX_K = 10000, INF = 1000000000;
3 struct Edge { int from, to, weight; };
4 struct HeapNode { Edge* edge; int depth; HeapNode* child[4]; }; // child[0..1] for heap G
    , child[2..3] for heap out edge
5
6 int n, m, k, s, t; Edge* edge[MAX_M];
7 int dist[MAX_N]; Edge* prev[MAX_N];
8 vector<Edge*> graph[MAX_N]; vector<Edge*> graphR[MAX_N];
9 HeapNode* nullNode; HeapNode* heapTop[MAX_N];
10
11 HeapNode* createHeap(HeapNode* curNode, HeapNode* newNode) {
12     if (curNode == nullNode) return newNode; HeapNode* rootNode = new HeapNode;
13     memcpy(rootNode, curNode, sizeof(HeapNode));
14     if (newNode->edge->weight < curNode->edge->weight) {
15         rootNode->edge = newNode->edge; rootNode->child[2] = newNode->child[2]; rootNode->
            child[3] = newNode->child[3];
16         newNode->edge = curNode->edge; newNode->child[2] = curNode->child[2]; newNode->child
            [3] = curNode->child[3];
17     } if (rootNode->child[0]->depth < rootNode->child[1]->depth) rootNode->child[0] =
        createHeap(rootNode->child[0], newNode);
18     else rootNode->child[1] = createHeap(rootNode->child[1], newNode);
19     rootNode->depth = max(rootNode->child[0]->depth, rootNode->child[1]->depth) + 1;
20     return rootNode;
21 }
22 bool heapNodeMoreThan(HeapNode* node1, HeapNode* node2) { return node1->edge->weight >
    node2->edge->weight; }
23
24 int main() {
25     scanf("%d%d%d", &n, &m, &k); scanf("%d%d", &s, &t); s--, t--;
26     while (m--) { Edge* newEdge = new Edge;
27         int i, j, w; scanf("%d%d%d", &i, &j, &w);
28         i--, j--; newEdge->from = i; newEdge->to = j; newEdge->weight = w;
29         graph[i].push_back(newEdge); graphR[j].push_back(newEdge);
30     }
31     //Dijkstra

```

```

32 queue<int> dfsOrder; memset(dist, -1, sizeof(dist));
33 typedef pair<int, pair<int, Edge*>> DijkstraQueueItem;
34 priority_queue<DijkstraQueueItem, vector<DijkstraQueueItem>, greater<DijkstraQueueItem>
    > dq;
35 dq.push(make_pair(0, make_pair(t, (Edge*) NULL)));
36 while (!dq.empty()) {
37     int d = dq.top().first; int i = dq.top().second.first;
38     Edge* edge = dq.top().second.second; dq.pop();
39     if (dist[i] != -1) continue;
40     dist[i] = d; prev[i] = edge; dfsOrder.push(i);
41     for_each(it, graphR[i]) dq.push(make_pair(d + (*it)->weight, make_pair((*it)->from, *
        it)));
42 }
43 //Create edge heap
44 nullNode = new HeapNode; nullNode->depth = 0; nullNode->edge = new Edge; nullNode->edge
    ->weight = INF;
45 fill(nullNode->child, nullNode->child + 4, nullNode);
46 while (!dfsOrder.empty()) {
47     int i = dfsOrder.front(); dfsOrder.pop();
48     if (prev[i] == NULL) heapTop[i] = nullNode;
49     else heapTop[i] = heapTop[prev[i]->to];
50     vector<HeapNode*> heapNodeList;
51     for_each(it, graph[i]) { int j = (*it)->to; if (dist[j] == -1) continue;
52         (*it)->weight += dist[j] - dist[i]; if (prev[i] != *it) {
53             HeapNode* curNode = new HeapNode;
54             fill(curNode->child, curNode->child + 4, nullNode);
55             curNode->depth = 1; curNode->edge = *it;
56             heapNodeList.push_back(curNode);
57         }
58     } if (!heapNodeList.empty()) { //Create heap out
59         make_heap(heapNodeList.begin(), heapNodeList.end(), heapNodeMoreThan);
60         int size = heapNodeList.size();
61         for (int p = 0; p < size; p++) {
62             heapNodeList[p]->child[2] = 2 * p + 1 < size ? heapNodeList[2 * p + 1] : nullNode
                ;
63             heapNodeList[p]->child[3] = 2 * p + 2 < size ? heapNodeList[2 * p + 2] : nullNode
                ;
64         } heapTop[i] = createHeap(heapTop[i], heapNodeList.front());
65     }
66 } //Walk on DAG
67 typedef pair<long long, HeapNode*> DAGQueueItem;
68 priority_queue<DAGQueueItem, vector<DAGQueueItem>, greater<DAGQueueItem> > aq;
69 if (dist[s] == -1) printf("NO\n");
70 else { printf("%d\n", dist[s]);
71     if (heapTop[s] != nullNode) aq.push(make_pair(dist[s] + heapTop[s]->edge->weight,
        heapTop[s]));
72 } k--; while (k--) {
73     if (aq.empty()) { printf("NO\n"); continue; }
74     long long d = aq.top().first; HeapNode* curNode = aq.top().second; aq.pop();

```

```

75     printf("%I64d\n", d);
76     if (heapTop[curNode->edge->to] != nullNode)
77         aq.push(make_pair(d + heapTop[curNode->edge->to]->edge->weight, heapTop[curNode->
            edge->to]));
78     for (int i = 0; i < 4; i++) if (curNode->child[i] != nullNode)
79         aq.push(make_pair(d - curNode->edge->weight + curNode->child[i]->edge->weight,
            curNode->child[i]));
80 } return 0;
81 }

```

4.16 K 短路 (不允许重复)

```

1 int Num[10005][205], Path[10005][205], dev[10005], from[10005], value[10005], dist[205],
    Next[205], Graph[205][205];
2 int N, M, K, s, t, tot, cnt; bool forbid[205], hasNext[10005][205];
3 struct cmp {
4     bool operator()(const int &a, const int &b) {
5         int *i, *j; if (value[a] != value[b]) return value[a] > value[b];
6         for (i = Path[a], j = Path[b]; (*i) == (*j); i++, j++);
7         return (*i) > (*j);
8     }
9 };
10 void Check(int idx, int st, int *path, int &res) {
11     int i, j; for (i = 0; i < N; i++) dist[i] = 1000000000, Next[i] = t;
12     dist[t] = 0; forbid[t] = true; j = t;
13     for ( ; ) {
14         for (i = 0; i < N; i++) if (!forbid[i] && (i != st || !hasNext[idx][j]) && (dist[j] +
            Graph[i][j] < dist[i] || (dist[j] + Graph[i][j] == dist[i] && j < Next[i])))
15             Next[i] = j, dist[i] = dist[j] + Graph[i][j];
16         j = -1; for (i = 0; i < N; i++) if (!forbid[i] && (j == -1 || dist[i] < dist[j])) j =
            i;
17         if (j == -1) break; forbid[j] = 1; if (j == st) break;
18     } res += dist[st]; for (i = st; i != t; i = Next[i], path++) (*path) = i; (*path) = i;
19 }
20 int main() {
21     int i, j, k, l;
22     while (scanf("%d%d%d%d", &N, &M, &K, &s, &t) && N) {
23         priority_queue<int, vector<int>, cmp> Q;
24         for (i = 0; i < N; i++) for (j = 0; j < N; j++) Graph[i][j] = 1000000000;
25         for (i = 0; i < M; i++) { scanf("%d%d", &j, &k, &l); Graph[j - 1][k - 1] = 1; }
26         s--; t--;
27         memset(forbid, false, sizeof(forbid)); memset(hasNext[0], false, sizeof(hasNext[0]));
28         Check(0, s, Path[0], value[0]); dev[0] = 0; from[0] = 0; Num[0][0] = 0; Q.push(0);
29         cnt = 1; tot = 1;
30         for (i = 0; i < K; i++) {
31             if (Q.empty()) break; l = Q.top(); Q.pop();
32             for (j = 0; j <= dev[l]; j++) Num[l][j] = Num[from[l]][j];
33             for (; Path[l][j] != t; j++) {

```

```

34     memset(hasNext[tot], false, sizeof(hasNext[tot])); Num[l][j] = tot++;
35 } for (j = 0; Path[l][j] != t; j++) hasNext[Num[l][j]][Path[l][j + 1]] = true;
36 for (j = dev[l]; Path[l][j] != t; j++) {
37     memset(forbid, false, sizeof(forbid)); value[cnt] = 0;
38     for (k = 0; k < j; k++) {
39         forbid[Path[l][k]] = true;
40         Path[cnt][k] = Path[l][k];
41         value[cnt] += Graph[Path[l][k]][Path[l][k + 1]];
42     } Check(Num[l][j], Path[l][j], &Path[cnt][j], value[cnt]);
43     if (value[cnt] > 2000000) continue;
44     dev[cnt] = j; from[cnt] = 1; Q.push(cnt); cnt++;
45 }
46 }
47 if (i < K || value[l] > 2000000) printf("None\n");
48 else {
49     for (i = 0; Path[l][i] != t; i++) printf("%d-", Path[l][i] + 1);
50     printf("%d\n", t + 1);
51 }
52 } return 0;
53 }

```

4.17 小知识

- 平面图: 一定存在一个度小于等于 5 的点. $E \leq 3V - 6$. 欧拉公式: $V + F - E = 1 + \text{连通块数}$

- 图连通度:

- k - 连通 (k -connected): 对于任意一对结点都至少存在结点各不相同的 k 条路
- 点连通度 ($vertex\ connectivity$): 把图变成非连通图所需删除的最少点数
- Whitney 定理: 一个图是 k - 连通的当且仅当它的点连通度至少为 k

- Lindstroem-Gessel-Viennot Lemma: 给定一个图的 n 个起点和 n 个终点, 令 A_{ij} = 第 i 个起点到第 j 个终点的路径条数, 则从起点到终点的不相交路径条数为 $\det(A)$

- 欧拉回路与树形图的联系: 对于出度等于入度的连通图 $s(G) = t_i(G) \prod_{j=1}^n (d^+(v_j) - 1)!$

- 密度子图: 给定无向图, 选取点集及其导出子图, 最大化 $W_e + P_v$ (点权可负).

$$\begin{aligned}
 - (S, u) &= U, (u, T) = U - 2P_u - D_u, (u, v) = (v, u) = W_e \\
 - \text{ans} &= \frac{Un - C[S, T]}{2}, \text{解集为 } S - \{s\}
 \end{aligned}$$

- 最大权闭合图: 选 a 则 a 的后继必须被选

$$\begin{aligned}
 - P_u > 0, (S, u) &= P_u, P_u < 0, (u, T) = -P_u \\
 - \text{ans} &= \sum_{P_u > 0} P_u - C[S, T], \text{解集为 } S - \{s\}
 \end{aligned}$$

- 判定边是否属于最小割:

- 可能属于最小割: (u, v) 不属于同一 SCC
- 一定在所有最小割中: (u, v) 不属于同一 SCC, 且 S, u 在同一 SCC, u, T 在同一 SCC

- 图同构 Hash: $F_t(i) = (F_{t-1}(i) \times A + \sum_{i \rightarrow j} F_{t-1}(j) \times B + \sum_{j \leftarrow i} F_{t-1}(j) \times C + D \times (i = a)) \pmod{P}$, 枚举点 a , 迭代 K 次后求得的 $F_k(a)$ 就是 a 点所对应的 Hash 值.

5 数学

5.1 单纯形 Cpp

$$\max \{cx | Ax \leq b, x \geq 0\}$$

```

1  const int MAXN = 11000, MAXM = 1100;
2  // here MAXN is the MAX number of conditions, MAXM is the MAX number of vars
3
4  int aveli[MAXN], avacnt;
5  double A[MAXN][MAXM];
6  double b[MAXN], c[MAXN];
7  double* simplex(int n, int m) {
8  // here n is the number of conditions, m is the number of vars
9      m++;
10     int r = n, s = m - 1;
11     static double D[MAXN + 2][MAXM + 1];
12     static int ix[MAXN + MAXM];
13     for (int i = 0; i < n + m; i++) ix[i] = i;
14     for (int i = 0; i < n; i++) {
15         for (int j = 0; j < m - 1; j++) D[i][j] = -A[i][j];
16         D[i][m - 1] = 1;
17         D[i][m] = b[i];
18         if (D[r][m] > D[i][m]) r = i;
19     }
20     for (int j = 0; j < m - 1; j++) D[n][j] = c[j];
21     D[n + 1][m - 1] = -1;
22     for (double d; ; ) {
23         if (r < n) {
24             int t = ix[s]; ix[s] = ix[r + m]; ix[r + m] = t;
25             D[r][s] = 1.0 / D[r][s];
26             for (int j = 0; j <= m; j++) if (j != s) D[r][j] *= -D[r][s];
27             avacnt = 0;
28             for (int i = 0; i <= m; ++i)
29                 if (fabs(D[r][i]) > EPS)
30                     aveli[avacnt++] = i;
31             for (int i = 0; i <= n + 1; i++) if (i != r) {
32                 if (fabs(D[i][s]) < EPS) continue;
33                 double *cur1 = D[i], *cur2 = D[r], tmp = D[i][s];
34                 //for (int j = 0; j <= m; j++) if (j != s) cur1[j] += cur2[j] * tmp;

```

```

35         for(int j = 0; j < avacnt; ++j) if(aval[i][j] != s) cur1[aval[i][j]] += cur2[aval[i][j]] * tmp;
36         D[i][s] *= D[r][s];
37     }
38 }
39 r = -1; s = -1;
40 for (int j = 0; j < m; j++) if (s < 0 || ix[s] > ix[j]) {
41     if (D[n + 1][j] > EPS || D[n + 1][j] > -EPS && D[n][j] > EPS) s = j;
42 }
43 if (s < 0) break;
44 for (int i = 0; i < n; i++) if (D[i][s] < -EPS) {
45     if (r < 0 || (d = D[r][m] / D[r][s] - D[i][m] / D[i][s]) < -EPS
46         || d < EPS && ix[r + m] > ix[i + m])
47         r = i;
48 }
49 if (r < 0) return null; // 非有界
50 }
51 if (D[n + 1][m] < -EPS) return null; // 无法执行
52 static double x[MAXM - 1];
53 for (int i = m; i < n + m; i++) if (ix[i] < m - 1) x[ix[i]] = D[i - m][m];
54 return x; // 值为  $D[n][m]$ 
55 }

```

5.2 单纯形 Java

```

1 double[] simplex(double[][] A, double[] b, double[] c) {
2     int n = A.length, m = A[0].length + 1, r = n, s = m - 1;
3     double[][] D = new double[n + 2][m + 1];
4     int[] ix = new int[n + m];
5     for (int i = 0; i < n + m; i++) ix[i] = i;
6     for (int i = 0; i < n; i++) {
7         for (int j = 0; j < m - 1; j++) D[i][j] = -A[i][j];
8         D[i][m - 1] = 1; D[i][m] = b[i]; if (D[r][m] > D[i][m]) r = i;
9     }
10    for (int j = 0; j < m - 1; j++) D[n][j] = c[j];
11    D[n + 1][m - 1] = -1;
12    for (double d; ; ) {
13        if (r < n) {
14            int t = ix[s]; ix[s] = ix[r + m]; ix[r + m] = t; D[r][s] = 1.0 / D[r][s];
15            for (int j = 0; j <= m; j++) if (j != s) D[r][j] *= -D[r][s];
16            for (int i = 0; i <= n + 1; i++) if (i != r) {
17                for (int j = 0; j <= m; j++) if (j != s) D[i][j] += D[r][j] * D[i][s];
18                D[i][s] *= D[r][s];
19            }
20        } r = -1; s = -1;
21        for (int j = 0; j < m; j++) if (s < 0 || ix[s] > ix[j]) {
22            if (D[n + 1][j] > EPS || D[n + 1][j] > -EPS && D[n][j] > EPS) s = j;
23        }

```

```

24        if (s < 0) break;
25        for (int i = 0; i < n; i++) if (D[i][s] < -EPS) {
26            if (r < 0 || (d = D[r][m] / D[r][s] - D[i][m] / D[i][s]) < -EPS
27                || d < EPS && ix[r + m] > ix[i + m])
28                r = i;
29        }
30        if (r < 0) return null; // 非有界
31    } if (D[n + 1][m] < -EPS) return null; // 无法执行
32    double[] x = new double[m - 1];
33    for (int i = m; i < n + m; i++) if (ix[i] < m - 1) x[ix[i]] = D[i - m][m];
34    return x; // 值为  $D[n][m]$ 
35 }

```

5.3 高斯消元

```

1 #define Zero(x) (fabs(x) <= EPS)
2 bool GaussElimination(double G[MAXN][MAXM], int N, int M) {
3     int rb = 1; memset(res, 0, sizeof(res));
4     Rep(i_th, 1, N) { int maxRow = 0;
5         Rep(row, rb, N) if (!Zero(G[row][i_th]))
6             if (!maxRow || fabs(G[row][i_th]) > fabs(G[maxRow][i_th]))
7                 maxRow = row;
8         if (!maxRow) continue;
9         swapRow(G[rb], G[maxRow]);
10        maxRow = rb++;
11        Rep(row, 1, N) if (row != maxRow && !Zero(G[row][i_th])) {
12            double coef = G[row][i_th] / G[maxRow][i_th];
13            Rep(col, 0, M) G[row][col] -= coef * G[maxRow][col];
14        }
15    }
16    Rep(row, 1, N) if (!Zero(G[row][0])) {
17        int i_th = 1;
18        for ( ; i_th <= M; ++i_th) if (!Zero(G[row][i_th])) break;
19        if (i_th > N) return false;
20        res[i_th] = G[row][0] / G[row][i_th];
21    }
22    return true;
23 }

```

5.4 FFT

```

1 namespace FFT {
2     #define mul(a, b) (Complex(a.x * b.x - a.y * b.y, a.x * b.y + a.y * b.x))
3     struct Complex {}; // something omitted
4     void FFT(Complex P[], int n, int oper) {
5         for (int i = 1, j = 0; i < n - 1; i++) {

```

```

6     for (int s = n; j ^= s >= 1, ~j & s; );
7     if (i < j) swap(P[i], P[j]);
8 }
9 for (int d = 0; (1 << d) < n; d++) {
10     int m = 1 << d, m2 = m * 2;
11     double p0 = PI / m * oper;
12     Complex unit_p0(cos(p0), sin(p0));
13     for (int i = 0; i < n; i += m2) {
14         Complex unit(1.0, 0.0);
15         for (int j = 0; j < m; j++) {
16             Complex &P1 = P[i + j + m], &P2 = P[i + j];
17             Complex t = mul(unit, P1);
18             P1 = Complex(P2.x - t.x, P2.y - t.y);
19             P2 = Complex(P2.x + t.x, P2.y - t.y);
20             unit = mul(unit, unit_p0);
21         }}}
22 vector<int> doFFT(const vector<int> &a, const vector<int> &b) {
23     vector<int> ret(max(0, (int) a.size() + (int) b.size() - 1), 0);
24     static Complex A[MAXB], B[MAXB], C[MAXB];
25     int len = 1; while (len < (int)ret.size()) len *= 2;
26     for (int i = 0; i < len; i++) A[i] = i < (int)a.size() ? a[i] : 0;
27     for (int i = 0; i < len; i++) B[i] = i < (int)b.size() ? b[i] : 0;
28     FFT(A, len, 1); FFT(B, len, 1);
29     for (int i = 0; i < len; i++) C[i] = mul(A[i], B[i]);
30     FFT(C, len, -1);
31     for (int i = 0; i < (int)ret.size(); i++)
32         ret[i] = (int) (C[i].x / len + 0.5);
33     return ret;
34 }
35 }

```

5.5 整数 FFT

```

1 namespace FFT {
2 // 替代方案: 23068673(= 11 * 221 + 1), 原根为 3
3 const int MOD = 786433, PRIMITIVE_ROOT = 10; // 3 * 218 + 1
4 const int MAXB = 1 << 20;
5 int getMod(int downLimit) { // 或者现场自己找一个 MOD
6     for (int c = 3; ; ++c) { int t = (c << 21) | 1;
7         if (t >= downLimit && isPrime(t)) return t;
8     }}
9 int modInv(int a) { return a <= 1 ? a : (long long) (MOD - MOD / a) * modInv(MOD % a) %
    MOD; }
10 void NTT(int P[], int n, int oper) {
11     for (int i = 1, j = 0; i < n - 1; i++) {
12         for (int s = n; j ^= s >= 1, ~j & s; );
13         if (i < j) swap(P[i], P[j]);
14     }

```

```

15     for (int d = 0; (1 << d) < n; d++) {
16         int m = 1 << d, m2 = m * 2;
17         long long unit_p0 = powMod(PRIMITIVE_ROOT, (MOD - 1) / m2);
18         if (oper < 0) unit_p0 = modInv(unit_p0);
19         for (int i = 0; i < n; i += m2) {
20             long long unit = 1;
21             for (int j = 0; j < m; j++) {
22                 int &P1 = P[i + j + m], &P2 = P[i + j];
23                 int t = unit * P1 % MOD;
24                 P1 = (P2 - t + MOD) % MOD; P2 = (P2 + t) % MOD;
25                 unit = unit * unit_p0 % MOD;
26             }}}
27 vector<int> mul(const vector<int> &a, const vector<int> &b) {
28     vector<int> ret(max(0, (int) a.size() + (int) b.size() - 1), 0);
29     static int A[MAXB], B[MAXB], C[MAXB];
30     int len = 1; while (len < (int)ret.size()) len <= 1;
31     for (int i = 0; i < len; i++) A[i] = i < (int)a.size() ? a[i] : 0;
32     for (int i = 0; i < len; i++) B[i] = i < (int)b.size() ? b[i] : 0;
33     NTT(A, len, 1); NTT(B, len, 1);
34     for (int i = 0; i < len; i++) C[i] = (long long) A[i] * B[i] % MOD;
35     NTT(C, len, -1); for (int i = 0, inv = modInv(len); i < (int)ret.size(); i++) ret[i]
        = (long long) C[i] * inv % MOD;
36     return ret;
37 }
38 }

```

5.6 扩展欧几里得

$$ax + by = g = \gcd(x, y)$$

```

1 void exgcd(LL x, LL y, LL &a0, LL &b0, LL &g) {
2     LL a1 = b0 = 0, b1 = a0 = 1, t;
3     while (y != 0) {
4         t = a0 - x / y * a1, a0 = a1, a1 = t;
5         t = b0 - x / y * b1, b0 = b1, b1 = t;
6         t = x % y, x = y, y = t;
7     } if (x < 0) a0 = -a0, b0 = -b0, x = -x;
8     g = x;
9 }

```

5.7 线性同余方程

- 中国剩余定理: 设 m_1, m_2, \dots, m_k 两两互素, 则同余方程组 $x \equiv a_i \pmod{m_i}$ for $i = 1, 2, \dots, k$ 在 $[0, M = m_1 m_2 \dots m_k]$ 内有唯一解. 记 $M_i = M/m_i$, 找出 p_i 使得 $M_i p_i \equiv 1 \pmod{m_i}$, 记 $e_i = M_i p_i$, 则 $x \equiv e_1 a_1 + e_2 a_2 + \dots + e_k a_k \pmod{M}$
- 多变元线性同余方程组: 方程的形式为 $a_1 x_1 + a_2 x_2 + \dots + a_n x_n + b \equiv 0 \pmod{m}$, 令 $d = (a_1, a_2, \dots, a_n, m)$, 有解的充要条件是 $d|b$, 解的个数为 $m^{n-1}d$

5.8 Miller-Rabin 素性测试

```

1 bool test(LL n, int base) {
2     LL m = n - 1, ret = 0; int s = 0;
3     for ( ; m % 2 == 0; ++s) m >>= 1; ret = pow_mod(base, m, n);
4     if (ret == 1 || ret == n - 1) return true;
5     for (--s; s >= 0; --s) {
6         ret = multiply_mod(ret, ret, n); if (ret == n - 1) return true;
7     } return false;
8 }
9 LL special[7] = {
10     1373653LL,      25326001LL,
11     3215031751LL,   25000000000LL,
12     2152302898747LL, 3474749660383LL, 341550071728321LL};
13 /*
14  * n < 2047          test[] = {2}
15  * n < 1,373,653     test[] = {2, 3}
16  * n < 9,080,191     test[] = {31, 73}
17  * n < 25,326,001    test[] = {2, 3, 5}
18  * n < 4,759,123,141 test[] = {2, 7, 61}
19  * n < 1,122,004,669,633 test[] = {2, 13, 23, 1662803}
20  * n < 2,152,302,898,747 test[] = {2, 3, 5, 7, 11}
21  * n < 3,474,749,660,383 test[] = {2, 3, 5, 7, 11, 13}
22  * n < 341,550,071,728,321 test[] = {2, 3, 5, 7, 11, 13, 17}
23  * n < 3,825,123,056,546,413,051 test[] = {2, 3, 5, 7, 11, 13, 17, 19, 23}
24  */
25 bool is_prime(LL n) {
26     if (n < 2) return false;
27     if (n < 4) return true;
28     if (!test(n, 2) || !test(n, 3)) return false;
29     if (n < special[0]) return true;
30     if (!test(n, 5)) return false;
31     if (n < special[1]) return true;
32     if (!test(n, 7)) return false;
33     if (n == special[2]) return false;
34     if (n < special[3]) return true;
35     if (!test(n, 11)) return false;
36     if (n < special[4]) return true;
37     if (!test(n, 13)) return false;
38     if (n < special[5]) return true;
39     if (!test(n, 17)) return false;
40     if (n < special[6]) return true;
41     return test(n, 19) && test(n, 23) && test(n, 29) && test(n, 31) && test(n, 37);
42 }

```

5.9 PollardRho

```

1 LL pollardRho(LL n, LL seed) {
2     LL x, y, head = 1, tail = 2; x = y = random() % (n - 1) + 1;
3     for ( ; ; ) {
4         x = addMod(multiplyMod(x, x, n), seed, n);
5         if (x == y) return n; LL d = gcd(myAbs(x - y), n);
6         if (1 < d && d < n) return d;
7         if (++head == tail) y = x, tail <= 1;
8     } vector<LL> divisors;
9     void factorize(LL n) { // 需要保证 n > 1
10         if (isPrime(n)) divisors.push_back(n);
11         else { LL d = n;
12             while (d >= n) d = pollardRho(n, random() % (n - 1) + 1);
13             factorize(n / d); factorize(d);
14         }

```

5.10 多项式求根

```

1 const double error = 1e-12;
2 const double infi = 1e+12;
3 int n; double a[10], x[10];
4 double f(double a[], int n, double x) {
5     double tmp = 1, sum = 0;
6     for (int i = 0; i <= n; i++) sum = sum + a[i] * tmp, tmp = tmp * x;
7     return sum;
8 }
9 double binary(double l, double r, double a[], int n) {
10     int sl = sign(f(a, n, l)), sr = sign(f(a, n, r));
11     if (sl == 0) return l; if (sr == 0) return r;
12     if (sl * sr > 0) return infi;
13     while (r - l > error) {
14         double mid = (l + r) / 2;
15         int ss = sign(f(a, n, mid));
16         if (ss == 0) return mid;
17         if (ss * sl > 0) l = mid; else r = mid;
18     } return l;
19 }
20 void solve(int n, double a[], double x[], int &nx) {
21     if (n == 1) { x[1] = -a[0] / a[1]; nx = 1; return; }
22     double da[10], dx[10]; int ndx;
23     for (int i = n; i >= 1; i--) da[i - 1] = a[i] * i;
24     solve(n - 1, da, dx, ndx); nx = 0;
25     if (ndx == 0) {
26         double tmp = binary(-infi, infi, a, n);
27         if (tmp < infi) x[++nx] = tmp; return;
28     } double tmp = binary(-infi, dx[1], a, n);
29     if (tmp < infi) x[++nx] = tmp;
30     for (int i = 1; i <= ndx - 1; i++) {
31         tmp = binary(dx[i], dx[i + 1], a, n);

```

```

32     if (tmp < infi) x[++nx] = tmp;
33 } tmp = binary(dx[ndx], infi, a, n);
34 if (tmp < infi) x[++nx] = tmp;
35 }
36 int main() {
37     scanf("%d", &n);
38     for (int i = n; i >= 0; i--) scanf("%lf", &a[i]);
39     int nx; solve(n, a, x, nx);
40     for (int i = 1; i <= nx; i++) printf("%.6f\n", x[i]);
41     return 0;
42 }

```

5.11 线性递推

$$\text{for } a_{i+n} = (\sum_{j=0}^{n-1} k_j a_{i+j}) + d, a_m = (\sum_{i=0}^{n-1} c_i a_i) + c_n d$$

```

1 vector<int> recFormula(int n, int k[], int m) {
2     vector<int> c(n + 1, 0);
3     if (m < n) c[m] = 1;
4     else {
5         static int a[MAX_K * 2 + 1];
6         vector<int> b = recFormula(n, k, m >> 1);
7         for (int i = 0; i < n + n; ++i) a[i] = 0;
8         int s = m & 1;
9         for (int i = 0; i < n; i++) {
10             for (int j = 0; j < n; j++) a[i + j + s] += b[i] * b[j];
11             c[n] += b[i];
12         } c[n] = (c[n] + 1) * b[n];
13         for (int i = n * 2 - 1; i >= n; i--) {
14             int add = a[i]; if (add == 0) continue;
15             for (int j = 0; j < n; j++) a[i - n + j] += k[j] * add;
16             c[n] += add;
17         } for (int i = 0; i < n; ++i) c[i] = a[i];
18     } return c;
19 }

```

5.12 原根

原根 g : g 是模 n 简化剩余系构成的乘法群的生成元. 模 n 有原根的充要条件是 $n = 2, 4, p^n, 2p^n$, 其中 p 是奇质数, n 是正整数

```

1 vector<int> findPrimitiveRoot(int N) {
2     if (N <= 4) return vector<int>(1, max(1, N - 1));
3     static int factor[100];
4     int phi = N, totF = 0;
5     { // check no solution and calculate phi
6         int M = N, k = 0;
7         if (~M & 1) M >>= 1, phi >>= 1;

```

```

8         if (~M & 1) return vector<int>(0);
9         for (int d = 3; d * d <= M; ++d) if (M % d == 0) {
10             if (++k > 1) return vector<int>(0);
11             for (phi -= phi / d; M % d == 0; M /= d);
12         } if (M > 1) {
13             if (++k > 1) return vector<int>(0); phi -= phi / M;
14         }
15     } { // factorize phi
16         int M = phi;
17         for (int d = 2; d * d <= M; ++d) if (M % d == 0) {
18             for (; M % d == 0; M /= d); factor[++totF] = d;
19         } if (M > 1) factor[++totF] = M;
20     } vector<int> ans;
21     for (int g = 2; g <= N; ++g) if (Gcd(g, N) == 1) {
22         bool good = true;
23         for (int i = 1; i <= totF && good; ++i)
24             if (powMod(g, phi / factor[i], N) == 1) good = false;
25         if (!good) continue;
26         for (int i = 1, gp = g; i <= phi; ++i, gp = (LL)gp * g % N)
27             if (Gcd(i, phi) == 1) ans.push_back(gp);
28         break;
29     } sort(ans.begin(), ans.end());
30     return ans;
31 }

```

5.13 离散对数

$A^x \equiv B \pmod{C}$, 对非质数 C 也适用.

```

1 int modLog(int A, int B, int C) {
2     static pii baby[MAX_SQRT_C + 11];
3     int d = 0; LL k = 1, D = 1; B %= C;
4     for (int i = 0; i < 100; ++i, k = k * A % C) // [0, log C]
5         if (k == B) return i;
6     for (int g; ; ++d) {
7         g = gcd(A, C); if (g == 1) break;
8         if (B % g != 0) return -1;
9         B /= g; C /= g; D = (A / g * D) % C;
10    } int m = (int) ceil(sqrt((double) C)); k = 1;
11    for (int i = 0; i <= m; ++i, k = k * A % C) baby[i] = pii(k, i);
12    sort(baby, baby + m + 1); // [0, m]
13    int n = unique(baby, baby + m + 1, equalFirst) - baby, am = powMod(A, m, C);
14    for (int i = 0; i <= m; ++i) {
15        LL e, x, y; exgcd(D, C, x, y, e); e = x * B % C;
16        if (e < 0) e += C;
17        if (e >= 0) {
18            int k = lower_bound(baby, baby + n, pii(e, -1)) - baby;
19            if (baby[k].first == e) return i * m + baby[k].second + d;

```

```
20     } D = D * am % C;
21   } return -1;
22 }
```

5.14 平方剩余

- Legendre Symbol: 对奇质数 p , $(\frac{a}{p}) = \begin{cases} 1 & \text{是平方剩余} \\ -1 & \text{是非平方剩余} = a^{\frac{p-1}{2}} \pmod p \\ 0 & a \equiv 0 \pmod p \end{cases}$
- 若 p 是奇质数, $(\frac{-1}{p}) = 1$ 当且仅当 $p \equiv 1 \pmod 4$
- 若 p 是奇质数, $(\frac{2}{p}) = 1$ 当且仅当 $p \equiv \pm 1 \pmod 8$
- 若 p, q 是奇素数且互质, $(\frac{p}{q})(\frac{q}{p}) = (-1)^{\frac{p-1}{2} \times \frac{q-1}{2}}$
- Jacobi Symbol: 对奇数 $n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}$, $(\frac{a}{n}) = (\frac{a}{p_1})^{\alpha_1} (\frac{a}{p_2})^{\alpha_2} \cdots (\frac{a}{p_k})^{\alpha_k}$
- Jacobi Symbol 为 -1 则一定不是平方剩余, 所有平方剩余的 Jacobi Symbol 都是 1, 但 1 不一定是平方剩余

$ax^2 + bx + c \equiv 0 \pmod p$, 其中 $a \not\equiv 0 \pmod p$, 且 p 是质数

```
1 inline int normalize(LL a, int P) { a %= P; return a < 0 ? a + P : a; }
2 vector<int> QuadraticResidue(LL a, LL b, LL c, int P) {
3     int h, t; LL r1, r2, delta, pb = 0;
4     a = normalize(a, P); b = normalize(b, P); c = normalize(c, P);
5     if (P == 2) { vector<int> res;
6         if (c % P == 0) res.push_back(0);
7         if ((a + b + c) % P == 0) res.push_back(1);
8         return res;
9     } delta = b * rev(a + a, P) % P;
10    a = normalize(-c * rev(a, P) + delta * delta, P);
11    if (powMod(a, P / 2, P) + 1 == P) return vector<int>(0);
12    for (t = 0, h = P / 2; h % 2 == 0; ++t, h /= 2);
13    r1 = powMod(a, h / 2, P);
14    if (t > 0) { do b = random() % (P - 2) + 2;
15        while (powMod(b, P / 2, P) + 1 != P); }
16    for (int i = 1; i <= t; ++i) {
17        LL d = r1 * r1 % P * a % P;
18        for (int j = 1; j <= t - i; ++j) d = d * d % P;
19        if (d + 1 == P) r1 = r1 * pb % P; pb = pb * pb % P;
20    } r1 = a * r1 % P; r2 = P - r1;
21    r1 = normalize(r1 - delta, P); r2 = normalize(r2 - delta, P);
22    if (r1 > r2) swap(r1, r2); vector<int> res(1, r1);
23    if (r1 != r2) res.push_back(r2);
24    return res;
25 }
```

5.15 N 次剩余

- 若 p 为奇质数, a 为 p 的 n 次剩余的充要条件是 $a^{\frac{p-1}{n}} \equiv 1 \pmod p$.

$x^N \equiv a \pmod p$, 其中 p 是质数

```
1 vector<int> solve(int p, int N, int a) {
2     if ((a %= p) == 0) return vector<int>(1, 0);
3     int g = findPrimitiveRoot(p), m = modLog(g, a, p); // g ^ m = a (mod p)
4     if (m == -1) return vector<int>(0);
5     LL B = p - 1, x, y, d; exgcd(N, B, x, y, d);
6     if (m % d != 0) return vector<int>(0);
7     vector<int> ret; x = (x * (m / d) % B + B) % B; // g ^ B mod p = g ^ (p - 1) mod p = 1
8     for (int i = 0, delta = B / d; i < d; ++i) {
9         x = (x + delta) % B; ret.push_back((int)powMod(g, x, p));
10    } sort(ret.begin(), ret.end());
11    ret.resize(unique(ret.begin(), ret.end()) - ret.begin());
12    return ret;
13 }
```

5.16 Pell 方程

$$\begin{pmatrix} x_k \\ y_k \end{pmatrix} = \begin{pmatrix} x_1 & dy_1 \\ y_1 & x_1 \end{pmatrix}^{k-1} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$$

```
1 pair<ULL, ULL> Pell(int n) {
2     static ULL p[50] = {0, 1}, q[50] = {1, 0}, g[50] = {0, 0}, h[50] = {0, 1}, a[50];
3     ULL t = a[2] = Sqrt(n);
4     for (int i = 2; ; ++i) {
5         g[i] = -g[i - 1] + a[i] * h[i - 1];
6         h[i] = (n - g[i] * g[i]) / h[i - 1];
7         a[i + 1] = (g[i] + t) / h[i];
8         p[i] = a[i] * p[i - 1] + p[i - 2];
9         q[i] = a[i] * q[i - 1] + q[i - 2];
10        if (p[i] * p[i] - n * q[i] * q[i] == 1) return make_pair(p[i], q[i]);
11    } return make_pair(-1, -1);
12 }
```

5.17 Romberg 积分

```
1 template <class T> double Romberg(const T&f, double a, double b, double eps = 1e-8) {
2     vector<double> t; double h = b - a, last, now; int k = 1, i = 1;
3     t.push_back(h * (f(a) + f(b)) / 2); // 梯形
4     do {
5         last = t.back(); now = 0; double x = a + h / 2;
6         for (int j = 0; j < k; ++j, x += h) now += f(x);
7         now = (t[0] + h * now) / 2; double k1 = 4.0 / 3.0, k2 = 1.0 / 3.0;
8         for (int j = 0; j < i; ++j, k1 = k2 + 1) {
```



```
9      double tmp = k1 * now - k2 * t[j];
10      t[j] = now; now = tmp; k2 /= 4 * k1 - k2; // 防止溢出
11      } t.push_back(now); k *= 2; h /= 2; ++i;
12  } while (fabs(last - now) > eps);
13  return t.back();
14 }
```

5.18 公式

5.18.1 级数与三角

- $\sum_{k=1}^n k^3 = (\frac{n(n+1)}{2})^2$
- $\sum_{k=1}^n k^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$
- $\sum_{k=1}^n k^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$
- $\sum_{k=1}^n k(k+1) = \frac{n(n+1)(n+2)}{3}$
- $\sum_{k=1}^n k(k+1)(k+2) = \frac{n(n+1)(n+2)(n+3)}{4}$
- $\sum_{k=1}^n k(k+1)(k+2)(k+3) = \frac{n(n+1)(n+2)(n+3)(n+4)}{5}$
- 错排: $D_n = n!(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + \frac{(-1)^n}{n!}) = (n-1)(D_{n-2} - D_{n-1})$
- $\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$
- $\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$
- $\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$
- $\tan \alpha \pm \tan \beta = \frac{\sin(\alpha \pm \beta)}{\cos \alpha \cos \beta}$
- $\sin \alpha + \sin \beta = 2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$
- $\sin \alpha - \sin \beta = 2 \cos \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$
- $\cos \alpha + \cos \alpha = 2 \cos \frac{\alpha + \beta}{2} \cos \frac{\alpha - \beta}{2}$
- $\cos \alpha - \cos \beta = -2 \sin \frac{\alpha + \beta}{2} \sin \frac{\alpha - \beta}{2}$
- $\cos n\alpha = \binom{n}{0} \cos^n \alpha - \binom{n}{2} \cos^{n-2} \alpha \sin^2 \alpha + \binom{n}{4} \cos^{n-4} \alpha \sin^4 \alpha \dots$
- $\sin n\alpha = \binom{n}{1} \cos^{n-1} \alpha \sin \alpha - \binom{n}{3} \cos^{n-3} \alpha \sin^3 \alpha + \binom{n}{5} \cos^{n-5} \alpha \sin^5 \alpha \dots$
- $\sum_{n=1}^N \cos nx = \frac{\sin(N+\frac{1}{2})x - \sin \frac{x}{2}}{2 \sin \frac{x}{2}}$
- $\sum_{n=1}^N \sin nx = \frac{-\cos(N+\frac{1}{2})x + \cos \frac{x}{2}}{2 \sin \frac{x}{2}}$

- $\int_0^{\frac{\pi}{2}} \sin^n x dx = \begin{cases} \frac{(n-1)!!}{n!!} \times \frac{\pi}{2} & n \text{ 是偶数} \\ \frac{(n-1)!!}{n!!} & n \text{ 是奇数} \end{cases}$
- $\int_0^{+\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$
- $\int_0^{+\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$
- 傅里叶级数: 设周期为 $2T$. 函数分段连续. 在不连续点的值为左右极限的平均数.
 - $a_n = \frac{1}{T} \int_{-T}^T f(x) \cos \frac{n\pi}{T} x dx$
 - $b_n = \frac{1}{T} \int_{-T}^T f(x) \sin \frac{n\pi}{T} x dx$
 - $f(x) = \frac{a_0}{2} + \sum_{n=1}^{+\infty} (a_n \cos \frac{n\pi}{T} x + b_n \sin \frac{n\pi}{T} x)$
- Beta 函数: $B(p, q) = \int_0^1 x^{p-1} (1-x)^{q-1} dx$
 - 定义域 $(0, +\infty) \times (0, +\infty)$, 在定义域上连续
 - $B(p, q) = B(q, p) = \frac{q-1}{p+q-1} B(p, q-1) = 2 \int_0^{\frac{\pi}{2}} \cos^{2p-1} \phi \sin^{2p-1} \phi d\phi = \int_0^{+\infty} \frac{t^{q-1}}{(1+t)^{p+q}} dt = \int_0^1 \frac{t^{p-1} + t^{q-1}}{(1+t)^{(p+q)}} dt$
 - $B(\frac{1}{2}, \frac{1}{2}) = \pi$
- Gamma 函数: $\Gamma = \int_0^{+\infty} x^{s-1} e^{-x} dx$
 - 定义域 $(0, +\infty)$, 在定义域上连续
 - $\Gamma(1) = 1, \Gamma(\frac{1}{2}) = \sqrt{\pi}$
 - $\Gamma(s) = (s-1)\Gamma(s-1)$
 - $B(p, q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$
 - $\Gamma(s)\Gamma(1-s) = \frac{\pi}{\sin \pi s}$ for $s > 0$
 - $\Gamma(s)\Gamma(s+\frac{1}{2}) = 2\sqrt{\pi} \frac{\Gamma(s)}{2^{2s-1}}$ for $0 < s < 1$
- 积分: 平面图形面积、曲线弧长、旋转体体积、旋转曲面面积 $y = f(x), \int_a^b f(x) dx, \int_a^b \sqrt{1+f'^2(x)} dx,$
 $\pi \int_a^b f^2(x) dx, 2\pi \int_a^b |f(x)| \sqrt{1+f'^2(x)} dx$
- $x = x(t), y = y(t), t \in [T_1, T_2], \int_{T_1}^{T_2} |y(t)x'(t)| dt, \int_{T_1}^{T_2} \sqrt{x'^2(t) + y'^2(t)} dt, \pi \int_{T_1}^{T_2} |x'(t)| y^2(t) dt,$
 $2\pi \int_{T_1}^{T_2} |y(t)| \sqrt{x'^2(t) + y'^2(t)} dt,$

$$\begin{aligned}r &= r(\theta), \theta \in [\alpha, \beta], & \frac{1}{2} \int_{\alpha}^{\beta} r^2(\theta) \mathrm{d}\theta, & \int_{\alpha}^{\beta} \sqrt{r^2(\theta) + r'^2(\theta)} \mathrm{d}\theta, & \frac{2}{3} \pi \int_{\alpha}^{\beta} r^3(\theta) \sin \theta \mathrm{d}\theta, \\ & 2\pi \int_{\alpha}^{\beta} r(\theta) \sin \theta \sqrt{r^2(\theta) + r'^2(\theta)} \mathrm{d}\theta\end{aligned}$$

5.18.2 三次方程求根公式

对一元三次方程 $x^3 + px + q = 0$, 令

$$\begin{aligned}A &= \sqrt[3]{-\frac{q}{2} + \sqrt{(\frac{q}{2})^2 + (\frac{p}{3})^3}} \\ B &= \sqrt[3]{-\frac{q}{2} - \sqrt{(\frac{q}{2})^2 + (\frac{p}{3})^3}} \\ \omega &= \frac{(-1 + \mathrm{i}\sqrt{3})}{2}\end{aligned}$$

则 $x_j = A\omega^j + B\omega^{2j}$ ($j = 0, 1, 2$).
当求解 $ax^3 + bx^2 + cx + d = 0$ 时, 令 $x = y - \frac{b}{3a}$, 再求解 y , 即转化为 $y^3 + py + q = 0$ 的形式. 其中,

$$\begin{aligned}p &= \frac{b^2 - 3ac}{3a^2} \\ q &= \frac{2b^3 - 9abc + 27a^2d}{27a^3}\end{aligned}$$

卡尔丹判别法: 令 $\Delta = (\frac{q}{2})^2 + (\frac{p}{3})^3$. 当 $\Delta > 0$ 时, 有一个实根和一对共轭虚根; 当 $\Delta = 0$ 时, 有三个实根, 其中两个相等; 当 $\Delta < 0$ 时, 有三个不相等的实根.

5.18.3 椭圆

- 椭圆 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, 其中离心率 $e = \frac{c}{a}, c = \sqrt{a^2 - b^2}$; 焦点参数 $p = \frac{b^2}{a}$
- 椭圆上 (x, y) 点处的曲率半径为 $R = a^2b^2(\frac{x^2}{a^4} + \frac{y^2}{b^4})^{\frac{3}{2}} = \frac{(r_1r_2)^{\frac{3}{2}}}{ab}$, 其中 r_1 和 r_2 分别为 (x, y) 与两焦点 F_1 和 F_2 的距离.

$$L_{AM} = a \int_0^{\arccos \frac{x}{a}} \sqrt{1 - e^2 \cos^2 t} \mathrm{d}t = a \int_{\arccos \frac{x}{a}}^{\frac{\pi}{2}} \sqrt{1 - e^2 \sin^2 t} \mathrm{d}t$$

- 椭圆的周长 $L = 4a \int_0^{\frac{\pi}{2}} \sqrt{1 - e^2 \sin^2 t} \mathrm{d}t = 4aE(e, \frac{\pi}{2})$, 其中

$$E(e, \frac{\pi}{2}) = \frac{\pi}{2} [1 - (\frac{1}{2})^2 e^2 - (\frac{1 \times 3}{2 \times 4})^2 \frac{e^4}{3} - (\frac{1 \times 3 \times 5}{2 \times 4 \times 6})^2 \frac{e^6}{5} - \dots]$$

- 设椭圆上点 $M(x, y), N(x, -y), x, y > 0, A(a, 0)$, 原点 $O(0, 0)$, 扇形 OAM 的面积 $S_{OAM} = \frac{1}{2} ab \arccos \frac{x}{a}$, 弓形 MAN 的面积 $S_{MAN} = ab \arccos \frac{x}{a} - xy$.

- 需要 5 个点才能确定一个圆锥曲线.

- 设 θ 为 (x, y) 点关于椭圆中心的极角, r 为 (x, y) 到椭圆中心的距离, 椭圆极坐标方程:

$$x = r \cos \theta, y = r \sin \theta, r^2 = \frac{b^2 a^2}{b^2 \cos^2 \theta + a^2 \sin^2 \theta}$$

5.18.4 抛物线

- 标准方程 $y^2 = 2px$, 曲率半径 $R = \frac{(p + 2x)^{\frac{3}{2}}}{\sqrt{p}}$

- 弧长: 设 $M(x, y)$ 是抛物线上一点, 则 $L_{OM} = \frac{p}{2} [\sqrt{\frac{2x}{p} (1 + \frac{2x}{p})} + \ln(\sqrt{\frac{2x}{p}} + \sqrt{1 + \frac{2x}{p}})]$

- 弓形面积: 设 M, D 是抛物线上两点, 且分居一, 四象限. 做一条平行于 MD 且与抛物线相切的直线 L . 若 M 到 L 的距离为 h . 则有 $S_{MOD} = \frac{2}{3} MD \cdot h$.

5.18.5 重心

- 半径 r , 圆心角为 θ 的扇形的重心与圆心的距离为 $\frac{4r \sin \frac{\theta}{2}}{3\theta}$

- 半径 r , 圆心角为 θ 的圆弧的重心与圆心的距离为 $\frac{4r \sin^3 \frac{\theta}{2}}{3(\theta - \sin \theta)}$

- 椭圆上半部分的重心与圆心的距离为 $\frac{4b}{3\pi}$

- 抛物线中弓形 MOD 的重心满足 $CQ = \frac{2}{5} PQ$, P 是直线 L 与抛物线的切点, Q 在 MD 上且 PQ 平行 x 轴, C 是重心

5.18.6 向量恒等式

- $\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} \times \vec{a}) = \vec{c} \cdot (\vec{a} \times \vec{b})$
- $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{c} \times \vec{b}) \times \vec{a} = \vec{b} (\vec{a} \cdot \vec{c}) - \vec{c} (\vec{a} \cdot \vec{b})$

5.18.7 常用几何公式

- 三角形的五心
 - 重心 $\vec{G} = \frac{\vec{A} + \vec{B} + \vec{C}}{3}$
 - 内心 $\vec{I} = \frac{a\vec{A} + b\vec{B} + c\vec{C}}{a + b + c}$, $R = \frac{2S}{a + b + c}$
 - 外心 $x = \frac{\vec{A} + \vec{B} - \frac{\vec{B}\vec{C} \cdot \vec{A}\vec{C}}{AB \times BC} \vec{AB}^T}{2}$, $y = \frac{\vec{A} + \vec{B} + \frac{\vec{B}\vec{C} \cdot \vec{A}\vec{C}}{AB \times BC} \vec{AB}^T}{2}$, $R = \frac{abc}{4S}$
 - 垂心 $\vec{H} = 3\vec{G} - 2\vec{O}$
 - 旁心 (三个) $\frac{-a\vec{A} + b\vec{B} + c\vec{C}}{-a + b + c}$
- 四边形: 设 D_1, D_2 为对角线, M 为对角线中点连线, A 为对角线夹角
 - $a^2 + b^2 + c^2 + d^2 = D_1^2 + D_2^2 + 4M^2$
 - $S = \frac{1}{2} D_1 D_2 \sin A$
 - $ac + bd = D_1 D_2$ (内接四边形适用)
 - Bretschneider 公式: $S = \sqrt{(p - a)(p - b)(p - c)(p - d) - abcd \cos^2(\frac{\theta}{2})}$, 其中 θ 为对角和

- 棱锥:
 - 体积 $V = \frac{1}{3}Ah$, A 为底面积, h 为高
 - (对正棱锥) 侧面积 $S = \frac{1}{2}lp$, l 为斜高, p 为底面周长

- 棱台:
 - 体积 $V = \frac{(A_1 + A_2 + \sqrt{A_1A_2}) \cdot h}{3}$, A_1, A_2 分别为上下底面面积, h 为高
 - (对正棱台) 侧面积 $S = \frac{1}{2}(p_1 + p_2) \cdot l$, p_1, p_2 为上下底面周长, l 为斜高.

5.18.8 树的计数

- 有根数计数: 令 $S_{n,j} = \sum_{1 \leq i \leq n/j} a_{n+1-ij} = S_{n-j,j} + a_{n+1-j}$
于是, $n+1$ 个结点的有根数的总数为 $a_{n+1} = \frac{\sum_{1 \leq j \leq n} j \cdot a_j \cdot S_{n,j}}{n}$
附: $a_1 = 1, a_2 = 1, a_3 = 2, a_4 = 4, a_5 = 9, a_6 = 20, a_9 = 286, a_{11} = 1842$
- 无根树计数: 当 n 是奇数时, 则有 $a_n - \sum_{1 \leq i \leq \frac{n}{2}} a_i a_{n-i}$ 种不同的无根树
当 n 是偶数时, 则有 $a_n - \sum_{1 \leq i \leq \frac{n}{2}} a_i a_{n-i} + \frac{1}{2} a_{\frac{n}{2}} (a_{\frac{n}{2}} + 1)$ 种不同的无根树
- Matrix-Tree 定理: 对任意图 G , 设 $\text{mat}[i][i] = i$ 的度数, $\text{mat}[i][j] = i$ 与 j 之间边数的相反数, 则 $\text{mat}[i][j]$ 的任意余子式的行列式就是该图的生成树个数

5.19 小知识

- 勾股数: 设正整数 n 的质因数分解为 $n = \prod p_i^{a_i}$, 则 $x^2 + y^2 = n$ 有整数解的充要条件是 n 中不存在形如 $p_i \equiv 3 \pmod{4}$ 且指数 a_i 为奇数的质因数 p_i . $(\frac{a-b}{2})^2 + ab = (\frac{a+b}{2})^2$.
- 素勾股数: 若 m 和 n 互质, 而且 m 和 n 中有一个是偶数, 则 $a = m^2 - n^2, b = 2mn, c = m^2 + n^2$, 则 a, b, c 是素勾股数.
- Stirling 公式: $n! \approx \sqrt{2\pi n} (\frac{n}{e})^n$
- Pick 定理: 简单多边形, 不自交, 顶点如果全是整点. 则: 严格在多边形内部的整点数 + $\frac{1}{2}$ 在边上的整点数 $- 1 =$ 面积
- Mersenne 素数: p 是素数且 $2^p - 1$ 的数是素数. (10000 以内的 p 有: 2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127, 521, 607, 1279, 2203, 2281, 3217, 4253, 4423, 9689, 9941)
- 序列差分表: 差分表的第 0 条对角线确定原序列. 设原序列为 h_i , 第 0 条对角线为 $c_0, c_1, \dots, c_p, 0, 0, \dots$ 有这样两个公式: $h_n = \binom{n}{0}c_0 + \binom{n}{1}c_1 + \dots + \binom{n}{p}c_p, \sum_{k=0}^n h_k = \binom{n+1}{1}c_0 + \binom{n+1}{2}c_2 + \dots + \binom{n+1}{p+1}c_p$
- GCD: $\text{gcd}(2^a - 1, 2^b - 1) = 2^{\text{gcd}(a,b)} - 1$

- Fermat 分解算法: 从 $t = \sqrt{n}$ 开始, 依次检查 $t^2 - n, (t+1)^2 - n, (t+2)^2 - n, \dots$, 直到出现一个平方数 y , 由于 $t^2 - y^2 = n$, 因此分解得 $n = (t-y)(t+y)$. 显然, 当两个因数很接近时这个方法能很快找到结果, 但如果遇到一个素数, 则需要检查 $\frac{n+1}{2} - \sqrt{n}$ 个整数
- 牛顿迭代: $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$
- 球与盒子的动人故事: (n 个球, m 个盒子, S 为第二类斯特林数)

- 球同, 盒同, 无空: dp
- 球同, 盒同, 可空: dp
- 球同, 盒不同, 无空: $\binom{n-1}{m-1}$
- 球同, 盒不同, 可空: $\binom{n+m-1}{n-1}$
- 球不同, 盒同, 无空: $S(n, m)$
- 球不同, 盒同, 可空: $\sum_{k=1}^m S(n, k)$
- 球不同, 盒不同, 无空: $m!S(n, m)$
- 球不同, 盒不同, 可空: m^n

- 组合数奇偶性: 若 $(n \& m) = m$, 则 $\binom{n}{m}$ 为奇数, 否则为偶数

- 格雷码 $G(x) = x \otimes (x \gg 1)$

- Fibonacci 数:

- $F_0 = F_1 = 1, F_i = F_{i-1} + F_{i-2}, F_{-i} = (-1)^{i-1} F_i$
 - $F_i = \frac{1}{\sqrt{5}}((\frac{1+\sqrt{5}}{2})^n - (\frac{1-\sqrt{5}}{2})^n)$
 - $\text{gcd}(F_n, F_m) = F_{\text{gcd}(n,m)}$
 - $F_{i+1}F_i - F_i^2 = (-1)^i$
 - $F_{n+k} = F_kF_{n+1} + F_{k-1}F_n$

- 第一类 Stirling 数: $[n_k]$ 代表第一类无符号 Stirling 数, 代表将 n 阶置换群中有 k 个环的置换个数; $s(n, k)$ 代表有符号型, $s(n, k) = (-1)^{n-k} [n_k]$.

- $(x)^{(n)} = \sum_{k=0}^n [n_k] x^k, (x)_n = \sum_{k=0}^n s(n, k) x^k$
 - $[n_k] = n[n_{k-1}^{n-1}] + [n_{k-1}^{n-1}], [0_0] = 1, [n_0] = [0_n] = 0$
 - $[n_{n-2}] = \frac{1}{4}(3n-1)\binom{n}{3}, [n_{n-3}] = \binom{n}{2}\binom{n}{4}$
 - $\sum_{k=0}^a [n_k] = n! - \sum_{k=0}^n [n_{k+a+1}]$
 - $\sum_{p=k}^n [n_p] \binom{p}{k} = [n_{k+1}^{n+1}]$

- 第二类 Stirling 数: $\{n_k\} = S(n, k)$ 代表 n 个不同的球, 放到 k 个相同的盒子里, 盒子非空.

$$- \{n_k\} = \frac{1}{k!} \sum_{j=0}^k (-1)^j \binom{k}{j} (k-j)^n$$

$$- \{n+1_k\} = k \{n_k\} + \{n_{k-1}\}, \{0_0\} = 1, \{n_0\} = \{0_n\} = 0$$

$$- \text{奇偶性: } (n-k) \& \frac{k-1}{2} == 0$$

- Bell 数: B_n 代表将 n 个元素划分成若干个非空集合的方案数

$$- B_0 = B_1 = 1, B_n = \sum_{k=0}^{n-1} \binom{n-1}{k} B_k$$

$$- B_n = \sum_{k=0}^n \{n_k\}$$

$$- \text{Bell 三角形: } a_{1,1} = 1, a_{n,1} = a_{n-1,n-1}, a_{n,m} = a_{n,m-1} + a_{n-1,m-1}, B_n = a_{n,1}$$

$$- \text{对质数 } p, B_{n+p} \equiv B_n + B_{n+1} \pmod{p}$$

$$- \text{对质数 } p, B_{n+p^m} \equiv m B_n + B_{n+1} \pmod{p}$$

$$- \text{对质数 } p, \text{ 模的周期一定是 } \frac{p^p-1}{p-1} \text{ 的约数, } p \leq 101 \text{ 时就是这个值}$$

$$- \text{从 } B_0 \text{ 开始, 前几项是 } 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975 \cdots$$

- Bernoulli 数

$$- B_0 = 1, B_1 = \frac{1}{2}, B_2 = \frac{1}{6}, B_4 = -\frac{1}{30}, B_6 = \frac{1}{42}, B_8 = B_4, B_{10} = \frac{5}{66}$$

$$- \sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k n^{m+1-k}$$

$$- B_m = 1 - \sum_{k=0}^{m-1} \binom{m}{k} \frac{B_k}{m-k+1}$$

- 完全数: x 是偶完全数等价于 $x = 2^{n-1}(2^n - 1)$, 且 $2^n - 1$ 是质数.

6 其他

6.1 Extended LIS

```
1 int G[MAXN][MAXN];
2 void insertYoung(int v) {
3     for (int x = 1, y = INT_MAX; ; ++x) {
4         Down(y, *G[x]); while (y > 0 && G[x][y] >= v) --y;
5         if (++y > *G[x]) { ++*G[x]; G[x][y] = v; break; }
6         else swap(G[x][y], v);
7     }
8 }
9 int solve(int N, int seq[]) {
10     Rep(i, 1, N) *G[i] = 0;
11     Rep(i, 1, N) insertYoung(seq[i]);
```

```
12     printf("%d\n", *G[1] + *G[2]);
13     return 0;
14 }
```

6.2 生成 nCk

```
1 void nCk(int n, int k) {
2     for (int comb = (1 << k) - 1; comb < (1 << n); ) {
3         int x = comb & -comb, y = comb + x;
4         comb = (((comb & ~y) / x) >> 1) | y;
5     }
6 }
```

6.3 nextPermutation

```
1 boolean nextPermutation(int[] is) {
2     int n = is.length;
3     for (int i = n - 1; i > 0; i--) {
4         if (is[i - 1] < is[i]) {
5             int j = n; while (is[i - 1] >= is[--j]);
6             swap(is, i - 1, j); // swap is[i - 1], is[j]
7             rev(is, i, n); // reverse is[i, n)
8             return true;
9         }
10    } rev(is, 0, n);
11    return false;
12 }
```

6.4 Josephus 数与逆 Josephus 数

```
1 int josephus(int n, int m, int k) { int x = -1;
2     for (int i = n - k + 1; i <= n; i++) x = (x + m) % i; return x;
3 }
4 int invJosephus(int n, int m, int x) {
5     for (int i = n; ; i--) { if (x == i) return n - i; x = (x - m % i + i) % i; }
6 }
```

6.5 表达式求值

```
1 inline int getLevel(char ch) {
2     switch (ch) { case '+': case '-': return 0; case '*': return 1; } return -1;
3 }
4 int evaluate(char *&p, int level) {
```

```

5   int res;
6   if (level == 2) {
7       if (*p == '(') ++p, res = evaluate(p, 0);
8       else res = isdigit(*p) ? *p - '0' : value[*p - 'a'];
9       ++p; return res;
10  } res = evaluate(p, level + 1);
11  for (int next; *p && getLevel(*p) == level; ) {
12      char op = *p++; next = evaluate(p, level + 1);
13      switch (op) {
14          case '+': res += next; break;
15          case '-': res -= next; break;
16          case '*': res *= next; break;
17      }
18  } return res;
19 }
20 int makeEvaluation(char *str) { char *p = str; return evaluate(p, 0); }

```

6.6 曼哈顿最小生成树

```

1  const int INF = 1000000005;
2  struct TreeEdge {
3      int x, y, z; void make(int _x, int _y, int _z) { x = _x; y = _y; z = _z; }
4  } data[maxn * 4];
5  int n, x[maxn], y[maxn], px[maxn], py[maxn], id[maxn], tree[maxn], node[maxn], val[maxn],
    fa[maxn];
6  bool operator < (const TreeEdge& x, const TreeEdge& y) { return x.z < y.z; }
7  bool cmp1(int a, int b) { return x[a] < x[b]; }
8  bool cmp2(int a, int b) { return y[a] < y[b]; }
9  bool cmp3(int a, int b) { return (y[a] - x[a] < y[b] - x[b] || (y[a] - x[a] == y[b] - x[b]
10     ] && y[a] > y[b])); }
11 bool cmp4(int a, int b) { return (y[a] - x[a] > y[b] - x[b] || (y[a] - x[a] == y[b] - x[b]
12     ] && x[a] > x[b])); }
13 bool cmp5(int a, int b) { return (x[a] + y[a] > x[b] + y[b] || (x[a] + y[a] == x[b] + y[b]
14     ] && x[a] < x[b])); }
15 bool cmp6(int a, int b) { return (x[a] + y[a] < x[b] + y[b] || (x[a] + y[a] == x[b] + y[b]
16     ] && y[a] > y[b])); }
17 void Change_X() {
18     for (int i = 0; i < n; ++i) val[i] = x[i];
19     for (int i = 0; i < n; ++i) id[i] = i;
20     sort(id, id + n, cmp1);
21     int cntM = 1, last = val[id[0]]; px[id[0]] = 1;
22     for (int i = 1; i < n; ++i) {
23         if (val[id[i]] > last) ++cntM, last = val[id[i]];
24         px[id[i]] = cntM;
25     }
26 }
27 void Change_Y() {
28     for (int i = 0; i < n; ++i) val[i] = y[i];

```

```

25     for (int i = 0; i < n; ++i) id[i] = i;
26     sort(id, id + n, cmp2);
27     int cntM = 1, last = val[id[0]]; py[id[0]] = 1;
28     for (int i = 1; i < n; ++i) {
29         if (val[id[i]] > last)
30             ++cntM, last = val[id[i]];
31         py[id[i]] = cntM;
32     }
33 }
34 inline int Cost(int a, int b) { return abs(x[a] - x[b]) + abs(y[a] - y[b]); }
35 int find(int x) { return (fa[x] == x) ? x : (fa[x] = find(fa[x])); }
36 int main() {
37     for (int i = 0; i < n; ++i) scanf("%d%d", x + i, y + i);
38     Change_X(); Change_Y();
39     int cntE = 0; for (int i = 0; i < n; ++i) id[i] = i;
40     sort(id, id + n, cmp3);
41     for (int i = 1; i <= n; ++i) tree[i] = INF, node[i] = -1;
42     for (int i = 0; i < n; ++i) {
43         int Min = INF, Tnode = -1;
44         for (int k = py[id[i]]; k <= n; k += k & (-k))
45             if (tree[k] < Min) Min = tree[k], Tnode = node[k];
46         if (Tnode >= 0) data[cntE++].make(id[i], Tnode, Cost(id[i], Tnode));
47         int tmp = x[id[i]] + y[id[i]];
48         for (int k = py[id[i]]; k; k -= k & (-k))
49             if (tmp < tree[k]) tree[k] = tmp, node[k] = id[i];
50     } sort(id, id + n, cmp4);
51     for (int i = 1; i <= n; ++i) tree[i] = INF, node[i] = -1;
52     for (int i = 0; i < n; ++i) {
53         int Min = INF, Tnode = -1;
54         for (int k = px[id[i]]; k <= n; k += k & (-k))
55             if (tree[k] < Min) Min = tree[k], Tnode = node[k];
56         if (Tnode >= 0) data[cntE++].make(id[i], Tnode, Cost(id[i], Tnode));
57         int tmp = x[id[i]] + y[id[i]];
58         for (int k = px[id[i]]; k; k -= k & (-k))
59             if (tmp < tree[k]) tree[k] = tmp, node[k] = id[i];
60     }
61     sort(id, id + n, cmp5);
62     for (int i = 1; i <= n; ++i) tree[i] = INF, node[i] = -1;
63     for (int i = 0; i < n; ++i) {
64         int Min = INF, Tnode = -1;
65         for (int k = px[id[i]]; k; k -= k & (-k))
66             if (tree[k] < Min) Min = tree[k], Tnode = node[k];
67         if (Tnode >= 0) data[cntE++].make(id[i], Tnode, Cost(id[i], Tnode));
68         int tmp = -x[id[i]] + y[id[i]];
69         for (int k = px[id[i]]; k <= n; k += k & (-k))
70             if (tmp < tree[k]) tree[k] = tmp, node[k] = id[i];
71     } sort(id, id + n, cmp6);
72     for (int i = 1; i <= n; ++i) tree[i] = INF, node[i] = -1;
73     for (int i = 0; i < n; ++i) {

```

```

74     int Min = INF, Tnode = -1;
75     for (int k = py[id[i]]; k <= n; k += k & (-k))
76         if (tree[k] < Min) Min = tree[k], Tnode = node[k];
77     if (Tnode >= 0) data[cntE++].make(id[i], Tnode, Cost(id[i], Tnode));
78     int tmp = -x[id[i]] + y[id[i]];
79     for (int k = py[id[i]]; k; k -= k & (-k))
80         if (tmp < tree[k]) tree[k] = tmp, node[k] = id[i];
81 }
82 long long Ans = 0; sort(data, data + cntE);
83 for (int i = 0; i < n; ++i) fa[i] = i;
84 for (int i = 0; i < cntE; ++i) if (find(data[i].x) != find(data[i].y)) {
85     Ans += data[i].z;
86     fa[fa[data[i].x]] = fa[data[i].y];
87 } cout << Ans << endl;
88 }

```

6.7 直线下的整点个数

求 $\sum_{i=0}^{n-1} \lfloor \frac{a+bi}{m} \rfloor$

```

1 LL count(LL n, LL a, LL b, LL m) {
2     if (b == 0) return n * (a / m);
3     if (a >= m) return n * (a / m) + count(n, a % m, b, m);
4     if (b >= m) return (n - 1) * n / 2 * (b / m) + count(n, a, b % m, m);
5     return count((a + b * n) / m, (a + b * n) % m, m, b);
6 }

```

6.8 Java 多项式

```

1 class Polynomial {
2     final static Polynomial ZERO = new Polynomial(new int[] { 0 });
3     final static Polynomial ONE = new Polynomial(new int[] { 1 });
4     final static Polynomial X = new Polynomial(new int[] { 0, 1 });
5     int[] coef;
6     static Polynomial valueOf(int val) { return new Polynomial(new int[] { val }); }
7     Polynomial(int[] coef) { this.coef = Arrays.copyOf(coef, coef.length); }
8     Polynomial add(Polynomial o, int mod); // omitted
9     Polynomial subtract(Polynomial o, int mod); // omitted
10    Polynomial multiply(Polynomial o, int mod); // omitted
11    Polynomial scale(int o, int mod); // omitted
12    public String toString() {
13        int n = coef.length; String ret = "";
14        for (int i = n - 1; i > 0; --i) if (coef[i] != 0)
15            ret += coef[i] + "x^" + i + "+";
16        return ret + coef[0];
17    }
18    static Polynomial lagrangeInterpolation(int[] x, int[] y, int mod) {

```

```

19        int n = x.length; Polynomial ret = Polynomial.ZERO;
20        for (int i = 0; i < n; ++i) {
21            Polynomial poly = Polynomial.valueOf(y[i]);
22            for (int j = 0; j < n; ++j) if (i != j) {
23                poly = poly.multiply(
24                    Polynomial.X.subtract(Polynomial.valueOf(x[j]), mod), mod);
25                poly = poly.scale(powMod(x[i] - x[j] + mod, mod - 2, mod), mod);
26            } ret = ret.add(poly, mod);
27        } return ret;
28    }
29 }

```

6.9 long long 乘法取模

```

1 LL multiplyMod(LL a, LL b, LL P) { // 需要保证 a 和 b 非负
2     LL t = (a * b - LL((long double)a / P * b + 1e-3) * P) % P;
3     return t < 0 : t + P : t;
4 }

```

6.10 重复覆盖

```

1 namespace DLX {
2     struct node { int x, y; node *l, *r, *u, *d; } base[MAX * MAX], *top, *head;
3     typedef node *link;
4     int row, col, nGE, ans, stamp, cntc[MAX], vis[MAX];
5     vector<link> eachRow[MAX], eachCol[MAX];
6     inline void addElement(int x, int y) {
7         top->x = x; top->y = y; top->l = top->r = top->u = top->d = NULL;
8         eachRow[x].push_back(top); eachCol[y].push_back(top++);
9     }
10    void init(int _row, int _col, int _nGE) {
11        row = _row; col = _col; nGE = _nGE; top = base; stamp = 0;
12        for (int i = 0; i <= col; ++i) vis[i] = 0;
13        for (int i = 0; i <= row; ++i) eachRow[i].clear();
14        for (int i = 0; i <= col; ++i) eachCol[i].clear();
15        for (int i = 0; i <= col; ++i) addElement(0, i);
16        head = eachCol[0].front();
17    }
18    void build() {
19        for (int i = 0; i <= row; ++i) {
20            vector<link> &v = eachRow[i];
21            sort(v.begin(), v.end(), cmpByY);
22            int s = v.size();
23            for (int j = 0; j < s; ++j) {
24                link l = v[j], r = v[(j + 1) % s]; l->r = r; r->l = l;
25            }

```

```

26     }
27     for (int i = 0; i <= col; ++i) {
28         vector<link> &v = eachCol[i];
29         sort(v.begin(), v.end(), cmpByX);
30         int s = v.size();
31         for (int j = 0; j < s; ++j) {
32             link u = v[j], d = v[(j + 1) % s]; u->d = d; d->u = u;
33         }
34     } for (int i = 0; i <= col; ++i) cntc[i] = (int) eachCol[i].size() - 1;
35 }
36 void removeExact(link c) {
37     c->l->r = c->r; c->r->l = c->l;
38     for (link i = c->d; i != c; i = i->d)
39         for (link j = i->r; j != i; j = j->r) {
40             j->d->u = j->u; j->u->d = j->d; --cntc[j->y];
41         }
42 }
43 void resumeExact(link c) {
44     for (link i = c->u; i != c; i = i->u)
45         for (link j = i->l; j != i; j = j->l) {
46             j->d->u = j; j->u->d = j; ++cntc[j->y];
47         }
48     c->l->r = c; c->r->l = c;
49 }
50 void removeRepeat(link c) {
51     for (link i = c->d; i != c; i = i->d) {
52         i->l->r = i->r; i->r->l = i->l;
53     }
54 }
55 void resumeRepeat(link c) {
56     for (link i = c->u; i != c; i = i->u) {
57         i->l->r = i; i->r->l = i;
58     }
59 }
60 int calcH() {
61     int y, res = 0; ++stamp;
62     for (link c = head->r; (y = c->y) <= row && c != head; c = c->r) {
63         if (vis[y] != stamp) {
64             vis[y] = stamp; ++res;
65             for (link i = c->d; i != c; i = i->d)
66                 for (link j = i->r; j != i; j = j->r) vis[j->y] = stamp;
67         }
68     } return res;
69 }
70 void DFS(int dep) { if (dep + calcH() >= ans) return;
71     if (head->r->y > nGE || head->r == head) {
72         if (ans > dep) ans = dep; return;
73     } link c = NULL;
74     for (link i = head->r; i->y <= nGE && i != head; i = i->r)

```

```

75         if (!c || cntc[i->y] < cntc[c->y]) c = i;
76     for (link i = c->d; i != c; i = i->d) {
77         removeRepeat(i);
78         for (link j = i->r; j != i; j = j->r) if (j->y <= nGE) removeRepeat(j);
79         for (link j = i->r; j != i; j = j->r) if (j->y > nGE) removeExact(base + j->y);
80         DFS(dep + 1);
81         for (link j = i->l; j != i; j = j->l) if (j->y > nGE) resumeExact(base + j->y);
82         for (link j = i->l; j != i; j = j->l) if (j->y <= nGE) resumeRepeat(j);
83         resumeRepeat(i);
84     }
85 }
86 int solve() { build(); ans = INF; DFS(0); return ans; }
87 }

```

6.11 星期几判定

```

1 int getDay(int y, int m, int d) {
2     if (m <= 2) m += 12, y--;
3     if (y < 1752 || (y == 1752 && m < 9) || (y == 1752 && m == 9 && d < 3))
4         return (d + 2 * m + 3 * (m + 1) / 5 + y + y / 4 + 5) % 7 + 1;
5     return (d + 2 * m + 3 * (m + 1) / 5 + y + y / 4 - y / 100 + y / 400) % 7 + 1;
6 }

```

6.12 LCSequence Fast

```

1 ULL *a, *b, *s, c, d;
2 for (i = 0, a = appear[(int)B[k]], b = row[max(k - 1, 0)], s = X; i < bitSetLen; ++i)
3     *s++ = *a++ | *b++; // X = row[i - 1] or appear[ B[i] ]
4 for (i = 0, a = dp, c = d = 0; i < bitSetLen; ++a, c = d, ++i)
5     d = *a >> 63, *a = -((*a << 1) + c); // row[i] = -((row[i] << 1) + 1)
6 for (i = 0, a = dp, b = X, c = 0; i < bitSetLen; ++a, ++b, ++i)
7     d = *b + c, c = (*a >= -d), *a += d; // row[i] += X
8 for (i = 0, a = dp, b = X; i < bitSetLen; ++a, ++b, ++i)
9     *a = (*a ^ *b) & *b; // row[i] = X and (row[i] xor X)

```

7 Templates

7.1 builtin 系列

- int `__builtin_ffs` (unsigned int x) 返回 x 的最后一位 1 的是从后向前第几位, 比如 7368(1110011001000) 返回 4.
- int `__builtin_clz` (unsigned int x) 返回前导的 0 的个数.
- int `__builtin_ctz` (unsigned int x) 返回后面的 0 的个数, 和 `__builtin_clz` 相对.

- `int __builtin_popcount (unsigned int x)` 返回二进制表示中 1 的个数.

- `int __builtin_parity (unsigned int x)` 返回 x 的奇偶校验位, 也就是 x 的 1 的个数模 2 的结果.

7.2 泰勒级数

$$\begin{aligned}
 \frac{1}{1-x} &= 1 + x + x^2 + x^3 + x^4 + \cdots &= \sum_{i=0}^{\infty} x^i \\
 \frac{1}{1-cx} &= 1 + cx + c^2x^2 + c^3x^3 + \cdots &= \sum_{i=0}^{\infty} c^i x^i \\
 \frac{1}{1-x^n} &= 1 + x^n + x^{2n} + x^{3n} + \cdots &= \sum_{i=0}^{\infty} x^{ni} \\
 \frac{x}{(1-x)^2} &= x + 2x^2 + 3x^3 + 4x^4 + \cdots &= \sum_{i=0}^{\infty} ix^i \\
 \sum_{k=0}^n \left\{ \begin{matrix} n \\ k \end{matrix} \right\} \frac{k!z^k}{(1-z)^{k+1}} &= x + 2^n x^2 + 3^n x^3 + 4^n x^4 + \cdots &= \sum_{i=0}^{\infty} i^n x^i \\
 e^x &= 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \cdots &= \sum_{i=0}^{\infty} \frac{x^i}{i!} \\
 \ln(1+x) &= x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots &= \sum_{i=1}^{\infty} (-1)^{i+1} \frac{x^i}{i} \\
 \ln \frac{1}{1-x} &= x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \cdots &= \sum_{i=1}^{\infty} \frac{x^i}{i} \\
 \sin x &= x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \cdots &= \sum_{i=0}^{\infty} (-1)^i \frac{x^{2i+1}}{(2i+1)!} \\
 \cos x &= 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \cdots &= \sum_{i=0}^{\infty} (-1)^i \frac{x^{2i}}{(2i)!} \\
 \tan^{-1} x &= x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \cdots &= \sum_{i=0}^{\infty} (-1)^i \frac{x^{2i+1}}{(2i+1)} \\
 (1+x)^n &= 1 + nx + \frac{n(n-1)}{2}x^2 + \cdots &= \sum_{i=0}^{\infty} \binom{n}{i} x^i \\
 \frac{1}{(1-x)^{n+1}} &= 1 + (n+1)x + \binom{n+2}{2}x^2 + \cdots &= \sum_{i=0}^{\infty} \binom{i+n}{i} x^i \\
 \frac{x}{e^x - 1} &= 1 - \frac{1}{2}x + \frac{1}{12}x^2 - \frac{1}{720}x^4 + \cdots &= \sum_{i=0}^{\infty} \frac{B_i x^i}{i!}
 \end{aligned}$$

$$\begin{aligned}
 \frac{1}{2x}(1 - \sqrt{1-4x}) &= 1 + x + 2x^2 + 5x^3 + \cdots &= \sum_{i=0}^{\infty} \frac{1}{i+1} \binom{2i}{i} x^i \\
 \frac{1}{\sqrt{1-4x}} &= 1 + 2x + 6x^2 + 20x^3 + \cdots &= \sum_{i=0}^{\infty} \binom{2i}{i} x^i \\
 \frac{1}{\sqrt{1-4x}} \left(\frac{1 - \sqrt{1-4x}}{2x} \right)^n &= 1 + (2+n)x + \binom{4+n}{2}x^2 + \cdots &= \sum_{i=0}^{\infty} \binom{2i+n}{i} x^i \\
 \frac{1}{1-x} \ln \frac{1}{1-x} &= x + \frac{3}{2}x^2 + \frac{11}{6}x^3 + \frac{25}{12}x^4 + \cdots &= \sum_{i=1}^{\infty} H_i x^i \\
 \frac{1}{2} \left(\ln \frac{1}{1-x} \right)^2 &= \frac{1}{2}x^2 + \frac{3}{4}x^3 + \frac{11}{24}x^4 + \cdots &= \sum_{i=2}^{\infty} \frac{H_{i-1} x^i}{i} \\
 \frac{x}{1-x-x^2} &= x + x^2 + 2x^3 + 3x^4 + \cdots &= \sum_{i=0}^{\infty} F_i x^i \\
 \frac{F_n x}{1 - (F_{n-1} + F_{n+1})x - (-1)^n x^2} &= F_n x + F_{2n} x^2 + F_{3n} x^3 + \cdots &= \sum_{i=0}^{\infty} F_{ni} x^i
 \end{aligned}$$

7.3 积分表

- $d(\tan x) = \sec^2 x dx$
- $d(\cot x) = \csc^2 x dx$
- $d(\sec x) = \tan x \sec x dx$
- $d(\csc x) = -\cot x \csc x dx$
- $d(\arcsin x) = \frac{1}{\sqrt{1-x^2}} dx$
- $d(\arccos x) = \frac{-1}{\sqrt{1-x^2}} dx$
- $d(\arctan x) = \frac{1}{1+x^2} dx$
- $d(\operatorname{arccot} x) = \frac{-1}{1+x^2} dx$
- $d(\operatorname{arcsec} x) = \frac{1}{x\sqrt{1-x^2}} dx$
- $d(\operatorname{arccsc} x) = \frac{-1}{u\sqrt{1-x^2}} dx$

$$\bullet \int cu \, dx = c \int u \, dx$$

$$\bullet \int (u + v) \, dx = \int u \, dx + \int v \, dx$$

$$\bullet \int x^n \, dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1$$

$$\bullet \int \frac{1}{x} dx = \ln x$$

$$\bullet \int e^x \, dx = e^x$$

$$\bullet \int \frac{dx}{1+x^2} = \arctan x$$

$$\bullet \int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$$

$$\bullet \int \sin x \, dx = -\cos x$$

$$\bullet \int \cos x \, dx = \sin x$$

$$\bullet \int \tan x \, dx = -\ln |\cos x|$$

$$\bullet \int \cot x \, dx = \ln |\cos x|$$

$$\bullet \int \sec x \, dx = \ln |\sec x + \tan x|$$

$$\bullet \int \csc x \, dx = \ln |\csc x + \cot x|$$

$$\bullet \int \arcsin \frac{x}{a} dx = \arcsin \frac{x}{a} + \sqrt{a^2 - x^2}, \quad a > 0$$

$$\bullet \int \arccos \frac{x}{a} dx = \arccos \frac{x}{a} - \sqrt{a^2 - x^2}, \quad a > 0$$

$$\bullet \int \arctan \frac{x}{a} dx = x \arctan \frac{x}{a} - \frac{a}{2} \ln(a^2 + x^2), \quad a > 0$$

$$\bullet \int \sin^2(ax) dx = \frac{1}{2a}(ax - \sin(ax) \cos(ax))$$

$$\bullet \int \cos^2(ax) dx = \frac{1}{2a}(ax + \sin(ax) \cos(ax))$$

$$\bullet \int \sec^2 x \, dx = \tan x$$

$$\bullet \int \csc^2 x \, dx = -\cot x$$

$$\bullet \int \sin^n x \, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x \, dx$$

$$\bullet \int \cos^n x \, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x \, dx$$

$$\bullet \int \tan^n x \, dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x \, dx, \quad n \neq 1$$

$$\bullet \int \cot^n x \, dx = -\frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x \, dx, \quad n \neq 1$$

$$\bullet \int \sec^n x \, dx = \frac{\tan x \sec^{n-1} x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x \, dx, \quad n \neq 1$$

$$\bullet \int \csc^n x \, dx = -\frac{\cot x \csc^{n-1} x}{n-1} + \frac{n-2}{n-1} \int \csc^{n-2} x \, dx, \quad n \neq 1$$

$$\bullet \int \sinh x \, dx = \cosh x$$

$$\bullet \int \cosh x \, dx = \sinh x$$

$$\bullet \int \tanh x \, dx = \ln |\cosh x|$$

$$\bullet \int \coth x \, dx = \ln |\sinh x|$$

$$\bullet \int \operatorname{sech} x \, dx = \arctan \sinh x$$

$$\bullet \int \operatorname{csch} x \, dx = \ln \left| \tanh \frac{x}{2} \right|$$

$$\bullet \int \sinh^2 x \, dx = \frac{1}{4} \sinh(2x) - \frac{1}{2} x$$

$$\bullet \int \cosh^2 x \, dx = \frac{1}{4} \sinh(2x) + \frac{1}{2} x$$

$$\bullet \int \operatorname{sech}^2 x \, dx = \tanh x$$

$$\bullet \int \operatorname{arcsinh} \frac{x}{a} dx = x \operatorname{arcsinh} \frac{x}{a} - \sqrt{x^2 + a^2}, \quad a > 0$$

$$\bullet \int \operatorname{arctanh} \frac{x}{a} dx = x \operatorname{arctanh} \frac{x}{a} + \frac{a}{2} \ln |a^2 - x^2|$$

$$\bullet \int \operatorname{arccosh} \frac{x}{a} dx = \begin{cases} x \operatorname{arccosh} \frac{x}{a} - \sqrt{x^2 + a^2}, & \text{if } \operatorname{arccosh} \frac{x}{a} > 0 \text{ and } a > 0 \\ x \operatorname{arccosh} \frac{x}{a} + \sqrt{x^2 + a^2}, & \text{if } \operatorname{arccosh} \frac{x}{a} < 0 \text{ and } a > 0 \end{cases}$$

$$\bullet \int \frac{dx}{\sqrt{a^2 + x^2}} = \ln \left(x + \sqrt{a^2 + x^2} \right), \quad a > 0$$

$$\bullet \int \frac{dx}{a^2 + x^2} = \frac{1}{a} \operatorname{arctan} \frac{x}{a}, \quad a > 0$$

$$\bullet \int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \operatorname{arcsin} \frac{x}{a}, \quad a > 0$$

$$\bullet \int (a^2 - x^2)^{3/2} dx = \frac{x}{8} (5a^2 - 2x^2) \sqrt{a^2 - x^2} + \frac{3a^4}{8} \operatorname{arcsin} \frac{x}{a}, \quad a > 0$$

$$\bullet \int \frac{dx}{\sqrt{a^2 - x^2}} = \operatorname{arcsin} \frac{x}{a}, \quad a > 0$$

$$\bullet \int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right|$$

$$\bullet \int \frac{dx}{(a^2 - x^2)^{3/2}} = \frac{x}{a^2 \sqrt{a^2 - x^2}}$$

$$\bullet \int \sqrt{a^2 \pm x^2} dx = \frac{x}{2} \sqrt{a^2 \pm x^2} \pm \frac{a^2}{2} \ln \left| x + \sqrt{a^2 \pm x^2} \right|$$

$$\bullet \int \frac{dx}{\sqrt{x^2 - a^2}} = \ln \left| x + \sqrt{x^2 - a^2} \right|, \quad a > 0$$

$$\bullet \int \frac{dx}{ax^2 + bx} = \frac{1}{a} \ln \left| \frac{x}{a+bx} \right|$$

$$\bullet \int x \sqrt{a+bx} dx = \frac{2(3bx-2a)(a+bx)^{3/2}}{15b^2}$$

$$\bullet \int \frac{\sqrt{a+bx}}{x} dx = 2\sqrt{a+bx} + a \int \frac{1}{x\sqrt{a+bx}} dx$$

$$\bullet \int \frac{x}{\sqrt{a+bx}} dx = \frac{1}{\sqrt{2}} \ln \left| \frac{\sqrt{a+bx} - \sqrt{a}}{\sqrt{a+bx} + \sqrt{a}} \right|, \quad a > 0$$

$$\bullet \int \frac{\sqrt{a^2 - x^2}}{x} dx = \sqrt{a^2 - x^2} - a \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right|$$

$$\bullet \int x \sqrt{a^2 - x^2} dx = -\frac{1}{3} (a^2 - x^2)^{3/2}$$

$$\bullet \int x^2 \sqrt{a^2 - x^2} dx = \frac{x}{8} (2x^2 - a^2) \sqrt{a^2 - x^2} + \frac{a^4}{8} \operatorname{arcsin} \frac{x}{a}, \quad a > 0$$

$$\bullet \int \frac{dx}{\sqrt{a^2 - x^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right|$$

$$\bullet \int \frac{x dx}{\sqrt{a^2 - x^2}} = -\sqrt{a^2 - x^2}$$

$$\bullet \int \frac{x^2 dx}{\sqrt{a^2 - x^2}} = -\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \operatorname{arcsin} \frac{x}{a}, \quad a > 0$$

$$\bullet \int \frac{\sqrt{a^2 + x^2}}{x} dx = \sqrt{a^2 + x^2} - a \ln \left| \frac{a + \sqrt{a^2 + x^2}}{x} \right|$$

$$\bullet \int \frac{\sqrt{x^2 - a^2}}{x} dx = \sqrt{x^2 - a^2} - a \operatorname{arccos} \frac{a}{|x|}, \quad a > 0$$

$$\bullet \int x \sqrt{x^2 \pm a^2} dx = \frac{1}{3} (x^2 \pm a^2)^{3/2}$$

$$\bullet \int \frac{dx}{x \sqrt{x^2 + a^2}} = \frac{1}{a} \ln \left| \frac{x}{a + \sqrt{a^2 + x^2}} \right|$$

$$\bullet \int \frac{dx}{x \sqrt{x^2 - a^2}} = \frac{1}{a} \operatorname{arccos} \frac{a}{|x|}, \quad a > 0$$

$$\bullet \int \frac{dx}{x^2 \sqrt{x^2 \pm a^2}} = \mp \frac{\sqrt{x^2 \pm a^2}}{a^2 x}$$

$$\bullet \int \frac{x dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}$$

$$\bullet \int \frac{\sqrt{x^2 \pm a^2}}{x^4} dx = \mp \frac{(x^2 + a^2)^{3/2}}{3a^2 x^3}$$

$$\bullet \int \frac{dx}{ax^2 + bx + c} = \begin{cases} \frac{1}{\sqrt{b^2 - 4ac}} \ln \left| \frac{2ax + b - \sqrt{b^2 - 4ac}}{2ax + b + \sqrt{b^2 - 4ac}} \right|, & \text{if } b^2 > 4ac \\ \frac{2}{\sqrt{4ac - b^2}} \operatorname{arctan} \frac{2ax + b}{\sqrt{4ac - b^2}}, & \text{if } b^2 < 4ac \end{cases}$$

$$\bullet \int \frac{\mathrm{d} x}{\sqrt{a x^2+b x+c}}=\left\{\begin{array}{l} \frac{1}{\sqrt{a}} \ln \left|2 a x+b+2 \sqrt{a} \sqrt{a x^2+b x+c}\right|, \text { if } a>0 \\ \frac{1}{\sqrt{-a}} \arcsin \frac{-2 a x-b}{\sqrt{b^2-4 a c}}, \text { if } a<0 \end{array}\right.$$

$$\bullet \int \sqrt{a x^2+b x+c} \mathrm{d} x=\frac{2 a x+b}{4 a} \sqrt{a x^2+b x+c}+\frac{4 a x-b^2}{8 a} \int \frac{\mathrm{d} x}{\sqrt{a x^2+b x+c}}$$

$$\bullet \int \frac{x \mathrm{d} x}{\sqrt{a x^2+b x+c}}=\frac{\sqrt{a x^2+b x+c}}{a}-\frac{b}{2 a} \int \frac{\mathrm{d} x}{\sqrt{a x^2+b x+c}}$$

$$\bullet \int \frac{\mathrm{d} x}{x \sqrt{a x^2+b x+c}}=\left\{\begin{array}{l} \frac{-1}{\sqrt{c}} \ln \left|\frac{2 \sqrt{c} \sqrt{a x^2+b x+c}+b x+2 c}{x}\right|, \text { if } c>0 \\ \frac{1}{\sqrt{-c}} \arcsin \frac{b x+2 c}{|x| \sqrt{b^2-4 a c}}, \text { if } c<0 \end{array}\right.$$

$$\bullet \int x^3 \sqrt{x^2+a^2} \mathrm{d} x=\left(\frac{1}{3} x^2-\frac{2}{15} a^2\right)\left(x^2+a^2\right)^{3 / 2}$$

$$\bullet \int x^n \sin (a x) \mathrm{d} x=-\frac{1}{a} x^n \cos (a x)+\frac{n}{a} \int x^{n-1} \cos (a x) \mathrm{d} x$$

$$\bullet \int x^n \cos (a x) \mathrm{d} x=\frac{1}{a} x^n \sin (a x)-\frac{n}{a} \int x^{n-1} \sin (a x) \mathrm{d} x$$

$$\bullet \int x^n e^{a x} \mathrm{d} x=\frac{x^n e^{a x}}{a}-\frac{n}{a} \int x^{n-1} e^{a x} \mathrm{d} x$$

$$\bullet \int x^n \ln (a x) \mathrm{d} x=x^{n+1}\left(\frac{\ln (a x)}{n+1}-\frac{1}{(n+1)^2}\right)$$

$$\bullet \int x^n(\ln a x)^m \mathrm{d} x=\frac{x^{n+1}}{n+1}(\ln a x)^m-\frac{m}{n+1} \int x^n(\ln a x)^{m-1} \mathrm{d} x$$

7.4 Eclipse 配置

Exec=env UBUNTU_MENUPROXY= /opt/eclipse/eclipse
preference general keys 把 word completion 设置成 alt+c, 把 content assistant 设置成 alt + /

7.5 C++

```
1 #pragma comment(linker, "/STACK:10240000")
2 #include <stdio>
3 #include <stdlib>
4 #include <string>
5 #include <iostream>
```

```
6 #include <algorithm>
7 #define Rep(i, a, b) for(int i = (a); i <= (b); ++i)
8 #define Foru(i, a, b) for(int i = (a); i < (b); ++i)
9 using namespace std;
10 typedef long long LL;
11 typedef pair<int, int> pii;
12 namespace BufferedReader {
13     char buff[MAX_BUFFER + 5], *ptr = buff, c; bool flag;
14     bool nextChar(char &c) {
15         if ( (c = *ptr++) == 0 ) {
16             int tmp = fread(buff, 1, MAX_BUFFER, stdin);
17             buff[tmp] = 0; if (tmp == 0) return false;
18             ptr = buff; c = *ptr++;
19         } return true;
20     }
21     bool nextUnsignedInt(unsigned int &x) {
22         for (;){if (!nextChar(c)) return false; if ('0'<=c && c<='9') break;}
23         for (x=c-'0'; nextChar(c); x = x * 10 + c - '0') if (c < '0' || c > '9') break;
24         return true;
25     }
26     bool nextInt(int &x) {
27         for (;;) { if (!nextChar(c)) return false; if (c=='-' || ('0'<=c && c<='9')) break; }
28         for ((c=='-') ? (x=0,flag=true) : (x=c-'0',flag=false); nextChar(c); x=x*10+c-'0')
29             if (c<'0' || c>'9') break;
30         if (flag) x=-x; return true;
31     }
32 };
33 #endif
```

7.6 Java

```
1 import java.io.*;
2 import java.util.*;
3 import java.math.*;
4
5 public class Main {
6     public void solve() {}
7     public void run() {
8         tokenizer = null; out = new PrintWriter(System.out);
9         in = new BufferedReader(new InputStreamReader(System.in));
10        solve();
11        out.close();
12    }
13    public static void main(String[] args) {
14        new Main().run();
15    }
16    public StringTokenizer tokenizer;
17    public BufferedReader in;
```

```
18     public PrintWriter out;
19     public String next() {
20         while (tokenizer == null || !tokenizer.hasMoreTokens()) {
21             try { tokenizer = new StringTokenizer(in.readLine()); }
22             catch (IOException e) { throw new RuntimeException(e); }
23         } return tokenizer.nextToken();
24     }
25 }
```

7.7 vim 配置

在.bashrc 中加入 export CXXFLAGS="-Wall -Wconversion -Wextra -g3"

```
1  set nu ru nobk cindent si
2  set mouse=a sw=4 sts=4 ts=4
3  set hlsearch incsearch
4  set whichwrap=b,s,<,>,[,]
5  syntax on
6
7  nmap <C-A> ggVG
8  vmap <C-C> "+y
9
10 autocmd BufNewFile *.cpp_0r~/Templates/cpp.cpp
11 map<F9>:!g++_o%<-Wall-Wconversion-Wextra-g3<CR>
12 map<F5>:!./%<CR>
13 map<F8>:!./%<_%.in<CR>
14
15 map<F3>:vnew%<.in<CR>
16 map<F4>:!(gedit_%&<CR>
```