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## 1 计算几何

### 1.1 二维计算几何基本操作

```

1 const double PI = 3.14159265358979323846264338327950288;
2 double arcSin(const double &a) {
3     return (a <= -1.0) ? (-PI / 2) : ((a >= 1.0) ? (PI / 2) : (asin(a))); }
4 double arcCos(const double &a) {
5     return (a <= -1.0) ? (PI) : ((a >= 1.0) ? (0) : (acos(a))); }
6 struct point { double x, y; // something omitted
7     point rot(const double &a) const { // counter-clockwise
8         return point(x * cos(a) - y * sin(a), x * sin(a) + y * cos(a)); }
9     point rot90() const { return point(-y, x); } // counter-clockwise
10    point project(const point &p1, const point &p2) const {
11        const point &q = *this; return p1 + (p2 - p1) * (dot(p2 - p1, q - p1) / (p2 - p1).norm()); }
12    bool onSeg(const point &a, const point &b) const { // a, b inclusive
13        const point &c = *this; return sign(dot(a - c, b - c)) <= 0 && sign(dot(b - a, c - a)) == 0; }
14    double distLP(const point &p1, const point &p2) const { // dist from *this to line p1->p2
15        const point &q = *this; return fabs(det(p2 - p1, q - p1)) / (p2 - p1).len(); }
16    double distSP(const point &p1, const point &p2) const { // dist from *this to segment [p1, p2]
17        const point &q = *this;
18        if (dot(p2 - p1, q - p1) < EPS) return (q - p1).len();
19        if (dot(p1 - p2, q - p2) < EPS) return (q - p2).len();
20        return distLP(p1, p2);
21    }
22    bool inAngle(const point &p1, const point &p2) const { // det(p1, p2) > 0
23        const point &q = *this; return det(p1, q) > -EPS && det(p2, q) < EPS;
24    }
25 };
26 bool lineIntersect(const point &a, const point &b, const point &c, const point &d, point &e) {
27     double s1 = det(c - a, d - a), s2 = det(d - b, c - b);
28     if (!sign(s1 + s2)) return false; e = (b - a) * (s1 / (s1 + s2)) + a; return true;
29 }
30 int segIntersectCheck(const point &a, const point &b, const point &c, const point &d, point &o) {
31     static double s1, s2, s3, s4;
32     static int iCnt;
33     int d1 = sign(s1 = det(b - a, c - a)), d2 = sign(s2 = det(b - a, d - a));
34     int d3 = sign(s3 = det(d - c, a - c)), d4 = sign(s4 = det(d - c, b - c));
35     if ((d1 ^ d2) == -2 && (d3 ^ d4) == -2) {
36         o = (c * s2 - d * s1) / (s2 - s1); return true;
37     } iCnt = 0;
38     if (d1 == 0 && c.onSeg(a, b)) o = c, ++iCnt;
39     if (d2 == 0 && d.onSeg(a, b)) o = d, ++iCnt;
40     if (d3 == 0 && a.onSeg(c, d)) o = a, ++iCnt;
41     if (d4 == 0 && b.onSeg(c, d)) o = b, ++iCnt;
42     return iCnt ? 2 : 0; // 不相交返回 0, 严格相交返回 1, 非严格相交返回 2
43 }
44 struct circle {
45     point o; double r, rSqure;
46     bool inside(const point &a) { return (a - o).len() < r + EPS; } // 非严格
47     bool contain(const circle &b) const { return sign(b.r + (o - b.o).len() - r) <= 0; } // 非严格
48     bool disjunct(const circle &b) const { return sign(b.r + r - (o - b.o).len()) <= 0; } // 非严格
49     int isCL(const point &p1, const point &p2, point &a, point &b) const {
50         double x = dot(p1 - o, p2 - p1), y = (p2 - p1).norm();
51         double d = x * x - y * ((p1 - o).norm() - rSqure);
52         if (d < -EPS) return 0; if (d < 0) d = 0;
53         point q1 = p1 - (p2 - p1) * (x / y);
54         point q2 = (p2 - p1) * (sqrt(d) / y);
55         a = q1 - q2; b = q1 + q2; return q2.len() < EPS ? 1 : 2;
56     }
57     int tanCP(const point &p, point &a, point &b) const { // 返回切点, 注意可能与 p 重合
58         double x = (p - o).norm(), d = x - rSqure;
59         if (d < -EPS) return 0; if (d < 0) d = 0;
60         point q1 = (p - o) * (rSqure / x), q2 = ((p - o) * (-r * sqrt(d) / x)).rot90();
61         a = o + (q1 - q2); b = o + (q1 + q2); return q2.len() < EPS ? 1 : 2;
62     }
63 };
64 bool checkCrossCS(const circle &cir, const point &p1, const point &p2) { // 非严格
65     const point &c = cir.o; const double &r = cir.r;
66     return c.distSP(p1, p2) < r + EPS && (r < (c - p1).len() + EPS || r < (c - p2).len() + EPS);
67 }
68 bool checkCrossCC(const circle &cir1, const circle &cir2) { // 非严格
69     const double &r1 = cir1.r, &r2 = cir2.r, d = (cir1.o - cir2.o).len();
70     return d < r1 + r2 + EPS && fabs(r1 - r2) < d + EPS;
71 }
72 int isCC(const circle &cir1, const circle &cir2, point &a, point &b) {
73     const point &c1 = cir1.o, &c2 = cir2.o;
74     double x = (c1 - c2).norm(), y = ((cir1.rSqure - cir2.rSqure) / x + 1) / 2;
75     double d = cir1.rSqure / x - y * y;
76     if (d < -EPS) return 0; if (d < 0) d = 0;
77     point q1 = c1 + (c2 - c1) * y, q2 = ((c2 - c1) * sqrt(d)).rot90();
78     a = q1 - q2; b = q1 + q2; return q2.len() < EPS ? 1 : 2;
79 }
80 vector<pair<point, point>> tanCC(const circle &cir1, const circle &cir2) {

```

```

81 // 注意: 如果只有三条切线, 即 s1 = 1, s2 = 1, 返回的切线可能重复, 切点没有问题
82 vector<pair<point, point>> list;
83 if (cir1.contain(cir2) || cir2.contain(cir1)) return list;
84 const point &c1 = cir1.o, &c2 = cir2.o;
85 double r1 = cir1.r, r2 = cir2.r; point p, a1, b1, a2, b2; int s1, s2;
86 if (sign(r1 - r2) == 0) {
87     p = c2 - c1; p = (p * (r1 / p.len())).rot90();
88     list.push_back(make_pair(c1 + p, c2 + p)); list.push_back(make_pair(c1 - p, c2 - p));
89 } else {
90     p = (c2 * r1 - c1 * r2) / (r1 - r2);
91     s1 = cir1.tanCP(p, a1, b1); s2 = cir2.tanCP(p, a2, b2);
92     if (s1 >= 1 && s2 >= 1)
93         list.push_back(make_pair(a1, a2)), list.push_back(make_pair(b1, b2));
94     p = (c1 * r2 + c2 * r1) / (r1 + r2);
95     s1 = cir1.tanCP(p, a1, b1); s2 = cir2.tanCP(p, a2, b2);
96     if (s1 >= 1 && s2 >= 1)
97         list.push_back(make_pair(a1, a2)), list.push_back(make_pair(b1, b2));
98     return list;
99 }
100 bool distConvexPin(const point &p1, const point &p2, const point &p3, const point &p4, const point &q) {
101     point o12 = (p1 - p2).rot90(), o23 = (p2 - p3).rot90(), o34 = (p3 - p4).rot90();
102     return ((q - p1).inAngle(o12, o23) || (q - p3).inAngle(o23, o34)
103         || ((q - p2).inAngle(o23, p3 - p2) && (q - p3).inAngle(p2 - p3, o23)));
104 }
105 double distConvexP(int n, point ps[], const point &q) { // 外部点到多边形的距离
106     int left = 0, right = n; while (right - left > 1) { int mid = (left + right) / 2;
107         if (distConvexPin(ps[left + n - 1] % n, ps[left], ps[mid], ps[(mid + 1) % n], q))
108             right = mid; else left = mid;
109     } return q.distSP(ps[left], ps[right % n]);
110 }
111 double areaCT(const circle &cir, point pa, point pb) {
112     pa = pa - cir.o; pb = pb - cir.o; double R = cir.r;
113     if (pa.len() < pb.len()) swap(pa, pb); if (pb.len() < EPS) return 0;
114     point pc = pb - pa; double a = pa.len(), b = pb.len(), c = pc.len(), S, h, theta;
115     double cosB = dot(pb, pc) / b / c, B = acos(cosB);
116     double cosC = dot(pa, pb) / a / b, C = acos(cosC);
117     if (b > R) {
118         S = C * 0.5 * R * R; h = b * a * sin(C) / c;
119         if (h < R && B < PI * 0.5) S -= acos(h / R) * R * R - h * sqrt(R * R - h * h);
120     } else if (a > R) {
121         theta = PI - B - asin(sin(B) / R * b);
122         S = 0.5 * b * R * sin(theta) + (C - theta) * 0.5 * R * R;
123     } else S = 0.5 * sin(C) * b * a;
124     return S;
125 }
126 circle minCircle(const point &a, const point &b) {
127     return circle((a + b) * 0.5, (b - a).len() * 0.5);
128 }
129 circle minCircle(const point &a, const point &b, const point &c) { // 钝角三角形没有被考虑
130     double a2((b - c).norm()), b2((a - c).norm()), c2((a - b).norm());
131     if (b2 + c2 <= a2 + EPS) return minCircle(b, c);
132     if (a2 + c2 <= b2 + EPS) return minCircle(a, c);
133     if (a2 + b2 <= c2 + EPS) return minCircle(a, b);
134     double A = 2.0 * (a.x - b.x), B = 2.0 * (a.y - b.y);
135     double D = 2.0 * (a.x - c.x), E = 2.0 * (a.y - c.y);
136     double C = a.norm() - b.norm(), F = a.norm() - c.norm();
137     point p((C * E - B * F) / (A * E - B * D), (A * F - C * D) / (A * E - B * D));
138     return circle(p, (p - a).len());
139 }
140 circle minCircle(point P[], int N) { // 1-based
141     if (N == 1) return circle(P[1], 0.0);
142     random_shuffle(P + 1, P + N + 1); circle O = minCircle(P[1], P[2]);
143     Rep(i, 1, N) if(!O.inside(P[i])) { O = minCircle(P[1], P[i]);
144         Foru(j, 1, i) if(!O.inside(P[j])) { O = minCircle(P[i], P[j]);
145             Foru(k, 1, j) if(!O.inside(P[k])) O = minCircle(P[i], P[j], P[k]); }
146     } return O;
147 }

```

### 1.2 圆的面积模板

```

1 struct Event { point p; double alpha; int add; // 构造函数省略
2     bool operator < (const Event &other) const { return alpha < other.alpha; } };
3 void circleKCover(circle *c, int N, double *area) { // area[k]: 至少被覆盖 k 次
4     static bool overlap[MAXN][MAXN], g[MAXN][MAXN];
5     Rep(i, 0, N + 1) area[i] = 0.0; Rep(i, 1, N) Rep(j, 1, N) overlap[i][j] = c[i].contain(c[j]);
6     Rep(i, 1, N) Rep(j, 1, N) g[i][j] = !(overlap[i][j] || overlap[j][i] || c[i].disjunct(c[j]));
7     Rep(i, 1, N) { static Event events[MAXN * 2 + 1]; int totE = 0, cnt = 1;
8         Rep(j, 1, N) if (j != i && overlap[j][i]) ++cnt;
9         Rep(j, 1, N) if (j != i && g[i][j]) {
10             circle &a = c[i], &b = c[j]; double l = (a.o - b.o).norm();
11             double s = ((a.r - b.r) * (a.r + b.r) / l + 1) * 0.5;
12             double t = sqrt(-l - sqr(a.r - b.r)) * (1 - sqrt(a.r + b.r)) / (1 * l * 4.0);

```

```

13     point dir = b.o - a.o, nDir = point(-dir.y, dir.x);
14     point aa = a.o + dir * s + nDir * t;
15     point bb = a.o + dir * s - nDir * t;
16     double A = atan2(aa.y - a.o.y, aa.x - a.o.x);
17     double B = atan2(bb.y - a.o.y, bb.x - a.o.x);
18     events[totE++] = Event(bb, B, 1); events[totE++] = Event(aa, A, -1); if (B > A) ++cnt;
19 } if (totE == 0) { area[cnt] += PI * c[i].rSquare; continue; }
20 sort(events, events + totE); events[totE] = events[0];
21 Foru(j, 0, totE) {
22     cnt += events[j].add; area[cnt] += 0.5 * det(events[j].p, events[j + 1].p);
23     double theta = events[j + 1].alpha - events[j].alpha; if (theta < 0) theta += 2.0 * PI;
24     area[cnt] += 0.5 * c[i].rSquare * (theta - sin(theta));
25 }

```

### 1.3 多边形相关

```

1 struct Polygon { // stored in [0, n)
2     int n; point ps[MAXN];
3     Polygon cut(const point &a, const point &b) {
4         static Polygon res; static point o; res.n = 0;
5         for (int i = 0; i < n; ++i) {
6             int s1 = sign(det(ps[i] - a, b - a));
7             int s2 = sign(det(ps[(i + 1) % n] - a, b - a));
8             if (s1 <= 0) res.ps[res.n++] = ps[i];
9             if (s1 * s2 < 0) {
10                 lineIntersect(a, b, ps[i], ps[(i + 1) % n], o);
11                 res.ps[res.n++] = o;
12             }
13         } return res;
14     }
15     bool contain(const point &p) const { // 1 if on border or inner, 0 if outer
16         static point A, B; int res = 0;
17         for (int i = 0; i < n; ++i) {
18             A = ps[i]; B = ps[(i + 1) % n];
19             if (p.onSeg(A, B)) return 1;
20             if (sign(A.y - B.y) <= 0) swap(A, B);
21             if (sign(p.y - A.y) > 0) continue;
22             if (sign(p.y - B.y) <= 0) continue;
23             res += (int)(sign(det(B - p, A - p)) > 0);
24         } return res & 1;
25     }
26     #define qs(x) (ps[x] - ps[0])
27     bool convexContain(point p) const { // counter-clockwise
28         point q = qs(n - 1); p = p - ps[0];
29         if (!p.inAngle(qs(1), q)) return false;
30         int L = 0, R = n - 1;
31         while (L + 1 < R) { int M((L + R) >> 1);
32             if (p.inAngle(qs(M), q)) L = M; else R = M;
33         } if (L == 0) return false; point l(qs(L)), r(qs(R));
34         return sign( fabs(det(l, p)) + fabs(det(p, r)) + fabs(det(r - l, p - l)) - det(l, r) ) == 0;
35     }
36     #undef qs
37     double isPLAtan2(const point &a, const point &b) {
38         double k = (b - a).alpha(); if (k < 0) k += 2 * PI;
39         return k;
40     }
41     point isPL_Get(const point &a, const point &b, const point &s1, const point &s2) {
42         double k1 = det(b - a, s1 - a), k2 = det(b - a, s2 - a);
43         if (sign(k1) == 0) return s1;
44         if (sign(k2) == 0) return s2;
45         return (s1 * k2 - s2 * k1) / (k2 - k1);
46     }
47     int isPL_Dic(const point &a, const point &b, int l, int r) {
48         int s = (det(b - a, ps[l] - a) < 0) ? -1 : 1;
49         while (l <= r) {
50             int mid = (l + r) / 2;
51             if (det(b - a, ps[mid] - a) * s <= 0) r = mid - 1;
52             else l = mid + 1;
53         }
54         return r + 1;
55     }
56     int isPL_Find(double k, double w[]) {
57         if (k <= w[0] || k > w[n - 1]) return 0;
58         int l = 0, r = n - 1, mid;
59         while (l <= r) {
60             mid = (l + r) / 2;
61             if (w[mid] >= k) r = mid - 1;
62             else l = mid + 1;
63         } return r + 1;
64     }
65     bool isPL(const point &a, const point &b, point &cp1, point &cp2) { // O(logN)
66         static double w[MAXN * 2]; // pay attention to the array size

```

```

67     for (int i = 0; i <= n; ++i) ps[i + n] = ps[i];
68     for (int i = 0; i < n; ++i) w[i] = w[i + n] = isPLAtan2(ps[i], ps[i + 1]);
69     int i = isPL_Find(isPLAtan2(a, b), w);
70     int j = isPL_Find(isPLAtan2(b, a), w);
71     double k1 = det(b - a, ps[i] - a), k2 = det(b - a, ps[j] - a);
72     if (sign(k1) * sign(k2) > 0) return false; // no intersection
73     if (sign(k1) == 0 || sign(k2) == 0) { // intersect with a point or a line in the convex
74         if (sign(k1) == 0) {
75             if (sign(det(b - a, ps[i + 1] - a)) == 0) cp1 = ps[i], cp2 = ps[i + 1];
76             else cp1 = cp2 = ps[i];
77             return true;
78         }
79         if (sign(k2) == 0) {
80             if (sign(det(b - a, ps[j + 1] - a)) == 0) cp1 = ps[j], cp2 = ps[j + 1];
81             else cp1 = cp2 = ps[j];
82         }
83         return true;
84     }
85     if (i > j) swap(i, j);
86     int x = isPL_Dic(a, b, i, j), y = isPL_Dic(a, b, j, i + n);
87     cp1 = isPL_Get(a, b, ps[x - 1], ps[x]);
88     cp2 = isPL_Get(a, b, ps[y - 1], ps[y]);
89     return true;
90 }
91 double getI(const point &o) const {
92     if (n <= 2) return 0;
93     point G(0.0, 0.0);
94     double S = 0.0, I = 0.0;
95     for (int i = 0; i < n; ++i) {
96         const point &x = ps[i], &y = ps[(i + 1) % n];
97         double d = det(x, y);
98         G = G + (x + y) * d / 3.0;
99         S += d;
100     } G = G / S;
101     for (int i = 0; i < n; ++i) {
102         point x = ps[i] - G, y = ps[(i + 1) % n] - G;
103         I += fabs(det(x, y)) * (x.norm() + dot(x, y) + y.norm());
104     }
105     return I = I / 12.0 + fabs(S * 0.5) * (0 - G).norm();
106 }
107

```

### 1.4 半平面交

```

1 struct Border {
2     point p1, p2; double alpha;
3     Border() : p1(), p2(), alpha(0.0) {}
4     Border(const point &a, const point &b): p1(a), p2(b), alpha( atan2(p2.y - p1.y, p2.x - p1.x) ) {}
5     bool operator == (const Border &b) const { return sign(alpha - b.alpha) == 0; }
6     bool operator < (const Border &b) const {
7         int c = sign(alpha - b.alpha); if (c != 0) return c > 0;
8         return sign(det(b.p2 - b.p1, p1 - b.p1)) >= 0;
9     }
10 };
11 point isBorder(const Border &a, const Border &b) { // a and b should not be parallel
12     point is; lineIntersect(a.p1, a.p2, b.p1, b.p2, is); return is;
13 }
14 bool checkBorder(const Border &a, const Border &b, const Border &me) {
15     point is; lineIntersect(a.p1, a.p2, b.p1, b.p2, is);
16     return sign(det(me.p2 - me.p1, is - me.p1)) > 0;
17 }
18 double HPI(int N, Border border[]) {
19     static Border que[MAXN * 2 + 1]; static point ps[MAXN];
20     int head = 0, tail = 0, cnt = 0; // [head, tail)
21     sort(border, border + N); N = unique(border, border + N) - border;
22     for (int i = 0; i < N; ++i) {
23         Border &cur = border[i];
24         while (head + 1 < tail && !checkBorder(que[tail - 2], que[tail - 1], cur)) --tail;
25         while (head + 1 < tail && !checkBorder(que[head], que[head + 1], cur)) ++head;
26         que[tail++] = cur;
27     } while (head + 1 < tail && !checkBorder(que[tail - 2], que[tail - 1], que[head])) --tail;
28     while (head + 1 < tail && !checkBorder(que[head], que[head + 1], que[tail - 1])) ++head;
29     if (tail - head <= 2) return 0.0;
30     Foru(i, head, tail) ps[cnt++] = isBorder(que[i], que[(i + 1 == tail) ? (head) : (i + 1)]);
31     double area = 0; Foru(i, 0, cnt) area += det(ps[i], ps[(i + 1) % cnt]);
32     return fabs(area * 0.5); // or (-area * 0.5)
33 }

```

### 1.5 最大面积空凸包

```

1 inline bool toUpRight(const point &a, const point &b) {
2     int c = sign(b.y - a.y); if (c > 0) return true;
3     return c == 0 && sign(b.x - a.x) > 0;
4 }
5 inline bool cmpByPolarAngle(const point &a, const point &b) { // counter-clockwise, shorter first if they
6     share the same polar angle
7     int c = sign(det(a, b)); if (c != 0) return c > 0;
8     return sign(b.len() - a.len()) > 0;
9 }
10 double maxEmptyConvexHull(int N, point p[]) {
11     static double dp[MAXN][MAXN];
12     static point vec[MAXN];
13     static int seq[MAXN]; // empty triangles formed with (0,0), vec[o], vec[seq[i]]
14     double ans = 0.0;
15     Rep(o, 1, N) {
16         int totVec = 0;
17         Rep(i, 1, N) if (toUpRight(p[o], p[i])) vec[++totVec] = p[i] - p[o];
18         sort(vec + 1, vec + totVec + 1, cmpByPolarAngle);
19         Rep(i, 1, totVec) Rep(j, 1, totVec) dp[i][j] = 0.0;
20         Rep(k, 2, totVec) {
21             int i = k - 1;
22             while (i > 0 && sign( det(vec[k], vec[i]) ) == 0) --i;
23             int totSeq = 0;
24             for (int j; i > 0; i = j) {
25                 seq[++totSeq] = i;
26                 for (j = i - 1; j > 0 && sign(det(vec[i] - vec[k], vec[j] - vec[k])) > 0; --j);
27                 double v = det(vec[i], vec[k]) * 0.5;
28                 if (j > 0) v += dp[i][j];
29                 dp[k][i] = v;
30                 cMax(ans, v);
31             } for (int i = totSeq - 1; i >= 1; --i) cMax( dp[k][ seq[i] ], dp[k][seq[i] + 1] );
32         } return ans;
33     }

```

## 1.6 最近点对

```

1 int N; point p[maxn];
2 bool cmpByX(const point &a, const point &b) { return sign(a.x - b.x) < 0; }
3 bool cmpByY(const int &a, const int &b) { return p[a].y < p[b].y; }
4 double minimalDistance(point *c, int n, int *ys) {
5     double ret = 1e+20;
6     if (n < 20) {
7         Foru(i, 0, n) Foru(j, i + 1, n) cMin(ret, (c[i] - c[j]).len());
8     } sort(ys, ys + n, cmpByY); return ret;
9     static int mergeTo[maxn];
10    int mid = n / 2; double xmid = c[mid].x;
11    ret = min(minimalDistance(c, mid, ys), minimalDistance(c + mid, n - mid, ys + mid));
12    merge(ys, ys + mid, ys + mid, ys + n, mergeTo, cmpByY);
13    copy(mergeTo, mergeTo + n, ys);
14    Foru(i, 0, n) {
15        while (i < n && sign(fabs(p[ys[i]].x - xmid) - ret) > 0) ++i;
16        int cnt = 0;
17        Foru(j, i + 1, n)
18            if (sign(p[ys[j]].y - p[ys[i]].y - ret) > 0) break;
19        else if (sign(fabs(p[ys[j]].x - xmid) - ret) <= 0) {
20            ret = min(ret, (p[ys[i]] - p[ys[j]]).len());
21            if (++cnt >= 10) break;
22        }
23    } return ret;
24 }
25 double work() {
26     sort(p, p + n, cmpByX); Foru(i, 0, n) ys[i] = i; return minimalDistance(p, n, ys);
27 }

```

## 1.7 凸包与点集直径

```

1 vector<point> convexHull(int n, point ps[]) { // counter-clockwise, strict
2     static point qs[MAXN * 2];
3     sort(ps, ps + n, cmpByXY);
4     if (n <= 2) return vector(ps, ps + n);
5     int k = 0;
6     for (int i = 0; i < n; qs[k++] = ps[i++])
7         while (k > 1 && det(qs[k - 1] - qs[k - 2], ps[i] - qs[k - 1]) < EPS) --k;
8     for (int i = n - 2, t = k; i >= 0; qs[k++] = ps[i--])
9         while (k > t && det(qs[k - 1] - qs[k - 2], ps[i] - qs[k - 1]) < EPS) --k;
10    return vector<point>(qs, qs + k);

```

```

11 }
12 double convexDiameter(int n, point ps[]) {
13     if (n < 2) return 0; if (n == 2) return (ps[1] - ps[0]).len();
14     double k, ans = 0;
15     for (int x = 0, y = 1, nx, ny; x < n; ++x) {
16         for(nx = (x == n - 1) ? (0) : (x + 1); ; y = ny) {
17             ny = (y == n - 1) ? (0) : (y + 1);
18             if ( sign(k = det(ps[nx] - ps[x], ps[ny] - ps[y])) <= 0) break;
19             ans = max(ans, (ps[x] - ps[y]).len());
20             if (sign(k) == 0) ans = max(ans, (ps[x] - ps[ny]).len());
21         } return ans;
22     }

```

## 1.8 Farmland

```

1 struct node { int begin[MAXN], *end; } a[MAXN]; // 按对 p[i] 的极角的 atan2 值排序
2 bool check(int n, point p[], int b1, int b2, bool vis[MAXN][MAXN]) {
3     static pii l[MAXN * 2 + 1]; static bool used[MAXN];
4     int tp(0), *k, p, p1, p2; double area(0.0);
5     for (l[0] = pii(b1, b2); ; ) {
6         vis[p1 = l[tp].first][p2 = l[tp].second] = true;
7         area += det(p[p1], p[p2]);
8         for (k = a[p2].begin; k != a[p2].end; ++k) if (*k == p1) break;
9         k = (k == a[p2].begin) ? (a[p2].end - 1) : (k - 1);
10        if ((l[++tp] = pii(p2, *k)) == l[0]) break;
11    } if (sign(area) < 0 || tp < 3) return false;
12    Rep(i, 1, n) used[i] = false;
13    for (int i = 0; i < tp; ++i) if (used[p = l[i].first]) return false; else used[p] = true;
14    return true; // a face with tp vertices
15 }
16 int countFaces(int n, point p[]) {
17     static bool vis[MAXN][MAXN]; int ans = 0;
18     Rep(x, 1, n) Rep(y, 1, n) vis[x][y] = false;
19     Rep(x, 1, n) for (int *itr = a[x].begin; itr != a[x].end; ++itr) if (!vis[x][*itr])
20         if (check(n, p, x, *itr, vis)) ++ans;
21     return ans;
22 }

```

## 1.9 Voronoi 图

不能有重点, 点数应当不小于 2

```

1 #define Oi(e) ((e)->oi)
2 #define Dt(e) ((e)->dt)
3 #define On(e) ((e)->on)
4 #define Op(e) ((e)->op)
5 #define Dn(e) ((e)->dn)
6 #define Dp(e) ((e)->dp)
7 #define Other(e, p) ((e)->oi == p ? (e)->dt : (e)->oi)
8 #define Next(e, p) ((e)->oi == p ? (e)->on : (e)->dn)
9 #define Prev(e, p) ((e)->oi == p ? (e)->op : (e)->dp)
10 #define V(p1, p2, u, v) (u = p2->x - p1->x, v = p2->y - p1->y)
11 #define C2(u1, v1, u2, v2) (u1 * v2 - v1 * u2)
12 #define C3(p1, p2, p3) ((p2->x - p1->x) * (p3->y - p1->y) - (p2->y - p1->y) * (p3->x - p1->x))
13 #define Dot(u1, v1, u2, v2) (u1 * u2 + v1 * v2)
14 #define dis(a,b) (sqrt((a->x - b->x) * (a->x - b->x) + (a->y - b->y) * (a->y - b->y)))
15 const int maxn = 110024;
16 const int aix = 4;
17 const double eps = 1e-7;
18 int n, M, k;
19 struct gEdge {
20     int u, v; double w;
21     bool operator <(const gEdge &e1) const { return w < e1.w - eps; }
22 } E[aix * maxn], MST[maxn];
23 struct point {
24     double x, y; int index; edge *in;
25     bool operator <(const point &p1) const { return x < p1.x - eps || (abs(x - p1.x) <= eps && y < p1.y - eps); }
26 };
27 struct edge { point *oi, *dt; edge *on, *op, *dn, *dp; };
28
29 point p[maxn], *Q[maxn];
30 edge mem[aix * maxn], *elist[aix * maxn];
31 int nfree;
32 void Alloc_memory() { nfree = aix * n; edge *e = mem; for (int i = 0; i < nfree; i++) elist[i] = e++; }
33 void Splice(edge *a, edge *b, point *v) {
34     edge *next;
35     if (Oi(a) == v) next = On(a), On(a) = b; else next = Dn(a), Dn(a) = b;
36     if (Oi(next) == v) Op(next) = b; else Dp(next) = b;

```

```

37 if (O1(b) == v) On(b) = next, Op(b) = a; else Dn(b) = next, Dp(b) = a;
38 }
39 edge *Make_edge(point *u, point *v) {
40     edge *e = elist[--nfree];
41     e->on = e->op = e->dn = e->dp = e; e->oi = u; e->dt = v;
42     if (!u->in) u->in = e;
43     if (!v->in) v->in = e;
44     return e;
45 }
46 edge *Join(edge *a, point *u, edge *b, point *v, int side) {
47     edge *e = Make_edge(u, v);
48     if (side == 1) {
49         if (O1(a) == u) Splice(Op(a), e, u);
50         else Splice(Dp(a), e, u);
51         Splice(b, e, v);
52     } else {
53         Splice(a, e, u);
54         if (O1(b) == v) Splice(Op(b), e, v);
55         else Splice(Dp(b), e, v);
56     } return e;
57 }
58 void Remove(edge *e) {
59     point *u = O1(e), *v = Dt(e);
60     if (u->in == e) u->in = e->on;
61     if (v->in == e) v->in = e->dn;
62     if (O1(e->on) == u) e->on->op = e->op; else e->on->dp = e->op;
63     if (O1(e->op) == u) e->op->on = e->on; else e->op->dn = e->on;
64     if (O1(e->dn) == v) e->dn->op = e->dp; else e->dn->dp = e->dp;
65     if (O1(e->dp) == v) e->dp->on = e->dn; else e->dp->dn = e->dn;
66     elist[nfree++] = e;
67 }
68 void Low_tangent(edge *e_l, point *o_l, edge *e_r, point *o_r, edge **l_low, point **OL, edge **r_low,
69     point **OR) {
70     for (point *d_l = Other(e_l, o_l), *d_r = Other(e_r, o_r); ; )
71         if (C3(o_l, o_r, d_l) < -eps) e_l = Prev(e_l, d_l), o_l = d_l, d_l = Other(e_l, o_l);
72         else if (C3(o_l, o_r, d_r) < -eps) e_r = Next(e_r, d_r), o_r = d_r, d_r = Other(e_r, o_r);
73         else break;
74     *OL = o_l, *OR = o_r; *l_low = e_l, *r_low = e_r;
75 }
76 void Merge(edge *lr, point *s, edge *rl, point *u, edge **tangent) {
77     double l1, l2, l3, l4, r1, r2, r3, r4, cot_L, cot_R, u1, v1, u2, v2, n1, cot_n, P1, cot_P;
78     point *O, *D, *OR, *OL; edge *B, *L, *R;
79     Low_tangent(lr, s, rl, u, &L, &OL, &R, &OR);
80     for (*tangent = B = Join(L, OL, R, OR, 0), O = OL, D = OR; ; ) {
81         edge *El = Next(B, O), *Er = Prev(B, D), *next, *prev;
82         point *l = Other(El, O), *r = Other(Er, D);
83         V(l, 0, l1, l2); V(l, D, l3, l4); V(r, 0, r1, r2); V(r, D, r3, r4);
84         double c1 = C2(l1, l2, l3, l4), cr = C2(r1, r2, r3, r4);
85         bool BL = c1 > eps, BR = cr > eps;
86         if (!BL && !BR) break;
87         if (BL) {
88             double dl = Dot(l1, l2, l3, l4);
89             for (cot_L = dl / c1; ; Remove(El), El = next, cot_L = cot_n) {
90                 next = Next(El, O); V(Other(next, O), 0, u1, v1); V(Other(next, O), D, u2, v2);
91                 n1 = C2(u1, v1, u2, v2); if (!n1 > eps) break;
92                 cot_n = Dot(u1, v1, u2, v2) / n1;
93                 if (cot_n > cot_L) break;
94             }
95         } if (BR) {
96             double dr = Dot(r1, r2, r3, r4);
97             for (cot_R = dr / cr; ; Remove(Er), Er = prev, cot_R = cot_P) {
98                 prev = Prev(Er, D); V(Other(prev, D), 0, u1, v1); V(Other(prev, D), D, u2, v2);
99                 P1 = C2(u1, v1, u2, v2); if (!P1 > eps) break;
100                 cot_P = Dot(u1, v1, u2, v2) / P1;
101                 if (cot_P > cot_R) break;
102             }
103         } if (!BL || (BL && BR && cot_R < cot_L)) B = Join(B, O, Er, r, 0), D = r;
104         else B = Join(El, l, B, D, 0), O = l;
105     }
106 }
107 void Divide(int s, int t, edge **L, edge **R) {
108     edge *a, *b, *c, *l1, *l_r, *r1, *rr, *tangent;
109     int n = t - s + 1;
110     if (n == 2) *L = *R = Make_edge(Q[s], Q[t]);
111     else if (n == 3) {
112         a = Make_edge(Q[s], Q[s + 1]), b = Make_edge(Q[s + 1], Q[t]);
113         Splice(a, b, Q[s + 1]);
114         double v = C3(Q[s], Q[s + 1], Q[t]);
115         if (v > eps) c = Join(a, Q[s], b, Q[t], 0), *L = a, *R = b;
116         else if (v < -eps) c = Join(a, Q[s], b, Q[t], 1), *L = c, *R = c;
117         else *L = a, *R = b;
118     } else if (n > 3) {
119         int split = (s + t) / 2;
120         Divide(s, split, &l1, &l_r); Divide(split + 1, t, &r1, &rr);
121         Merge(l_r, Q[split], r1, Q[split + 1], &tangent);
122         if (O1(tangent) == Q[s]) l1 = tangent;

```

```

123     if (Dt(tangent) == Q[t]) rr = tangent;
124     *L = l1; *R = rr;
125 }
126 }
127 void Make_Graph() {
128     edge *start, *e; point *u, *v;
129     for (int i = 0; i < n; i++) {
130         start = e = (u = &p[i])->in;
131         do { v = Other(e, u);
132             if (u < v) E[M++] .u = (u - p, v - p, dis(u, v)); // M < a1x * maxn
133             } while ((e = Next(e, u)) != start);
134     }
135 }
136 int b[maxn];
137 int Find(int x) { while (x != b[x]) { b[x] = b[b[x]]; x = b[x]; } return x; }
138 void Kruskal() {
139     memset(b, 0, sizeof(b)); sort(E, E + M);
140     for (int i = 0; i < n; i++) b[i] = i;
141     for (int i = 0, kk = 0; i < M && kk < n - 1; i++) {
142         int m1 = Find(E[i].u), m2 = Find(E[i].v);
143         if (m1 != m2) b[m1] = m2, MST[kk++] = E[i];
144     }
145 }
146 void solve() {
147     scanf("%d", &n);
148     for (int i = 0; i < n; i++) scanf("%lf%lf", &p[i].x, &p[i].y), p[i].index = i, p[i].in = NULL;
149     Alloc_memory(); sort(p, p + n);
150     for (int i = 0; i < n; i++) Q[i] = p + i;
151     edge *L, *R; Divide(0, n - 1, &L, &R);
152     M = 0; Make_Graph(); Kruskal();
153 }
154 int main() { solve(); return 0; }

```

## 1.10 四边形双费马点

```

1 typedef complex<double> Tpoint;
2 const double eps = 1e-8;
3 const double sqrt3 = sqrt(3.0);
4 bool cmp(const Tpoint &a, const Tpoint &b) {
5     return a.real() < b.real() - eps || (a.real() < b.real() + eps && a.imag() < b.imag());
6 }
7 Tpoint rotate(const Tpoint &a, const Tpoint &b, const Tpoint &c) {
8     Tpoint d = b - a; d = Tpoint(-d.imag(), d.real());
9     if (Sign(cross(a, b, c)) == Sign(cross(a, b, a + d))) d *= -1.0;
10    return unit(d);
11 }
12 Tpoint p[10], a[10], b[10];
13 int N, T;
14 double totlen(const Tpoint &p, const Tpoint &a, const Tpoint &b, const Tpoint &c) {
15     return abs(p - a) + abs(p - b) + abs(p - c);
16 }
17 double fermat(const Tpoint &x, const Tpoint &y, const Tpoint &z, Tpoint &cp) {
18     a[0] = a[3] = x; a[1] = a[4] = y; a[2] = a[5] = z;
19     double len = 1e100, len2;
20     for (int i = 0; i < 3; i++) {
21         len2 = totlen(a[i], x, y, z);
22         if (len2 < len) len = len2, cp = a[i];
23     }
24     for (int i = 0; i < 3; i++) {
25         b[i] = rotate(a[i + 1], a[i], a[i + 2]);
26         b[i] = (a[i + 1] + a[i]) / 2.0 + b[i] * (abs(a[i + 1] - a[i]) * sqrt3 / 2.0);
27     }
28     b[3] = b[0];
29     Tpoint cp2 = intersect(b[0], a[2], b[1], a[3]);
30     len2 = totlen(cp2, x, y, z);
31     if (len2 < len) len = len2, cp = cp2;
32     return len;
33 }
34 double getans(const Tpoint &a) {
35     double len = 0; for (int i = 0; i < N; i++) len += abs(a - p[i]);
36     return len;
37 }
38 double mindist(const Tpoint &p, const Tpoint &a, const Tpoint &b, const Tpoint &c, const Tpoint &d) {
39     return min(min(abs(p - a), abs(p - b)), min(abs(p - c), abs(p - d)));
40 }
41 int main() {
42     N = 4;
43     for (cin >> T; T; T--) {
44         double ret = 1e100, len_cur, len_before, len1, len2, len;
45         Tpoint cp, cp1, cp2;
46         Foru(i, 0, N) cin >> p[i];
47         Foru(i, 0, N) ret = min(ret, getans(p[i]));

```

```

48 Foru(i, 1, N) Foru(j, 1, N) if (j != i) Foru(k, 1, N) if (k != i && k != j) {
49     cMin(ret, abs(p[0] - p[i]) + abs(p[j] - p[k]))
50     + min( min(abs(p[0] - p[j]), abs(p[0] - p[k])),
51           min(abs(p[i] - p[j]), abs(p[i] - p[k]))
52     ));
53     ret = min(ret, getans(intersect(p[0], p[i], p[j], p[k])));
54 }
55 Foru(i, 0, N) Foru(j, i + 1, N) Foru(k, j + 1, N) {
56     double len = feramat(p[i], p[j], p[k], cp);
57     ret = min(ret, len + mindist(p[6 - i - j - k], p[i], p[j], p[k], cp));
58 }
59 sort(p, p + N, cmp);
60 for (int i = 1; i < N; i++) {
61     cp1 = (p[0] + p[i]) / 2.0;
62     int j, k;
63     for (j = 1; j < N && j == i; j++);
64     for (k = 6 - i - j, len_before = 1e100; ; ) {
65         len1 = feramat(cp1, p[j], p[k], cp2);
66         len1 = feramat(cp2, p[0], p[i], cp1);
67         len = len1 + abs(cp2 - p[j]) + abs(cp2 - p[k]);
68         if (len < len_before - (1e-6)) len_before = len;
69         else break;
70     } ret = min(ret, len_before);
71 } printf("%.4f\n", ret);
72 }
73 return 0;
74 }

```

### 1.11 三维计算几何基本操作

```

1 struct point { double x, y, z; // something omitted
2     friend point det(const point &a, const point &b) {
3         return point(a.y * b.z - a.z * b.y, a.z * b.x - a.x * b.z, a.x * b.y - a.y * b.x);
4     }
5     friend double mix(const point &a, const point &b, const point &c) {
6         return a.x * b.y * c.z + a.y * b.z * c.x + a.z * b.x * c.y - a.z * b.y * c.x - a.x * b.z * c.y - a.y
          * b.x * c.z;
7     }
8     double distLP(const point &p1, const point &p2) const {
9         return det(p2 - p1, *this - p1).len() / (p2 - p1).len();
10    }
11    double distFP(const point &p1, const point &p2, const point &p3) const {
12        point n = det(p2 - p1, p3 - p1); return fabs( dot(n, *this - p1) / n.len() );
13    }
14 };
15 double distLL(const point &p1, const point &p2, const point &q1, const point &q2) {
16     point p = q1 - p1, u = p2 - p1, v = q2 - q1;
17     double d = u.norm() * v.norm() - dot(u, v) * dot(u, v);
18     if (sign(d) == 0) return p1.distLP(q1, q2);
19     double s = (dot(p, u) * v.norm() - dot(p, v) * dot(u, v)) / d;
20     return (p1 + u * s).distLP(q1, q2);
21 }
22 double distSS(const point &p1, const point &p2, const point &q1, const point &q2) {
23     point p = q1 - p1, u = p2 - p1, v = q2 - q1;
24     double d = u.norm() * v.norm() - dot(u, v) * dot(u, v);
25     if (sign(d) == 0) return min( min((p1 - q1).len(), (p1 - q2).len()),
26                                   min((p2 - q1).len(), (p2 - q2).len()));
27     double s1 = (dot(p, u) * v.norm() - dot(p, v) * dot(u, v)) / d;
28     double s2 = (dot(p, v) * u.norm() - dot(p, u) * dot(u, v)) / d;
29     if (s1 < 0.0) s1 = 0.0; if (s1 > 1.0) s1 = 1.0;
30     if (s2 < 0.0) s2 = 0.0; if (s2 > 1.0) s2 = 1.0;
31     point r1 = p1 + u * s1; point r2 = q1 + v * s2;
32     return (r1 - r2).len();
33 }
34 bool isFL(const point &p, const point &o, const point &q1, const point &q2, point &res) {
35     double a = dot(o, q2 - p), b = dot(o, q1 - p), d = a - b;
36     if (sign(d) == 0) return false;
37     res = (q1 * a - q2 * b) / d;
38     return true;
39 }
40 bool isFF(const point &p1, const point &o1, const point &p2, const point &o2, point &a, point &b) {
41     point e = det(o1, o2), v = det(o1, e);
42     double d = dot(o2, v); if (sign(d) == 0) return false;
43     point q = p1 + v * (dot(o2, p2 - p1) / d);
44     a = q; b = q + e;
45     return true;
46 }

```

### 1.12 凸多面体切割

```

1 vector<vector<point>> > convexCut(const vector<vector<point>> > &pss, const point &p, const point &o) {
2     vector<vector<point>> > res;
3     vector<point> sec;
4     for (unsigned itr = 0, size = pss.size(); itr < size; ++itr) {
5         const vector<point> &ps = pss[itr];
6         int n = ps.size();
7         vector<point> qs;
8         bool dif = false;
9         for (int i = 0; i < n; ++i) {
10             int d1 = sign( dot(o, ps[i] - p) );
11             int d2 = sign( dot(o, ps[(i + 1) % n] - p) );
12             if (d1 <= 0) qs.push_back(ps[i]);
13             if (d1 * d2 < 0) {
14                 point q;
15                 isFL(p, o, ps[i], ps[(i + 1) % n], q); // must return true
16                 qs.push_back(q);
17                 sec.push_back(q);
18             }
19             if (d1 == 0) sec.push_back(ps[i]);
20             else dif = true;
21             dif |= dot(o, det(ps[(i + 1) % n] - ps[i], ps[(i + 2) % n] - ps[i])) < -EPS;
22         }
23         if (!qs.empty() && dif)
24             res.insert(res.end(), qs.begin(), qs.end());
25     }
26     if (!sec.empty()) {
27         vector<point> tmp( convexHull2D(sec, o) );
28         res.insert(res.end(), tmp.begin(), tmp.end());
29     }
30     return res;
31 }
32
33 vector<vector<point>> > initConvex() {
34     vector<vector<point>> > pss(6, vector<point>(4));
35     pss[0][0] = pss[1][0] = pss[2][0] = point(-INF, -INF, -INF);
36     pss[0][3] = pss[1][1] = pss[5][2] = point(-INF, -INF, INF);
37     pss[0][1] = pss[2][3] = pss[4][2] = point(-INF, INF, -INF);
38     pss[0][2] = pss[5][3] = pss[4][1] = point(-INF, INF, INF);
39     pss[1][3] = pss[2][1] = pss[3][1] = point( INF, -INF, -INF);
40     pss[1][2] = pss[5][1] = pss[3][3] = point( INF, -INF, INF);
41     pss[2][2] = pss[4][3] = pss[3][1] = point( INF, INF, -INF);
42     pss[5][0] = pss[4][0] = pss[3][0] = point( INF, INF, INF);
43     return pss;
44 }

```

### 1.13 三维凸包

不能有重点

```

1 namespace ConvexHull3D {
2     #define volume(a, b, c, d) (mix(ps[b] - ps[a], ps[c] - ps[a], ps[d] - ps[a]))
3     vector<Facet> getHull(int n, point ps[]) {
4         static int mark[MAXN][MAXN], a, b, c; int stamp = 0; bool exist = false;
5         vector<Facet> facet; random_shuffle(ps, ps + n);
6         for (int i = 2; i < n && !exist; i++) {
7             point ndir = det(ps[0] - ps[i], ps[1] - ps[i]);
8             if (ndir.len() < EPS) continue;
9             swap(ps[i], ps[2]); for (int j = i + 1; j < n && !exist; j++)
10                 if (sign(volume(0, 1, 2, j)) != 0) {
11                     exist = true; swap(ps[j], ps[3]);
12                     facet.push_back(Facet(0, 1, 2)); facet.push_back(Facet(0, 2, 1));
13                 }
14         }
15         if (!exist) return ConvexHull2D(n, ps);
16         for (int i = 0; i < n; ++i) for (int j = 0; j < n; ++j) mark[i][j] = 0;
17         stamp = 0; for (int v = 3; v < n; ++v) {
18             vector<Facet> tmp; ++stamp;
19             for (unsigned i = 0; i < facet.size(); i++) {
20                 a = facet[i].a; b = facet[i].b; c = facet[i].c;
21                 if (sign(volume(v, a, b, c)) < 0)
22                     mark[a][b] = mark[a][c] = mark[b][a] = mark[b][c] = mark[c][a] = mark[c][b] = stamp;
23                 else tmp.push_back(facet[i]);
24             } facet = tmp;
25             for (unsigned i = 0; i < tmp.size(); i++) {
26                 a = facet[i].a; b = facet[i].b; c = facet[i].c;
27                 if (mark[a][b] == stamp) facet.push_back(Facet(b, a, v));
28                 if (mark[b][c] == stamp) facet.push_back(Facet(c, b, v));
29                 if (mark[c][a] == stamp) facet.push_back(Facet(a, c, v));
30             }
31         } return facet;
32     }
33     #undef volume
34 }

```

```

34 namespace Gravity {
35     using ConvexHull3D::Facet;
36     point findG(point ps[], const vector<Facet> &facet) {
37         double ws = 0; point res(0.0, 0.0, 0.0), o = ps[ facet[0].a ];
38         for (int i = 0, size = facet.size(); i < size; ++i) {
39             const point &a = ps[ facet[i].a ], &b = ps[ facet[i].b ], &c = ps[ facet[i].c ];
40             point p = (a + b + c + o) * 0.25; double w = mix(a - o, b - o, c - o);
41             ws += w; res = res + p * w;
42         } res = res / ws;
43         return res;
44     }
45 }

```

## 1.14 球面点表面点距离

```

1 double distOnBall(double lat1, double long1, double lat2, double long2, double R) {
2     lat1 *= PI / 180; long1 *= PI / 180;
3     lat2 *= PI / 180; long2 *= PI / 180;
4     double x1 = cos(lat1) * sin(long1);
5     double y1 = cos(lat1) * cos(long1);
6     double z1 = sin(lat1);
7     double x2 = cos(lat2) * sin(long2);
8     double y2 = cos(lat2) * cos(long2);
9     double z2 = sin(lat2);
10    double theta = acos(x1 * x2 + y1 * y2 + z1 * z2);
11    return R * theta;
12 }

```

## 1.15 长方体表面点距离

```

1 int r;
2 void turn(int i, int j, int x, int y, int z, int x0, int y0, int L, int W, int H) {
3     if (z == 0) r = min(r, x * x + y * y);
4     else {
5         if (i >= 0 && i < 2) turn(i + 1, j, x0 + L + z, y, x0 + L - x, x0 + L, y0, H, W, L);
6         if (j >= 0 && j < 2) turn(i, j + 1, x, y0 + W + z, y0 + W - y, x0, y0 + W, L, H, W);
7         if (i <= 0 && i > -2) turn(i - 1, j, x0 - z, y, x - x0, x0 - H, y0, H, W, L);
8         if (j <= 0 && j > -2) turn(i, j - 1, x, y0 - z, y - y0, x0, y0 - H, L, H, W);
9     }
10 }
11 int calc(int L, int H, int W, int x1, int y1, int z1, int x2, int y2, int z2) {
12     if (z1 != 0 && z1 != H)
13         if (y1 == 0 || y1 == W) swap(y1, z1), swap(y2, z2), swap(W, H);
14         else swap(x1, z1), swap(x2, z2), swap(L, H);
15     if (z1 == H) z1 = 0, z2 = H - z2;
16     r = INF; turn(0, 0, x2 - x1, y2 - y1, z2, -x1, -y1, L, W, H);
17     return r;
18 }

```

## 1.16 最小覆盖球

```

1 namespace MinBall {
2     int outCnt;
3     point out[4], res;
4     double radius;
5     void ball() {
6         static point q[3];
7         static double m[3][3], sol[3], L[3], det;
8         int i, j;
9         res = point(0.0, 0.0, 0.0);
10        radius = 0.0;
11        switch (outCnt) {
12            case 1:
13                res = out[0];
14                break;
15            case 2:
16                res = (out[0] + out[1]) * 0.5;
17                radius = (res - out[0]).norm();
18                break;
19            case 3:
20                q[0] = out[1] - out[0];
21                q[1] = out[2] - out[0];
22                for (i = 0; i < 2; ++i)

```

```

23                for (j = 0; j < 2; ++j)
24                    m[i][j] = dot(q[i], q[j]) * 2.0;
25                for (i = 0; i < 2; ++i)
26                    sol[i] = dot(q[i], q[i]);
27                det = m[0][0] * m[1][1] - m[0][1] * m[1][0];
28                if (sign(det) == 0)
29                    return;
30                L[0] = (sol[0] * m[1][1] - sol[1] * m[0][1]) / det;
31                L[1] = (sol[1] * m[0][0] - sol[0] * m[1][0]) / det;
32                res = out[0] + q[0] * L[0] + q[1] * L[1];
33                radius = (res - out[0]).norm();
34                break;
35            case 4:
36                q[0] = out[1] - out[0];
37                q[1] = out[2] - out[0];
38                q[2] = out[3] - out[0];
39                for (i = 0; i < 3; ++i)
40                    for (j = 0; j < 3; ++j)
41                        m[i][j] = dot(q[i], q[j]) * 2;
42                for (i = 0; i < 3; ++i)
43                    sol[i] = dot(q[i], q[i]);
44                det = m[0][0] * m[1][1] * m[2][2] + m[0][1] * m[1][2] * m[2][0]
45                    + m[0][2] * m[2][1] * m[1][0] - m[0][2] * m[1][1] * m[2][0]
46                    - m[0][1] * m[1][0] * m[2][2] - m[0][0] * m[1][2] * m[2][1];
47                if (sign(det) == 0)
48                    return;
49                for (j = 0; j < 3; ++j) {
50                    for (i = 0; i < 3; ++i)
51                        m[i][j] = sol[i];
52                    L[j] = (m[0][0] * m[1][1] * m[2][2] + m[0][1] * m[1][2] * m[2][0]
53                        + m[0][2] * m[2][1] * m[1][0] - m[0][2] * m[1][1] * m[2][0]
54                        - m[0][1] * m[1][0] * m[2][2] - m[0][0] * m[1][2] * m[2][1])
55                        / det;
56                    for (i = 0; i < 3; ++i)
57                        m[i][j] = dot(q[i], q[j]) * 2;
58                }
59                res = out[0];
60                for (i = 0; i < 3; ++i)
61                    res += q[i] * L[i];
62                radius = (res - out[0]).norm();
63            }
64        }
65    }
66    void minball(int n, point pt[]) {
67        ball();
68        if (outCnt < 4)
69            for (int i = 0; i < n; ++i)
70                if ((res - pt[i]).norm() > +radius + EPS) {
71                    out[outCnt] = pt[i];
72                    ++outCnt;
73                    minball(i, pt);
74                    --outCnt;
75                    if (i > 0) {
76                        point Tt = pt[i];
77                        memmove(&pt[i], &pt[0], sizeof(point) * i);
78                        pt[0] = Tt;
79                    }
80                }
81    }
82    pair<point, double> main(int npoint, point pt[]) { // 0-based
83        random_shuffle(pt, pt + npoint);
84        radius = -1;
85        for (int i = 0; i < npoint; i++) {
86            if ((res - pt[i]).norm() > EPS + radius) {
87                outCnt = i;
88                out[0] = pt[i];
89                minball(i, pt);
90            }
91        }
92        return make_pair(res, sqrt(radius));
93    }
94 }
95 }

```

## 1.17 三维向量操作矩阵

- 绕单位向量  $u = (u_x, u_y, u_z)$  右手方向旋转  $\theta$  度的矩阵:

$$\begin{bmatrix} \cos \theta + u_x^2(1 - \cos \theta) & u_x u_y(1 - \cos \theta) - u_z \sin \theta & u_x u_z(1 - \cos \theta) + u_y \sin \theta \\ u_y u_x(1 - \cos \theta) + u_z \sin \theta & \cos \theta + u_y^2(1 - \cos \theta) & u_y u_z(1 - \cos \theta) - u_x \sin \theta \\ u_z u_x(1 - \cos \theta) - u_y \sin \theta & u_z u_y(1 - \cos \theta) + u_x \sin \theta & \cos \theta + u_z^2(1 - \cos \theta) \end{bmatrix}$$

- $$= \cos \theta I + \sin \theta \begin{bmatrix} 0 & -u_z & u_y \\ u_z & 0 & -u_x \\ -u_y & u_x & 0 \end{bmatrix} + (1 - \cos \theta) \begin{bmatrix} u_x^2 & u_x u_y & u_x u_z \\ u_y u_x & u_y^2 & u_y u_z \\ u_z u_x & u_z u_y & u_z^2 \end{bmatrix}$$
- 点  $a$  绕单位向量  $u = (u_x, u_y, u_z)$  右手方向旋转  $\theta$  度的对应点为  $a' = a \cos \theta + (u \times a) \sin \theta + (u \otimes u)a(1 - \cos \theta)$
  - 关于向量  $v$  作对称变换的矩阵  $H = I - 2 \frac{vv^T}{v^T v}$ ,
  - 点  $a$  对称点:  $a' = a - 2 \frac{v^T a}{v^T v} \cdot v$

1.18 立体角

对于任意一个四面体  $OABC$ , 从  $O$  点观察  $\triangle ABC$  的立体角  $\tan \frac{\Omega}{2} = \frac{\text{mix}(\vec{a}, \vec{b}, \vec{c})}{|a||b||c| + (\vec{a} \cdot \vec{b})|c| + (\vec{a} \cdot \vec{c})|b| + (\vec{b} \cdot \vec{c})|a|}$ .

2 数据结构

2.1 动态凸包 (只支持插入)

```
1 #define x first // upperHull ← (x,y)
2 #define y second // lowerHull ← (x,-y)
3 typedef map<int, int> mii;
4 typedef map<int, int>::iterator mit;
5 struct point { point(const mit &p): x(p->first), y(p->second) {} };
6 inline bool checkInside(mii &a, const point &p) { // border inclusive
7     int x = p.x, y = p.y; mit p1 = a.lower_bound(x);
8     if (p1 == a.end()) return false; if (p1->x == x) return y <= p1->y;
9     if (p1 == a.begin()) return false; mit p2(p1--);
10    return sign(det(p - point(p1), point(p2) - p)) >= 0;
11 } inline void addPoint(mii &a, const point &p) { // no collinear points
12     int x = p.x, y = p.y; mit pnt = a.insert(make_pair(x, y)).first, p1, p2;
13     for (pnt->y = y; ; a.erase(p2)) {
14         p1 = pnt; if (++p1 == a.end()) break;
15         p2 = p1; if (++p1 == a.end()) break;
16         if (det(point(p2) - p, point(p1) - p) < 0) break;
17     } for ( ; ; a.erase(p2)) {
18         if ((p1 = pnt) == a.begin()) break;
19         if (--p1 == a.begin()) break; p2 = p1--;
20         if (det(point(p2) - p, point(p1) - p) > 0) break;
21     }
22 }
```

2.2 Rope 用法

```
1 #include <ext/rope>
2 using __gnu_cxx::crope; using __gnu_cxx::rope;
3 a = b.substr(from, len); // [from, from + len)
4 a = b.substr(from); // [from, from]
5 b.c_str(); // might lead to memory leaks
6 b.delete_c_str(); // delete the c_str that created before
7 a.insert(p, str); // insert str before position p
8 a.erase(i, n); // erase [i, i + n)
```

2.3 Treap

```
1 struct node { int key, prio, size; node *ch[2]; } base[MAXN], *top, *root, *null, nil;
2 typedef node *tree;
3 tree newNode(int key) {
4     static int seed = 3312;
5     top->key = key; top->prio = seed = int(seed * 48271LL % 2147483647);
6     top->size = 1; top->ch[0] = top->ch[1] = null; return top++;
7 }
8 void Rotate(tree &x, int d) {
9     tree y = x->ch[!d]; x->ch[!d] = y->ch[d]; y->ch[d] = x; y->size = x->size;
10    x->size = x->ch[0]->size + 1 + x->ch[1]->size; x = y;
```

```
11 }
12 void Insert(tree &t, int key) {
13     if (t == null) t = newNode(key);
14     else { int d = t->key < key; Insert(t->ch[d], key); ++t->size;
15         if (t->ch[d]->prio < t->prio) Rotate(t, !d);
16     }
17 }
18 void Delete(tree &t, int key) {
19     if (t->key != key) { Delete(t->ch[t->key < key], key); --t->size; }
20     else if (t->ch[0] == null) t = t->ch[1];
21     else if (t->ch[1] == null) t = t->ch[0];
22     else { int d = t->ch[0]->prio < t->ch[1]->prio;
23         Rotate(t, d); Delete(t->ch[d], key); --t->size;
24     }
25 }
```

2.4 可持久化 Treap

```
1 inline bool randomBySize(int a, int b) {
2     static long long seed = 1;
3     return (seed = seed * 48271 % 2147483647) * (a + b) < 2147483647LL * a;
4 }
5 tree merge(tree x, tree y) {
6     if (x == null) return y; if (y == null) return x;
7     tree t = NULL;
8     if (randomBySize(x->size, y->size)) t = newNode(x), t->r = merge(x->r, y);
9     else t = newNode(y), t->l = merge(x, y->l);
10    update(t); return t;
11 }
12 void splitByKey(tree t, int k, tree &l, tree &r) { // [-∞, k)[k, +∞fty)
13     if (t == null) l = r = null;
14     else if (t->key < k) l = newNode(t), splitByKey(t->r, k, l->r, r), update(l);
15     else r = newNode(t), splitByKey(t->l, k, l, r->l), update(r);
16 }
17 void splitBySize(tree t, int k, tree &l, tree &r) { // [1, k)[k, +∞)
18     static int s; if (t == null) l = r = null;
19     else if ((s = t->l->size + 1) < k) l = newNode(t), splitBySize(t->r, k - s, l->r, r), update(l);
20     else r = newNode(t), splitBySize(t->l, k, l, r->l), update(r);
21 }
```

2.5 左偏树

```
1 tree merge(tree a, tree b) {
2     if (a == null) return b;
3     if (b == null) return a;
4     if (a->key > b->key) swap(a, b);
5     a->rc = merge(a->rc, b);
6     a->rc->fa = a;
7     if (a->lc->dist < a->rc->dist) swap(a->lc, a->rc);
8     a->dist = a->rc->dist + 1;
9     return a;
10 }
11 void erase(tree t) {
12     tree x = t->fa, y = merge(t->lc, t->rc);
13     if (y != null) y->fa = x;
14     if (x == null) root = y;
15     else
16         for ((x->lc == t ? x->lc : x->rc) = y; x != null; y = x, x = x->fa) {
17             if (x->lc->dist < x->rc->dist) swap(x->lc, x->rc);
18             if (x->rc->dist + 1 == x->dist) return;
19             x->dist = x->rc->dist + 1;
20         }
21 }
```

2.6 Link-Cut Tree

```
1 struct node { int rev; node *pre, *ch[2]; } base[MAXN], nil, *null;
2 typedef node *tree;
3 #define isRoot(x) (x->pre->ch[0] != x && x->pre->ch[1] != x)
4 #define isRight(x) (x->pre->ch[1] == x)
5 inline void MakeRev(tree t) { if (t != null) { t->rev ^= 1; swap(t->ch[0], t->ch[1]); } }
6 inline void PushDown(tree t) { if (t->rev) { MakeRev(t->ch[0]); MakeRev(t->ch[1]); t->rev = 0; } }
7 inline void Rotate(tree x) {
```



```

8   tree y = x->pre; PushDown(y); PushDown(x);
9   int d = isRight(x);
10  if (!isRoot(y)) y->pre->ch[isRight(y)] = x; x->pre = y->pre;
11  if ((y->ch[d] = x->ch[d]) != null) y->ch[d]->pre = y;
12  x->ch[!d] = y; y->pre = x; Update(y);
13
14  inline void Splay(tree x) {
15      PushDown(x); for (tree y; !isRoot(x); Rotate(x)) {
16          y = x->pre; if (!isRoot(y)) Rotate(isRight(x) != isRight(y) ? x : y);
17      } Update(x);
18  }
19  inline void Splay(tree x, tree to) {
20      PushDown(x); for (tree y; (y = x->pre) != to; Rotate(x)) if (y->pre != to)
21          Rotate(isRight(x) != isRight(y) ? x : y);
22      Update(x);
23  }
24  inline tree Access(tree t) {
25      tree last = null; for (; t != null; last = t, t = t->pre) Splay(t), t->ch[1] = last, Update(t);
26      return last;
27  }
28  inline void MakeRoot(tree t) { Access(t); Splay(t); MakeRev(t); }
29  inline tree FindRoot(tree t) { Access(t); Splay(t); tree last = null;
30      for (; t != null; last = t, t = t->ch[0]) PushDown(t); Splay(last); return last;
31  }
32  inline void Join(tree x, tree y) { MakeRoot(y); y->pre = x; }
33  inline void Cut(tree t) { Access(t); Splay(t); t->ch[0]->pre = null; t->ch[0] = null; Update(t); }
34  inline void Cut(tree x, tree y) {
35      tree upper = (Access(x), Access(y));
36      if (upper == x) { Splay(x); y->pre = null; x->ch[1] = null; Update(x); }
37      else if (upper == y) { Access(x); Splay(y); x->pre = null; y->ch[1] = null; Update(y); }
38      else assert(0); // impossible to happen
39  }
40  inline int Query(tree a, tree b) { // query the cost in path a <-> b, lca inclusive
41      Access(a); tree c = Access(b); // c is lca
42      int v1 = c->ch[1]->maxCost; Access(a);
43      int v2 = c->ch[1]->maxCost;
44      return max(max(v1, v2), c->cost);
45  }
46  void Init() {
47      null = &nil; null->ch[0] = null->ch[1] = null->pre = null; null->rev = 0;
48      Rep(1, 1, N) { node &n = base[i]; n.rev = 0; n.pre = n.ch[0] = n.ch[1] = null; }
49  }

```

## 2.7 K-D Tree Nearest

```

1  struct Point { int x, y; };
2  struct Rectangle {
3      int lx, rx, ly, ry;
4      void set(const Point &p) { lx = rx = p.x; ly = ry = p.y; }
5      void merge(const Point &o) {
6          lx = min(lx, o.x); rx = max(rx, o.x); ly = min(ly, o.y); ry = max(ry, o.y);
7      } void merge(const Rectangle &o) {
8          lx = min(lx, o.lx); rx = max(rx, o.rx); ly = min(ly, o.ly); ry = max(ry, o.ry);
9      } LL dist(const Point &p) {
10         LL res = 0;
11         if (p.x < lx) res += sqr(lx - p.x); else if (p.x > rx) res += sqr(p.x - rx);
12         if (p.y < ly) res += sqr(ly - p.y); else if (p.y > ry) res += sqr(p.y - ry);
13         return res;
14     }
15 };
16 struct Node { int child[2]; Point p; Rectangle rect; };
17 const int MAX_N = 111111;
18 const LL INF = 100000000;
19 int n, m, tot, root, LL result;
20 Point a[MAX_N], p; Node tree[MAX_N];
21 int build(int s, int t, bool d) {
22     int k = ++tot, mid = (s + t) >> 1;
23     nth_element(a + s, a + mid, a + t, d ? cmpXY : cmpYX);
24     tree[k].p = a[mid]; tree[k].rect.set(a[mid]); tree[k].child[0] = tree[k].child[1] = 0;
25     if (s < mid)
26         tree[k].child[0] = build(s, mid, d ^ 1), tree[k].rect.merge(tree[tree[k].child[0]].rect);
27     if (mid + 1 < t)
28         tree[k].child[1] = build(mid + 1, t, d ^ 1), tree[k].rect.merge(tree[tree[k].child[1]].rect);
29     return k;
30 }
31 int insert(int root, bool d) {
32     if (root == 0) {
33         tree[++tot].p = p; tree[tot].rect.set(p); tree[tot].child[0] = tree[tot].child[1] = 0;
34         return tot;
35     } tree[root].rect.merge(p);
36     if ((d && cmpXY(p, tree[root].p)) || (!d && cmpYX(p, tree[root].p))) {
37         tree[root].child[0] = insert(tree[root].child[0], d ^ 1);
38     } else tree[root].child[1] = insert(tree[root].child[1], d ^ 1);
39 }

```

```

39     return root;
40 }
41 void query(int k, bool d) {
42     if (tree[k].rect.dist(p) >= result) return;
43     cMin(result, dist(tree[k].p, p));
44     if ((d && cmpXY(p, tree[k].p)) || (!d && cmpYX(p, tree[k].p))) {
45         if (tree[k].child[0]) query(tree[k].child[0], d ^ 1);
46         if (tree[k].child[1]) query(tree[k].child[1], d ^ 1);
47     } else {
48         if (tree[k].child[1]) query(tree[k].child[1], d ^ 1);
49         if (tree[k].child[0]) query(tree[k].child[0], d ^ 1);
50     }
51 }
52 void example(int n) {
53     root = tot = 0; scan(a); root = build(0, n, 0); // init, a[0...n-1]
54     scan(p); root = insert(root, 0); // insert
55     scan(p); result = INF; ans = query(root, 0); // query
56 }

```

## 2.8 K-D Tree Farthest

输入  $n$  个点, 对每个询问  $px, py, k$ , 输出  $k$  远点的编号

```

1  struct Point { int x, y, id; };
2  struct Rectangle {
3      int lx, rx, ly, ry;
4      void set(const Point &p) { lx = rx = p.x; ly = ry = p.y; }
5      void merge(const Rectangle &o) {
6          lx = min(lx, o.lx); rx = max(rx, o.rx); ly = min(ly, o.ly); ry = max(ry, o.ry);
7      }
8      LL dist(const Point &p) { LL res = 0;
9          res += max(sqr(rx - p.x), sqr(lx - p.x));
10         res += max(sqr(ry - p.y), sqr(ly - p.y));
11         return res;
12     }
13 }; struct Node { Point p; Rectangle rect; };
14 const int MAX_N = 111111;
15 const LL INF = 1LL << 60;
16 int n, m;
17 Point a[MAX_N], b[MAX_N];
18 Node tree[MAX_N * 3];
19 Point p; // p is the query point
20 pair<LL, int> result[22];
21 void build(int k, int s, int t, bool d) {
22     int mid = (s + t) >> 1;
23     nth_element(a + s, a + mid, a + t, d ? cmpX : cmpY);
24     tree[k].p = a[mid];
25     tree[k].rect.set(a[mid]);
26     if (s < mid)
27         build(k << 1, s, mid, d ^ 1), tree[k].rect.merge(tree[k << 1].rect);
28     if (mid + 1 < t)
29         build(k << 1 | 1, mid + 1, t, d ^ 1), tree[k].rect.merge(tree[k << 1 | 1].rect);
30 }
31 void query(int k, int s, int t, bool d, int kth) {
32     if (tree[k].rect.dist(p) < result[kth].first) return;
33     pair<LL, int> tmp(dist(tree[k].p, p), -tree[k].p.id);
34     for (int i = 1; i <= kth; i++) if (tmp > result[i]) {
35         for (int j = kth + 1; j > i; j--) result[j] = result[j - 1]; result[i] = tmp;
36         break;
37     }
38     int mid = (s + t) >> 1;
39     if ((d && cmpX(p, tree[k].p)) || (!d && cmpY(p, tree[k].p))) {
40         if (mid + 1 < t) query(k << 1 | 1, mid + 1, t, d ^ 1, kth);
41         if (s < mid) query(k << 1, s, mid, d ^ 1, kth);
42     } else {
43         if (s < mid) query(k << 1, s, mid, d ^ 1, kth);
44         if (mid + 1 < t) query(k << 1 | 1, mid + 1, t, d ^ 1, kth);
45     }
46 }
47 void example(int n) {
48     scan(a); build(1, 0, n, 0); // init, a[0...n-1]
49     scan(p, k); // query
50     Rep(j, 1, k) result[j].first = -1;
51     query(1, 0, n, 0, k); ans = -result[k].second + 1;
52 }

```

## 2.9 树链剖分

```

1 int N, fa[MAXN], dep[MAXN], que[MAXN], size[MAXN], own[MAXN];
2 int LCA(int x, int y) { if (x == y) return x;
3   for ( ; ; x = fa[own[x]]) {
4     if (dep[x] < dep[y]) swap(x, y); if (own[x] == own[y]) return y;
5     if (dep[own[x]] < dep[own[y]]) swap(x, y);
6   } return -1;
7 }
8 void Decomposition() {
9   static int path[MAXN]; int x, y, a, next, head = 0, tail = 0, cnt; // BFS omitted
10  for (int i = 1; i <= N; ++i) if (own[a = que[i]] == -1)
11    for (x = a, cnt = 0; ; x = next) { next = -1; own[x] = a; path[++cnt] = x;
12      for (edge e(fir[x]); e; e = e->next) if ( (y = e->to) != fa[x] )
13        if (next == -1 || size[y] > size[next]) next = y;
14      if (next == -1) { tree[a].init(cnt, path); break; }
15    }
16 }

```

## 3 字符串相关

### 3.1 Manacher

```

1 // len[i] : the max length of palindrome whose mid point is (i / 2)
2 void Manacher(int n, char cs[], int len[]) { // 0-based, len[] must be double sized
3   for (int i = 0; i < n + n; ++i) len[i] = 0;
4   for (int i = 0, j = 0, k; i < n * 2; i += k, j = max(j - k, 0)) {
5     while (i - j >= 0 && i + j + 1 < n * 2 && cs[(i - j) / 2] == cs[(i + j + 1) / 2]) j++;
6     len[i] = j; for (k = 1; i - k >= 0 && j - k >= 0 && len[i - k] != j - k; k++)
7       len[i + k] = min(len[i - k], j - k);
8   }
9 }

```

### 3.2 KMP

$next[i] = \max\{len|A[0 \dots len - 1] = A \text{ 的第 } i \text{ 位向前或后的长度为 } len \text{ 的串}\}$

$ext[i] = \max\{len|A[0 \dots len - 1] = B \text{ 的第 } i \text{ 位向前或后的长度为 } len \text{ 的串}\}$

```

1 void KMP(char *a, int la, char *b, int lb, int *next, int *ext) {
2   --a; --b; --next; --ext;
3   for (int i = 2, j = next[1] = 0; i <= la; i++) {
4     while (j && a[j + 1] != a[i]) j = next[j]; if (a[j + 1] == a[i]) ++j; next[i] = j;
5   } for (int i = 1, j = 0; i <= lb; ++i) {
6     while (j && a[j + 1] != b[i]) j = next[j]; if (a[j + 1] == b[i]) ++j; ext[i] = j;
7     if (j == la) j = next[j];
8   }
9 } void ExKMP(char *a, int la, char *b, int lb, int *next, int *ext) {
10  next[0] = la; for (int &j = next[1] = 0; j + 1 < la && a[j] == a[j + 1]; ++j);
11  for (int i = 2, k = 1; i < la; ++i) {
12    int p = k + next[k], l = next[i - k]; if (l < p - i) next[i] = l;
13    else for (int &j = next[k = i] = max(0, p - i); i + j < la && a[j] == a[i + j]; ++j);
14  } for (int &j = ext[0] = 0; j < la && j < lb && a[j] == b[j]; ++j);
15  for (int i = 1, k = 0; i < lb; ++i) {
16    int p = k + ext[k], l = next[i - k]; if (l < p - i) ext[i] = l;
17    else for (int &j = ext[k = i] = max(0, p - i); j < la && i + j < lb && a[j] == b[i + j]; ++j);
18  }
19 }

```

### 3.3 后缀自动机

```

1 struct node { int len; node *fa, *go[26]; } base[MAXNODE], *top = base, *root, *que[MAXNODE];
2 typedef node *tree;
3 inline tree newNode(int len) {
4   top->len = len; top->fa = NULL; memset(top->go, 0, sizeof(top->go)); return top++;
5 } inline tree newNode(int len, tree fa, tree *go) {
6   top->len = len; top->fa = fa; memcpy(top->go, go, sizeof(top->go)); return top++;
7 } void construct(char *A, int N) {
8   tree p = root = newNode(0), q, up, fa;
9   for (int i = 0; i < N; ++i) {
10    int w = A[i] - 'a'; up = p; p = newNode(i + 1);
11    for ( ; up && !up->go[w]; up = up->fa) up->go[w] = p;
12    if (!up) p->fa = root;
13    else { q = up->go[w];

```

```

14      if (up->len + 1 == q->len) p->fa = q;
15      else { fa = newNode(up->len + 1, q->fa, q->go);
16        for (p->fa = q->fa = fa; up && up->go[w] == q; up = up->fa) up->go[w] = fa;
17      }
18    }
19  } static int cnt[MAXLEN]; memset(cnt, 0, sizeof(int) * (N + 1));
20  for (tree i(base); i != top; ++i) ++cnt[i->len];
21  Rep(i, 1, N) cnt[i] += cnt[i - 1];
22  for (tree i(base); i != top; ++i) Q[ cnt[i->len]-- ] = i;
23 }

```

### 3.4 后缀数组

待排序的字符串串在  $r[0 \dots n - 1]$  中, 最大值小于  $m$ .

$r[0 \dots n - 2] > 0, r[n - 1] = 0$ .

结果放在  $sa[0 \dots n - 1]$ .

```

1 namespace SuffixArrayDoubling {
2   int wa[MAXN], wb[MAXN], wv[MAXN], ws[MAXN];
3   int cmp(int *r, int a, int b, int l) { return r[a] == r[b] && r[a + l] == r[b + l]; }
4   void da(int *r, int *sa, int n, int m) { // the last char must be '$'
5     int i, j, p, *x = wa, *y = wb, *t;
6     for (i = 0; i < m; i++) ws[i] = 0;
7     for (i = 0; i < n; i++) ws[x[i]] = r[i]++;
8     for (i = 1; i < m; i++) ws[i] += ws[i - 1];
9     for (i = n - 1; i >= 0; i--) sa[-ws[x[i]]] = i;
10    for (j = 1, p = 1; p < n; j *= 2, m = p) {
11      for (p = 0; i = n - j; i < n; i++) y[p++] = i;
12      for (i = 0; i < n; i++) if (sa[i] >= j) y[p++] = sa[i] - j;
13      for (i = 0; i < n; i++) wv[i] = x[y[i]];
14      for (i = 0; i < m; i++) ws[i] = 0;
15      for (i = 0; i < n; i++) ws[wv[i]]++;
16      for (i = 1; i < m; i++) ws[i] += ws[i - 1];
17      for (i = n - 1; i >= 0; i--) sa[-ws[wv[i]]] = y[i];
18      for (t = x, x = y, y = t, p = 1, x[sa[0]] = 0, i = 1; i < n; i++)
19        x[sa[i]] = cmp(y, sa[i - 1], sa[i], j) ? p - 1 : p++;
20    }
21  } namespace CalcHeight {
22    int rank[MAXN], height[MAXN]; // if you add '$', remove
23    void calheight(int *r, int *sa, int n) { // it before call this function
24      int i, j, k = 0; for (i = 1; i <= n; i++) rank[sa[i]] = i;
25      for (i = 0; i < n; height[rank[i++]] = k)
26        for (k ? k-- : 0, j = sa[rank[i] - 1]; r[i + k] == r[j + k]; k++);
27    }

```

### 3.5 环串最小表示

```

1 int minimalRepresentation(int N, char *s) { // s must be double-sized and 0-based
2   int i, j, k, l; for (i = 0; i < N; ++i) s[i + N] = s[i]; s[N + N] = 0;
3   for (i = 0, j = 1; j < N; ) {
4     for (k = 0; k < N && s[i + k] == s[j + k]; ++k);
5     if (k >= N) break; if (s[i + k] < s[j + k]) j += k + 1;
6     else l = i + k, i = j, j = max(l, j) + 1;
7   } return i; // [i, i + N) is the minimal representation
8 }

```

## 4 图论

### 4.1 带花树

```

1 namespace Blossom {
2   int n, head, tail, S, T, lca;
3   int match[MAXN], Q[MAXN], pred[MAXN], label[MAXN], inq[MAXN], inb[MAXN];
4   vector<int> link[MAXN];
5   inline void push(int x) { Q[ tail++ ] = x; inq[x] = true; }
6   int findCommonAncestor(int x, int y) {
7     static bool inPath[MAXN]; for (int i = 0; i < n; ++i) inPath[i] = 0;
8     for ( ; x = pred[ match[x] ] ) { x = label[x]; inPath[x] = true; if (x == S) break; }
9     for ( ; y = pred[ match[y] ] ) { y = label[y]; if (inPath[y]) break; } return y;
10  }
11  void resetTrace(int x, int lca) {

```

```

12 while (label[x] != lca) { int y = match[x]; inb[ label[x] ] = inb[ label[y] ] = true;
13   x = pred[y]; if (label[x] != lca) pred[x] = y; }
14 void blossomContract(int x, int y) {
15   lca = findCommonAncestor(x, y);
16   Foru(i, 0, n) inb[i] = 0; resetTrace(x, lca); resetTrace(y, lca);
17   if (label[x] != lca) pred[x] = y; if (label[y] != lca) pred[y] = x;
18   Foru(i, 0, n) if (inb[ label[i] ]) { label[i] = lca; if (!inq[i]) push(i); }
19 }
20 bool findAugmentingPath() {
21   Foru(i, 0, n) pred[i] = -1, label[i] = i, inq[i] = 0;
22   int x, y, z; head = tail = 0;
23   for (push(S); head < tail; ) for (int i = (int)link[x = Q[head++]].size() - 1; i >= 0; --i) {
24     y = link[x][i]; if (label[x] == label[y] || x == match[y]) continue;
25     if (y == S || (match[y] >= 0 && pred[ match[y] ] >= 0)) blossomContract(x, y);
26     else if (pred[y] == -1) {
27       pred[y] = x; if (match[y] >= 0) push(match[y]);
28     }
29     for (x = y; x >= 0; x = z) {
30       y = pred[x], z = match[y]; match[x] = y, match[y] = x;
31     } return true; } } return false;
32 }
33 int findMaxMatching() {
34   int ans = 0; Foru(i, 0, n) match[i] = -1;
35   for (S = 0; S < n; ++S) if (match[S] == -1) if (findAugmentingPath()) ++ans;
36   return ans;
37 }
38 }

```

## 4.2 最大流

```

1 namespace Maxflow {
2   int h[MAXNODE], vh[MAXNODE], S, T, Ncnt; edge cur[MAXNODE], pe[MAXNODE];
3   void init(int _S, int _T, int _Ncnt) { S = _S; T = _T; Ncnt = _Ncnt; }
4   int maxflow() {
5     static int Q[MAXNODE]; int x, y, augc, flow = 0, head = 0, tail = 0; edge e;
6     Rep(i, 0, Ncnt) cur[i] = fir[i]; Rep(i, 0, Ncnt) h[i] = INF; Rep(i, 0, Ncnt) vh[i] = 0;
7     for (Q[head++] = T, h[T] = 0; head < tail; ) {
8       x = Q[head]; ++vh[ h[x] ];
9       for (e = fir[x]; e; e = e->next) if (e->op->c)
10         if (h[y = e->to] >= INF) h[y] = h[x] + 1, Q[head++] = y;
11     } for (x = S; h[S] < Ncnt; ) {
12       for (e = cur[x]; e; e = e->next) if (e->c)
13         if (h[y = e->to] + 1 == h[x]) { cur[x] = pe[y] = e; x = y; break; }
14       if (!e) {
15         if (--vh[ h[x] ] == 0) break; h[x] = Ncnt; cur[x] = NULL;
16         for (e = fir[x]; e; e = e->next) if (e->c)
17           if ( cMin( h[x], h[e->to] + 1 ) ) cur[x] = e;
18         ++vh[ h[x] ];
19         if (x != S) x = pe[x]->op->to;
20       } else if (x == T) { augc = INF;
21         for (x = T; x != S; x = pe[x]->op->to) cMin(augc, pe[x]->c);
22         for (x = T; x != S; x = pe[x]->op->to) {
23           pe[x]->c -= augc; pe[x]->op->c += augc;
24         } flow += augc;
25       }
26     } return flow;
27 }
28 }

```

## 4.3 最高标号预流推进

```

1 namespace Network {
2   int S, T, Ncnt, hsize, heap[MAXN], h[MAXN], inq[MAXN], Q[MAXN], vh[MAXN * 2 + 1];
3   LL E[MAXN]; edge cur[MAXN];
4   inline void pushFlow(int x, int y, edge e) {
5     int d = (int)min(E[x], (LL)e->c);
6     E[x] -= d; e->c -= d; E[y] += d; e->op->c += d;
7   } inline bool heapCmp(int x, int y) { return h[x] < h[y]; }
8   inline void hpush(int x) {
9     inq[x] = true; heap[++hsize] = x; push_heap(heap + 1, heap + hsize + 1, heapCmp);
10   } inline void hpop(int x) {
11     inq[x] = false; pop_heap(heap + 1, heap + hsize + 1, heapCmp); --hsize;
12   } LL maxFlow() {
13     int head = 0, tail = 0, x, y, h0;
14     memset(h, 63, sizeof(int) * (Ncnt + 1));
15     memset(vh, 0, sizeof(int) * (2 * Ncnt + 2));
16     memset(E, 0, sizeof(LL) * (Ncnt + 1));
17     memset(inq, 0, sizeof(int) * (Ncnt + 1));

```

```

18 memcopy(cur, fir, sizeof(edge) * (Ncnt + 1));
19 for (Q[head++] = T, h[T] = 0; head < tail; )
20   for (edge e(fir[x = Q[head++]]); e; e = e->next) if (e->op->c)
21     if (h[y = e->to] >= INF) h[y] = h[x] + 1, Q[head++] = y;
22 if (h[S] >= Ncnt) return 0;
23 h[S] = Ncnt; E[S] = LL_INF;
24 for (int i = 1; i <= Ncnt; ++i) if (h[i] <= Ncnt) ++vh[ h[i] ];
25 hsize = 0;
26 for (edge e(fir[S]); e; e = e->next) if (e->c && h[y = e->to] < Ncnt) {
27   pushFlow(S, y, e); if (!inq[y] && y != S && y != T) hpush(y);
28 } while (hsize) {
29   bool good = false;
30   for (edge &e(cur[x = heap[1]]); e; e = e->next) if (e->c)
31     if (h[x] == h[y = e->to] + 1) {
32       good = true; pushFlow(x, y, e); if (E[x] == 0) hpop(x);
33       if (inq[y] == false && y != S && y != T) hpush(y);
34       break;
35     }
36   if (!good) { // relabel
37     hpop(x); --vh[ h0 = h[x] ];
38     int &minH = h[x] = INF; cur[x] = NULL;
39     for (edge e(fir[x]); e; e = e->next) if (e->c)
40       if ( cMin(minH, h[e->to] + 1 ) ) cur[x] = fir[x];
41     hpush(x); ++vh[ h[x] ];
42     if (vh[h0] == 0 && h0 < Ncnt) {
43       hsize = 0;
44       for (int i = 1; i <= Ncnt; ++i) {
45         if (h[i] > h0 && h[i] < Ncnt) --vh[ h[i] ], ++vh[ h[i] = Ncnt + 1 ];
46         if (i != S && i != T && E[i]) heap[++hsize] = i;
47       } make_heap(heap + 1, heap + hsize + 1, heapCmp);
48     }
49   }
50 } return E[T];
51 }
52 }

```

## 4.4 KM

```

1 int N, Tcnt, w[MAXN][MAXN], slack[MAXN];
2 int lx[MAXN], linkx[MAXN], visy[MAXN], ly[MAXN], linky[MAXN], visx[MAXN]; // 初值全为 0
3 bool DFS(int x) { visx[x] = Tcnt;
4   Rep(y, 1, N) if (visy[y] != Tcnt) { int t = lx[x] + ly[y] - w[x][y];
5     if (t == 0) { visy[y] = Tcnt;
6       if (!linky[y] || DFS(linky[y])) { linkx[x] = y; linky[y] = x; return true; }
7     } else cMin(slack[y], t);
8   } return false;
9 } void KM() {
10   Tcnt = 0; Rep(x, 1, N) Rep(y, 1, N) cMax(lx[x], w[x][y]);
11   Rep(S, 1, N) { Rep(i, 1, N) slack[i] = INF;
12     for (++Tcnt; !DFS(S); ++Tcnt) { int d = INF;
13       Rep(y, 1, N) if (visy[y] != Tcnt) cMin(d, slack[y]);
14       Rep(x, 1, N) if (visx[x] == Tcnt) lx[x] -= d;
15       Rep(y, 1, N) if (visy[y] == Tcnt) ly[y] += d; else slack[y] -= d;
16     }
17   }
18 }

```

## 4.5 2-SAT 与 Kosaraju

注意 Kosaraju 需要建反图

```

1 namespace SCC {
2   int code[MAXN * 2], seq[MAXN * 2], sCnt;
3   void DFS_1(int x) { code[x] = 1;
4     for (edge e(fir[x]); e; e = e->next) if (code[e->to] == -1) DFS_1(e->to);
5     seq[++sCnt] = x;
6   } void DFS_2(int x) { code[x] = sCnt;
7     for (edge e(fir2[x]); e; e = e->next) if (code[e->to] == -1) DFS_2(e->to); }
8   void SCC(int N) {
9     sCnt = 0; for (int i = 1; i <= N; ++i) code[i] = -1;
10    for (int i = 1; i <= N; ++i) if (code[i] == -1) DFS_1(i);
11    sCnt = 0; for (int i = 1; i <= N; ++i) code[i] = -1;
12    for (int i = N; i >= 1; --i) if (code[seq[i]] == -1) {
13      ++sCnt; DFS_2(seq[i]); }
14  }
15 } // true - 2s - 1
16 // false - 2s
17 bool TwoSat() { SCC::SCC(N + N);

```

```
18 // if code[2i - 1] = code[2i]: no solution
19 // if code[2i - 1] > code[2i]: i selected. else i not selected
20 }
```

## 4.6 全局最小割 Stoer-Wagner

```
1 int minCut(int N, int G[MAXN][MAXN]) { // 0-based
2     static int weight[MAXN], used[MAXN]; int ans = INT_MAX;
3     while (N > 1) {
4         for (int i = 0; i < N; ++i) used[i] = false; used[0] = true;
5         for (int i = 0; i < N; ++i) weight[i] = G[i][0];
6         int S = -1, T = 0;
7         for (int _r = 2; _r <= N; ++_r) { // N - 1 selections
8             int x = -1;
9             for (int i = 0; i < N; ++i) if (!used[i])
10                 if (x == -1 || weight[i] > weight[x]) x = i;
11             for (int i = 0; i < N; ++i) weight[i] += G[x][i];
12             S = T; T = x; used[x] = true;
13             } ans = min(ans, weight[T]);
14             for (int i = 0; i < N; ++i) G[i][S] += G[i][T], G[S][i] += G[i][T];
15             G[S][S] = 0; --N;
16             for (int i = 0; i <= N; ++i) swap(G[i][T], G[i][N]);
17             for (int i = 0; i < N; ++i) swap(G[T][i], G[N][i]);
18         } return ans;
19 }
```

## 4.7 Hopcroft-Karp

```
1 int N, M, level[MAXN], matchX[MAXN], matchY[MAXN];
2 bool used[MAXN];
3 bool DFS(int x) {
4     used[x] = true; for (edge e(fir[x]); e; e = e->next) {
5         int y = e->to, z = matchY[y];
6         if (z == -1 || (!used[z] && level[x] < level[z] && DFS(z))) {
7             matchX[x] = y; matchY[y] = x; return true;
8         }
9     } return false;
10 }
11 int maxMatch() {
12     for (int i = 0; i < N; ++i) used[i] = false;
13     for (int i = 0; i < N; ++i) matchX[i] = -1;
14     for (int i = 0; i < M; ++i) matchY[i] = -1;
15     for (int i = 0; i < N; ++i) level[i] = -1;
16     int match = 0, d;
17     for ( ; ; match += d) {
18         static int Q[MAXN * 2 + 1];
19         int head = 0, tail = d = 0;
20         for (int x = 0; x < N; ++x) level[x] = -1;
21         for (int x = 0; x < N; ++x) if (matchX[x] == -1)
22             level[x] = 0, Q[++tail] = x;
23         while (head < tail)
24             for (edge e(fir[x = Q[++head]]); e; e = e->next) {
25                 int y = e->to, z = matchY[y];
26                 if (z != -1 && level[z] < 0) level[z] = level[x] + 1, Q[++tail] = z;
27             }
28         for (int x = 0; x < N; ++x) used[x] = false;
29         for (int x = 0; x < N; ++x) if (matchX[x] == -1) if (DFS(x)) ++d;
30         if (d == 0) break;
31     } return match;
32 }
```

## 4.8 欧拉路

```
1 vector<int> eulerianWalk(int N, int S) {
2     static int res[MAXM], stack[MAXN]; static edge cur[MAXN];
3     int rcnt = 0, top = 0, x; for (int i = 1; i <= N; ++i) cur[i] = fir[i];
4     for (stack[top++] = S; top; ) {
5         for (x = stack[--top]; ; ) {
6             edge &e = cur[x]; if (e == NULL) break;
7             stack[top++] = x; x = e->to; e = e->next;
8             // 对于无向图需要删掉反向边
9         } res[rcnt++] = x;
10    } reverse(res, res + rcnt); return vector<int>(res, res + rcnt);
11 }
```

## 4.9 稳定婚姻

```
1 namespace StableMatching {
2     int pairM[MAXN], pairW[MAXN], p[MAXN];
3     // init: pairM[0..n - 1] = pairW[0..n - 1] = -1, p[0..n - 1] = 0
4     void stableMatching(int n, int orderM[MAXN][MAXN], int preferW[MAXN][MAXN]) {
5         for (int i = 0; i < n; ++i) while (pairM[i] < 0) {
6             int w = orderM[i][p[i]++], m = pairW[w];
7             if (m == -1) pairM[i] = w, pairW[w] = i;
8             else if (preferW[w][i] < preferW[w][m])
9                 pairM[m] = -1, pairM[i] = w, pairW[w] = i, i = m;
10        }
11    }
12 }
```

## 4.10 最大团搜索

```
1 namespace MaxClique { // 1-based
2     int g[MAXN][MAXN], len[MAXN], list[MAXN][MAXN], mc[MAXN], ans, found;
3     void DFS(int size) {
4         if (len[size] == 0) { if (size > ans) ans = size, found = true; return; }
5         for (int k = 0; k < len[size] && !found; ++k) {
6             if (size + len[size] - k <= ans) break;
7             int i = list[size][k]; if (size + mc[i] <= ans) break;
8             for (int j = k + 1; len[size + 1] = 0; j < len[size]; ++j) if (g[i][list[size][j]])
9                 list[size + 1][len[size + 1]++] = list[size][j];
10            DFS(size + 1);
11        }
12    }
13    int work(int n) {
14        mc[n] = ans = 1; for (int i = n - 1; i; --i) { found = false; len[i] = 0;
15            for (int j = i + 1; j <= n; ++j) if (g[i][j]) list[i][len[i]++] = j;
16            DFS(i); mc[i] = ans;
17        } return ans;
18    }
19 }
```

## 4.11 最小树形图

```
1 namespace EdmondsAlgorithm { // O(ElogE + V^2) !!! 0-based !!!
2     struct enode { int from, c, key, delta, dep; enode *ch[2], *next;
3     } ebase[maxm], *etop, *fir[maxn], nil, *null, *inEdge[maxn], *chs[maxn];
4     typedef enode *edge; typedef enode *tree;
5     int n, m, setFa[maxn], deg[maxn], que[maxn];
6     inline void pushDown(tree x) { if (x->delta) {
7         x->ch[0]->key += x->delta; x->ch[0]->delta += x->delta;
8         x->ch[1]->key += x->delta; x->ch[1]->delta += x->delta; x->delta = 0;
9     } }
10    tree merge(tree x, tree y) {
11        if (x == null) return y; if (y == null) return x;
12        if (x->key > y->key) swap(x, y); pushDown(x); x->ch[1] = merge(x->ch[1], y);
13        if (x->ch[0]->dep < x->ch[1]->dep) swap(x->ch[0], x->ch[1]);
14        x->dep = x->ch[1]->dep + 1; return x;
15    }
16    void addEdge(int u, int v, int w) {
17        etop->from = u; etop->c = w; etop->delta = etop->dep = 0;
18        etop->next = fir[v]; etop->ch[0] = etop->ch[1] = null;
19        fir[v] = etop; inEdge[v] = merge(inEdge[v], etop++);
20    }
21    void deleteMin(tree &r) { pushDown(r); r = merge(r->ch[0], r->ch[1]); }
22    int findSet(int x) { return setFa[x] == x ? x : setFa[x] = findSet(setFa[x]); }
23    void clear(int V, int E) {
24        null = &nil; null->ch[0] = null->ch[1] = null; null->dep = -1;
25        n = V; m = E; etop = ebase; Foru(i, 0, V) fir[i] = NULL; Foru(i, 0, V) inEdge[i] = null;
26    }
27    int solve(int root) { int res = 0, head, tail;
28        for (int i = 0; i < n; ++i) setFa[i] = i;
29        for ( ; ; ) { memset(deg, 0, sizeof(int) * n); chs[root] = inEdge[root];
30            for (int i = 0; i < n; ++i) if (i != root && setFa[i] == i) {
31                while (findSet(inEdge[i]->from) == findSet(i)) deleteMin(inEdge[i]);
32                ++deg[ findSet(chs[i] = inEdge[i]->from) ];
33            }
34            for (int i = head = tail = 0; i < n; ++i)
35                if (i != root && setFa[i] == i && deg[i] == 0) que[tail++] = i;
36            while (head < tail) {
37                int x = findSet(chs[que[head++]]->from);
```

```

38     if (--deg[x] == 0) que[tail++] = x;
39     } bool found = false;
40     for (int i = 0; i < n; ++i) if (i != root && setFa[i] == i && deg[i] > 0) {
41         int j = i; tree temp = null; found = true;
42         do {setFa[j] = findSet(chs[j]->from)} = i;
43         deleteMin(inEdge[j]); res += chs[j]->key;
44         inEdge[j]->key -= chs[j]->key; inEdge[j]->delta -= chs[j]->key;
45         temp = merge(temp, inEdge[j]);
46         } while (j != i; inEdge[i] = temp;
47     } if (!found) break;
48     } for (int i = 0; i < n; ++ i) if (i != root && setFa[i] == i) res += chs[i]->key;
49     return res;
50 }
51 }
52 namespace ChuLiu { //  $O(V^3)$  !!! 1-based !!!
53 int n, used[maxn], pass[maxn], eg[maxn], more, que[maxn], g[maxn][maxn];
54 void combine(int id, int &sum) { int tot = 0, from, i, j, k;
55     for ( ; id != 0 && !pass[id]; id = eg[id]) que[tot++] = id, pass[id] = 1;
56     for (from = 0; from < tot && que[from] != id; from++);
57     if (from == tot) return; more = 1;
58     for (i = from; i < tot; i++) {
59         sum += g[eg[que[i]]][que[i]]; if (i == from) continue;
60         for (j = used[que[i]] = 1; j <= n; j++) if (!used[j])
61             if (g[que[i]][j] < g[id][j]) g[id][j] = g[que[i]][j];
62     }
63     for (i = 1; i <= n; i++) if (!used[i] && i != id)
64         for (j = from; j < tot; j++) {
65             k = que[j]; if (g[i][id] > g[i][k] - g[eg[k]][k])
66                 g[i][id] = g[i][k] - g[eg[k]][k];
67         }
68 }
69 void clear(int V) { n = V; Rep(i, 1, V) Rep(j, 1, V) g[i][j] = inf; }
70 int solve(int root) {
71     int i, j, k, sum = 0; memset(used, 0, sizeof(int) * (n + 1));
72     for (more = 1; more; ) {
73         more = 0; memset(eg, 0, sizeof(int) * (n + 1));
74         for (i = 1; i <= n; i++) if (!used[i] && i != root) {
75             for (j = 1, k = 0; j <= n; j++) if (!used[j] && i != j)
76                 if (k == 0 || g[j][i] < g[k][i]) k = j;
77             eg[i] = k;
78         } memset(pass, 0, sizeof(int) * (n + 1));
79         for (i = 1; i <= n; i++) if (!used[i] && !pass[i] && i != root)
80             combine(i, sum);
81     } for (i = 1; i <= n; i++) if (!used[i] && i != root) sum += g[eg[i]][i];
82     return sum;
83 }
84 }

```

## 4.12 离线动态最小生成树

$O(Q \log^2 Q)$ .  $(qx[i], qy[i])$  表示将编号为  $qx[i]$  的边的权值改为  $qy[i]$ , 删除一条边相当于将其权值改为  $\infty$ , 加入一条边相当于将其权值从  $\infty$  变成某个值.

```

1  const int maxn = 100000 + 5;
2  const int maxm = 1000000 + 5;
3  const int maxq = 1000000 + 5;
4  const int qsize = maxm + 3 * maxq;
5  int n, m, Q, x[qsize], y[qsize], z[qsize], qx[maxq], qy[maxq], a[maxn], *tz;
6  int kx[maxn], ky[maxn], kt, vd[maxn], id[maxm], app[maxm];
7  bool extra[maxn];
8  void init() {
9      scanf("%d%d", &n, &m); for (int i = 0; i < m; i++) scanf("%d%d%d", x + i, y + i, z + i);
10     scanf("%d", &Q); for (int i = 0; i < Q; i++) { scanf("%d%d", qx + i, qy + i); qx[i]--; }
11 }
12 int find(int x) {
13     int root = x, next; while (a[root]) root = a[root];
14     while ((next = a[x]) != 0) a[x] = root, x = next; return root;
15 }
16 inline bool cmp(const int &a, const int &b) { return tz[a] < tz[b]; }
17 void solve(int *qx, int *qy, int Q, int n, int *x, int *y, int *z, int m, long long ans) {
18     int ri, rj;
19     if (Q == 1) {
20         for (int i = 1; i <= n; i++) a[i] = 0; z[qx[0]] = qy[0];
21         for (int i = 0; i < m; i++) id[i] = i;
22         tz = z; sort(id, id + m, cmp);
23         for (int i = 0; i < m; i++) {
24             ri = find(x[id[i]]); rj = find(y[id[i]]);
25             if (ri != rj) ans += z[id[i]], a[ri] = rj;
26         } printf("%I64d\n", ans);
27         return;
28     } int tm = kt = 0, n2 = 0, m2 = 0;
29     for (int i = 1; i <= n; i++) a[i] = 0;

```

```

30     for (int i = 0; i < Q; i++) {
31         ri = find(x[qx[i]]); rj = find(y[qx[i]]); if (ri != rj) a[ri] = rj;
32     }
33     for (int i = 0; i < m; i++) extra[i] = true;
34     for (int i = 0; i < Q; i++) extra[qx[i]] = false;
35     for (int i = 0; i < m; i++) if (extra[i]) id[tm++] = i;
36     tz = z; sort(id, id + tm, cmp);
37     for (int i = 0; i < tm; i++) {
38         ri = find(x[id[i]]); rj = find(y[id[i]]);
39         if (ri != rj)
40             a[ri] = rj, ans += z[id[i]], kx[kt] = x[id[i]], ky[kt] = y[id[i]], kt++;
41     }
42     for (int i = 1; i <= n; i++) a[i] = 0;
43     for (int i = 0; i < kt; i++) a[find(kx[i])] = find(ky[i]);
44     for (int i = 1; i <= n; i++) if (a[i] == 0) vd[i] = ++n2;
45     for (int i = 1; i <= n; i++) if (a[i] != 0) vd[i] = vd[find(i)];
46     int *Nx = x + m, *Ny = y + m, *Nz = z + m;
47     for (int i = 0; i < m; i++) app[i] = -1;
48     for (int i = 0; i < Q; i++)
49         if (app[qx[i]] == -1)
50             Nx[m2] = vd[x[qx[i]]], Ny[m2] = vd[y[qx[i]]], Nz[m2] = z[qx[i]], app[qx[i]] = m2, m2++;
51     for (int i = 0; i < Q; i++) {
52         z[qx[i]] = qy[i];
53         qx[i] = app[qx[i]];
54     }
55     for (int i = 1; i <= n2; i++) a[i] = 0;
56     for (int i = 0; i < tm; i++) {
57         ri = find(vd[x[id[i]]]); rj = find(vd[y[id[i]]]);
58         if (ri != rj)
59             a[ri] = rj, Nx[m2] = vd[x[id[i]]], Ny[m2] = vd[y[id[i]]], Nz[m2] = z[id[i]], m2++;
60     }
61     int mid = Q / 2;
62     solve(qx, qy, mid, n2, Nx, Ny, Nz, m2, ans);
63     solve(qx + mid, qy + mid, Q - mid, n2, Nx, Ny, Nz, m2, ans);
64 }
65 void work() { if (Q) solve(qx, qy, Q, n, x, y, z, m, 0); }
66 int main() { init(); work(); return 0; }

```

## 4.13 弦图

- 任何一个弦图都至少有一个单纯点, 不是完全图的弦图至少有两个不相邻的单纯点.

- 设第  $i$  个点在弦图的完美消除序列第  $p(i)$  个. 令  $N(v) = \{w | w \text{ 与 } v \text{ 相邻且 } p(w) > p(v)\}$  弦图的极大团一定是  $v \cup N(v)$  的形式.

- 弦图最多有  $n$  个极大团.

- 设  $next(v)$  表示  $N(v)$  中最前的点. 令  $w^*$  表示所有满足  $A \in B$  的  $w$  中最后的一个点. 判断  $v \cup N(v)$  是否为极大团, 只需判断是否存在一个  $w$ , 满足  $Next(w) = v$  且  $|N(v)| + 1 \leq |N(w)|$  即可.

- 最小染色: 完美消除序列从后往前依次给每个点染色, 给每个点染上可以染的最小的颜色. (团数 = 色数)

- 最大独立集: 完美消除序列从前往后能选就选.

- 最小团覆盖: 设最大独立集为  $\{p_1, p_2, \dots, p_t\}$ , 则  $\{p_1 \cup N(p_1), \dots, p_t \cup N(p_t)\}$  为最小团覆盖. (最大独立集数 = 最小团覆盖数)

```

1  class Chordal { // 1-Based, G is the Graph, must be sorted before call Check_Chordal
2  public: // Construct will sort it automatically
3      int v[Maxn], id[Maxn]; bool inseq[Maxn]; priority_queue<pair<int, int> > pq;
4      vector<int> Construct_Perfect_Elimination_Sequence(vector<int> *G, int n) { //  $O(m + n \log n)$ 
5          vector<int> seq(n + 1, 0);
6          for (int i = 0; i <= n; ++i) inseq[i] = false, sort(G[i].begin(), G[i].end()), v[i] = 0;
7          int cur = n; pair<int, int> Mx; while (!pq.empty()) pq.pop(); pq.push(make_pair(0, 1));
8          for (int i = n; i >= 1; --i) {
9              while (!pq.empty() && (Mx = pq.top(), inseq[Mx.second] || Mx.first != v[Mx.second])) pq.pop();
10             id[Mx.second] = cur;
11             int x = seq[cur--] = Mx.second, sz = (int)G[Mx.second].size(); inseq[x] = true;
12             for (int j = 0; j < sz; ++j) {
13                 int y = G[x][j]; if (!inseq[y]) pq.push(make_pair(++v[y], y));
14             }
15         } return seq;
16     }
17     bool Check_Chordal(vector<int> *G, vector<int> &seq, int n) { //  $O(n + m \log n)$ , plz gen seq first

```

```

18 bool isChordal = true;
19 for (int i = n - 1; i >= 1 && isChordal; --i) {
20     int x = seq[i], sz, y = -1;
21     if ((sz = (int)G[x].size()) == 0) continue;
22     for(int j = 0; j < sz; ++j) {
23         if (id[G[x][j]] < i) continue;
24         if (y == -1 || id[y] > id[G[x][j]]) y = G[x][j];
25     } if (y == -1) continue;
26     for (int j = 0; j < sz; ++j) {
27         int y1 = G[x][j]; if (id[y1] < i) continue;
28         if (y1 == y || binary_search(G[y].begin(), G[y].end(), y1)) continue;
29         isChordal = false; break;
30     }
31 } return isChordal;
32 }
33 };

```

#### 4.14 K 短路 (允许重复)

```

1 #define for_each(it, v) for (vector<Edge*>::iterator it = (v).begin(); it != (v).end(); ++it)
2 const int MAX_N = 10000;
3 const int MAX_M = 50000;
4 const int MAX_K = 10000;
5 const int INF = 1000000000;
6
7 struct Edge {
8     int from, to, weight;
9 };
10 struct HeapNode {
11     Edge* edge;
12     int depth;
13     HeapNode* child[4];
14     //child[0..1] for heap G
15     //child[2..3] for heap out edge
16 };
17
18 int n, m, k, s, t;
19 Edge* edge[MAX_M];
20 int dist[MAX_N];
21 Edge* prev[MAX_N];
22 vector<Edge*> graph[MAX_N];
23 vector<Edge*> graphR[MAX_N];
24 HeapNode* nullNode;
25 HeapNode* heapTop[MAX_N];
26
27 HeapNode* createHeap(HeapNode* curNode, HeapNode* newNode) {
28     if (curNode == nullNode) return newNode;
29     HeapNode* rootNode = new HeapNode;
30     memcpy(rootNode, curNode, sizeof(HeapNode));
31     if (newNode->edge->weight < curNode->edge->weight) {
32         rootNode->edge = newNode->edge;
33         rootNode->child[2] = newNode->child[2];
34         rootNode->child[3] = newNode->child[3];
35         newNode->edge = curNode->edge;
36         newNode->child[2] = curNode->child[2];
37         newNode->child[3] = curNode->child[3];
38     }
39     if (rootNode->child[0]->depth < rootNode->child[1]->depth)
40         rootNode->child[0] = createHeap(rootNode->child[0], newNode);
41     else
42         rootNode->child[1] = createHeap(rootNode->child[1], newNode);
43     rootNode->depth = max(rootNode->child[0]->depth, rootNode->child[1]->depth) + 1;
44     return rootNode;
45 }
46
47 bool heapNodeMoreThan(HeapNode* node1, HeapNode* node2) {
48     return node1->edge->weight > node2->edge->weight;
49 }
50
51 int main() {
52     scanf("%d%d%d", &n, &m, &k);
53     scanf("%d%d", &s, &t);
54     s--; t--;
55     while (m--) {
56         Edge* newEdge = new Edge;
57         int i, j, w;
58         scanf("%d%d%d", &i, &j, &w);
59         i--; j--;
60         newEdge->from = i;
61         newEdge->to = j;
62         newEdge->weight = w;
63         graph[i].push_back(newEdge);
64         graphR[j].push_back(newEdge);

```

```

65     }
66
67     //Dijkstra
68     queue<int> dfsOrder;
69
70     memset(dist, -1, sizeof(dist));
71     typedef pair<int, pair<int, Edge*>> DijkstraQueueItem;
72     priority_queue<DijkstraQueueItem, vector<DijkstraQueueItem>, greater<DijkstraQueueItem>> dq;
73     dq.push(make_pair(0, make_pair(t, (Edge*) NULL)));
74     while (!dq.empty()) {
75         int d = dq.top().first;
76         int i = dq.top().second.first;
77         Edge* edge = dq.top().second.second;
78         dq.pop();
79         if (dist[i] != -1) continue;
80         dist[i] = d;
81         prev[i] = edge;
82         dfsOrder.push(i);
83         for_each(it, graphR[i]) dq.push(make_pair(d + (*it)->weight, make_pair((*it)->from, *it)));
84     }
85
86     //Create edge heap
87     nullNode = new HeapNode;
88     nullNode->depth = 0;
89     nullNode->edge = new Edge;
90     nullNode->edge->weight = INF;
91     fill(nullNode->child, nullNode->child + 4, nullNode);
92
93     while (!dfsOrder.empty()) {
94         int i = dfsOrder.front();
95         dfsOrder.pop();
96
97         if (prev[i] == NULL)
98             heapTop[i] = nullNode;
99         else
100             heapTop[i] = heapTop[prev[i]->to];
101
102         vector<HeapNode*> heapNodeList;
103         for_each(it, graph[i]) {
104             int j = (*it)->to;
105             if (dist[j] == -1)
106                 continue;
107             (*it)->weight += dist[j] - dist[i];
108             if (prev[j] != *it) {
109                 HeapNode* curNode = new HeapNode;
110                 fill(curNode->child, curNode->child + 4, nullNode);
111                 curNode->depth = 1;
112                 curNode->edge = *it;
113                 heapNodeList.push_back(curNode);
114             }
115         }
116
117         if (!heapNodeList.empty()) { //Create heap out
118             make_heap(heapNodeList.begin(), heapNodeList.end(), heapNodeMoreThan);
119             int size = heapNodeList.size();
120             for (int p = 0; p < size; p++) {
121                 heapNodeList[p]->child[2] = 2 * p + 1 < size ? heapNodeList[2 * p + 1] : nullNode;
122                 heapNodeList[p]->child[3] = 2 * p + 2 < size ? heapNodeList[2 * p + 2] : nullNode;
123             }
124             heapTop[i] = createHeap(heapTop[i], heapNodeList.front());
125         }
126     }
127
128     //Walk on DAG
129     typedef pair<long long, HeapNode*> DAGQueueItem;
130     priority_queue<DAGQueueItem, vector<DAGQueueItem>, greater<DAGQueueItem>> aq;
131     if (dist[s] == -1)
132         printf("NO\n");
133     else {
134         printf("%d\n", dist[s]);
135         if (heapTop[s] != nullNode)
136             aq.push(make_pair(dist[s] + heapTop[s]->edge->weight, heapTop[s]));
137     }
138     k--;
139     while (k--) {
140         if (aq.empty()) {
141             printf("NO\n");
142             continue;
143         }
144         long long d = aq.top().first;
145         HeapNode* curNode = aq.top().second;
146         aq.pop();
147         printf("%I64d\n", d);
148         if (heapTop[curNode->edge->to] != nullNode)
149             aq.push(make_pair(d + heapTop[curNode->edge->to]->edge->weight, heapTop[curNode->edge->to]));
150         for (int i = 0; i < 4; i++)
151             if (curNode->child[i] != nullNode)

```

```

152         aq.push(make_pair(d - curNode->edge->weight + curNode->child[i]->edge->weight, curNode->child[i])
153         );
154     }
155     return 0;
156 }

```

#### 4.15 K 短路 (不允许重复)

```

1  int Num[10005][205], Path[10005][205], dev[10005];
2  int from[10005], value[10005], dist[205], Next[205], Graph[205][205];
3  bool forbid[205];
4  bool hasNext[10005][205];
5  int N, M, K, s, t;
6  int tot, cnt;
7
8  struct cmp {
9      bool operator()(const int &a, const int &b) {
10         int *i, *j;
11         if (value[a] != value[b])
12             return value[a] > value[b];
13         for (i = Path[a], j = Path[b]; (*i) == (*j); i++, j++);
14         return (*i) > (*j);
15     }
16 };
17
18 void Check(int idx, int st, int *path, int &res) {
19     int i, j;
20     for (i = 0; i < N; i++) {
21         dist[i] = 1000000000;
22         Next[i] = t;
23     }
24     dist[t] = 0;
25     forbid[t] = true;
26     j = t;
27     while (1) {
28         for (i = 0; i < N; i++)
29             if (!forbid[i] && (i != st || !hasNext[idx][j]) && (dist[j] + Graph[i][j] < dist[i] || (dist[j] +
30                 Graph[i][j] == dist[i] && j < Next[i]))) {
31                 Next[i] = j;
32                 dist[i] = dist[j] + Graph[i][j];
33             }
34             j = -1;
35             for (i = 0; i < N; i++)
36                 if (!forbid[i] && (j == -1 || dist[i] < dist[j])) j = i;
37             if (j == -1) break;
38             forbid[j] = 1;
39             if (j == st) break;
40         }
41         res += dist[st];
42         for (i = st; i != t; i = Next[i], path++) (*path) = i;
43         (*path) = i;
44     }
45
46 int main() {
47     int i, j, k, l;
48     while (scanf("%d%d%d%d%d", &N, &M, &K, &s, &t) && N) {
49         priority_queue<int, vector<int>, cmp> Q;
50         for (i = 0; i < N; i++)
51             for (j = 0; j < N; j++)
52                 Graph[i][j] = 1000000000;
53         for (i = 0; i < M; i++) {
54             scanf("%d%d%d", &j, &k, &l);
55             Graph[j - 1][k - 1] = 1;
56         }
57         s--;
58         t--;
59         memset(forbid, false, sizeof(forbid));
60         memset(hasNext[0], false, sizeof(hasNext[0]));
61         Check(0, s, Path[0], value[0]);
62         dev[0] = 0;
63         from[0] = 0;
64         Num[0][0] = 0;
65         Q.push(0);
66         cnt = 1;
67         tot = 1;
68         for (i = 0; i < K; i++) {
69             if (Q.empty()) break;
70             l = Q.top(); Q.pop();
71             for (j = 0; j <= dev[l]; j++)
72                 Num[l][j] = Num[from[l]][j];
73             for (; Path[l][j] != t; j++) {

```

```

73         memset(hasNext[tot], false, sizeof(hasNext[tot]));
74         Num[l][j] = tot++;
75     }
76     for (j = 0; Path[l][j] != t; j++)
77         hasNext[Num[l][j]][Path[l][j] + 1] = true;
78     for (j = dev[l]; Path[l][j] != t; j++) {
79         memset(forbid, false, sizeof(forbid));
80         value[cnt] = 0;
81         for (k = 0; k < j; k++) {
82             forbid[Path[l][k]] = true;
83             Path[cnt][k] = Path[l][k];
84             value[cnt] += Graph[Path[l][k]][Path[l][k] + 1];
85         }
86         Check(Num[l][j], Path[l][j], &Path[cnt][j], value[cnt]);
87
88         if (value[cnt] > 2000000)
89             continue;
90         dev[cnt] = j;
91         from[cnt] = l;
92         Q.push(cnt);
93         cnt++;
94     }
95 }
96
97 if (i < K || value[l] > 2000000)
98     printf("None\n");
99 else {
100     for (i = 0; Path[l][i] != t; i++)
101         printf("%d-", Path[l][i] + 1);
102     printf("%d\n", t + 1);
103 }
104 }
105 return 0;
106 }

```

#### 4.16 小知识

- 平面图: 一定存在一个度小于等于 5 的点.  $E \leq 3V - 6$ . 欧拉公式:  $V + F - E = 1 + \text{连通块数}$

- 图连通度:

- $k$ - 连通 ( $k$ -connected): 对于任意一对结点都至少存在结点各不相同的  $k$  条路
- 点连通度 ( $vertex\ connectivity$ ): 把图变成非连通图所需删除的最少点数
- Whitney 定理: 一个图是  $k$ - 连通的当且仅当它的点连通度至少为  $k$

- Lindstroem-Gessel-Viennot Lemma: 给定一个图的  $n$  个起点和  $n$  个终点, 令  $A_{ij}$  = 第  $i$  个起点到第  $j$  个终点的路径条数, 则从起点到终点的不相交路径条数为  $\det(A)$

- 欧拉回路与树形图的联系: 对于出度等于入度的连通图  $s(G) = t_i(G) \prod_{j=1}^n (d^+(v_j) - 1)!$

- 密度子图: 给定无向图, 选取点集及其导出子图, 最大化  $W_e + P_v$  (点权可负).

$$\begin{aligned}
 - (S, u) &= U, (u, T) = U - 2P_u - D_u, (u, v) = (v, u) = W_e \\
 - \text{ans} &= \frac{Un - C[S, T]}{2}, \text{解集为 } S - \{s\}
 \end{aligned}$$

- 最大权闭合图: 选  $a$  则  $a$  的后继必须被选

$$\begin{aligned}
 - P_u &> 0, (S, u) = P_u, P_u < 0, (u, T) = -P_u \\
 - \text{ans} &= \sum_{P_u > 0} P_u - C[S, T], \text{解集为 } S - \{s\}
 \end{aligned}$$

- 判定边是否属于最小割:

- 可能属于最小割:  $(u, v)$  不属于同一 SCC
- 一定在所有最小割中:  $(u, v)$  不属于同一 SCC, 且  $S, u$  在同一 SCC,  $u, T$  在同一 SCC

## 5 数学

### 5.1 单纯形 Cpp

$$\max \{cx | Ax \leq b, x \geq 0\}$$

```

1 const int MAXN = 11000, MAXM = 1100;
2 // here MAXN is the MAX number of conditions, MAXM is the MAX number of vars
3
4 int avali[MAXN], avacnt;
5 double A[MAXN][MAXM];
6 double b[MAXN], c[MAXN];
7 double* simplex(int n, int m) {
8     // here n is the number of conditions, m is the number of vars
9     m++;
10    int r = n, s = m - 1;
11    static double D[MAXN + 2][MAXM + 1];
12    static int ix[MAXN + MAXM];
13    for (int i = 0; i < n + m; i++) ix[i] = i;
14    for (int i = 0; i < n; i++) {
15        for (int j = 0; j < m - 1; j++) D[i][j] = -A[i][j];
16        D[i][m - 1] = 1;
17        D[i][m] = b[i];
18        if (D[r][m] > D[i][m]) r = i;
19    }
20    for (int j = 0; j < m - 1; j++) D[n][j] = c[j];
21    D[n + 1][m - 1] = -1;
22    for (double d; ; ) {
23        if (r < n) {
24            int t = ix[s]; ix[s] = ix[r + m]; ix[r + m] = t;
25            D[r][s] = 1.0 / D[r][s];
26            for (int j = 0; j <= m; j++) if (j != s) D[r][j] *= -D[r][s];
27            avacnt = 0;
28            for (int i = 0; i <= m; ++i)
29                if (fabs(D[r][i]) > EPS)
30                    avali[avacnt++] = i;
31            for (int i = 0; i <= n + 1; i++) if (i != r) {
32                if (fabs(D[i][s]) < EPS) continue;
33                double *cur1 = D[i], *cur2 = D[r], tmp = D[i][s];
34                //for (int j = 0; j <= m; j++) if (j != s) cur1[j] += cur2[j] * tmp;
35                for (int j = 0; j < avacnt; ++j) if (aval[i][j] != s) cur1[aval[i][j]] += cur2[aval[i][j]] * tmp;
36                D[i][s] *= D[r][s];
37            }
38            r = -1; s = -1;
39            for (int j = 0; j < m; j++) if (s < 0 || ix[s] > ix[j]) {
40                if (D[n + 1][j] > EPS || D[n + 1][j] > -EPS && D[n][j] > EPS) s = j;
41            }
42            if (s < 0) break;
43            for (int i = 0; i < n; i++) if (D[i][s] < -EPS) {
44                if (r < 0 || (d = D[r][m] / D[r][s] - D[i][m] / D[i][s]) < -EPS
45                    || d < EPS && ix[r + m] > ix[i + m])
46                    r = i;
47            }
48            if (r < 0) return null; // 非有界
49        }
50        if (D[n + 1][m] < -EPS) return null; // 无法执行
51        static double x[MAXN - 1];
52        for (int i = m; i < n + m; i++) if (ix[i] < m - 1) x[ix[i]] = D[i - m][m];
53        return x; // 值为 D[n][m]
54    }
55 }

```

### 5.2 单纯形 Java

```

1 double[] simplex(double[][] A, double[] b, double[] c) {
2     int n = A.length, m = A[0].length + 1, r = n, s = m - 1;
3     double[][] D = new double[n + 2][m + 1];
4     int[] ix = new int[n + m];
5     for (int i = 0; i < n + m; i++) ix[i] = i;
6     for (int i = 0; i < n; i++) {
7         for (int j = 0; j < m - 1; j++) D[i][j] = -A[i][j];
8         D[i][m - 1] = 1; D[i][m] = b[i]; if (D[r][m] > D[i][m]) r = i;
9     }
10    for (int j = 0; j < m - 1; j++) D[n][j] = c[j];
11    D[n + 1][m - 1] = -1;
12    for (double d; ; ) {
13        if (r < n) {
14            int t = ix[s]; ix[s] = ix[r + m]; ix[r + m] = t; D[r][s] = 1.0 / D[r][s];
15            for (int j = 0; j <= m; j++) if (j != s) D[r][j] *= -D[r][s];
16            for (int i = 0; i <= n + 1; i++) if (i != r) {

```

```

17                for (int j = 0; j <= m; j++) if (j != s) D[i][j] += D[r][j] * D[i][s];
18                D[i][s] *= D[r][s];
19            }
20            r = -1; s = -1;
21            for (int j = 0; j < m; j++) if (s < 0 || ix[s] > ix[j]) {
22                if (D[n + 1][j] > EPS || D[n + 1][j] > -EPS && D[n][j] > EPS) s = j;
23            }
24            if (s < 0) break;
25            for (int i = 0; i < n; i++) if (D[i][s] < -EPS) {
26                if (r < 0 || (d = D[r][m] / D[r][s] - D[i][m] / D[i][s]) < -EPS
27                    || d < EPS && ix[r + m] > ix[i + m])
28                    r = i;
29            }
30            if (r < 0) return null; // 非有界
31        } if (D[n + 1][m] < -EPS) return null; // 无法执行
32        double[] x = new double[m - 1];
33        for (int i = m; i < n + m; i++) if (ix[i] < m - 1) x[ix[i]] = D[i - m][m];
34        return x; // 值为 D[n][m]
35    }

```

### 5.3 FFT

```

1 namespace FFT {
2     #define mul(a, b) (Complex(a.x * b.x - a.y * b.y, a.x * b.y + a.y * b.x))
3     struct Complex { }; // something omitted
4     void FFT(Complex P[], int n, int oper) {
5         for (int i = 1, j = 0; i < n - 1; i++) {
6             for (int s = n; j ^= s >= 1, ~j & s; );
7             if (i < j) swap(P[i], P[j]);
8         }
9         for (int d = 0; (1 << d) < n; d++) {
10            int m = 1 << d, m2 = m * 2;
11            double p0 = PI / m * oper;
12            Complex unit_p0(cos(p0), sin(p0));
13            for (int i = 0; i < n; i += m2) {
14                Complex unit(1.0, 0.0);
15                for (int j = 0; j < m; j++) {
16                    Complex &P1 = P[i + j + m], &P2 = P[i + j];
17                    Complex t = mul(unit, P1);
18                    P1 = Complex(P2.x - t.x, P2.y - t.y);
19                    P2 = Complex(P2.x + t.x, P2.y - t.y);
20                    unit = mul(unit, unit_p0);
21                }
22            }
23        }
24        vector<int> doFFT(const vector<int> &a, const vector<int> &b) {
25            vector<int> ret(max(0, (int) a.size() + (int) b.size() - 1), 0);
26            static Complex A[MAXB], B[MAXB], C[MAXB];
27            int len = 1; while ((len < (int) ret.size()) len *= 2;
28            for (int i = 0; i < len; i++) A[i] = i < (int) a.size() ? a[i] : 0;
29            for (int i = 0; i < len; i++) B[i] = i < (int) b.size() ? b[i] : 0;
30            FFT(A, len, 1); FFT(B, len, 1);
31            for (int i = 0; i < len; i++) C[i] = mul(A[i], B[i]);
32            FFT(C, len, -1);
33            for (int i = 0; i < (int) ret.size(); i++)
34                ret[i] = (int) (C[i].x / len + 0.5);
35            return ret;
36        }
37    }
38 }

```

### 5.4 整数 FFT

```

1 namespace FFT {
2     // 替代方案: 23068673 (= 11 * 221 + 1), 原根为 3
3     const int MOD = 786433, PRIMITIVE_ROOT = 10; // 3 * 218 + 1
4     const int MAXB = 1 << 20;
5     int getMod(int downLimit) { // 或者现场自己找一个 MOD
6         for (int c = 3; ; ++c) { int t = (c << 21) | 1;
7             if (t >= downLimit && isPrime(t)) return t;
8         }
9     }
10    int modInv(int a) { return a <= 1 ? a : (long long) (MOD - MOD / a) * modInv(MOD % a) % MOD; }
11    void NTT(int P[], int n, int oper) {
12        for (int i = 1, j = 0; i < n - 1; i++) {
13            for (int s = n; j ^= s >= 1, ~j & s; );
14            if (i < j) swap(P[i], P[j]);
15        }
16        for (int d = 0; (1 << d) < n; d++) {
17            int m = 1 << d, m2 = m * 2;
18            long long unit_p0 = powMod(PRIMITIVE_ROOT, (MOD - 1) / m2);

```



```
18     if (oper < 0) unit_p0 = modInv(unit_p0);
19     for (int i = 0; i < n; i += m2) {
20         long long unit = 1;
21         for (int j = 0; j < m; j++) {
22             int &P1 = P[i + j + m], &P2 = P[i + j];
23             int t = unit * P1 % MOD;
24             P1 = (P2 - t + MOD) % MOD; P2 = (P2 + t) % MOD;
25             unit = unit * unit_p0 % MOD;
26         }
27     }
28     vector<int> mul(const vector<int> &a, const vector<int> &b) {
29         vector<int> ret(max(0, (int) a.size() + (int) b.size() - 1), 0);
30         static int A[MAXB], B[MAXB], C[MAXB];
31         int len = 1; while (len < (int)ret.size()) len <= 1;
32         for (int i = 0; i < len; i++) A[i] = i < (int)a.size() ? a[i] : 0;
33         for (int i = 0; i < len; i++) B[i] = i < (int)b.size() ? b[i] : 0;
34         NTT(A, len, 1); NTT(B, len, 1);
35         for (int i = 0; i < len; i++) C[i] = (long long) A[i] * B[i] % MOD;
36         NTT(C, len, -1); for (int i = 0, inv = modInv(len); i < (int)ret.size(); i++) ret[i] = (long long) C[i]
37             * inv % MOD;
38     }
39 }
```

5.5 扩展欧几里得

$ax + by = g = \gcd(x, y)$

```
1 void exgcd(LL x, LL y, LL &a0, LL &b0, LL &g) {
2     LL a1 = b0 = 0, b1 = a0 = 1, t;
3     while (y != 0) {
4         t = a0 - x / y * a1, a0 = a1, a1 = t;
5         t = b0 - x / y * b1, b0 = b1, b1 = t;
6         t = x % y, x = y, y = t;
7     } if (x < 0) a0 = -a0, b0 = -b0, x = -x;
8     g = x;
9 }
```

5.6 线性同余方程

- 中国剩余定理: 设  $m_1, m_2, \dots, m_k$  两两互素, 则同余方程组  $x \equiv a_i \pmod{m_i}$  for  $i = 1, 2, \dots, k$  在  $[0, M = m_1 m_2 \dots m_k]$  内有唯一解. 记  $M_i = M/m_i$ , 找出  $p_i$  使得  $M_i p_i \equiv 1 \pmod{m_i}$ , 记  $e_i = M_i p_i$ , 则  $x \equiv e_1 a_1 + e_2 a_2 + \dots + e_k a_k \pmod{M}$
- 多变元线性同余方程组: 方程的形式为  $a_1 x_1 + a_2 x_2 + \dots + a_n x_n + b \equiv 0 \pmod{m}$ , 令  $d = (a_1, a_2, \dots, a_n, m)$ , 有解的充要条件是  $d|b$ , 解的个数为  $m^{n-1}d$

5.7 Miller-Rabin 素性测试

```
1 bool test(LL n, int base) {
2     LL m = n - 1, ret = 0; int s = 0;
3     for ( ; m % 2 == 0; ++s) m >>= 1; ret = pow_mod(base, m, n);
4     if (ret == 1 || ret == n - 1) return true;
5     for (--s; s > 0; --s) {
6         ret = multiply_mod(ret, ret, n); if (ret == n - 1) return true;
7     } return false;
8 }
9 LL special[7] = {
10     1373653LL,      25326001LL,
11     3215031751LL,   250000000000LL,
12     2152302898747LL, 3474749660383LL, 341550071728321LL};
13 /*
14 * n < 2047          test[] = {2}
15 * n < 1,373,653     test[] = {2, 3}
16 * n < 9,080,191     test[] = {31, 73}
17 * n < 25,326,001    test[] = {2, 3, 5}
18 * n < 4,759,123,141 test[] = {2, 7, 61}
19 * n < 1,122,004,669,633 test[] = {2, 13, 23, 1662803}
20 * n < 2,152,302,898,747 test[] = {2, 3, 5, 7, 11}
21 * n < 3,474,749,660,383 test[] = {2, 3, 5, 7, 11, 13}
22 * n < 341,550,071,728,321 test[] = {2, 3, 5, 7, 11, 13, 17}
23 * n < 3,825,123,056,546,413,051 test[] = {2, 3, 5, 7, 11, 13, 17, 19, 23}
24 */
25 bool is_prime(LL n) {
```

```
26     if (n < 2) return false;
27     if (n < 4) return true;
28     if (!test(n, 2) || !test(n, 3)) return false;
29     if (n < special[0]) return true;
30     if (!test(n, 5)) return false;
31     if (n < special[1]) return true;
32     if (!test(n, 7)) return false;
33     if (n == special[2]) return false;
34     if (n < special[3]) return true;
35     if (!test(n, 11)) return false;
36     if (n < special[4]) return true;
37     if (!test(n, 13)) return false;
38     if (n < special[5]) return true;
39     if (!test(n, 17)) return false;
40     if (n < special[6]) return true;
41     return test(n, 19) && test(n, 23) && test(n, 29) && test(n, 31) && test(n, 37);
42 }
```

5.8 PollardRho

```
1 LL pollardRho(LL n, LL seed) {
2     LL x, y, head = 1, tail = 2; x = y = random() % (n - 1) + 1;
3     for ( ; ) {
4         x = addMod(multiplyMod(x, x, n), seed, n);
5         if (x == y) return n; LL d = gcd(myAbs(x - y), n);
6         if (1 < d && d < n) return d;
7         if (++head == tail) y = x, tail <= 1;
8     } vector<LL> divisors;
9     void factorize(LL n) { // 需要保证 n > 1
10         if (isPrime(n)) divisors.push_back(n);
11         else { LL d = n;
12             while (d >= n) d = pollardRho(n, random() % (n - 1) + 1);
13             factorize(n / d); factorize(d);
14         }
15     }
```

5.9 多项式求根

```
1 const double error = 1e-12;
2 const double infi = 1e+12;
3 int n; double a[10], x[10];
4 double f(double a[], int n, double x) {
5     double tmp = 1, sum = 0;
6     for (int i = 0; i <= n; i++) sum = sum + a[i] * tmp, tmp = tmp * x;
7     return sum;
8 }
9 double binary(double l, double r, double a[], int n) {
10     int sl = sign(f(a, n, l)), sr = sign(f(a, n, r));
11     if (sl == 0) return l; if (sr == 0) return r;
12     if (sl * sr > 0) return infi;
13     while (r - l > error) {
14         double mid = (l + r) / 2;
15         int ss = sign(f(a, n, mid));
16         if (ss == 0) return mid;
17         if (ss * sl > 0) l = mid; else r = mid;
18     } return l;
19 }
20 void solve(int n, double a[], double x[], int &nx) {
21     if (n == 1) { x[1] = -a[0] / a[1]; nx = 1; return; }
22     double da[10], dx[10]; int ndx;
23     for (int i = n; i >= 1; i--) da[i - 1] = a[i] * i;
24     solve(n - 1, da, dx, ndx); nx = 0;
25     if (ndx == 0) {
26         double tmp = binary(-infi, infi, a, n);
27         if (tmp < infi) x[++nx] = tmp; return;
28     } double tmp = binary(-infi, dx[1], a, n);
29     if (tmp < infi) x[++nx] = tmp;
30     for (int i = 1; i <= ndx - 1; i++) {
31         tmp = binary(dx[i], dx[i + 1], a, n);
32         if (tmp < infi) x[++nx] = tmp;
33     } tmp = binary(dx[ndx], infi, a, n);
34     if (tmp < infi) x[++nx] = tmp;
35 }
36 int main() {
37     scanf("%d", &n);
38     for (int i = n; i >= 0; i--) scanf("%lf", &a[i]);
39     int nx; solve(n, a, x, nx);
40     for (int i = 1; i <= nx; i++) printf("%.6f\n", x[i]);
41 }
```

```
42 } }
```

5.10 线性递推

for  $a_{i+n} = (\sum_{i=0}^{n-1} k_j a_{i+j}) + d, a_m = (\sum_{i=0}^{n-1} c_i a_i) + c_n d$

```
1 vector<int> recFormula(int n, int k[], int m) {
2     vector<int> c(n + 1, 0);
3     if (m < n) c[m] = 1;
4     else {
5         static int a[MAX_K * 2 + 1];
6         vector<int> b = recFormula(n, k, m >> 1);
7         for (int i = 0; i < n + n; ++i) a[i] = 0;
8         int s = m & 1;
9         for (int i = 0; i < n; i++) {
10             for (int j = 0; j < n; j++) a[i + j + s] += b[i] * b[j];
11             c[n] += b[i];
12             c[n] = (c[n] + 1) * b[n];
13             for (int i = n * 2 - 1; i >= n; i--) {
14                 int add = a[i]; if (add == 0) continue;
15                 for (int j = 0; j < n; j++) a[i - n + j] += k[j] * add;
16                 c[n] += add;
17             } for (int i = 0; i < n; ++i) c[i] = a[i];
18         } return c;
19     }
```

5.11 原根

原根  $g$ :  $g$  是模  $n$  简化剩余系构成的乘法群的生成元. 模  $n$  有原根的充要条件是  $n = 2, 4, p^n, 2p^n$ , 其中  $p$  是奇质数,  $n$  是正整数

```
1 vector<int> findPrimitiveRoot(int N) {
2     if (N <= 4) return vector<int>(1, max(1, N - 1));
3     static int factor[100];
4     int phi = N, totF = 0;
5     { // check no solution and calculate phi
6         int M = N, k = 0;
7         if (~M & 1) M >>= 1, phi >>= 1;
8         if (~M & 1) return vector<int>(0);
9         for (int d = 3; d * d <= M; ++d) if (M % d == 0) {
10             if (++k > 1) return vector<int>(0);
11             for (phi -= phi / d; M % d == 0; M /= d);
12         } if (M > 1) {
13             if (++k > 1) return vector<int>(0); phi -= phi / M;
14         }
15     } // factorize phi
16     int M = phi;
17     for (int d = 2; d * d <= M; ++d) if (M % d == 0) {
18         for ( ; M % d == 0; M /= d); factor[++totF] = d;
19     } if (M > 1) factor[++totF] = M;
20     vector<int> ans;
21     for (int g = 2; g <= N; ++g) if (Gcd(g, N) == 1) {
22         bool good = true;
23         for (int i = 1; i <= totF && good; ++i)
24             if (powMod(g, phi / factor[i], N) == 1) good = false;
25         if (!good) continue;
26         for (int i = 1, gp = g; i <= phi; ++i, gp = (LL)gp * g % N)
27             if (Gcd(i, phi) == 1) ans.push_back(gp);
28         break;
29     } sort(ans.begin(), ans.end());
30     return ans;
31 }
```

5.12 离散对数

$A^x \equiv B \pmod C$ , 对非质数  $C$  也适用.

```
1 int modLog(int A, int B, int C) {
2     static pii baby[MAX_SQRT_C + 1];
3     int d = 0; LL k = 1, D = 1; B %= C;
4     for (int i = 0; i < 100; ++i, k = k * A % C) // [0, log C]
5         if (k == B) return i;
6     for (int g; ; ++d) {
7         g = gcd(A, C); if (g == 1) break;
```

```
8         if (B % g != 0) return -1;
9         B /= g; C /= g; D = (A / g * D) % C;
10    } int m = (int) ceil(sqrt((double) C)); k = 1;
11    for (int i = 0; i <= m; ++i, k = k * A % C) baby[i] = pii(k, i);
12    sort(baby, baby + m + 1); // [0, m]
13    int n = unique(baby, baby + m + 1, equalFirst) - baby, am = powMod(A, m, C);
14    for (int i = 0; i <= m; ++i) {
15        LL e, x, y; exgcd(D, C, x, y, e); e = x * B % C;
16        if (e < 0) e += C;
17        if (e >= 0) {
18            int k = lower_bound(baby, baby + n, pii(e, -1)) - baby;
19            if (baby[k].first == e) return i * m + baby[k].second + d;
20        } D = D * am % C;
21    }
22    } return -1;
```

5.13 平方剩余

• Legendre Symbol: 对奇质数  $p, (\frac{a}{p}) = \begin{cases} 1 & \text{是平方剩余} \\ -1 & \text{是非平方剩余} = a^{\frac{p-1}{2}} \pmod p \\ 0 & a \equiv 0 \pmod p \end{cases}$

• 若  $p$  是奇质数,  $(\frac{-1}{p}) = 1$  当且仅当  $p \equiv 1 \pmod 4$

• 若  $p$  是奇质数,  $(\frac{2}{p}) = 1$  当且仅当  $p \equiv \pm 1 \pmod 8$

• 若  $p, q$  是奇素数且互质,  $(\frac{p}{q})(\frac{q}{p}) = (-1)^{\frac{p-1}{2} \times \frac{q-1}{2}}$

• Jacobi Symbol: 对奇数  $n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_k^{\alpha_k}, (\frac{a}{n}) = (\frac{a}{p_1})^{\alpha_1} (\frac{a}{p_2})^{\alpha_2} \cdots (\frac{a}{p_k})^{\alpha_k}$

• Jacobi Symbol 为  $-1$  则一定不是平方剩余, 所有平方剩余的 Jacobi Symbol 都是 1, 但 1 不一定是平方剩余

$ax^2 + bx + c \equiv 0 \pmod p$ , 其中  $a \not\equiv 0 \pmod p$ , 且  $p$  是质数

```
1 inline int normalize(LL a, int P) { a %= P; return a < 0 ? a + P : a; }
2 vector<int> QuadraticResidue(LL a, LL b, LL c, int P) {
3     int h, t; LL r1, r2, delta, pb = 0;
4     a = normalize(a, P); b = normalize(b, P); c = normalize(c, P);
5     if (P == 2) { vector<int> res;
6         if (c % P == 0) res.push_back(0);
7         if ((a + b + c) % P == 0) res.push_back(1);
8         return res;
9     } delta = b * rev(a + a, P) % P;
10    a = normalize(-c * rev(a, P) + delta * delta, P);
11    if (powMod(a, P / 2, P) + 1 == P) return vector<int>(0);
12    for (t = 0, h = P / 2; h % 2 == 0; ++t, h /= 2);
13    r1 = powMod(a, h / 2, P);
14    if (t > 0) { do b = random() % (P - 2) + 2;
15        while (powMod(b, P / 2, P) + 1 != P); }
16    for (int i = 1; i <= t; ++i) {
17        LL d = r1 * r1 % P * a % P;
18        for (int j = 1; j <= t - i; ++j) d = d * d % P;
19        if (d + 1 == P) r1 = r1 * pb % P; pb = pb * pb % P;
20    } r1 = a * r1 % P; r2 = P - r1;
21    r1 = normalize(r1 - delta, P); r2 = normalize(r2 - delta, P);
22    if (r1 > r2) swap(r1, r2); vector<int> res(1, r1);
23    if (r1 != r2) res.push_back(r2);
24    return res;
25 }
```

5.14 N 次剩余

• 若  $p$  为奇质数,  $a$  为  $p$  的  $n$  次剩余的充要条件是  $a^{\frac{p-1}{a, p-1}} \equiv 1 \pmod p$ .

$x^N \equiv a \pmod p$ , 其中  $p$  是质数

```
1 vector<int> solve(int p, int N, int a) {
2     if ((a % p) == 0) return vector<int>(1, 0);
3     int g = findPrimitiveRoot(p), m = modLog(g, a, p); // g ^ m = a (mod p)
4     if (m == -1) return vector<int>(0);
5     LL B = p - 1, x, y, d; exgcd(N, B, x, y, d);
6     if (m % d != 0) return vector<int>(0);
7     vector<int> ret; x = (x * (m / d) % B + B) % B; // g ^ B mod p = g ^ (p - 1) mod p = 1
8     for (int i = 0, delta = B / d; i < d; ++i) {
9         x = (x + delta) % B; ret.push_back((int)powMod(g, x, p));
10    } sort(ret.begin(), ret.end());
11    ret.resize(unique(ret.begin(), ret.end()) - ret.begin());
12    return ret;
13 }
```

5.15 Pell 方程

$$\begin{pmatrix} x_k \\ y_k \end{pmatrix} = \begin{pmatrix} x_1 & dy_1 \\ y_1 & x_1 \end{pmatrix}^{k-1} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$$

```
1 pair<ULL, ULL> Pell(int n) {
2     static ULL p[50] = {0, 1}, q[50] = {1, 0}, g[50] = {0, 0}, h[50] = {0, 1}, a[50];
3     ULL t = a[2] = Sqrt(n);
4     for (int i = 2; ; ++i) {
5         g[i] = -g[i - 1] + a[i] * h[i - 1];
6         h[i] = (n - g[i] * g[i]) / h[i - 1];
7         a[i + 1] = (g[i] + t) / h[i];
8         p[i] = a[i] * p[i - 1] + p[i - 2];
9         q[i] = a[i] * q[i - 1] + q[i - 2];
10        if (p[i] * p[i] - n * q[i] * q[i] == 1) return make_pair(p[i], q[i]);
11    } return make_pair(-1, -1);
12 }
```

5.16 Romberg 积分

```
1 template <class T> double Romberg(const T&f, double a, double b, double eps = 1e-8) {
2     vector<double> t; double h = b - a, last, now; int k = 1, i = 1;
3     t.push_back(h * (f(a) + f(b)) / 2); // 梯形
4     do {
5         last = t.back(); now = 0; double x = a + h / 2;
6         for (int j = 0; j < k; ++j, x += h) now += f(x);
7         now = (t[0] + h * now) / 2; double k1 = 4.0 / 3.0, k2 = 1.0 / 3.0;
8         for (int j = 0; j < i; ++j, k1 = k2 + 1) {
9             double tmp = k1 * now - k2 * t[j];
10            t[j] = now; now = tmp; k2 /= 4 * k1 - k2; // 防止溢出
11        } t.push_back(now); k *= 2; h /= 2; ++i;
12    } while (fabs(last - now) > eps);
13    return t.back();
14 }
```

5.17 公式

5.17.1 级数与三角

- $\sum_{k=1}^n k^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$
- $\sum_{k=1}^n k^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$
- 错排:  $D_n = n!(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + \frac{(-1)^n}{n!}) = (n-1)(D_{n-2} - D_{n-1})$
- $\tan \alpha \pm \tan \beta = \frac{\sin(\alpha \pm \beta)}{\cos \alpha \cos \beta}$
- $\cos n\alpha = \binom{n}{0} \cos^n \alpha - \binom{n}{2} \cos^{n-2} \alpha \sin^2 \alpha + \binom{n}{4} \cos^{n-4} \alpha \sin^4 \alpha \cdots$
- $\sin n\alpha = \binom{n}{1} \cos^{n-1} \alpha \sin \alpha - \binom{n}{3} \cos^{n-3} \alpha \sin^3 \alpha + \binom{n}{5} \cos^{n-5} \alpha \sin^5 \alpha \cdots$

- $\sum_{n=1}^N \cos nx = \frac{\sin(N+\frac{1}{2})x - \sin \frac{x}{2}}{2 \sin \frac{x}{2}}$
- $\sum_{n=1}^N \sin nx = \frac{-\cos(N+\frac{1}{2})x + \cos \frac{x}{2}}{2 \sin \frac{x}{2}}$
- $\int_0^{\frac{\pi}{2}} \sin^n x dx = \begin{cases} \frac{(n-1)!!}{n!!} \times \frac{\pi}{2} & n \text{是偶数} \\ \frac{(n-1)!!}{n!!} & n \text{是奇数} \end{cases}$

$$\int_0^{+\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$$

$$\int_0^{+\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$$

- 傅里叶级数: 设周期为  $2T$ . 函数分段连续. 在不连续点的值为左右极限的平均数.

$$- a_n = \frac{1}{T} \int_{-T}^T f(x) \cos \frac{n\pi}{T} x dx$$

$$- b_n = \frac{1}{T} \int_{-T}^T f(x) \sin \frac{n\pi}{T} x dx$$

$$- f(x) = \frac{a_0}{2} + \sum_{n=1}^{+\infty} (a_n \cos \frac{n\pi}{T} x + b_n \sin \frac{n\pi}{T} x)$$

- Beta 函数:  $B(p, q) = \int_0^1 x^{p-1} (1-x)^{q-1} dx$

- 定义域  $(0, +\infty) \times (0, +\infty)$ , 在定义域上连续

$$- B(p, q) = B(q, p) = \frac{q-1}{p+q-1} B(p, q-1) = 2 \int_0^{\frac{\pi}{2}} \cos^{2p-1} \phi \sin^{2p-1} \phi d\phi = \int_0^{+\infty} \frac{t^{q-1}}{(1+t)^{p+q}} dt = \int_0^1 \frac{t^{p-1} + t^{q-1}}{(1+t)^{(p+q)}} dt$$

$$- B(\frac{1}{2}, \frac{1}{2}) = \pi$$

- Gamma 函数:  $\Gamma = \int_0^{+\infty} x^{s-1} e^{-x} dx$

- 定义域  $(0, +\infty)$ , 在定义域上连续

$$- \Gamma(1) = 1, \Gamma(\frac{1}{2}) = \sqrt{\pi}$$

$$- \Gamma(s) = (s-1)\Gamma(s-1)$$

$$- B(p, q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$$

$$- \Gamma(s)\Gamma(1-s) = \frac{\pi}{\sin \pi s} \text{ for } s > 0$$

$$- \Gamma(s)\Gamma(s + \frac{1}{2}) = 2\sqrt{\pi} \frac{\Gamma(s)}{2^{2s-1}} \text{ for } 0 < s < 1$$

- 积分：平面图形面积、曲线弧长、旋转体体积、旋转曲面面积
$$y=f(x),\int_a^bf(x)\mathrm{d}x,\int_a^b\sqrt{1+f'^2(x)}\mathrm{d}x,$$
$$\pi\int_a^bf^2(x)\mathrm{d}x,2\pi\int_a^b|f(x)|\sqrt{1+f'^2(x)}\mathrm{d}x$$
$$x=x(t),y=y(t),t\in[T_1,T_2],\int_{T_1}^{T_2}|y(t)x'(t)|\mathrm{d}t,\int_{T_1}^{T_2}\sqrt{x'^2(t)+y'^2(t)}\mathrm{d}t,\pi\int_{T_1}^{T_2}|x'(t)|y^2(t)\mathrm{d}t,$$
$$2\pi\int_{T_1}^{T_2}|y(t)|\sqrt{x'^2(t)+y'^2(t)}\mathrm{d}t,$$
$$r=r(\theta),\theta\in[\alpha,\beta],\frac{1}{2}\int_{\alpha}^{\beta}r^2(\theta)\mathrm{d}\theta,\int_{\alpha}^{\beta}\sqrt{r^2(\theta)+r'^2(\theta)}\mathrm{d}\theta,\frac{2}{3}\pi\int_{\alpha}^{\beta}r^3(\theta)\sin\theta\mathrm{d}\theta,$$
$$2\pi\int_{\alpha}^{\beta}r(\theta)\sin\theta\sqrt{r^2(\theta)+r'^2(\theta)}\mathrm{d}\theta$$

5.17.2 三次方程求根公式

对一元三次方程  $x^3+px+q=0$ , 令

$$A=\sqrt[3]{-\frac{q}{2}+\sqrt{(\frac{q}{2})^2+(\frac{p}{3})^3}}$$
$$B=\sqrt[3]{-\frac{q}{2}-\sqrt{(\frac{q}{2})^2+(\frac{p}{3})^3}}$$
$$\omega=\frac{(-1+\mathrm{i}\sqrt{3})}{2}$$

则  $x_j=A\omega^j+B\omega^{2j}$  ( $j=0,1,2$ ).  
当求解  $ax^3+bx^2+cx+d=0$  时, 令  $x=y-\frac{b}{3a}$ , 再求解  $y$ , 即转化为  $y^3+py+q=0$  的形式. 其中,

$$p=\frac{b^2-3ac}{3a^2}$$
$$q=\frac{2b^3-9abc+27a^2d}{27a^3}$$

卡尔丹判别法: 令  $\Delta=(\frac{q}{2})^2+(\frac{p}{3})^3$ . 当  $\Delta>0$  时, 有一个实根和一对共轭虚根; 当  $\Delta=0$  时, 有三个实根, 其中两个相等; 当  $\Delta<0$  时, 有三个不相等的实根.

5.17.3 椭圆

- 椭圆  $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ , 其中离心率  $e=\frac{c}{a},c=\sqrt{a^2-b^2}$ ; 焦点参数  $p=\frac{b^2}{a}$
- 椭圆上  $(x,y)$  点处的曲率半径为  $R=a^2b^2(\frac{x^2}{a^4}+\frac{y^2}{b^4})^{\frac{3}{2}}=\frac{(r_1r_2)^{\frac{3}{2}}}{ab}$ , 其中  $r_1$  和  $r_2$  分别为  $(x,y)$  与两焦点  $F_1$  和  $F_2$  的距离.
- 椭圆的周长  $L=4a\int_0^{\frac{\pi}{2}}\sqrt{1-e^2\sin^2t}\mathrm{d}t=4aE(e,\frac{\pi}{2})$ , 其中

$$E(e,\frac{\pi}{2})=\frac{\pi}{2}[1-(\frac{1}{2})^2e^2-(\frac{1\times3}{2\times4})^2\frac{e^4}{3}-(\frac{1\times3\times5}{2\times4\times6})^2\frac{e^6}{5}-\cdots$$

- 设椭圆上点  $M(x,y),N(x,-y),x,y>0,A(a,0)$ , 原点  $O(0,0)$ , 扇形  $OAM$  的面积  $S_{OAM}=\frac{1}{2}ab\arccos\frac{x}{a}$ , 弓形  $MAN$  的面积  $S_{MAN}=ab\arccos\frac{x}{a}-xy$ .

- 设  $\theta$  为  $(x,y)$  点关于椭圆中心的极角,  $r$  为  $(x,y)$  到椭圆中心的距离, 椭圆极坐标方程:

$$x=r\cos\theta,y=r\sin\theta,r^2=\frac{b^2a^2}{b^2\cos^2\theta+a^2\sin^2\theta}$$

5.17.4 抛物线

- 标准方程  $y^2=2px$ , 曲率半径  $R=\frac{(p+2x)^{\frac{3}{2}}}{\sqrt{p}}$
- 弧长: 设  $M(x,y)$  是抛物线上一点, 则  $L_{OM}=\frac{p}{2}[\sqrt{\frac{2x}{p}(1+\frac{2x}{p})}+\ln(\sqrt{\frac{2x}{p}}+\sqrt{1+\frac{2x}{p}})]$
- 弓形面积: 设  $M,D$  是抛物线上两点, 且分居一, 四象限. 做一条平行于  $MD$  且与抛物线相切的直线  $L$ . 若  $M$  到  $L$  的距离为  $h$ . 则有  $S_{MOD}=\frac{2}{3}MD\cdot h$ .

5.17.5 重心

- 半径  $r$ , 圆心角为  $\theta$  的扇形的重心与圆心的距离为  $\frac{4r\sin\frac{\theta}{2}}{3\theta}$
- 半径  $r$ , 圆心角为  $\theta$  的圆弧的重心与圆心的距离为  $\frac{4r\sin^3\frac{\theta}{2}}{3(\theta-\sin\theta)}$
- 椭圆上半部分的重心与圆心的距离为  $\frac{4b}{3\pi}$
- 抛物线中弓形  $MOD$  的重心满足  $CQ=\frac{2}{5}PQ$ ,  $P$  是直线  $L$  与抛物线的切点,  $Q$  在  $MD$  上且  $PQ$  平行  $x$  轴,  $C$  是重心

5.17.6 向量恒等式

- $\vec{a}\cdot(\vec{b}\times\vec{c})=\vec{b}\cdot(\vec{c}\times\vec{a})=\vec{c}\cdot(\vec{a}\times\vec{b})$
- $\vec{a}\times(\vec{b}\times\vec{c})=(\vec{c}\times\vec{b})\times\vec{a}=\vec{b}(\vec{a}\cdot\vec{c})-\vec{c}(\vec{a}\cdot\vec{b})$

5.17.7 常用几何公式

- 三角形的五心
  - 重心  $\vec{G}=\frac{\vec{A}+\vec{B}+\vec{C}}{3}$
  - 内心  $\vec{I}=\frac{a\vec{A}+b\vec{B}+c\vec{C}}{a+b+c},R=\frac{2S}{a+b+c}$
  - 外心  $x=\frac{\vec{A}+\vec{B}-\frac{\vec{B}\vec{C}\cdot\vec{A}\vec{C}}{\vec{A}\vec{B}\times\vec{B}\vec{C}}\vec{A}\vec{B}^T}{2},y=\frac{\vec{A}+\vec{B}+\frac{\vec{B}\vec{C}\cdot\vec{A}\vec{C}}{\vec{A}\vec{B}\times\vec{B}\vec{C}}\vec{A}\vec{B}^T}{2},R=\frac{abc}{4S}$
  - 垂心  $\vec{H}=3\vec{G}-2\vec{O}$
  - 旁心 (三个)  $\frac{-a\vec{A}+b\vec{B}+c\vec{C}}{-a+b+c}$
- 四边形: 设  $D_1,D_2$  为对角线,  $M$  为对角线中点连线,  $A$  为对角线夹角
  - $a^2+b^2+c^2+d^2=D_1^2+D_2^2+4M^2$

- $S = \frac{1}{2}D_1D_2\sin A$
- $ac + bd = D_1D_2$  (内接四边形适用)
- Bretschneider 公式:  $S = \sqrt{(p-a)(p-b)(p-c)(p-d) - abcd\cos^2(\frac{\theta}{2})}$ , 其中  $\theta$  为对角

5.17.8 树的计数

- 有根数计数: 令  $S_{n,j} = \sum_{1 \leq i \leq n/j} a_{n+1-i,j} = S_{n-j,j} + a_{n+1-j}$   
于是,  $n + 1$  个结点的有根数的总数为  $a_{n+1} = \frac{\sum_{1 \leq j \leq n} j \cdot a_j \cdot S_{n,j}}{n}$   
附:  $a_1 = 1, a_2 = 1, a_3 = 2, a_4 = 4, a_5 = 9, a_6 = 20, a_9 = 286, a_{11} = 1842$

- 无根树计数: 当  $n$  是奇数时, 则有  $a_n - \sum_{1 \leq i \leq \frac{n}{2}} a_i a_{n-i}$  种不同的无根树  
当  $n$  是偶数时, 则有  $a_n - \sum_{1 \leq i \leq \frac{n}{2}} a_i a_{n-i} + \frac{1}{2} a_{\frac{n}{2}} (a_{\frac{n}{2}} + 1)$  种不同的无根树
- Matrix-Tree 定理: 对任意图  $G$ , 设  $\text{mat}[i][i] = i$  的度数,  $\text{mat}[i][j] = i$  与  $j$  之间边数的相反数, 则  $\text{mat}[i][j]$  的任意余子式的行列式就是该图的生成树个数

5.18 小知识

- 勾股数: 设正整数  $n$  的质因数分解为  $n = \prod p_i^{a_i}$ , 则  $x^2 + y^2 = n$  有整数解的充要条件是  $n$  中不存在形如  $p_i \equiv 3 \pmod{4}$  且指数  $a_i$  为奇数的质因数  $p_i$ .  $(\frac{a-b}{2})^2 + ab = (\frac{a+b}{2})^2$ .
- 素勾股数: 若  $m$  和  $n$  互质, 而且  $m$  和  $n$  中有一个是偶数, 则  $a = m^2 - n^2, b = 2mn, c = m^2 + n^2$ , 则  $a, b, c$  是素勾股数.
- Stirling 公式:  $n! \approx \sqrt{2\pi n}(\frac{n}{e})^n$
- Pick 定理: 简单多边形, 不自交, 顶点如果全是整点. 则: 严格在多边形内部的整点数 +  $\frac{1}{2}$  在边上的整点数  $- 1 =$  面积
- Mersenne 素数:  $p$  是素数且  $2^p - 1$  的数是素数. (10000 以内的  $p$  有: 2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127, 521, 607, 1279, 2203, 2281, 3217, 4253, 4423, 9689, 9941)
- Fermat 分解算法: 从  $t = \sqrt{n}$  开始, 依次检查  $t^2 - n, (t + 1)^2 - n, (t + 2)^2 - n, \dots$ , 直到出现一个平方数  $y$ , 由于  $t^2 - y^2 = n$ , 因此分解得  $n = (t - y)(t + y)$ . 显然, 当两个因数很接近时这个方法能很快找到结果, 但如果遇到一个素数, 则需要检查  $\frac{n+1}{2} - \sqrt{n}$  个整数
- 牛顿迭代:  $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$
- 球与盒子的动人故事: ( $n$  个球,  $m$  个盒子,  $S$  为第二类斯特林数)
  1. 球同, 盒同, 无空: dp
  2. 球同, 盒同, 可空: dp
  3. 球同, 盒不同, 无空:  $\binom{n-1}{m-1}$
  4. 球同, 盒不同, 可空:  $\binom{n+m-1}{n-1}$
  5. 球不同, 盒同, 无空:  $S(n, m)$
  6. 球不同, 盒同, 可空:  $\sum_{k=1}^m S(n, k)$

7. 球不同, 盒不同, 无空:  $m!S(n, m)$
8. 球不同, 盒不同, 可空:  $m^n$

- 组合数奇偶性: 若  $(n \& m) = m$ , 则  $\binom{n}{m}$  为奇数, 否则为偶数

- 格雷码  $G(x) = x \otimes (x \gg 1)$

- Fibonacci 数:
  - $F_0 = F_1 = 1, F_i = F_{i-1} + F_{i-2}, F_{-i} = (-1)^{i-1} F_i$
  - $F_i = \frac{1}{\sqrt{5}}((\frac{1 + \sqrt{5}}{2})^n - (\frac{1 - \sqrt{5}}{2})^n)$
  - $\gcd(F_n, F_m) = F_{\gcd(n, m)}$
  - $F_{i+1}F_i - F_i^2 = (-1)^i$
  - $F_{n+k} = F_k F_{n+1} + F_{k-1} F_n$
- 第一类 Stirling 数:  $[n_k]$  代表第一类无符号 Stirling 数, 代表将  $n$  阶置换群中有  $k$  个环的置换个数;  $s(n, k)$  代表有符号型,  $s(n, k) = (-1)^{n-k} [n_k]$ .

$$\begin{aligned} - (x)^{(n)} &= \sum_{k=0}^n [n_k] x^k, (x)_n = \sum_{k=0}^n s(n, k) x^k \\ - [n] &= n [n-1_k] + [n-1]_1, [0] = 1, [n] = [n]_0 = 0 \\ - [n-2] &= \frac{1}{4}(3n-1) \binom{n}{3}, [n-3] = \binom{n}{2} \binom{n}{4} \\ - \sum_{k=0}^a [n_k] &= n! - \sum_{k=0}^n [k+a+1] \\ - \sum_{p=k}^n [p] \binom{p}{k} &= [k+1]^{n+1} \end{aligned}$$

- 第二类 Stirling 数:  $\{n_k\} = S(n, k)$  代表  $n$  个不同的球, 放到  $k$  个相同的盒子里, 盒子非空.

$$\begin{aligned} - \{n_k\} &= \frac{1}{k!} \sum_{j=0}^k (-1)^j \binom{k}{j} (k-j)^n \\ - \{n+1_k\} &= k \{n_k\} + \{n_{k-1}\}, \{0\} = 1, \{n_0\} = \{n\}_0 = 0 \\ - \text{奇偶性: } (n-k) \&\frac{k-1}{2} &= 0 \end{aligned}$$

- Bell 数:  $B_n$  代表将  $n$  个元素划分成若干个非空集合的方案数

$$\begin{aligned} - B_0 &= B_1 = 1, B_n = \sum_{k=0}^{n-1} \binom{n-1}{k} B_k \\ - B_n &= \sum_{k=0}^n \{n_k\} \\ - \text{Bell 三角形: } a_{1,1} &= 1, a_{n,1} = a_{n-1,n-1}, a_{n,m} = a_{n,m-1} + a_{n-1,m-1}, B_n = a_{n,1} \\ - \text{对质数 } p, B_{n+p} &\equiv B_n + B_{n+1} \pmod{p} \\ - \text{对质数 } p, B_{n+p^m} &\equiv m B_n + B_{n+1} \pmod{p} \\ - \text{对质数 } p, \text{ 模的周期一定是 } \frac{p^p-1}{p-1} \text{ 的约数, } p \leq 101 \text{ 时就是这个值} \end{aligned}$$

– 从  $B_0$  开始, 前几项是 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975...

- Bernoulli 数
  - $B_0 = 1, B_1 = \frac{1}{2}, B_2 = \frac{1}{6}, B_4 = -\frac{1}{30}, B_6 = \frac{1}{42}, B_8 = B_4, B_{10} = \frac{5}{66}$
  - $$-\sum_{k=1}^n k^m = \frac{1}{m+1} \sum_{k=0}^m \binom{m+1}{k} B_k n^{m+1-k}$$
  - $$-B_m = 1 - \sum_{k=0}^{m-1} \binom{m}{k} \frac{B_k}{m-k+1}$$
- 完全数:  $x$  是偶完全数等价于  $x = 2^{n-1}(2^n - 1)$ , 且  $2^n - 1$  是质数.

## 6 其他

### 6.1 Extended LIS

```
1 int G[MAXN][MAXN];
2 void insertYoung(int v) {
3     for (int x = 1, y = INT_MAX; ; ++x) {
4         Down(y, *G[x]); while (y > 0 && G[x][y] >= v) --y;
5         if (++y > *G[x]) { ++*G[x]; G[x][y] = v; break; }
6         else swap(G[x][y], v);
7     }
8 }
9 int solve(int N, int seq[]) {
10     Rep(i, 1, N) *G[i] = 0;
11     Rep(i, 1, N) insertYoung(seq[i]);
12     printf("%d\n", *G[1] + *G[2]);
13     return 0;
14 }
```

### 6.2 生成 nCk

```
1 void nCk(int n, int k) {
2     for (int comb = (1 << k) - 1; comb < (1 << n); ) {
3         int x = comb & -comb, y = comb + x;
4         comb = ((comb & -y) / x) >> 1 | y;
5     }
6 }
```

### 6.3 nextPermutation

```
1 boolean nextPermutation(int[] is) {
2     int n = is.length;
3     for (int i = n - 1; i > 0; i--) {
4         if (is[i - 1] < is[i]) {
5             int j = n; while (is[i - 1] >= is[--j]);
6             swap(is, i - 1, j); // swap is[i - 1], is[j]
7             rev(is, i, n); // reverse is[i, n)
8             return true;
9         }
10    } rev(is, 0, n);
11    return false;
12 }
```

### 6.4 Josephus 数与逆 Josephus 数

```
1 int josephus(int n, int m, int k) { int x = -1;
2     for (int i = n - k + 1; i <= n; i++) x = (x + m) % i; return x;
3 }
4 int invJosephus(int n, int m, int x) {
5     for (int i = n; ; i--) { if (x == i) return n - i; x = (x - m % i + i) % i; }
6 }
```

### 6.5 表达式求值

```
1 inline int getLevel(char ch) {
2     switch (ch) { case '+': case '-': return 0; case '*': return 1; } return -1;
3 }
4 int evaluate(char *p, int level) {
5     int res;
6     if (level == 2) {
7         if (*p == '(') ++p, res = evaluate(p, 0);
8         else res = isdigit(*p) ? *p - '0' : value[*p - 'a'];
9         ++p; return res;
10    } res = evaluate(p, level + 1);
11    for (int next; *p && getLevel(*p) == level; ) {
12        char op = *p++; next = evaluate(p, level + 1);
13        switch (op) {
14            case '+': res += next; break;
15            case '-': res -= next; break;
16            case '*': res *= next; break;
17        }
18    } return res;
19 }
20 int makeEvaluation(char *str) { char *p = str; return evaluate(p, 0); }
```

### 6.6 直线下的整点个数

```
求  $\sum_{i=0}^{n-1} \lfloor \frac{a+bi}{m} \rfloor$ 
```

```
1 LL count(LL n, LL a, LL b, LL m) {
2     if (b == 0) return n * (a / m);
3     if (a >= m) return n * (a / m) + count(n, a % m, b, m);
4     if (b >= m) return (n - 1) * n / 2 * (b / m) + count(n, a, b % m, m);
5     return count((a + b * n) / m, (a + b * n) % m, m, b);
6 }
```

### 6.7 Java 多项式

```
1 class Polynomial {
2     final static Polynomial ZERO = new Polynomial(new int[] { 0 });
3     final static Polynomial ONE = new Polynomial(new int[] { 1 });
4     final static Polynomial X = new Polynomial(new int[] { 0, 1 });
5     int[] coef;
6     static Polynomial valueOf(int val) { return new Polynomial(new int[] { val }); }
7     Polynomial(int[] coef) { this.coef = Arrays.copyOf(coef, coef.length); }
8     Polynomial add(Polynomial o, int mod); // omitted
9     Polynomial subtract(Polynomial o, int mod); // omitted
10    Polynomial multiply(Polynomial o, int mod); // omitted
11    Polynomial scale(int o, int mod); // omitted
12    public String toString() {
13        int n = coef.length; String ret = "";
14        for (int i = n - 1; i > 0; --i) if (coef[i] != 0)
15            ret += coef[i] + "x" + i + "+";
16        return ret + coef[0];
17    }
18    static Polynomial lagrangeInterpolation(int[] x, int[] y, int mod) {
19        int n = x.length; Polynomial ret = Polynomial.ZERO;
20        for (int i = 0; i < n; ++i) {
21            Polynomial poly = Polynomial.valueOf(y[i]);
22            for (int j = 0; j < n; ++j) if (i != j) {
23                poly = poly.multiply(
24                    Polynomial.X.subtract(Polynomial.valueOf(x[j]), mod), mod);
25                poly = poly.scale(powMod(x[i] - x[j] + mod, mod - 2, mod), mod);
26            } ret = ret.add(poly, mod);
27        } return ret;
28    }
29 }
```

### 6.8 long long 乘法取模

```
1 LL multiplyMod(LL a, LL b, LL P) { // 需要保证 a 和 b 非负
2     LL t = (a * b - LL((long double)a / P * b + 1e-3) * P) % P;
3     return t < 0 : t + P : t;
4 }
```

6.9 重复覆盖

```
1 struct node { int x, y; node *l, *r, *u, *d; } base[MAX * MAX], *top, *head;
2 typedef node *link;
3 int row, col, nGE, ans, stamp, cntc[MAX], vis[MAX];
4 void removeExact(link c) { c->l->r = c->r; c->r->l = c->l;
5   for (link i = c->d; i != c; i = i->d)
6     for (link j = i->r; j != i; j = j->r) j->d->u = j->u, j->u->d = j->d, --cntc[j->y];
7 }
8 void resumeExact(link c) {
9   for (link i = c->u; i != c; i = i->u)
10     for (link j = i->l; j != i; j = j->l) j->d->u = j, j->u->d = j, ++cntc[j->y];
11   c->l->r = c; c->r->l = c;
12 }
13 void removeRepeat(link c) { for (link i = c->d; i != c; i = i->d) i->l->r = i->r, i->r->l = i->l; }
14 void resumeRepeat(link c) { for (link i = c->u; i != c; i = i->u) i->l->r = i; i->r->l = i; }
15 int calcH() { int y, res = 0; ++stamp;
16   for (link c = head->r; (y = c->y) <= row && i != head; c = c->r) if (vis[y] != stamp) {
17     vis[y] = stamp; ++res; for (link i = c->d; i != c; i = i->d)
18       for (link j = i->r; j != i; j = j->r) vis[j->y] = stamp;
19   } return res;
20 }
21 void DFS(int dep) { if (dep + calcH() >= ans) return;
22   if (head->r->y > nGE || head->r == head) { if (ans > dep) ans = dep; return; }
23   link c = NULL;
24   for (link i = head->r; i->y <= nGE && i != head; i = i->r)
25     if (!c || cntc[i->y] < cntc[c->y]) c = i;
26   for (link i = c->d; i != c; i = i->d) {
27     removeRepeat(i);
28     for (link j = i->r; j != i; j = j->r) if (j->y <= nGE) removeRepeat(j);
29     for (link j = i->r; j != i; j = j->r) if (j->y > nGE) removeExact(base + j->y);
30     DFS(dep + 1);
31     for (link j = i->l; j != i; j = j->l) if (j->y > nGE) resumeExact(base + j->y);
32     for (link j = i->l; j != i; j = j->l) if (j->y <= nGE) resumeRepeat(j);
33     resumeRepeat(i);
34   }
35 }
```

6.10 星期几判定

```
1 int getDay(int y, int m, int d) {
2   if (m <= 2) m += 12, y--;
3   if (y < 1752 || (y == 1752 && m < 9) || (y == 1752 && m == 9 && d < 3))
4     return (d + 2 * m + 3 * (m + 1) / 5 + y + y / 4 + 5) % 7 + 1;
5   return (d + 2 * m + 3 * (m + 1) / 5 + y + y / 4 - y / 100 + y / 400) % 7 + 1;
6 }
```

6.11 LCSequence Fast

```
1 ULL *a, *b, *s, c, d;
2 for (i = 0, a = appear[(int)B[k]], b = row[max(k - 1, 0)], s = X; i < bitSetLen; ++i)
3   *s++ = *a++ | *b++; // X = row[i - 1] or appear[ B[i] ]
4 for (i = 0, a = dp, c = d = 0; i < bitSetLen; ++a, c = d, ++i)
5   d = *a >> 63, *a = -(((*a << 1) + c); // row[i] = -(row[i] << 1) + 1)
6 for (i = 0, a = dp, b = X, c = 0; i < bitSetLen; ++a, ++b, ++i)
7   d = *b + c, c = (*a >= -d), *a += d; // row[i] += X
8 for (i = 0, a = dp, b = X; i < bitSetLen; ++a, ++b, ++i)
9   *a = (*a ^ *b) & *b; // row[i] = X and (row[i] xor X)
```

7 Templates

7.1 vim 配置

在.bashrc 中加入 export CXXFLAGS=“-Wall -Wconversion -Wextra -g3”

```
1 set nu ru nobk cindent si
2 set mouse=aw sw=4 sts=4 ts=4
3 set hlsearch incsearch
4 set whichwrap=b,s,<,>,[,]
5 syntax on
6
```

```
7 nmap <C-A> ggVG
8 vmap <C-C> "+y
9
10 autocmd BufNewFile *.cpp _0r_~/Templates/cpp.cpp
11 map<F9>_:!g++_%-o_%-Wall_-Wconversion_-Wextra_-g3_<CR>
12 map<F5>_:!./_%_<CR>
13 map<F8>_:!./_%_<_<_%.in_<CR>
14
15 map<F3>_:vnew_%.in_<CR>
16 map<F4>_:!(gedit_%%&)<CR>
```

7.2 C++

```
1 #pragma comment(linker, "/STACK:10240000")
2 #include <cstdio>
3 #include <cstdlib>
4 #include <cstring>
5 #include <iostream>
6 #include <algorithm>
7 #define Rep(i, a, b) for(int i = (a); i <= (b); ++i)
8 #define Foru(i, a, b) for(int i = (a); i < (b); ++i)
9 using namespace std;
10 typedef long long LL;
11 typedef pair<int, int> pii;
12 namespace BufferedReader {
13   char buff[MAX_BUFFER + 5], *ptr = buff, c; bool flag;
14   bool nextChar(char &c) {
15     if ( (c = *ptr++) == 0 ) {
16       int tmp = fread(buff, 1, MAX_BUFFER, stdin);
17       buff[tmp] = 0; if (tmp == 0) return false;
18       ptr = buff; c = *ptr++;
19     } return true;
20   }
21   bool nextUnsignedInt(unsigned int &x) {
22     for (;){if (!nextChar(c)) return false; if ('0'<=c && c<='9') break;}
23     for (x=c-'0'; nextChar(c); x = x * 10 + c - '0') if (c < '0' || c > '9') break;
24     return true;
25   }
26   bool nextInt(int &x) {
27     for (;){ if (!nextChar(c)) return false; if (c=='-' || ('0'<=c && c<='9')) break; }
28     for ((c=='-') ? (x=0,flag=true) : (x=c-'0',flag=false); nextChar(c); x=x*10+c-'0')
29       if (c<'0' || c>'9') break;
30     if (flag) x=-x; return true;
31   }
32 };
33 #endif
```

7.3 Java

```
1 import java.io.*;
2 import java.util.*;
3 import java.math.*;
4
5 public class Main {
6   public void solve() {}
7   public void run() {
8     tokenizer = null; out = new PrintWriter(System.out);
9     in = new BufferedReader(new InputStreamReader(System.in));
10    solve();
11    out.close();
12  }
13  public static void main(String[] args) {
14    new Main().run();
15  }
16  public StringTokenizer tokenizer;
17  public BufferedReader in;
18  public PrintWriter out;
19  public String next() {
20    while (tokenizer == null || !tokenizer.hasMoreTokens()) {
21      try { tokenizer = new StringTokenizer(in.readLine()); }
22      catch (IOException e) { throw new RuntimeException(e); }
23    } return tokenizer.nextToken();
24  }
25 }
```

## 7.4 Eclipse 配置

Exec=env UBUNTU\_MENUPROXY= /opt/eclipse/eclipse

preference general keys 把 word completion 设置成 alt+c, 把 content assistant 设置成 alt + /

## 7.5 泰勒级数

$$\begin{aligned}
 \frac{1}{1-x} &= 1 + x + x^2 + x^3 + x^4 + \cdots & &= \sum_{i=0}^{\infty} x^i \\
 \frac{1}{1-cx} &= 1 + cx + c^2x^2 + c^3x^3 + \cdots & &= \sum_{i=0}^{\infty} c^i x^i \\
 \frac{1}{1-x^n} &= 1 + x^n + x^{2n} + x^{3n} + \cdots & &= \sum_{i=0}^{\infty} x^{ni} \\
 \frac{x}{(1-x)^2} &= x + 2x^2 + 3x^3 + 4x^4 + \cdots & &= \sum_{i=0}^{\infty} ix^i \\
 \sum_{k=0}^n \left\{ \begin{matrix} n \\ k \end{matrix} \right\} \frac{k!z^k}{(1-z)^{k+1}} &= x + 2^n x^2 + 3^n x^3 + 4^n x^4 + \cdots & &= \sum_{i=0}^{\infty} i^n x^i \\
 e^x &= 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \cdots & &= \sum_{i=0}^{\infty} \frac{x^i}{i!} \\
 \ln(1+x) &= x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \cdots & &= \sum_{i=1}^{\infty} (-1)^{i+1} \frac{x^i}{i} \\
 \ln \frac{1}{1-x} &= x + \frac{1}{2}x^2 + \frac{1}{3}x^3 + \frac{1}{4}x^4 + \cdots & &= \sum_{i=1}^{\infty} \frac{x^i}{i} \\
 \sin x &= x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \cdots & &= \sum_{i=0}^{\infty} (-1)^i \frac{x^{2i+1}}{(2i+1)!} \\
 \cos x &= 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \cdots & &= \sum_{i=0}^{\infty} (-1)^i \frac{x^{2i}}{(2i)!} \\
 \tan^{-1} x &= x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \cdots & &= \sum_{i=0}^{\infty} (-1)^i \frac{x^{2i+1}}{(2i+1)} \\
 (1+x)^n &= 1 + nx + \frac{n(n-1)}{2}x^2 + \cdots & &= \sum_{i=0}^{\infty} \binom{n}{i} x^i \\
 \frac{1}{(1-x)^{n+1}} &= 1 + (n+1)x + \binom{n+2}{2}x^2 + \cdots & &= \sum_{i=0}^{\infty} \binom{i+n}{i} x^i \\
 \frac{x}{e^x - 1} &= 1 - \frac{1}{2}x + \frac{1}{12}x^2 - \frac{1}{720}x^4 + \cdots & &= \sum_{i=0}^{\infty} \frac{B_i x^i}{i!} \\
 \frac{1}{2x}(1 - \sqrt{1-4x}) &= 1 + x + 2x^2 + 5x^3 + \cdots & &= \sum_{i=0}^{\infty} \frac{1}{i+1} \binom{2i}{i} x^i
 \end{aligned}$$

$$\begin{aligned}
 \frac{1}{\sqrt{1-4x}} &= 1 + 2x + 6x^2 + 20x^3 + \cdots & &= \sum_{i=0}^{\infty} \binom{2i}{i} x^i \\
 \frac{1}{\sqrt{1-4x}} \left( \frac{1 - \sqrt{1-4x}}{2x} \right)^n &= 1 + (2+n)x + \binom{4+n}{2}x^2 + \cdots & &= \sum_{i=0}^{\infty} \binom{2i+n}{i} x^i \\
 \frac{1}{1-x} \ln \frac{1}{1-x} &= x + \frac{3}{2}x^2 + \frac{11}{6}x^3 + \frac{25}{12}x^4 + \cdots & &= \sum_{i=1}^{\infty} H_i x^i \\
 \frac{1}{2} \left( \ln \frac{1}{1-x} \right)^2 &= \frac{1}{2}x^2 + \frac{3}{4}x^3 + \frac{11}{24}x^4 + \cdots & &= \sum_{i=2}^{\infty} \frac{H_{i-1} x^i}{i} \\
 \frac{x}{1-x-x^2} &= x + x^2 + 2x^3 + 3x^4 + \cdots & &= \sum_{i=0}^{\infty} F_i x^i \\
 \frac{F_n x}{1 - (F_{n-1} + F_{n+1})x - (-1)^n x^2} &= F_n x + F_{2n} x^2 + F_{3n} x^3 + \cdots & &= \sum_{i=0}^{\infty} F_{ni} x^i
 \end{aligned}$$

## 7.6 积分表

- $d(\tan x) = \sec^2 x dx$
- $d(\cot x) = \csc^2 x dx$
- $d(\sec x) = \tan x \sec x dx$
- $d(\csc x) = -\cot x \csc x dx$
- $d(\arcsin x) = \frac{1}{\sqrt{1-x^2}} dx$
- $d(\arccos x) = \frac{-1}{\sqrt{1-x^2}} dx$
- $d(\arctan x) = \frac{1}{1+x^2} dx$
- $d(\operatorname{arccot} x) = \frac{-1}{1+x^2} dx$
- $d(\operatorname{arcsec} x) = \frac{1}{x\sqrt{1-x^2}} dx$
- $d(\operatorname{arccsc} x) = \frac{-1}{u\sqrt{1-x^2}} dx$
- $\int cu \, dx = c \int u \, dx$
- $\int (u+v) \, dx = \int u \, dx + \int v \, dx$
- $\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1$
- $\int \frac{1}{x} dx = \ln x$



- $\int e^x dx = e^x$
- $\int \frac{dx}{1+x^2} = \arctan x$
- $\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx$
- $\int \sin x dx = -\cos x$
- $\int \cos x dx = \sin x$
- $\int \tan x dx = -\ln |\cos x|$
- $\int \cot x dx = \ln |\cos x|$
- $\int \sec x dx = \ln |\sec x + \tan x|$
- $\int \csc x dx = \ln |\csc x + \cot x|$
- $\int \arcsin \frac{x}{a} dx = \arcsin \frac{x}{a} + \sqrt{a^2 - x^2}, \quad a > 0$
- $\int \arccos \frac{x}{a} dx = \arccos \frac{x}{a} - \sqrt{a^2 - x^2}, \quad a > 0$
- $\int \arctan \frac{x}{a} dx = x \arctan \frac{x}{a} - \frac{a}{2} \ln(a^2 + x^2), \quad a > 0$
- $\int \sin^2(ax) dx = \frac{1}{2a}(ax - \sin(ax) \cos(ax))$
- $\int \cos^2(ax) dx = \frac{1}{2a}(ax + \sin(ax) \cos(ax))$
- $\int \sec^2 x dx = \tan x$
- $\int \csc^2 x dx = -\cot x$
- $\int \sin^n x dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} \int \sin^{n-2} x dx$
- $\int \cos^n x dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} \int \cos^{n-2} x dx$
- $\int \tan^n x dx = \frac{\tan^{n-1} x}{n-1} - \int \tan^{n-2} x dx, \quad n \neq 1$
- $\int \cot^n x dx = -\frac{\cot^{n-1} x}{n-1} - \int \cot^{n-2} x dx, \quad n \neq 1$
- $\int \sec^n x dx = \frac{\tan x \sec^{n-1} x}{n-1} + \frac{n-2}{n-1} \int \sec^{n-2} x dx, \quad n \neq 1$
- $\int \csc^n x dx = -\frac{\cot x \csc^{n-1} x}{n-1} + \frac{n-2}{n-1} \int \csc^{n-2} x dx, \quad n \neq 1$

- $\int \sinh x dx = \cosh x$
- $\int \cosh x dx = \sinh x$
- $\int \tanh x dx = \ln |\cosh x|$
- $\int \coth x dx = \ln |\sinh x|$
- $\int \operatorname{sech} x dx = \arctan \sinh x$
- $\int \operatorname{csch} x dx = \ln \left| \tanh \frac{x}{2} \right|$
- $\int \sinh^2 x dx = \frac{1}{4} \sinh(2x) - \frac{1}{2} x$
- $\int \cosh^2 x dx = \frac{1}{4} \sinh(2x) + \frac{1}{2} x$
- $\int \operatorname{sech}^2 x dx = \tanh x$
- $\int \operatorname{arcsinh} \frac{x}{a} dx = x \operatorname{arcsinh} \frac{x}{a} - \sqrt{x^2 + a^2}, \quad a > 0$
- $\int \operatorname{arctanh} \frac{x}{a} dx = x \operatorname{arctanh} \frac{x}{a} + \frac{a}{2} \ln |a^2 - x^2|$
- $\int \operatorname{arccosh} \frac{x}{a} dx = \begin{cases} x \operatorname{arccosh} \frac{x}{a} - \sqrt{x^2 + a^2}, & \text{if } \operatorname{arccosh} \frac{x}{a} > 0 \text{ and } a > 0 \\ x \operatorname{arccosh} \frac{x}{a} + \sqrt{x^2 + a^2}, & \text{if } \operatorname{arccosh} \frac{x}{a} < 0 \text{ and } a > 0 \end{cases}$
- $\int \frac{dx}{\sqrt{a^2 + x^2}} = \ln \left( x + \sqrt{a^2 + x^2} \right), \quad a > 0$
- $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan \frac{x}{a}, \quad a > 0$
- $\int \sqrt{a^2 - x^2} dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin \frac{x}{a}, \quad a > 0$
- $\int (a^2 - x^2)^{3/2} dx = \frac{x}{8} (5a^2 - 2x^2) \sqrt{a^2 - x^2} + \frac{3a^4}{8} \arcsin \frac{x}{a}, \quad a > 0$
- $\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin \frac{x}{a}, \quad a > 0$
- $\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right|$
- $\int \frac{dx}{(a^2 - x^2)^{3/2}} = \frac{x}{a^2 \sqrt{a^2 - x^2}}$
- $\int \sqrt{a^2 \pm x^2} dx = \frac{x}{2} \sqrt{a^2 \pm x^2} \pm \frac{a^2}{2} \ln \left| x + \sqrt{a^2 \pm x^2} \right|$
- $\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln \left| x + \sqrt{x^2 - a^2} \right|, \quad a > 0$

- $\int \frac{dx}{ax^2 + bx} = \frac{1}{a} \ln \left| \frac{x}{a + bx} \right|$
- $\int x\sqrt{a + bx} dx = \frac{2(3bx - 2a)(a + bx)^{3/2}}{15b^2}$
- $\int \frac{\sqrt{a + bx}}{x} dx = 2\sqrt{a + bx} + a \int \frac{1}{x\sqrt{a + bx}} dx$
- $\int \frac{x}{\sqrt{a + bx}} dx = \frac{1}{\sqrt{2}} \ln \left| \frac{\sqrt{a + bx} - \sqrt{a}}{\sqrt{a + bx} + \sqrt{a}} \right|, \quad a > 0$
- $\int \frac{\sqrt{a^2 - x^2}}{x} dx = \sqrt{a^2 - x^2} - a \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right|$
- $\int x\sqrt{a^2 - x^2} dx = -\frac{1}{3}(a^2 - x^2)^{3/2}$
- $\int x^2\sqrt{a^2 - x^2} dx = \frac{x}{8}(2x^2 - a^2)\sqrt{a^2 - x^2} + \frac{a^4}{8}\arcsin \frac{x}{a}, \quad a > 0$
- $\int \frac{dx}{\sqrt{a^2 - x^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right|$
- $\int \frac{x dx}{\sqrt{a^2 - x^2}} = -\sqrt{a^2 - x^2}$
- $\int \frac{x^2 dx}{\sqrt{a^2 - x^2}} = -\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\arcsin \frac{x}{a}, \quad a > 0$
- $\int \frac{\sqrt{a^2 + x^2}}{x} dx = \sqrt{a^2 + x^2} - a \ln \left| \frac{a + \sqrt{a^2 + x^2}}{x} \right|$
- $\int \frac{\sqrt{x^2 - a^2}}{x} dx = \sqrt{x^2 - a^2} - a \arccos \frac{a}{|x|}, \quad a > 0$
- $\int x\sqrt{x^2 \pm a^2} dx = \frac{1}{3}(x^2 \pm a^2)^{3/2}$
- $\int \frac{dx}{x\sqrt{x^2 + a^2}} = \frac{1}{a} \ln \left| \frac{x}{a + \sqrt{a^2 + x^2}} \right|$
- $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \arccos \frac{a}{|x|}, \quad a > 0$
- $\int \frac{dx}{x^2\sqrt{x^2 \pm a^2}} = \mp \frac{\sqrt{x^2 \pm a^2}}{a^2 x}$
- $\int \frac{x dx}{\sqrt{x^2 \pm a^2}} = \sqrt{x^2 \pm a^2}$
- $\int \frac{\sqrt{x^2 \pm a^2}}{x^4} dx = \mp \frac{(x^2 + a^2)^{3/2}}{3a^2 x^3}$

- $\int \frac{dx}{ax^2 + bx + c} = \begin{cases} \frac{1}{\sqrt{b^2 - 4ac}} \ln \left| \frac{2ax + b - \sqrt{b^2 - 4ac}}{2ax + b + \sqrt{b^2 - 4ac}} \right|, & \text{if } b^2 > 4ac \\ \frac{2}{\sqrt{4ac - b^2}} \arctan \frac{2ax + b}{\sqrt{4ac - b^2}}, & \text{if } b^2 < 4ac \end{cases}$
- $\int \frac{dx}{\sqrt{ax^2 + bx + c}} = \begin{cases} \frac{1}{\sqrt{a}} \ln \left| 2ax + b + 2\sqrt{a}\sqrt{ax^2 + bx + c} \right|, & \text{if } a > 0 \\ \frac{1}{\sqrt{-a}} \arcsin \frac{-2ax - b}{\sqrt{b^2 - 4ac}}, & \text{if } a < 0 \end{cases}$
- $\int \sqrt{ax^2 + bx + c} dx = \frac{2ax + b}{4a} \sqrt{ax^2 + bx + c} + \frac{4ac - b^2}{8a} \int \frac{dx}{\sqrt{ax^2 + bx + c}}$
- $\int \frac{x dx}{\sqrt{ax^2 + bx + c}} = \frac{\sqrt{ax^2 + bx + c}}{a} - \frac{b}{2a} \int \frac{dx}{\sqrt{ax^2 + bx + c}}$
- $\int \frac{dx}{x\sqrt{ax^2 + bx + c}} = \begin{cases} -\frac{1}{\sqrt{c}} \ln \left| \frac{2\sqrt{c}\sqrt{ax^2 + bx + c} + bx + 2c}{x} \right|, & \text{if } c > 0 \\ \frac{1}{\sqrt{-c}} \arcsin \frac{bx + 2c}{|x|\sqrt{b^2 - 4ac}}, & \text{if } c < 0 \end{cases}$
- $\int x^3 \sqrt{x^2 + a^2} dx = \left( \frac{1}{3}x^2 - \frac{2}{15}a^2 \right) (x^2 + a^2)^{3/2}$
- $\int x^n \sin(ax) dx = -\frac{1}{a}x^n \cos(ax) + \frac{n}{a} \int x^{n-1} \cos(ax) dx$
- $\int x^n \cos(ax) dx = \frac{1}{a}x^n \sin(ax) - \frac{n}{a} \int x^{n-1} \sin(ax) dx$
- $\int x^n e^{ax} dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} dx$
- $\int x^n \ln(ax) dx = x^{n+1} \left( \frac{\ln(ax)}{n+1} - \frac{1}{(n+1)^2} \right)$
- $\int x^n (\ln ax)^m dx = \frac{x^{n+1}}{n+1} (\ln ax)^m - \frac{m}{n+1} \int x^n (\ln ax)^{m-1} dx$