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28 1 计算几何**28 1.1 二维计算几何基本操作**

```

29 1 const double PI = 3.14159265358979323846264338327950288;
29 2 double arcSin(const double &a) {
29 3     return (a <= -1.0) ? (-PI / 2) : ((a >= 1.0) ? (PI / 2) : (asin(a)));
29 4 }
30 5 double arcCos(const double &a) {
30 6     return (a <= -1.0) ? (PI) : ((a >= 1.0) ? (0) : (acos(a)));
30 7 }
30 8 struct point { double x, y; // something omitted
30 9     point rot(const double &a) const { // counter-clockwise
30 10         return point(x * cos(a) - y * sin(a), x * sin(a) + y * cos(a));
30 11     }
30 12     point rot90() const { return point(-y, x); } // counter-clockwise
30 13     point project(const point &p1, const point &p2) const {
32 14         const point &q = *this; return p1 + (p2 - p1) * (dot(p2 - p1, q - p1) / (p2 - p1).norm())
32 15         ;
32 16     }
32 17     bool onSeg(const point &a, const point &b) const { // a, b inclusive
32 18         const point &c = *this; return sign(dot(a - c, b - c)) <= 0 && sign(det(b - a, c - a)) ==
32 19         0;
32 20     }
32 21     double distLP(const point &p1, const point &p2) const { // dist from *this to line p1->p2
32 22         const point &q = *this; return fabs(det(p2 - p1, q - p1)) / (p2 - p1).len();
32 23     }
32 24     double distSP(const point &p1, const point &p2) const { // dist from *this to segment [p1, p2]
32 25         const point &q = *this;
32 26         if (dot(p2 - p1, q - p1) < EPS) return (q - p1).len();
32 27         if (dot(p1 - p2, q - p2) < EPS) return (q - p2).len();
32 28         return distLP(p1, p2);
32 29     }
32 30     bool inAngle(const point &p1, const point &p2) const { // det(p1, p2) ≥ 0
32 31         const point &q = *this; return det(p1, q) > -EPS && det(p2, q) < EPS;
32 32     }
32 33 };
32 34 bool lineIntersect(const point &a, const point &b, const point &c, const point &d, point &e)
32 35 {
32 36     double s1 = det(c - a, d - a), s2 = det(d - b, c - b);
32 37     if (!sign(s1 + s2)) return false;
32 38     e = (b - a) * (s1 / (s1 + s2)) + a;
32 39     return true;
32 40 }
32 41 bool segIntersect(const point &a, const point &b, const point &c, const point &d, point &e) {
32 42     double s1 = det(c - a, d - a), s2 = det(d - b, c - b);
32 43     if (!sign(s1 + s2)) return false;
32 44     e = (b - a) * (s1 / (s1 + s2)) + a;
32 45     return e.onSeg(a, b) && e.onSeg(c, d);
32 46 }
32 47 int segIntersectCheck(const point &a, const point &b, const point &c, const point &d, point &
32 48 o) {

```

```

45 static double s1, s2, s3, s4;
46 static int iCnt;
47 int d1 = sign(s1 = det(b - a, c - a)), d2 = sign(s2 = det(b - a, d - a));
48 int d3 = sign(s3 = det(d - c, a - c)), d4 = sign(s4 = det(d - c, b - c));
49 if ((d1 ^ d2) == -2 && (d3 ^ d4) == -2) {
50     o = (c * s2 - d * s1) / (s2 - s1); return true;
51 } iCnt = 0;
52 if (d1 == 0 && c.onSeg(a, b)) o = c, ++iCnt;
53 if (d2 == 0 && d.onSeg(a, b)) o = d, ++iCnt;
54 if (d3 == 0 && a.onSeg(c, d)) o = a, ++iCnt;
55 if (d4 == 0 && b.onSeg(c, d)) o = b, ++iCnt;
56 return iCnt ? 2 : 0; // 不相交返回 0, 严格相交返回 1, 非严格相交返回 2
57 }
58 struct circle {
59     point o; double r, rSqure;
60     circle() : o(), r(0.0), rSqure(0.0) {}
61     circle(const point &o, const double &r) : o(o), r(r), rSqure(r * r) {}
62     bool inside(const point &a) { return (a - o).len() < r + EPS; } // 非严格
63     bool contain(const circle &b) const { return sign(b.r + (o - b.o).len() - r) <= 0; } // 非严格
64     bool disjunct(const circle &b) const { return sign(b.r + r - (o - b.o).len()) <= 0; } // 非严格
65     int isCL(const point &p1, const point &p2, point &a, point &b) const {
66         double x = dot(p1 - o, p2 - p1), y = (p2 - p1).norm();
67         double d = x * x - y * ((p1 - o).norm() - rSqure);
68         if (d < -EPS) return 0; if (d < 0) d = 0;
69         point q1 = p1 - (p2 - p1) * (x / y);
70         point q2 = (p2 - p1) * (sqrt(d) / y);
71         a = q1 - q2; b = q1 + q2;
72         return q2.len() < EPS ? 1 : 2;
73     }
74     int tanCP(const point &p, point &a, point &b) const { // 返回切点, 注意可能与 p 重合
75         double x = (p - o).norm(), d = x - rSqure;
76         if (d < -EPS) return 0; if (d < 0) d = 0;
77         point q1 = (p - o) * (rSqure / x), q2 = ((p - o) * (-r * sqrt(d) / x)).rot90();
78         a = o + (q1 - q2); b = o + (q1 + q2);
79         return q2.len() < EPS ? 1 : 2;
80     }
81 };
82 bool checkCrossCS(const circle &cir, const point &p1, const point &p2) { // 非严格
83     const point &c = cir.o; const double &r = cir.r;
84     return c.distSP(p1, p2) < r + EPS && (r < (c - p1).len() + EPS || r < (c - p2).len() + EPS)
85     ;
86 }
87 bool checkCrossCC(const circle &cir1, const circle &cir2) { // 非严格
88     double &r1 = cir1.r, &r2 = cir2.r, d = (cir1.o - cir2.o).len();
89     return d < r1 + r2 + EPS && fabs(r1 - r2) < d + EPS;
90 }
91 int isCC(const circle &cir1, const circle &cir2, point &a, point &b) {
92     const point &c1 = cir1.o, &c2 = cir2.o;
93     double x = (c1 - c2).norm(), y = ((cir1.rSqure - cir2.rSqure) / x + 1) / 2;
94     double d = cir1.rSqure / x - y * y;
95     if (d < -EPS) return 0; if (d < 0) d = 0;

```

```

95     point q1 = c1 + (c2 - c1) * y, q2 = ((c2 - c1) * sqrt(d)).rot90();
96     a = q1 - q2; b = q1 + q2;
97     return q2.len() < EPS ? 1 : 2;
98 }
99 vector<pair<point, point>> tanCC(const circle &cir1, const circle &cir2) {
100     // 注意: 如果只有三条切线, 即 s1 = 1, s2 = 1, 返回的切线可能重复, 切点没有问题
101     vector<pair<point, point>> list;
102     if (cir1.contain(cir2) || cir2.contain(cir1)) return list;
103     const point &c1 = cir1.o, &c2 = cir2.o;
104     double r1 = cir1.r, r2 = cir2.r;
105     point p, a1, b1, a2, b2; int s1, s2;
106     if (sign(r1 - r2) == 0) {
107         p = c2 - c1;
108         p = (p * (r1 / p.len())).rot90();
109         list.push_back(make_pair(c1 + p, c2 + p));
110         list.push_back(make_pair(c1 - p, c2 - p));
111     } else {
112         p = (c2 * r1 - c1 * r2) / (r1 - r2);
113         s1 = cir1.tanCP(p, a1, b1);
114         s2 = cir2.tanCP(p, a2, b2);
115         if (s1 >= 1 && s2 >= 1) {
116             list.push_back(make_pair(a1, a2));
117             list.push_back(make_pair(b1, b2));
118         }
119         p = (c1 * r2 + c2 * r1) / (r1 + r2);
120         s1 = cir1.tanCP(p, a1, b1);
121         s2 = cir2.tanCP(p, a2, b2);
122         if (s1 >= 1 && s2 >= 1) {
123             list.push_back(make_pair(a1, a2));
124             list.push_back(make_pair(b1, b2));
125         } return list;
126     }
127 bool distConvexPIn(const point &p1, const point &p2, const point &p3, const point &p4, const
128     point &q) {
129     point o12 = (p1 - p2).rot90(), o23 = (p2 - p3).rot90(), o34 = (p3 - p4).rot90();
130     return (q - p1).inAngle(o12, o23) || (q - p3).inAngle(o23, o34)
131     || ((q - p2).inAngle(o23, p3 - p2) && (q - p3).inAngle(p2 - p3, o23));
132 }
133 double distConvexP(int n, point ps[], const point &q) {
134     int left = 0, right = n;
135     while (right - left > 1) {
136         int mid = (left + right) / 2;
137         if (distConvexPIn(ps[(left + n - 1) % n], ps[left], ps[mid], ps[(mid + 1) % n], q))
138             right = mid;
139         else left = mid;
140     } return q.distSP(ps[left], ps[right % n]);
141 }
142 double areaCT(const circle &cir, point pa, point pb) {
143     pa = pa - cir.o; pb = pb - cir.o;
144     double R = cir.r;
145     if (pa.len() < pb.len()) swap(pa, pb);
146     if (pb.len() < EPS) return 0;
147     point pc = pb - pa;

```

```

147 double a = pa.len(), b = pb.len(), c = pc.len();
148 double cosB = dot(pb, pc) / b / c, B = acos(cosB);
149 double cosC = dot(pa, pb) / a / b, C = acos(cosC);
150 double S, h, theta;
151 if (b > R) {
152     S = C * 0.5 * R * R;
153     h = b * a * sin(C) / c;
154     if (h < R && B < PI * 0.5) S -= acos(h / R) * R * R - h * sqrt(R * R - h * h);
155 } else if (a > R) {
156     theta = PI - B - asin(sin(B) / R * b);
157     S = 0.5 * b * R * sin(theta) + (C - theta) * 0.5 * R * R;
158 } else S = 0.5 * sin(C) * b * a;
159 return S;
160 }
161 circle minCircle(const point &a, const point &b) {
162     return circle((a + b) * 0.5, (b - a).len() * 0.5);
163 }
164
165 circle minCircle(const point &a, const point &b, const point &c) { // 钝角三角形没有被考虑
166     double a2((b - c).norm()), b2((a - c).norm()), c2((a - b).norm());
167     if (b2 + c2 <= a2 + EPS) return minCircle(b, c);
168     if (a2 + c2 <= b2 + EPS) return minCircle(a, c);
169     if (a2 + b2 <= c2 + EPS) return minCircle(a, b);
170     double A = 2.0 * (a.x - b.x), B = 2.0 * (a.y - b.y);
171     double D = 2.0 * (a.x - c.x), E = 2.0 * (a.y - c.y);
172     double C = a.norm() - b.norm();
173     double F = a.norm() - c.norm();
174     point p((C * E - B * F) / (A * E - B * D), (A * F - C * D) / (A * E - B * D));
175     return circle(p, (p - a).len());
176 }
177 circle minCircle(point P[], int N) { // 1-based
178     if (N == 1) return circle(P[1], 0.0);
179     random_shuffle(P + 1, P + N + 1);
180     circle O = minCircle(P[1], P[2]);
181     Rep(i, 1, N) if(!O.inside(P[i])) { O = minCircle(P[1], P[i]);
182         Foru(j, 1, i) if(!O.inside(P[j])) { O = minCircle(P[i], P[j]);
183             Foru(k, 1, j) if(!O.inside(P[k])) O = minCircle(P[i], P[j], P[k]);
184         }
185     } return O;
186 }

```

1.2 圆的面积模板

```

1 struct Event {
2     point p; double alpha; int add;
3     Event(): p(), alpha(0.0), add(0) {}
4     Event(const point &p, const double &alpha, const int &add): p(p), alpha(alpha), add(add) {}
5     bool operator < (const Event &other) const { return alpha < other.alpha; }
6 };
7 void circleKCover(circle *c, int N, double *area) { // area[k] : covered not less than k
            times

```

```

8 static bool overlap[MAXN][MAXN], g[MAXN][MAXN];
9 Rep(i, 0, N + 1) area[i] = 0.0;
10 Rep(i, 1, N) Rep(j, 1, N) overlap[i][j] = c[i].contain(c[j]);
11 Rep(i, 1, N) Rep(j, 1, N) g[i][j] = !(overlap[i][j] || overlap[j][i] || c[i].disjunct(c[j])
12 );
13 Rep(i, 1, N) {
14     static Event events[MAXN * 2 + 1];
15     int totE = 0, cnt = 1;
16     Rep(j, 1, N) if (j != i && overlap[j][i]) ++cnt;
17     Rep(j, 1, N) if (j != i && g[i][j]) {
18         circle &a = c[i], &b = c[j];
19         double l = (a.o - b.o).norm();
20         double s = ((a.r - b.r) * (a.r + b.r) / l + 1) * 0.5;
21         double t = sqrt(-(1 - sqrt(a.r - b.r)) * (1 - sqrt(a.r + b.r)) / (1 * 1 * 4.0));
22         point dir = b.o - a.o, nDir = point(-dir.y, dir.x);
23         point aa = a.o + dir * s + nDir * t;
24         point bb = a.o + dir * s - nDir * t;
25         double A = atan2(aa.y - a.o.y, aa.x - a.o.x);
26         double B = atan2(bb.y - a.o.y, bb.x - a.o.x);
27         events[totE++] = Event(bb, B, 1);
28         events[totE++] = Event(aa, A, -1);
29         if (B > A) ++cnt;
30     }
31     if (totE == 0) { area[cnt] += PI * c[i].r2; continue; }
32     sort(events, events + totE); events[totE] = events[0];
33     Foru(j, 0, totE) {
34         cnt += events[j].add;
35         area[cnt] += 0.5 * det(events[j].p, events[j + 1].p);
36         double theta = events[j + 1].alpha - events[j].alpha;
37         if (theta < 0) theta += 2.0 * PI;
38         area[cnt] += 0.5 * c[i].r2 * (theta - sin(theta));
39     }
40 }

```

1.3 多边形相关

```

1 struct Polygon { // stored in [0, n)
2     int n; point list[MAXN];
3     Polygon cut(const point &a, const point &b) {
4         static Polygon res;
5         static point o;
6         res.n = 0;
7         for (int i = 0; i < n; ++i) {
8             int s1 = sign(det(list[i] - a, b - a));
9             int s2 = sign(det(list[(i + 1) % n] - a, b - a));
10            if (s1 <= 0) res.list[res.n++] = list[i];
11            if (s1 * s2 < 0) {
12                lineIntersect(a, b, list[i], list[(i + 1) % n], o);
13                res.list[res.n++] = o;
14            }

```

```

15     } return res;
16 }
17 bool contain(const point &p) const { // 1 if on border or inner, 0 if outer
18     static point A, B;
19     int res = 0;
20     for (int i = 0; i < n; ++i) {
21         A = list[i]; B = list[(i + 1) % n];
22         if (p.onSeg(A, B)) return 1;
23         if (sign(A.y - B.y) <= 0) swap(A, B);
24         if (sign(p.y - A.y) > 0) continue;
25         if (sign(p.y - B.y) <= 0) continue;
26         res += (int)(sign(det(B - p, A - p)) > 0);
27     }
28     return res & 1;
29 }
30 bool convexContain(const point &p) const { // sort by polar angle
31     for (int i = 1; i < n; ++i) list[i] = list[i] - list[0];
32     point q = p - list[0];
33     if (sign(det(list[1], q)) < 0 || sign(det(list[n - 1], q)) > 0) return false;
34     int l = 2, r = n - 1;
35     while (l <= r) {
36         int mid = (l + r) >> 1;
37         double d1 = sign(det(list[mid], q)), d2 = sign(det(list[mid - 1], q));
38         if (d1 <= 0) {
39             if (d2 <= 0) {
40                 if (sign(det(q - list[mid - 1], list[mid] - list[mid - 1])) <= 0) <= 0)
41                     return true;
42             } else r = mid - 1;
43         } else l = mid + 1;
44     }
45     return false;
46 }
47 double isPLAtan2(const point &a, const point &b) {
48     double k = (b - a).alpha(); if (k < 0) k += 2 * PI;
49     return k;
50 }
51 point isPL_Get(const point &a, const point &b, const point &s1, const point &s2) {
52     double k1 = det(b - a, s1 - a), k2 = det(b - a, s2 - a);
53     if (sign(k1) == 0) return s1;
54     if (sign(k2) == 0) return s2;
55     return (s1 * k2 - s2 * k1) / (k2 - k1);
56 }
57 int isPL_Dic(const point &a, const point &b, int l, int r) {
58     int s = (det(b - a, list[l] - a) < 0) ? -1 : 1;
59     while (l <= r) {
60         int mid = (l + r) / 2;
61         if (det(b - a, list[mid] - a) * s <= 0) r = mid - 1;
62         else l = mid + 1;
63     }
64     return r + 1;
65 }
66 int isPL_Find(double k, double w[]) {
67     if (k <= w[0] || k > w[n - 1]) return 0;

```

```

68     int l = 0, r = n - 1, mid;
69     while (l <= r) {
70         mid = (l + r) / 2;
71         if (w[mid] >= k) r = mid - 1;
72         else l = mid + 1;
73     } return r + 1;
74 }
75 bool isPL(const point &a, const point &b, point &cp1, point &cp2) { // O(logN)
76     static double w[MAXN * 2]; // pay attention to the array size
77     for (int i = 0; i <= n; ++i) list[i + n] = list[i];
78     for (int i = 0; i < n; ++i) w[i] = w[i + n] = isPLAtan2(list[i], list[i + 1]);
79     int i = isPL_Find(isPLAtan2(a, b), w);
80     int j = isPL_Find(isPLAtan2(b, a), w);
81     double k1 = det(b - a, list[i] - a), k2 = det(b - a, list[j] - a);
82     if (sign(k1) * sign(k2) > 0) return false; // no intersection
83     if (sign(k1) == 0 || sign(k2) == 0) { // intersect with a point or a line in the convex
84         if (sign(k1) == 0) {
85             if (sign(det(b - a, list[i + 1] - a)) == 0) cp1 = list[i], cp2 = list[i + 1];
86             else cp1 = cp2 = list[i];
87             return true;
88         }
89         if (sign(k2) == 0) {
90             if (sign(det(b - a, list[j + 1] - a)) == 0) cp1 = list[j], cp2 = list[j + 1];
91             else cp1 = cp2 = list[j];
92         }
93         return true;
94     }
95     if (i > j) swap(i, j);
96     int x = isPL_Dic(a, b, i, j), y = isPL_Dic(a, b, j, i + n);
97     cp1 = isPL_Get(a, b, list[x - 1], list[x]);
98     cp2 = isPL_Get(a, b, list[y - 1], list[y]);
99     return true;
100 }
101 double getI(const point &O) const {
102     if (n <= 2) return 0;
103     point G(0.0, 0.0);
104     double S = 0.0, I = 0.0;
105     for (int i = 0; i < n; ++i) {
106         const point &x = list[i], &y = list[(i + 1) % n];
107         double d = det(x, y);
108         G = G + (x + y) * d / 3.0;
109         S += d;
110     } G = G / S;
111     for (int i = 0; i < n; ++i) {
112         point x = list[i] - G, y = list[(i + 1) % n] - G;
113         I += fabs(det(x, y)) * (x.norm() + dot(x, y) + y.norm());
114     }
115     return I = I / 12.0 + fabs(S * 0.5) * (O - G).norm();
116 }
117 };

```

1.4 半平面交

```

1 struct Border {
2     point p1, p2; double alpha;
3     Border() : p1(), p2(), alpha(0.0) {}
4     Border(const point &a, const point &b): p1(a), p2(b), alpha( atan2(p2.y - p1.y, p2.x - p1.x) ) {}
5     bool operator == (const Border &b) const { return sign(alpha - b.alpha) == 0; }
6     bool operator < (const Border &b) const {
7         int c = sign(alpha - b.alpha); if (c != 0) return c > 0;
8         return sign(det(b.p2 - b.p1, p1 - b.p1)) >= 0;
9     }
10 };
11 point isBorder(const Border &a, const Border &b) { // a and b should not be parallel
12     point is; lineIntersect(a.p1, a.p2, b.p1, b.p2, is); return is;
13 }
14 bool checkBorder(const Border &a, const Border &b, const Border &me) {
15     point is; lineIntersect(a.p1, a.p2, b.p1, b.p2, is);
16     return sign(det(me.p2 - me.p1, is - me.p1)) > 0;
17 }
18 double HPI(int N, Border border[]) {
19     static Border que[MAXN * 2 + 1]; static point ps[MAXN];
20     int head = 0, tail = 0, cnt = 0; // [head, tail)
21     sort(border, border + N); N = unique(border, border + N) - border;
22     for (int i = 0; i < N; ++i) {
23         Border &cur = border[i];
24         while (head + 1 < tail && !checkBorder(que[tail - 2], que[tail - 1], cur)) --tail;
25         while (head + 1 < tail && !checkBorder(que[head], que[head + 1], cur)) ++head;
26         que[tail++] = cur;
27     } while (head + 1 < tail && !checkBorder(que[tail - 2], que[tail - 1], que[head])) --tail;
28     while (head + 1 < tail && !checkBorder(que[head], que[head + 1], que[tail - 1])) ++head;
29     if (tail - head <= 2) return 0.0;
30     Foru(i, head, tail) ps[cnt++] = isBorder(que[i], que[(i + 1 == tail) ? (head) : (i + 1)]);
31     double area = 0; Foru(i, 0, cnt) area += det(ps[i], ps[(i + 1) % cnt]);
32     return fabs(area * 0.5); // or (-area * 0.5)
33 }

```

1.5 最大面积空凸包

```

1 inline bool toUpRight(const point &a, const point &b) {
2     int c = sign(b.y - a.y); if (c > 0) return true;
3     return c == 0 && sign(b.x - a.x) > 0;
4 }
5 inline bool cmpByPolarAngle(const point &a, const point &b) { // counter-clockwise, shorter first
6     if they share the same polar angle
7     int c = sign(det(a, b)); if (c != 0) return c > 0;
8     return sign(b.len() - a.len()) > 0;
9 }
10 double maxEmptyConvexHull(int N, point p[]) {
11     static double dp[MAXN][MAXN];
12     static point vec[MAXN];

```

```

12 static int seq[MAXN]; // empty triangles formed with (0,0), vec[o], vec[seq[i]]
13 double ans = 0.0;
14 Rep(o, 1, N) {
15     int totVec = 0;
16     Rep(i, 1, N) if (toUpRight(p[o], p[i])) vec[++totVec] = p[i] - p[o];
17     sort(vec + 1, vec + totVec + 1, cmpByPolarAngle);
18     Rep(i, 1, totVec) Rep(j, 1, totVec) dp[i][j] = 0.0;
19     Rep(k, 2, totVec) {
20         int i = k - 1;
21         while (i > 0 && sign( det(vec[k], vec[i]) ) == 0) --i;
22         int totSeq = 0;
23         for (int j; i > 0; i = j) {
24             seq[++totSeq] = i;
25             for (j = i - 1; j > 0 && sign(det(vec[i] - vec[k], vec[j] - vec[k])) > 0; --j);
26             double v = det(vec[i], vec[k]) * 0.5;
27             if (j > 0) v += dp[i][j];
28             dp[k][i] = v;
29             cMax(ans, v);
30         } for (int i = totSeq - 1; i >= 1; --i) cMax( dp[k][ seq[i] ], dp[k][seq[i + 1]] );
31     }
32 } return ans;
33 }

```

1.6 最近点对

```

1 int N; point p[maxn];
2 bool cmpByX(const point &a, const point &b) { return sign(a.x - b.x) < 0; }
3 bool cmpByY(const int &a, const int &b) { return p[a].y < p[b].y; }
4 double minimalDistance(point *c, int n, int *ys) {
5     double ret = 1e+20;
6     if (n < 20) {
7         Foru(i, 0, n) Foru(j, i + 1, n) cMin(ret, (c[i] - c[j]).len() );
8         sort(ys, ys + n, cmpByY); return ret;
9     } static int mergeTo[maxn];
10     int mid = n / 2; double xmid = c[mid].x;
11     ret = min(minimalDistance(c, mid, ys), minimalDistance(c + mid, n - mid, ys + mid));
12     merge(ys, ys + mid, ys + mid, ys + n, mergeTo, cmpByY);
13     copy(mergeTo, mergeTo + n, ys);
14     Foru(i, 0, n) {
15         while (i < n && sign(fabs(p[ys[i]].x - xmid) - ret) > 0) ++i;
16         int cnt = 0;
17         Foru(j, i + 1, n)
18             if (sign(p[ys[j]].y - p[ys[i]].y - ret) > 0) break;
19             else if (sign(fabs(p[ys[j]].x - xmid) - ret) <= 0) {
20                 ret = min(ret, (p[ys[i]] - p[ys[j]]).len());
21                 if (++cnt >= 10) break;
22             }
23     } return ret;
24 }
25 double work() {
26     sort(p, p + n, cmpByX); Foru(i, 0, n) ys[i] = i; return minimalDistance(p, n, ys);

```

27 }

```
21     return ans;
22 }
```

1.7 凸包与点集直径

```
1 vector<point> convexHull(int n, point ps[]) { // counter-clockwise, strict
2     static point qs[MAXN * 2];
3     sort(ps, ps + n, cmpByXY);
4     if (n <= 2) return vector(ps, ps + n);
5     int k = 0;
6     for (int i = 0; i < n; qs[k++] = ps[i++])
7         while (k > 1 && det(qs[k - 1] - qs[k - 2], ps[i] - qs[k - 1]) < EPS) --k;
8     for (int i = n - 2, t = k; i >= 0; qs[k++] = ps[i--])
9         while (k > t && det(qs[k - 1] - qs[k - 2], ps[i] - qs[k - 1]) < EPS) --k;
10    return vector<point>(qs, qs + k);
11 }
12 double convexDiameter(int n, point ps[]) {
13     if (n < 2) return 0; if (n == 2) return (ps[1] - ps[0]).len();
14     double k, ans = 0;
15     for (int x = 0, y = 1, nx, ny; x < n; ++x) {
16         for(nx = (x == n - 1) ? (0) : (x + 1); ; y = ny) {
17             ny = (y == n - 1) ? (0) : (y + 1);
18             if (sign(k = det(ps[nx] - ps[x], ps[ny] - ps[y])) <= 0) break;
19             ans = max(ans, (ps[x] - ps[y]).len());
20             if (sign(k) == 0) ans = max(ans, (ps[x] - ps[ny]).len());
21         } return ans;
22 }
```

1.8 Farmland

```
1 struct node { int begin[MAXN], *end; } a[MAXN]; // 按对 p[i] 的极角的 atan2 值排序
2 bool check(int n, point p[], int b1, int b2, bool vis[MAXN][MAXN]) {
3     static pii l[MAXN * 2 + 1]; static bool used[MAXN];
4     int tp(0), *k, p, p1, p2; double area(0.0);
5     for (l[0] = pii(b1, b2); ; ) {
6         vis[p1 = l[tp].first][p2 = l[tp].second] = true;
7         area += det(p[p1], p[p2]);
8         for (k = a[p2].begin; k != a[p2].end; ++k) if (*k == p1) break;
9         k = (k == a[p2].begin) ? (a[p2].end - 1) : (k - 1);
10        if ((l[++tp] = pii(p2, *k)) == l[0]) break;
11    } if (sign(area) < 0 || tp < 3) return false;
12    Rep(i, 1, n) used[i] = false;
13    for (int i = 0; i < tp; ++i) if (used[p = l[i].first]) return false; else used[p] = true;
14    return true; // a face with tp vertices
15 }
16 int countFaces(int n, point p[]) {
17     static bool vis[MAXN][MAXN]; int ans = 0;
18     Rep(x, 1, n) Rep(y, 1, n) vis[x][y] = false;
19     Rep(x, 1, n) for (int *itr = a[x].begin; itr != a[x].end; ++itr) if (!vis[x][*itr])
20         if (check(n, p, x, *itr, vis)) ++ans;
```

1.9 Voronoi 图

不能有重点

```
1 #define Oi(e) ((e)->oi)
2 #define Dt(e) ((e)->dt)
3 #define On(e) ((e)->on)
4 #define Op(e) ((e)->op)
5 #define Dn(e) ((e)->dn)
6 #define Dp(e) ((e)->dp)
7 #define Other(e, p) ((e)->oi == p ? (e)->dt : (e)->oi)
8 #define Next(e, p) ((e)->oi == p ? (e)->on : (e)->dn)
9 #define Prev(e, p) ((e)->oi == p ? (e)->op : (e)->dp)
10 #define V(p1, p2, u, v) (u = p2->x - p1->x, v = p2->y - p1->y)
11 #define C2(u1, v1, u2, v2) (u1 * v2 - v1 * u2)
12 #define C3(p1, p2, p3) ((p2->x - p1->x) * (p3->y - p1->y) - (p2->y - p1->y) * (p3->x - p1->x))
13 #define Dot(u1, v1, u2, v2) (u1 * u2 + v1 * v2)
14 #define dis(a, b) (sqrt((a->x - b->x) * (a->x - b->x) + (a->y - b->y) * (a->y - b->y)))
15 const int maxn = 110024;
16 const int aix = 4;
17 const double eps = 1e-7;
18 int n, M, k;
19 struct gEdge {
20     int u, v; double w;
21     bool operator <(const gEdge &e1) const { return w < e1.w - eps; }
22 } E[aix * maxn], MST[maxn];
23 struct point {
24     double x, y; int index; edge *in;
25     bool operator <(const point &p1) const { return x < p1.x - eps || (abs(x - p1.x) <= eps &&
26         y < p1.y - eps); }
27 };
28 struct edge { point *oi, *dt; edge *on, *op, *dn, *dp; };
29 point p[maxn], *Q[maxn];
30 edge mem[aix * maxn], *elist[aix * maxn];
31 int nfree;
32 void Alloc_memory() { nfree = aix * n; edge *e = mem; for (int i = 0; i < nfree; i++) elist[i] = e++; }
33 void Splice(edge *a, edge *b, point *v) {
34     edge *next;
35     if (Oi(a) == v) next = On(a), On(a) = b; else next = Dn(a), Dn(a) = b;
36     if (Oi(next) == v) Op(next) = b; else Dp(next) = b;
37     if (Oi(b) == v) On(b) = next, Op(b) = a; else Dn(b) = next, Dp(b) = a;
38 }
39 edge *Make_edge(point *u, point *v) {
40     edge *e = elist[--nfree];
41     e->on = e->op = e->dn = e->dp = e; e->oi = u; e->dt = v;
```

```

42   if (!u->in) u->in = e;
43   if (!v->in) v->in = e;
44   return e;
45 }
46 edge *Join(edge *a, point *u, edge *b, point *v, int side) {
47   edge *e = Make_edge(u, v);
48   if (side == 1) {
49     if (Oi(a) == u) Ssplice(Op(a), e, u);
50     else Ssplice(Dp(a), e, u);
51     Ssplice(b, e, v);
52   } else {
53     Ssplice(a, e, u);
54     if (Oi(b) == v) Ssplice(Op(b), e, v);
55     else Ssplice(Dp(b), e, v);
56   } return e;
57 }
58 void Remove(edge *e) {
59   point *u = Oi(e), *v = Dt(e);
60   if (u->in == e) u->in = e->on;
61   if (v->in == e) v->in = e->dn;
62   if (Oi(e->on) == u) e->on->op = e->op; else e->on->dp = e->op;
63   if (Oi(e->op) == u) e->op->on = e->on; else e->op->dn = e->on;
64   if (Oi(e->dn) == v) e->dn->op = e->dp; else e->dn->dp = e->dp;
65   if (Oi(e->dp) == v) e->dp->on = e->dn; else e->dp->dn = e->dn;
66   elist[Infree++] = e;
67 }
68 void Low_tangent(edge *e_l, point *o_l, edge *e_r, point *o_r, edge **l_low, point **OL, edge
   **r_low, point **OR) {
69   for (point *d_l = Other(e_l, o_l), *d_r = Other(e_r, o_r); ; )
70     if (C3(o_l, o_r, d_l) < -eps) e_l = Prev(e_l, d_l), o_l = d_l, d_l = Other(e_l, o_l)
       ;
71     else if (C3(o_l, o_r, d_r) < -eps) e_r = Next(e_r, d_r), o_r = d_r, d_r = Other(e_r, o_r)
       ;
72     else break;
73   *OL = o_l, *OR = o_r; *l_low = e_l, *r_low = e_r;
74 }
75 void Merge(edge *lr, point *s, edge *rl, point *u, edge **tangent) {
76   double l1, l2, l3, l4, r1, r2, r3, r4, cot_L, cot_R, u1, v1, u2, v2, n1, cot_n, P1, cot_P;
77   point *O, *D, *OR, *OL; edge *B, *L, *R;
78   Low_tangent(lr, s, rl, u, &L, &OL, &R, &OR);
79   for (*tangent = B = Join(L, OL, R, OR, 0), O = OL, D = OR; ; ) {
80     edge *El = Next(B, O), *Er = Prev(B, D), *next, *prev;
81     point *l = Other(El, O), *r = Other(Er, D);
82     V(l, O, l1, l2); V(l, D, l3, l4); V(r, O, r1, r2); V(r, D, r3, r4);
83     double c1 = C2(l1, l2, l3, l4), cr = C2(r1, r2, r3, r4);
84     bool BL = c1 > eps, BR = cr > eps;
85     if (!BL && !BR) break;
86     if (BL) {
87       double d1 = Dot(l1, l2, l3, l4);
88       for (cot_L = d1 / c1; ; Remove(El), El = next, cot_L = cot_n) {
89         next = Next(El, O); V(Other(next, O), O, u1, v1); V(Other(next, O), D, u2, v2);
90         n1 = C2(u1, v1, u2, v2); if (!(n1 > eps)) break;
91         cot_n = Dot(u1, v1, u2, v2) / n1;

```

```

92       if (cot_n > cot_L) break;
93     }
94   } if (BR) {
95     double dr = Dot(r1, r2, r3, r4);
96     for (cot_R = dr / cr; ; Remove(Er), Er = prev, cot_R = cot_P) {
97       prev = Prev(Er, D); V(Other(prev, D), O, u1, v1); V(Other(prev, D), D, u2, v2);
98       P1 = C2(u1, v1, u2, v2); if (!(P1 > eps)) break;
99       cot_P = Dot(u1, v1, u2, v2) / P1;
100      if (cot_P > cot_R) break;
101    }
102  } l = Other(El, O); r = Other(Er, D);
103  if (!BL || (BL && BR && cot_R < cot_L)) B = Join(B, O, Er, r, 0), D = r;
104  else B = Join(El, l, B, D, 0), O = l;
105 }
106 }
107 void Divide(int s, int t, edge **L, edge **R) {
108   edge *a, *b, *c, *ll, *lr, *rl, *rr, *tangent;
109   int n = t - s + 1;
110   if (n == 2) *L = *R = Make_edge(Q[s], Q[t]);
111   else if (n == 3) {
112     a = Make_edge(Q[s], Q[s + 1]), b = Make_edge(Q[s + 1], Q[t]);
113     Ssplice(a, b, Q[s + 1]);
114     double v = C3(Q[s], Q[s + 1], Q[t]);
115     if (v > eps) c = Join(a, Q[s], b, Q[t], 0), *L = a, *R = b;
116     else if (v < -eps) c = Join(a, Q[s], b, Q[t], 1), *L = c, *R = c;
117     else *L = a, *R = b;
118   } else if (n > 3) {
119     int split = (s + t) / 2;
120     Divide(s, split, &ll, &lr); Divide(split + 1, t, &rl, &rr);
121     Merge(lr, Q[split], rl, Q[split + 1], &tangent);
122     if (Oi(tangent) == Q[s]) ll = tangent;
123     if (Dt(tangent) == Q[t]) rr = tangent;
124     *L = ll; *R = rr;
125   }
126 }
127 void Make_Graph() {
128   edge *start, *e; point *u, *v;
129   for (int i = 0; i < n; i++) {
130     start = e = (u = &p[i])->in;
131     do { v = Other(e, u);
132         if (u < v) E[M++] = (u - p, v - p, dis(u, v)); // M < aix * maxn
133       } while ((e = Next(e, u)) != start);
134   }
135 }
136 int b[maxn];
137 int Find(int x) { while (x != b[x]) { b[x] = b[b[x]]; x = b[x]; } return x; }
138 void Kruskal() {
139   memset(b, 0, sizeof(b)); sort(E, E + M);
140   for (int i = 0; i < n; i++) b[i] = i;
141   for (int i = 0, kk = 0; i < M && kk < n - 1; i++) {
142     int m1 = Find(E[i].u), m2 = Find(E[i].v);
143     if (m1 != m2) b[m1] = m2, MST[kk++] = E[i];
144   }

```



```

145 }
146 void solve() {
147     scanf("%d", &n);
148     for (int i = 0; i < n; i++) scanf("%lf%lf", &p[i].x, &p[i].y), p[i].index = i, p[i].in =
        NULL;
149     Alloc_memory(); sort(p, p + n);
150     for (int i = 0; i < n; i++) Q[i] = p + i;
151     edge *L, *R; Divide(0, n - 1, &L, &R);
152     M = 0; Make_Graph(); Kruskal();
153 }
154 int main() { solve(); return 0; }

```

```

37     res = (q1 * a - q2 * b) / d;
38     return true;
39 }
40 bool isFF(const point &p1, const point &o1, const point &p2, const point &o2, point &a, point
    &b) {
41     point e = det(o1, o2), v = det(o1, e);
42     double d = dot(o2, v); if (sign(d) == 0) return false;
43     point q = p1 + v * (dot(o2, p2 - p1) / d);
44     a = q; b = q + e;
45     return true;
46 }

```

1.10 三维计算几何基本操作

```

1 struct point { double x, y, z; // something omitted
2     friend point det(const point &a, const point &b) {
3         return point(a.y * b.z - a.z * b.y, a.z * b.x - a.x * b.z, a.x * b.y - a.y * b.x);
4     }
5     friend double mix(const point &a, const point &b, const point &c) {
6         return a.x * b.y * c.z + a.y * b.z * c.x + a.z * b.x * c.y - a.z * b.y * c.x - a.x * b.z
            * c.y - a.y * b.x * c.z;
7     }
8     double distLP(const point &p1, const point &p2) const {
9         return det(p2 - p1, *this - p1).len() / (p2 - p1).len();
10    }
11    double distFP(const point &p1, const point &p2, const point &p3) const {
12        point n = det(p2 - p1, p3 - p1); return fabs( dot(n, *this - p1) / n.len() );
13    }
14 };
15 double distLL(const point &p1, const point &p2, const point &q1, const point &q2) {
16     point p = q1 - p1, u = p2 - p1, v = q2 - q1;
17     double d = u.norm() * v.norm() - dot(u, v) * dot(u, v);
18     if (sign(d) == 0) return p1.distLP(q1, q2);
19     double s = (dot(p, u) * v.norm() - dot(p, v) * dot(u, v)) / d;
20     return (p1 + u * s).distLP(q1, q2);
21 }
22 double distSS(const point &p1, const point &p2, const point &q1, const point &q2) {
23     point p = q1 - p1, u = p2 - p1, v = q2 - q1;
24     double d = u.norm() * v.norm() - dot(u, v) * dot(u, v);
25     if (sign(d) == 0) return min( min((p1 - q1).len(), (p1 - q2).len()),
26         min((p2 - q1).len(), (p2 - q2).len()));
27     double s1 = (dot(p, u) * v.norm() - dot(p, v) * dot(u, v)) / d;
28     double s2 = (dot(p, v) * u.norm() - dot(p, u) * dot(u, v)) / d;
29     if (s1 < 0.0) s1 = 0.0; if (s1 > 1.0) s1 = 1.0;
30     if (s2 < 0.0) s2 = 0.0; if (s2 > 1.0) s2 = 1.0;
31     point r1 = p1 + u * s1; point r2 = q1 + v * s2;
32     return (r1 - r2).len();
33 }
34 bool isFL(const point &p, const point &o, const point &q1, const point &q2, point &res) {
35     double a = dot(o, q2 - p), b = dot(o, q1 - p), d = a - b;
36     if (sign(d) == 0) return false;

```

1.11 凸多面体切割

```

1 vector<vector<point>> > convexCut(const vector<vector<point>> > &pss, const point &p, const
    point &o) {
2     vector<vector<point>> > res;
3     vector<point> sec;
4     for (unsigned itr = 0, size = pss.size(); itr < size; ++itr) {
5         const vector<point> &ps = pss[itr];
6         int n = ps.size();
7         vector<point> qs;
8         bool dif = false;
9         for (int i = 0; i < n; ++i) {
10             int d1 = sign( dot(o, ps[i] - p) );
11             int d2 = sign( dot(o, ps[(i + 1) % n] - p) );
12             if (d1 <= 0) qs.push_back(ps[i]);
13             if (d1 * d2 < 0) {
14                 point q;
15                 isFL(p, o, ps[i], ps[(i + 1) % n], q); // must return true
16                 qs.push_back(q);
17                 sec.push_back(q);
18             }
19             if (d1 == 0) sec.push_back(ps[i]);
20             else dif = true;
21             dif |= dot(o, det(ps[(i + 1) % n] - ps[i], ps[(i + 2) % n] - ps[i])) < -EPS;
22         }
23         if (!qs.empty() && dif)
24             res.insert(res.end(), qs.begin(), qs.end());
25     }
26     if (!sec.empty()) {
27         vector<point> tmp( convexHull2D(sec, o) );
28         res.insert(res.end(), tmp.begin(), tmp.end());
29     }
30     return res;
31 }
32
33 vector<vector<point>> > initConvex() {
34     vector<vector<point>> > pss(6, vector<point>(4));
35     pss[0][0] = pss[1][0] = pss[2][0] = point(-INF, -INF, -INF);
36     pss[0][3] = pss[1][1] = pss[5][2] = point(-INF, -INF, INF);

```

```

37 pss[0][1] = pss[2][3] = pss[4][2] = point(-INF, INF, -INF);
38 pss[0][2] = pss[5][3] = pss[4][1] = point(-INF, INF, INF);
39 pss[1][3] = pss[2][1] = pss[3][2] = point(INF, -INF, -INF);
40 pss[1][2] = pss[5][1] = pss[3][3] = point(INF, -INF, INF);
41 pss[2][2] = pss[4][3] = pss[3][1] = point(INF, INF, -INF);
42 pss[5][0] = pss[4][0] = pss[3][0] = point(INF, INF, INF);
43 return pss;
44 }

```

1.12 三维凸包

不能有重点

```

1 namespace ConvexHull3D {
2     const int MAXN = 1033;
3     struct Facet {
4         int a, b, c;
5         Facet(int a, int b, int c): a(a), b(b), c(c) {}
6     };
7     #define volume(a, b, c, d) (mix(ps[b] - ps[a], ps[c] - ps[a], ps[d] - ps[a]))
8     vector<Facet> getHull(int n, point ps[]) {
9         static int mark[MAXN][MAXN];
10        int a, b, c, stamp = 0;
11        bool exist = false;
12        vector<Facet> facet;
13
14        random_shuffle(ps, ps + n);
15        facet.clear();
16        for (int i = 2; i < n && !exist; i++) {
17            point ndir = det(ps[0] - ps[i], ps[1] - ps[i]);
18            if (ndir.len() < EPS) continue;
19            swap(ps[i], ps[2]);
20            for (int j = i + 1; j < n; j++)
21                if (sign(volume(0, 1, 2, j)) != 0) {
22                    swap(ps[j], ps[3]);
23                    facet.push_back(Facet(0, 1, 2));
24                    facet.push_back(Facet(0, 2, 1));
25                    exist = true;
26                    break;
27                }
28        }
29        if (!exist) return vector<int>(); // All points are in the same plane
30        for (int i = 0; i < n; ++i)
31            for (int j = 0; j < n; ++j)
32                mark[i][j] = 0;
33        stamp = 0;
34        for (int v = 3; v < n; ++v) {
35            vector<Facet> tmp;
36            ++stamp;
37            for (unsigned i = 0; i < facet.size(); i++) {
38                a = facet[i].a;

```

```

39                b = facet[i].b;
40                c = facet[i].c;
41                if (sign(volume(v, a, b, c)) < 0)
42                    mark[a][b] = mark[a][c] =
43                    mark[b][a] = mark[b][c] =
44                    mark[c][a] = mark[c][b] = stamp;
45                else
46                    tmp.push_back(facet[i]);
47            }
48            facet = tmp;
49            for (unsigned i = 0; i < tmp.size(); i++) {
50                a = facet[i].a;
51                b = facet[i].b;
52                c = facet[i].c;
53                if (mark[a][b] == stamp)
54                    facet.push_back(Facet(b, a, v));
55                if (mark[b][c] == stamp)
56                    facet.push_back(Facet(c, b, v));
57                if (mark[c][a] == stamp)
58                    facet.push_back(Facet(a, c, v));
59            }
60        }
61        return facet;
62    }
63    #undef volume
64 }
65 namespace Gravity {
66     using ConvexHull3D::Facet;
67     point findG(point ps[], const vector<Facet> &facet) {
68         double ws = 0;
69         point res(0.0, 0.0, 0.0);
70         point o = ps[ facet[0].a ];
71         for (int i = 0, size = facet.size(); i < size; ++i) {
72             const point &a = ps[ facet[i].a ];
73             const point &b = ps[ facet[i].b ];
74             const point &c = ps[ facet[i].c ];
75             point p = (a + b + c + o) * 0.25;
76             double w = mix(a - o, b - o, c - o);
77             ws += w;
78             res = res + p * w;
79         }
80         res = res / ws;
81         return res;
82     }
83 }

```

1.13 球面点表面点距离

```

1 double distOnBall(double lati1, double longi1, double lati2, double longi2, double R) {
2     lati1 *= PI / 180; longi1 *= PI / 180;
3     lati2 *= PI / 180; longi2 *= PI / 180;

```

```

4   double x1 = cos(lati1) * sin(longi1);
5   double y1 = cos(lati1) * cos(longi1);
6   double z1 = sin(lati1);
7   double x2 = cos(lati2) * sin(longi2);
8   double y2 = cos(lati2) * cos(longi2);
9   double z2 = sin(lati2);
10  double theta = acos(x1 * x2 + y1 * y2 + z1 * z2);
11  return R * theta;
12 }

```

1.14 长方体表面点距离

```

1   int r;
2   void turn(int i, int j, int x, int y, int z, int x0, int y0, int L, int W, int H) {
3       if (z == 0) r = min(r, x * x + y * y);
4       else {
5           if (i >= 0 && i < 2) turn(i + 1, j, x0 + L + z, y, x0 + L - x, x0 + L, y0, H, W, L);
6           if (j >= 0 && j < 2) turn(i, j + 1, x, y0 + W + z, y0 + W - y, x0, y0 + W, L, H, W);
7           if (i <= 0 && i > -2) turn(i - 1, j, x0 - z, y, x - x0, x0 - H, y0, H, W, L);
8           if (j <= 0 && j > -2) turn(i, j - 1, x, y0 - z, y - y0, x0, y0 - H, L, H, W);
9       }
10  }
11  int calc(int L, int H, int W, int x1, int y1, int z1, int x2, int y2, int z2) {
12      if (z1 != 0 && z1 != H)
13          if (y1 == 0 || y1 == W) swap(y1, z1), swap(y2, z2), swap(W, H);
14      else
15          swap(x1, z1), swap(x2, z2), swap(L, H);
16      if (z1 == H) z1 = 0, z2 = H - z2;
17      r = INF; turn(0, 0, x2 - x1, y2 - y1, z2, -x1, -y1, L, W, H);
18      return r;
19  }

```

1.15 最小覆盖球

```

1   namespace MinBall {
2       int outCnt;
3       point out[4], res;
4       double radius;
5       void ball() {
6           static point q[3];
7           static double m[3][3], sol[3], L[3], det;
8           int i, j;
9           res = point(0.0, 0.0, 0.0);
10          radius = 0.0;
11          switch (outCnt) {
12              case 1:
13                  res = out[0];
14                  break;
15              case 2:
16                  res = (out[0] + out[1]) * 0.5;

```

```

17          radius = (res - out[0]).norm();
18          break;
19          case 3:
20              q[0] = out[1] - out[0];
21              q[1] = out[2] - out[0];
22              for (i = 0; i < 2; ++i)
23                  for (j = 0; j < 2; ++j)
24                      m[i][j] = dot(q[i], q[j]) * 2.0;
25              for (i = 0; i < 2; ++i)
26                  sol[i] = dot(q[i], q[i]);
27              det = m[0][0] * m[1][1] - m[0][1] * m[1][0];
28              if (sign(det) == 0)
29                  return;
30              L[0] = (sol[0] * m[1][1] - sol[1] * m[0][1]) / det;
31              L[1] = (sol[1] * m[0][0] - sol[0] * m[1][0]) / det;
32              res = out[0] + q[0] * L[0] + q[1] * L[1];
33              radius = (res - out[0]).norm();
34              break;
35          case 4:
36              q[0] = out[1] - out[0];
37              q[1] = out[2] - out[0];
38              q[2] = out[3] - out[0];
39              for (i = 0; i < 3; ++i)
40                  for (j = 0; j < 3; ++j)
41                      m[i][j] = dot(q[i], q[j]) * 2;
42              for (i = 0; i < 3; ++i)
43                  sol[i] = dot(q[i], q[i]);
44              det = m[0][0] * m[1][1] * m[2][2] + m[0][1] * m[1][2] * m[2][0]
45                  + m[0][2] * m[1][0] * m[2][1] - m[0][2] * m[1][1] * m[2][0]
46                  - m[0][1] * m[1][0] * m[2][2] - m[0][0] * m[1][2] * m[2][1];
47              if (sign(det) == 0)
48                  return;
49              for (j = 0; j < 3; ++j) {
50                  for (i = 0; i < 3; ++i)
51                      m[i][j] = sol[i];
52                  L[j] = (m[0][0] * m[1][1] * m[2][2] + m[0][1] * m[1][2] * m[2][0]
53                      + m[0][2] * m[1][0] * m[2][1] - m[0][2] * m[1][1] * m[2][0]
54                      - m[0][1] * m[1][0] * m[2][2] - m[0][0] * m[1][2] * m[2][1])
55                      / det;
56                  for (i = 0; i < 3; ++i)
57                      m[i][j] = dot(q[i], q[j]) * 2;
58              }
59              res = out[0];
60              for (i = 0; i < 3; ++i)
61                  res += q[i] * L[i];
62              radius = (res - out[0]).norm();
63          }
64      }
65
66      void minball(int n, point pt[]) {
67          ball();
68          if (outCnt < 4)
69              for (int i = 0; i < n; ++i)

```

```

70     if ((res - pt[i]).norm() > +radius + EPS) {
71         out[outCnt] = pt[i];
72         ++outCnt;
73         minball(i, pt);
74         --outCnt;
75         if (i > 0) {
76             point Tt = pt[i];
77             memmove(&pt[1], &pt[0], sizeof(point) * i);
78             pt[0] = Tt;
79         }
80     }
81 }
82
83 pair<point, double> main(int npoint, point pt[]) { // 0-based
84     random_shuffle(pt, pt + npoint);
85     radius = -1;
86     for (int i = 0; i < npoint; i++) {
87         if ((res - pt[i]).norm() > EPS + radius) {
88             outCnt = 1;
89             out[0] = pt[i];
90             minball(i, pt);
91         }
92     }
93     return make_pair(res, sqrt(radius));
94 }
95 }

```

1.16 三维向量操作矩阵

- 绕单位向量 $u = (u_x, u_y, u_z)$ 右手方向旋转 θ 度的矩阵:

$$\begin{bmatrix} \cos \theta + u_x^2(1 - \cos \theta) & u_x u_y(1 - \cos \theta) - u_z \sin \theta & u_x u_z(1 - \cos \theta) + u_y \sin \theta \\ u_y u_x(1 - \cos \theta) + u_z \sin \theta & \cos \theta + u_y^2(1 - \cos \theta) & u_y u_z(1 - \cos \theta) - u_x \sin \theta \\ u_z u_x(1 - \cos \theta) - u_y \sin \theta & u_z u_y(1 - \cos \theta) + u_x \sin \theta & \cos \theta + u_z^2(1 - \cos \theta) \end{bmatrix}$$

$$= \cos \theta I + \sin \theta \begin{bmatrix} 0 & -u_z & u_y \\ u_z & 0 & -u_x \\ -u_y & u_x & 0 \end{bmatrix} + (1 - \cos \theta) \begin{bmatrix} u_x^2 & u_x u_y & u_x u_z \\ u_y u_x & u_y^2 & u_y u_z \\ u_z u_x & u_z u_y & u_z^2 \end{bmatrix}$$

- 点 a 绕单位向量 $u = (u_x, u_y, u_z)$ 右手方向旋转 θ 度的对应点为 $a' = a \cos \theta + (u \times a) \sin \theta + (u \otimes u)a(1 - \cos \theta)$

- 关于向量 v 作对称变换的矩阵 $H = I - 2 \frac{vv^T}{v^T v}$,

- 点 a 对称点: $a' = a - 2 \frac{v^T a}{v^T v} \cdot v$

1.17 立体角

对于任意一个四面体 $OABC$, 从 O 点观察 $\triangle ABC$ 的立体角 $\tan \frac{\Omega}{2} = \frac{\text{mix}(\vec{a}, \vec{b}, \vec{c})}{|a||b||c| + (\vec{a} \cdot \vec{b})|c| + (\vec{a} \cdot \vec{c})|b| + (\vec{b} \cdot \vec{c})|a|}$.

2 数据结构

2.1 动态凸包 (只支持插入)

```

1 #define x first // upperHull ← (x, y)
2 #define y second // lowerHull ← (x, -y)
3 typedef map<int, int> mii;
4 typedef map<int, int>::iterator mit;
5 struct point { point(const mit &p): x(p->first), y(p->second) {} };
6 inline bool checkInside(mii &a, const point &p) { // border inclusive
7     int x = p.x, y = p.y; mit p1 = a.lower_bound(x);
8     if (p1 == a.end()) return false; if (p1->x == x) return y <= p1->y;
9     if (p1 == a.begin()) return false; mit p2(p1--);
10    return sign(det(p - point(p1), point(p2) - p)) >= 0;
11 } inline void addPoint(mii &a, const point &p) { // no collinear points
12     int x = p.x, y = p.y; mit pnt = a.insert(make_pair(x, y)).first, p1, p2;
13     for (pnt->y = y; ; a.erase(p2)) {
14         p1 = pnt; if (++p1 == a.end()) break;
15         p2 = p1; if (++p1 == a.end()) break;
16         if (det(point(p2) - p, point(p1) - p) < 0) break;
17     } for ( ; ; a.erase(p2)) {
18         if ((p1 = pnt) == a.begin()) break;
19         if (--p1 == a.begin()) break; p2 = p1--;
20         if (det(point(p2) - p, point(p1) - p) > 0) break;
21     }
22 }

```

2.2 Rope 用法

```

1 #include <ext/rope>
2 using __gnu_cxx::crope; using __gnu_cxx::rope;
3 a = b.substr(from, len); // [from, from + len)
4 a = b.substr(from); // [from, from)
5 b.c_str(); // might lead to memory leaks
6 b.delete_c_str(); // delete the c_str that created before
7 a.insert(p, str); // insert str before position p
8 a.erase(i, n); // erase [i, i + n)

```

2.3 可持久化 Treap

```

1 inline bool randomBySize(int a, int b) {
2     static long long seed = 1;
3     return (seed = seed * 48271 % 2147483647) * (a + b) < 2147483647LL * a;
4 }
5 tree merge(tree x, tree y) {
6     if (x == null) return y; if (y == null) return x;
7     tree t = NULL;
8     if (randomBySize(x->size, y->size)) t = newNode(x), t->r = merge(x->r, y);
9     else t = newNode(y), t->l = merge(x, y->l);
10    update(t); return t;
11 }
12 void splitByKey(tree t, int k, tree &l, tree &r) { //  $[-\infty, k)[k, +\infty)$ 
13     if (t == null) l = r = null;
14     else if (t->key < k) l = newNode(t), splitByKey(t->r, k, l->r, r), update(l);
15     else r = newNode(t), splitByKey(t->l, k, l, r->l), update(r);
16 }
17 void splitBySize(tree t, int k, tree &l, tree &r) { //  $[1, k)[k, +\infty)$ 
18     static int s; if (t == null) l = r = null;
19     else if ((s = t->l->size + 1) < k) l = newNode(t), splitBySize(t->r, k - s, l->r, r),
20         update(l);
21     else r = newNode(t), splitBySize(t->l, k, l, r->l), update(r);
22 }

```

2.4 左偏树

```

1 tree merge(tree a, tree b) {
2     if (a == null) return b;
3     if (b == null) return a;
4     if (a->key > b->key) swap(a, b);
5     a->rc = merge(a->rc, b);
6     a->rc->fa = a;
7     if (a->lc->dist < a->rc->dist) swap(a->lc, a->rc);
8     a->dist = a->rc->dist + 1;
9     return a;
10 }
11 void erase(tree t) {
12     tree x = t->fa, y = merge(t->lc, t->rc);
13     if (y != null) y->fa = x;
14     if (x == null) root = y;
15     else
16     for ((x->lc == t ? x->lc : x->rc) = y; x != null; y = x, x = x->fa) {
17         if (x->lc->dist < x->rc->dist) swap(x->lc, x->rc);
18         if (x->rc->dist + 1 == x->dist) return;
19         x->dist = x->rc->dist + 1;
20     }
21 }

```

2.5 Link-Cut Tree

```

1 struct node { int rev; node *pre, *ch[2]; } base[MAXN], nil, *null;
2 typedef node *tree;
3 #define isRoot(x) (x->pre->ch[0] != x && x->pre->ch[1] != x)
4 #define isRight(x) (x->pre->ch[1] == x)
5 inline void MakeRev(tree t) { if (t != null) { t->rev ^= 1; swap(t->ch[0], t->ch[1]); } }
6 inline void PushDown(tree t) { if (t->rev) { MakeRev(t->ch[0]); MakeRev(t->ch[1]); t->rev =
7     0; } }
8 inline void Rotate(tree x) {
9     tree y = x->pre; PushDown(y); PushDown(x);
10    int d = isRight(x);
11    if (!isRoot(y)) y->pre->ch[isRight(y)] = x; x->pre = y->pre;
12    if ((y->ch[d] = x->ch[!d]) != null) y->ch[d]->pre = y;
13    x->ch[!d] = y; y->pre = x; Update(y);
14 }
15 inline void Splay(tree x) {
16     PushDown(x); for (tree y; !isRoot(x); Rotate(x)) {
17         y = x->pre; if (!isRoot(y)) Rotate(isRight(x) != isRight(y) ? x : y);
18     } Update(x);
19 }
20 inline void Splay(tree x, tree to) {
21     PushDown(x); for (tree y; (y = x->pre) != to; Rotate(x)) if (y->pre != to)
22         Rotate(isRight(x) != isRight(y) ? x : y);
23     Update(x);
24 }
25 inline tree Access(tree t) {
26     tree last = null; for (; t != null; last = t, t = t->pre) Splay(t), t->ch[1] = last, Update(
27         t);
28     return last;
29 }
30 inline void MakeRoot(tree t) { Access(t); Splay(t); MakeRev(t); }
31 inline tree FindRoot(tree t) { Access(t); Splay(t); tree last = null;
32     for (; t != null; last = t, t = t->ch[0]) PushDown(t); Splay(last); return last;
33 }
34 inline void Join(tree x, tree y) { MakeRoot(y); y->pre = x; }
35 inline void Cut(tree t) { Access(t); Splay(t); t->ch[0]->pre = null; t->ch[0] = null; Update(t
36     ); }
37 inline void Cut(tree x, tree y) {
38     tree upper = (Access(x), Access(y));
39     if (upper == x) { Splay(x); y->pre = null; x->ch[1] = null; Update(x); }
40     else if (upper == y) { Access(x); Splay(y); x->pre = null; y->ch[1] = null; Update(y); }
41     else assert(0); // impossible to happen
42 }
43 inline int Query(tree a, tree b) { // query the cost in path a <-> b, lca inclusive
44     Access(a); tree c = Access(b); // c is lca
45     int v1 = c->ch[1]->maxCost; Access(a);
46     int v2 = c->ch[1]->maxCost;
47     return max(max(v1, v2), c->cost);
48 }
49 void Init() {
50     null = &nil; null->ch[0] = null->ch[1] = null->pre = null; null->rev = 0;
51 }

```

```

48 Rep(i, 1, N) { node &n = base[i]; n.rev = 0; n.pre = n.ch[0] = n.ch[1] = null; }
49 }

```

2.6 K-D Tree Nearest

```

1 struct Point { int x, y; };
2 struct Rectangle {
3     int lx, rx, ly, ry;
4     void set(const Point &p) { lx = rx = p.x; ly = ry = p.y; }
5     void merge(const Point &o) {
6         lx = min(lx, o.x); rx = max(rx, o.x); ly = min(ly, o.y); ry = max(ry, o.y);
7     } void merge(const Rectangle &o) {
8         lx = min(lx, o.lx); rx = max(rx, o.rx); ly = min(ly, o.ly); ry = max(ry, o.ry);
9     } LL dist(const Point &p) {
10         LL res = 0;
11         if (p.x < lx) res += sqr(lx - p.x); else if (p.x > rx) res += sqr(p.x - rx);
12         if (p.y < ly) res += sqr(ly - p.y); else if (p.y > ry) res += sqr(p.y - ry);
13         return res;
14     }
15 };
16 struct Node { int child[2]; Point p; Rectangle rect; };
17 const int MAX_N = 111111;
18 const LL INF = 100000000;
19 int n, m, tot, root; LL result;
20 Point a[MAX_N], p; Node tree[MAX_N];
21 int build(int s, int t, bool d) {
22     int k = ++tot, mid = (s + t) >> 1;
23     nth_element(a + s, a + mid, a + t, d ? cmpXY : cmpYX);
24     tree[k].p = a[mid]; tree[k].rect.set(a[mid]); tree[k].child[0] = tree[k].child[1] = 0;
25     if (s < mid)
26         tree[k].child[0] = build(s, mid, d ^ 1), tree[k].rect.merge(tree[tree[k].child[0]].rect);
27     if (mid + 1 < t)
28         tree[k].child[1] = build(mid + 1, t, d ^ 1), tree[k].rect.merge(tree[tree[k].child[1]].rect);
29     return k;
30 }
31 int insert(int root, bool d) {
32     if (root == 0) {
33         tree[++tot].p = p; tree[tot].rect.set(p); tree[tot].child[0] = tree[tot].child[1] = 0;
34         return tot;
35     } tree[root].rect.merge(p);
36     if ((d && cmpXY(p, tree[root].p)) || (!d && cmpYX(p, tree[root].p)))
37         tree[root].child[0] = insert(tree[root].child[0], d ^ 1);
38     else tree[root].child[1] = insert(tree[root].child[1], d ^ 1);
39     return root;
40 }
41 void query(int k, bool d) {
42     if (tree[k].rect.dist(p) >= result) return;
43     cMin(result, dist(tree[k].p, p));
44     if ((d && cmpXY(p, tree[k].p)) || (!d && cmpYX(p, tree[k].p))) {

```

```

45         if (tree[k].child[0]) query(tree[k].child[0], d ^ 1);
46         if (tree[k].child[1]) query(tree[k].child[1], d ^ 1);
47     } else {
48         if (tree[k].child[1]) query(tree[k].child[1], d ^ 1);
49         if (tree[k].child[0]) query(tree[k].child[0], d ^ 1);
50     }
51 }
52 void example(int n) {
53     root = tot = 0; scan(a); root = build(0, n, 0); // init, a[0...n-1]
54     scan(p); root = insert(root, 0); // insert
55     scan(p); result = INF; ans = query(root, 0); // query
56 }

```

2.7 K-D Tree Farthest

输入 n 个点, 对每个询问 px, py, k , 输出 k 远点的编号

```

1 struct Point { int x, y, id; };
2 struct Rectangle {
3     int lx, rx, ly, ry;
4     void set(const Point &p) { lx = rx = p.x; ly = ry = p.y; }
5     void merge(const Rectangle &o) {
6         lx = min(lx, o.lx); rx = max(rx, o.rx); ly = min(ly, o.ly); ry = max(ry, o.ry);
7     }
8     LL dist(const Point &p) { LL res = 0;
9         res += max(sqr(rx - p.x), sqr(lx - p.x));
10        res += max(sqr(ry - p.y), sqr(ly - p.y));
11        return res;
12    }
13 }; struct Node { Point p; Rectangle rect; };
14 const int MAX_N = 111111;
15 const LL INF = 1LL << 60;
16 int n, m;
17 Point a[MAX_N], b[MAX_N];
18 Node tree[MAX_N * 3];
19 Point p; // p is the query point
20 pair<LL, int> result[22];
21 void build(int k, int s, int t, bool d) {
22     int mid = (s + t) >> 1;
23     nth_element(a + s, a + mid, a + t, d ? cmpX : cmpY);
24     tree[k].p = a[mid];
25     tree[k].rect.set(a[mid]);
26     if (s < mid)
27         build(k << 1, s, mid, d ^ 1), tree[k].rect.merge(tree[k << 1].rect);
28     if (mid + 1 < t)
29         build(k << 1 | 1, mid + 1, t, d ^ 1), tree[k].rect.merge(tree[k << 1 | 1].rect);
30 }
31 void query(int k, int s, int t, bool d, int kth) {
32     if (tree[k].rect.dist(p) < result[kth].first) return;
33     pair<LL, int> tmp(dist(tree[k].p, p), -tree[k].p.id);
34     for (int i = 1; i <= kth; i++) if (tmp > result[i]) {

```

```

35     for (int j = kth + 1; j > i; j--) result[j] = result[j - 1]; result[i] = tmp;
36     break;
37 }
38 int mid = (s + t) >> 1;
39 if ((d && cmpX(p, tree[k].p)) || (!d && cmpY(p, tree[k].p))) {
40     if (mid + 1 < t) query(k << 1 | 1, mid + 1, t, d ^ 1, kth);
41     if (s < mid) query(k << 1, s, mid, d ^ 1, kth);
42 } else {
43     if (s < mid) query(k << 1, s, mid, d ^ 1, kth);
44     if (mid + 1 < t) query(k << 1 | 1, mid + 1, t, d ^ 1, kth);
45 }
46 }
47 void example(int n) {
48     scan(a); build(1, 0, n, 0); // init, a[0...n-1]
49     scan(p, k); // query
50     Rep(j, 1, k) result[j].first = -1;
51     query(1, 0, n, 0, k); ans = -result[k].second + 1;
52 }

```

2.8 树链剖分

```

1 int N, fa[MAXN], dep[MAXN], Q[MAXN], size[MAXN], own[MAXN];
2 int LCA(int x, int y) { if (x == y) return x;
3     for ( ; x = fa[own[x]]; ) {
4         if (dep[x] < dep[y]) swap(x, y); if (own[x] == own[y]) return y;
5         if (dep[own[x]] < dep[own[y]]) swap(x, y);
6     } return -1;
7 }
8 void Decomposition() {
9     static int path[MAXN]; int x, y, a, next, head = 0, tail = 0, cnt;
10    for (fa[1] = -1, dep[1] = 0, Q[++tail] = 1; head < tail; )
11        for (edge e(fir[x = Q[++head]]); e; e = e->next) if ( (y = e->to) != fa[x] )
12            fa[y] = x, dep[y] = dep[x] + 1, Q[++tail] = y;
13    for (int i = N; i >= 1; --i) { int &res = size[x = Q[i]] = 1;
14        for (edge e(fir[x]); e; e = e->next) if ( (y = e->to) != fa[x] ) res += size[y];
15    } for (int i = 1; i <= N; ++i) own[i] = -1;
16    for (int i = 1; i <= N; ++i) if (own[a = Q[i]] == -1)
17        for (x = a, cnt = 0; ; x = next) {
18            next = -1; own[x] = a; path[++cnt] = x;
19            for (edge e(fir[x]); e; e = e->next) if ( (y = e->to) != fa[x] )
20                if (next == -1 || size[y] > size[next]) next = y;
21            if (next == -1) { tree[a].init(cnt, path); break; }
22        }
23 }

```

3 字符串相关

3.1 Manacher

```

1 // len[i] : the max length of palindrome whose mid point is (i / 2)
2 void Manacher(int n, char cs[], int len[]) { // 0-based, len[] must be double sized
3     for (int i = 0; i < n + n; ++i) len[i] = 0;
4     for (int i = 0, j = 0, k; i < n * 2; i += k, j = max(j - k, 0)) {
5         while (i - j >= 0 && i + j + 1 < n * 2 && cs[(i - j) / 2] == cs[(i + j + 1) / 2]) j++;
6         len[i] = j;
7         for (k = 1; i - k >= 0 && j - k >= 0 && len[i - k] != j - k; k++)
8             len[i + k] = min(len[i - k], j - k);
9     }
10 }

```

3.2 KMP

$next[i] = \max\{len|A[0 \dots len - 1] = A \text{ 的第 } i \text{ 位向前或后的长度为 } len \text{ 的串}\}$

$ext[i] = \max\{len|A[0 \dots len - 1] = B \text{ 的第 } i \text{ 位向前或后的长度为 } len \text{ 的串}\}$

```

1 void KMP(char *a, int la, char *b, int lb, int *next, int *ext) {
2     --a; --b; --next; --ext;
3     for (int i = 2, j = next[1] = 0; i <= la; i++) {
4         while (j && a[j + 1] != a[i]) j = next[j];
5         if (a[j + 1] == a[i]) ++j; next[i] = j;
6     } for (int i = 1, j = 0; i <= lb; ++i) {
7         while (j && a[j + 1] != b[i]) j = next[j];
8         if (a[j + 1] == b[i]) ++j; ext[i] = j;
9         if (j == la) j = next[j];
10    }
11 }
12 void ExKMP(char *a, int la, char *b, int lb, int *next, int *ext) {
13     next[0] = la; for (int &j = next[1] = 0; j + 1 < la && a[j] == a[j + 1]; ++j);
14     for (int i = 2, k = 1; i <= la; ++i) {
15         int p = k + next[k], l = next[i - k]; if (1 < p - i) next[i] = l;
16         else for (int &j = next[k = i] = max(0, p - i); i + j < la && a[j] == a[i + j]; ++j);
17     } for (int &j = ext[0] = 0; j < la && j < lb && a[j] == b[j]; ++j);
18     for (int i = 1, k = 0; i <= lb; ++i) {
19         int p = k + ext[k], l = next[i - k]; if (1 < p - i) ext[i] = l;
20         else for (int &j = ext[k = i] = max(0, p - i); j < la && i + j < lb && a[j] == b[i + j];
21             ++j);
22     }
23 }

```

3.3 Aho-Corasick 自动机

```

1 void construct() {
2     static tree Q[MAX_NODE]; int head = 0, tail = 0;
3     for (root->fail = root, Q[++tail] = root; head < tail; ) {
4         tree x = Q[++head];
5         // if (x->fail->danger) x->danger = true;
6         Rep(d, 0, sigma - 1) if (!x->next[d])

```

```

7     x->next[d] = (x == root) ? (root) : (x->fail->next[d]);
8     else {
9         x->next[d]->fail = (x == root) ? (root) : (x->fail->next[d]);
10        Q[++tail] = x->next[d];
11    }
12 }
13 }

```

3.4 后缀自动机

```

1 struct node { int len; node *fa, *go[26]; } base[MAXNODE], *top = base, *root, *que[MAXNODE];
2 typedef node *tree;
3 inline tree newNode(int len) {
4     top->len = len; top->fa = NULL; memset(top->go, 0, sizeof(top->go)); return top++;
5 } inline tree newNode(int len, tree fa, tree *go) {
6     top->len = len; top->fa = fa; memcpy(top->go, go, sizeof(top->go)); return top++;
7 } void construct(char *A, int N) {
8     tree p = root = newNode(0), q, up, fa;
9     for (int i = 0; i < N; ++i) {
10         int w = A[i] - 'a'; up = p; p = newNode(i + 1);
11         for (; up && !up->go[w]; up = up->fa) up->go[w] = p;
12         if (!up) p->fa = root;
13         else { q = up->go[w];
14             if (up->len + 1 == q->len) p->fa = q;
15             else { fa = newNode(up->len + 1, q->fa, q->go);
16                 for (p->fa = q->fa = fa; up && up->go[w] == q; up = up->fa) up->go[w] = fa;
17             }
18         }
19     } static int cnt[MAXLEN]; memset(cnt, 0, sizeof(int) * (N + 1));
20     for (tree i(base); i != top; ++i) ++cnt[i->len];
21     Rep(i, 1, N) cnt[i] += cnt[i - 1];
22     for (tree i(base); i != top; ++i) Q[cnt[i->len]--] = i;
23 }

```

3.5 后缀数组

待排序的字符串放在 $r[0 \dots n-1]$ 中, 最大值小于 m .

$r[0 \dots n-2] > 0, r[n-1] = 0$.

结果放在 $sa[0 \dots n-1]$.

```

1 namespace SuffixArrayDoubling {
2     int wa[MAXN], wb[MAXN], wv[MAXN], ws[MAXN];
3     int cmp(int *r, int a, int b, int l) {
4         return r[a] == r[b] && r[a + 1] == r[b + 1];
5     }
6     void da(int *r, int *sa, int n, int m) {
7         int i, j, p, *x = wa, *y = wb, *t;
8         for (i = 0; i < m; i++) ws[i] = 0;
9         for (i = 0; i < n; i++) ws[x[i]] = r[i]++;

```

```

10         for (i = 1; i < m; i++) ws[i] += ws[i - 1];
11         for (i = n - 1; i >= 0; i--) sa[--ws[x[i]]] = i;
12         for (j = 1, p = 1; p < n; j *= 2, m = p) {
13             for (p = 0, i = n - j; i < n; i++) y[p++] = i;
14             for (i = 0; i < n; i++) if (sa[i] >= j) y[p++] = sa[i] - j;
15             for (i = 0; i < n; i++) wv[i] = x[y[i]];
16             for (i = 0; i < m; i++) ws[i] = 0;
17             for (i = 0; i < n; i++) ws[wv[i]]++;
18             for (i = 1; i < m; i++) ws[i] += ws[i - 1];
19             for (i = n - 1; i >= 0; i--) sa[--ws[wv[i]]] = y[i];
20             for (t = x, x = y, y = t, p = 1, x[sa[0]] = 0, i = 1; i < n; i++)
21                 x[sa[i]] = cmp(y, sa[i - 1], sa[i], j) ? p - 1 : p++;
22         }
23     }
24 }
25 namespace SuffixArrayDC3 { // r 与 sa 大小需 3 倍
26     #define F(x) ((x) / 3 + ((x) % 3 == 1 ? 0 : tb))
27     #define G(x) ((x) < tb ? (x) * 3 + 1 : ((x) - tb) * 3 + 2)
28     int wa[MAXN], wb[MAXN], wv[MAXN], ws[MAXN];
29     int c0(int *r, int a, int b) {
30         return r[a] == r[b] && r[a + 1] == r[b + 1] && r[a + 2] == r[b + 2];
31     }
32     int c12(int k, int *r, int a, int b) {
33         if (k == 2) return r[a] < r[b] || (r[a] == r[b] && c12(1, r, a + 1, b + 1));
34         else return r[a] < r[b] || (r[a] == r[b] && wv[a + 1] < wv[b + 1]);
35     }
36     void sort(int *r, int *a, int *b, int n, int m) {
37         for (int i = 0; i < n; i++) wv[i] = r[a[i]];
38         for (int i = 0; i < m; i++) ws[i] = 0;
39         for (int i = 0; i < n; i++) ws[wv[i]]++;
40         for (int i = 1; i < m; i++) ws[i] += ws[i - 1];
41         for (int i = n - 1; i >= 0; i--) b[--ws[wv[i]]] = a[i];
42     }
43     void dc3(int *r, int *sa, int n, int m) {
44         int i, j, *rn = r + n, *san = sa + n, ta = 0, tb = (n + 1) / 3, tbc = 0, p;
45         r[n] = r[n + 1] = 0;
46         for (i = 0; i < n; i++) if (i % 3 != 0) wa[tbc++] = i;
47         sort(r + 2, wa, wb, tbc, m);
48         sort(r + 1, wb, wa, tbc, m);
49         sort(r, wa, wb, tbc, m);
50         for (p = 1, rn[F(wb[0])] = 0, i = 1; i < tbc; i++)
51             rn[F(wb[i])] = c0(r, wb[i - 1], wb[i]) ? p - 1 : p++;
52         if (p < tbc) dc3(rn, san, tbc, p);
53         else for (i = 0; i < tbc; i++) san[rn[i]] = i;
54         for (i = 0; i < tbc; i++) if (san[i] < tb) wb[ta++] = san[i] * 3;
55         if (n % 3 == 1) wb[ta++] = n - 1;
56         sort(r, wb, wa, ta, m);
57         for (i = 0; i < tbc; i++) wv[wb[i]] = G(san[i]) = i;
58         for (i = 0, j = 0, p = 0; i < ta && j < tbc; p++)
59             sa[p] = c12(wb[j] % 3, r, wa[i], wb[j]) ? wa[i++] : wb[j++];
60         for (; i < ta; p++) sa[p] = wa[i++];
61         for (; j < tbc; p++) sa[p] = wb[j++];
62     }

```



```

63 #undef F
64 #undef G
65 }
66 namespace CalcHeight {
67     int rank[MAXN], height[MAXN];
68     void calheight(int *r, int *sa, int n) {
69         int i, j, k = 0;
70         for (i = 1; i <= n; i++) rank[sa[i]] = i;
71         for (i = 0; i < n; height[rank[i++]] = k)
72             for (k ? k-- : 0, j = sa[rank[i] - 1]; r[i + k] == r[j + k]; k++);
73     }
74 }

```

3.6 环串最小表示

```

1 int minimalRepresentation(int N, char *s) { // s must be double-sized and 0-based
2     int i, j, k, l;
3     for (i = 0; i < N; ++i) s[i + N] = s[i]; s[N + N] = 0;
4     for (i = 0, j = 1; j < N; ) {
5         for (k = 0; k < N && s[i + k] == s[j + k]; ++k);
6         if (k >= N) break;
7         if (s[i + k] < s[j + k]) j += k + 1;
8         else l = i + k, i = j, j = max(l, j) + 1;
9     } return i; // [i, i + N) is the minimal representation
10 }

```

4 图论

4.1 带花树

```

1 namespace Blossom {
2     int n, head, tail, S, T, lca;
3     int match[MAXN], Q[MAXN], pred[MAXN], label[MAXN], inq[MAXN], inb[MAXN];
4     vector<int> link[MAXN];
5     void init(int _n) { n = _n; for (int i = 0; i < n; ++i) link[i].clear(); }
6     inline void push(int x) { Q[tail++] = x; inq[x] = true; }
7     int findCommonAncestor(int x, int y) {
8         static bool inPath[MAXN];
9         for (int i = 0; i < n; ++i) inPath[i] = 0;
10        for ( ; ; x = pred[ match[x] ]) {
11            x = label[x]; inPath[x] = true; if (x == S) break;
12        }
13        for ( ; ; y = pred[ match[y] ]) {
14            y = label[y]; if (inPath[y]) break;
15        } return y;
16    }
17    void resetTrace(int x, int lca) {
18        while (label[x] != lca) {

```

```

19        int y = match[x]; inb[ label[x] ] = inb[ label[y] ] = true;
20        x = pred[y]; if (label[x] != lca) pred[x] = y;
21    }
22 }
23 void blossomContract(int x, int y) {
24     lca = findCommonAncestor(x, y);
25     for (int i = 0; i < n; ++i) inb[i] = 0;
26     resetTrace(x, lca); resetTrace(y, lca);
27     if (label[x] != lca) pred[x] = y;
28     if (label[y] != lca) pred[y] = x;
29     for (int i = 0; i < n; ++i) if (inb[ label[i] ]) {
30         label[i] = lca; if (!inq[i]) push(i);
31     }
32 }
33 bool findAugmentingPath() {
34     for (int i = 0; i < n; ++i) pred[i] = -1;
35     for (int i = 0; i < n; ++i) label[i] = i;
36     for (int i = 0; i < n; ++i) inq[i] = 0;
37     int x, y, z; head = tail = 0;
38     for (push(S); head < tail; )
39         for (int i = (int)link[x = Q[head++]].size() - 1; i >= 0; --i) {
40             y = link[x][i]; if (label[x] == label[y] || x == match[y]) continue;
41             if (y == S || (match[y] >= 0 && pred[ match[y] ] >= 0))
42                 blossomContract(x, y);
43             else if (pred[y] == -1) {
44                 pred[y] = x;
45                 if (match[y] >= 0) push(match[y]);
46                 else { // augment
47                     for (x = y; x >= 0; x = z) {
48                         y = pred[x], z = match[y]; match[x] = y, match[y] = x;
49                     } return true;
50                 }
51             }
52         }
53     return false;
54 }
55 int findMaxMatching() {
56     int ans = 0;
57     for (int i = 0; i < n; ++i) match[i] = -1;
58     for (S = 0; S < n; ++S) if (match[S] == -1)
59         if (findAugmentingPath()) ++ans;
60     return ans;
61 }
62 }

```

4.2 最大流

```

1 namespace Maxflow {
2     int h[MAXNODE], vh[MAXNODE], S, T, Ncnt; edge cur[MAXNODE], pe[MAXNODE];
3     void init(int _S, int _T, int _Ncnt) { S = _S; T = _T; Ncnt = _Ncnt; }
4     int maxflow() {

```

```

5 static int Q[MAXNODE]; int x, y, augc, flow = 0, head = 0, tail = 0; edge e;
6 Rep(i, 0, Ncnt) cur[i] = fir[i]; Rep(i, 0, Ncnt) h[i] = INF; Rep(i, 0, Ncnt) vh[i] = 0;
7 for (Q[++tail] = T, h[T] = 0; head < tail; ) {
8     x = Q[++head]; ++vh[ h[x] ];
9     for (e = fir[x]; e; e = e->next) if (e->op->c)
10         if (h[y = e->to] >= INF) h[y] = h[x] + 1, Q[++tail] = y;
11 } for (x = S; h[S] < Ncnt; ) {
12     for (e = cur[x]; e; e = e->next) if (e->c)
13         if (h[y = e->to] + 1 == h[x]) { cur[x] = pe[y] = e; x = y; break; }
14     if (!e) {
15         if (--vh[ h[x] ] == 0) break; h[x] = Ncnt; cur[x] = NULL;
16         for (e = fir[x]; e; e = e->next) if (e->c)
17             if ( cMin( h[x], h[e->to] + 1 ) ) cur[x] = e;
18         ++vh[ h[x] ];
19         if (x != S) x = pe[x]->op->to;
20     } else if (x == T) { augc = INF;
21         for (x = T; x != S; x = pe[x]->op->to) cMin(augc, pe[x]->c);
22         for (x = T; x != S; x = pe[x]->op->to) {
23             pe[x]->c -= augc; pe[x]->op->c += augc;
24             } flow += augc;
25     }
26 } return flow;
27 }
28 }

```

4.3 KM

```

1 namespace KM {
2     int n, m, ans; // left & right
3     int L[MAX_LEFT], R[MAX_RIGHT], v[MAX_RIGHT];
4     bool bx[MAX_LEFT], by[MAX_RIGHT];
5     bool find(int x) {
6         bx[x] = true; for (edge e(fir[x]); e; e = e->next) {
7             int y = e->to, c = e->c;
8             if (!by[y] && L[x] + R[y] == c) {
9                 by[y] = true;
10                 if (!v[y] || find(v[y])) { v[y] = x; return true; }
11             }
12         } return false;
13     }
14     int km() {
15         memset(L, 0, sizeof(L)); memset(R, 0, sizeof(R)); memset(v, 0, sizeof(v));
16         for (int x = 1; x <= n; ++x) for (edge e(fir[x]); e; e = e->next)
17             L[x] = max(L[x], e->c);
18         ans = 0;
19         for (int i = 1; i <= min(n, m); ++i) for ( ; ; ) {
20             memset(bx, 0, sizeof(bx)); memset(by, 0, sizeof(by));
21             if (find(i)) break; int Min = INF;
22             for (int x = 1; x <= n; ++x) if (bx[x])
23                 for (edge e(fir[x]); e; e = e->next) {
24                     if (!by[e->to]) Min = min(Min, L[x] + R[e->to] - e->c);

```

```

25         }
26         for (int x = 1; x <= n; ++x) if (bx[x]) L[x] -= Min;
27         for (int y = 1; y <= m; ++y) if (by[y]) R[y] += Min;
28     }
29     for (int x = 1; x <= n; ++x) for (edge e(fir[x]); e; e = e->next)
30         if (v[e->to] == x) ans += e->c;
31     return ans;
32 }
33 }

```

4.4 2-SAT 与 Kosaraju

注意 Kosaraju 需要建反图

```

1 namespace SCC {
2     int code[MAXN * 2], seq[MAXN * 2], sCnt;
3     void DFS_1(int x) {
4         code[x] = 1;
5         for (edge e(fir[x]); e; e = e->next) if (code[e->to] == -1) DFS_1(e->to);
6         seq[++sCnt] = x;
7     }
8     void DFS_2(int x) {
9         code[x] = sCnt;
10        for (edge e(fir2[x]); e; e = e->next) if (code[e->to] == -1) DFS_2(e->to);
11    }
12    void SCC(int N) {
13        sCnt = 0; for (int i = 1; i <= N; ++i) code[i] = -1;
14        for (int i = 1; i <= N; ++i) if (code[i] == -1) DFS_1(i);
15        sCnt = 0; for (int i = 1; i <= N; ++i) code[i] = -1;
16        for (int i = N; i >= 1; --i) if (code[seq[i]] == -1) {
17            ++sCnt; DFS_2(seq[i]); }
18    }
19 }
20 // true    - 2i - 1
21 // false   - 2i
22 bool TwoSat() {
23     using SCC::code;
24     SCC::SCC(N + N);
25     for (int i = 1; i <= N; ++i) if (code[i + i - 1] == code[i + i]) return false;
26     for (int i = 1; i <= N; ++i)
27         if (code[i + i - 1] > code[i + i]) // i : selected
28             else // i : not selected
29         return true;
30 }

```

4.5 全局最小割 Stoer-Wagner

```

1 int minCut(int N, int G[MAXN][MAXN]) { // 0-based
2     static int weight[MAXN], used[MAXN]; int ans = INT_MAX;

```

```

3 while (N > 1) {
4     for (int i = 0; i < N; ++i) used[i] = false; used[0] = true;
5     for (int i = 0; i < N; ++i) weight[i] = G[i][0];
6     int S = -1, T = 0;
7     for (int _r = 2; _r <= N; ++_r) { // N - 1 selections
8         int x = -1;
9         for (int i = 0; i < N; ++i) if (!used[i])
10             if (x == -1 || weight[i] > weight[x]) x = i;
11         for (int i = 0; i < N; ++i) weight[i] += G[x][i];
12         S = T; T = x; used[x] = true;
13     } ans = min(ans, weight[T]);
14     for (int i = 0; i < N; ++i) G[i][S] += G[i][T], G[S][i] += G[i][T];
15     G[S][S] = 0; --N;
16     for (int i = 0; i <= N; ++i) swap(G[i][T], G[i][N]);
17     for (int i = 0; i < N; ++i) swap(G[T][i], G[N][i]);
18     return ans;
19 }

```

4.6 欧拉路

```

1 vector<int> eulerianWalk(int N, int S) {
2     static int res[MAXN], stack[MAXN];
3     static edge cur[MAXN];
4     int rcnt = 0, top = 0;
5     for (int i = 1; i <= N; ++i) cur[i] = fir[i];
6     for (stack[top++] = S; top; ) {
7         int x = stack[--top];
8         for ( ; ; ) {
9             edge &e = cur[x];
10            if (e == NULL) break;
11            stack[top++] = x;
12            x = e->to;
13            e = e->next; // the opposite edge must be banned if this is a bidirectional graph
14        }
15        res[rcnt++] = x;
16    }
17    reverse(res, res + rcnt);
18    return vector<int>(res, res + rcnt);
19 }

```

4.7 稳定婚姻

```

1 namespace StableMatching {
2     int pairM[MAXN], pairW[MAXN], p[MAXN];
3     // init: pairM[0...n-1] = pairW[0...n-1] = -1, p[0...n-1] = 0
4     void stableMatching(int n, int orderM[MAXN][MAXN], int preferW[MAXN][MAXN]) {
5         for (int i = 0; i < n; i++) while (pairM[i] < 0) {
6             int w = orderM[i][p[i]++], m = pairW[w];
7             if (m == -1) pairM[i] = w, pairW[w] = i;

```

```

8             else if (preferW[w][i] < preferW[w][m])
9                 pairM[m] = -1, pairM[i] = w, pairW[w] = i, i = m;
10         }
11     }
12 }

```

4.8 最大团搜索

```

1 namespace MaxClique { // 1-based
2     int g[MAXN][MAXN], len[MAXN], list[MAXN][MAXN], mc[MAXN], ans, found;
3     void DFS(int size) {
4         if (len[size] == 0) { if (size > ans) ans = size, found = true; return; }
5         for (int k = 0; k < len[size] && !found; ++k) {
6             if (size + len[size] - k <= ans) break;
7             int i = list[size][k]; if (size + mc[i] <= ans) break;
8             for (int j = k + 1, len[size + 1] = 0; j < len[size]; ++j) if (g[i][list[size][j]])
9                 list[size + 1][len[size + 1]++] = list[size][j];
10            DFS(size + 1);
11        }
12    }
13    int work(int n) {
14        mc[n] = ans = 1; for (int i = n - 1; i; --i) { found = false; len[i] = 0;
15            for (int j = i + 1; j <= n; ++j) if (g[i][j]) list[i][len[i]++] = j;
16            DFS(i); mc[i] = ans;
17        } return ans;
18    }
19 }

```

4.9 最小树形图

```

1 namespace EdmondsAlgorithm { // O(E log E + V^2) !!! 0-based !!!
2     struct enode { int from, c, key, delta, dep; enode *ch[2], *next;
3     } ebase[maxm], *etop, *fir[maxn], nil, *null, *inEdge[maxn], *chs[maxn];
4     typedef enode *edge; typedef enode *tree;
5     int n, m, setFa[maxn], deg[maxn], que[maxn];
6     inline void pushDown(tree x) { if (x->delta) {
7         x->ch[0]->key += x->delta; x->ch[0]->delta += x->delta;
8         x->ch[1]->key += x->delta; x->ch[1]->delta += x->delta; x->delta = 0;
9     }}
10    tree merge(tree x, tree y) {
11        if (x == null) return y; if (y == null) return x;
12        if (x->key > y->key) swap(x, y); pushDown(x); x->ch[1] = merge(x->ch[1], y);
13        if (x->ch[0]->dep < x->ch[1]->dep) swap(x->ch[0], x->ch[1]);
14        x->dep = x->ch[1]->dep + 1; return x;
15    }
16    void addEdge(int u, int v, int w) {
17        etop->from = u; etop->c = etop->key = w; etop->delta = etop->dep = 0;
18        etop->next = fir[v]; etop->ch[0] = etop->ch[1] = null;
19        fir[v] = etop; inEdge[v] = merge(inEdge[v], etop++);

```

```

20 }
21 void deleteMin(tree &r) { pushDown(r); r = merge(r->ch[0], r->ch[1]); }
22 int findSet(int x) { return setFa[x] == x ? x : setFa[x] = findSet(setFa[x]); }
23 void clear(int V, int E) {
24     null = &nil; null->ch[0] = null->ch[1] = null; null->dep = -1;
25     n = V; m = E; etop = ebase; Foru(i, 0, V) fir[i] = NULL; Foru(i, 0, V) inEdge[i] = null;
26 }
27 int solve(int root) { int res = 0, head, tail;
28     for (int i = 0; i < n; ++i) setFa[i] = i;
29     for ( ; ; ) { memset(deg, 0, sizeof(int) * n); chs[root] = inEdge[root];
30         for (int i = 0; i < n; ++i) if (i != root && setFa[i] == i) {
31             while (findSet(inEdge[i]->from) == findSet(i)) deleteMin(inEdge[i]);
32             ++deg[ findSet((chs[i] = inEdge[i])->from) ];
33         }
34         for (int i = head = tail = 0; i < n; ++i)
35             if (i != root && setFa[i] == i && deg[i] == 0) que[tail++] = i;
36         while (head < tail) {
37             int x = findSet(chs[que[head++]]->from);
38             if (--deg[x] == 0) que[tail++] = x;
39         } bool found = false;
40         for (int i = 0; i < n; ++i) if (i != root && setFa[i] == i && deg[i] > 0) {
41             int j = i; tree temp = null; found = true;
42             do {setFa[j] = findSet(chs[j]->from)} = i;
43             deleteMin(inEdge[j]); res += chs[j]->key;
44             inEdge[j]->key -= chs[j]->key; inEdge[j]->delta -= chs[j]->key;
45             temp = merge(temp, inEdge[j]);
46             } while (j != i); inEdge[i] = temp;
47         } if (!found) break;
48     } for (int i = 0; i < n; ++ i) if (i != root && setFa[i] == i) res += chs[i]->key;
49     return res;
50 }
51 }
52 namespace ChuLiu { // O(V ^ 3) !!! 1-based !!!
53     int n, used[maxn], pass[maxn], eg[maxn], more, que[maxn], g[maxn][maxn];
54     void combine(int id, int &sum) { int tot = 0, from, i, j, k;
55         for ( ; id != 0 && !pass[id]; id = eg[id]) que[tot++] = id, pass[id] = 1;
56         for (from = 0; from < tot && que[from] != id; from++);
57         if (from == tot) return; more = 1;
58         for (i = from; i < tot; i++) {
59             sum += g[eg[que[i]]][que[i]]; if (i == from) continue;
60             for (j = used[que[i]] = 1; j <= n; j++) if (!used[j])
61                 if (g[que[i]][j] < g[id][j]) g[id][j] = g[que[i]][j];
62         }
63         for (i = 1; i <= n; i++) if (!used[i] && i != id)
64             for (j = from; j < tot; j++) {
65                 k = que[j]; if (g[i][id] > g[i][k] - g[eg[k]][k])
66                     g[i][id] = g[i][k] - g[eg[k]][k];
67             }
68     }
69 void clear(int V) { n = V; Rep(i, 1, V) Rep(j, 1, V) g[i][j] = inf; }
70 int solve(int root) {
71     int i, j, k, sum = 0; memset(used, 0, sizeof(int) * (n + 1));
72     for (more = 1; more; ) {

```

```

73         more = 0; memset(eg, 0, sizeof(int) * (n + 1));
74         for (i = 1; i <= n; i++) if (!used[i] && i != root) {
75             for (j = 1, k = 0; j <= n; j++) if (!used[j] && i != j)
76                 if (k == 0 || g[j][i] < g[k][i]) k = j;
77             eg[i] = k;
78         } memset(pass, 0, sizeof(int) * (n + 1));
79         for (i = 1; i <= n; i++) if (!used[i] && !pass[i] && i != root)
80             combine(i, sum);
81     } for (i = 1; i <= n; i++) if (!used[i] && i != root) sum += g[eg[i]][i];
82     return sum;
83 }
84 }

```

4.10 离线动态最小生成树

$O(Q \log^2 Q)$. $(qx[i], qy[i])$ 表示将编号为 $qx[i]$ 的边的权值改为 $qy[i]$, 删除一条边相当于将其权值改为 ∞ , 加入一条边相当于将其权值从 ∞ 变成某个值.

```

1  const int maxn = 100000 + 5;
2  const int maxm = 1000000 + 5;
3  const int maxq = 1000000 + 5;
4  const int qsize = maxm + 3 * maxq;
5  int n, m, Q, x[qsize], y[qsize], z[qsize], qx[maxq], qy[maxq], a[maxn], *tz;
6  int kx[maxn], ky[maxn], kt, vd[maxn], id[maxm], app[maxm];
7  bool extra[maxm];
8  void init() {
9      scanf("%d%d", &n, &m); for (int i = 0; i < m; i++) scanf("%d%d%d", x + i, y + i, z + i);
10     scanf("%d", &Q); for (int i = 0; i < Q; i++) { scanf("%d%d", qx + i, qy + i); qx[i]--; }
11 }
12 int find(int x) {
13     int root = x, next;
14     while (a[root]) root = a[root];
15     while ((next = a[x]) != 0) a[x] = root, x = next;
16     return root;
17 }
18 inline bool cmp(const int &a, const int &b) { return tz[a] < tz[b]; }
19 void solve(int *qx, int *qy, int Q, int n, int *x, int *y, int *z, int m, long long ans) {
20     int ri, rj;
21     if (Q == 1) {
22         for (int i = 1; i <= n; i++) a[i] = 0; z[qx[0]] = qy[0];
23         for (int i = 0; i < m; i++) id[i] = i;
24         tz = z; sort(id, id + m, cmp);
25         for (int i = 0; i < m; i++) {
26             ri = find(x[id[i]]); rj = find(y[id[i]]);
27             if (ri != rj) ans += z[id[i]], a[ri] = rj;
28         } printf("%I64d\n", ans);
29         return;
30     } int tm = kt = 0, n2 = 0, m2 = 0;
31     for (int i = 1; i <= n; i++) a[i] = 0;
32     for (int i = 0; i < Q; i++) {
33         ri = find(x[qx[i]]); rj = find(y[qx[i]]);

```

```

34     if (ri != rj) a[ri] = rj;
35 }
36 for (int i = 0; i < m; i++) extra[i] = true;
37 for (int i = 0; i < Q; i++) extra[qx[i]] = false;
38 for (int i = 0; i < m; i++) if (extra[i]) id[tm++] = i;
39 tz = z; sort(id, id + tm, cmp);
40 for (int i = 0; i < tm; i++) {
41     ri = find(x[id[i]]); rj = find(y[id[i]]);
42     if (ri != rj)
43         a[ri] = rj, ans += z[id[i]], kx[kt] = x[id[i]], ky[kt] = y[id[i]], kt++;
44 }
45 for (int i = 1; i <= n; i++) a[i] = 0;
46 for (int i = 0; i < kt; i++) a[find(kx[i])] = find(ky[i]);
47 for (int i = 1; i <= n; i++) if (a[i] == 0) vd[i] = ++n2;
48 for (int i = 1; i <= n; i++) if (a[i] != 0) vd[i] = vd[find(i)];
49 int *Nx = x + m, *Ny = y + m, *Nz = z + m;
50 for (int i = 0; i < m; i++) app[i] = -1;
51 for (int i = 0; i < Q; i++)
52     if (app[qx[i]] == -1)
53         Nx[m2] = vd[x[qx[i]]], Ny[m2] = vd[y[qx[i]]], Nz[m2] = z[qx[i]], app[qx[i]] = m2, m2++;
54 for (int i = 0; i < Q; i++) {
55     z[qx[i]] = qy[i];
56     qx[i] = app[qx[i]];
57 }
58 for (int i = 1; i <= n2; i++) a[i] = 0;
59 for (int i = 0; i < tm; i++) {
60     ri = find(vd[x[id[i]]]); rj = find(vd[y[id[i]]]);
61     if (ri != rj)
62         a[ri] = rj, Nx[m2] = vd[x[id[i]]], Ny[m2] = vd[y[id[i]]], Nz[m2] = z[id[i]], m2++;
63 }
64 int mid = Q / 2;
65 solve(qx, qy, mid, n2, Nx, Ny, Nz, m2, ans);
66 solve(qx + mid, qy + mid, Q - mid, n2, Nx, Ny, Nz, m2, ans);
67 }
68 void work() { if (Q) solve(qx, qy, Q, n, x, y, z, m, 0); }
69 int main() { init(); work(); return 0; }

```

4.11 小知识

- 平面图:
 - 平面图一定存在一个度小于等于 5 的点
 - $E \leq 3V - 6$
 - 欧拉公式: $V + F - E = 1 + \text{连通块数}$
- 图连通度:
 - k -连通 (k -connected): 对于任意一对结点都至少存在结点各不相同的 k 条路
 - 点连通度 ($\text{vertex connectivity}$): 把图变成非连通图所需删除的最少点数

3. Whitney 定理: 一个图是 k -连通的当且仅当它的点连通度至少为 k

- Lindstroem-Gessel-Viennot Lemma: 给定一个图的 n 个起点和 n 个终点, 令 A_{ij} = 第 i 个起点到第 j 个终点的路径条数, 则从起点到终点的不相交路径条数为 $\det(A)$
- 欧拉回路与树形图的联系: 对于出度等于入度的连通图 $s(G) = t_i(G) \prod_{j=1}^n (d^+(v_j) - 1)!$
- 密度子图: 给定无向图, 选取点集及其导出子图, 最大化 $W_e + P_v$ (点权可负).

$$-(S, u) = U$$

$$-(u, T) = U - 2P_u - D_u$$

$$-(u, v) = (v, u) = W_e$$

$$-\text{ans} = \frac{Un - C[S, T]}{2}, \text{解集为 } S - \{s\}$$

- 最大权闭合图: 选 a 则 a 的后继必须被选

$$-P_u > 0, (S, u) = P_u$$

$$-P_u < 0, (u, T) = -P_u$$

$$-\text{ans} = \sum_{P_u > 0} P_u - C[S, T], \text{解集为 } S - \{s\}$$

- 判定边是否属于最小割:

— 可能属于最小割: (u, v) 不属于同一 SCC

— 一定在所有最小割中: (u, v) 不属于同一 SCC, 且 S, u 在同一 SCC, u, T 在同一 SCC

5 数学

5.1 单纯形 Java

$$\max \{cx \mid Ax \leq b, x \geq 0\}$$

```

1 double[] simplex(double[][] A, double[] b, double[] c) {
2     int n = A.length, m = A[0].length + 1, r = n, s = m - 1;
3     double[][] D = new double[n + 2][m + 1];
4     int[] ix = new int[n + m];
5     for (int i = 0; i < n + m; i++) ix[i] = i;
6     for (int i = 0; i < n; i++) {
7         for (int j = 0; j < m - 1; j++) D[i][j] = -A[i][j];
8         D[i][m - 1] = 1;
9         D[i][m] = b[i];
10        if (D[r][m] > D[i][m]) r = i;
11    }
12    for (int j = 0; j < m - 1; j++) D[n][j] = c[j];
13    D[n + 1][m - 1] = -1;

```

```

14 for (double d; ; ) {
15     if (r < n) {
16         int t = ix[s]; ix[s] = ix[r + m]; ix[r + m] = t;
17         D[r][s] = 1.0 / D[r][s];
18         for (int j = 0; j <= m; j++) if (j != s) D[r][j] *= -D[r][s];
19         for (int i = 0; i <= n + 1; i++) if (i != r) {
20             for (int j = 0; j <= m; j++) if (j != s) D[i][j] += D[r][j] * D[i][s];
21             D[i][s] *= D[r][s];
22         }
23     }
24     r = -1; s = -1;
25     for (int j = 0; j < m; j++) if (s < 0 || ix[s] > ix[j]) {
26         if (D[n + 1][j] > EPS || D[n + 1][j] > -EPS && D[n][j] > EPS) s = j;
27     }
28     if (s < 0) break;
29     for (int i = 0; i < n; i++) if (D[i][s] < -EPS) {
30         if (r < 0 || (d = D[r][m] / D[r][s] - D[i][m] / D[i][s]) < -EPS
31             || d < EPS && ix[r + m] > ix[i + m])
32             r = i;
33     }
34     if (r < 0) return null; // 非有界
35 } if (D[n + 1][m] < -EPS) return null; // 无法执行
36 double[] x = new double[m - 1];
37 for (int i = m; i < n + m; i++) if (ix[i] < m - 1) x[ix[i]] = D[i - m][m];
38 return x; // 值为 D[n][m]
39 }

```

5.2 FFT

```

1 namespace FFT {
2     #define mul(a, b) (Complex(a.x * b.x - a.y * b.y, a.x * b.y + a.y * b.x))
3     struct Complex {}; // something omitted
4     void FFT(Complex P[], int n, int oper) {
5         for (int i = 1, j = 0; i < n - 1; i++) {
6             for (int s = n; j ^= s >= 1, -j & s; );
7             if (i < j) swap(P[i], P[j]);
8         }
9         for (int d = 0; (1 << d) < n; d++) {
10             int m = 1 << d, m2 = m * 2;
11             double p0 = PI / m * oper;
12             Complex unit_p0(cos(p0), sin(p0));
13             for (int i = 0; i < n; i += m2) {
14                 Complex unit(1.0, 0.0);
15                 for (int j = 0; j < m; j++) {
16                     Complex &P1 = P[i + j + m], &P2 = P[i + j];
17                     Complex t = mul(unit, P1);
18                     P1 = Complex(P2.x - t.x, P2.y - t.y);
19                     P2 = Complex(P2.x + t.x, P2.y - t.y);
20                     unit = mul(unit, unit_p0);
21                 }
22             }
23         }
24     }
25     vector<int> doFFT(const vector<int> &a, const vector<int> &b) {

```

```

23     vector<int> ret(max(0, (int) a.size() + (int) b.size() - 1), 0);
24     static Complex A[MAXB], B[MAXB], C[MAXB];
25     int len = 1; while (len < (int)ret.size()) len *= 2;
26     for (int i = 0; i < len; i++) A[i] = i < (int)a.size() ? a[i] : 0;
27     for (int i = 0; i < len; i++) B[i] = i < (int)b.size() ? b[i] : 0;
28     FFT(A, len, 1); FFT(B, len, 1);
29     for (int i = 0; i < len; i++) C[i] = mul(A[i], B[i]);
30     FFT(C, len, -1);
31     for (int i = 0; i < (int)ret.size(); i++)
32         ret[i] = (int) (C[i].x / len + 0.5);
33     return ret;
34 }
35 }

```

5.3 整数 FFT

```

1 namespace FFT {
2     // 替代方案: 23068673(= 11 * 221 + 1), 原根为 3
3     const int MOD = 786433, PRIMITIVE_ROOT = 10; // 3 * 218 + 1
4     const int MAXB = 1 << 20;
5     int getMod(int downLimit) { // 或者现场自己找一个 MOD
6         for (int c = 3; ; ++c) { int t = (c << 21) | 1;
7             if (t >= downLimit && isPrime(t)) return t;
8         }
9     }
10     int modInv(int a) { return a <= 1 ? a : (long long) (MOD - MOD / a) * modInv(MOD % a) % MOD; }
11     void NTT(int P[], int n, int oper) {
12         for (int i = 1, j = 0; i < n - 1; i++) {
13             for (int s = n; j ^= s >= 1, -j & s; );
14             if (i < j) swap(P[i], P[j]);
15         }
16         for (int d = 0; (1 << d) < n; d++) {
17             int m = 1 << d, m2 = m * 2;
18             long long unit_p0 = powMod(PRIMITIVE_ROOT, (MOD - 1) / m2);
19             if (oper < 0) unit_p0 = modInv(unit_p0);
20             for (int i = 0; i < n; i += m2) {
21                 long long unit = 1;
22                 for (int j = 0; j < m; j++) {
23                     int &P1 = P[i + j + m], &P2 = P[i + j];
24                     int t = unit * P1 % MOD;
25                     P1 = (P2 - t + MOD) % MOD; P2 = (P2 + t) % MOD;
26                     unit = unit * unit_p0 % MOD;
27                 }
28             }
29         }
30         vector<int> mul(const vector<int> &a, const vector<int> &b) {
31             vector<int> ret(max(0, (int) a.size() + (int) b.size() - 1), 0);
32             static int A[MAXB], B[MAXB], C[MAXB];
33             int len = 1; while (len < (int)ret.size()) len <= 1;
34             for (int i = 0; i < len; i++) A[i] = i < (int)a.size() ? a[i] : 0;
35             for (int i = 0; i < len; i++) B[i] = i < (int)b.size() ? b[i] : 0;
36             NTT(A, len, 1); NTT(B, len, 1);
37             for (int i = 0; i < len; i++) C[i] = (long long) A[i] * B[i] % MOD;

```

```

35     NTT(C, len, -1); for (int i = 0, inv = modInv(len); i < (int)ret.size(); i++) ret[i] = (
        long long) C[i] * inv % MOD;
36     return ret;
37 }
38 }

```

5.4 扩展欧几里得

$$ax + by = g = \gcd(x, y)$$

```

1 void exgcd(LL x, LL y, LL &a0, LL &b0, LL &g) {
2     LL a1 = b0 = 0, b1 = a0 = 1, t;
3     while (y != 0) {
4         t = a0 - x / y * a1, a0 = a1, a1 = t;
5         t = b0 - x / y * b1, b0 = b1, b1 = t;
6         t = x % y, x = y, y = t;
7     } if (x < 0) a0 = -a0, b0 = -b0, x = -x;
8     g = x;
9 }

```

5.5 线性同余方程

- 中国剩余定理: 设 m_1, m_2, \dots, m_k 两两互素, 则同余方程组 $x \equiv a_i \pmod{m_i}$ for $i = 1, 2, \dots, k$ 在 $[0, M = m_1 m_2 \dots m_k)$ 内有唯一解. 记 $M_i = M/m_i$, 找出 p_i 使得 $M_i p_i \equiv 1 \pmod{m_i}$, 记 $e_i = M_i p_i$, 则 $x \equiv e_1 a_1 + e_2 a_2 + \dots + e_k a_k \pmod{M}$
- 多元线性同余方程组: 方程的形式为 $a_1 x_1 + a_2 x_2 + \dots + a_n x_n + b \equiv 0 \pmod{m}$, 令 $d = (a_1, a_2, \dots, a_n, m)$, 有解的充要条件是 $d|b$, 解的个数为 $m^{n-1}d$

5.6 Miller-Rabin 素性测试

```

1 bool test(LL n, int base) {
2     LL m = n - 1, ret = 0; int s = 0;
3     for ( ; m % 2 == 0; ++s) m >>= 1; ret = pow_mod(base, m, n);
4     if (ret == 1 || ret == n - 1) return true;
5     for (--s; s >= 0; --s) {
6         ret = multiply_mod(ret, ret, n); if (ret == n - 1) return true;
7     } return false;
8 }
9 LL special[7] = {
10     1373653LL,      25326001LL,
11     3215031751LL,   250000000000LL,
12     2152302898747LL, 3474749660383LL, 341550071728321LL;
13 /*
14  * n < 2047          test[] = {2}
15  * n < 1,373,653     test[] = {2, 3}
16  * n < 9,080,191     test[] = {31, 73}

```

```

17  * n < 25,326,001     test[] = {2, 3, 5}
18  * n < 4,759,123,141  test[] = {2, 7, 61}
19  * n < 1,122,004,669,633 test[] = {2, 13, 23, 1662803}
20  * n < 2,152,302,898,747 test[] = {2, 3, 5, 7, 11}
21  * n < 3,474,749,660,383 test[] = {2, 3, 5, 7, 11, 13}
22  * n < 341,550,071,728,321 test[] = {2, 3, 5, 7, 11, 13, 17}
23  * n < 3,825,123,056,546,413,051 test[] = {2, 3, 5, 7, 11, 13, 17, 19, 23}
24  */
25 bool is_prime(LL n) {
26     if (n < 2) return false;
27     if (n < 4) return true;
28     if (!test(n, 2) || !test(n, 3)) return false;
29     if (n < special[0]) return true;
30     if (!test(n, 5)) return false;
31     if (n < special[1]) return true;
32     if (!test(n, 7)) return false;
33     if (n == special[2]) return false;
34     if (n < special[3]) return true;
35     if (!test(n, 11)) return false;
36     if (n < special[4]) return true;
37     if (!test(n, 13)) return false;
38     if (n < special[5]) return true;
39     if (!test(n, 17)) return false;
40     if (n < special[6]) return true;
41     return test(n, 19) && test(n, 23) && test(n, 29) && test(n, 31) && test(n, 37);
42 }

```

5.7 PollardRho

```

1 LL pollardRho(LL n, LL seed) {
2     LL x, y, head = 1, tail = 2; x = y = random() % (n - 1) + 1;
3     for ( ; ; ) {
4         x = addMod(multiplyMod(x, x, n), seed, n);
5         if (x == y) return n; LL d = gcd(myAbs(x - y), n);
6         if (1 < d && d < n) return d;
7         if (++head == tail) y = x, tail <= 1;
8     } vector<LL> divisors;
9     void factorize(LL n) { // 需要保证 n > 1
10         if (isPrime(n)) divisors.push_back(n);
11         else { LL d = n;
12             while (d >= n) d = pollardRho(n, random() % (n - 1) + 1);
13             factorize(n / d); factorize(d);
14         }

```

5.8 多项式求根

```

1 const double error = 1e-12;
2 const double infi = 1e+12;
3 int n; double a[10], x[10];

```

```

4 double f(double a[], int n, double x) {
5     double tmp = 1, sum = 0;
6     for (int i = 0; i <= n; i++) sum = sum + a[i] * tmp, tmp = tmp * x;
7     return sum;
8 }
9 double binary(double l, double r, double a[], int n) {
10    int sl = sign(f(a, n, l)), sr = sign(f(a, n, r));
11    if (sl == 0) return l; if (sr == 0) return r;
12    if (sl * sr > 0) return infi;
13    while (r - l > error) {
14        double mid = (l + r) / 2;
15        int ss = sign(f(a, n, mid));
16        if (ss == 0) return mid;
17        if (ss * sl > 0) l = mid; else r = mid;
18    } return l;
19 }
20 void solve(int n, double a[], double x[], int &nx) {
21    if (n == 1) { x[1] = -a[0] / a[1]; nx = 1; return; }
22    double da[10], dx[10]; int ndx;
23    for (int i = n; i >= 1; i--) da[i - 1] = a[i] * i;
24    solve(n - 1, da, dx, ndx); nx = 0;
25    if (ndx == 0) {
26        double tmp = binary(-infi, infi, a, n);
27        if (tmp < infi) x[++nx] = tmp; return;
28    } double tmp = binary(-infi, dx[1], a, n);
29    if (tmp < infi) x[++nx] = tmp;
30    for (int i = 1; i <= ndx - 1; i++) {
31        tmp = binary(dx[i], dx[i + 1], a, n);
32        if (tmp < infi) x[++nx] = tmp;
33    } tmp = binary(dx[ndx], infi, a, n);
34    if (tmp < infi) x[++nx] = tmp;
35 }
36 int main() {
37     scanf("%d", &n);
38     for (int i = n; i >= 0; i--) scanf("%lf", &a[i]);
39     int nx; solve(n, a, x, nx);
40     for (int i = 1; i <= nx; i++) printf("%.6f\n", x[i]);
41     return 0;
42 }

```

5.9 线性递推

$$\text{for } a_{i+n} = (\sum_{j=0}^{n-1} k_j a_{i+j}) + d, a_m = (\sum_{i=0}^{n-1} c_i a_i) + c_n d$$

```

1 vector<int> recFormula(int n, int k[], int m) {
2     vector<int> c(n + 1, 0);
3     if (m < n) c[m] = 1;
4     else {
5         static int a[MAX_K * 2 + 1];
6         vector<int> b = recFormula(n, k, m >> 1);
7         for (int i = 0; i < n + n; ++i) a[i] = 0;

```

```

8         int s = m & 1;
9         for (int i = 0; i < n; i++) {
10             for (int j = 0; j < n; j++) a[i + j + s] += b[i] * b[j];
11             c[n] += b[i];
12         } c[n] = (c[n] + 1) * b[n];
13         for (int i = n * 2 - 1; i >= n; i--) {
14             int add = a[i]; if (add == 0) continue;
15             for (int j = 0; j < n; j++) a[i - n + j] += k[j] * add;
16             c[n] += add;
17         } for (int i = 0; i < n; ++i) c[i] = a[i];
18     } return c;
19 }

```

5.10 原根

原根 g : g 是模 n 简化剩余系构成的乘法群的生成元. 模 n 有原根的充要条件是 $n = 2, 4, p^n, 2p^n$, 其中 p 是奇质数, n 是正整数

```

1 vector<int> findPrimitiveRoot(int N) {
2     if (N <= 4) return vector<int>(1, max(1, N - 1));
3     static int factor[100];
4     int phi = N, totF = 0;
5     { // check no solution and calculate phi
6         int M = N, k = 0;
7         if (~M & 1) M >>= 1, phi >>= 1;
8         if (~M & 1) return vector<int>(0);
9         for (int d = 3; d * d <= M; ++d) if (M % d == 0) {
10             if (++k > 1) return vector<int>(0);
11             for (phi -= phi / d; M % d == 0; M /= d);
12         } if (M > 1) {
13             if (++k > 1) return vector<int>(0); phi -= phi / M;
14         }
15     } { // factorize phi
16         int M = phi;
17         for (int d = 2; d * d <= M; ++d) if (M % d == 0) {
18             for (; M % d == 0; M /= d); factor[++totF] = d;
19         } if (M > 1) factor[++totF] = M;
20     } vector<int> ans;
21     for (int g = 2; g <= N; ++g) if (Gcd(g, N) == 1) {
22         bool good = true;
23         for (int i = 1; i <= totF && good; ++i)
24             if (powMod(g, phi / factor[i], N) == 1) good = false;
25         if (!good) continue;
26         for (int i = 1, gp = g; i <= phi; ++i, gp = (LL)gp * g % N)
27             if (Gcd(i, phi) == 1) ans.push_back(gp);
28         break;
29     } sort(ans.begin(), ans.end());
30     return ans;
31 }

```


5.11 离散对数

$A^x \equiv B \pmod{C}$, 对非质数 C 也适用.

```
1 int modLog(int A, int B, int C) {
2     static pii baby[MAX_SQRT_C + 11];
3     int d = 0; LL k = 1, D = 1; B %= C;
4     for (int i = 0; i < 100; ++i, k = k * A % C) // [0, log C]
5         if (k == B) return i;
6     for (int g; ; ++d) {
7         g = gcd(A, C); if (g == 1) break;
8         if (B % g != 0) return -1;
9         B /= g; C /= g; D = (A / g * D) % C;
10    } int m = (int) ceil(sqrt((double) C)); k = 1;
11    for (int i = 0; i <= m; ++i, k = k * A % C) baby[i] = pii(k, i);
12    sort(baby, baby + m + 1); // [0, m]
13    int n = unique(baby, baby + m + 1, equalFirst) - baby, am = powMod(A, m, C);
14    for (int i = 0; i <= m; ++i) {
15        LL e, x, y; exgcd(D, C, x, y, e); e = x * B % C;
16        if (e < 0) e += C;
17        if (e >= 0) {
18            int k = lower_bound(baby, baby + n, pii(e, -1)) - baby;
19            if (baby[k].first == e) return i * m + baby[k].second + d;
20        } D = D * am % C;
21    } return -1;
22 }
```

5.12 平方剩余

- Legrendre Symbol: 对奇质数 p , $\left(\frac{a}{p}\right) = \begin{cases} 1 & \text{是平方剩余} \\ -1 & \text{是非平方剩余} \\ 0 & a \equiv 0 \pmod{p} \end{cases} = a^{\frac{p-1}{2}} \pmod{p}$

- 若 p 是奇质数, $\left(\frac{-1}{p}\right) = 1$ 当且仅当 $p \equiv 1 \pmod{4}$
- 若 p 是奇质数, $\left(\frac{2}{p}\right) = 1$ 当且仅当 $p \equiv \pm 1 \pmod{8}$
- 若 p, q 是奇素数且互质, $\left(\frac{p}{q}\right)\left(\frac{q}{p}\right) = (-1)^{\frac{p-1}{2} \times \frac{q-1}{2}}$
- Jacobi Symbol: 对奇数 $n = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_k^{\alpha_k}$, $\left(\frac{a}{n}\right) = \left(\frac{a}{p_1}\right)^{\alpha_1} \left(\frac{a}{p_2}\right)^{\alpha_2} \dots \left(\frac{a}{p_k}\right)^{\alpha_k}$
- Jacobi Symbol 为 -1 则一定不是平方剩余, 所有平方剩余的 Jacobi Symbol 都是 1, 但 1 不一定是平方剩余

$ax^2 + bx + c \equiv 0 \pmod{p}$, 其中 $a \not\equiv 0 \pmod{p}$, 且 p 是质数

```
1 inline int normalize(LL a, int P) { a %= P; return a < 0 ? a + P : a; }
2 vector<int> QuadraticResidue(LL a, LL b, LL c, int P) {
3     int h, t; LL r1, r2, delta, pb = 0;
```

```
4     a = normalize(a, P); b = normalize(b, P); c = normalize(c, P);
5     if (P == 2) { vector<int> res;
6         if (c % P == 0) res.push_back(0);
7         if ((a + b + c) % P == 0) res.push_back(1);
8         return res;
9     } delta = b * rev(a + a, P) % P;
10    a = normalize(-c * rev(a, P) + delta * delta, P);
11    if (powMod(a, P / 2, P) + 1 == P) return vector<int>(0);
12    for (t = 0, h = P / 2; h % 2 == 0; ++t, h /= 2);
13    r1 = powMod(a, h / 2, P);
14    if (t > 0) { do b = random() % (P - 2) + 2;
15        while (powMod(b, P / 2, P) + 1 != P); }
16    for (int i = 1; i <= t; ++i) {
17        LL d = r1 * r1 % P * a % P;
18        for (int j = 1; j <= t - i; ++j) d = d * d % P;
19        if (d + 1 == P) r1 = r1 * pb % P; pb = pb * pb % P;
20    } r1 = a * r1 % P; r2 = P - r1;
21    r1 = normalize(r1 - delta, P); r2 = normalize(r2 - delta, P);
22    if (r1 > r2) swap(r1, r2); vector<int> res(1, r1);
23    if (r1 != r2) res.push_back(r2);
24    return res;
25 }
```

5.13 N 次剩余

- 若 p 为奇质数, a 为 p 的 n 次剩余的充要条件是 $a^{\frac{p-1}{n}} \equiv 1 \pmod{p}$.

$x^N \equiv a \pmod{p}$, 其中 p 是质数

```
1 vector<int> solve(int p, int N, int a) {
2     if ((a % p) == 0) return vector<int>(1, 0);
3     int g = findPrimitiveRoot(p), m = modLog(g, a, p); // g ^ m = a (mod p)
4     if (m == -1) return vector<int>(0);
5     LL B = p - 1, x, y, d; exgcd(N, B, x, y, d);
6     if (m % d != 0) return vector<int>(0);
7     vector<int> ret; x = (x * (m / d) % B + B) % B; // g ^ B mod p = g ^ (p - 1) mod p = 1
8     for (int i = 0, delta = B / d; i < d; ++i) {
9         x = (x + delta) % B; ret.push_back((int)powMod(g, x, p));
10    } sort(ret.begin(), ret.end());
11    ret.resize(unique(ret.begin(), ret.end()) - ret.begin());
12    return ret;
13 }
```

5.14 Romberg 积分

```
1 template <class T> double Romberg(const T&f, double a, double b, double eps = 1e-8) {
2     vector<double> t; double h = b - a, last, now; int k = 1, i = 1;
3     t.push_back(h * (f(a) + f(b)) / 2); // 梯形
4     do {
```

```

5   last = t.back(); now = 0; double x = a + h / 2;
6   for (int j = 0; j < k; ++j, x += h) now += f(x);
7   now = (t[0] + h * now) / 2; double k1 = 4.0 / 3.0, k2 = 1.0 / 3.0;
8   for (int j = 0; j < i; ++j, k1 = k2 + 1) {
9       double tmp = k1 * now - k2 * t[j];
10      t[j] = now; now = tmp; k2 /= 4 * k1 - k2; // 防止溢出
11  } t.push_back(now); k *= 2; h /= 2; ++i;
12  } while (fabs(last - now) > eps);
13  return t.back();
14  }

```

5.15 公式

5.15.1 级数与三角

- $\sum_{k=1}^n k^3 = \left(\frac{n(n+1)}{2}\right)^2$
- $\sum_{k=1}^n k^4 = \frac{n(n+1)(2n+1)(3n^2+3n-1)}{30}$
- $\sum_{k=1}^n k^5 = \frac{n^2(n+1)^2(2n^2+2n-1)}{12}$
- $\sum_{k=1}^n k(k+1) = \frac{n(n+1)(n+2)}{3}$
- $\sum_{k=1}^n k(k+1)(k+2) = \frac{n(n+1)(n+2)(n+3)}{4}$
- $\sum_{k=1}^n k(k+1)(k+2)(k+3) = \frac{n(n+1)(n+2)(n+3)(n+4)}{5}$
- 错排: $D_n = n!(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + \frac{(-1)^n}{n!}) = (n-1)(D_{n-2} - D_{n-1})$
- $\sin(\alpha \pm \beta) = \sin \alpha \cos \beta \pm \cos \alpha \sin \beta$
- $\cos(\alpha \pm \beta) = \cos \alpha \cos \beta \mp \sin \alpha \sin \beta$
- $\tan(\alpha \pm \beta) = \frac{\tan \alpha \pm \tan \beta}{1 \mp \tan \alpha \tan \beta}$
- $\tan \alpha \pm \tan \beta = \frac{\sin(\alpha \pm \beta)}{\cos \alpha \cos \beta}$
- $\sin \alpha + \sin \beta = 2 \sin \frac{\alpha+\beta}{2} \cos \frac{\alpha-\beta}{2}$
- $\sin \alpha - \sin \beta = 2 \cos \frac{\alpha+\beta}{2} \sin \frac{\alpha-\beta}{2}$
- $\cos \alpha + \cos \beta = 2 \cos \frac{\alpha+\beta}{2} \cos \frac{\alpha-\beta}{2}$
- $\cos \alpha - \cos \beta = -2 \sin \frac{\alpha+\beta}{2} \sin \frac{\alpha-\beta}{2}$
- $\cos n\alpha = \binom{n}{0} \cos^n \alpha - \binom{n}{2} \cos^{n-2} \alpha \sin^2 \alpha + \binom{n}{4} \cos^{n-4} \alpha \sin^4 \alpha \dots$
- $\sin n\alpha = \binom{n}{1} \cos^{n-1} \alpha \sin \alpha - \binom{n}{3} \cos^{n-3} \alpha \sin^3 \alpha + \binom{n}{5} \cos^{n-5} \alpha \sin^5 \alpha \dots$

- $\sum_{n=1}^N \cos nx = \frac{\sin(N+\frac{1}{2})x - \sin \frac{x}{2}}{2 \sin \frac{x}{2}}$
- $\sum_{n=1}^N \sin nx = \frac{-\cos(N+\frac{1}{2})x + \cos \frac{x}{2}}{2 \sin \frac{x}{2}}$
- $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} \dots$ for $x \in (-\infty, +\infty)$
- $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} \dots$ for $x \in (-\infty, +\infty)$
- $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} \dots$ for $x \in (-\infty, +\infty)$
- $\arcsin x = x + \sum_{n=1}^{\infty} \frac{(2n-1)!!}{(2n)!!} \frac{x^{2n+1}}{2n+1}$ for $x \in [-1, 1]$
- $\arccos x = \frac{\pi}{2} - \sum_{n=1}^{\infty} \frac{(2n-1)!!}{(2n)!!} \frac{x^{2n+1}}{2n+1}$ for $x \in [-1, 1]$
- $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} \dots$ for $x \in [-1, 1]$
- $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} \dots$ for $x \in (-1, 1]$
- $\int_0^{\frac{\pi}{2}} \sin^n x dx = \begin{cases} \frac{(n-1)!!}{n!!} \times \frac{\pi}{2} & n \text{ 是偶数} \\ \frac{(n-1)!!}{n!!} & n \text{ 是奇数} \end{cases}$
- $\int_0^{+\infty} \frac{\sin x}{x} dx = \frac{\pi}{2}$
- $\int_0^{+\infty} e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$
- 傅里叶级数: 设周期为 $2T$. 函数分段连续. 在不连续点的值为左右极限的平均数.

$$- a_n = \frac{1}{T} \int_{-T}^T f(x) \cos \frac{n\pi}{T} x dx$$

$$- b_n = \frac{1}{T} \int_{-T}^T f(x) \sin \frac{n\pi}{T} x dx$$

$$- f(x) = \frac{a_0}{2} + \sum_{n=1}^{+\infty} (a_n \cos \frac{n\pi}{T} x + b_n \sin \frac{n\pi}{T} x)$$

$$\bullet \text{ Beta 函数: } B(p, q) = \int_0^1 x^{p-1} (1-x)^{q-1} dx$$

- 定义域 $(0, +\infty) \times (0, +\infty)$, 在定义域上连续

$$- B(p, q) = B(q, p) = \frac{q-1}{p+q-1} B(p, q-1) = 2 \int_0^{\frac{\pi}{2}} \cos^{2p-1} \phi \sin^{2q-1} \phi d\phi = \int_0^{+\infty} \frac{t^{q-1}}{(1+t)^{p+q}} dt = \int_0^1 \frac{t^{p-1} + t^{q-1}}{(1+t)^{p+q}} dt$$

$$-B\left(\frac{1}{2}, \frac{1}{2}\right) = \pi$$

- Gamma 函数: $\Gamma = \int_0^{+\infty} x^{s-1} e^{-x} dx$
 - 定义域 $(0, +\infty)$, 在定义域上连续
 - $\Gamma(1) = 1, \Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$
 - $\Gamma(s) = (s-1)\Gamma(s-1)$
 - $B(p, q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$
 - $\Gamma(s)\Gamma(1-s) = \frac{\pi}{\sin \pi s}$ for $s > 0$
 - $\Gamma(s)\Gamma\left(s + \frac{1}{2}\right) = 2\sqrt{\pi} \frac{\Gamma(s)}{2^{2s-1}}$ for $0 < s < 1$

	$y = f(x)$	$x = x(t), y = y(t), t \in [T_1, T_2]$	$r = r(\theta), \theta \in [\alpha, \beta]$
平面图形面积	$\int_a^b f(x) dx$	$\int_{T_1}^{T_2} y(t)x'(t) dt$	$\frac{1}{2} \int_{\alpha}^{\beta} r^2(\theta) d\theta$
曲线弧长	$\int_a^b \sqrt{1+f'^2(x)} dx$	$\int_{T_1}^{T_2} \sqrt{x'^2(t) + y'^2(t)} dt$	$\int_{\alpha}^{\beta} \sqrt{r^2(\theta) + r'^2(\theta)} d\theta$
旋转体体积	$\pi \int_a^b f^2(x) dx$	$\pi \int_{T_1}^{T_2} x'(t) y^2(t) dt$	$\frac{2}{3} \pi \int_{\alpha}^{\beta} r^3(\theta) \sin \theta d\theta$
旋转曲面面积	$2\pi \int_a^b f(x) \sqrt{1+f'^2(x)} dx$	$2\pi \int_{T_1}^{T_2} y(t) \sqrt{x'^2(t) + y'^2(t)} dt$	$2\pi \int_{\alpha}^{\beta} r(\theta) \sin \theta \sqrt{r^2(\theta) + r'^2(\theta)} d\theta$

5.15.2 三次方程求根公式

对一元三次方程 $x^3 + px + q = 0$, 令

$$A = \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}$$

$$B = \sqrt[3]{-\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}}$$

$$\omega = \frac{(-1 + i\sqrt{3})}{2}$$

则 $x_j = A\omega^j + B\omega^{2j}$ ($j = 0, 1, 2$).

当求解 $ax^3 + bx^2 + cx + d = 0$ 时, 令 $x = y - \frac{b}{3a}$, 再求解 y , 即转化为 $y^3 + py + q = 0$ 的形式. 其中,

$$p = \frac{b^2 - 3ac}{3a^2}$$

$$q = \frac{2b^3 - 9abc + 27a^2d}{27a^3}$$

卡尔丹判别法: 令 $\Delta = \left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3$. 当 $\Delta > 0$ 时, 有一个实根和一对共轭虚根; 当 $\Delta = 0$ 时, 有三个实根, 其中两个相等; 当 $\Delta < 0$ 时, 有三个不相等的实根.

5.15.3 椭圆

- 椭圆 $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, 其中离心率 $e = \frac{c}{a}, c = \sqrt{a^2 - b^2}$; 焦点参数 $p = \frac{b^2}{a}$
- 椭圆上 (x, y) 点处的曲率半径为 $R = a^2 b^2 \left(\frac{x^2}{a^4} + \frac{y^2}{b^4}\right)^{\frac{3}{2}} = \frac{(r_1 r_2)^{\frac{3}{2}}}{ab}$, 其中 r_1 和 r_2 分别为 (x, y) 与两焦点 F_1 和 F_2 的距离. 设点 A 和点 M 的坐标分别为 $(a, 0)$ 和 (x, y) , 则 AM 的弧长为

$$L_{AM} = a \int_0^{\arccos \frac{x}{a}} \sqrt{1 - e^2 \cos^2 t} dt = a \int_{\arccos \frac{x}{a}}^{\frac{\pi}{2}} \sqrt{1 - e^2 \sin^2 t} dt$$

- 椭圆的周长 $L = 4a \int_0^{\frac{\pi}{2}} \sqrt{1 - e^2 \sin^2 t} dt = 4aE(e, \frac{\pi}{2})$, 其中

$$E(e, \frac{\pi}{2}) = \frac{\pi}{2} \left[1 - \left(\frac{1}{2}\right)^2 e^2 - \left(\frac{1 \times 3}{2 \times 4}\right)^2 \frac{e^4}{3} - \left(\frac{1 \times 3 \times 5}{2 \times 4 \times 6}\right)^2 \frac{e^6}{5} - \dots \right]$$

- 设椭圆上点 $M(x, y), N(x, -y), x, y > 0, A(a, 0)$, 原点 $O(0, 0)$, 扇形 OAM 的面积 $S_{OAM} = \frac{1}{2} ab \arccos \frac{x}{a}$, 弓形 MAN 的面积 $S_{MAN} = ab \arccos \frac{x}{a} - xy$.

- 需要 5 个点才能确定一个圆锥曲线.

- 设 θ 为 (x, y) 点关于椭圆中心的极角, r 为 (x, y) 到椭圆中心的距离, 椭圆极坐标方程:

$$x = r \cos \theta, y = r \sin \theta, r^2 = \frac{b^2 a^2}{b^2 \cos^2 \theta + a^2 \sin^2 \theta}$$

5.15.4 抛物线

- 标准方程 $y^2 = 2px$, 曲率半径 $R = \frac{(p + 2x)^{\frac{3}{2}}}{\sqrt{p}}$
- 弧长: 设 $M(x, y)$ 是抛物线上一点, 则 $L_{OM} = \frac{p}{2} \left[\sqrt{\frac{2x}{p} \left(1 + \frac{2x}{p}\right)} + \ln \left(\sqrt{\frac{2x}{p}} + \sqrt{1 + \frac{2x}{p}} \right) \right]$
- 弓形面积: 设 M, D 是抛物线上两点, 且分居一, 四象限. 做一条平行于 MD 且与抛物线相切的直线 L . 若 M 到 L 的距离为 h . 则有 $S_{MOD} = \frac{2}{3} MD \cdot h$.

5.15.5 重心

- 半径 r , 圆心角为 θ 的扇形的重心与圆心的距离为 $\frac{4r \sin \frac{\theta}{2}}{3\theta}$
- 半径 r , 圆心角为 θ 的圆弧的重心与圆心的距离为 $\frac{4r \sin^3 \frac{\theta}{2}}{3(\theta - \sin \theta)}$
- 椭圆上半部分的重心与圆心的距离为 $\frac{4b}{3\pi}$
- 抛物线中弓形 MOD 的重心满足 $CQ = \frac{2}{5} PQ$, P 是直线 L 与抛物线的切点, Q 在 MD 上且 PQ 平行 x 轴, C 是重心

5.15.6 向量恒等式

- $\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} \times \vec{a}) = \vec{c} \cdot (\vec{a} \times \vec{b})$
- $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{c} \times \vec{b}) \times \vec{a} = \vec{b}(\vec{a} \cdot \vec{c}) - \vec{c}(\vec{a} \cdot \vec{b})$

5.15.7 常用几何公式

- 三角形的五心
 - 重心 $\vec{G} = \frac{\vec{A} + \vec{B} + \vec{C}}{3}$
 - 内心 $\vec{I} = \frac{a\vec{A} + b\vec{B} + c\vec{C}}{a+b+c}$, $R = \frac{2S}{a+b+c}$
 - 外心 $\vec{O} = \frac{\vec{A} + \vec{B} + \frac{\vec{A}\vec{C} \cdot \vec{B}\vec{C}}{\vec{AB} \times \vec{BC}} \vec{AB}^T}{2}$, $R = \frac{abc}{4S}$
 - 垂心 $\vec{H} = 3\vec{G} - 2\vec{O} = \vec{C} + \frac{\vec{B}\vec{C} \cdot \vec{A}\vec{C}}{\vec{BC} \times \vec{AC}} \vec{AB}^T$
 - 旁心 (三个) $\frac{-a\vec{A} + b\vec{B} + c\vec{C}}{-a+b+c}$
- 四边形: 设 D_1, D_2 为对角线, M 为对角线中点连线, A 为对角线夹角
 - $a^2 + b^2 + c^2 + d^2 = D_1^2 + D_2^2 + 4M^2$
 - $S = \frac{1}{2}D_1D_2 \sin A$
 - $ac + bd = D_1D_2$ (内接四边形适用)
 - Bretschneider 公式: $S = \sqrt{(p-a)(p-b)(p-c)(p-d) - abcd \cos^2(\frac{\theta}{2})}$, 其中 θ 为对角和
- 棱锥:
 - 体积 $V = \frac{1}{3}Ah$, A 为底面积, h 为高
 - (对正棱锥) 侧面积 $S = \frac{1}{2}lp$, l 为斜高, p 为底面周长
- 棱台:
 - 体积 $V = \frac{(A_1 + A_2 + \sqrt{A_1A_2}) \cdot h}{3}$, A_1, A_2 分别为上下底面面积, h 为高
 - (对正棱台) 侧面积 $S = \frac{1}{2}(p_1 + p_2) \cdot l$, p_1, p_2 为上下底面周长, l 为斜高.

5.15.8 树的计数

- 有根数计数: 令 $S_{n,j} = \sum_{1 \leq i \leq n/j} a_{n+1-ij} = S_{n-j,j} + a_{n+1-j}$

于是, $n+1$ 个结点的有根数的总数为 $a_{n+1} = \frac{\sum_{1 \leq j \leq n} j \cdot a_j \cdot S_{n,j}}{n}$

附: $a_1 = 1, a_2 = 1, a_3 = 2, a_4 = 4, a_5 = 9, a_6 = 20, a_9 = 286, a_{11} = 1842$

- 无根树计数: 当 n 是奇数时, 则有 $a_n - \sum_{1 \leq i \leq \frac{n}{2}} a_i a_{n-i}$ 种不同的无根树
- 当 n 是偶数时, 则有 $a_n - \sum_{1 \leq i \leq \frac{n}{2}} a_i a_{n-i} + \frac{1}{2} a_{\frac{n}{2}} (a_{\frac{n}{2}} + 1)$ 种不同的无根树

- Matrix-Tree 定理: 对任意图 G , 设 $\text{mat}[i][i] = i$ 的度数, $\text{mat}[i][j] = i$ 与 j 之间边数的相反数, 则 $\text{mat}[i][j]$ 的任意余子式的行列式就是该图的生成树个数

5.16 小知识

- 勾股数: 设正整数 n 的质因数分解为 $n = \prod p_i^{a_i}$, 则 $x^2 + y^2 = n$ 有整数解的充要条件是 n 中不存在形如 $p_i \equiv 3 \pmod{4}$ 且指数 a_i 为奇数的质因数 p_i
- 勾股数 2:

```

a[0] := 0;
s := 0;
for i := 1 to n - 2 do
  begin
    a[i] := a[i - 1] + 1;
    s := s + sqr(a[i]);
  end;
{=====s + sqr(a[n-1]) + sqr(a[n]) = k^2=====}
a[n - 1] := a[n - 2];
repeat
  a[n - 1] := a[n - 1] + 1;
until odd(s + sqr(a[n - 1])) and (a[n - 1] > 2);
a[n] := (s + sqr(a[n - 1]) - 1) shr 1;

```

知道 s 和 $a[n-1]$ 后, 直接求了 $a[n]$. 神奇了点.

其实, 有当 n 为奇数: $n^2 + \lfloor \frac{n^2-1}{2} \rfloor^2 = \lfloor \frac{n^2+1}{2} \rfloor^2$

若:

$$a = k \cdot (s^2 - t^2)$$

$$b = 2 \cdot k \cdot s \cdot t$$

$$c = k \cdot (s^2 + t^2)$$

$$\text{则 } c^2 = a^2 + b^2.$$

- Stirling 公式: $n! \approx \sqrt{2\pi n} (\frac{n}{e})^n$

- Pick 定理: 简单多边形, 不自交, 顶点如果全是整点. 则: 严格在多边形内部的整点数 + $\frac{1}{2}$ 在边上的整点数 $- 1 =$ 面积
- Mersenne 素数: p 是素数且 $2^p - 1$ 的数是素数. (10000 以内的 p 有: 2, 3, 5, 7, 13, 17, 19, 31, 61, 89, 107, 127, 521, 607, 1279, 2203, 2281, 3217, 4253, 4423, 9689, 9941)
- 序列差分表: 差分表的第 0 条对角线确定原序列. 设原序列为 h_i , 第 0 条对角线为 $c_0, c_1, \dots, c_p, 0, 0, \dots$. 有这样两个公式: $h_n = \binom{n}{0}c_0 + \binom{n}{1}c_1 + \dots + \binom{n}{p}c_p$, $\sum_{k=0}^n h_k = \binom{n+1}{1}c_0 + \binom{n+1}{2}c_1 + \dots + \binom{n+1}{p+1}c_p$
- Fibonacci 数相关:
 - $\gcd(F_n, F_m) = F_{\gcd(n, m)}$
 - 如果 a 是 b 的倍数, 那么 F_a 是 F_b 的倍数
- GCD: $\gcd(2^a - 1, 2^b - 1) = 2^{\gcd(a, b)} - 1$
- Fermat 分解算法: 从 $t = \sqrt{n}$ 开始, 依次检查 $t^2 - n, (t+1)^2 - n, (t+2)^2 - n, \dots$, 直到出现一个平方数 y , 由于 $t^2 - y^2 = n$, 因此分解得 $n = (t-y)(t+y)$. 显然, 当两个因数很接近时这个方法能很快找到结果, 但如果遇到一个素数, 则需要检查 $\frac{n+1}{2} - \sqrt{n}$ 个整数
- 牛顿迭代: $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$
- 球与盒子的动人故事: (n 个球, m 个盒子, S 为第二类斯特林数)
 1. 球同, 盒同, 无空: dp
 2. 球同, 盒同, 可空: dp
 3. 球同, 盒不同, 无空: $\binom{n-1}{m-1}$
 4. 球同, 盒不同, 可空: $\binom{n+m-1}{n-1}$
 5. 球不同, 盒同, 无空: $S(n, m)$
 6. 球不同, 盒同, 可空: $\sum_{k=1}^m S(n, k)$
 7. 球不同, 盒不同, 无空: $m!S(n, m)$
 8. 球不同, 盒不同, 可空: m^n
- 组合数:
 - Lucas 定理: 对质数 p , 设 $n = n_k p^k + n_{k-1} p^{k-1} + \dots + n_1 p + n_0, m = m_k p^k + m_{k-1} p^{k-1} + \dots + m_1 p + m_0$, 则 $\binom{n}{m} \equiv \prod \binom{n_i}{m_i} \pmod{p}$
 - 组合数判断奇偶性: 若 $(n \& m) = m$, 则 $\binom{n}{m}$ 为奇数, 否则为偶数
- 格雷码 $G(x) = x \otimes (x \gg 1)$

- Bell 数: B_n 代表将 n 个元素划分成若干个非空集合的方案数

- $B_0 = B_1 = 1, B_n = \sum_{k=0}^{n-1} \binom{n-1}{k} B_k$
- $B_n = \sum_{k=0}^n \{n \atop k\}$
- Bell 三角形: $a_{1,1} = 1, a_{n,1} = a_{n-1,n-1}, a_{n,m} = a_{n,m-1} + a_{n-1,m-1}, B_n = a_{n,1}$
- 对质数 $p, B_{n+p} \equiv B_n + B_{n+1} \pmod{p}$
- 对质数 $p, B_{n+p^m} \equiv m B_n + B_{n+1} \pmod{p}$
- 对质数 p , 模的周期一定是 $\frac{p^p-1}{p-1}$ 的约数, $p \leq 101$ 时就是这个值
- 从 B_0 开始, 前几项是 1, 1, 2, 5, 15, 52, 203, 877, 4140, 21147, 115975 \dots

6 其他

6.1 Extended LIS

```

1 int G[MAXN][MAXN];
2 void insertYoung(int v) {
3     for (int x = 1, y = INT_MAX; ; ++x) {
4         Down(y, *G[x]); while (y > 0 && G[x][y] >= v) --y;
5         if (++y > *G[x]) { ++*G[x]; G[x][y] = v; break; }
6         else swap(G[x][y], v);
7     }
8 }
9 int solve(int N, int seq[]) {
10     Rep(i, 1, N) *G[i] = 0;
11     Rep(i, 1, N) insertYoung(seq[i]);
12     printf("%d\n", *G[1] + *G[2]);
13     return 0;
14 }

```

6.2 生成 nCk

```

1 void nCk(int n, int k) {
2     for (int comb = (1 << k) - 1; comb < (1 << n); ) {
3         int x = comb & -comb, y = comb + x;
4         comb = (((comb & -y) / x) >> 1) | y;
5     }
6 }

```

6.3 nextPermutation

```

1 boolean nextPermutation(int[] is) {
2     int n = is.length;
3     for (int i = n - 1; i > 0; i--) {
4         if (is[i - 1] < is[i]) {
5             int j = n; while (is[i - 1] >= is[--j]);
6             swap(is, i - 1, j); // swap is[i - 1], is[j]
7             rev(is, i, n); // reverse is[i, n)
8             return true;
9         }
10    } rev(is, 0, n);
11    return false;
12 }

```

6.4 Josephus 数与逆 Josephus 数

```

1 int josephus(int n, int m, int k) { int x = -1;
2     for (int i = n - k + 1; i <= n; i++) x = (x + m) % i;
3     return x;
4 }
5 int invJosephus(int n, int m, int x) {
6     for (int i = n; ; i--) { if (x == i) return n - i;
7         x = (x - m % i + i) % i;
8     }
9 }

```

6.5 表达式求值

```

1 inline int getLevel(char ch) {
2     switch (ch) { case '+': case '-': return 0; case '*': return 1; } return -1;
3 }
4 int evaluate(char *&p, int level) {
5     int res;
6     if (level == 2) {
7         if (*p == '(') ++p, res = evaluate(p, 0);
8         else res = isdigit(*p) ? *p - '0' : value[*p - 'a'];
9         ++p; return res;
10    } res = evaluate(p, level + 1);
11    for (int next; *p && getLevel(*p) == level; ) {
12        char op = *p++; next = evaluate(p, level + 1);
13        switch (op) {
14            case '+': res += next; break;
15            case '-': res -= next; break;
16            case '*': res *= next; break;
17        }
18    } return res;
19 }
20 int makeEvaluation(char *str) { char *p = str; return evaluate(p, 0); }

```

6.6 曼哈顿最小生成树

```

1 const int INF = 1000000005;
2 struct TreeEdge {
3     int x, y, z; void make(int _x, int _y, int _z) { x = _x; y = _y; z = _z; }
4 } data[maxn * 4];
5 int n, x[maxn], y[maxn], px[maxn], py[maxn], id[maxn], tree[maxn], node[maxn], val[maxn], fa[
    maxn];
6 bool operator < (const TreeEdge& x, const TreeEdge& y) { return x.z < y.z; }
7 bool cmp1(int a, int b) { return x[a] < x[b]; }
8 bool cmp2(int a, int b) { return y[a] < y[b]; }
9 bool cmp3(int a, int b) { return (y[a] - x[a] < y[b] - x[b] || (y[a] - x[a] == y[b] - x[b] &&
    y[a] > y[b])); }
10 bool cmp4(int a, int b) { return (y[a] - x[a] > y[b] - x[b] || (y[a] - x[a] == y[b] - x[b] &&
    x[a] > x[b])); }
11 bool cmp5(int a, int b) { return (x[a] + y[a] > x[b] + y[b] || (x[a] + y[a] == x[b] + y[b] &&
    x[a] < x[b])); }
12 bool cmp6(int a, int b) { return (x[a] + y[a] < x[b] + y[b] || (x[a] + y[a] == x[b] + y[b] &&
    y[a] > y[b])); }
13 void Change_X() {
14     for (int i = 0; i < n; ++i) val[i] = x[i];
15     for (int i = 0; i < n; ++i) id[i] = i;
16     sort(id, id + n, cmp1);
17     int cntM = 1, last = val[id[0]]; px[id[0]] = 1;
18     for (int i = 1; i < n; ++i) {
19         if (val[id[i]] > last) ++cntM, last = val[id[i]];
20         px[id[i]] = cntM;
21     }
22 }
23 void Change_Y() {
24     for (int i = 0; i < n; ++i) val[i] = y[i];
25     for (int i = 0; i < n; ++i) id[i] = i;
26     sort(id, id + n, cmp2);
27     int cntM = 1, last = val[id[0]]; py[id[0]] = 1;
28     for (int i = 1; i < n; ++i) {
29         if (val[id[i]] > last)
30             ++cntM, last = val[id[i]];
31         py[id[i]] = cntM;
32     }
33 }
34 inline int Cost(int a, int b) { return abs(x[a] - x[b]) + abs(y[a] - y[b]); }
35 int find(int x) { return (fa[x] == x) ? x : (fa[x] = find(fa[x])); }
36 int main() {
37     for (int i = 0; i < n; ++i) scanf("%d%d", x + i, y + i);
38     Change_X(); Change_Y();
39     int cntE = 0; for (int i = 0; i < n; ++i) id[i] = i;
40     sort(id, id + n, cmp3);
41     for (int i = 1; i <= n; ++i) tree[i] = INF, node[i] = -1;
42     for (int i = 0; i < n; ++i) {
43         int Min = INF, Tnode = -1;
44         for (int k = py[id[i]]; k <= n; k += k & (-k))
45             if (tree[k] < Min) Min = tree[k], Tnode = node[k];

```

```

46     if (Tnode >= 0) data[cntE++].make(id[i], Tnode, Cost(id[i], Tnode));
47     int tmp = x[id[i]] + y[id[i]];
48     for (int k = py[id[i]]; k; k -= k & (-k))
49         if (tmp < tree[k]) tree[k] = tmp, node[k] = id[i];
50 } sort(id, id + n, cmp4);
51 for (int i = 1; i <= n; ++i) tree[i] = INF, node[i] = -1;
52 for (int i = 0; i < n; ++i) {
53     int Min = INF, Tnode = -1;
54     for (int k = px[id[i]]; k <= n; k += k & (-k))
55         if (tree[k] < Min) Min = tree[k], Tnode = node[k];
56     if (Tnode >= 0) data[cntE++].make(id[i], Tnode, Cost(id[i], Tnode));
57     int tmp = x[id[i]] + y[id[i]];
58     for (int k = px[id[i]]; k; k -= k & (-k))
59         if (tmp < tree[k]) tree[k] = tmp, node[k] = id[i];
60 }
61 sort(id, id + n, cmp5);
62 for (int i = 1; i <= n; ++i) tree[i] = INF, node[i] = -1;
63 for (int i = 0; i < n; ++i) {
64     int Min = INF, Tnode = -1;
65     for (int k = px[id[i]]; k; k -= k & (-k))
66         if (tree[k] < Min) Min = tree[k], Tnode = node[k];
67     if (Tnode >= 0) data[cntE++].make(id[i], Tnode, Cost(id[i], Tnode));
68     int tmp = -x[id[i]] + y[id[i]];
69     for (int k = px[id[i]]; k <= n; k += k & (-k))
70         if (tmp < tree[k]) tree[k] = tmp, node[k] = id[i];
71 } sort(id, id + n, cmp6);
72 for (int i = 1; i <= n; ++i) tree[i] = INF, node[i] = -1;
73 for (int i = 0; i < n; ++i) {
74     int Min = INF, Tnode = -1;
75     for (int k = py[id[i]]; k <= n; k += k & (-k))
76         if (tree[k] < Min) Min = tree[k], Tnode = node[k];
77     if (Tnode >= 0) data[cntE++].make(id[i], Tnode, Cost(id[i], Tnode));
78     int tmp = -x[id[i]] + y[id[i]];
79     for (int k = py[id[i]]; k; k -= k & (-k))
80         if (tmp < tree[k]) tree[k] = tmp, node[k] = id[i];
81 }
82 long long Ans = 0; sort(data, data + cntE);
83 for (int i = 0; i < n; ++i) fa[i] = i;
84 for (int i = 0; i < cntE; ++i) if (find(data[i].x) != find(data[i].y)) {
85     Ans += data[i].z;
86     fa[fa[data[i].x]] = fa[data[i].y];
87 } cout << Ans << endl;
88 }

```

6.7 直线下的整点个数

$$\text{求 } \sum_{i=0}^{n-1} \lfloor \frac{a+bi}{m} \rfloor$$

```

1 LL count(LL n, LL a, LL b, LL m) {
2     if (b == 0) return n * (a / m);
3     if (a >= m) return n * (a / m) + count(n, a % m, b, m);

```

```

4     if (b >= m) return (n - 1) * n / 2 * (b / m) + count(n, a, b % m, m);
5     return count((a + b * n) / m, (a + b * n) % m, m, b);
6 }

```

6.8 Java 分数类

```

1 class Polynomial {
2     final static Polynomial ZERO = new Polynomial(new int[] { 0 });
3     final static Polynomial ONE = new Polynomial(new int[] { 1 });
4     final static Polynomial X = new Polynomial(new int[] { 0, 1 });
5     int[] coef;
6     static Polynomial valueOf(int val) { return new Polynomial(new int[] { val }); }
7     Polynomial(int[] coef) { this.coef = Arrays.copyOf(coef, coef.length); }
8     Polynomial add(Polynomial o, int mod); // omitted
9     Polynomial subtract(Polynomial o, int mod); // omitted
10    Polynomial multiply(Polynomial o, int mod); // omitted
11    Polynomial scale(int o, int mod); // omitted
12    public String toString() {
13        int n = coef.length; String ret = "";
14        for (int i = n - 1; i > 0; --i) if (coef[i] != 0)
15            ret += coef[i] + "x" + i + "+";
16        return ret + coef[0];
17    }
18    static Polynomial lagrangeInterpolation(int[] x, int[] y, int mod) {
19        int n = x.length; Polynomial ret = Polynomial.ZERO;
20        for (int i = 0; i < n; ++i) {
21            Polynomial poly = Polynomial.valueOf(y[i]);
22            for (int j = 0; j < n; ++j) if (i != j) {
23                poly = poly.multiply(
24                    Polynomial.X.subtract(Polynomial.valueOf(x[j]), mod), mod);
25                poly = poly.scale(powMod(x[i] - x[j] + mod, mod - 2, mod), mod);
26            } ret = ret.add(poly, mod);
27        } return ret;
28    }
29 }

```

6.9 long long 乘法取模

```

1 LL multiplyMod(LL a, LL b, LL P) { // 需要保证 a 和 b 非负
2     LL t = (a * b - LL((long double)a / P * b + 1e-3) * P) % P;
3     return t < 0 : t + P : t;
4 }

```

6.10 重复覆盖

```

1 namespace DLX {
2     struct node { int x, y; node *l, *r, *u, *d; } base[MAX * MAX], *top, *head;

```

```

3  typedef node *link;
4  int row, col, nGE, ans, stamp, cntc[MAX], vis[MAX];
5  vector<link> eachRow[MAX], eachCol[MAX];
6  inline void addElement(int x, int y) {
7      top->x = x; top->y = y; top->l = top->r = top->u = top->d = NULL;
8      eachRow[x].push_back(top); eachCol[y].push_back(top++);
9  }
10 void init(int _row, int _col, int _nGE) {
11     row = _row; col = _col; nGE = _nGE; top = base; stamp = 0;
12     for (int i = 0; i <= col; ++i) vis[i] = 0;
13     for (int i = 0; i <= row; ++i) eachRow[i].clear();
14     for (int i = 0; i <= col; ++i) eachCol[i].clear();
15     for (int i = 0; i <= col; ++i) addElement(0, i);
16     head = eachCol[0].front();
17 }
18 void build() {
19     for (int i = 0; i <= row; ++i) {
20         vector<link> &v = eachRow[i];
21         sort(v.begin(), v.end(), cmpByY);
22         int s = v.size();
23         for (int j = 0; j < s; ++j) {
24             link l = v[j], r = v[(j + 1) % s]; l->r = r; r->l = l;
25         }
26     }
27     for (int i = 0; i <= col; ++i) {
28         vector<link> &v = eachCol[i];
29         sort(v.begin(), v.end(), cmpByX);
30         int s = v.size();
31         for (int j = 0; j < s; ++j) {
32             link u = v[j], d = v[(j + 1) % s]; u->d = d; d->u = u;
33         }
34     } for (int i = 0; i <= col; ++i) cntc[i] = (int) eachCol[i].size() - 1;
35 }
36 void removeExact(link c) {
37     c->l->r = c->r; c->r->l = c->l;
38     for (link i = c->d; i != c; i = i->d)
39         for (link j = i->r; j != i; j = j->r) {
40             j->d->u = j->u; j->u->d = j->d; --cntc[j->y];
41         }
42 }
43 void resumeExact(link c) {
44     for (link i = c->u; i != c; i = i->u)
45         for (link j = i->l; j != i; j = j->l) {
46             j->d->u = j; j->u->d = j; ++cntc[j->y];
47         }
48     c->l->r = c; c->r->l = c;
49 }
50 void removeRepeat(link c) {
51     for (link i = c->d; i != c; i = i->d) {
52         i->l->r = i->r; i->r->l = i->l;
53     }
54 }
55 void resumeRepeat(link c) {

```

```

56     for (link i = c->u; i != c; i = i->u) {
57         i->l->r = i; i->r->l = i;
58     }
59 }
60 int calcH() {
61     int y, res = 0; ++stamp;
62     for (link c = head->r; (y = c->y) <= row && c != head; c = c->r) {
63         if (vis[y] != stamp) {
64             vis[y] = stamp; ++res;
65             for (link i = c->d; i != c; i = i->d)
66                 for (link j = i->r; j != i; j = j->r) vis[j->y] = stamp;
67         }
68     } return res;
69 }
70 void DFS(int dep) { if (dep + calcH() >= ans) return;
71     if (head->r->y > nGE || head->r == head) {
72         if (ans > dep) ans = dep; return;
73     } link c = NULL;
74     for (link i = head->r; i->y <= nGE && i != head; i = i->r)
75         if (!c || cntc[i->y] < cntc[c->y]) c = i;
76     for (link i = c->d; i != c; i = i->d) {
77         removeRepeat(i);
78         for (link j = i->r; j != i; j = j->r) if (j->y <= nGE) removeRepeat(j);
79         for (link j = i->r; j != i; j = j->r) if (j->y > nGE) removeExact(base + j->y);
80         DFS(dep + 1);
81         for (link j = i->l; j != i; j = j->l) if (j->y > nGE) resumeExact(base + j->y);
82         for (link j = i->l; j != i; j = j->l) if (j->y <= nGE) resumeRepeat(j);
83         resumeRepeat(i);
84     }
85 }
86 int solve() { build(); ans = INF; DFS(0); return ans; }
87 }

```

6.11 星期几判定

```

1  int getDay(int y, int m, int d) {
2      if (m <= 2) m += 12, y--;
3      if (y < 1752 || (y == 1752 && m < 9) || (y == 1752 && m == 9 && d < 3))
4          return (d + 2 * m + 3 * (m + 1) / 5 + y + y / 4 + 5) % 7 + 1;
5      return (d + 2 * m + 3 * (m + 1) / 5 + y + y / 4 - y / 100 + y / 400) % 7 + 1;
6  }

```

6.12 LCSequence Fast

```

1  ULL *a, *b, *s, c, d;
2  for (i = 0, a = appear[(int)B[k]], b = row[max(k - 1, 0)], s = X; i < bitSetLen; ++i)
3      *s++ = *a++ | *b++; // X = row[i - 1] or appear[ B[i] ]
4  for (i = 0, a = dp, c = d = 0; i < bitSetLen; ++a, c = d, ++i)
5      d = *a >> 63, *a = ~(*a << 1) + c; // row[i] = ~(row[i] << 1) + 1

```



```

6 for (i = 0, a = dp, b = X, c = 0; i < bitSetLen; ++a, ++b, ++i)
7   d = *b + c, c = (*a >= -d), *a += d;    // row[i] += X
8 for (i = 0, a = dp, b = X; i < bitSetLen; ++a, ++b, ++i)
9   *a = (*a ^ *b) & *b;                  // row[i] = X and (row[i] xor X)

```

7 Templates

7.1 vim 配置

```

1 set nu ru nobk ci si
2 set mouse=a sw=4 sts=4 ts=4
3 set hlsearch incsearch
4 set whichwrap=b,s,<,>[,]
5 syntax on
6
7 nmap <C-A> ggVG
8 vmap <C-C> "+y
9
10 autocmd BufNewFile *.cpp _Or_ ~/Templates/cpp.cpp
11 map<F9>_ :!g++_ -o_ <_Wall_ -g3_<CR>
12 map<F5>_ :!./_%<CR>
13 map<F8>_ :!./_%<_<.in_<CR>
14
15 map<F3>_ :vnew_%<.in_<CR>
16 map<F4>_ :!(gedit_%&)<CR>

```

7.2 C++

```

1 #pragma comment(linker, "/STACK:10240000")
2 #include <cstdio>
3 #include <cstdlib>
4 #include <cstring>
5 #include <iostream>
6 #include <algorithm>
7 #define Rep(i, a, b) for(int i = (a); i <= (b); ++i)
8 #define Foru(i, a, b) for(int i = (a); i < (b); ++i)
9 using namespace std;
10 typedef long long LL;
11 typedef pair<int, int> pii;
12 #ifdef MAX_BUFFER
13 namespace BufferedReader {
14   char buff[MAX_BUFFER + 5], *ptr = buff, c; bool flag;
15   bool nextChar(char &c) {
16     if ( (c = *ptr++) == 0 ) {
17       int tmp = fread(buff, 1, MAX_BUFFER, stdin);
18       buff[tmp] = 0;
19       if (tmp == 0) return false;
20       ptr = buff; c = *ptr++;

```

```

21   } return true;
22 }
23 bool nextUnsignedInt(unsigned int &x) {
24   for ( ; ; ) {
25     if ( !nextChar(c) ) return false;
26     if ('0' <= c && c <= '9') break;
27   }
28   for (x = c - '0'; ; x = x * 10 + c - '0') {
29     if ( !nextChar(c) ) break;
30     if (c < '0' || c > '9') break;
31   } return true;
32 }
33 bool nextInt(int &x) {
34   for ( ; ; ) {
35     if ( !nextChar(c) ) return false;
36     if (c == '-' || ('0' <= c && c <= '9')) break;
37   }
38   for ((c == '-') ? (x = 0, flag = true) : (x = c - '0' : flag = false); ; ) {
39     if ( !nextChar(c) ) break;
40     if (c < '0' || c > '9') break;
41     x = x * 10 + c - '0';
42   } if (flag) x = -x;
43   return true;
44 }
45 };
46 #endif

```

7.3 Java

```

1 import java.io.*;
2 import java.util.*;
3 import java.math.*;
4
5 class Node implements Comparable<Node> {
6   int key;
7   public int compareTo(Node o) {
8     if (key != o.key) return key < o.key ? -1 : 1;
9     return 0;
10  }
11  public boolean equals(Object o) { return false; }
12  public String toString() { return ""; }
13  public int hashCode() { return key; }
14 }
15
16 class MyComparator implements Comparator<Node> {
17   public int compare(Node a, Node b) {
18     if (a.key != b.key)
19       return a.key < b.key ? -1 : 1;
20     return 0;
21   }
22 }

```

```
23
24 public class Main {
25
26     public void solve() {
27         PriorityQueue<Integer> Q = new PriorityQueue<Integer>();
28         Q.offer(1); Q.poll(); Q.peek(); Q.size();
29
30         HashMap<Node, Integer> dict = new HashMap<Node, Integer>();
31         dict.entrySet(); dict.put(new Node(), 0); dict.containsKey(new Node());
32         //Map.Entry e = (Map.Entry)it.next(); e.getValue(); e.getKey();
33
34         HashSet<Node> h = new HashSet<Node>();
35         h.contains(new Node()); h.add(new Node()); h.remove(new Node());
36
37         Random rand = new Random();
38         rand.nextInt(); rand.nextDouble();
39
40         int temp = 0;
41         BigInteger a = BigInteger.ZERO, b = new BigInteger("1"), c = BigInteger.valueOf(2);
42         a.remainder(b); a.modPow(b, c); a.pow(temp); a.intValue();
43         a.isProbablePrime(temp); // 1 - 1 / 2 ^ certainty
44         a.nextProbablePrime();
45
46         Arrays.asList(array);
47         Arrays.sort(array, fromIndex, toIndex, comparator);
48         Arrays.fill(array, fromIndex, toIndex, value);
49         Arrays.binarySearch(array, key, comparator); // found ? index : -(insertPoint) - 1
50         Arrays.equals(array, array2);
51         Collection.toArray(arrayType[]);
52
53         Collections.copy(dest, src);
54         Collections.fill(collection, value);
55         Collections.max(collection, comparator);
56         Collections.replaceAll(list, oldValue, newValue);
57         Collections.reverse(list);
58         Collections.reverseOrder();
59         Collections.rotate(list, distance); // ----->
60         Collections.shuffle(list); // random_shuffle
61
62     }
63
64     public void run() {
65         tokenizer = null;
66         reader = new BufferedReader(new InputStreamReader(System.in));
67         out = new PrintWriter(System.out);
68
69         solve();
70
71         out.close();
72     }
73
74     public static void main(String[] args) {
75         new Main().run();
```

```
76     }
77
78     public StringTokenizer tokenizer;
79     public BufferedReader reader;
80     public PrintWriter out;
81
82     public String next() {
83         while (tokenizer == null || !tokenizer.hasMoreTokens()) {
84             try {
85                 tokenizer = new StringTokenizer(reader.readLine());
86             }
87             catch (IOException e) {
88                 throw new RuntimeException(e);
89             }
90         }
91         return tokenizer.nextToken();
92     }
93
94 }
```