Improper Integrals are Undefined

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1 Introduction

Improper integrals are undefined.

2 Proof

Improper integrals are undefined by the definition of the integral. Let's first give integral definition:

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i) * \frac{b-a}{n}$$

Here dx is an infinitesimal and corresponds to $\frac{b-a}{n}$ where n reaches ∞.
An improper integral is:

$$\lim_{b\to\infty} \int_a^b f(x)\,dx \; = \int_a^\infty f(x)\,dx$$

Then the definition of an improper integral will be:

$$\lim_{b\to\infty} \int_a^b f(x)\,dx = \lim_{b\to\infty} \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i) * \frac{b-a}{n}$$

Then we have:

$$\sum_{i=1}^{\infty} f(x_i) * \frac{\infty - a}{\infty} \; = \sum_{i=1}^{\infty} f(x_i) * \frac{\infty}{\infty}$$

Here ∞/∞ does not correspond to dx or any other mathematical entity and is an indeterminate form. Therefore, improper integrals are undefined.

Improper integrals are extensively used in physics, other engineering and math disciplines but they are fundamentally wrong. Integrals are defined and exist within a finite interval.

Intuitively, an infinite number of infinitesimals (dx) are used to compute a finite integral. Therefore, you can't compute an infinite interval.