

# Improper Integrals are Undefined

Ugur Buyukdurak

February 2025

## 1 Introduction

Improper integrals are undefined.

## 2 Proof

Improper integrals are undefined by the definition of the integral. Let's first give integral definition:

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) * \frac{b-a}{n}$$

Here  $dx$  is an infinitesimal and corresponds to  $\frac{b-a}{n}$  where  $n$  reaches  $\infty$ .  
An improper integral is:

$$\lim_{b \rightarrow \infty} \int_a^b f(x) dx = \int_a^\infty f(x) dx$$

Then the definition of an improper integral will be:

$$\lim_{b \rightarrow \infty} \int_a^b f(x) dx = \lim_{b \rightarrow \infty} \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) * \frac{b-a}{n}$$

Then we have:

$$\sum f(x_i) * (\infty - a) / \infty = \sum f(x_i) * \infty / \infty$$

Here  $\infty/\infty$  does not correspond to  $dx$  and is an indeterminate form. Therefore, improper integrals do not exist.