Finite Space Construction

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1   Introduction

The former teachings of mathematical spaces taught us that spaces can be both finite and infinite. For example, we are told that the real number line extends infinitely while constituting an infinite space. This paper says that when a space is constructed by the smallest entity, a point, which is finite, then finite spaces of all dimensions are well-defined, whereas infinite spaces are undefined. Hence, this paper is going to show you how an infinite number of points constitutes a finite space and that infinite spaces are undefined.

2   Proof
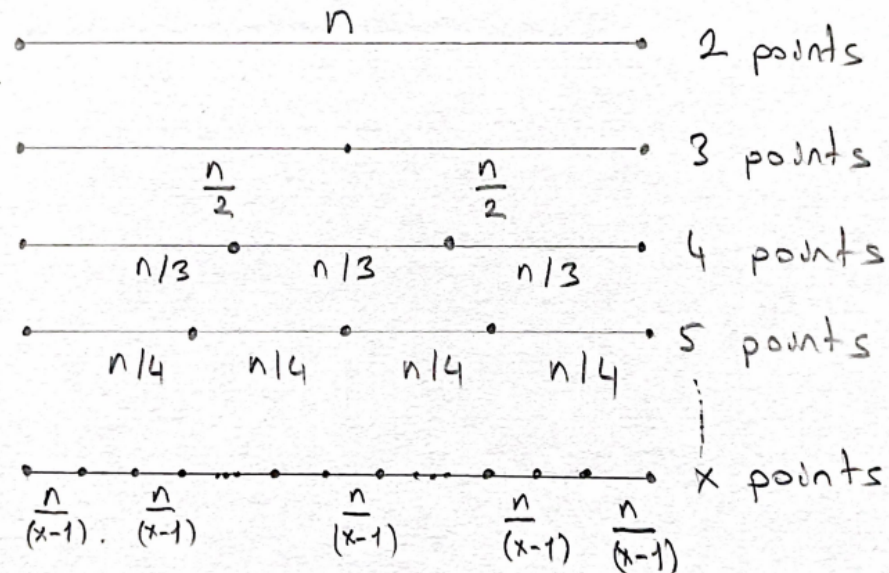
As seen in the final illustration at the bottom of the picture above, when there are x number of points equally distributed along the line, there are $(x-1)$ equal divisions, and the length of each division is $n/(x-1)$. We are trying to close the gap between any two consecutive points in order to construct a finite continuous space; therefore, we have the following formula:

$$\lim_{x\to\infty} n/(x-1) = 0$$

The above statement tells us that as the number of points equally distributed on the line reaches infinity, $(x-1)$ approaches infinity, thus making the gaps between all consecutive points infinitely approach zero in length - which is zero, which means that if there is an infinite number of points between any two points, they constitute a continuous finite line. You will see that if there is a finite number of points, the gaps never become zero in length, which means that a finite number of points cannot constitute a continuous finite line.

3   Results

We can show that, unlike finite spaces, infinite spaces cannot be formally constructed using the proof, that is, if you replace n with ∞ it becomes

$$\lim_{x\to\infty} \infty/(x-1) = \infty/\infty$$

which is an indeterminate form. According to the formula, in order to constitute a space of any kind, whether finite or infinite, gaps have to close. However, the existing indeterminate form says nothing about whether gaps close or not. Therefore, technically speaking, infinite spaces are undefined.