

Improper Integrals are Undefined

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1 Introduction

Improper integrals are undefined.

2 Proof

Improper integrals are undefined by the definition of the integral. Let's first give integral definition:

$$\int_a^b f(x) dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) * \frac{b-a}{n}$$

Here dx is an infinitesimal and corresponds to $\frac{b-a}{n}$ where n reaches ∞ .
An improper integral is:

$$\lim_{b \rightarrow \infty} \int_a^b f(x) dx = \int_a^\infty f(x) dx$$

Then the definition of an improper integral will be:

$$\lim_{b \rightarrow \infty} \int_a^b f(x) dx = \lim_{b \rightarrow \infty} \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i) * \frac{b-a}{n}$$

Then we have:

$$\sum f(x_i) * (\infty - a)/\infty = \sum f(x_i) * \infty/\infty$$

Here ∞/∞ does not correspond to dx or any other mathematical entity and is an indeterminate form. Therefore, improper integrals are undefined.