About Approaching Infinitely

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1 Introduction

This paper is going to mention a detail that caught my eye, but about which there is no official explanation. Formal limit definition is a bit odd here, and it has to do with limits, and $0.\overline{9}$ being equal 1.

2 Idea

For example,

$$\lim_{x \to 1} (x-1)(x+1)/(x-1) = \lim_{x \to 1} x+1 = 1+1 = 2$$

Here, we are generally taught that you can divide by (x-1) because x is infinitely approaching 1, while not 1. But the same bastards say $0.\overline{9} = 1$ (including with rigorous proofs), which is exactly the same thing as the limit that approaches 1. You could then say,

$$(0.\overline{9}-1)(x+1)/((0.\overline{9}-1)) = (0-1)/(x+1)/0$$

which would technically be undefined because you cannot divide by zero (but limit does).

You see that $0.\overline{9}$ is same as $\lim_{x \to 1^-}$ where the former is 1 and the latter is not considered 1 during division operation.

I don't know you, but it does not make sense to me.