About Approaching Infinitely

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1 Introduction

This paper is going to mention a detail that caught my eye, but about which there is no official explanation. The formal limit definition is a bit odd here, and it has to do with limits, and $0.\overline{9}$ being equal to 1.

2 Idea

For example,

$$\lim_{x \to \ 1^-} (x-1)(x+1)/(x-1) = \lim_{x \to \ 1^-} x+1 = 1+1 = 2$$

Here, we are generally taught that you can divide by (x-1) because x is infinitely approaching 1, while not being 1. But the same bastards say $0.\overline{9} = 1$ (with rigorous proofs), which is exactly the same thing as the limit that approaches 1 from left. You could then say,

$$(0.\overline{9}-1)(x+1)/(0.\overline{9}-1) = (1-1)(1+1)/(1-1) = 0 * 2/0 = 0/0$$

which would technically be undefined because you cannot divide by zero (but limit does).

You see that $0.\overline{9}$ is same as $\lim_{x \to \ 1^-}$ where the former is 1 and the latter is not considered 1 during division operation.

There seems to be inconsistencies in definitions and concepts when approaching infinitely.