Finite Space Construction

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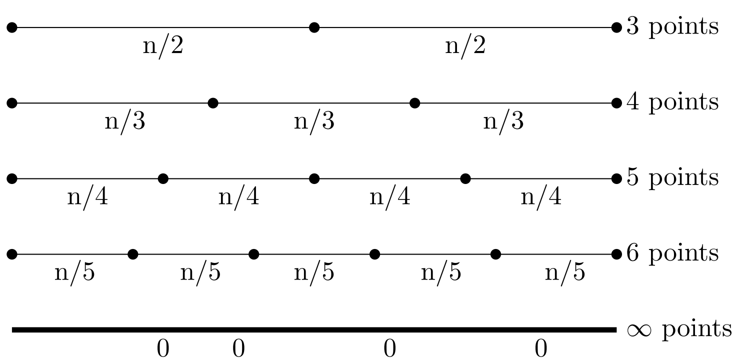
May 2025

1 Introduction

The former teachings of mathematical spaces taught us that spaces can be both finite and infinite. For example, we are told that the real number line extends infinitely while constituting an infinite space. This paper says that when a space is constructed by the smallest entity, a point, which is finite, then finite spaces of all dimensions are well-defined, whereas infinite spaces are undefined. Hence, this paper is going to show you how an infinite number of points constitutes a finite space and that you cannot formally construct an infinite space from an infinite number of points.

2 Proof



As seen in the illustration above, as you keep adding points, the gaps will become equally smaller but will never become zero in length. However, you need to make the gaps zero in order to construct a continuous space, and only way to do this is to use the limit. But you first need a function for that.

When there is x number of points that are equally distributed on the line, there is $x-1$ number of divisions and the length of each division is $n/(x-1)$. We are trying to fully close the gaps between all consecutive points to construct a finite continuous space; therefore, we use the limit:

$$\lim_{x\to\infty} \frac{n}{x-1} = 0$$

The above statement is a function that models the behavior in the illustration and tells us that as the number of points equally distributed on the line, x, reaches infinity, $(x-1)$ approaches infinity, thus making the gaps between all consecutive points infinitely approach zero in length - which is effectively zero, which means that as the number of points reaches infinity between any two points, they constitute a continuous finite line. You will see from the formula that if there is a finite number of points, the gaps never become zero in length, which means that a finite number of points cannot constitute a continuous finite line.

3 Results

We can show that, unlike finite spaces, infinite spaces cannot be formally constructed using the proof, that is, if you replace n with ∞ it becomes

$$\lim_{x\to\infty} \frac{\infty}{x-1} = \frac{\infty}{\infty}$$

which is an indeterminate form. According to the formula, to constitute a space of any kind, finite or infinite, gaps must fully close. However, the existing indeterminate form says nothing about the distance between all consecutive points because ∞/∞ does not mean a definite value, be it 0 or something else. It can be 0, any finite value, or infinity due to the nature of the definition of indeterminate form. That does not tell us anything about the distance between points. Furthermore, existing mathematics says that further evaluation is needed to decide the limit, but in this case further evaluation is not possible. Therefore, technically speaking, infinite spaces are undefined. Intuitively, this makes sense because the number of points reaches infinity between two ever-expanding boundaries. Can you then tell whether the gaps between all consecutive points close or not? The other way to look at it is that an infinite number of points make up a continuous finite space, therefore, you cannot build an infinite space with an infinite number of points. 2-D and 3-D spaces are built from the ground up in the same way as in the line example. Therefore, outer space as we perceive it is finite.

This paper says that spaces always have an end. That means that you cannot get infinitely smaller because that would violate the property of space that it has an end.

For us to be able to say that an infinite space exists, we should have been able to show:

$$\lim_{x\to\infty} \frac{\infty}{x-1} = \frac{\infty}{\infty} = 0$$

But such a form **does not exist.**

Based on the previous proof, if a line consists of points, then it cannot be infinite. If a line is infinite, then it cannot consist of points (so an infinite real number line cannot consist of points, thus it cannot represent individual numbers, but this is a paradox).