Documentation for `Boltzmannsolver`

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`Boltzmannsolver` is a program which numerically solves a system ordinary differential equations (ODE's). These non-linear ODE's, known as *Boltzmann equations*, describe how the density of particles change as the universe expands. The primary goal of `Boltzmannsolver` is to compute the present density of *dark matter*, for the class of models depicted in Fig. 1. For additional details see http://arxiv.org/abs/1502.05406 and references therein.

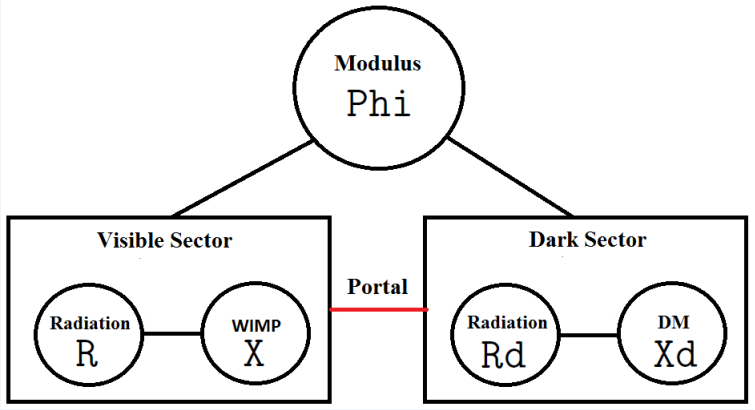

Figure 1: Fig. 1: Schematic depection of model

Each circle in Fig. 1 represents a set of particles, while the lines represent interactions between particles. The dependent variables of the ODE system are the densities of the particles in Fig. 1, denoted in the code as `Phi, R, X , Rd, Xd.` The independent variable A is known as the *scale factor*, and represents the expansion of the universe ($A =$ increases as the universe expands). To determine present day dark matter density, we must determine `Xd` for the present day value of A.

The resulting ODE system is *stiff*, as the solutions tend to vary by several orders of magnitude in the region of interest. Even with numerical ODE solvers specialized for stiff systems, `MATLAB` has difficulty solving the entire system. Boltzmannsolver instead adopts an iterative procedure:

1. Set `Rd` and `Xd` to zero, and solve for `Phi, X` and `R.`
2. Solve for `Rd` and `Xd,` using solution for `Phi, X` and `R` from previous step.
3. Update solution for `Phi, X, R` using solution for `Rd` and `Xd` from previous step.
4. Repeat steps 2 and 3 until final value of `Xd` converges.

The code outputs a plot of `Phi, X` and `Xd` (normalized to their maximum values) as a function of $\log A$. In addition, `Boltzmannsolver` prints the predicted present day dark matter density as a fraction of the observed dark matter density. If this fraction is greater than 1, the model predicts too much dark matter!