

Start with $c=0$, i.e. $x_{n+1} = Rx_n$

Assume x_0 is an eigenvector corresponding to eigenvalue λ with $|\lambda| \geq 1$.

$$\Rightarrow x_1 = Rx_0 = Rv = \lambda v = \lambda x_0, \text{ where } v \text{ is eigenvector}$$

$$\Rightarrow x_2 = Rx_1 = R(\lambda v) = \lambda^2 v = \lambda^2 x_0$$

$$\Rightarrow x_n = \lambda^n x_0$$

Given $|\lambda| \geq 1$, λ^n doesn't converge to 0.

$\Rightarrow x_n$ only converge to a finite $\lambda^n x_0$ when $x_0 = 0$

More general, $c \neq 0$

the constant c doesn't change the convergence fact above unless $c=0$,

\Rightarrow for any non-zero c , if $\rho(R) \geq 1$, $x_{n+1} = Rx_n + c$ doesn't converge