Q3:

We can use the Central Limit Theorem to calculate the confidence interval with one set of payoffs generated from a single Monte Carlo simulation.

Assume we already have the set of payoffs from a single run of the Monte Carlo pricer.

1. Calculate the Mean and Variance of the payoffs. Denote as μ and variance σ of these payoffs
2. Compute the standard error (SE) as
   $SE = \frac{\sigma}{\sqrt{N}}$,
   where N is the number of simulated paths
3. Use Z-Score of 95% Confidence 1.96
4. Compute the Confidence Interval as
   $$Lower = \mu - 1.96 * SE$$
   $$Upper = \mu + 1.9 * SE$$
5. Discount the upper and lower bound into present values using the risk-free rate. The discounted bound is the 95% confidence interval for the option's current value

Use the Finite Difference Method to approximate the delta.

1. Increase the initial price of the underlying asset by a small amount, denote as δS. Re-run the Monte Carlo Simulation to get V_{up}
2. Decrease the initial price of the underlying asset by a small amount, same amont of δS. Re-run the Monte Carlo Simulation to get V_{down}
3. Calculate delta using
   $$\Delta = \frac{V_{up} - V_{down}}{2 * \delta S}$$

This only requires running Monte Carlo pricer twice