Start with $c=0$, i.e. $x_{n+1} = Rx_n$

Assume x_0 is an eigenvector corresponding to eigenvalue λ with $|\lambda| \geq 1$.

$\Rightarrow x_1 = Rx_0 = Rv = \lambda v = \lambda x_0$, where v is eigenvector

$\Rightarrow x_2 = Rx_1 = R(\lambda v) = \lambda^2 v = \lambda^2 x_0$

$\Rightarrow x_n = \lambda^n x_0$

1° if $\lambda \neq 1$ and $|\lambda| \geq 1$, $\{x_n\}$ does not converge

2° if $\lambda = 1$, we choose $c = v$ as well

$\therefore x_{n+1} = Rx_n + v$, $x_0 = v$

Thus $x_n = (n+1)\cdot v$, prove by induction

[1] $x_0 = (0+1)\cdot v = v$

[2] Given $x_k = (k+1)v$, $x_{k+1} = Rx_k + v = R((k+1)v) + v = (k+1)\cdot Rv + v = (k+2)v$ #

So $\{x_n\}$ does not converge

In conclusion, when $\rho(R) \geq 1$, there exist $c \in \mathbb{R}^n$, $x_0 \in \mathbb{R}^n$,

s.t. $\{x_n\}$ does not converge