# Pareto Distribution: An Overview

Pareto distribution is a high tailed, with finite mean and infinite variance continuous probability distribution used for modelling on and off period durations of a voice source. Its PDF and CDF are as follows [1]:

$$\begin{aligned} f_X(x) &= \frac{k x_m^k}{x^{k+1}} \\ F_X(x) &= 1 - \left(\frac{x_m}{x}\right)^k \end{aligned} \tag{1}$$

where $k$ is the shape parameter of the distribution and $x_m$ is the minimum value of the distribution. If $1 < k < 2$, mean of the distribution is finite but its variance is infinite [2]. Recommended values for k values when modelling a voice source are $k_{on} = 1.9$ and $k_{off} = 1.25$ [3].

To generate pareto distributed random numbers, one can pick a random number from a uniform distribution of interval $[0 : 1]$, say r, then perform the following calculation [1]:

$$x = x_m(1 - r)^{-1/k} \tag{2}$$

Mean value of the distribution is:

$$E[X] = \frac{k x_m}{(k-1)} \qquad for\ \ k > 1 \tag{3}$$

Variance the distribution is:

$$Var[X] = \frac{k x_m^k}{(k-1)^2(k-2)} \qquad for\ \ k > 2 \tag{4}$$

Given that mean value of the distribution is $\mu$, by using Equation3 one can easily derive $k$ as follows:

$$k = \frac{\mu}{\mu - x_m} \qquad for\ \ x_m > 0 \tag{5}$$

Also note that when the distribution is high tailed (finite mean, infinite variance), $k$ must be less than 2. Therefore, from Equation5 $\mu$ must be greater than $2 \times x_m$.

# References


[1] B. Khan A. Al-Fuqaha M. Guizani, A. Rayes. *Network Modelling and Simulation*. WILEY, 2010.

[2] I-Hui Li. Effects of ON-OFF Variability in Two-State Pareto Traffic Models on Multimedia Application Transmission Performance. In *IEEE Fourth International Conference on Networked Computing and Advanced Information Management*, pages 1–6, 2008.

[3] L. Tedesco, A. Mello, L. Giacomet, N. Calazans, and F. Moraes. Application Driven Traffic Modeling for NoCs. *ACM SBCCI*, 2006.