BEP Walls for Collaborative Spectrum Sensing

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***Abstract*—The main focus of this paper is to present a performance limitation of collaborative spectrum sensing in cognitive radios with imperfect reporting channels. We consider hard decision (HD) based cooperative sensing (CS), in which each SU sends a one-bit binary decision corresponding to the absence or the presence of primary user (PU) to a fusion center (FC). Each SU sends the hard decision over a reporting channel that may cause bit errors. The effect of reporting channel errors is modeled through the widely used bit error probability (BEP). The FC fuses the local binary decisions from all the SUs to make a final decision. Counting rule or K-out-of-N fusion rule is considered for CS and its performance is studied using analytical tools and simulations. Under the constraints on the error probabilities of false alarm and missed detection, a performance limitation in the form of a BEP wall is shown to exist for the counting rule. If the BEP of the reporting channel is above the BEP wall value, then constraints on the cooperative detection performance cannot be met at the FC irrespective of the received signal quality on the listening channel or the sensing time at the SUs. Expressions for the BEP walls are presented for K-out-of-N fusion rules in terms of the error probabilities at the FC and the number of SUs collaborating. The BEP wall values are shown to be sufficiently low to be of practical importance.**

***Index Terms*—Cooperative Detection, Boolean Fusion Rule, Imperfect Reporting Channels, Performance Analysis.**

I. Introduction

Distributed detection has been a topic of great interest in radar, sensor network and cognitive radio research communities [1]-[4]. Cooperative detection provides significant gains through spatial diversity. It improves the detector performance, increases the coverage in a cognitive radio network, and facilitates simpler detectors [5]-[6].

Using hard local decisions for distributed detection facilitates easy implementation and reduces the communication cost at the expense of information loss. Performance of hard decision (HD) based cooperative detection in the presence of reporting channel errors has been well studied in the distributed detection literature [7]-[9] and in the cognitive radio literature [10]. However, in our opinion, the fundamental performance limitation issues have not received sufficient attention. Establishing the limitations of a fusion rule is an important topic as it helps in designing practical detectors and communication protocols between the detectors and the fusion center (FC). For example, the SNR wall is a performance limitation that needs to be taken into account if energy detection is employed [11]. In this paper, we establish a performance limitation in the form of a Bit Error Probability (BEP) wall for counting rule or 'K-out-of-N' rule in the presence of reporting channel errors with constraints on the probabilities of missed detection and false alarm at the FC.

In [8],[9], the performance analysis is carried out for different modulations schemes like Frequency Shift Keying (FSK), On/Off keying (OOK), Phase Shift Keying (PSK) in the presence of different channel conditions. However, as our aim is to show the performance limitation of HD based CS, we use a more general and a more widely applicable model of BEP.

In [10], the performance limitation of OR rule is studied in the presence of channel errors under the constraint on false alarm. In this paper, we establish the performance limitation for K-out-of-N fusion rule, which is a more general class of fusion rule and includes widely used OR, AND, and MAJORITY Boolean fusion rules. In addition, they are optimal under mild assumptions [3].

Detectors are normally designed such that they achieve a given Receiver Operating Characteristic (ROC) constraints for the SNR range of interest. Therefore it is practical to have constraints on both the error probabilities, i.e., false alarm probability and missed detection probability for cooperative sensing (CS). In this paper, we take into account both the constraints instead of only false alarm probability as was done in [10].

Contributions of this paper are

- Existence of a BEP wall is shown for the 'K-out-of-N' fusion rule in the presence of reporting channel errors with constraints on the probabilities of missed detection and false alarm at the FC. *If the reporting channel BEP is above the BEP wall value, it is impossible to satisfy the imposed performance constraints on the detector error probabilities at the FC irrespective of the SNR on the listening channel or the sensing time at the SUs.*
- Expressions for the BEP walls in terms of the false alarm probability and the missed detection probability at the FC are given. It is shown that the BEP wall exists even for BEP values as low as 10^{-3} and is therefore important to take into account.

Most of the related work in the literature (see [7]-[9] and references therein) concentrate on how different reporting channel conditions affect CS performance and on the design of optimal fusion rules to improve the performance. In this paper, we find the worst case reporting channel conditions where the collaborative spectrum sensing scheme will fail to meet the performance constraints. In such conditions, even improving the quality of sensor decisions (by increasing SNR on the listening channel or increasing sensing time) cannot help the CS to meet the performance constraint.

This paper is organized as follows. In Section II, the performance of CS based on HDs with imperfect reporting channels is presented and the BEP wall phenomenon is explained. Section III presents the simulation results. This is followed by Section IV, which concludes the paper.

II. Cooperative Sensing with Imperfect Reporting Channels

As shown in the Fig. 1, we consider a scenario in which N secondary users (SUs) are cooperating to detect a primary user transmission and identify underutilized spectrum. For convenience, we assume that each sensor is experiencing the same average SNR on the listening channel and has an identical structure. Each SU employs a Neyman-Pearson detector for constraining the false alarm rate while maximizing the probability of detection. For HD based CS, the n^{th} SU makes a hard decision u_n and sends it to the FC

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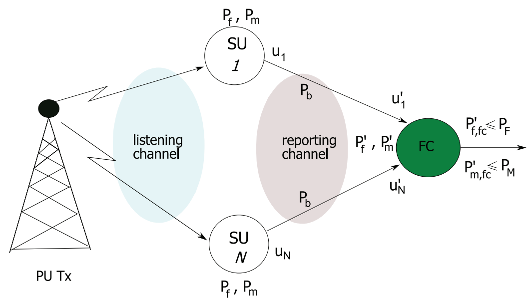

Figure 1. Considered Scenario: Secondary users (SUs) cooperate to detect a primary user (PU). SUs send hard decisions (HDs) to the fusion center (FC) over a reporting channel with bit error probability P_b. FC uses the K-out-of-N rule to make a final decision.

on the erroneous reporting channel. Because of the channel errors, the error probabilities for each SU change at the FC. We assume that the channel errors are independent and identically distributed (*i.i.d.*) for all the reporting channels between the SUs and the FC. The FC uses the K-out-of-N rule to fuse the HDs from the local detectors to arrive at a final decision. Following is a list of symbols used in the paper:

- P_d, P_f, P_m: Probabilities of detection, false alarm, and missed detection respectively at each SU. Note $P_m = 1 - P_d$.
- P_b: BEP of the reporting channel for each SU.
- u_n: hard decision sent by the n^{th} SU to the FC.
- u'_n: hard decision received by the FC from the n^{th} SU.
- P'_d, P'_f, P'_m: Probabilities of detection, false alarm, and missed detection respectively at the FC for each SU. Note $P_m = 1 - P_d$.
- $P'_{m,fc}, P'_{f,fc}$: Probabilities of missed detection and false alarm respectively at the FC for CS.

Because of the channel errors, the error probabilities for each SU change when their decisions arrive at the FC and are given by [9]

$$\begin{aligned} P'_f &= (1-P_b)P_f + P_b(1-P_f), \quad \text{and} \\ P'_m &= P_b(1-P_m) + (1-P_b)P_m. \end{aligned} \tag{1}$$

Similarly, the probability of detection for each SU at the FC is given by

$$P'_d = (1-P_b)P_d + P_b(1-P_d). \tag{2}$$

For the K-out-of-N fusion rule, the overall probability of detection for CS at the FC is given by [4]

$$\begin{aligned} P'_{d,fc} &= \sum_{k=K}^{N} \binom{N}{k} (P'_d)^k (1-P'_d)^{N-k} \\ &= 1 - \mathcal{B}(K-1, N, P'_d). \end{aligned} \tag{3}$$

where $\mathcal{B}(k,n,p)$ is the binomial cumulative distribution function (CDF) with parameters k, n, and p and is given by

$$\mathcal{B}(k,n,p) = \sum_{i=0}^{k} \binom{n}{i} (p)^i (1-p)^{n-i}. \tag{4}$$

Therefore, the missed detection probability for the K-out-of-N fusion rule at the FC is

$$\begin{aligned} P'_{m,fc} &= 1 - P'_{d,fc} \\ &= \mathcal{B}(K-1, N, 1-P'_m). \end{aligned} \tag{5}$$

Similarly, the false alarm probability for the K-out-of-N fusion rule at the FC is

$$\begin{aligned} P'_{f,fc} &= \sum_{k=K}^{N} \binom{N}{k} (P'_f)^k (1-P'_f)^{N-k} \\ &= 1 - \mathcal{B}(K-1, N, P'_f). \end{aligned} \tag{6}$$

A. Constraint on False Alarm Probability

Suppose that we want to set a constraint on the false alarm probability at the FC as $P'_{f,fc} \leq P_F$. Substituting (6) in the constraint $P'_{f,fc} \leq P_F$ and rearranging, we get the following inequality [12]

$$P_b \leq \frac{\mathcal{B}^{-1}(K-1, N, 1-P_F) - P_f}{1-2P_f}, \tag{7}$$

under the reasonable assumption that $P_f < 0.5$. See [12] for the detailed derivation. In (7), the fact is used that $\mathcal{B}(k,n,p)$ is a strictly decreasing bijective function of p [13] and therefore $\mathcal{B}^{-1}(k,n,p)$ exists and is unique. Next, the assumption that $P_f > 0.5$ results in the redundant constraint $P_b \geq 0$ while the assumption that $P_f = 0.5$ results in another redundant constraint $P_b \leq \infty$. Rearranging (7), we get

$$P_f \leq \frac{\mathcal{B}^{-1}(K-1, N, 1-P_F) - P_b}{1-2P_b} \triangleq P_{fb}. \tag{8}$$

In other words, for a given BEP of the reporting channel, the false alarm probability at each SU should be less than P_{fb} so that the constraint on the false alarm probability at the FC can be satisfied. Using (8) and the fact that $P_f \geq 0$ leads us to the following constraint on the BEP of the reporting channels

$$P_b \leq \mathcal{B}^{-1}(K-1, N, 1-P_F) \triangleq P_{bf} \tag{9}$$

under the reasonable assumption of $P_b < 0.5$. As $P_b \rightarrow P_{bf}$, $P_{fb} \rightarrow 0$. This corresponds to the case when threshold of the Neyman-Pearson detector at the SUs is infinity. This means that the SUs disregard the received signal and declare H_0. For $P_b \geq P_{bf}$, it is impossible to maintain the constraint on false alarm probability at the FC.

B. Constraint on Missed Detection Probability

Suppose that we want to set a constraint on the missed detection probability at the FC as $P'_{m,fc} \leq P_M$. Substituting (5) in the constraint $P'_{m,fc} \leq P_M$ and rearranging, we get the following inequality [12]

$$P_b \leq \frac{1 - \mathcal{B}^{-1}(K-1, N, P_M) - P_m}{1-2P_m}, \tag{10}$$

under the reasonable assumption that $P_m < 0.5$. The assumption that $P_m > 0.5$ results in the redundant constraint $P_b \geq 0$ while the assumption that $P_f = 0.5$ results in another redundant constraint $P_b \leq \infty$. See [12] for the detailed derivation.

Rearranging (10), we get

$$P_m \leq \frac{1 - \mathcal{B}^{-1}(K-1, N, P_M) - P_b}{1-2P_b} \triangleq P_{mb}. \tag{11}$$

In other words, for a given BEP of the reporting channel, the missed detection probability at each SU should be less than P_{mb} so that the constraint on the missed detection probability at the FC can be satisfied. Using (11) and the fact that $P_m \geq 0$ leads us to the following constraint on the BEP of the reporting channels

$$P_b \leq 1 - \mathcal{B}^{-1}(K-1, N, P_M) \triangleq P_{bm} \tag{12}$$

under the reasonable assumption of $P_b < 0.5$. As $P_b \rightarrow P_{bm}$, $P_{mb} \rightarrow 0$. This corresponds to the case when either the threshold at

Table I
THEORETICAL $P_{b,wall}$ VALUES FOR THE K-OUT-OF-N FUSION RULES FOR DIFFERENT VALUES OF (P_M, P_F, N).

$(P_M, P_F, N) \downarrow; K \rightarrow$	1	2	3	4	5
$(0.01, 0.01, 5)$	0.002	0.033	0.105	0.033	0.002
$(0.01, 0.05, 5)$	0.01	0.076	0.105	0.033	0.002
$(0.05, 0.01, 5)$	0.002	0.033	0.105	0.076	0.01

the SUs is zero or the SNR at the SU corresponding to the listening channel tends to infinity. Therefore if the channel BEP is more than P_{bm} it is impossible to satisfy the constraint on the missed detection probability at the FC.

C. BEP Wall

Now suppose that we have constraints on both the error probabilities. Consider the following cases:

- $P_b \rightarrow P_{bf}$: We have already seen that as $P_b \rightarrow P_{bf}$, the required threshold values of the Neyman-Pearson detectors at the SUs tend to infinity so that the false alarm probability constraint is met. Therefore to meet the missed detection constraint, the SNRs required at the SUs on the listening channels will tend to infinity even for the case $P_b \ll P_{bm}$.
- $P_b \rightarrow P_{bm}$: Again, we know that as $P_b \rightarrow P_{bm}$, the required SNRs at the SUs on the listening channels tend to infinity so that the constraints on missed detection is met. Note that when $P_b \rightarrow P_{bm}$, setting the threshold to zero is not a viable option as it will increase the false alarm probability at the FC even for $P_b \ll P_{bf}$.

Therefore for $P_b \geq P_{bf}$ or $P_b \geq P_{bm}$, it is impossible to meet the constraints on both the error probabilities irrespective of the SNR value at the SUs. This is the BEP wall phenomenon.

We define the BEP wall as the worst case channel BEP value above which it is impossible to achieve the error probabilities at the FC irrespective of SNR on the listening channels or the sensing time at the SUs. The minimum of the two BEP values P_{bf} and P_{mf} will determine the BEP wall values. Therefore if we denote the BEP wall as $P_{b,wall}$, then the expression for the BEP wall for the K-out-of-N rule is given by [12]

$$\begin{aligned} P_{b,wall} &= \min(P_{bf}, P_{bm}) \\ &= \min\Big(\mathcal{B}^{-1}(K-1, N, 1-P_F), \\ &\qquad 1-\mathcal{B}^{-1}(K-1, N, P_M)\Big). \end{aligned} \tag{13}$$

Table I presents theoretical $P_{b,wall}$ values for different values of P_M, and P_F with $N = 5$. It can be seen that for various values of P_F and P_M, the theoretical BEP wall values for most of the K-out-of-N rules are significantly low (in the range of 10^{-3}-10^{-1}). Therefore it is important that the BEP wall phenomenon is taken into account for the K-out-of-N fusion rules in the presence of imperfect reporting channels.

III. SIMULATION RESULTS

For the simulations, the listening and reporting channels are *i.i.d.* additive white Gaussian noise (AWGN) channels, the primary user signal is an orthogonal frequency division multiplexing (OFDM) signal, and the local detector is an autocorrelation detector [6]. The useful symbol length and the cyclic prefix length in an OFDM symbol is 32 and 8 symbols, respectively, and the number of OFDM symbols used for sensing is 100. For the simulation results, we plot SNR Loss in dB as a function of P_b for different values of K. In this paper, SNR Loss is defined as the additional amount of SNR required at all the SUs to meet the same constraints on the false alarm probability and missed detection probability with erroneous reporting channels as compared to the case of error-free channels.

Fig. 2 shows the SNR Loss vs. P_b curves for the K-out-of-N fusion rule for $P_F = 0.01$, $P_M = 0.01$, and $N = 5$. There are three distinct regions of BEP values corresponding to each of the curves:

- Region I: In this region, the effect of channel errors is negligible and the performance of the fusion rule is similar to the case of error-free reporting channels. It is apparent from the results that the MAJORITY fusion rule has a longer range of BEP values which belong to Region I among the K-out-of-N fusion rules.
- Region II: In this region, the effect of channel errors is clearly seen as the SNR Loss increases considerably with an increase in the BEP of the reporting channels.
- Region III: In this region, SNR Loss $\rightarrow \infty$ as the channel BEP reaches the BEP wall. If the BEP of the reporting channel is above the BEP wall value, then irrespective of the received signal quality on the listening channel or the sensing time at the SUs, constraints on the detector performance cannot be met at the FC.

The existence of BEP wall for the K-out-of-N fusion rules is clearly seen. The simulated BEP wall values are close to the theoretical BEP wall values in Table I. OR ($K = 1$) and AND ($K = N$) fusion rules have very low BEP wall values of 2×10^{-3} and are very sensitive to the reporting channel errors. On the other hand, the MAJORITY ($K = 3$) rule has the highest BEP wall value of 1.05×10^{-1} and is the most robust among the K-out-of-N rules against the reporting channel errors.

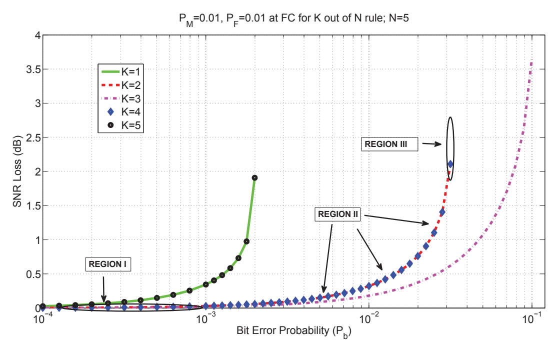

Figure 2. SNR Loss in dB vs. P_b for the K-out-of-N fusion rules for $P_M = 0.01$, $P_F = 0.01$ and $N = 5$. The SNR Loss is the required increase in the local SNR for maintaining the same error levels at the FC as in error-free reporting channel case. The BEP wall phenomenon is clearly observed.

Fig. 3 shows the SNR Loss vs. P_b curves for the K-out-of-N fusion rule for $P_F = 0.05$, $P_M = 0.01$, and $N = 5$. It can be seen from the Table I and the Fig. 3 that the theoretical and the simulated values for the BEP walls are sufficiently close. Note from the Figs. 2 and 3 that there is a rightward shift in the SNR Loss curves for $K = 1, 2$ when P_F increases from 0.01 to 0.05 while the curves for $K = 3, 4, 5$ do not shift. This means that the P_{bf} is more dominant for $K = 1, 2$ as compared to P_{bm}. Therefore the fusion rules with $K = 1$ and $K = 2$ are more sensitive to the false alarm probability constraint among the K-out-of-N fusion rules.

Fig. 4 depicts the SNR Loss vs. P_b curves for the K-out-of-N fusion rule for $P_F = 0.01$ and $P_M = 0.05$. Again, we see that the theoretical values in Table I and the simulated values in Fig. 4 for the BEP walls are sufficiently close. Moreover, the observations are similar to Fig. 3 except that the curves affected by the increase of P_M

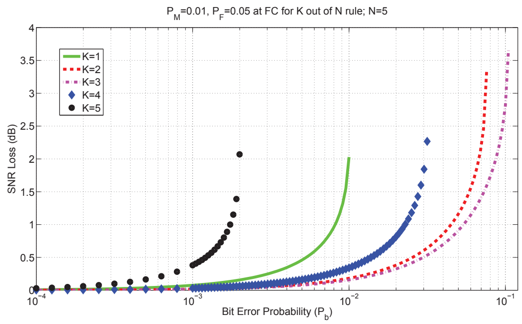

Figure 3. SNR Loss in dB vs. P_b for the K-out-of-N fusion rules for $P_M = 0.01$, $P_F = 0.05$ and $N = 5$. The fusion rules with $K = 1$ and $K = 2$ are sensitive to the false alarm probability constraint.

from 0.01 to 0.05 are the curves for $K = 4, 5$ instead of the curves $K = 1, 2$. Consequently, the P_{bm} is more dominant for $K = 4, 5$ than P_{bf} and thus they are the most sensitive to the missed detection probability constraint among the K-out-of-N fusion rules.

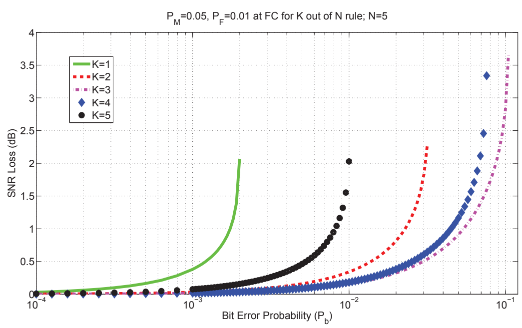

Figure 4. SNR Loss in dB vs. P_b for the K-out-of-N fusion rules for $P_M = 0.05$, $P_F = 0.01$ and $N = 5$. The fusion rules with $K = 4$ and $K = 5$ are sensitive to the missed detection probability constraint.

Fig. 5 shows the SNR Loss vs. P_b curves for the OR ($K = 1$), MAJORITY ($K = \lceil N/2 \rceil$), and AND ($K = N$) fusion rules for $N = 5$ and $N = 10$ for given $P_F = 0.01$ and $P_M = 0.01$. It can be seen that as the number of collaborating SUs N increase, the SNR Loss curves for OR and AND fusion rules have shifted left. Therefore there is degradation in the performance of cooperating sensing for OR and AND fusion rules in the presence of channel errors. In addition, the BEP wall values decrease with an increase in the number of collaborating SUs. This is in contrast to the general belief that cooperation always improves the performance of cooperative sensing. Therefore the performance of OR and AND fusion rules is severely limited in the presence of channel errors. On the other hand, the performance of the MAJORITY rule improves with an increase in N and also the BEP wall value increases. Therefore the MAJORITY rule is more robust against the reporting channel errors.

IV. Conclusion

In this paper, we have demonstrated the performance limitations of HD based CS with imperfect reporting channels. The K-out-of-N fusion rule is employed for fusing the HDs at the FC. Channel errors are modeled through a widely applicable BEP model. We have shown the existence of a BEP wall phenomenon for HD based CS. If the BEP of the reporting channel is higher than the BEP wall, then it is impossible to satisfy the constraints on error probabilities at the FC irrespective of the SNR on the listening channel or the sensing time at the SUs. We have also derived closed form expressions for the BEP wall in terms of false alarm probability, missed detection probability and the number of SUs. For the considered cases, the BEP wall values are in the range of $10^{-3} - 10^{-1}$. These values of BEP wall are sufficiently low to be of practical importance in designing communication systems with non-ideal reporting channels. It is also shown that the widely used OR and AND fusion rules are very susceptible to the reporting channel errors while the MAJORITY rule is more robust among K-out-of-N rules. Detailed derivations, further properties, and results for the BEP wall can be found in [12].

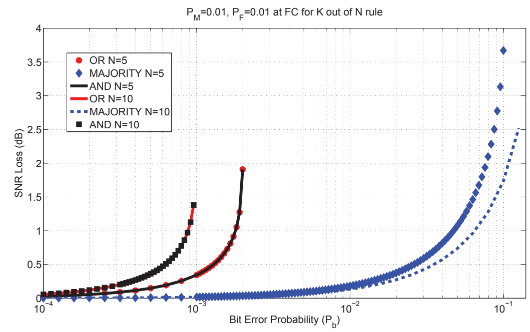

Figure 5. SNR Loss in dB vs. P_b for the K-out-of-N fusion rules for $N = 5$ and $N = 10$ for given $P_M = 0.01$, and $P_F = 0.01$. Cooperation hurts the performance of OR and AND fusion rules in the presence of reporting channel errors. On the other hand, cooperation helps improve the performance of collaborative sensing for the MAJORITY fusion rule.

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