Limites fundamentais

Para $a_n \to \infty$ e $x \in \mathbb{R}$:

$$\left(1+\frac{x}{a_n}\right)^{a_n} \to e^x.$$

Para $\varepsilon_n \to 0$:

$$\frac{\sin \varepsilon_n}{\varepsilon_n} \to 1 \qquad\qquad \frac{1-\cos \varepsilon_n}{\varepsilon_n^2} \to \frac{1}{2}$$

$$\frac{a^{\varepsilon_n}-1}{\varepsilon_n} \to \ln a, \quad a>0 \qquad\qquad \frac{(1+\varepsilon_n)^a-1}{\varepsilon_n} \to a, \quad a \in \mathbb{R}$$

$$\frac{\ln(1+\varepsilon_n)}{\varepsilon_n} \to 1.$$

Velocidade dos infinitos em ordem crescente

$$(\ln n)^\alpha,\ \alpha>0 \qquad n^\alpha,\ \alpha>0 \qquad a^n,\ a>1 \qquad n! \qquad n^n.$$

Séries

Para $\alpha, \beta \in \mathbb{R}$:

$$\sum_{n=1}^{\infty} \frac{1}{n^\alpha} < +\infty \ \Leftrightarrow\ \alpha > 1$$

$$\sum_{n=2}^{\infty} \frac{1}{n^\alpha (\ln n)^\beta} < +\infty \ \Leftrightarrow\ \alpha > 1 \text{ ou } \begin{cases} \alpha = 1 \\ \beta > 1. \end{cases}$$

Polinômio de Taylor

Forma geral do polinômio de Taylor de ordem n da função f no ponto $x_0 \in \mathbb{R}$:

$$P_n(x-x_0) = \sum_{i=0}^{n} \frac{f^{(i)}(x_0)}{i!}(x-x_0)^i.$$

Desenvolvimento de Taylor das funções elementares na origem:

$$e^x = 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \cdots + \frac{1}{n!}x^n + o(x^n)$$

$$\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \cdots + \frac{(-1)^{n+1}}{n}x^n + o(x^n)$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots + x^n + o(x^n)$$

$$\sin(x) = x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \cdots + \frac{(-1)^n}{(2n+1)!}x^{2n+1} + o(x^{2n+2})$$
$$\cos(x) = 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \cdots + \frac{(-1)^n}{(2n)!}x^{2n} + o(x^{2n+1})$$
$$\tan(x) = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \frac{62}{2835}x^9 + o(x^{10})$$
$$\arctan(x) = x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \cdots + \frac{(-1)^n}{2n+1}x^{2n+1} + o(x^{2n+2})$$
$$\sinh(x) = x + \frac{1}{3!}x^3 + \frac{1}{5!}x^5 + \cdots + \frac{1}{(2n+1)!}x^{2n+1} + o(x^{2n+2})$$
$$\cosh(x) = 1 + \frac{1}{2!}x^2 + \frac{1}{4!}x^4 + \cdots + \frac{1}{(2n)!}x^{2n} + o(x^{2n+1})$$
$$\tanh(x) = x - \frac{1}{3}x^3 + \frac{2}{15}x^5 - \frac{17}{315}x^7 + \frac{62}{2835}x^9 + o(x^{10})$$
$$\operatorname{arctanh}(x) = x + \frac{1}{3}x^3 + \frac{1}{5}x^5 + \cdots + \frac{1}{2n+1}x^{2n+1} + o(x^{2n+2})$$
$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3 + \cdots + \binom{\alpha}{n}x^n + o(x^n), \quad \alpha \in \mathbb{R}.$$

Funções analíticas

Para $x \in \mathbb{R}$:

$e^x = \sum_{n=0}^{+\infty} \frac{x^n}{n!}$ $\qquad$ $\sin(x) = \sum_{n=0}^{+\infty} \frac{(-1)^n}{(2n+1)!}x^{2n+1}$

$\cos(x) = \sum_{n=0}^{+\infty} \frac{(-1)^n}{(2n)!}x^{2n}$ $\qquad$ $\sinh(x) = \sum_{n=0}^{+\infty} \frac{1}{(2n+1)!}x^{2n+1}$

$\cosh(x) = \sum_{n=0}^{+\infty} \frac{1}{(2n)!}x^{2n}$.

Para $x \in (-1,1)$:

$\frac{1}{1-x} = \sum_{n=0}^{+\infty} x^n$ $\qquad$ $(1+x)^\alpha = \sum_{n=0}^{+\infty} \binom{\alpha}{n}x^n, \quad \alpha \in \mathbb{R}$

$\operatorname{arctanh}(x) = \sum_{n=0}^{+\infty} \frac{1}{2n+1}x^{2n+1}$.

Para $x \in (-1,1]$: $\qquad$ $\ln(1+x) = \sum_{n=1}^{+\infty} \frac{(-1)^{n+1}}{n}x^n$.

Para $x \in [-1,1]$: $\qquad$ $\arctan(x) = \sum_{n=0}^{+\infty} \frac{(-1)^n}{2n+1}x^{2n+1}$.