1. Sequências

Calcule os seguintes limites:

(1) $$\lim_{n\to+\infty} \frac{\sqrt[3]{n}+\sqrt[6]{n}}{\sqrt{n}-1}$$

(2) $$\lim_{n\to+\infty} \frac{n!-e^n}{2e^n+n^{10}+\log n}$$

(3) $$\lim_{n\to+\infty} \frac{2^n-\cos(n)}{2^{n+1}-n^2}$$

(4) $$\lim_{n\to+\infty} \sqrt[n]{n}$$

(5) $$\lim_{n\to+\infty} \Big(1-\frac{1}{n^2}\Big)^{(n^3)}$$

(6) $$\lim_{n\to+\infty} \frac{2^n-3^n}{1+3^n}$$

(7) $$\lim_{n\to+\infty} n^2\Big(1-\cos\frac{n+1}{2n^2-1}\Big)$$

(8) $$\lim_{n\to+\infty} \frac{n^{n-2}+(n-2)^n}{4(n^n)-3(n!)}$$

(9) $$\lim_{n\to+\infty} \frac{n^6+\log n+2^n}{3^n+n^4+\log^5 n}\sin(n)$$

(10) $$\lim_{n\to+\infty} \Big(n\log\Big(1+\frac{3}{n}\Big)+\frac{1}{\sqrt{n}}\log\Big(1+\frac{1}{n}\Big)\Big)$$

(11) $$\lim_{n\to+\infty} n\big(3^{\frac{n+1}{n^2+1}}-1\big)$$

(12) $$\lim_{n\to+\infty} \frac{n!}{\sqrt{n^n}}$$

(13) $$\lim_{n\to+\infty} \frac{\log(n+1)}{\log\ n}$$

(14) $$\lim_{n\to+\infty} \Big(\frac{n^3-1}{3n^3+\log n+2}\Big)^{\frac{n}{2}}$$

(15) $$\lim_{n\to+\infty} (\sqrt{n+2}-\sqrt{n})$$

(16) $$\lim_{n\to+\infty} \Big(\sqrt[3]{\frac{n^2-1}{n^2+1}}-1\Big)n^2$$

(17) $$\lim_{n\to+\infty} \Big(\frac{n+2}{n+3}\Big)^n\frac{1}{n}$$

(18) $$(*)\lim_{n\to+\infty} \frac{\log((n+5)!)-\log(n!+5)}{\log(2n^6+\cos(n\pi))}$$

(19) $$(*)\lim_{n\to+\infty} \frac{n^{n+1}+3(n+1)^{n+1}}{n^n+n!}\sin\frac{\pi}{n}$$

(20) $$(*)\lim_{n\to+\infty}\frac{(n!)^{n-1}-((n-1)!)^n}{((n-1)!\,(n-10))^{n-1}}.$$

Demonstre que as seguintes sequências não têm limite:

(1) $\cos(n^2\pi)$ (2) $(-1)^n\dfrac{3n+2}{n+4}$.

2. Séries

Estabeleça o caráter das seguintes séries:

(1) $$\sum_{n=1}^{+\infty}\frac{n^2+n}{n^3\log^2 n+\log^4 n}$$

(2) $$\sum_{n=1}^{+\infty}\Big(n-n\cos\frac{1}{n}\Big)$$

(3) $$\sum_{n=1}^{+\infty}\frac{\log n+\log\log n}{\log^2 n+3\log^4\log n}\sin\frac{\pi}{n}$$

(4) $$\sum_{n=0}^{+\infty}\sqrt[7]{\frac{n^6+n^3}{n^{15}+1}}$$

(5) $$\sum_{n=0}^{+\infty}\sqrt[5]{\frac{n^4+n^3}{n^9+1}}$$

(6) $$\sum_{n=0}^{+\infty}\frac{(n!)^2}{(2n)!}$$

(7) $$\sum_{n=1}^{+\infty}\Big(n^2-n^2\cos\frac{1}{n^2}\Big)$$

(8) $$\sum_{n=1}^{+\infty}\Big(\frac{2n!+n^2}{3n!-e^n-1}\Big)^n$$

(9) $$\sum_{n=1}^{+\infty}\frac{n^5+1}{e^{(n^2)}}$$

(10) $$\sum_{n=1}^{+\infty}\Big(\frac{\log\log n}{\log n}\Big)^n$$

(11) $$\sum_{n=1}^{+\infty}(-1)^n\frac{1}{\log^2 n}$$

(12) $$\sum_{n=1}^{+\infty}(-1)^n\frac{1}{n\log^2 n-\log^5 n+\sin(n!)}$$

(13) $$\sum_{n=1}^{+\infty} n^2(e^{\frac{1}{n^2+n-1}} - 1)$$

(14) $$\sum_{n=1}^{+\infty} n^2 e^{-\sqrt[3]{n}}$$

(15) $$\sum_{n=1}^{+\infty} \sqrt{n}\log\Big(\frac{2n^2+3}{2n^2+2}\Big)$$

(16) $$\sum_{n=1}^{+\infty} \frac{\sin(n^4)+\sqrt[3]{n^5}}{\sqrt[3]{n^5}\log(n^n+n!)}$$

(17) $$\sum_{n=1}^{+\infty} \frac{\cos(n\pi)}{\sqrt{n}}$$

(18) $$\sum_{n=1}^{+\infty} \frac{2^n-n^2}{5^n+n^5+\log n}$$

(19) $$(*)\sum_{n=1}^{+\infty} \Big(\frac{1}{n^3\log n}\Big)^{\frac{n}{7}\sin\frac{2}{n}}$$

(20) $$(*)\sum_{n=1}^{+\infty} \Big(\frac{1}{n^5\log^2 n}\Big)^{\frac{n}{4}\sin\frac{1}{n}}.$$

3. Polinômio de Taylor

1. Calcule o polinômio de Taylor das seguintes funções até a ordem n especificada no ponto x_0 especificado.

(1) $y = \log(1+3x)$, $n = 3$, $x_0 = 0$.
(2) $y = \cos(x^2)$, $n = 10$, $x_0 = 0$.
(3) $y = e^x$, $n = 3$, $x_0 = -1$.
(4) $y = \sin x$, $n = 5$, $x_0 = \frac{\pi}{2}$.
(5) $y = \sqrt{1+x} - \sqrt{1-x}$, $n = 3$, $x_0 = 0$.
(6) $y = e^{(x^3)} - 1 - \sin(x^3)$, $n = 12$, $x_0 = 0$.
(7) $y = (e^{3x} - 1)\sin(2x)$, $n = 4$, $x_0 = 0$.
(8) $y = (e^{-x} - 1)^3$, $n = 4$, $x_0 = 0$.
(9) $y = 2 + x + 3x^2 - x^3$, $n = 2$, $x_0 = 1$.
(10) $y = \log x$, $n = 3$, $x_0 = 2$.
(11) $y = \frac{e^{2x}+e^{-2x}}{2}$, $n = 4$, $x_0 = 0$.
(12) $y = \frac{1}{\cos(2x)}$, $n = 3$, $x_0 = 0$.
(13) $y = \log(1+\sin x)$, $n = 3$, $x_0 = 0$.
(14) $y = \log(\cos x)$, $n = 4$, $x_0 = 0$.
(15) $y = \frac{1}{1+x+x^2}$, $n = 4$, $x_0 = 0$.

2. Calcule os seguintes limites:

(1) $$\lim_{x\to 0} \frac{e^x - 1 + \log(1-x)}{\tan x - x}$$

(2) $$\lim_{x\to 0} \frac{e^{(x^2)} - \cos x - \frac{3}{2}x^2}{x^4}$$

(3) $$\lim_{x\to 0} \Big(\frac{1}{x} - \frac{\cos x}{x^3} + \frac{\sin x}{x^4}\Big)$$

(4) $$\lim_{x\to 0} \frac{\sin(x)\log(1+x) - x^2}{1-\cos(x^{\frac{3}{2}})}$$

(5) $$\lim_{n\to\infty} \Big(n - n^2 \log\Big(1 + \sin\frac{1}{n}\Big)\Big)$$

(6) $$\lim_{n\to\infty} \frac{\log(1+\frac{1}{n}) - \frac{1}{n}}{1-\cos\frac{1}{n}}$$

(7) $$\lim_{n\to\infty} \frac{\sin\frac{1}{n^2} - \frac{1}{n^2}}{\sin\frac{1}{n} - \frac{1}{n}}$$

(8) $$\lim_{n\to\infty} n^2\Big(e^{(\frac{1}{n}+\frac{1}{n^2})} - 1 - \frac{1}{n} - \frac{2}{n^2}\Big).$$

3. Estabeleça o caráter das seguintes séries:

(1) $$\sum_{n=1}^{+\infty} \frac{n}{\log^2 n}\Big(e^{\frac{1}{n}} - 1 - \frac{1}{n}\Big)$$

(2) $$\sum_{n=1}^{+\infty} \log n\Big(\sin\frac{1}{n} - \frac{1}{n}\Big)$$

(3) $$\sum_{n=1}^{+\infty} \log\Big(\cos\frac{2}{n} + \frac{2}{n^2}\Big)$$

(4) $$\sum_{n=1}^{+\infty} n^2\Big(\frac{1}{1+\sin\frac{1}{n}} - 1 + \frac{1}{n} - \frac{1}{n^2}\Big).$$