1 Strategic Games

Definition 1.1: Strategic Game

A **strategic game** is a model of interacting decision-makers.

- a set of **players** (decision-makers)
- for each player, a set of **actions**
- for each player, **preferences** over the set of action profiles (list of all the players' actions)

Example- *Animals fighting over some prey*:

- **Players**: Animals
- **Actions**: Concession times
- **Preferences**: A reflection of whether an animal wins or loses

It is convenient to specify the players' preferences by giving *payoff functions* that represent them. With only *ordinal* significance (e.g., $a = 1, b = 1, c = 10$), the only conclusion that can be derived is that a player prefers, for instance, c to b and b to a.

In a strategic game, time is absent from the model. In other words, a player chooses their actions the same time as the other player so the actions are chosen once and for all.

2 The Prisoner's Dilemma

About: Two suspects in a major crime are held in separate cells. There is enough evidence to convict each of them of a minor offense, but not enough evidence to convict either of them of the major crime unless one of them acts as an informer against the other (finks). If they **both stay quiet**, each will be convicted of the minor offense and spend *one year in prison.* If one and only **one of them finks**, she will be *freed* and used as a witness against the other, who will spend *four years in prison.* If they **both fink**, each will spend *three years in prison.*

- **Players**: The two suspects
- **Actions**: Each player's set of actions is {Quiet, Fink}
- **S1 Preferences**: $(Fink, Quiet)$: Freed, $(Quiet, Quiet)$: 1 year in prison, $(Fink, Fink)$: 3 years in prison, $(Quiet, Fink)$: 4 years in prison
- **S2 Preferences**: $(Quiet, Fink)$: 4 years in prison, $(Quiet, Quiet)$: 1 year in prison, $(Fink, Fink)$: 3 years in prison, $(Fink, Quiet)$: Freed

Suspect 1's Payoff function:

$$u_1(Fink, Quiet) > u_1(Quiet, Quiet) > u_1(Fink, Fink) > u_1(Quiet, Fink) \tag{1}$$

Suspect 2's Payoff function:

$$u_2(Quiet, Fink) > u_2(Quiet, Quiet) > u_2(Fink, Fink) > u_2(Fink, Quiet) \tag{2}$$

The *Prisoner's Dilemma* models a situation in which there are gains from cooperation (*Quiet* than *Fink*) but each player has an incentive to "free-ride" (*Fink*).

3 Bach or Stravinsky?

About: Two people wish to go out together. Two concerts are available: one of music by Bach, and one of music by Stravinsky. One per- son prefers Bach and the other prefers Stravinsky. If they go to **different concerts**, each of them is *equally unhappy listening to the music of either composer*.

4 Matching Pennies

About: Two people choose, simultaneously, whether to show the **Head or the Tail** of a coin. If they show the **same side**, *person 2 pays person 1 a dollar*; if they show **different sides**, *person 1 pays person 2 a dollar*. Each person cares only about the amount of money she receives, and (naturally!) prefers to receive more than less.

5 Stag Hunt

About: Each of a group of hunters has two options: she may remain attentive to the pursuit of a stag, or catch a hare. If all hunters pursue the stag, they catch it and share it equally; if any hunter devotes her energy to catching a hare, the stag escapes, and the hare belongs to the defecting hunter alone. Each hunter prefers a share of the stag to a hare.