See Hammack's *Book of Proof* pages 3-13, 173-180, 194-198

1 Sets

Definition 1.1: Set

A **set** is a collection of things.

Definition 1.2: Elements

Elements are things in the *set*.

Example:

- {A, B, C, ..., Z}

A *set* can be **infinite** (e.g., whole numbers) or **finite** (e.g., the alphabet).

Two sets are **equal** if they contain the same elements:

- $\{1, 2, 3, 4\} = \{4, 3, 2, 1\}$

1.1 Notation

- A-Z: stands for sets (e.g. $A = \{1, 2, 3, 4\}$)
- ϵ: in (e.g. $2 \ \epsilon \ A$)
- $\mathbb{N}$: set of natural numbers (positive whole numbers)
- $\mathbb{Z}$: set of integers
- $\mathbb{R}$: set of all real numbers
- $|X|$: cardinality (i.e., number of elements of a *finite set*)
- $\emptyset$: empty set ($|\emptyset| = 0$)
- $\{expression : rule\}$: set-builder notation

1.2 The Cartesian Product

Definition 1.3: Ordered Pair

An **ordered pair** is a list of two things (i.e., numbers, letters, sets, etc.) enclosed by two parentheses.

Unlike *sets*, order matters for an ordered pair/list (same idea in programming).

Definition 1.4: Cartesian Product

The **cartesian product** is a set of all ordered pairs from a group of sets.

Example:

- $A = \{1, 3, 5\}$
- $B = \{2, 4\}$
- $A\ x\ B = \{(1, 2), (1, 4), (3, 2), (3, 4), (5, 2), (5, 4)\}$

Definition 1.5: Cartesian Powers

The **cartesian powers** states for any set A and positive integer n, the power $A^n is the cartesian product of$ A $with itself$ n $times.$

1.3 Subsets

Definition 1.6: Subset

A **subset** is if every element of B is in A (*superset*) (e.g., $A \supset B$, $B \subset A$) .

Every set is a subset of itself (excluding empty sets). The empty set is a subset of every set

2 Relations

3 Functions