Fintech 545
Assignment 3

Problem 1

Problem 1

Calculate and compare the expected value and standard deviation of price at time $t(P_t)$, given each of the 3 types of price returns, assuming $r_t - N(0, \sigma^2)$. Simulate each return equation using $r_t - N(0, \sigma^2)$ and show the mean and standard deviation match your expectations.

1.1 Calculate the mean and variance theoratically

1. Classical Brownian Motion

$$P_t = P_{t-1} + r_t$$

$$P_t = P_0 + \sum r_i$$

$$E(P_t) = E(P_0 + \sum r_i)$$

$$E(P_t) = P_0$$

$$var(P_1) = var(P_0 + r_i)$$

$$var(P_t) = t\sigma^2$$

1.1 Calculate the mean and variance theoratically

2. Arithmetic Return System

$$P_t = P_{t-1}(1 + r_t)$$

$$E(P_t) = E(P_{t-1} + P_{t-1}r_t)$$

$$E(P_t) = E(P_{t-1})$$

$$E(P_t) = P_{t-1}$$

$$var(P_t) = var(P_{t-1} + P_{t-1}r_t)$$

$$var(P_t) = P_{t-1}^2\sigma^2$$

1.1 Calculate the mean and variance theoratically

3. Geometric Brownian Motion

$$ln(P_t) = ln(P_{t-1}) + r_t$$

$$E(ln(P_t)) = E(ln(P_{t-1}) + r_t)$$

$$E(ln(P_t)) = E(ln(P_{t-1}))$$

$$E(ln(P_t)) = ln(P_0)$$

$$var(ln(P_1)) = var(ln(P_0) + r_t)$$

$$var(ln(P_1)) = \sigma^2$$

$$var(ln(P_t)) = t\sigma^2$$

1.2 Do the simulation

Implement the simulation functions for 3 types of price returns. Return the expected value and variance of simulations

```
def sim_CBM(t,P0,var,sim):
    result = []
    start = P0
    for _ in range(sim):
        for i in range(t):
            start+=np.random.normal(0,np.sqrt(var))
        result.append(start)
        start = P0
    return (np.sum(result)/len(result), np.var(result))
def sim_ARS(t,P0,sigma,sim):
    result = []
    start = P0
    for _ in range(sim):
        start *= (1+np.random.normal(0,np.sqrt(var)))
        result.append(start)
        start = P0
    return (np.sum(result)/len(result), np.var(result))
def sim_GBM(t,P0,sigma,sim):
    result = []
    start = np.log(P0)
    for _ in range(sim):
        for i in range(t):
            start+=np.random.normal(0,np.sqrt(var))
        result.append(start)
        start = np.log(P0)
    return (np.sum(result)/len(result), np.var(result))
```

Result

Theoratical calculation and simulation result matched

```
t = 100
P0= 100
var = 0.01
sim = 10000
print("The expectations of simulated and theoratical Classical Brownian Motion are", [sim_CBM(t,P0,var,sim)[0], P0])
print("The variance of simulated and theoratical Classical Brownian Motion are", [sim_CBM(t,P0,var,sim)[1], t*var])
print("The expectations of simulated and theoratical Arithmetric Return System are", [sim_ARS(t,P0,var,sim)[0], P0])
print("The variance of simulated and theoratical Arithmetric Return System are", [sim_ARS(t,P0,var,sim)[1], P0**2*va
print("The expectation of simulated and theoratical Geometric Brownian Motion are", [sim_GBM(t,P0,var,sim)[0], np.lo
print("The variance of simulated and theoratical Geometric Brownian Motion are", [sim_GBM(t,P0,var,sim)[1], t*var])
```

```
The expectations of simulated and theoratical Classical Brownian Motion are [100.0051898884819, 100]
The variance of simulated and theoratical Classical Brownian Motion are [0.9986879313439958, 1.0]
The expectations of simulated and theoratical Arithmetric Return System are [100.04759162793532, 100]
The variance of simulated and theoratical Arithmetric Return System are [99.40830591599008, 100.0]
The expectation of simulated and theoratical Geometric Brownian Motion are [4.616024282093369, 4.605170185988092]
The variance of simulated and theoratical Geometric Brownian Motion are [0.968106451236245, 1.0]
```

Problem 2

Problem 2

Implement a function similar to the "return_calculate()" in this week's code. Allow the user to specify the method of return calculation.

Use INTC.csv. Calculate the arithmetic returns for INTC. Remove the mean from the series so that the mean(INTC)=0

Calculate VaR 1. Using a normal distribution. 2. Using a normal distribution with an Exponentially Weighted variance ($\lambda = 0.94$) 3. Using a MLE fitted T distribution. 4. Using a Historic Simulation.

Compare the 4 values. Look at the empirical distribution of returns, in sample.

Download from Yahoo! Finance the prices since the end of the data in the CSV file(about 3 months). Look the empirical distribution of returns, out of sample.

Discuss the ability of these models to describe the risk in this stock

Problem 2

Implement "return_calculate()"

In [62]:
```
def return_calculate(price, method="DISCRETE"):
    price = price.pct_change().dropna()
    if method == "DISCRETE":
        return price
    elif method == "LOG":
        return np.log(price)
```

Read INTC data

In [63]:
```
data = pd.read_csv("INTC.csv")
data
```

Out[63]:

	Date	INTC
0	9/27/2021	52.915421
1	9/28/2021	52.276489
2	9/29/2021	51.782768
3	9/30/2021	51.579464
4	10/1/2021	52.140957
...	...	...

Problem 2

Calculate the arithmetic returns for INTC and remove the mean

In [64]:
```
returns = return_calculate(data['INTC'])
returns = returns - np.mean(returns)
returns
```

Out[64]:
```
1      -0.011228
2      -0.008598
3      -0.003080
4       0.011732
5      -0.006395
          ...
166    -0.007011
167     0.013565
168     0.031178
169     0.025455
170    -0.002072
Name: INTC, Length: 170, dtype: float64
```

Problem 2

1. Calculate VaR using a normal distribution

In [65]:
```
sim1 = np.random.normal(0,np.std(returns),size=10000)
-np.percentile(sim1,5)
```

Out[65]: 0.03592470271112527

2. Calculate VaR using a normal distribution with an Exponentially Weighted variance (λ=0.94)

In [66]:
```
def expo_weighted_cov(data,lam):
    weight = np.zeros(170)
    t = len(data)
    for i in range(t):
        weight[t-1-i]  = (1-lam)*lam**i
    weight = weight/sum(weight)
    norm_data = data - data.mean()
    return norm_data.T @ (np.diag(weight) @ norm_data)
var = expo_weighted_cov(returns,0.94)
sim2 = np.random.normal(0,np.sqrt(var),size=10000)
-np.percentile(sim2,5)
```

Out[66]: 0.03931295126922911

Problem 2

3. Calculate VaR using a MLE fitted T distribution

```
In [67]: from scipy import stats
         from scipy.optimize import minimize
         def MLE_T(p):
             return -1*np.sum(stats.t.logpdf(returns, df=p[0], loc = p[1],scale=p[2]))
         constraints=({"type":"ineq", "fun":lambda x: x[0]-1},
                      {"type":"ineq", "fun":lambda x: x[2]})
         df, loc, scale = minimize(MLE_T, x0 = (10,np.mean(returns),np.std(returns)),constraints=constraints).x

         sim3= stats.t(df=df, scale=scale).rvs(10000)
         -np.percentile(sim3,5)

Out[67]: 0.03318426205106343
```

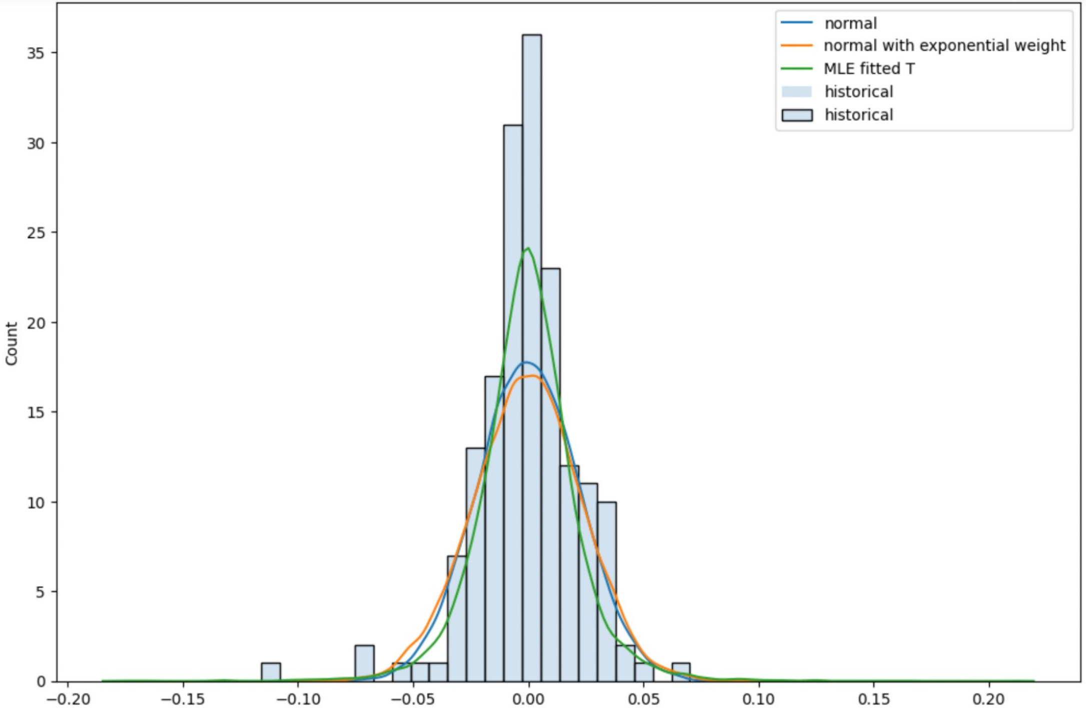
4. Calculate VaR using a Historic Simulation

```
In [68]: -np.percentile(returns,5)

Out[68]: 0.029574903865632305
```

Draw historical distributions

69]:
```
f, ax = plt.subplots(1, 1, figsize =(12,8))
sns.histplot(returns, ax = ax, label='historical', alpha = 0.2)
sns.distplot(sim1, ax = ax, hist=False, label='normal')
sns.distplot(sim2, ax = ax, hist=False, label='normal with exponential weight')
sns.distplot(sim3, ax = ax, hist=False, label='MLE fitted T')
ax.legend()
```



None of them fits very well with the historical data. The best fit distribution is MLE fitted T.

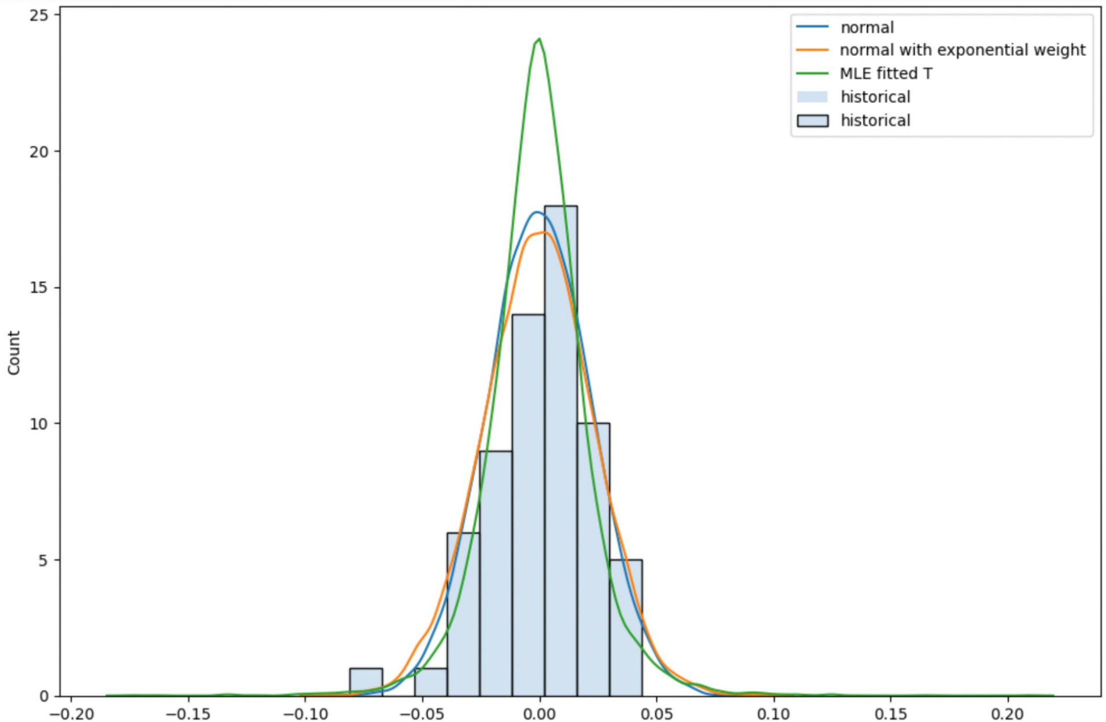
Download data from Yahoo Finance

```
import yfinance as yf
intc_df = yf.download('INTC', start='2022-05-31', end='2022-09-01', progress=False)
f_returns = return_calculate(intc_df["Adj Close"])
f_returns -= f_returns.mean()
```

Draw future distributions

```
f, ax = plt.subplots(1, 1, figsize = (12,8))
sns.histplot(f_returns, ax = ax, label='historical', alpha = 0.2)
sns.distplot(sim1, ax = ax, hist=False, label='normal')
sns.distplot(sim2, ax = ax, hist=False, label='normal witl
sns.distplot(sim3, ax = ax, hist=False, label='MLE fitted
ax.legend()
```

The normal and the normal with exponential weight distributions both fit the future distribution.



Problem 3

Problem 3 ¶

Using Portfolio.csv and DailyPrices.csv. Assume the expected return on all stocks is 0.

This file contains the stock holdings of 3 portfolios. You own each of these portfolios. Calculate the VaR of each portfolio as well as your total VaR(VaR of the total holdings).

Discuss your methods, why you chose those methods, and your results.

Problem 3

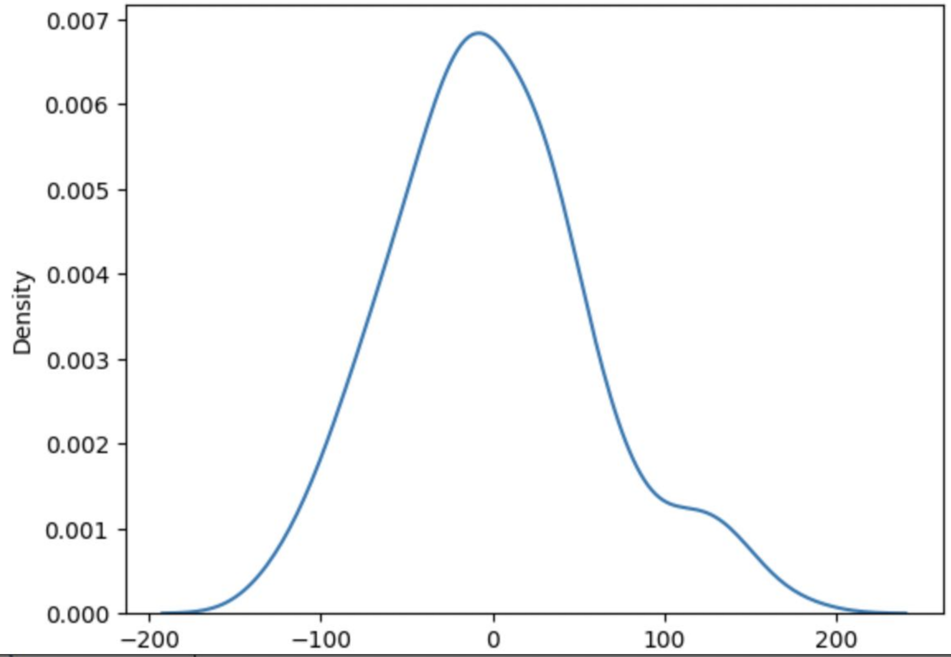
Since we only have half of the total stocks that are normality distributed, the historical VaR should be used.

```
p = pd.read_csv("Portfolio.csv")
dp = pd.read_csv("DailyPrices.csv")
s = 0
for i in p[p["Portfolio"]=="A"]['Stock']:
    if shapiro(return_calculate(dp.loc[:,i]))[1]>0.05:
        s+=1
print("the percentage of portfolio A that is normaly distributed is:", s/len(p[p["Portfolio"]=="A"]))
s=0
for i in p[p["Portfolio"]=="B"]['Stock']:
    if shapiro(return_calculate(dp.loc[:,i]))[1]>0.05:
        s+=1
print("the percentage of portfolio B that is normaly distributed is:", s/len(p[p["Portfolio"]=="B"]))
s=0
for i in p[p["Portfolio"]=="C"]['Stock']:
    if shapiro(return_calculate(dp.loc[:,i]))[1]>0.05:
        s+=1
print("the percentage of portfolio C that is normaly distributed is:", s/len(p[p["Portfolio"]=="C"]))
```

```
the percentage of portfolio A that is normaly distributed is: 0.5277777777777778
the percentage of portfolio B that is normaly distributed is: 0.46875
the percentage of portfolio C that is normaly distributed is: 0.59375
```

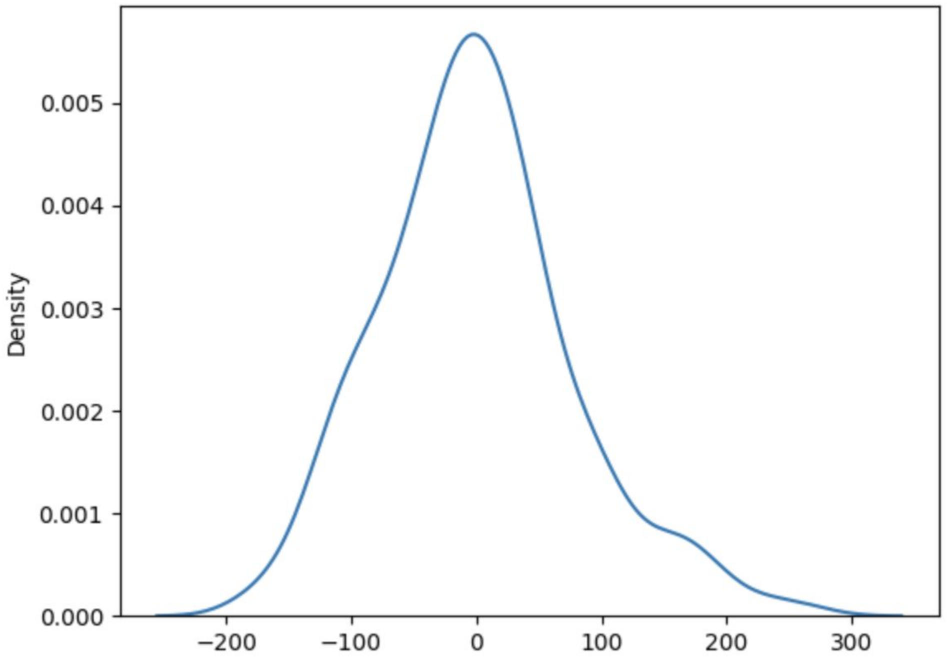
```python
portA_sim = []
for _ in range(200):
    value = 0
    for i in p[p["Portfolio"]=="A"]['Stock']:
        cp = dp.loc[:,i].iloc[-1]
        returns = return_calculate(dp.loc[:,i])
        value+=cp*(1+random.choice(list(returns)))
    portA_sim.append(value)
portA_sim -= np.mean(portA_sim)
sns.kdeplot(portA_sim)
print(-np.percentile(portA_sim,5))
```

```
89.0951498747323
```



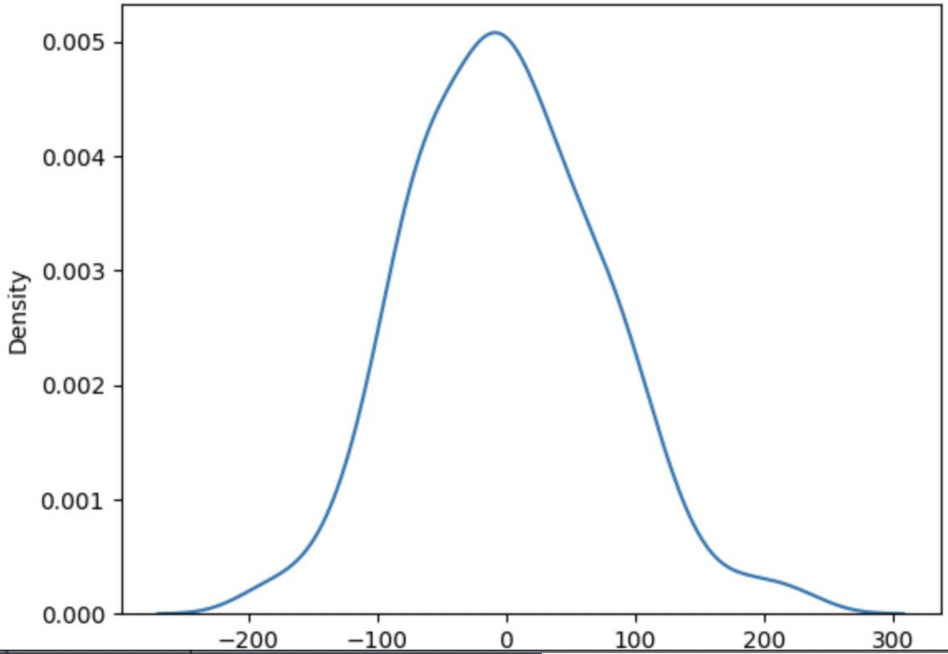
```python
portB_sim = []
for _ in range(200):
    value = 0
    for i in p[p["Portfolio"]=="B"]['Stock']:
        cp = dp.loc[:,i].iloc[-1]
        returns = return_calculate(dp.loc[:,i])
        value+=cp*(1+random.choice(list(returns)))
    portB_sim.append(value)
portB_sim -= np.mean(portB_sim)
sns.kdeplot(portB_sim)
print(-np.percentile(portB_sim,5))
```

```
118.3693363523469
```



```
portC_sim = []
for _ in range(200):
    value = 0
    for i in p[p["Portfolio"]=="C"]['Stock']:
        cp = dp.loc[:,i].iloc[-1]
        returns = return_calculate(dp.loc[:,i])
        value+=cp*(1+random.choice(list(returns)))
    portC_sim.append(value)
portC_sim -= np.mean(portC_sim)
sns.kdeplot(portC_sim)
print(-np.percentile(portC_sim,5))
```

```
113.03743521224605
```



```python
port_sim = []
for _ in range(200):
    value = 0
    for i in p['Stock']:
        cp = dp.loc[:,i].iloc[-1]
        returns = return_calculate(dp.loc[:,i])
        value+=cp*(1+random.choice(list(returns)))
    port_sim.append(value)
port_sim -= np.mean(port_sim)
sns.kdeplot(port_sim)
print(-np.percentile(port_sim,5))
```

```
202.7390290845502
```