Evaluation rules for $L23_{rr}$ (big-step semantics)

$$\frac{}{n \Downarrow n} \qquad \frac{}{\mathtt{T} \Downarrow \mathtt{T}} \qquad \frac{}{\mathtt{F} \Downarrow \mathtt{F}} \qquad \frac{}{() \Downarrow ()} \qquad \frac{}{[\lambda\ x\ \tau_1\ t_1] \Downarrow [\lambda\ x\ \tau_1\ t_1]}$$?

$$\frac{t_1 \Downarrow [\lambda\ x\ \tau_1\ t_1] \quad t_2 \Downarrow v_2 \quad [x \mapsto v_2]t_1 \Downarrow v_3}{(t_1\ t_2) \Downarrow v_3}$$

$$\frac{t_1 \Downarrow [\lambda\ f\ \tau_1\ t_{11}] \quad [f \mapsto (\mathtt{fix}\ [\lambda\ f\ \tau_1\ t_{11}])]t_{11} \Downarrow v_1}{(\mathtt{fix}\ t_1) \Downarrow v_1}$$

$$\frac{t_1 \Downarrow v_1 \quad [x \mapsto v_1]t_2 \Downarrow v_2}{\{x\ t_1\ t_2\} \Downarrow v_2} \qquad \frac{t_1 \Downarrow \mathtt{T} \quad t_2 \Downarrow v_2}{[t_1\ ?\ t_2 : t_3] \Downarrow v_2} \qquad \frac{t_1 \Downarrow \mathtt{F} \quad t_3 \Downarrow v_3}{[t_1\ ?\ t_2 : t_3] \Downarrow v_3}$$

$$\frac{t_1 \Downarrow n_1 \quad t_2 \Downarrow n_2}{[t_1 + t_2] \Downarrow n_1 {+} n_2} \qquad \frac{t_1 \Downarrow n_1 \quad t_2 \Downarrow n_2}{[t_1 - t_2] \Downarrow n_1 {-} n_2} \qquad \frac{t_1 \Downarrow n_1 \quad t_2 \Downarrow n_2}{[t_1 * t_2] \Downarrow n_1 {*} n_2}$$

$$\frac{t_1 \Downarrow n_1 \quad t_2 \Downarrow n_2}{[t_1 == t_2] \Downarrow n_1 {=} n_2} \qquad \frac{t_1 \Downarrow n_1 \quad t_2 \Downarrow n_2}{[t_1 < t_2] \Downarrow n_1 {<} n_2} \qquad \frac{t_1 \Downarrow \mathtt{T}}{!t_1 \Downarrow \mathtt{F}} \qquad \frac{t_1 \Downarrow \mathtt{F}}{!t_1 \Downarrow \mathtt{T}}$$

$$\frac{t_1 \Downarrow v_1\ \ldots\ t_k \Downarrow v_k}{[{\sim}\mathtt{l}_1\ t_1\ \ldots\ {\sim}\mathtt{l}_k\ t_k] \Downarrow [{\sim}\mathtt{l}_1\ v_1\ \ldots\ {\sim}\mathtt{l}_k\ v_k]} \qquad \frac{t_1 \Downarrow [\ldots\ {\sim}\mathtt{l}_i\ v_i\ \ldots]}{({\sim}\mathtt{l}_i\ t_1) \Downarrow v_i}$$

Typing rules for $L23_{rr}$

$$\frac{}{\Gamma \vdash n : \mathtt{I}} \qquad \frac{}{\Gamma \vdash \mathtt{T} : \mathtt{B}} \qquad \frac{}{\Gamma \vdash \mathtt{F} : \mathtt{B}} \qquad \frac{}{\Gamma \vdash () : \mathtt{U}}$$

$$\frac{\Gamma(x) = \tau_1}{\Gamma \vdash x : \tau_1} \qquad \frac{\Gamma, x : \tau_1 \vdash t_1 : \tau_2}{\Gamma \vdash \{\mathtt{lam}\ x\ \tau_1\ t_1\} : \tau_1 \to \tau_2}$$

$$\frac{\Gamma \vdash t_1 : \tau_1 \to \tau_2 \quad \Gamma \vdash t_2 : \tau_3 \quad \tau_3 <: \tau_1}{\Gamma(t_1\ t_2) : \tau_2} \qquad \frac{\Gamma \vdash t_1 : \tau_1 \to \tau_1}{\Gamma \vdash (\mathtt{fix}\ t_1) : \tau_1}$$

$$\frac{\Gamma \vdash t_1 : \tau_1 \quad \Gamma, x : \tau_1 \vdash t_2 : \tau_2}{\Gamma \vdash \{x\ t_1\ t_2\} : \tau_2}$$

$$\frac{\Gamma \vdash t_1 : \mathtt{B} \quad \Gamma \vdash t_2 : \tau_2 \quad \Gamma \vdash t_3 : \tau_3 \quad \tau_4 = s(\tau_2, \tau_3)}{\Gamma \vdash [t_1\ ?\ t_2 : t_3] : \tau_4}$$

$$\frac{\Gamma \vdash t_1 : \mathtt{I} \quad \Gamma \vdash t_2 : \mathtt{I}}{\Gamma \vdash [t_1 + t_2] : \mathtt{I}}$$

Likewise the other arithmetic operators.

$$\frac{\Gamma \vdash t_1 : \mathtt{I} \quad \Gamma \vdash t_2 : \mathtt{I}}{\Gamma \vdash [t_1 == t_2] : \mathtt{B}} \qquad \frac{\Gamma \vdash t_1 : \mathtt{I} \quad \Gamma \vdash t_2 : \mathtt{I}}{\Gamma \vdash [t_1 < t_2] : \mathtt{B}} \qquad \frac{\Gamma \vdash t_1 : \mathtt{B}}{\Gamma \vdash !t_1 : \mathtt{B}}$$

$$\frac{\Gamma \vdash t_1 : \tau_1 \quad \ldots \quad \Gamma \vdash t_k : \tau_k}{\Gamma \vdash [\sim\!\mathtt{l}_1\ t_1\ \ldots\ \sim\!\mathtt{l}_k\ t_k] : (\sim\!\mathtt{l}_1\ \tau_1\ \ldots\ \sim\!\mathtt{l}_k\ \tau_k)} \qquad \frac{\Gamma \vdash t_1 : (\ldots\ \sim\!\mathtt{l}_i\ \tau_i\ \ldots)}{\Gamma \vdash (\sim\!\mathtt{l}_i\ t_1) : \tau_i}$$

$$s(\tau_1, \tau_2) = \begin{cases} \tau_2 & \tau_1 <: \tau_2 \\ \tau_1 & \tau_2 <: \tau_1 \\ \text{fail} & \text{otherwise} \end{cases}$$