

Small-step evaluation rules for the $L23_\tau$ internal language

Corrections: The boxed rules for conditionals have been corrected; Z and SZ were exchanged in the t_1 position.

$$\begin{array}{c}
 \frac{t_1 \rightarrow t_1'}{\mathbf{S}t_1 \rightarrow \mathbf{S}t_1'} \quad \frac{}{[\mathbf{Z} + t_2] \rightarrow t_2} \quad \frac{}{[\mathbf{S}v_1 + t_2] \rightarrow [v_1 + \mathbf{S}t_2]} \quad \frac{t_1 \rightarrow t_1'}{[t_1 + t_2] \rightarrow [t_1' + t_2]} \\
 \\
 \frac{}{[\mathbf{Z} - v_2] \rightarrow \mathbf{Z}} \quad \frac{}{[v_1 - \mathbf{Z}] \rightarrow v_1} \quad \frac{}{[\mathbf{S}v_1 - \mathbf{S}v_2] \rightarrow [v_1 - v_2]} \\
 \\
 \frac{t_1 \rightarrow t_1'}{[t_1 - t_2] \rightarrow [t_1' - t_2]} \quad \frac{t_2 \rightarrow t_2'}{[v_1 - t_2] \rightarrow [v_1 - t_2']} \\
 \\
 \frac{}{[\mathbf{Z} < \mathbf{Z}] \rightarrow \mathbf{Z}} \quad \frac{}{[\mathbf{Z} < \mathbf{S}v_2] \rightarrow \mathbf{SZ}} \quad \frac{}{[v_1 < \mathbf{Z}] \rightarrow \mathbf{Z}} \quad \frac{}{[\mathbf{S}v_1 < \mathbf{S}v_2] \rightarrow [v_1 < v_2]} \\
 \\
 \frac{t_1 \rightarrow t_1'}{[t_1 < t_2] \rightarrow [t_1' < t_2]} \quad \frac{t_2 \rightarrow t_2'}{[v_1 < t_2] \rightarrow [v_1 < t_2']} \\
 \\
 \frac{}{[\mathbf{Z} == \mathbf{Z}] \rightarrow \mathbf{SZ}} \quad \frac{}{[\mathbf{Z} == \mathbf{S}v_2] \rightarrow \mathbf{Z}} \quad \frac{}{[\mathbf{S}v_1 == \mathbf{Z}] \rightarrow \mathbf{Z}} \quad \frac{}{[\mathbf{S}v_1 == \mathbf{S}v_2] \rightarrow [v_1 == v_2]} \\
 \\
 \frac{}{[(v_1, v_2) == (v_3, v_4)] \rightarrow [[v_1 == v_3]?[v_2 == v_4] : \mathbf{Z}]} \\
 \\
 \frac{t_1 \rightarrow t_1'}{[t_1 == t_2] \rightarrow [t_1' == t_2]} \quad \frac{t_2 \rightarrow t_2'}{[v_1 == t_2] \rightarrow [v_1 == t_2']} \\
 \\
 \boxed{\frac{}{[\mathbf{SZ} ? t_2 : t_3] \rightarrow t_2}} \quad \boxed{\frac{}{[\mathbf{Z} ? t_2 : t_3] \rightarrow t_3}} \quad \frac{t_1 \rightarrow t_1'}{[t_1 ? t_2 : t_3] \rightarrow [t_1' ? t_2 : t_3]} \\
 \\
 \frac{t_1 \rightarrow t_1'}{(t_1, t_2) \rightarrow (t_1', t_2)} \quad \frac{t_2 \rightarrow t_2'}{(v_1, t_2) \rightarrow (v_1, t_2')} \\
 \\
 \frac{t_1 \rightarrow t_1'}{1\#t_1 \rightarrow 1\#t_1'} \quad \frac{t_1 \rightarrow t_1'}{2\#t_1 \rightarrow 2\#t_1'} \quad \frac{}{1\#(v_1, v_2) \rightarrow v_1} \quad \frac{}{2\#(v_1, v_2) \rightarrow v_2}
 \end{array}$$