Dual Optimization for Newsvendor-like Problem

Chuwen

February 23, 2021

FMP

Recall FMP formulation:

$$f = \min_{x_{it}, u_{it}, \delta_t, \epsilon_t} \sum_t (b \cdot \delta_t + h \cdot \epsilon_t) \tag{1}$$

s.t.

$$\sum_i u_{it} + \delta_t - \epsilon_t = d_t, \quad \forall t \in T \tag{2}$$

$$s_{i,t+1} = s_{it} - \alpha_i u_{it} + \beta_i x_{i,t-\tau}, \quad \forall i \in I, t \in T \tag{3}$$

$$x_{it} + u_{i,t} \leq 1, \quad \forall i \in I, t \in T \tag{4}$$

$$x_{it} + x_{i\rho} + u_{i,\rho} \leq 1, \quad \forall i \in I, t \in T, \rho = t+1, ..., t+\tau \tag{5}$$

$$s_{it} \geq L, \quad \forall i \in I, t \in T \tag{6}$$

(3) - (6) can be expressed as $\Omega_i, \forall i \in I$

FMP: variant case and weighted summation

$$f = \min_{x_{it}, u_{it}, \delta_t, \epsilon_t} b^{\mathsf{T}}\delta + h^{\mathsf{T}}\epsilon \tag{7}$$

s.t.

$$\sum_i c_i u_{it} + \delta_t - \epsilon_t = d_t, \quad \forall t \in T \tag{8}$$

$$U_{i,\cdot}, S_{i,\cdot}, X_{i,\cdot} \in \Omega_i, \quad \forall i \in I \tag{9}$$

- b, h are time-variant
- $c_i \neq 1$.

FMP: Lagrangian relaxation

For the FMP, dual function:

$$\phi(\lambda) = -\sum_t \lambda_t d_t + \sum_i \min_{\Omega_i} c_i \sum_t \lambda_t u_{it}$$

It reduces to a set of low dimensional minimization problems for each $i \in I, \forall \lambda$
(Recall $\lambda_t \in [-b, h]$ and $\delta_t^\star = \epsilon_t^\star = 0$ else unbounded)

$$\min_{\Omega_i} c_i \sum_t \lambda_t \cdot u_{i,t} \tag{10}$$

(10) is the subproblem to be solved by dynamic programming.
(states: lifespan, action: work or start maintence)

FMP: subgradient method

At each iteration k:

- y_k solves $\phi(\lambda_k) = \min_y \lambda_k^T (y - b)$
- for FMP: $y_k = U_k^T c$, $U_k = (u_{it}^{(k)})$, solved from (10) by DP.
- update λ_k.

Results

Tests are done on FMP with random instances: rand(low, high) means random integer in [low, high]

- L = 2
- $d_t = \text{rand}(\frac{|I|}{2}, |I|), \forall t$
- $\tau_i = \text{rand}(2, 5), a_i = \text{rand}(2, 5), b_i = \text{rand}(5, 10), \forall i$
- $s_{i,0} = \text{rand}(5, 8), \forall i$
- $c = \text{rand}(1, 5)$

We consider following different parameters

i , invariant: $h_t = 2, p_t = 3, \forall t$ or variant: $h_t = \text{rand}(1, 5), p_t = \text{rand}(2, 6), \forall t$

ii , $c = 1$ or $c = \text{rand}(1, 5)$

so we have 4 test settings.

Results

The subgradient method (sg) stops at (maximum) 400 iterations, then compared with benchmarks by Gurobi (grb).

- lb_grb, f_{grb} are lower bound, primal value from grb.
- $f_{\text{grb\_lp}}$ is the relaxed LP value from grb
- t_grb, t_sg are runtime from grb and sg, respectively.
- ϕ_{sg} is the dual value (ϕ) by sg.
- z_{sg} is primal value in sg using averaged primal recovery, i.e.

$$z_{\text{sg}} = z(\bar{y}_k) = f(\bar{\delta}_k, \bar{\epsilon}_k)$$

- ϕ_{gap}, z_gap are relative gap from sg to grb for dual and primal values.

FMP: $c = 1$, invariant h, b

	lb_grb	f_{grb}	t_grb	t_sg	ϕ_{sg}	ϕ_{gap}	z_{sg}	z_gap
0	0.00	0.00	0.01	0.38	0.00	0.00%	0.02	inf%
1	0.00	0.00	0.00	0.33	-0.02	-inf%	2.33	inf%
2	3.00	3.00	0.00	0.24	3.00	0.00%	3.02	0.83%
3	0.00	0.00	0.00	0.33	0.00	0.00%	0.02	inf%
4	0.00	0.00	0.00	0.34	0.00	0.00%	0.01	inf%
5	0.00	0.00	0.00	0.33	0.00	0.00%	0.02	inf%
6	3.00	3.00	0.00	0.18	3.00	0.00%	3.02	0.57%
7	0.00	0.00	0.00	0.39	0.00	0.00%	0.06	inf%
8	0.00	0.00	0.00	0.31	0.00	0.00%	0.02	inf%
9	0.00	0.00	0.00	0.32	0.00	0.00%	0.01	inf%
10	0.00	0.00	0.01	0.38	0.00	0.00%	0.02	inf%
11	3.00	3.00	0.00	0.21	3.00	0.00%	3.03	1.10%
12	9.00	9.00	0.00	0.07	9.00	0.00%	9.04	0.50%
13	6.00	6.00	0.00	0.17	6.00	0.00%	6.03	0.50%
14	0.00	0.00	0.00	0.40	0.00	0.00%	0.05	inf%
15	3.00	3.00	0.00	0.29	3.00	0.00%	3.03	1.01%
16	9.00	9.00	0.00	0.06	9.00	0.00%	9.05	0.50%
17	9.00	9.00	0.00	0.11	9.00	0.00%	9.05	0.50%
18	0.00	0.00	0.00	0.50	0.00	0.00%	0.05	inf%
19	9.00	9.00	0.00	0.13	9.00	0.00%	9.05	0.50%

FMP: $c = 1$ variant h, b

	lb_grb	f_{grb}	t_grb	t_sg	ϕ_{sg}	ϕ_{gap}	z_{sg}	z_gap
0	0.00	0.00	0.00	0.37	-0.00	0.00%	3.58	inf%
1	0.00	0.00	0.00	0.33	-3.00	-inf%	8.94	inf%
2	0.00	0.00	0.00	0.40	-0.00	0.00%	0.07	inf%
3	2.00	2.00	0.00	0.29	2.00	0.00%	2.01	0.67%
4	0.00	0.00	0.00	0.32	-0.05	-inf%	1.05	inf%
5	0.00	0.00	0.00	0.32	-0.00	0.00%	0.16	inf%
6	0.00	0.00	0.00	0.33	-0.00	0.00%	0.10	inf%
7	0.00	0.00	0.00	0.45	-0.01	-inf%	2.20	inf%
8	0.00	0.00	0.00	0.33	0.00	0.00%	0.08	inf%
9	0.00	0.00	0.00	0.32	-2.00	-inf%	7.91	inf%
10	8.00	8.00	0.00	0.19	8.00	0.00%	8.02	0.31%
11	2.00	2.00	0.00	0.38	2.00	0.00%	4.04	101.85%
12	0.00	0.00	0.00	0.38	-0.03	-inf%	2.46	inf%
13	4.00	4.00	0.00	0.26	4.00	0.00%	4.04	1.10%
14	3.00	3.00	0.00	0.23	3.00	0.00%	3.03	1.01%
15	5.00	5.00	0.00	0.41	5.00	0.00%	5.06	1.18%
16	0.00	0.00	0.00	0.40	0.00	0.00%	0.02	inf%
17	0.00	0.00	0.00	0.37	-0.02	-inf%	1.97	inf%
18	3.00	3.00	0.00	0.34	3.00	0.00%	3.02	0.70%
19	14.00	14.00	0.00	0.07	14.00	0.00%	14.07	0.50%
20	4.00	4.00	0.00	0.39	4.00	0.00%	4.37	9.34%

FMP: $c \neq 1$ invariant h, b

	lb_grb	f_{grb}	t_grb	$f_{\text{grb\_lp}}$	t_sg	ϕ_{sg}	ϕ_{gap}	z_{sg}	z_gap
0	3.00	3.00	0.00	0.00	0.20	0.00	-100.00%	4.39	46.28%
1	3.00	3.00	0.00	0.00	0.08	3.00	0.00%	3.02	0.50%
2	6.00	6.00	0.00	0.00	0.24	0.00	-100.00%	11.52	92.05%
3	6.00	6.00	0.00	0.00	0.26	0.00	-100.00%	3.70	-38.30%
4	6.00	6.00	0.00	0.00	0.25	0.00	-100.00%	2.58	-56.93%
5	2.00	2.00	0.00	0.00	0.38	0.00	-100.00%	0.95	-52.43%
6	4.00	4.00	0.00	0.00	0.36	0.00	-100.00%	8.07	101.66%
7	2.00	2.00	0.00	0.00	0.22	0.00	-100.00%	0.95	-52.43%
8	0.00	0.00	0.00	0.00	0.17	0.00	nan%	0.02	inf%
9	0.00	0.00	0.00	0.00	0.23	0.00	nan%	5.08	inf%
10	8.00	8.00	0.00	0.00	0.17	3.00	-62.50%	3.02	-62.25%
11	4.00	4.00	0.01	0.00	0.28	0.00	-100.00%	0.10	-97.61%
12	6.00	6.00	0.00	0.00	0.36	0.00	-100.00%	2.96	-50.66%
13	0.00	0.00	0.00	0.00	0.23	0.00	nan%	0.03	inf%
14	6.00	6.00	0.00	0.00	0.39	0.00	-100.00%	2.59	-56.78%
15	3.00	3.00	0.00	0.75	0.29	3.00	0.00%	3.02	0.57%
16	8.00	8.00	0.00	0.00	0.23	3.00	-62.50%	3.02	-62.25%
17	8.00	8.00	0.00	0.00	0.23	3.00	-62.50%	3.02	-62.25%
18	8.00	8.00	0.00	0.00	0.37	0.00	-100.00%	5.62	-29.81%
19	9.00	9.00	0.00	0.00	0.31	0.00	-100.00%	4.73	-47.49%

FMP: $c \neq 1$ variant h, b

	lb_grb	f_{grb}	t_grb	f_{grb_lp}	t_sg	ϕ_{sg}	ϕ_{gap}	z_{sg}	z_gap
0	5.00	5.00	0.00	0.00	0.07	0.00	-100.00%	0.00	-100.00%
1	2.00	2.00	0.00	0.00	0.30	0.00	-100.00%	1.19	-40.71%
2	0.00	0.00	0.00	0.00	0.43	0.00	nan%	5.98	inf%
3	0.00	0.00	0.00	0.00	0.25	0.00	nan%	12.87	inf%
4	5.00	5.00	0.00	0.00	0.24	0.00	-100.00%	4.63	-7.35%
5	5.00	5.00	0.00	0.00	0.23	0.00	-100.00%	12.02	140.33%
6	4.00	4.00	0.00	0.00	0.23	0.00	-100.00%	0.85	-78.87%
7	0.00	0.00	0.00	0.00	0.29	0.00	nan%	2.05	inf%
8	2.00	2.00	0.00	0.00	0.23	0.00	-100.00%	3.03	51.44%
9	8.00	8.00	0.00	0.00	0.32	0.00	-100.00%	6.71	-16.18%
10	9.00	9.00	0.02	0.00	0.30	0.00	-100.00%	8.41	-6.59%
11	4.00	4.00	0.01	0.00	0.16	0.00	-100.00%	0.02	-99.45%
12	6.00	6.00	0.00	0.00	0.06	6.00	0.00%	6.03	0.50%
13	8.00	8.00	0.00	0.00	0.06	8.00	0.00%	8.04	0.50%
14	9.00	9.00	0.00	0.00	0.30	0.00	-100.00%	7.41	-17.66%
15	6.00	6.00	0.00	0.00	0.28	0.00	-100.00%	1.96	-67.34%
16	5.00	5.00	0.00	0.00	0.32	0.00	-100.00%	5.46	9.25%
17	11.00	11.00	0.00	0.00	0.52	0.00	-100.00%	10.94	-0.53%
18	12.00	12.00	0.00	0.00	0.25	0.00	-100.00%	1.98	-83.48%
19	14.00	14.00	0.00	0.00	0.00	14.00	0.00%	14.00	0.00%

Conclusion

- zero duality gap: $\phi^\star = f^\star$, $\phi^\star$ is the best bound by ϕ and $f^\star$ is the best primal value.
- can we bound the quality of heuristic for averaged solution? $\bar{z}_k = z(\bar{y}_k)$ converges to $\phi^\star$:

$$|\bar{z}_k - \phi^\star|$$

- for variant case, can we improve $\bar{z} = z(\bar{y})$?