Dual Optimization for Newsvendor-like Problem

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March 7, 2021

FMP: variant case and weighted summation

$$f = \min_{x_{it}, u_{it}, \delta_t, \epsilon_t} b^{\mathsf{T}} \delta + h^{\mathsf{T}} \epsilon \tag{1}$$

s.t.

$$\sum_i c_i u_{it} + \delta_t - \epsilon_t = d_t, \quad \forall t \in T \tag{2}$$

$$U_{i,\cdot}, S_{i,\cdot}, X_{i,\cdot} \in \Omega_i, \quad \forall i \in I \tag{3}$$

- b, h are time-variant
- $c_i \neq 1$.

FMP: Lagrangian relaxation

For the FMP, dual function:

$$\phi(\lambda) = -\sum_t \lambda_t d_t + \sum_i \min_{\Omega_i} c_i \sum_t \lambda_t u_{it}$$

It reduces to a set of low dimensional minimization problems for each $i \in I, \forall \lambda$
(Recall $\lambda_t \in [-b, h]$ and $\delta_t^\star = \epsilon_t^\star = 0$ else unbounded)

$$\min_{\Omega_i} c_i \sum_t \lambda_t \cdot u_{i,t} \tag{4}$$

(4) is the subproblem to be solved by dynamic programming.
(states: lifespan, action: work or start maintence)

FMP: subgradient method

At each iteration k:

- y_k solves $\phi(\lambda_k) = \min_y \lambda_k^{\mathsf{T}}(y - b)$
- for FMP: $y_k = U_k^{\mathsf{T}} c$, $U_k = (u_{it}^{(k)})$, solved from (4) by DP.
- update λ_k.

Small case: $|I| = 1, t = 2$

$c = 1$

$$\begin{aligned} \min \quad & 2\delta_0 + 3\delta_1 + 4\epsilon_0 + \epsilon_1 \\ s.t. \quad & u_0 + \delta_0 - \epsilon_0 = 1 \\ & u_1 + \delta_1 - \epsilon_1 = 0 \\ & 2u_0 + s_0 = 6 \\ & -6x_0 + 2u_1 - s_0 + s_1 = 0 \\ & x_0 + u_0 \leq 1 \\ & x_1 + u_1 \leq 1 \\ & s_0, s_1 \geq 2 \end{aligned}$$

dual

$$-2 \leq \lambda_0 \leq 4$$

$$-3 \leq \lambda_1 \leq 1$$

$$\min_u \lambda_0(u_0 - 1) + \lambda_1(u_1 - 0)$$

$c = 2$

$$\begin{aligned} \min \quad & 2\delta_0 + 3\delta_1 + 4\epsilon_0 + \epsilon_1 \\ s.t. \quad & 2u_0 + \delta_0 - \epsilon_0 = 1 \\ & 2u_1 + \delta_1 - \epsilon_1 = 0 \\ & 2u_0 + s_0 = 6 \\ & -6x_0 + 2u_1 - s_0 + s_1 = 0 \\ & x_0 + u_0 \leq 1 \\ & x_1 + u_1 \leq 1 \\ & s_0, s_1 \geq 2 \end{aligned}$$

$$\min_u \lambda_0(2u_0 - 1) + \lambda_1(2u_1 - 0)$$

Small case: $|I| = 1, t = 2$

results:

$$
\begin{aligned}
\textit{gurobi}: & \\
c = 1 & \qquad c = 2 \\
u^\star = [1, 0] & \qquad u^\star = [0, 0] \\
f^\star = 0 & \qquad f^\star = 2 \\
\textit{subgradient}: & \\
\lambda_k = [-2, 0] & \qquad \lambda_k = [-3.5203e^{-5}, 0] \\
\phi_k = 0 & \qquad \phi_k = 0 \\
u_k = [1, 0] & \qquad u_k = [0, 0] \\
\phi_k = f^\star & \qquad \phi_k \neq f^\star
\end{aligned}
$$

Small case, relax

results:

$$\begin{aligned} gurobi : u \in [0,1] & \\ c = 1 & \qquad\qquad c = 2 \\ u^\star = [1,0] & \qquad\qquad u^\star = [0.5,0] \\ f^\star = 0 & \qquad\qquad f^\star = 0 \\ gurobi : x \in [0,1] & \\ c = 1 & \qquad\qquad c = 2 \\ u^\star = [1,0] & \qquad\qquad u^\star = [0,0] \\ f^\star = 0 & \qquad\qquad f^\star = 2 \end{aligned}$$

Small case

The subgradient method (sg) stops at (maximum) 400 iterations, then compared with benchmarks by Gurobi (grb).

- lb_grb, f_{grb} are lower bound, primal value from grb.
- f^u_{relax}, f^x_{relax} primal value from grb for problem with relaxed u and x only, respectively: $f^x_{\text{relax}} = f^\star$?
- t_grb, t_sg are runtime from grb and sg, respectively.
- ϕ_{sg} is the dual value (ϕ) by sg.
- z_{sg} is primal value in sg using averaged primal recovery, i.e.

$$z_{\text{sg}} = z(\bar{y}_k) = f(\bar{\delta}_k, \bar{\epsilon}_k)$$

- ϕ_{gap}, z_gap are relative gap from sg to grb for dual and primal values.

FMP: $c = 1$ small case 5×5

	lb_grb	f_{grb}	t_grb	f^u_{relax}	f^x_{relax}	t_sg	ϕ_{sg}	ϕ_{gap}
0	34.00	34.00	0.01	32.50	34.00	1.26	34.00	0.00%
1	42.00	42.00	0.01	40.83	42.00	0.01	42.00	0.00%
2	84.00	84.00	0.01	78.25	84.00	0.01	84.00	0.00%
3	49.00	49.00	0.01	45.50	49.00	0.04	48.96	-0.08%
4	50.00	50.00	0.01	50.00	50.00	0.00	50.00	0.00%
5	112.00	112.00	0.01	106.17	112.00	0.01	112.00	0.00%
6	47.00	47.00	0.02	47.00	47.00	0.02	47.00	0.00%
7	69.00	69.00	0.03	67.33	69.00	0.04	68.98	-0.04%
8	41.00	41.00	0.02	34.25	41.00	0.64	41.00	0.00%
9	91.00	91.00	0.00	89.67	91.00	0.01	91.00	0.00%

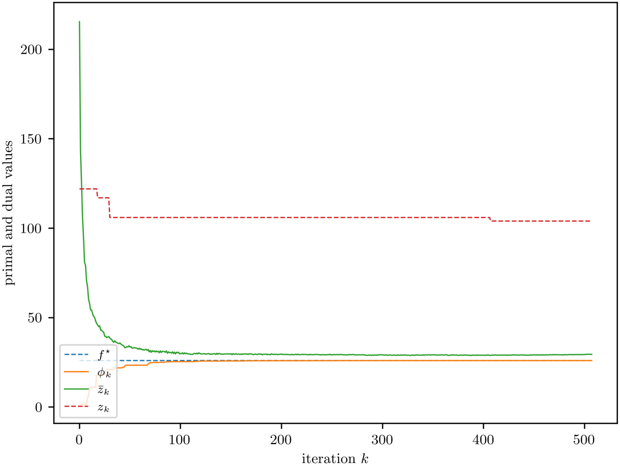
FMP: $c = 1$ large case 10×15

- $f^{x}_{\text{relax}} \neq f^{\star}$

	lb_grb	f_{grb}	t_grb	f^{u}_{relax}	f^{x}_{relax}	t_sg	ϕ_{sg}	ϕ_{gap}
0	467.00	467.00	0.59	460.25	455.00	3.97	467.00	0.00%
1	375.00	375.00	0.22	370.00	366.00	0.12	375.00	0.00%
2	158.00	158.00	0.67	155.92	158.00	4.73	158.00	0.00%
3	463.00	463.00	0.80	457.08	458.00	5.86	463.00	0.00%
4	370.00	370.00	0.11	367.17	363.00	4.46	370.00	0.00%
5	336.00	336.00	1.04	329.17	333.00	0.10	336.00	0.00%
6	350.00	350.00	0.74	344.42	348.00	3.93	350.00	0.00%
7	416.00	416.00	0.09	409.17	409.00	0.19	415.61	-0.09%
8	374.00	374.00	0.88	363.50	363.00	7.81	374.00	0.00%
9	343.00	343.00	1.30	330.67	336.00	0.24	342.67	-0.10%

FMP: $c = 1$ large case 10×15, primal solution

z_k is the value of best primal (recovery) solution, ϕ_k is the dual value, $f^\star$ is the benchmark optimal value by grb. there is a gap from z_k to $f^\star$



FMP: $c \neq 1$ small case 5×5

	lb_grb	f_{grb}	t_grb	f^{u}_{relax}	f^{x}_{relax}	t_sg	ϕ_{sg}	ϕ_{gap}
0	8.00	8.00	0.01	0.00	8.00	0.75	-0.00	-99.99%
1	9.00	9.00	0.01	0.00	9.00	0.68	8.00	-11.11%
2	55.00	55.00	0.02	52.50	55.00	1.33	54.98	-0.03%
3	11.00	11.00	0.03	5.00	11.00	1.06	10.00	-9.09%
4	7.00	7.00	0.02	0.00	7.00	0.85	4.00	-42.85%
5	20.00	20.00	0.03	12.33	20.00	1.08	20.00	-0.00%
6	22.00	22.00	0.01	20.00	22.00	0.01	22.00	0.00%
7	15.00	15.00	0.01	8.67	15.00	0.01	15.00	0.00%
8	14.00	14.00	0.03	6.00	14.00	1.01	10.00	-28.57%
9	18.00	18.00	0.02	13.00	18.00	0.68	18.00	-0.00%

Conclusion

- ? zero duality gap: $\phi^\star = f^\star$, $\phi^\star$ is the best bound by ϕ and $f^\star$ is the best primal value.
- ? can we bound the quality of heuristic for averaged solution? $\bar{z}_k = z(\bar{y}_k)$ converges to $\phi^\star$:

$$|\bar{z}_k - \phi^\star|$$

- ? can we improve $\bar{z} = z(\bar{y})$, round $\bar{z}$