Dual Optimization for Newsvendor-like Problem

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FMP: variant case and weighted summation

$$f = \min_{x_{it}, u_{it}, \delta_t, \epsilon_t} b^{\mathsf{T}}\delta + h^{\mathsf{T}}\epsilon \tag{1}$$

s.t.

$$\sum_i c_i u_{it} + \delta_t - \epsilon_t = d_t, \quad \forall t \in T \tag{2}$$

$$U_{i,\cdot}, S_{i,\cdot}, X_{i,\cdot} \in \Omega_i, \quad \forall i \in I \tag{3}$$

- b, h are time-variant
- $c_i \neq 1$.

FMP: Lagrangian relaxation

For the FMP, dual function:

$$\phi(\lambda) = -\sum_t \lambda_t d_t + \sum_i \min_{\Omega_i} c_i \sum_t \lambda_t u_{it}$$

It reduces to a set of low dimensional minimization problems for each $i \in I, \forall \lambda$
(Recall $\lambda_t \in [-b, h]$ and $\delta_t^\star = \epsilon_t^\star = 0$ else unbounded)

$$\min_{\Omega_i} c_i \sum_t \lambda_t \cdot u_{i,t} \tag{4}$$

(4) is the subproblem to be solved by dynamic programming.
(states: lifespan, action: work or start maintence)

FMP: subgradient method

At each iteration k:

- y_k solves $\phi(\lambda_k) = \min_y \lambda_k^\mathsf{T}(y - b)$
- for FMP: $y_k = U_k^\mathsf{T} c$, $U_k = (u_{it}^{(k)})$, solved from (4) by DP.
- update λ_k.

Small case: $|I| = 1, t = 2$

$c = 1$

$$\begin{aligned} \min \quad & 2\delta_0 + 3\delta_1 + 4\epsilon_0 + \epsilon_1 \\ s.t. \quad & u_0 + \delta_0 - \epsilon_0 = 1 \\ & u_1 + \delta_1 - \epsilon_1 = 0 \\ & 2u_0 + s_0 = 6 \\ & -6x_0 + 2u_1 - s_0 + s_1 = 0 \\ & x_0 + u_0 \leq 1 \\ & x_1 + u_1 \leq 1 \\ & s_0, s_1 \geq 2 \end{aligned}$$

dual

$$-2 \leq \lambda_0 \leq 4$$

$$-3 \leq \lambda_1 \leq 1$$

$$\min_u \lambda_0(u_0 - 1) + \lambda_1(u_1 - 0)$$

$c = 2$

$$\begin{aligned} \min \quad & 2\delta_0 + 3\delta_1 + 4\epsilon_0 + \epsilon_1 \\ s.t. \quad & 2u_0 + \delta_0 - \epsilon_0 = 1 \\ & 2u_1 + \delta_1 - \epsilon_1 = 0 \\ & 2u_0 + s_0 = 6 \\ & -6x_0 + 2u_1 - s_0 + s_1 = 0 \\ & x_0 + u_0 \leq 1 \\ & x_1 + u_1 \leq 1 \\ & s_0, s_1 \geq 2 \end{aligned}$$

$$\min_u \lambda_0(2u_0 - 1) + \lambda_1(2u_1 - 0)$$

Small case: $|I| = 1, t = 2$

results:

$$
\begin{aligned}
&\textit{gurobi}: & & \\
&c = 1 & & c = 2 \\
&u^\star = [1, 0] & & u^\star = [0, 0] \\
&f^\star = 0 & & f^\star = 2 \\
&\textit{subgradient}: & & \\
&\lambda_k = [-2, 0] & & \lambda_k = [-3.5203e^{-5}, 0] \\
&\phi_k = 0 & & \phi_k = 0 \\
&u_k = [1, 0] & & u_k = [0, 0] \\
&\phi_k = f^\star & & \phi_k \neq f^\star
\end{aligned}
$$

Small case

The subgradient method (sg) stops at (maximum) 400 iterations, then compared with benchmarks by Gurobi (grb).

- lb_grb, f_{grb} are lower bound, primal value from grb.
- t_grb, t_sg are runtime from grb and sg, respectively.
- ϕ_{sg} is the dual value (ϕ) by sg.
- z_{sg} is primal value in sg using averaged primal recovery, i.e.

$$z_{\text{sg}} = z(\bar{y}_k) = f(\bar{\delta}_k, \bar{\epsilon}_k)$$

- ϕ_{gap}, z_gap are relative gap from sg to grb for dual and primal values.

FMP: $c = 1$ small case 5×5

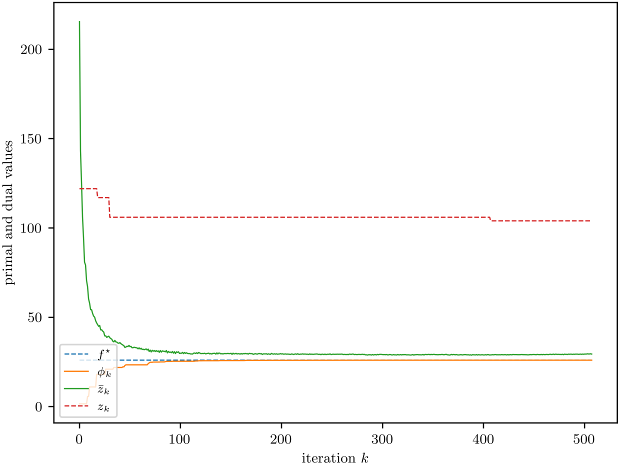
	lb_grb	f_{grb}	t_grb	t_sg	ϕ_{sg}	ϕ_{gap}
0	0.00	0.00	0.00	0.78	0.00	nan%
1	4.00	4.00	0.01	1.16	4.00	0.00%
2	0.00	0.00	0.00	0.66	0.00	nan%
3	4.00	4.00	0.01	1.09	4.00	0.00%
4	0.00	0.00	0.01	0.67	0.00	nan%
5	12.00	12.00	0.01	0.93	12.00	0.00%
6	2.00	2.00	0.01	1.61	2.00	0.00%
7	0.00	0.00	0.01	0.95	0.00	nan%
8	10.00	10.00	0.01	0.02	10.00	0.00%
9	0.00	0.00	0.00	0.79	0.00	nan%
10	8.00	8.00	0.01	3.63	8.00	0.00%
11	10.00	10.00	0.04	2.34	10.00	0.00%
12	4.00	4.00	0.02	2.59	4.00	0.00%
13	0.00	0.00	0.01	0.86	0.00	nan%
14	7.00	7.00	0.01	2.20	7.00	0.00%
15	4.00	4.00	0.01	1.55	4.00	0.00%
16	8.00	8.00	0.03	2.97	8.00	-0.00%
17	0.00	0.00	0.00	0.75	0.00	nan%
18	6.00	6.00	0.01	1.14	6.00	-0.00%
19	2.00	2.00	0.01	1.32	2.00	-0.00%

FMP: $c = 1$ large case 10×15

	lb_grb	f_{grb}	t_grb	t_sg	ϕ_{sg}	ϕ_{gap}
0	26.00	26.00	4.29	17.19	26.00	-0.00%
1	23.00	23.00	0.09	20.67	23.00	-0.00%
2	22.00	22.00	12.33	15.08	22.00	-0.00%
3	30.00	30.00	8.15	21.32	30.00	-0.00%
4	27.00	27.00	5.36	15.69	27.00	-0.00%
5	41.00	41.00	0.38	15.55	41.00	-0.00%
6	22.00	22.00	2.49	18.95	22.00	-0.00%
7	48.00	48.00	1.04	15.46	48.00	-0.00%
8	30.00	30.00	1.19	17.27	30.00	-0.00%
9	35.00	35.00	0.35	18.33	35.00	-0.00%
10	23.00	23.00	0.35	20.72	23.00	-0.00%
11	12.00	12.00	3.77	18.62	12.00	-0.00%
12	28.00	28.00	0.66	22.03	27.99	-0.03%
13	18.00	18.00	0.07	20.70	18.00	-0.01%
14	31.00	31.00	6.73	20.06	31.00	-0.00%
15	24.00	24.00	0.86	17.00	24.00	-0.00%
16	8.00	8.00	17.42	19.30	8.00	-0.01%
17	27.00	27.00	1.66	16.74	27.00	-0.00%
18	33.00	33.00	3.35	21.38	33.00	-0.00%
19	20.00	20.00	21.09	17.54	20.00	-0.00%

FMP: $c = 1$ large case 10×15, primal solution

z_k is the value of best primal (recovery) solution, ϕ_k is the dual value, $f^\star$ is the benchmark optimal value by grb. there is a gap from z_k to $f^\star$



FMP: $c \neq 1$ small case 5×5

	lb_grb	f_{grb}	t_grb	t_sg	ϕ_{sg}	ϕ_{gap}
0	0.00	0.00	0.00	0.00	0.00	0.00%
1	16.00	16.00	0.01	1.22	6.00	-62.50%
2	0.00	0.00	0.03	1.92	-0.00	-120.00%
3	6.00	6.00	0.02	1.05	4.00	-33.33%
4	16.00	16.00	0.04	1.14	16.00	-0.00%
5	0.00	0.00	0.01	1.25	-0.00	-80.00%
6	5.00	5.00	0.01	1.53	3.00	-40.00%
7	5.00	5.00	0.02	1.49	4.00	-20.00%
8	0.00	0.00	0.01	1.53	-0.00	-70.00%
9	0.00	0.00	0.02	1.48	-0.00	-50.00%
10	17.00	17.00	0.02	1.15	14.00	-17.65%
11	5.00	5.00	0.06	1.32	-0.00	-99.99%
12	1.00	1.00	0.03	1.19	-0.00	-100.02%
13	8.00	8.00	0.02	1.06	8.00	-0.01%
14	12.00	12.00	0.02	0.98	12.00	-0.00%
15	4.00	4.00	0.02	1.02	-0.00	-99.99%
16	2.00	2.00	0.03	1.70	-0.00	-99.99%
17	0.00	0.00	0.01	1.23	-0.00	-70.00%
18	0.00	0.00	0.01	1.28	-0.00	-60.00%
19	2.00	2.00	0.01	1.16	-0.00	-99.99%
20	0.00	0.00	0.02	1.38	-0.00	-70.00%

Conclusion

- ? zero duality gap: $\phi^\star = f^\star$, $\phi^\star$ is the best bound by ϕ and $f^\star$ is the best primal value.
- ? can we bound the quality of heuristic for averaged solution? $\bar{z}_k = z(\bar{y}_k)$ converges to $\phi^\star$:

$$|\bar{z}_k - \phi^\star|$$

- can we improve $\bar{z} = z(\bar{y})$? round $\bar{z}$