October 25, 2017

Matrix multiplication in softly quadratic time

1. Boolean matrix multiplication

Approach

We borrow ideas from Kaplan and Gupta; namely, we use idea of *union-intersection arithmetic* and *colored range counting*. However, instead of using *orthant decomposition* for three- or four-sided query via 3-d or 4-d segment tree, we use 2-d range tree.

We would have had to use time cubic in n for Kaplan's four-sided query approach. If we take advantage of output sensitivity, then we get time softly in $O(n^{2.5})$.

To avoid using orthant images of row/column points via orthant decomposition and 4-d segment tree, we use row/column points directly with 2-d range tree.

We rely heavily on taking advantage of point arrangement and query ranges being well-formed.

Boolean MM approach can be faster if output values are in $\{0, 1\}$ as opposed to integer. For example, in this case, we could just reduce to semi-ring flavor max-times to get time $O(n^2 \cdot \log(n))$, which is optimal as it matches lower bound by Raz.

In practice, we reduce to floating-point MM, with $m = m' = O(1)$.

This implies ω is softly two for Boolean MM for rings.

Running time

$O(n^2 \cdot \log^3(n))$

References

- Gupta et al. - Further results on generalized intersection searching problems: counting, reporting, and dynamization (1995)
- Kaplan et al. - Efficient colored orthogonal range counting (2008)
- Munro et al. - Range counting with distinct constraints (2015)
- Raz - On the complexity of matrix product (2002)