1 Problem Formulation

We model the true but unobserved observations $X_1, \dots X_{918}$ using a Bernoulli autoregressive process:

$$\begin{aligned} Y_t &= \nu + A^* X_{t-1} \\ X_t &\sim \text{Bernoulli}\left(\frac{1}{1+\exp(-Y_t)}\right). \end{aligned} \tag{1.1}$$

Here $X_t \in \{0,1\}^{77}$ is a vector indicating whether events occurred in each of the 77 community areas during week t. The vector $\nu \in \mathbb{R}^{77}$ is a constant bias term, and the matrix $A^* \in \mathbb{R}^{77\times 77}$ is the weighted adjacency matrix associated with the network we wish to estimate.

We do not have access to the true crime data, but instead observe $Z_1, \dots Z_{918}$, a corrupted version of (1.1) where only 75% of events are observed as follows:

$$\begin{aligned} W_{t,i} &\overset{iid}{\sim} \text{Bernoulli}(.75) \\ Z_t &= W_t \odot X_t. \end{aligned}$$

Here $\odot$ denotes the Hadamard product and $W_t \in \{0,1\}^{77}$ is a vector where each entry is independently drawn to be one with probability .75 and zero with probability .25.

2 Proposed Estimation

Given access to the full data $X_1, \dots, X_{918}$ we could estimate A^* using regularized MLE with the full data negative log-likelihood $L_X(A)$:

$$\widehat{A} = \arg\min_A L_X(A) + \lambda ||A||_{1,1} \tag{2.1}$$

Instead we observe the corrupted data $Z_1, \dots, Z_{918}$. A naive approach would be to ignore this corruption and estimate A^* using the full data likelihood as in Equation (2.1). This method is implemented in 'full data estimate network'. With the 'estimate network' routine we propose an alternative algorithm where we consider an unbiased estimator $L_Z(A)$ for the full data likelihood $L_X(A)$. In other words, we consider an estimator $L_Z(A)$ with the property

$$\mathbb{E}[L_Z(A)|X] = L_X(A).$$

We can then estimate A^* using this likelihood

$$\widehat{A} \in \arg\min_A L_Z(A) + \lambda ||A||_{1,1}. \tag{2.2}$$

The function $L_Z(A)$ is expensive to compute precisely, so the 'estimate network' routine considers a slight variant of Equation (2.2) using the degree two Taylor truncation of $L_Z(A)$.