COMP9313: Big Data Management

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Course web site: http://www.cse.unsw.edu.au/~cs9313/

Chapter 7: Mining Data Streams

Data Streams

In many data mining situations, we do not know the entire data set in advance

Stream Management is important when the input rate is controlled **externally:**

- Google queries
- Twitter or Facebook status updates

We can think of the **data** as **infinite** and **non-stationary** (the distribution changes over time)

Characteristics of Data Streams

Traditional DBMS: data stored in *finite, persistent data sets*

Data Streams: distributed, continuous, unbounded, rapid, time varying, noisy, . . .

Characteristics

- Huge volumes of continuous data, possibly infinite
- Fast changing and requires fast, real-time response
- Random access is expensive—single scan algorithm (can only have one look)
- Store only the summary of the data seen thus far

Massive Data Streams

Data is *continuously growing* faster than our ability to store or index it

There are 3 Billion Telephone Calls in US each day,
30 Billion emails daily, 1 Billion SMS, IMs

Scientific data: NASA's observation satellites generate billions of readings each per day

IP Network Traffic: up to 1 Billion packets per hour per router. Each ISP has many (hundreds) routers!

… …

The Stream Model

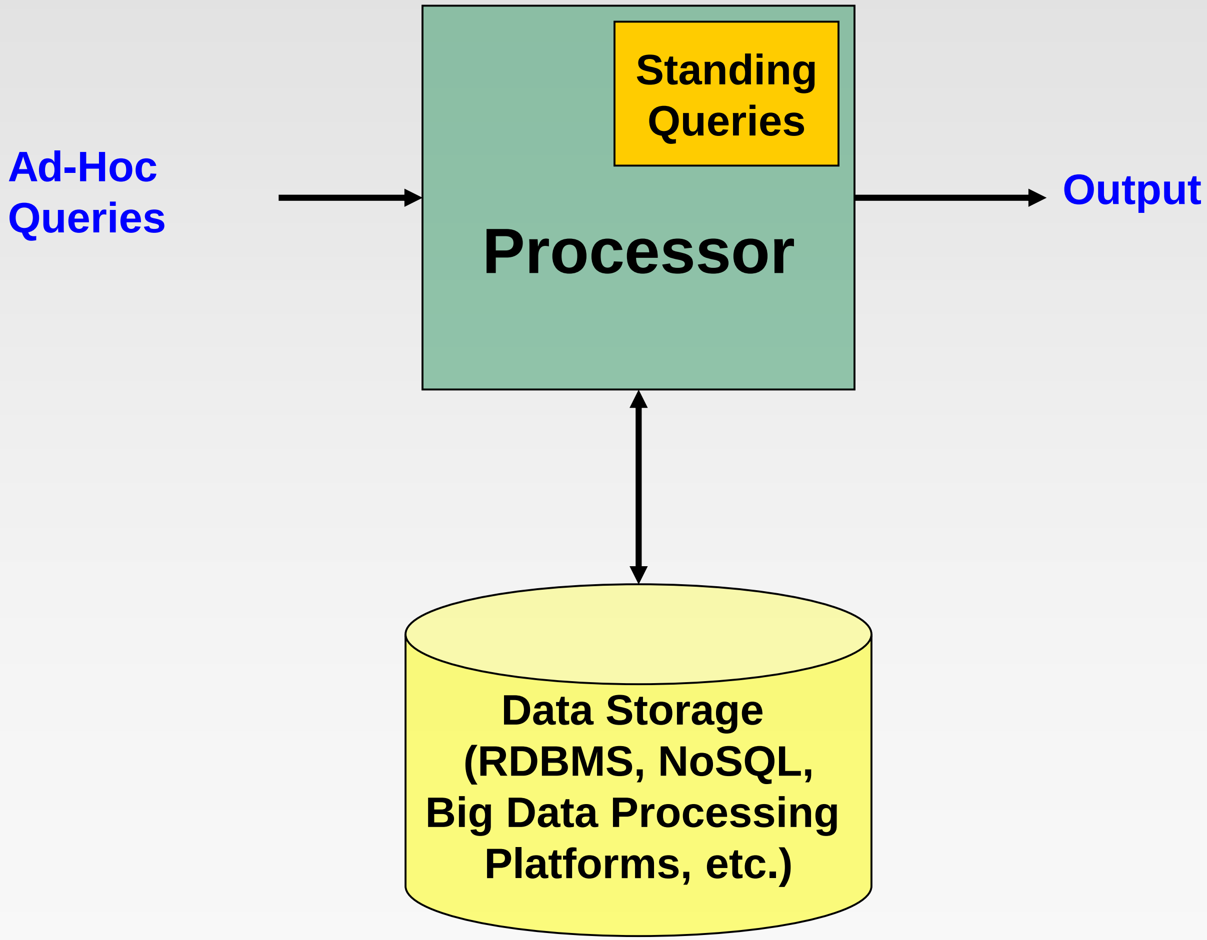
Input elements enter at a rapid rate, at one or more input ports (i.e., streams)

We call elements of the stream tuples

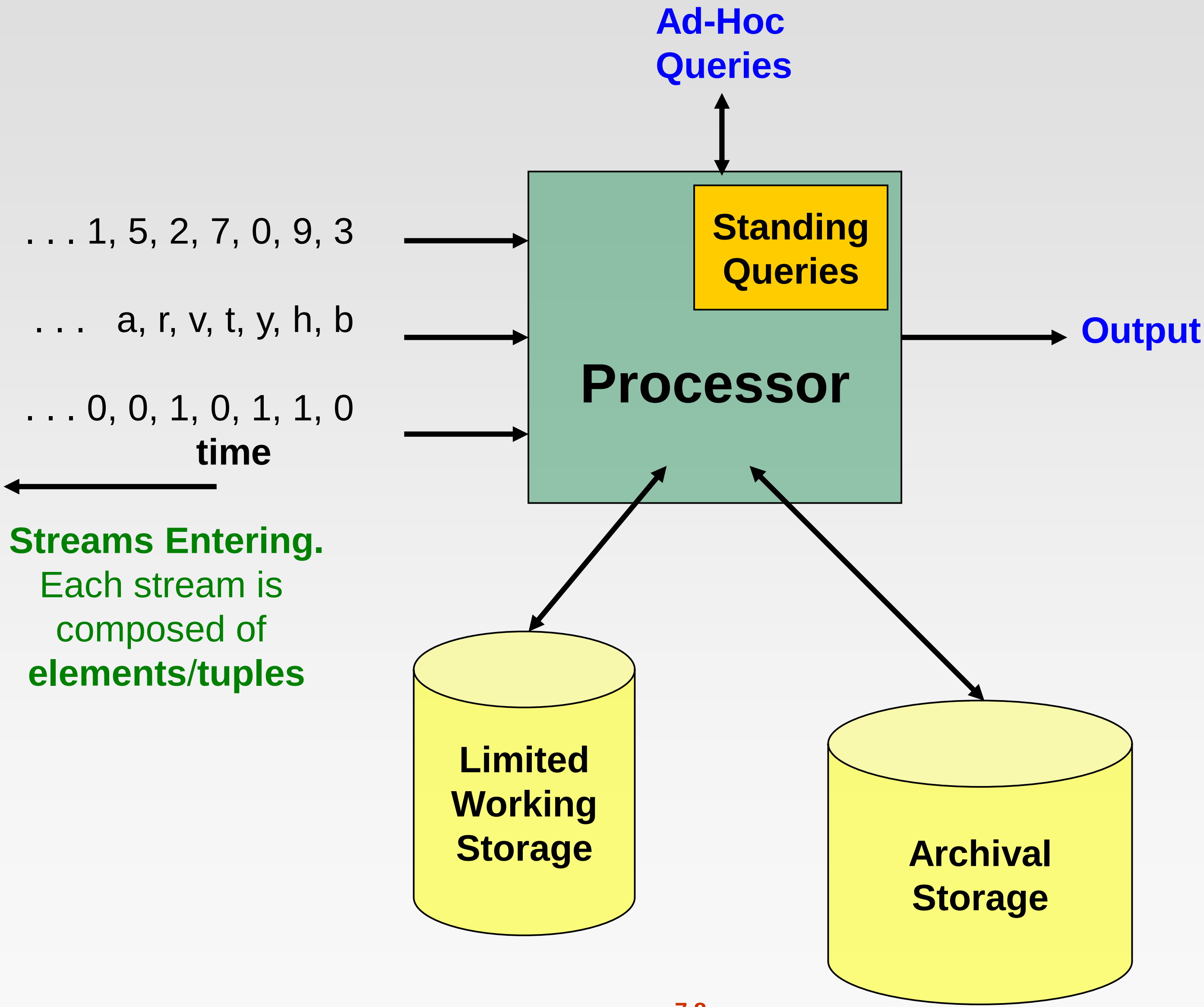
The system cannot store the entire stream accessibly

Q: How do you make critical calculations about the stream using a limited amount of memory?

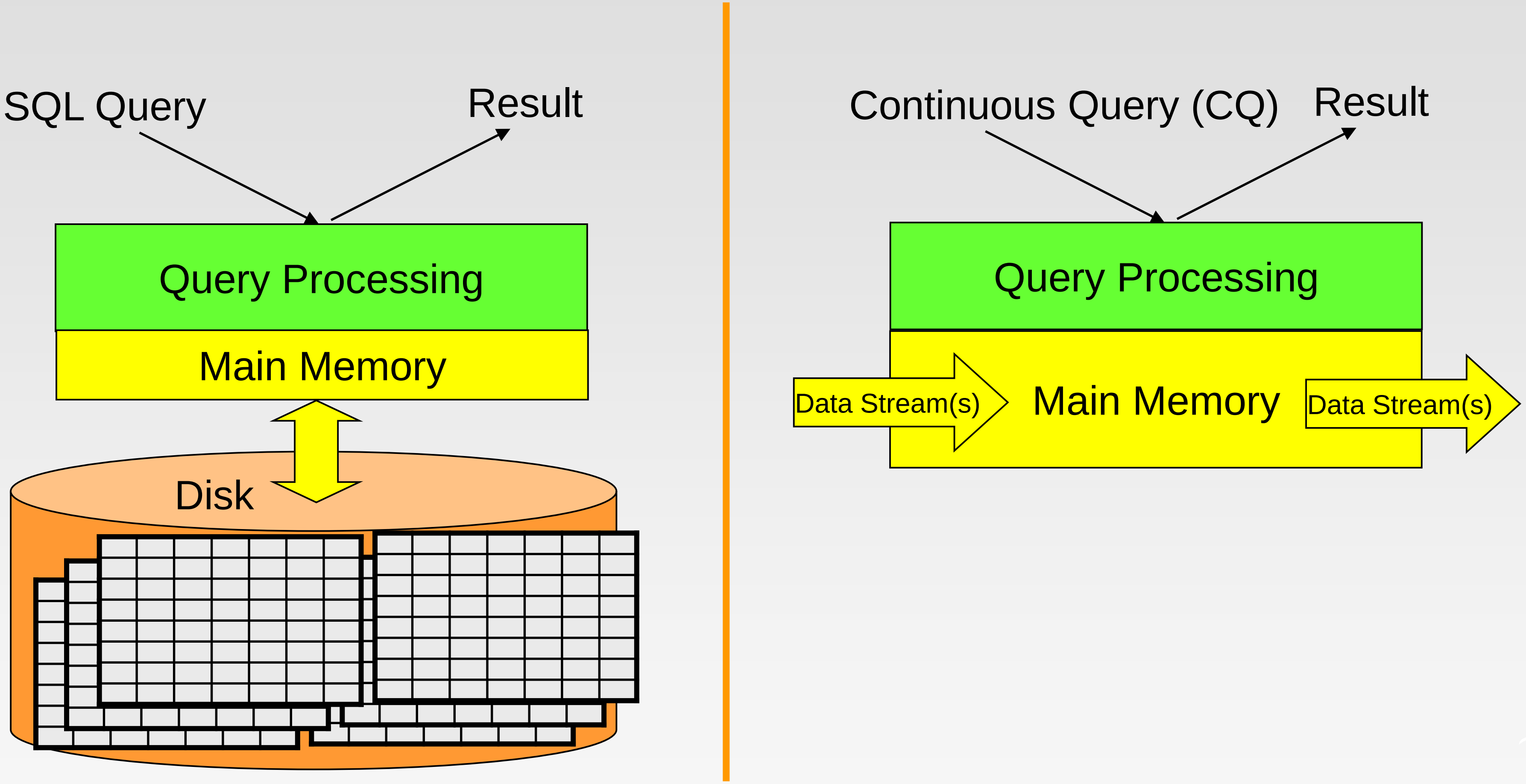
Database Management System (DBMS) Data Processing



General Data Stream Management System (DSMS) Processing Model



DBMS vs. DSMS #1



DBMS vs. DSMS #2

Traditional DBMS:

- stored sets of relatively static records with no pre-defined notion of time
- good for applications that require persistent data storage and complex querying

DSMS:

- support on-line analysis of rapidly changing data streams
- data stream: real-time, continuous, ordered (implicitly by arrival time or explicitly by timestamp) sequence of items, too large to store entirely, no ending
- continuous queries

DBMS vs. DSMS #3

DBMS

- Persistent relations (relatively static, stored)
- One-time queries
- Random access
- “Unbounded” disk store
- Only current state matters
- No real-time services
- Relatively low update rate
- Data at any granularity
- Assume precise data
- Access plan determined by query processor, physical DB design

DSMS

- Transient streams (on-line analysis)
- Continuous queries (CQs)
- Sequential access
- Bounded main memory
- Historical data is important
- Real-time requirements
- Possibly multi-GB arrival rate
- Data at fine granularity
- Data stale/imprecise
- Unpredictable/variable data arrival and characteristics

Problems on Data Streams

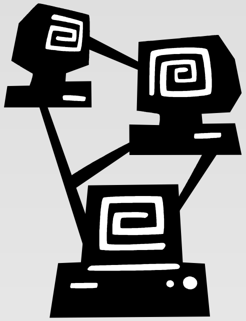
Types of queries one wants on answer on a data stream: (we'll learn these today)

- Sampling data from a stream
  - Construct a random sample
- Queries over sliding windows
  - Number of items of type x in the last *k* elements of the stream
- Filtering a data stream
  - Select elements with property x from the stream
- Counting distinct elements
  - Number of distinct elements in the last *k* elements of the stream

Applications

- Mining query streams
  - Google wants to know what queries are more frequent today than yesterday
- Mining click streams
  - Yahoo wants to know which of its pages are getting an unusual number of hits in the past hour
- Mining social network news feeds
  - E.g., look for trending topics on Twitter, Facebook
- Sensor Networks
  - Many sensors feeding into a central controller
- Telephone call records
  - Data feeds into customer bills as well as settlements between telephone companies
- IP packets monitored at a switch
  - Gather information for optimal routing

Example: IP Network Data



Networks are sources of massive data: the metadata per hour per IP router is gigabytes

Fundamental problem of data stream analysis:

Too much information to store or transmit

So process data as it arrives

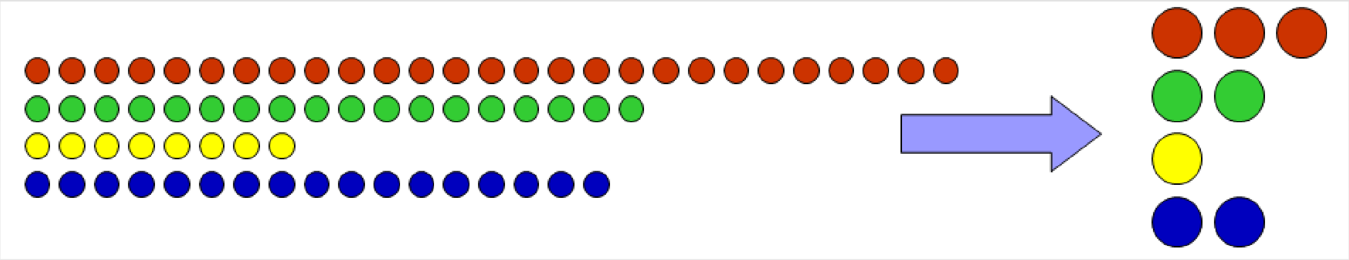
One pass, small space: the data stream approach

Approximate answers to many questions are OK, if there are guarantees of result quality

Part 1: Sampling Data Streams

Sampling from a Data Stream

Since we can not store the entire stream, one obvious approach is to store a **sample**



Two different problems:

- **(1)** Sample a **fixed proportion** of elements in the stream (say 1 in 10)
  - As the stream grows the sample also gets bigger
- **(2)** Maintain a **random sample of fixed size** over a potentially infinite stream
  - As the stream grows, the sample is of fixed size
  - At any "time" ***t*** we would like a random sample of ***s*** elements
    - **What is the property of the sample we want to maintain?** For all time steps ***t***, each of ***t*** elements seen so far has equal probability of being sampled

Sampling a Fixed Proportion

Problem 1: Sampling fixed proportion

Scenario: Search engine query stream

- **Stream of tuples:** (user, query, time)
- **Answer questions such as: How often did a user run the same query in a single days**
- Have space to store **1/10th** of query stream

Naïve solution:

- Generate a random integer in **[0..9]** for each query
- Store the query if the integer is **0**, otherwise discard

Problem with Naïve Approach

Simple question: What fraction of queries by an average search engine user are duplicates?

Suppose each user issues ***x*** queries once and ***d*** queries twice (total of ***x*+2*d*** queries)

- **Correct answer: *d*/(*x*+*d*)**

Proposed solution: We keep 10% of the queries

- Sample will contain ***x*/10** of the singleton queries and **2*d*/10** of the duplicate queries at least once
- But only ***d*/100** pairs of duplicates
  - **d/100** = **1/10 · 1/10 · d**
- Of ***d*** "duplicates" ***18d/100*** appear exactly once
  - **18d/100 = ((1/10 · 9/10)+(9/10 · 1/10)) · d**

So the sample-based answer is $\frac{\frac{d}{100}}{\frac{x}{10}+\frac{d}{100}+\frac{18d}{100}} = \frac{d}{10x+19d}$ **≠*d*/(*x*+*d*)**

Solution: Sample Users

Solution:

Pick **1/10th** of **users** and take all their searches in the sample

Use a hash function that hashes the user name or user id uniformly into 10 buckets

We hash each user name to one of ten buckets, 0 through 9

If the user hashes to bucket 0, then accept this search query for the sample, and if not, then not.

Generalized Problem and Solution

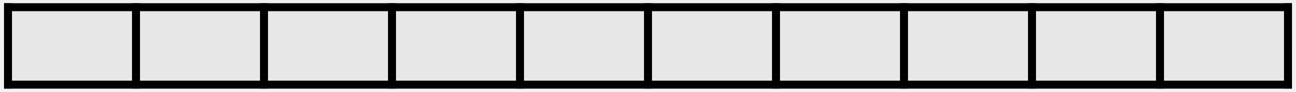
Problem: Give a data stream, take a sample of fraction a/b.

Stream of tuples with keys:

- Key is some subset of each tuple's components
  - e.g., tuple is (user, search, time); key is **user**
- Choice of key depends on application

To get a sample of a/b fraction of the stream:

- Hash each tuple's key uniformly into ***b*** buckets
- Pick the tuple if its hash value is at most ***a***



How to generate a 30% sample?

Hash into b=10 buckets, take the tuple if it hashes to one of the first 3 buckets

Maintaining a Fixed-size Sample

Problem 2: Fixed-size sample

Suppose we need to maintain a random sample S of size exactly s tuples

E.g., main memory size constraint

Why? Don't know length of stream in advance

Suppose at time *n* we have seen *n* items

Each item is in the sample *S* with equal prob. *s/n*

Note that the same item is treated as different tuples at different timestamps

How to think about the problem: say s = 2

Stream: a x c y z k c d e g…

At **n= 5,** each of the first 5 tuples is included in the sample **S** with equal prob.

At **n= 7,** each of the first 7 tuples is included in the sample **S** with equal prob.

Impractical solution would be to store all the *n* tuples seen so far and out of them pick *s* at random

Solution: Fixed Size Sample

Algorithm (a.k.a. Reservoir Sampling)

Store all the first ***s*** elements of the stream to ***S***

Suppose we have seen ***n-1*** elements, and now the ***n***th element arrives (***n* > *s***)

- With probability ***s/n***, keep the ***n***th element, else discard it
- If we picked the ***n***th element, then it replaces one of the ***s*** elements in the sample ***S***, picked uniformly at random

Claim: This algorithm maintains a sample ***S*** with the desired property:

After ***n*** elements, the sample contains each element seen so far with probability ***s/n***

Proof: By Induction

We prove this by induction:

Assume that after ***n*** elements, the sample contains each element seen so far with probability ***s/n***

We need to show that after seeing element ***n+1*** the sample maintains the property

- Sample contains each element seen so far with probability ***s/(n+1)***

Base case:

After we see **n=s** elements the sample **S** has the desired property

- Each out of **n=s** elements is in the sample with probability ***s/s* = *1***

Proof: By Induction

Inductive hypothesis: After ***n*** elements, the sample ***S*** contains each element seen so far with prob. ***s/n***

Now element *n+1* arrives

Inductive step: For elements already in ***S***, probability that the algorithm keeps it in ***S*** is:

$$\left(1-\frac{s}{n+1}\right)+\left(\frac{s}{n+1}\right)\left(\frac{s-1}{s}\right)=\frac{n}{n+1}$$

Element **n+1** discarded — Element **n+1** not discarded — Element in the sample not picked

So, at time ***n**,* tuples in ***S*** were there with prob. **s/n**

Time ***n→n+1**,* tuple stayed in ***S*** with prob. **n/(n+1)**

So prob. tuple is in ***S*** at time ***n+1*** $= \frac{s}{n} \cdot \frac{n}{n+1} = \frac{s}{n+1}$

Part 2: Querying Data Streams

Sliding Windows

A useful model of stream processing is that queries are about a ***window*** of length ***N*** – the ***N*** most recent elements received

Interesting case: ***N*** is so large that the data cannot be stored in memory, or even on disk

- Or, there are so many streams that windows for all cannot be stored

Amazon example:

- For every product **X** we keep 0/1 stream of whether that product was sold in the ***n***-th transaction
- We want answer queries, how many times have we sold **X** in the last ***k*** sales

Sliding Window: 1 Stream

Sliding window on a single stream:

N = 7

q w e r t y u i o p a $\boxed{\text{s d f g h j k}}$ l z x c v b n m

q w e r t y u i o p a s $\boxed{\text{d f g h j k l}}$ z x c v b n m

q w e r t y u i o p a s d $\boxed{\text{f g h j k l z}}$ x c v b n m

q w e r t y u i o p a s d f $\boxed{\text{g h j k l z x}}$ c v b n m

← Past Future →

Counting Bits (1)

Problem:

Given a stream of **0**s and **1**s

Be prepared to answer queries of the form:
How many 1s are in the last *k* bits? where ***k*** ≤ ***N***

Obvious solution:

Store the most recent ***N*** bits

- When new bit comes in, discard the ***N*+1st** bit

0 1 0 0 1 1 0 1 1 1 0 1 0 1 0 1 1 0 1 1 0 1 1 0

Suppose N=7

← Past Future →

Counting Bits (2)

You can not get an exact answer without storing the entire window

Real Problem:
What if we cannot afford to store *N* bits?

E.g., we're processing 1 billion streams and ***N* = 1 billion**

But we are happy with an approximate answer

0 1 0 0 1 1 0 1 1 1 0 1 0 1 0 1 1 ~~0 1 1 0 1 1 0~~

← Past   Future →

An attempt: Simple solution

Q: How many 1s are in the last ***N*** bits?

A simple solution that does not really solve our problem: **Uniformity Assumption**

N

0 1 0 0 1 1 1 0 0 0 1 0 1 0 0 1 0 0 0 1 0 1 1 0 1 1 0 1 1 1 0 0 1 0 1 0 1 1 0 0 1 1 0 1 0

Past Future

Maintain 2 counters:

- ***S***: number of 1s from the beginning of the stream
- ***Z***: number of 0s from the beginning of the stream

How many 1s are in the last ***N*** bits? $N \cdot \frac{S}{S+Z}$

But, what if stream is non-uniform?

- What if distribution changes over time?

The Datar-Gionis-Indyk-Motwani (DGIM) Algorithm

Maintaining Stream Statistics over Sliding Windows (SODA'02)

DGIM solution that does not assume uniformity

We store $\boldsymbol{O}(\log^2 \boldsymbol{N})$ bits per stream

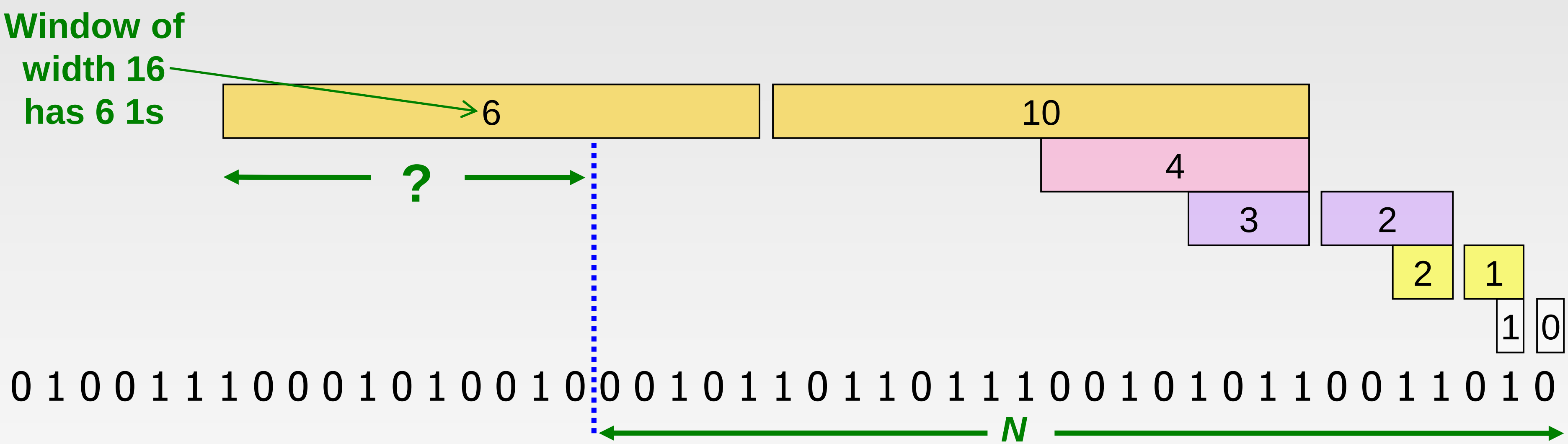
Solution gives approximate answer, never off by more than 50%

- Error factor can be reduced to any fraction > 0, with more complicated algorithm and proportionally more stored bits

Idea: Exponential Windows

Solution that doesn't (quite) work:

- Summarize **exponentially increasing** regions of the stream, looking backward
- Drop small regions if they begin at the same point as a larger region



We can reconstruct the count of the last ***N*** bits, except we are not sure how many of the last **6 1s** are included in the ***N***

What's Good?

Stores only $\boldsymbol{O}(\log^{\mathbf{2}} \boldsymbol{N})$ bits

$\boldsymbol{O}(\log \boldsymbol{N})$ counts of $\log_{\mathbf{2}}\boldsymbol{N}$ bits each

Easy update as more bits enter

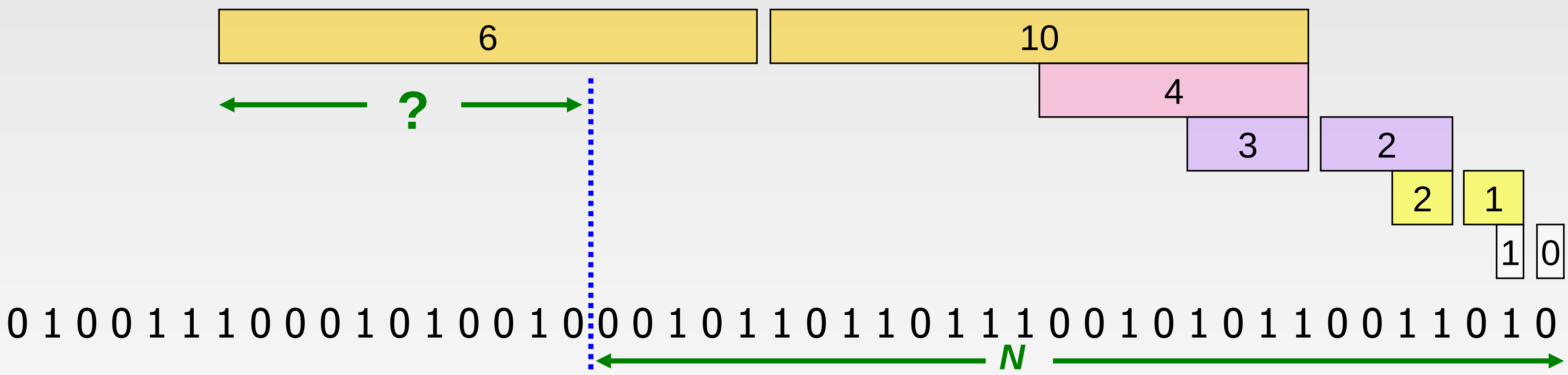
Error in count no greater than the number of **1s** in the "**unknown**" area

What's Not So Good?

As long as the **1s** are fairly evenly distributed, the error due to the unknown region is small – **no more than 50%**

But it could be that all the 1s are in the unknown area at the end

In that case, the error is unbounded!

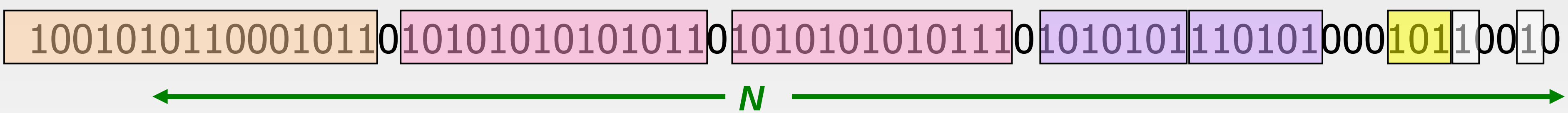


Fixup: DGIM Algorithm

Idea: Instead of summarizing fixed-length blocks, summarize blocks with specific number of **1s**:

Let the block ***sizes*** (number of **1s**) increase exponentially

When there are few 1s in the window, block sizes stay small, so errors are small



DGIM: Timestamps

Each bit in the stream has a timestamp, starting from **1**, **2,** …

Record timestamps modulo ***N*** (**the window size**), so we can represent any **relevant** timestamp in $\boldsymbol{O}(\log_{\mathbf{2}}\boldsymbol{N})$ bits

- E.g., given the windows size 40 (***N***), timestamp 123 will be recorded as 3, and thus the encoding is on 3 rather than 123

DGIM: Buckets

A bucket in the DGIM method is a record consisting of:

(A) The timestamp of its end [$O(\log N)$ bits]

(B) The number of 1s between its beginning and end [$O(\log\log N)$ bits]

Constraint on buckets:

Number of 1s must be a power of 2

That explains the $O(\log\log N)$ in (B) above

1001010110001011 0 1010101010101011 0 101010101010111 0 1010101 110101 000 101 1 00 1 0

Representing a Stream by Buckets

The right end of a bucket is always a position with a 1

Every position with a 1 is in some bucket

Either **one** or **two** buckets with the same **power-of-2 number** of **1s**

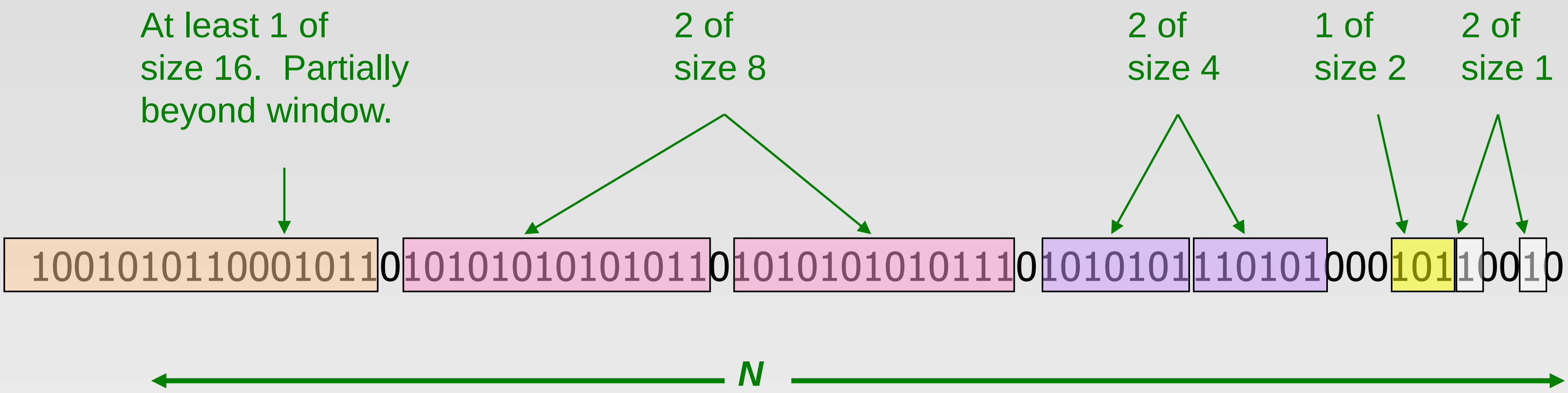
Buckets do not overlap in timestamps

Buckets are sorted by size

Earlier buckets are not smaller than later buckets

Buckets disappear when their end-time is **>** ***N*** time units in the past

Example: Bucketized Stream



Three properties of buckets that are maintained:

- Either **one** or **two** buckets with the same power-of-2 number of 1s
- Buckets do not overlap in timestamps
- Buckets are sorted by size

Updating Buckets

When a new bit comes in, drop the last (oldest) bucket if its end-time is prior to ***N*** time units before the current time

2 cases: Current bit is **0** or **1**

If the current bit is 0: no other changes are needed

If the current bit is 1:

(1) Create a new bucket of size 1, for just this bit

- End timestamp = current time

(2) If there are now three buckets of size 1, combine the oldest two into a bucket of size 2

(3) If there are now three buckets of size 2, combine the oldest two into a bucket of size 4

(4) And so on …

Example: Updating Buckets

Current state of the stream:

1001010110001011010101010101010110101010101011101010101110101000101100100 10

Bit of value 1 arrives

001010110001011010101010101010110101010101011101010101110101000101100101 01

Two white buckets get merged into a yellow bucket

0010101100010110101010101010101101010101010111010101011101010001011001 01

Next bit 1 arrives, new orange white is created, then 0 comes, then 1:

0101100010110101010101010101101010101010111010101011101010001011001 01101

Buckets get merged…

0101100010110101010101010101101010101010111010101011101010001011001 01101

State of the buckets after merging

0101100010110101010101010101101010101010111010101011101010001011001 01101

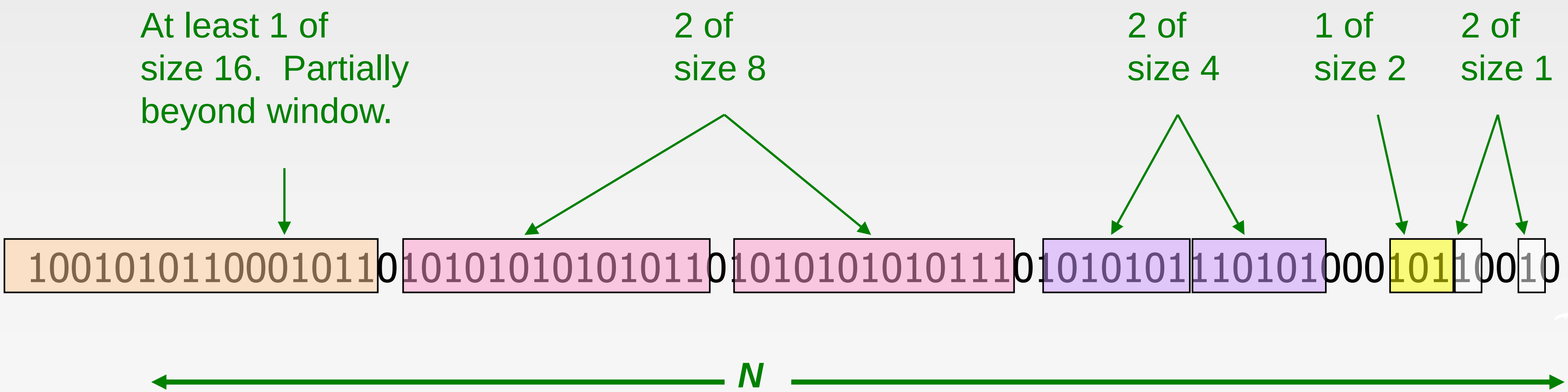
How to Query?

To estimate the number of 1s in the most recent N bits:

- Sum the sizes of all buckets but the last
  - (note “size” means the number of 1s in the bucket)
- Add half the size of the last bucket

Remember: We do not know how many 1s of the last bucket are still within the wanted window

Example:



Error Bound: Proof

Why is error 50%? Let's prove it!

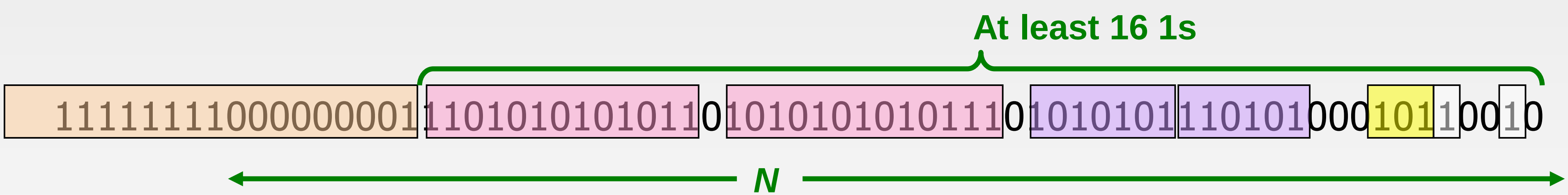
Suppose the last bucket has size $\mathbf{2^{r}}$

Then by assuming $\mathbf{2^{r-1}}$ (i.e., half) of its **1s** are still within the window, we make an error of at most $\mathbf{2^{r-1}}$

Since there is at least one bucket of each of the sizes less than $\mathbf{2^{r}}$, the true sum is at least

$$\mathbf{1 + 2 + 4 + .. + 2^{r-1} = 2^{r} - 1}$$

Thus, error at most **50%**



Further Reducing the Error

Instead of maintaining **1** or **2** of each size bucket, we allow either ***r*-1** or ***r*** buckets (***r* > 2**)

- Except for the largest size buckets; we can have any number between **1** and ***r*** of those

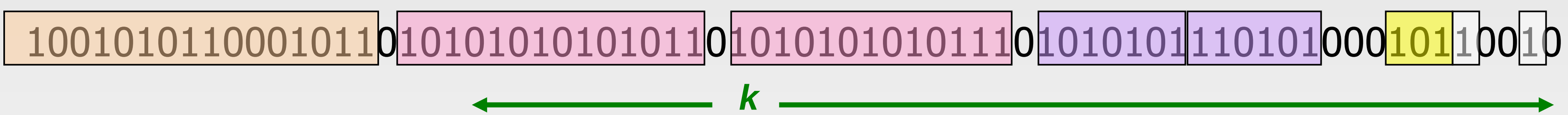
Error is at most *O(*1/*r)*

By picking ***r*** appropriately, we can tradeoff between number of bits we store and the error

Extensions (optional)

Can we use the same trick to answer queries **How many 1's in the last *k*?** where ***k* < *N***?

> **A:** Find earliest bucket **B** that at overlaps with ***k***.
> Number of **1s** is the **sum of sizes of more recent buckets + ½ size of B**



Can we handle the case where the stream is not bits, but integers, and we want the sum of the last *k* elements?

Extensions (optional)

Stream of positive integers

We want the sum of the last *k* elements

Amazon: Avg. price of last **k** sales

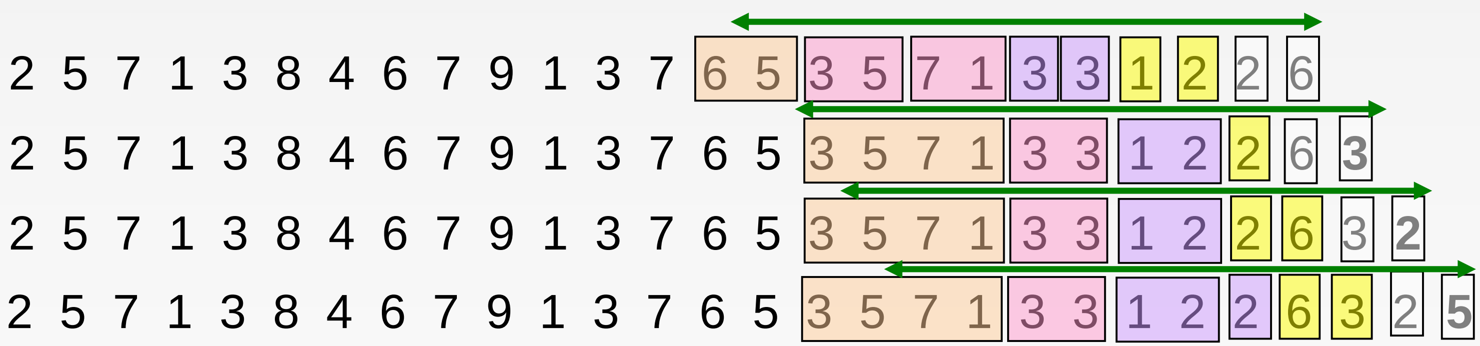
Solution:

(1) If you know all have at most *m* bits

- Treat ***m*** bits of each integer as a separate stream
- Use DGIM to count **1s** in each integer
- The sum is $= \sum_{i=0}^{m-1} c_i 2^i$ $\quad$ $\boldsymbol{c_i}$ …estimated count for **i-th** bit

(2) Use buckets to keep partial sums

- **Sum of elements in size *b* bucket is at most 2^b**



Idea: Sum in each bucket is at most $\mathbf{2^b}$ (unless bucket has only **1** integer)

Bucket sizes:

16	**8**	**4**	**2**	**1**

Part 3: Filtering Data Streams

Filtering Data Streams

Each element of data stream is a tuple

Given a list of keys **S**

Determine which tuples of stream are in *S*

Obvious solution: Hash table

- But suppose we **do not have enough memory** to store all of ***S*** in a hash table
  - E.g., we might be processing millions of filters on the same stream

Applications

Example: Email spam filtering

- We know 1 billion “good” email addresses
- If an email comes from one of these, it is **NOT** spam

Publish-subscribe systems

- You are collecting lots of messages (news articles)
- People express interest in certain sets of keywords
- Determine whether each message matches user’s interest

First Cut Solution (1)

Given a set of keys ***S*** that we want to filter

Create a **bit array *B*** of ***n*** bits, initially all ***0*s**

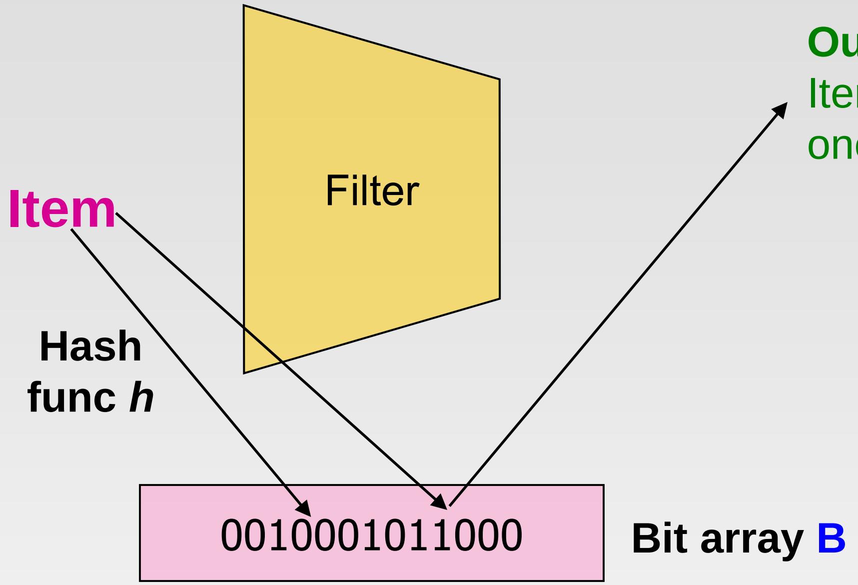
Choose a **hash function *h*** with range **[*0,n*)**

Hash each member of $s \in S$ to one of ***n*** buckets, and set that bit to **1**, i.e., ***B[h(s)]=1***

Hash each element ***a*** of the stream and output only those that hash to bit that was set to **1**

Output *a* if B[h(a)] == 1

First Cut Solution (2)



Output the item since it may be in *S*.
Item hashes to a bucket that at least one of the items in **S** hashed to.

Drop the item.
It hashes to a bucket set to **0** so it is surely not in ***S***.

Creates false positives but no false negatives

If the item is in ***S*** we surely output it, if not we may still output it

First Cut Solution (3)

|S| = 1 billion email addresses
|B|= 1GB = 8 billion bits

If the email address is in ***S***, then it surely hashes to a bucket that has the big set to **1**, so it always gets through (***no false negatives***)

- False negative: a result indicates that a condition failed, while it actually was successful

Approximately 1/8 of the bits are set to 1, so about 1/8th of the addresses not in S get through to the output (***false positives***)

- False positive: a result that indicates a given condition has been fulfilled, when it actually has not been fulfilled
- Actually, less than 1/8th, because more than one address might hash to the same bit
- Since the majority of emails are spam, eliminating 7/8th of the spam is a significant benefit

Analysis: Throwing Darts (1)

More accurate analysis for the number of **false positives**

Consider: If we throw ***m*** darts into ***n*** equally likely targets, **what is the probability that a target gets at least one dart?**

In our case:

Targets = bits/buckets

Darts = hash values of items

Analysis: Throwing Darts (2)

We have ***m*** darts, ***n*** targets

What is the probability that a target gets at least one dart?

Equals **1/e**
as **n** $\rightarrow\infty$

Equivalent

$$\boxed{1 - \underline{\boxed{\underline{(1 - 1/n)}^{\,n}}^{\,(m/n)}}}$$

$$\mathbf{1 - e^{-m/n}}$$

Probability some
target **X** not hit
by a dart

Probability at
least one dart
hits target **X**

Analysis: Throwing Darts (3)

Fraction of 1s in the array B

= probability of false positive = $1 - e^{-m/n}$

Example: **10^9** darts, **$8 \cdot 10^9$** targets

Fraction of **1s** in **B** = $1 - e^{-1/8} = 0.1175$

- Compare with our earlier estimate: **1/8 = 0.125**

Bloom Filter

Consider: **|S| = *m*, |B| = *n***

Use ***k*** independent hash functions $h_1, \ldots, h_k$

Initialization:

Set **B** to all **0s**

Hash each element $s \in S$ using each hash function h_i, set $B[h_i(s)] = 1$ (for each $i = 1, .., k$)

Run-time:

When a stream element with key ***x*** arrives

- If $B[h_i(x)] = 1$ **for all** $i = 1, \ldots, k$ then declare that ***x*** is in ***S***
  - That is, ***x*** hashes to a bucket set to **1** for every hash function $h_i(x)$
- Otherwise discard the element ***x***

Bloom Filter Example

Consider a Bloom filter of size m=10 and number of hash functions k=3. Let H(x) denote the result of the three hash functions.

The 10-bit array is initialized as below

0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0

Insert x_0 with $H(x_0) = \{1, 4, 9\}$

0	1	2	3	4	5	6	7	8	9
0	1	0	0	1	0	0	0	0	1

Insert x_1 with $H(x_1) = \{4, 5, 8\}$

0	1	2	3	4	5	6	7	8	9
0	1	0	0	1	1	0	0	1	1

Query y_0 with $H(y_0) = \{0, 4, 8\}$ => ???

Query y_1 with $H(y_1) = \{1, 5, 8\}$ => ??? **False positive!**

Another Example: https://llimllib.github.io/bloomfilter-tutorial/

Bloom Filter – Analysis

What fraction of the bit vector B are 1s?

Throwing $k \cdot m$ darts at n targets

So fraction of **1**s is $(1 - e^{-km/n})$

But we have k independent hash functions and we only let the element x through **if all** k hash element x to a bucket of value **1**

So, false **positive probability = $(1 - e^{-km/n})^k$**

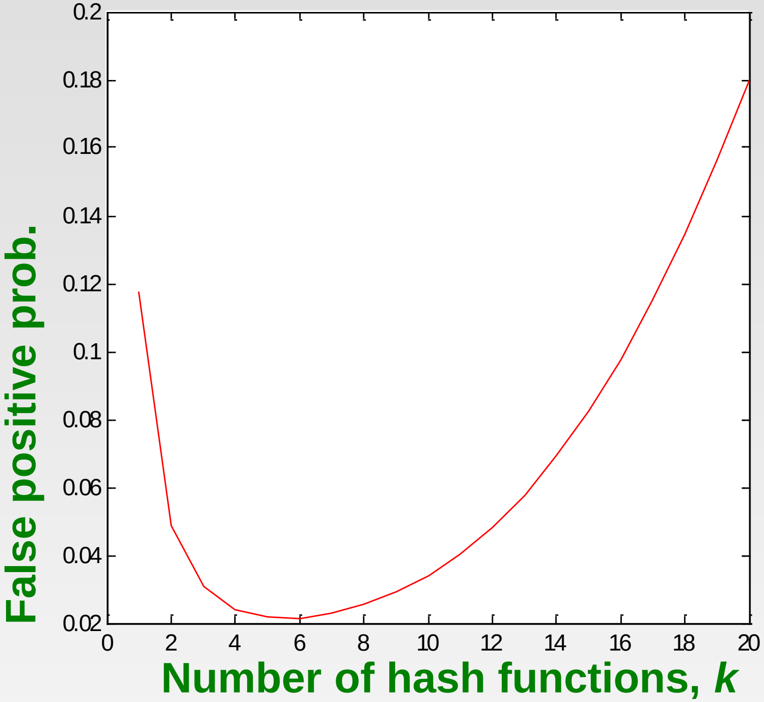
Bloom Filter – Analysis (2)

***m* = 1 billion, *n* = 8 billion**

k = 1: $(1 - e^{-1/8})$ = **0.1175**

k = 2: $(1 - e^{-1/4})^2$ = **0.0493**

What happens as we keep increasing *k*?



"Optimal" value of ***k***: ***n/m* ln(2)**

In our case: Optimal **k = 8 ln(2) = 5.54 ≈ 6**

- **Error at k = 6**: $(1 - e^{-1/6})^2$ = **0.0235**

Bloom Filter: Wrap-up

Bloom filters guarantee no false negatives, and use limited memory

Great for pre-processing before more expensive checks

Suitable for hardware implementation

Hash function computations can be parallelized

Is it better to have **1** big **B** or ***k*** small **B**s?

It is the same: $(1 - e^{-km/n})^k$ vs. $(1 - e^{-m/(n/k)})^k$

But keeping **1 big B** is simpler

Part 4: Counting Data Streams (Sketch)

Counting Distinct Elements

Problem:

- Data stream consists of a universe of elements chosen from a set of size ***N***
- Maintain a count of the number of distinct elements seen so far

Example:

Data stream: | 3 | 2 | 5 | 3 | 2 | 1 | 7 | 5 | 1 | 2 | 3 | 7 |

Number of distinct values: 5

Obvious approach: Maintain the set of elements seen so far

- That is, keep a hash table of all the distinct elements seen so far

Applications

How many different words are found among the Web pages being crawled at a site?

- Unusually low or high numbers could indicate artificial pages (spam?)

How many different Web pages does each customer request in a week?

How many distinct products have we sold in the last week?

Using Small Storage

Real problem: What if we do not have space to maintain the set of elements seen so far?

Estimate the count in an unbiased way

Accept that the count may have a little error, but limit the probability that the error is large

Sketches

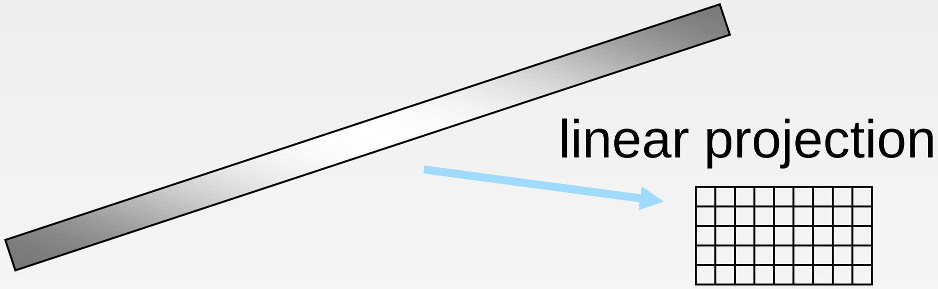
Sampling does not work!

- If a large fraction of items aren't sampled, don't know if they are all same or all different

Sketch: a technique takes advantage that the algorithm can "see" all the data even if it can't "remember" it all

Essentially, sketch is a linear transform of the input

- Model stream as defining a vector, sketch is result of multiplying stream vector by an (implicit) matrix



Flajolet-Martin Sketch

Probabilistic Counting Algorithms for Data Base Applications. 1985.

Pick a hash function ***h*** that maps each of the ***N*** elements to at least $\log_2 N$ bits

For each stream element ***a***, let ***r*(*a*)** be the number of trailing **0s** in ***h*(*a*)**

- **r(a)** = position of first 1 counting from the right
  - E.g., say ***h(a) = 12***, then ***12*** is ***1100*** in binary, so ***r(a) = 2***

Record *R* = the maximum *r*(*a*) seen

- $R = \max_a r(a)$, over all the items ***a*** seen so far

Estimated number of distinct elements = 2^R

Why It Works: Intuition

Very very rough and heuristic intuition why Flajolet-Martin works:

- ***h(a)*** hashes ***a*** with **equal prob.** to any of ***N*** values
- Then ***h(a)*** is a sequence of $\log_2 N$ bits, where 2^{-r} fraction of all ***a***s have a tail of ***r*** zeros
  - About 50% of ***a***s hash to *****0**
  - About 25% of ***a***s hash to ****00**
  - So, if we saw the longest tail of ***r=2*** (i.e., item hash ending ***100**) then we have probably seen **about *4*** distinct items so far
- So, it takes to hash about 2^r items before we see one with zero-suffix of length r

Why It Works: More formally

Formally, we will show that **probability of finding a tail of *r* zeros:**

Goes to 1 if $m \gg 2^r$

Goes to 0 if $m \ll 2^r$

where m is the number of distinct elements seen so far in the stream

Thus, 2^R will almost always be around *m!*

Why It Works: More formally

The probability that a given *h*(*a*) ends in at least *r* zeros is 2^{-r}

h(a) hashes elements uniformly at random

Probability that a random number ends in at least ***r*** zeros is $\mathbf{2^{-r}}$

Then, the probability of **NOT** seeing a tail of length ***r*** among ***m*** elements:

$$\boxed{\left(\boxed{1-2^{-r}}\right)^{m}}$$

Prob. all end in fewer than ***r*** zeros.

Prob. that given **h(a)** ends in fewer than ***r*** zeros

Why It Works: More formally

Note: $(1-2^{-r})^m = (1-2^{-r})^{2^r(m2^{-r})} \approx e^{-m2^{-r}}$

Prob. of NOT finding a tail of length *r* is:

If ***m* << *2^r***, then prob. tends to **1**

- $(1-2^{-r})^m \approx e^{-m2^{-r}} = 1$ as **$m/2^r \to 0$**
- So, the probability of finding a tail of length ***r*** tends to **0**

If ***m* >> *2^r***, then prob. tends to **0**

- $(1-2^{-r})^m \approx e^{-m2^{-r}} = 0$ as **$m/2^r \to \infty$**
- So, the probability of finding a tail of length ***r*** tends to **1**

Thus, 2^R will almost always be around *m!*

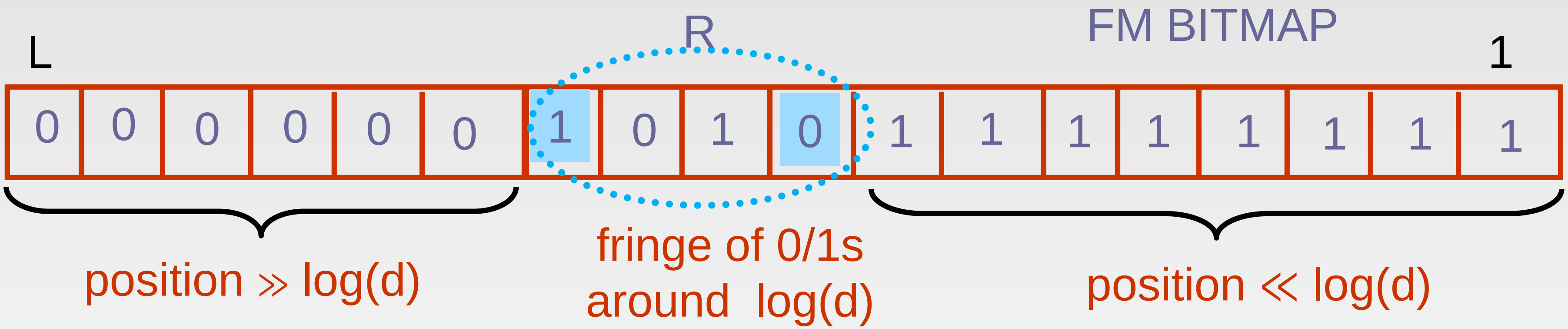
Flajolet-Martin Sketch

Maintain FM Sketch = bitmap array of L = $\log N$ bits

Initialize bitmap to all 0s

For each incoming value a, set FM[r(a)] = 1

If d distinct values, expect d/2 map to FM[1], d/4 to FM[2]…



Use the leftmost 1: R = $\mathbf{max_a\ r(a)}$

Use the rightmost 0: also an indicator of log(d)

- Estimate $d = c2^R$ for scaling constant c ≈ 1.3 (original paper)

Average many copies (different hash functions) improves accuracy

References

Chapter 4, Mining of Massive Datasets.

End of Chapter 7