

COMP9313: Big Data Management

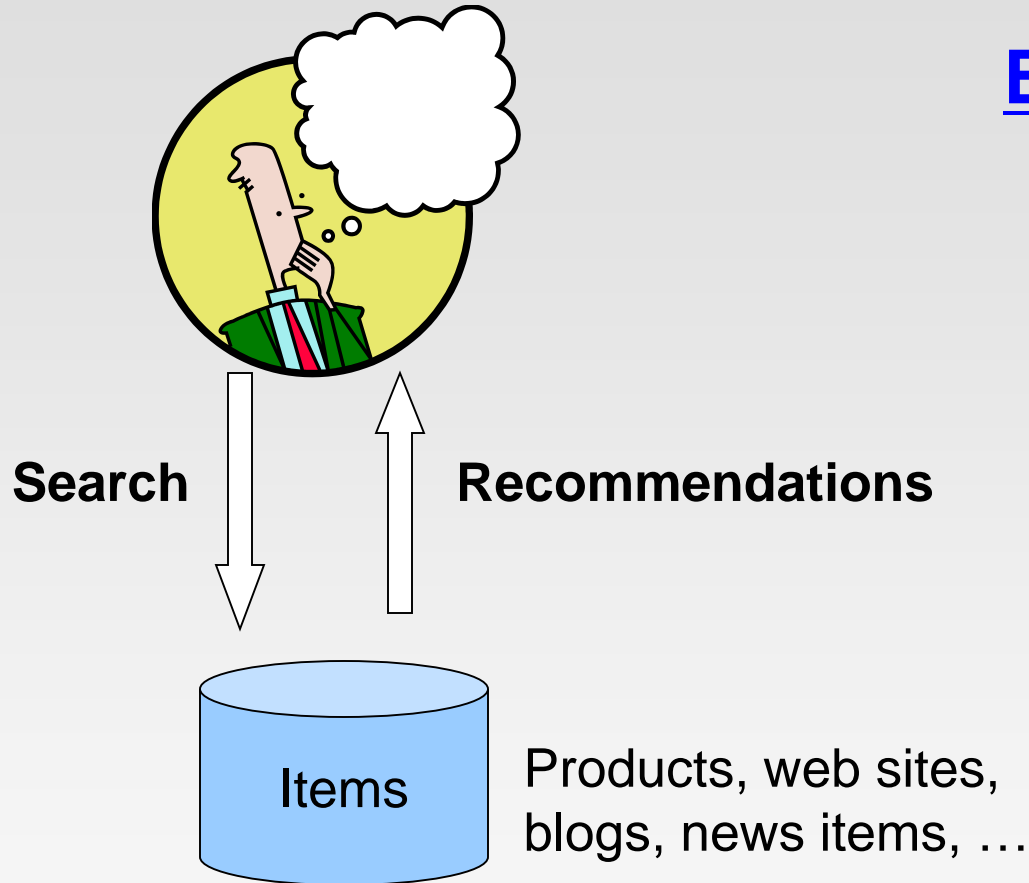


Lecturer: Xin Cao

Course web site: <http://www.cse.unsw.edu.au/~cs9313/>

Chapter 11: Recommender Systems

Recommendations



Examples:

amazon.com



movielens
helping you find the *right* movies

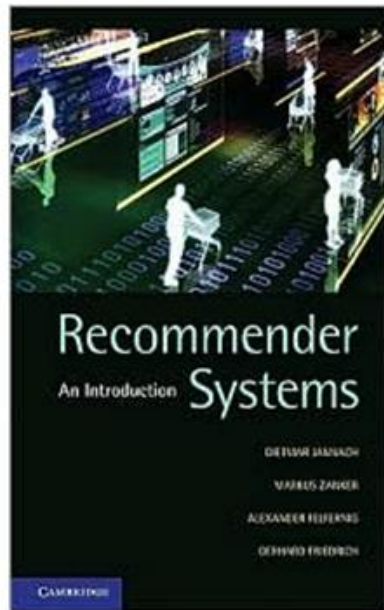
last.fm
the social music revolution

Google
News

YouTube

XBOX
LIVE

Recommender Systems



Recommender Systems: An Introduction

by [Dietmar Jannach](#), [Markus Zanker](#), [Alexander Felfernig](#), [Gerhard Friedrich](#)

AVERAGE CUSTOMER RATING:

☆☆☆☆☆ ([Be the first to review](#))



Registrieren, um sehen zu können, was deinen Freunden gefällt.

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Customers who bought this also bought



Recommender Systems

Application areas

Movie recommendation (Netflix)

Related product recommendation (Amazon)

Web page ranking (Google)

Social recommendation (Facebook)

... ..

Related hotels...



Hotel 41

1,170 Reviews

London, England

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★★★★☆ (53)



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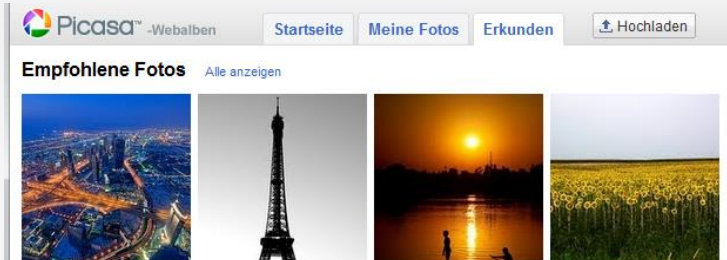
×



Senior Program Manager (f/m)

Johnson Controls - Germany-NW-Burscheid

×



Netflix Movie Recommendation

Training data

user	movie	date	score
1	21	5/7/02	1
1	213	8/2/04	5
2	345	3/6/01	4
2	123	5/1/05	4
2	768	7/15/02	3
3	76	1/22/01	5
4	45	8/3/00	4
5	568	9/10/05	1
5	342	3/5/03	2
5	234	12/28/00	2
6	76	8/11/02	5
6	56	6/15/03	4

Test data

user	movie	date	score
1	62	1/6/05	?
1	96	9/13/04	?
2	7	8/18/05	?
2	3	11/22/05	?
3	47	6/13/02	?
3	15	8/12/01	?
4	41	9/1/00	?
4	28	8/27/05	?
5	93	4/4/05	?
5	74	7/16/03	?
6	69	2/14/04	?
6	83	10/3/03	?

Why using Recommender Systems?

Value for the customer

- Find things that are interesting
- Narrow down the set of choices
- Help me explore the space of options
- Discover new things
- Entertainment
- ...

Value for the provider

- Additional and probably unique personalized service for the customer
- Increase trust and customer loyalty
- Increase sales, click through rates, conversion etc.
- Opportunities for promotion, persuasion
- Obtain more knowledge about customers
- ...

Recommender systems

RS seen as a function

Given:

User model (e.g. ratings, preferences, demographics, situational context)

Items (with or without description of item characteristics)

Find:

Relevance score. Used for ranking.

Finally:

Recommend items that are assumed to be relevant

But:

Remember that relevance might be context-dependent

Characteristics of the list itself might be important (diversity)

Formal Model

X = set of Customers

S = set of Items

Utility function $u: X \times S \rightarrow R$

R = set of ratings

R is a totally ordered set

e.g., 0-5 stars, real number in $[0,1]$

Utility Matrix

	Avatar	LOTR	Matrix	Pirates
Alice	1		0.2	
Bob		0.5		0.3
Carol	0.2		1	
David				0.4

Key Problems

Gathering “known” ratings for matrix

How to collect the data in the utility matrix

Extrapolate unknown ratings from the known ones

Mainly interested in high unknown ratings

- ▶ We are not interested in knowing what you don't like but what you like

Evaluating extrapolation methods

How to measure success/performance of recommendation methods

Gathering Ratings

Explicit

- Ask people to rate items

- Doesn't work well in practice – people can't be bothered

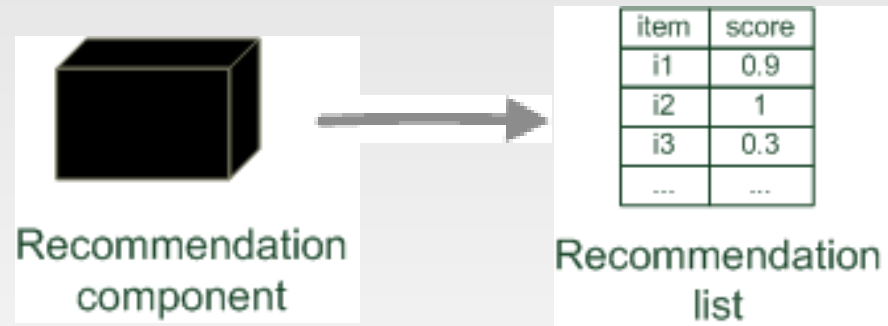
Implicit

- Learn ratings from user actions

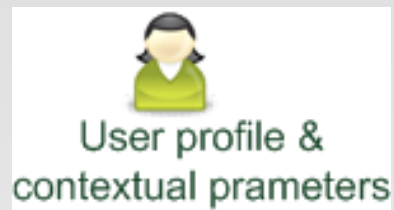
- ▶ E.g., purchase implies high rating

Paradigms of recommender systems

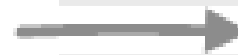
Recommender systems reduce information overload by estimating relevance



Paradigms of recommender systems



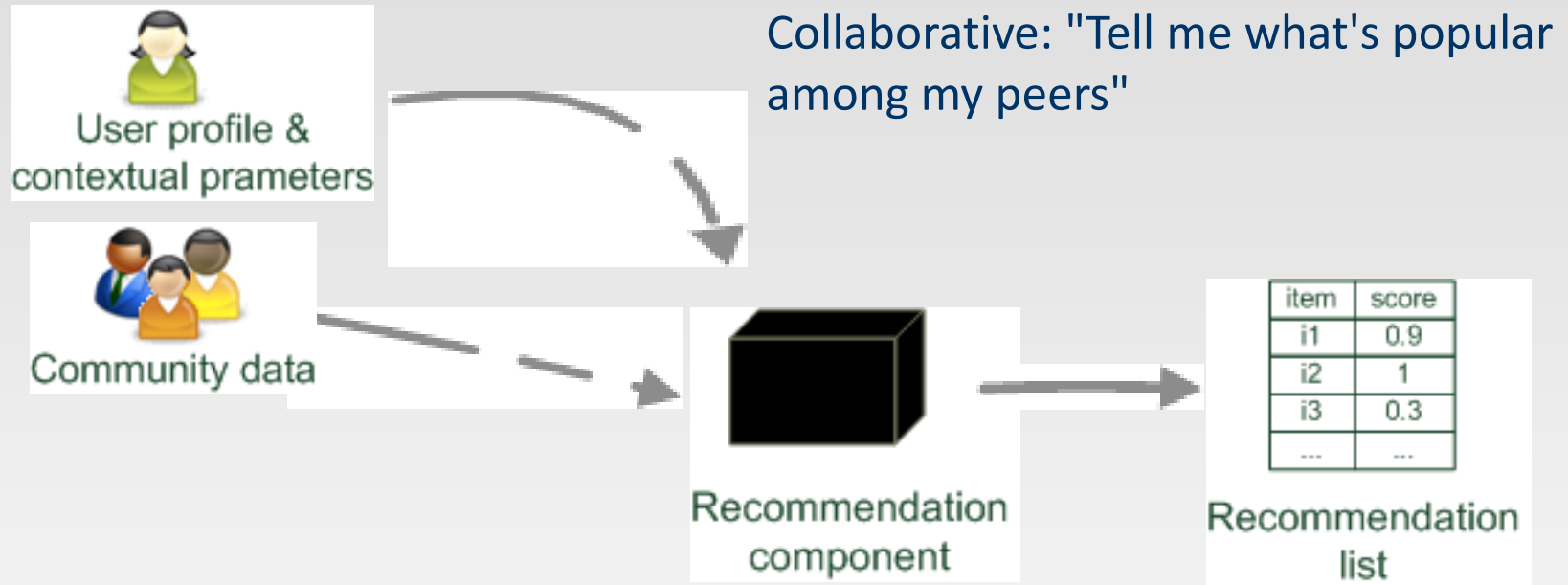
Personalized recommendations



item	score
i1	0.9
i2	1
i3	0.3
...	...

Recommendation
list

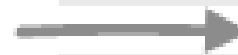
Paradigms of recommender systems



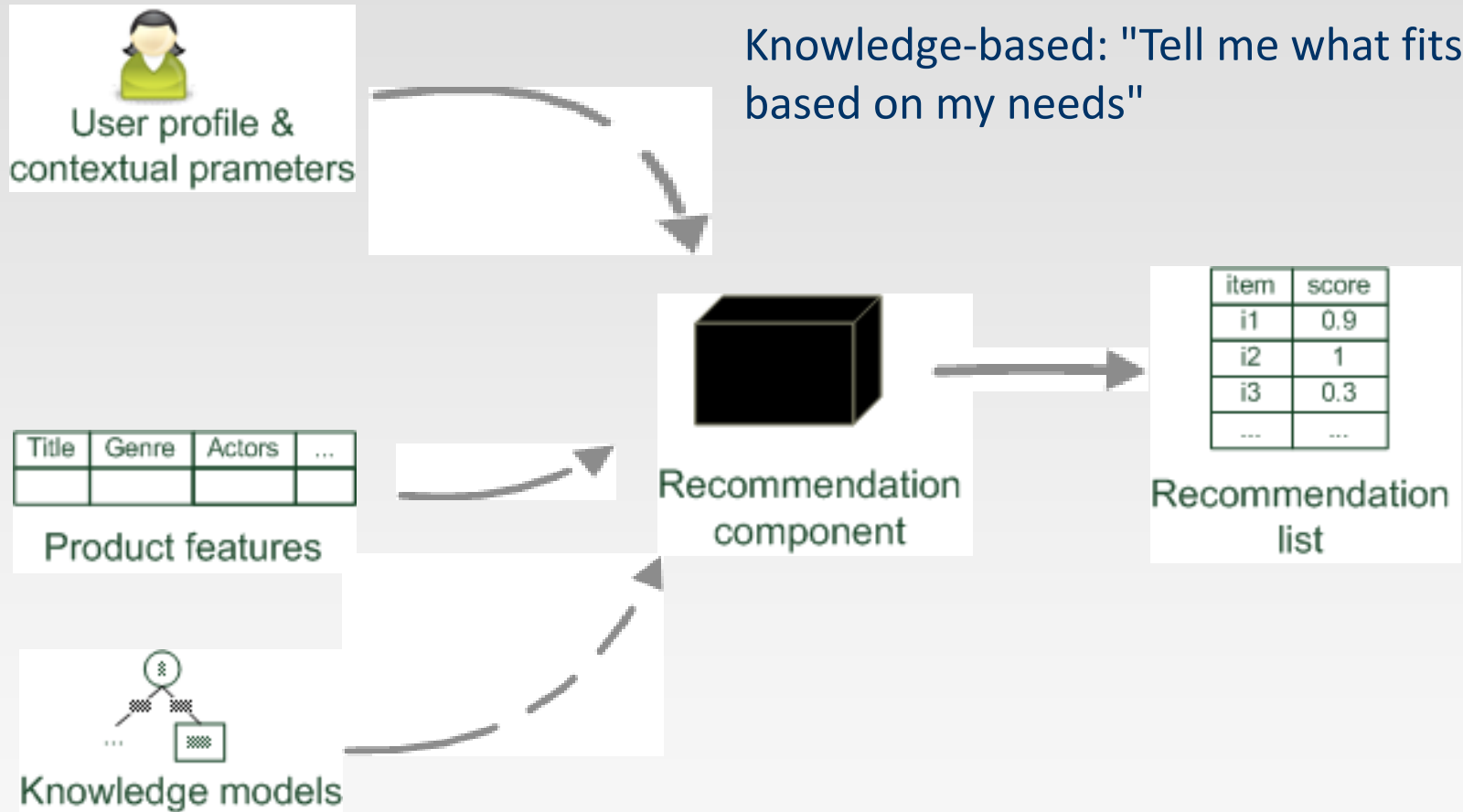
Paradigms of recommender systems



Content-based: "Show me more of the same what I've liked"

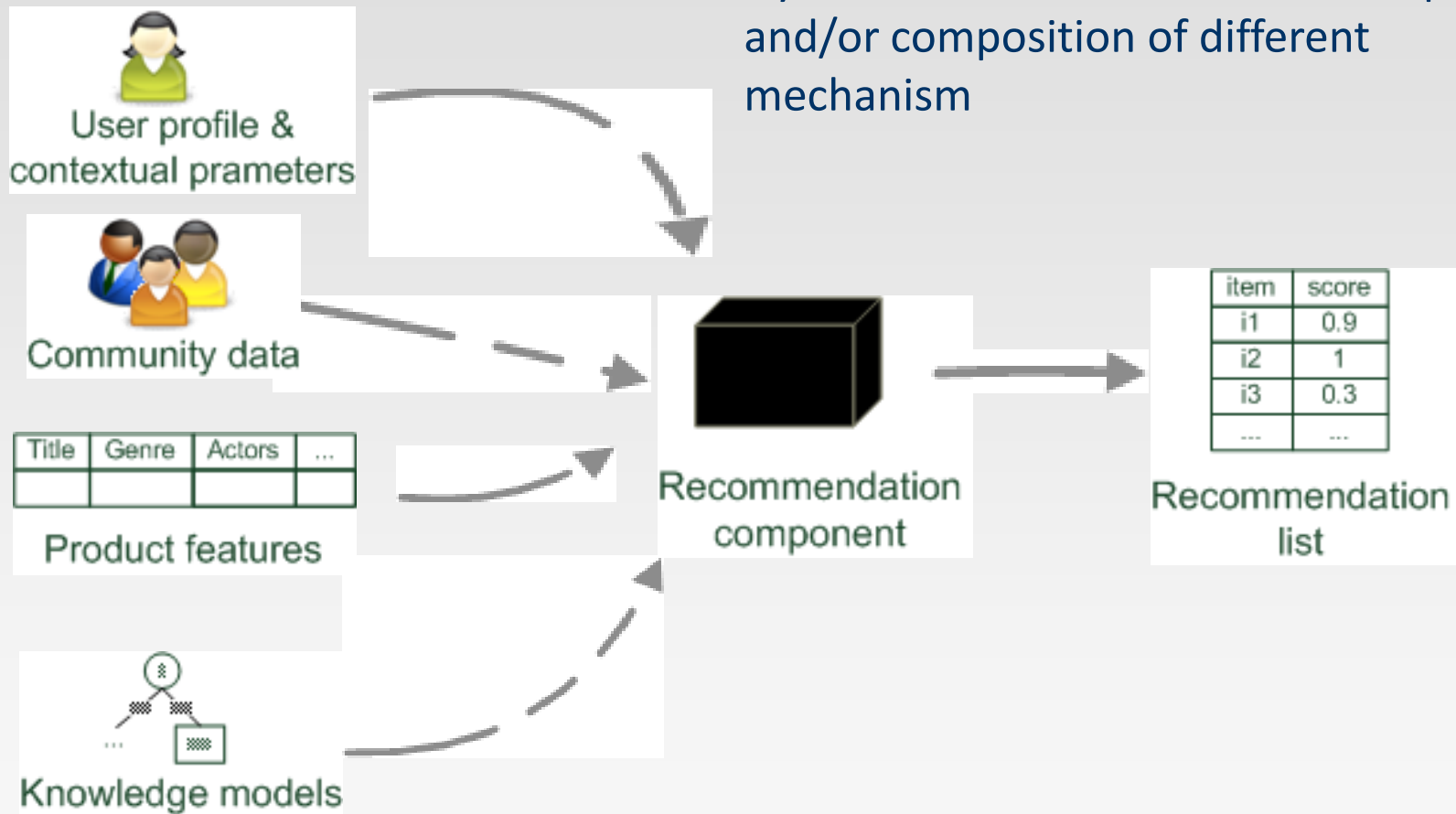


Paradigms of recommender systems

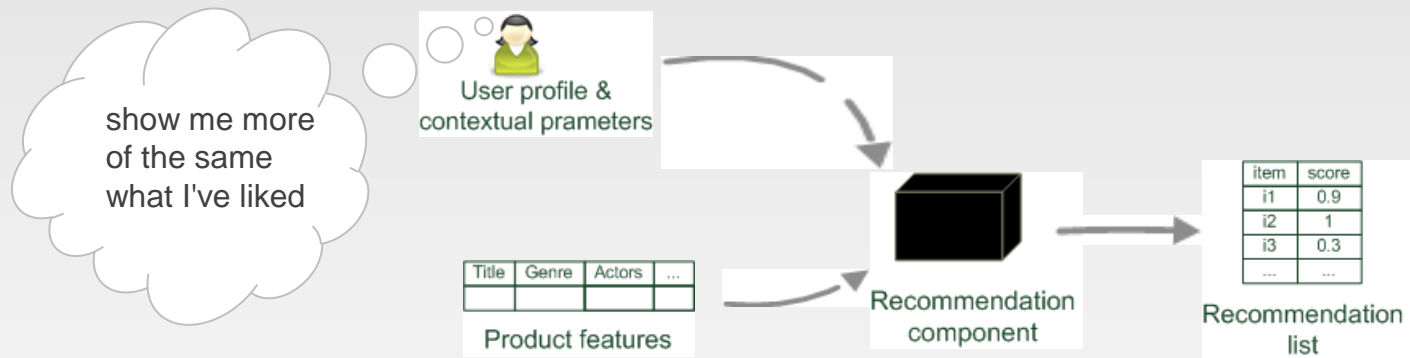


Paradigms of recommender systems

Hybrid: combinations of various inputs and/or composition of different mechanism



Content-based Recommendation



Content-based Recommendations

Main idea: Recommend items to customer x similar to previous items rated highly by x

What do we need:

Some information about the available items such as the genre ("content")

Some sort of *user profile* describing what the user likes (the preferences)

Example:

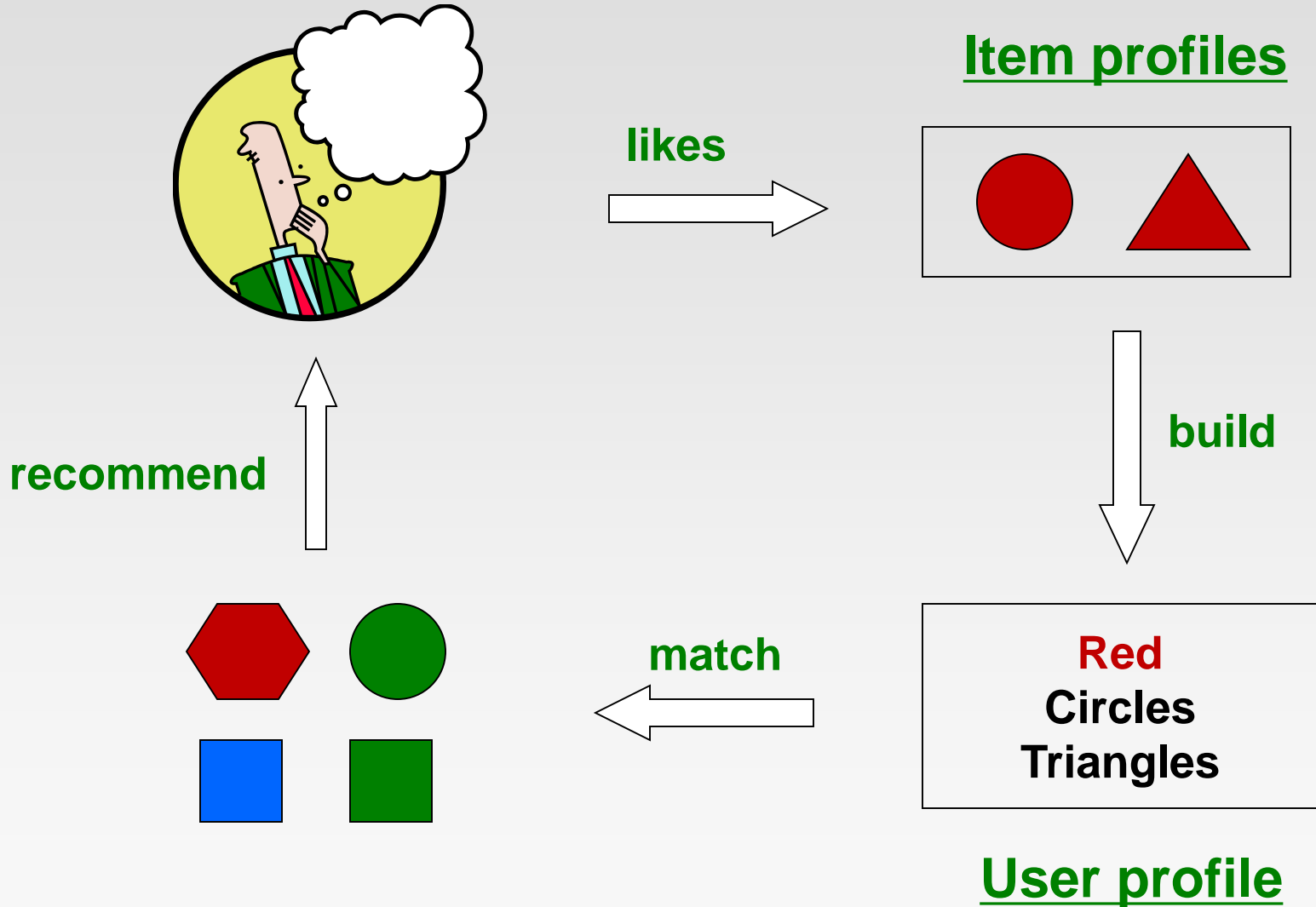
Movie recommendations:

- ▶ Recommend movies with same actor(s), director, genre, ...

Websites, blogs, news:

- ▶ Recommend other sites with “similar” content

Plan of Action



What is the “Content”?

Most CB-recommendation techniques were applied to recommending text documents.

Like web pages or newsgroup messages for example.

Content of items can also be represented as text documents.

With textual descriptions of their basic characteristics.

Structured: Each item is described by the same set of attributes



Title	Genre	Author	Type	Price	Keywords
The Night of the Gun	Memoir	David Carr	Paperback	29.90	Press and journalism, drug addiction, personal memoirs, New York
The Lace Reader	Fiction, Mystery	Brunonia Barry	Hardcover	49.90	American contemporary fiction, detective, historical
Into the Fire	Romance, Suspense	Suzanne Brockmann	Hardcover	45.90	American fiction, murder, neo-Nazism

Unstructured: free-text description.

Item Profiles

For each item, create an **item profile**

Profile is a set (vector) of features

Movies: author, title, actor, director,...

Text: Set of “important” words in document

How to pick important features?

Usual heuristic from text mining is **TF-IDF**
(Term frequency * Inverse Doc Frequency)

- ▶ **Term ... Feature**
- ▶ **Document ... Item**

Term-Frequency - Inverse Document Frequency (TF-IDF)

Compute the overall importance of keywords

Given a keyword i and a document j

$$TF-IDF(i,j) = TF(i,j) * IDF(i)$$

Term frequency (TF)

Let $freq(i,j)$ number of occurrences of keyword i in document j

Let $maxOthers(i,j)$ denote the highest number of occurrences of another keyword of j

$$TF(i,j) = \frac{freq(i,j)}{maxOthers(i,j)}$$

Inverse Document Frequency (IDF)

N : number of all recommendable documents

$n(i)$: number of documents in which keyword i appears

$$IDF(i) = \log \frac{N}{n(i)}$$

User Profiles and Prediction

User profile possibilities:

Weighted average of rated item profiles

Variation: weight by difference from average rating for item

...

Prediction heuristic:

Given user profile \mathbf{x} and item profile \mathbf{i} , estimate

$$u(\mathbf{x}, \mathbf{i}) = \cos(\mathbf{x}, \mathbf{i}) = \frac{\mathbf{x} \cdot \mathbf{i}}{||\mathbf{x}|| \cdot ||\mathbf{i}||}$$

Pros: Content-based Approach

+: No need for data on other users

+: Able to recommend to users with unique tastes

+: Able to recommend new & unpopular items

No first-rater problem

+: Able to provide explanations

Can provide explanations of recommended items by listing content-features that caused an item to be recommended

Cons: Content-based Approach

–: Finding the appropriate features is hard

E.g., images, movies, music

–: Recommendations for new users

How to build a user profile?

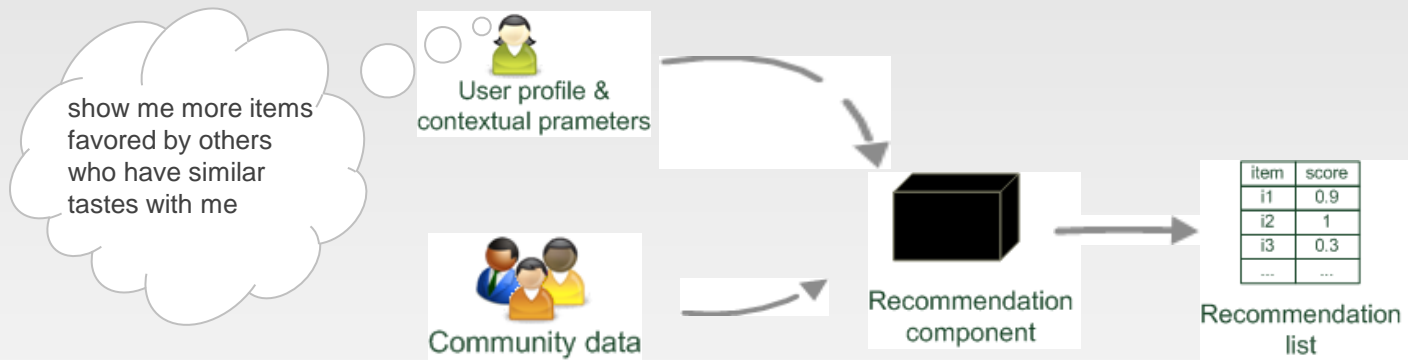
–: Overspecialization

Never recommends items outside user's content profile

People might have multiple interests

Unable to exploit quality judgments of other users

Collaborative Filtering



Collaborative Filtering (CF)

The most prominent approach to generate recommendations
used by large, commercial e-commerce sites
well-understood, various algorithms and variations exist
applicable in many domains (book, movies, DVDs, ..)

Approach

use the "wisdom of the crowd" to recommend items

Basic assumption and idea

Users give ratings to catalog items (implicitly or explicitly)

Customers who had similar tastes in the past, will have similar tastes in the future

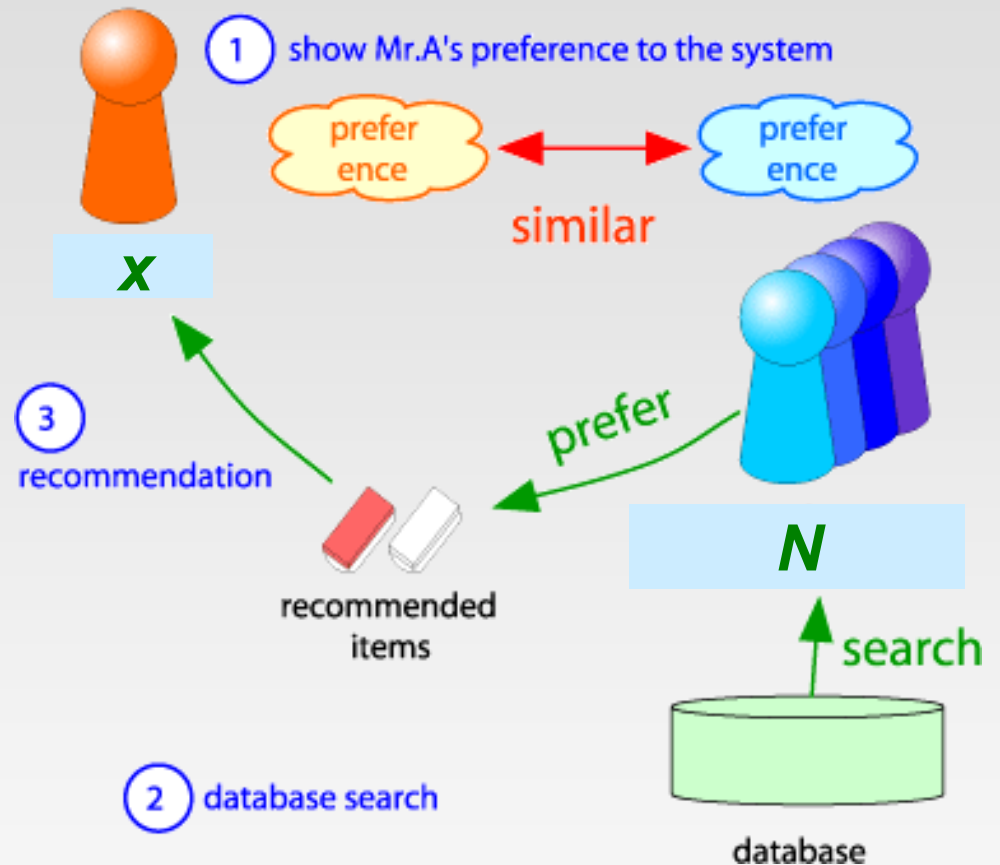


Collaborative Filtering

Consider user x

Find set N of other users whose ratings are “**similar**” to x ’s ratings

Estimate x ’s ratings based on ratings of users in N



User-based Nearest-Neighbor Collaborative Filtering

The basic technique

Given an "active user" (Alice) and an item i not yet seen by Alice

- ▶ find a set of users (peers/nearest neighbors) who liked the same items as Alice in the past **and** who have rated item i
- ▶ use, e.g. the average of their ratings to predict, if Alice will like item i
- ▶ do this for all items Alice has not seen and recommend the best-rated

Basic assumption and idea

If users had similar tastes in the past they will have similar tastes in the future

User preferences remain stable and consistent over time

User-based Nearest-Neighbor Collaborative Filtering

Example

A database of ratings of the current user, Alice, and some other users is given:

	Item1	Item2	Item3	Item4	Item5
Alice	5	3	4	4	?
User1	3	1	2	3	3
User2	4	3	4	3	5
User3	3	3	1	5	4
User4	1	5	5	2	1

Determine whether Alice will like or dislike *Item5*, which Alice has not yet rated or seen

User-based Nearest-Neighbor Collaborative Filtering

Some first questions

How do we measure similarity?

How many neighbors should we consider?

How do we generate a prediction from the neighbors' ratings?



	Item1	Item2	Item3	Item4	Item5
Alice	5	3	4	4	?
User1	3	1	2	3	3
User2	4	3	4	3	5
User3	3	3	1	5	4
User4	1	5	5	2	1

Finding “Similar” Users

Let r_x be the vector of user x 's ratings

$$r_x = [*, _, _, *, ***]$$

$$r_y = [*, _, **, **, _]$$

Jaccard similarity measure $\frac{||r_x \cap r_y||}{||r_x \cup r_y||}$

Problem: Ignores the value of the rating

Cosine similarity measure

$$\text{sim}(x, y) = \cos(r_x, r_y) = \frac{r_x \cdot r_y}{||r_x|| \cdot ||r_y||}$$

Problem: Treats missing ratings as “negative”

Pearson correlation coefficient

S_{xy} = items rated by both users x and y

$$\text{sim}(x, y) = \frac{\sum_{s \in S_{xy}} (r_{xs} - \bar{r}_x)(r_{ys} - \bar{r}_y)}{\sqrt{\sum_{s \in S_{xy}} (r_{xs} - \bar{r}_x)^2} \sqrt{\sum_{s \in S_{xy}} (r_{ys} - \bar{r}_y)^2}}$$

r_x, r_y as sets:

$$r_x = \{1, 4, 5\}$$

$$r_y = \{1, 3, 4\}$$

r_x, r_y as points:

$$r_x = \{1, 0, 0, 1, 3\}$$

$$r_y = \{1, 0, 2, 2, 0\}$$

$\bar{r}_x, \bar{r}_y \dots$ avg.
rating of x, y

Similarity Metric

Cosine similarity:

$$\text{sim}(x, y) = \frac{\sum_i r_{xi} \cdot r_{yi}}{\sqrt{\sum_i r_{xi}^2} \cdot \sqrt{\sum_i r_{yi}^2}}$$

	HP1	HP2	HP3	TW	SW1	SW2	SW3
A	4			5	1		
B	5	5	4				
C				2	4	5	
D		3					3

Intuitively we want: $\text{sim}(A, B) > \text{sim}(A, C)$

Jaccard similarity: $1/5 < 2/4$

Cosine similarity: $0.380 > 0.322$

$$\frac{4 \times 5}{\sqrt{4^2 + 5^2 + 1^2} \sqrt{5^2 + 5^2 + 4^2}} = 0.380$$

Considers missing ratings as “negative”

Solution: subtract the (row) mean

	HP1	HP2	HP3	TW	SW1	SW2	SW3
A	2/3			5/3	-7/3		
B	1/3	1/3	-2/3				
C				-5/3	1/3	4/3	
D		0					0

sim A,B vs. A,C:
 $0.092 > -0.559$


Notice cosine sim. is correlation when data is centered at 0

Similarity Metric (Cont')

A popular similarity measure in user-based CF: **Pearson correlation**

$$\text{sim}(x, y) = \frac{\sum_{s \in S_{xy}} (r_{xs} - \bar{r}_x)(r_{ys} - \bar{r}_y)}{\sqrt{\sum_{s \in S_{xy}} (r_{xs} - \bar{r}_x)^2} \sqrt{\sum_{s \in S_{xy}} (r_{ys} - \bar{r}_y)^2}}$$

Possible similarity values between -1 and 1;

	Item1	Item2	Item3	Item4	Item5	
Alice	5	3	4	4	?	
User1	3	1	2	3	3	
User2	4	3	4	3	5	
User3	3	3	1	5	4	
User4	1	5	5	2	1	

sim = 0,85

sim = 0,00

sim = 0,70

sim = -0,79

Rating Predictions

From similarity metric to recommendations:

Let r_x be the vector of user x 's ratings

Let N be the set of k users most similar to x who have rated item i

Prediction for item s of user x :

$$r_{xi} = \frac{1}{k} \sum_{y \in N} r_{yi}$$

$$r_{xi} = \frac{\sum_{y \in N} s_{xy} \cdot r_{yi}}{\sum_{y \in N} s_{xy}}$$

Other options?

Many other tricks possible...

Shorthand:

$$s_{xy} = \text{sim}(x, y)$$

CF: Common Practice

Before:

$$r_{xi} = \frac{\sum_{j \in N(i;x)} s_{ij} r_{xj}}{\sum_{j \in N(i;x)} s_{ij}}$$

Define **similarity** s_{ij} of users i and j

Select k nearest neighbors $N(i; x)$

Users most similar to i , that have ratings on x

Estimate rating r_{xi} as the weighted average:

$$r_{xi} = b_{xi} + \frac{\sum_{j \in N(i;x)} s_{ij} \cdot (r_{xj} - b_{xj})}{\sum_{j \in N(i;x)} s_{ij}}$$

baseline estimate for r_{xi}

$$b_{xi} = \mu + b_x + b_i$$

- μ = overall mean movie rating
- b_x = rating deviation of user x
= (avg. rating of user x) - μ
- b_i = rating deviation of movie i

Memory-based and Model-based Approaches

User-based CF is said to be "memory-based"

- the rating matrix is directly used to find neighbors / make predictions

- does not scale for most real-world scenarios

- large e-commerce sites have tens of millions of customers and millions of items

Model-based approaches

- based on an offline pre-processing or "model-learning" phase

- at run-time, only the learned model is used to make predictions

- models are updated / re-trained periodically

- large variety of techniques used

- model-building and updating can be computationally expensive

Item-Item Collaborative Filtering

So far: **User-user collaborative filtering**

Another view: Item-item

Basic idea:

- ▶ Use the similarity between items (and not users) to make predictions

For item i , find other similar items

Estimate rating for item i based on ratings for similar items

Can use same similarity metrics and prediction functions as in user-user model

$$r_{xi} = \frac{\sum_{j \in N(i;x)} s_{ij} \cdot r_{xj}}{\sum_{j \in N(i;x)} s_{ij}}$$

s_{ij} ... similarity of items i and j

r_{xj} ... rating of user u on item j

$N(i;x)$... set items rated by x similar to i

Item-Item Collaborative Filtering

Example:

Look for items that are similar to Item5

Take Alice's ratings for these items to predict the rating for Item5

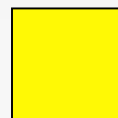
	Item1	Item2	Item3	Item4	Item5
Alice	5	3	4	4	?
User1	3	1	2	3	3
User2	4	3	4	3	5
User3	3	3	1	5	4
User4	1	5	5	2	1

Item-Item CF ($|N|=2$)

		users											
		1	2	3	4	5	6	7	8	9	10	11	12
movies	1	1		3			5			5		4	
	2			5	4			4			2	1	3
	3	2	4		1	2		3		4	3	5	
	4		2	4		5			4			2	
	5			4	3	4	2					2	5
	6	1		3		3			2			4	



- unknown rating



- rating between 1 to 5

Item-Item CF ($|N|=2$)

		users											
		1	2	3	4	5	6	7	8	9	10	11	12
movies	1	1		3		?	5			5		4	
	2			5	4			4			2	1	3
	3	2	4		1	2		3		4	3	5	
	4		2	4		5			4			2	
	5			4	3	4	2					2	5
	6	1		3		3			2			4	



- estimate rating of movie 1 by user 5

Item-Item CF ($|N|=2$)

		users												sim(1,m)
		1	2	3	4	5	6	7	8	9	10	11	12	
movies	1	1		3		?	5			5		4		1.00
	2			5	4			4			2	1	3	-0.18
	<u>3</u>	2	4		1	2		3		4	3	5		<u>0.41</u>
	4		2	4		5			4			2		-0.10
	5			4	3	4	2					2	5	-0.31
	<u>6</u>	1		3		3			2			4		<u>0.59</u>

Neighbor selection:

Identify movies similar to movie 1, rated by user 5

Here we use Pearson correlation as similarity:

1) Subtract mean rating m_i from each movie i

$$m_1 = (1+3+5+5+4)/5 = 3.6$$

row 1: [-2.6, 0, -0.6, 0, 0, 1.4, 0, 0, 1.4, 0, 0.4, 0]

2) Compute cosine similarities between rows

Item-Item CF ($|N|=2$)

		users												sim(1,m)
		1	2	3	4	5	6	7	8	9	10	11	12	
movies	1	1		3		?	5			5		4		1.00
	2			5	4			4			2	1	3	-0.18
	<u>3</u>	2	4		1	2		3		4	3	5		<u>0.41</u>
	4		2	4		5			4			2		-0.10
	5			4	3	4	2					2	5	-0.31
	<u>6</u>	1		3		3			2			4		<u>0.59</u>

Compute similarity weights:

$s_{1,3}=0.41$, $s_{1,6}=0.59$

Item-Item CF ($|N|=2$)

		users											
		1	2	3	4	5	6	7	8	9	10	11	12
movies	1	1		3		2.6	5			5		4	
	2			5	4			4			2	1	3
	<u>3</u>	2	4		1	2		3		4	3	5	
	4		2	4		5			4			2	
	5			4	3	4	2					2	5
	<u>6</u>	1		3		3			2			4	

Predict by taking weighted average:

$$r_{1.5} = (0.41 \cdot 2 + 0.59 \cdot 3) / (0.41 + 0.59) = 2.6$$

$$r_{ix} = \frac{\sum_{j \in N(i;x)} s_{ij} \cdot r_{jx}}{\sum s_{ij}}$$

Item-Item vs. User-User

	Avatar	LOTR	Matrix	Pirates
Alice	1		0.8	
Bob		0.5		0.3
Carol	0.9		1	0.8
David			1	0.4

In practice, it has been observed that item-item often works better than user-user

Why? Items are simpler, users have multiple tastes

More on Ratings – Explicit Ratings

Probably the most precise ratings

Most commonly used (1 to 5, 1 to 7 Likert response scales)

Main problems

Users not always willing to rate many items

- ▶ number of available ratings could be too small → sparse rating matrices → poor recommendation quality

How to stimulate users to rate more items?

More on Ratings – Implicit Ratings

Typically collected by the web shop or application in which the recommender system is embedded

When a customer buys an item, for instance, many recommender systems interpret this behavior as a positive rating

Clicks, page views, time spent on some page, demo downloads ...

Implicit ratings can be collected constantly and do not require additional efforts from the side of the user

Main problem

One cannot be sure whether the user behavior is correctly interpreted

For example, a user might not like all the books he or she has bought; the user also might have bought a book for someone else

Implicit ratings can be used in addition to explicit ones; question of correctness of interpretation

Collaborative Filtering: Complexity

Expensive step is finding k most similar customers: $O(|X|)$

Too expensive to do at runtime

Could pre-compute

Naïve pre-computation takes time $O(k \cdot |X|)$

— X ... set of customers

Ways of doing this:

Near-neighbor search in high dimensions (**LSH**)

Clustering

Dimensionality reduction

... ..

Supported by Hadoop: Apache Mahout

<https://mahout.apache.org/users/basics/algorithms.html>

What is a Good Recommendation in Practice?

Total sales numbers

Promotion of certain items

...

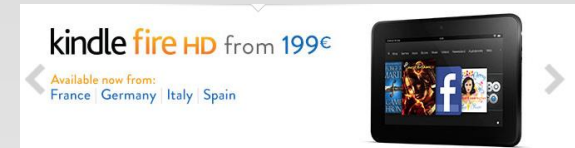
Click-through-rates

Interactivity on platform

...

Customer return rates

Customer satisfaction and loyalty



Evaluation

movies

users

1	3	4			
	3	5			5
		4	5		5
		3			
		3			
2			2		2
				5	
	2	1			1
	3			3	
1					

Evaluation

movies

users

1	3	4			
	3	5			5
		4	5		5
		3			
		3			
2			?		?
				?	
	2	1			?
	3			?	
1					

Test Data Set

Evaluating Predictions

Compare predictions with known ratings

Root-mean-square error (RMSE)

- ▶ $\sqrt{\sum_{xi} (r_{xi} - r_{xi}^*)^2}$ where r_{xi} is predicted, r_{xi}^* is the true rating of x on i

Precision at top 10:

- ▶ % of those in top 10

Rank Correlation:

- ▶ Spearman's *correlation* between system's and user's complete rankings

Another approach: 0/1 model

Coverage:

- ▶ Number of items/users for which system can make predictions

Precision:

- ▶ Accuracy of predictions

Receiver operating characteristic (ROC)

- ▶ Tradeoff curve between false positives and false negatives

The Netflix Prize

Training data

100 million ratings, 480,000 users, 17,770 movies

6 years of data: 2000-2005

Test data

Last few ratings of each user (2.8 million)

Evaluation criterion: Root Mean Square Error (RMSE) =

$$\frac{1}{|R|} \sqrt{\sum_{(i,x) \in R} (\hat{r}_{xi} - r_{xi})^2}$$

Netflix's system RMSE: 0.9514

Competition

2,700+ teams

\$1 million prize for 10% improvement on Netflix

The Netflix Utility Matrix R

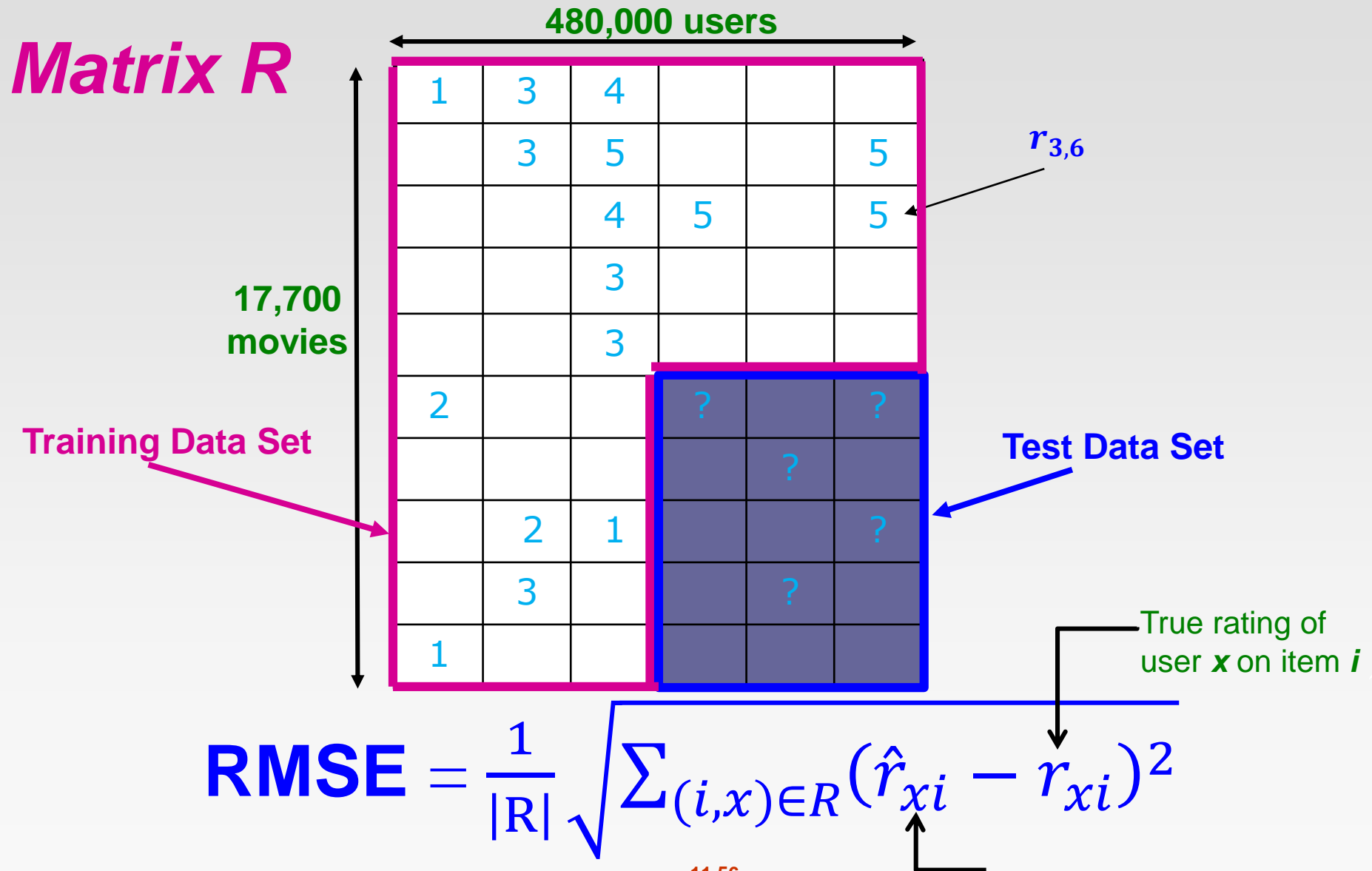
Matrix R

17,700
movies

480,000 users

1	3	4			
	3	5			5
		4	5		5
		3			
		3			
2			2		2
				5	
	2	1			1
	3			3	
1					

Utility Matrix R : Evaluation



BellKor Recommender System

The winner of the Netflix Challenge!

Multi-scale modeling of the data:

Combine top level, “regional” modeling of the data, with a refined, local view:

Global:

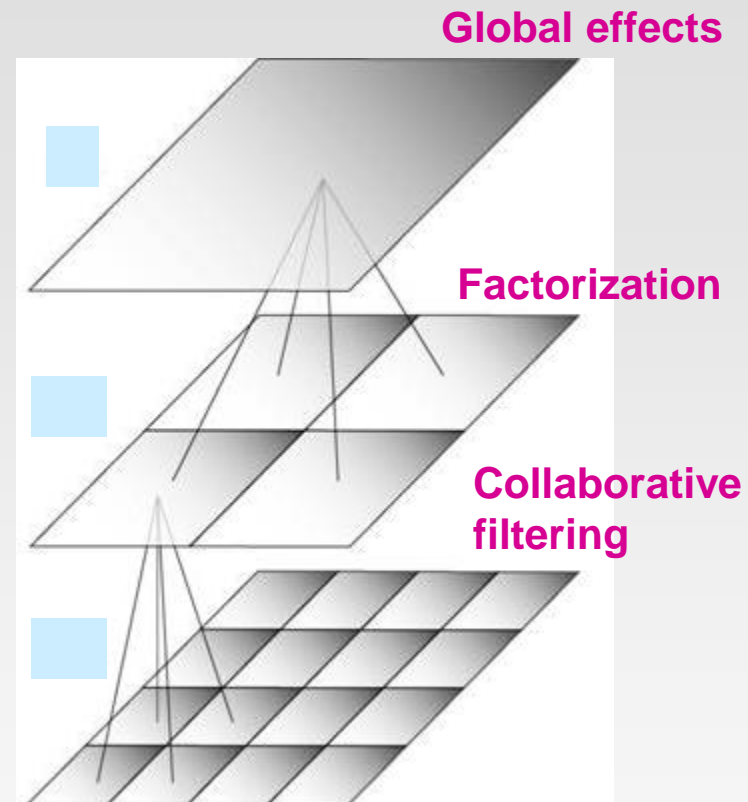
- ▶ Overall deviations of users/movies

Factorization:

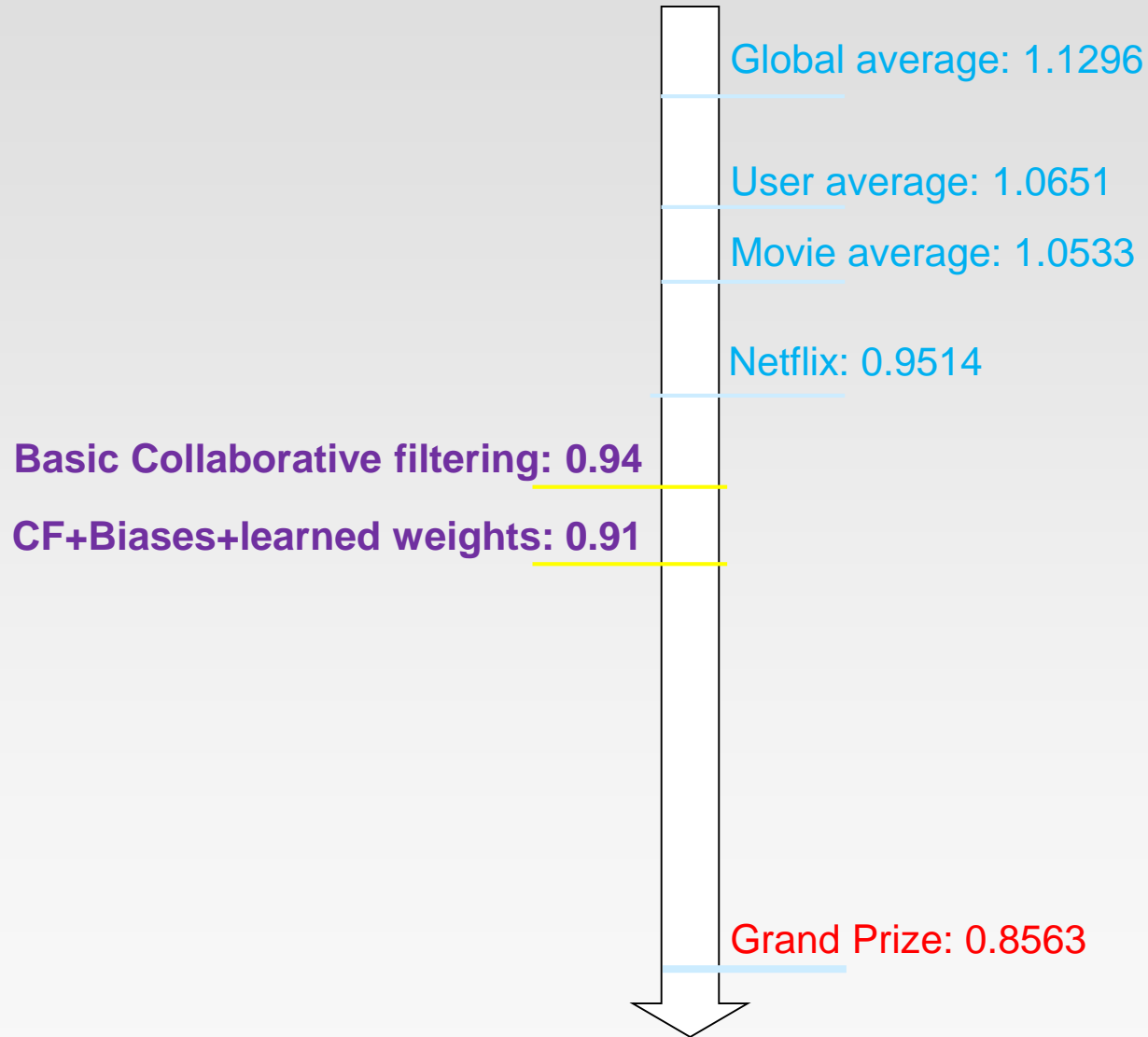
- ▶ Addressing “regional” effects

Collaborative filtering:

- ▶ Extract local patterns



Performance of Various Methods



Modeling Local & Global Effects

Global:

Mean movie rating: **3.7 stars**

The Sixth Sense is **0.5** stars above avg.

Joe rates **0.2** stars below avg.

⇒ **Baseline estimation:**

Joe will rate *The Sixth Sense* 4 stars

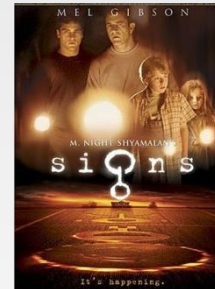


Local neighborhood (CF/NN):

Joe didn't like related movie *Signs*

⇒ **Final estimate:**

Joe will rate *The Sixth Sense* 3.8 stars



Modeling Local & Global Effects

In practice we get better estimates if we model deviations:

$$\hat{r}_{xi} = b_{xi} + \frac{\sum_{j \in N(i;x)} s_{ij} \cdot (r_{xj} - b_{xj})}{\sum_{j \in N(i;x)} s_{ij}}$$

baseline estimate for r_{xi}

$$b_{xi} = \mu + b_x + b_i$$

μ = overall mean rating

b_x = rating deviation of user x
= (avg. rating of user x) - μ

b_i = (avg. rating of movie i) - μ

Problems/Issues:

- 1) Similarity measures are “arbitrary”
- 2) Pairwise similarities neglect interdependencies among users
- 3) Taking a weighted average can be restricting

Solution: Instead of s_{ij} use w_{ij} that we estimate directly from data

Idea: Interpolation Weights w_{ij}

Use a **weighted sum** rather than **weighted avg.**:

$$\widehat{r_{xi}} = b_{xi} + \sum_{j \in N(i;x)} w_{ij}(r_{xj} - b_{xj})$$

A few notes:

$N(i; x)$... set of movies rated by user x that are similar to movie i

w_{ij} is the interpolation weight (some real number)

► We allow: $\sum_{j \in N(i,x)} w_{ij} \neq 1$

w_{ij} models interaction between pairs of movies (it does not depend on user x)

Idea: Interpolation Weights w_{ij}

$$\widehat{r_{xi}} = b_{xi} + \sum_{j \in N(i,x)} w_{ij} (r_{xj} - b_{xj})$$

How to set w_{ij} ?

Remember, error metric is: $\frac{1}{|R|} \sqrt{\sum_{(i,x) \in R} (\widehat{r_{xi}} - r_{xi})^2}$ or equivalently

SSE: $\sum_{(i,x) \in R} (\widehat{r_{xi}} - r_{xi})^2$

Find w_{ij} that minimize **SSE** on **training data**!

- Models relationships between item i and its neighbors j

w_{ij} can be **learned/estimated** based on \mathbf{x} and all other users that rated i

Recommendations via Optimization

Goal: Make good recommendations

Quantify goodness using **RMSE**:

Lower RMSE \Rightarrow better recommendations

Want to make good recommendations on items that user has not yet seen. **Can't really do this!**

Let's build a system such that it works well on known (user, item) ratings

And **hope** the system will also predict well the **unknown ratings**

Recommendations via Optimization

Idea: Let's set values w such that they work well on known (user, item) ratings

How to find such values w ?

Idea: Define an objective function and solve the optimization problem

Find w_{ij} that minimize **SSE on training data!**

$$J(w) = \sum_{x,i} \left(\underbrace{\left[b_{xi} + \sum_{j \in N(i;x)} w_{ij}(r_{xj} - b_{xj}) \right]}_{\text{Predicted rating}} - \underbrace{r_{xi}}_{\text{True rating}} \right)^2$$

Think of w as a vector of numbers

Interpolation Weights

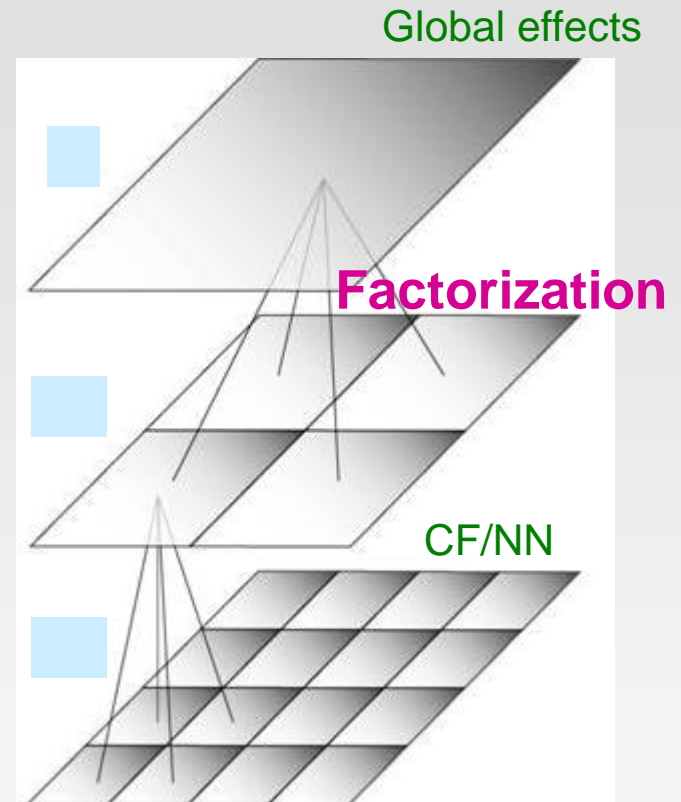
So far: $\widehat{r_{xi}} = b_{xi} + \sum_{j \in N(i;x)} w_{ij}(r_{xj} - b_{xj})$

Weights w_{ij} derived based on their role; **no use of an arbitrary similarity measure** ($w_{ij} \neq s_{ij}$)

Explicitly account for interrelationships among the neighboring movies

Next: Latent factor model

Extract “regional” correlations



More model-based approaches

Plethora of different techniques proposed in the last years, e.g.,

Matrix factorization techniques, statistics

- ▶ singular value decomposition, principal component analysis

Association rule mining

- ▶ compare: shopping basket analysis

Probabilistic models

- ▶ clustering models, Bayesian networks, probabilistic Latent Semantic Analysis

Various other machine learning approaches

Costs of pre-processing

Usually not discussed

Incremental updates possible?

Matrix Factorization

Informally, the SVD theorem (Golub and Kahan 1965) states that a given matrix M can be decomposed into a product of three matrices as follows

$$M = U \times \Sigma \times V^T$$

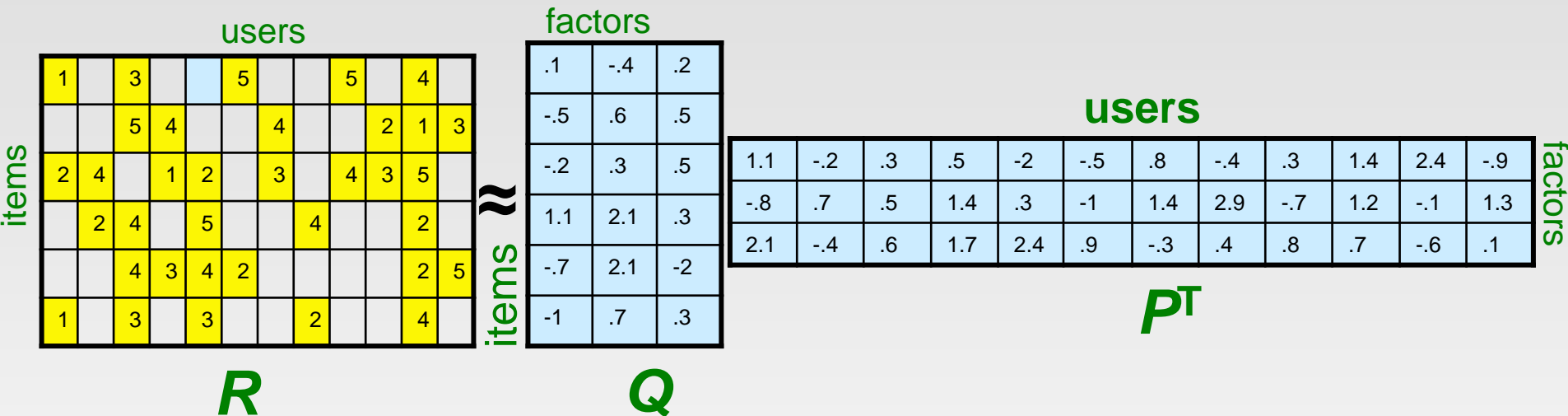
where U and V are called *left* and *right singular vectors* and the values of the diagonal of Σ are called the *singular values*

We can approximate the full matrix by observing only the most important features – those with the largest singular values

In the example, we calculate U , V , and Σ (with the help of some linear algebra software) but retain only the two most important features by taking only the first two columns of U and V^T

Matrix Factorization

“SVD” on Netflix data: $R \approx Q \cdot P^T$



For now let's assume we can approximate the rating matrix R as a product of “thin” $Q \cdot P^T$

R has missing entries but let's ignore that for now!

- Basically, we will want the reconstruction error to be small on known ratings and we don't care about the values on the missing ones

Ratings as Products of Factors

How to estimate the missing rating of user x for item i ?

users

items

1		3			5			5		4	
		5	4	2.4	4			2	1	3	
2	4		1	2		3		4	3	5	
	2	4		5			4			2	
		4	3	4	2					2	5
1		3		3			2			4	

≈

$$\hat{r}_{xi} = q_i \cdot p_x^T$$

$$= \sum_f q_{if} \cdot p_{xf}$$

q_i = row i of Q
 p_x = column x of P^T

items

factors

Q

.1	-.4	.2
-.5	.6	.5
-.2	.3	.5
1.1	2.1	.3
-.7	2.1	-2
-1	.7	.3

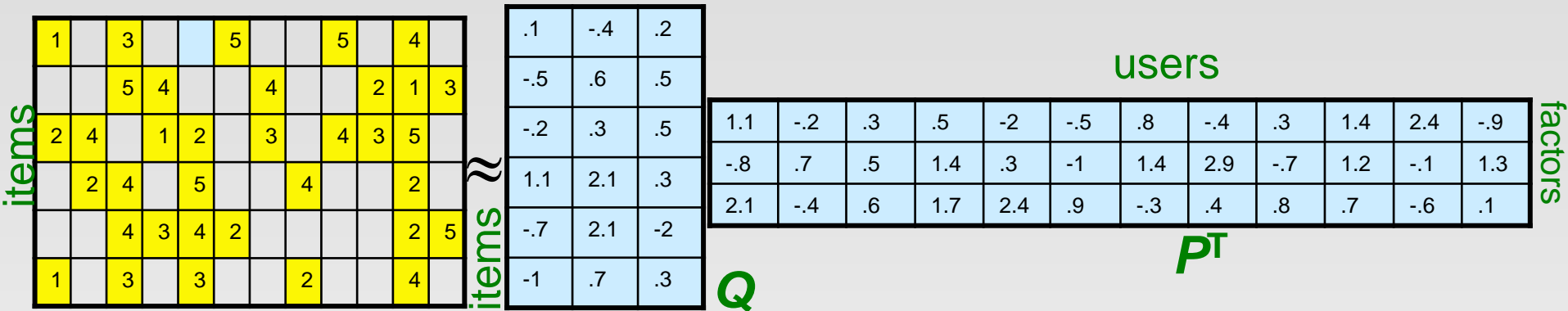
factors

users

P^T

1.1	-.2	.3	.5	-2	-.5	.8	-.4	.3	1.4	2.4	-.9
-.8	.7	.5	1.4	.3	-1	1.4	2.9	-.7	1.2	-.1	1.3
2.1	-.4	.6	1.7	2.4	.9	-.3	.4	.8	.7	-.6	.1

Matrix Factorization



SVD isn't defined when entries are missing!

Use specialized methods to find P, Q

$$\min_{P, Q} \sum_{(i, x) \in R} (r_{xi} - q_i \cdot p_x^T)^2$$

$$\hat{r}_{xi} = q_i \cdot p_x^T$$

Note:

- ▶ We don't require cols of P, Q to be orthogonal/unit length
- ▶ P, Q map users/movies to a latent space

Pros/Cons of Collaborative Filtering

- + Works for any kind of item

 - No feature selection needed

- Cold Start:

 - Need enough users in the system to find a match

- Sparsity:

 - The user/ratings matrix is sparse

 - Hard to find users that have rated the same items

- First rater:

 - Cannot recommend an item that has not been previously rated

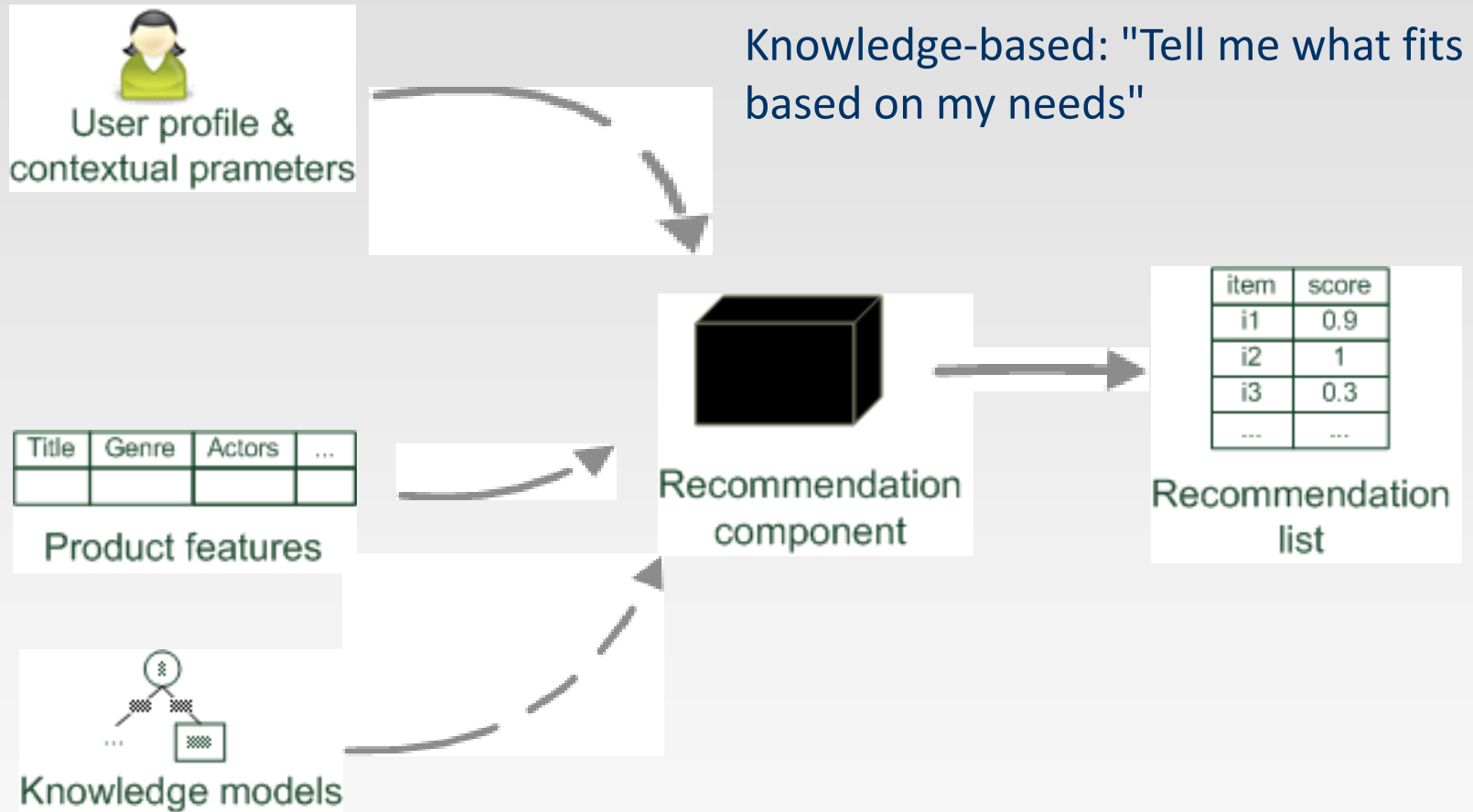
 - New items, Esoteric items

- Popularity bias:

 - Cannot recommend items to someone with unique taste

 - Tends to recommend popular items

Knowledge-Based Recommendation



Why do we need knowledge-based recommendation?

Products with low number of available ratings



Time span plays an important role

- Five-year-old ratings for computers

- User lifestyle or family situation changes

Customers want to define their requirements explicitly

- "The color of the car should be black"

Knowledge-based Recommendation

Constraint-based

- based on explicitly defined set of recommendation rules
- fulfill recommendation rules

Case-based

- based on different types of similarity measures
- retrieve items that are similar to specified requirements

Both approaches are similar in their **conversational** recommendation process

- users specify the requirements
- systems try to identify solutions
- if no solution can be found, users change requirements

Hybrid Methods

Implement two or more different recommenders and combine predictions

- Perhaps using a linear model

Add content-based methods to collaborative filtering

- Item profiles for new item problem

- Demographics to deal with new user problem

References

Chapter 9 of Mining of Massive Datasets.

Tutorial: Recommender Systems. IJCAI 2013.

End of Chapter 11