THE UNIVERSITY OF NEW SOUTH WALES

DEPARTMENT OF STATISTICS

Additional exercises for MATH5905, Statistical Inference

Part five: Higher order asymptotics

1. Consider n i.i.d. observations from Poisson (λ) distribution.

   i) Show that the cumulant generating function for a single observation is $K_X(t) = \lambda[e^t - 1]$.

   ii) Show that the one-term saddlepoint approximation for the density of the sum of n i.i.d. observations $Y = \sum_{i=1}^{n} X_i$ from this distribution is given by

   $$\hat{g}(y) = \frac{1}{\sqrt{2\pi}} e^{-n\lambda} \frac{e^y (n\lambda)^y}{y^{y+1/2}}, \; y = 0, 1, 2, \ldots.$$

   (Having in mind Stirling's approximation of $n! \approx \sqrt{2\pi n} e^{-n} n^n$ for large n, the above approximation coincides up to a normalising constant with the exact density (which is Poisson $(n\lambda)$) and the constant tends to 1 when $n \to \infty$.)

   Solution:

   1) Hint: Using the formula (27) from lecture 8 (Higher order asymptotics), get the saddlepoint density approximation of the mean $\bar{Y}$ and then use density transformation formula.