Query Complexity of Mastermind Variants

Aaron Berger Christopher Chute Matthew Stone

August 4, 2015

Abstract

We analyze variants of the popular board game Mastermind. In this two-player game, the codebreaker submits queries with the goal of identifying a hidden sequence, constructed at the beginning of the game by the codemaker. At each step, the codebreaker receives feedback in the form of "black" and "white" hits and incorporates the response into his next guess. We discuss asymptotics for the number of guesses needed to identify an unknown n-vector constructed from k possible colors. We look at strategies that receive two-color responses, as well as black hit-only responses. We consider both allowing and prohibiting repeated colors in the hidden sequence, and we analyze both adaptive and non-adaptive guessing strategies.

1 Introduction

Mastermind is a two-player board game that was invented in 1970, and variants on its basic four-spot, six-color structure have been studied extensively. In the original game, there are two players: The codemaker and the codebreaker. The codemaker initiates a game by constructing a 4-vector p from the six available colors, and the codebreaker attempts to guess the hidden vector in as few turns as possible. A turn consists of two parts: First the codebreaker submits a query vector q, which has the same form as a hidden vector. Second, the codemaker gives a two-part response:

1. Black Hits: The number of correct colors in the correct spot (the number of positions i such that $q_i = p_i$).

2. White Hits: The number of correct colors in the incorrect spot, and which have not been used in another hit.

In 1976, Donald Knuth presented a greedy "minimax" algorithm, and he showed via computer simulation that his algorithm always guesses the hidden vector in five turns or fewer.[1] Moreover, the minimax algorithm for Mastermind is optimal in the worst case—there is no algorithm that can guarantee to win in at most four turns.

[1] DK76

There are multiple natural extensions to the original Mastermind game. Varying the number of spots n and colors k, as well as the relationship between n and k, is one such extension. In this paper, we also consider both allowing and prohibiting repeated colors in vectors, and we examine both dual-color and black-only responses. We explore another degree of freedom between adaptive strategies, in which the codebreaker receives responses and adjusts the next guess accordingly, and non-adaptive strategies, in which all guesses are submitted at the beginning of the game and the hidden vector must be uniquely determined by the sequence of responses. [2]

1.1 Previous Work

We build on the work of Grebinski and Kucherov; Chvatal; Doerr, Spohel, Thomas, and Winzen; Erdos and Renyi; Knuth; Ko and Teng; Goodrich; and Bshouty. For a list of previously established bounds on the query complexity of mastermind variants, please see the appendix.

1.2 Our Contribution

2 Adaptive Strategies

3 Non-Adaptive Strategies

[2]Note that in a non-adaptive guessing strategy, our guesses need to *identify* the hidden vector rather than guess it outright. That is, a non-adaptive strategy wins if the sequence of responses to the guess vectors allows the codebreaker to distinguish between all possible hidden vectors. It is not necessary that a winning set of non-adaptive guesses includes the hidden vector itself.

References