Mastermind with No Repeats, $n = k$: Lower Bound for Guessing Strategies

- Previous Bound: $n - n/log(n)$

- Our Bound: There is no strategy that guarantees a win in $n - \log \log n$ turns.

Lower Bound on Guessing Effectiveness for Permutation Game

- Previous Work: None found

- Our work: In Mastermind with no repeats with n spots and k colors, for any set of possible remaining solutions, there exists a guess vector for which any response will eliminate at least $1/nk$ of the remaining solutions. This gives us the following asymptotic bounds:

    i. The minimax algorithm for Mastermind with n spots, k colors, and no repeats takes at most $O(n^2 k \log k)$ turns to find the hidden vector.
    ii. In the same game where $n = k$, the minimax algorithm takes at most $O(n^3 \log n)$ turns to find the hidden vector.

Extension of Previous Section to Repeated Colors

- Previous Work: None found

- Our Work: In a game of Mastermind with n spots and k colors, for any set of remaining solutions that can actually be achieved during a game of Mastermind, there is always a guess for which any response will eliminate at least $1/nk$ of the remaining solutions.
  This gives the same asymptotic bounds as above.

Optimality of Mastermind for $(n, k) = (4, 6)$

- Previous Work: This was presumably already done by Knuth, not confirmed yet.

- Our bound: It is not possible for a deterministic guessing strategy to guarantee finding the hidden vector in fewer than five turns.

Lower Bound for Non-Adaptive Strategies Using Only Black Hits

- Previous Work: This is an adaptation of a bound for Mastermind with repeats by [Doerr et al] to Mastermind without repeats.

- Our Work: Every non-adaptive strategy for Mastermind with no repeats must submit at least $O(n \log k)$ queries to uniquely identify all possible hidden vectors if given only black-hit responses.

Upper Bound on Non-Adaptive Strategies for Mastermind

- Previous Work: None found

- Our Work: There exists a set of at most nk queries which uniquely determines every possible hidden permutation (with or without repeats).

Non-Adaptive Lower Bound using Black and White Hits

- Previous Work: None found

- Our Work: For $k \geq n^2$, we get lower bound of $O(n \log k)$ turns for any non-adaptive strategy for Mastermind without repeats using both black and white hits.