

## Mastermind with No Repeats, $n = k$ : Lower Bound for Guessing Strategies

- Previous Bound:  $n - n/\log(n)$
- Our Bound: There is no strategy that guarantees a win in  $n - \log \log n$  turns.

## Lower Bound on Guessing Effectiveness for Permutation Game

- Previous Work: None found
- Our work: In Mastermind with no repeats with  $n$  spots and  $k$  colors, for any set of possible remaining solutions, there exists a guess vector for which any response will eliminate at least  $1/nk$  of the remaining solutions. This gives us the following asymptotic bounds:
  - i. The minimax algorithm for Mastermind with  $n$  spots,  $k$  colors, and no repeats takes at most  $O(n^2 k \log k)$  turns to find the hidden vector.
  - ii. In the same game where  $n = k$ , the minimax algorithm takes at most  $O(n^3 \log n)$  turns to find the hidden vector.

## Extension of Previous Section to Repeated Colors

- Previous Work: None found
- Our Work: In a game of Mastermind with  $n$  spots and  $k$  colors, for any set of remaining solutions that can actually be achieved during a game of Mastermind, there is always a guess for which any response will eliminate at least  $1/nk$  of the remaining solutions.  
This gives the same asymptotic bounds as above.

## Optimality of Mastermind for $(n, k) = (4, 6)$

- Previous Work: [Doerr et al] cite Knuth as having proved this, but that appears to be wrong. As far as we know, this has not been proved yet.
- Our bound: It is not possible for a deterministic guessing strategy to guarantee finding the hidden vector in fewer than five turns.

## Lower Bound for Non-Adaptive Strategies Using Only Black Hits

- Previous Work: This is an adaptation of a bound for Mastermind with repeats by [Doerr et al] to Mastermind without repeats.
- Our Work: Every non-adaptive strategy for Mastermind with no repeats must submit at least  $O(n \log k)$  queries to uniquely identify all possible hidden vectors if given only black-hit responses.

## Upper Bound on Non-Adaptive Strategies for Mastermind

- Previous Work: None found
- Our Work: There exists a set of at most  $nk$  queries which uniquely determines every possible hidden permutation (with or without repeats).

## Non-Adaptive Lower Bound using Black and White Hits

- Previous Work: None found
- Our Work: For  $k \geq n^2$ , we get lower bound of  $O(n \log k)$  turns for any non-adaptive strategy for Mastermind without repeats using both black and white hits.