This proof was inspired by [Grebinski and Kucherov (optimally...additive model)]. This holds for $k \geq n$.
Claim:

$$\sum_{i=0}^{\frac{x}{2}} \Pr[\mathrm{Eq}(q, v_1) = i \wedge \mathrm{Eq}(q, v_2) = i] \leq \left(1 - \frac{1}{k}\right)^x$$

Case 1: $x \leq k - 3$. We know

$$\begin{aligned}
\Pr[\mathrm{Eq}(q, v_1) = i \wedge \mathrm{Eq}(q, v_2) = i] &= \Pr[\mathrm{Eq}(q, v_2) = i] \cdot \Pr[\mathrm{Eq}(q, v_1) = i | \mathrm{Eq}(q, v_2) = i] \\
&= \binom{x}{i} \left(\frac{1}{k}\right)^i \left(\frac{k-1}{k}\right)^{x-i} \cdot \binom{x-i}{i} \left(\frac{1}{k-1}\right)^i \left(\frac{k-2}{k-1}\right)^{x-2i} \\
&= \frac{x!}{(k-2)^{2i}(i!)^2(x-2i)!} \left(\frac{k-2}{k}\right)^x \\
&\leq \frac{x^{2i}}{(k-2)^{2i}(i!)} \left(\frac{k-2}{k}\right)^x.
\end{aligned}$$

Now we have

$$\begin{aligned}
\sum_{i=0}^{\frac{x}{2}} \Pr[\mathrm{Eq}(q, v_1) = i \wedge \mathrm{Eq}(q, v_2) = i] \left(\frac{k}{k-1}\right)^x &\leq \sum_{i=0}^{\frac{x}{2}} \frac{x^{2i}}{(k-2)^{2i}(i!)} \left(\frac{k-2}{k}\right)^x \left(\frac{k}{k-1}\right)^x \\
&= \sum_{i=0}^{\frac{x}{2}} \frac{x^{2i}}{(k-2)^{2i}(i!)} \left(1 - \frac{1}{k-1}\right)^x \\
&< \left(1 - \frac{1}{k-1}\right)^x \sum_{i=0}^{\infty} \left(\frac{x^2}{(k-2)^2}\right)^i \frac{1}{i!} \\
&< e^{-\frac{x}{k-1}} \cdot e^{\frac{x^2}{(k-2)^2}} \\
&< 1 \quad \text{for } x \leq (k-3).
\end{aligned}$$

Multiplying both sides of this inequality by $(1 - 1/k)^x$ results in the claim.

Case 2: $k-2 \leq x \leq k$

$$\sum_{i=0}^{\lfloor x/2 \rfloor} \binom{x}{i}\binom{x-i}{i}(n-2)^{-2i} = \sum_{i=0}^{\lfloor x/2 \rfloor} \frac{\frac{x(x-1)\cdots(x-2i+1)}{(n-2)^{2i}}}{i!^2}$$

For $x = n-2, n-1, \text{or } n$:

For $i \geq 3$, $x(x-1)\cdots(x-2i+1) \leq n(n-1)\cdots(n-2i+1) \leq (n-2)^{2i}$. Thus,

$$\begin{aligned}\sum_{i=3}^{\lfloor x/2 \rfloor} \binom{x}{i}\binom{x-i}{i}(n-2)^{-2i} &\leq \sum_{i=3}^{\lfloor x/2 \rfloor} \frac{1}{(i!)^2} \\ &\leq \sum_{i=3}^{\infty} \frac{1}{(i!)^2} \\ &< .0296\end{aligned}$$

$$\begin{aligned}\sum_{i=0}^{2} \binom{x}{i}\binom{x-i}{i}(n-2)^{-2i} &= 1 + \frac{x(x-1)}{(n-2)^2} + \frac{x(x-1)(x-2)(x-3)}{4(n-2)^4} \\ &\leq 1 + \frac{n(n-1)}{(n-2)^2} + \frac{n(n-1)(n-2)(n-3)}{4(n-2)^4} \\ &< 2.5536 \text{ for } n \geq 14\end{aligned}$$

Thus putting the two together gives

$$\begin{aligned}\sum_{i=0}^{\lfloor x/2 \rfloor} \binom{x}{i}\binom{x-i}{i}(n-2)^{-2i} &\leq 2.5832 \\ &< \left(1 + \frac{1}{n-2}\right)^{n-2} \quad \text{for } n \geq 14 \\ &\leq \left(1 + \frac{1}{n-2}\right)^{x}\end{aligned}$$

So the sum of the probability that two vectors have the same response on all $s$ questions is:

$$\sum_{x=1}^{n}(\text{\# of reduced pairs that disagree in } x \text{ spots})(\text{Probability this pair is in the same bucket})$$

$$\begin{aligned}
&= \sum_{x=1}^{n} \binom{n}{x} n^x (n-1)^x \left(1 - \frac{1}{k}\right)^{s \cdot x} \\
&\leq \sum_{x=1}^{n} n^{3x} \left(1 - \frac{1}{k}\right)^{s \cdot x} \\
&= \sum_{x=1}^{n} n^{3x} \left(1 - \frac{1}{k}\right)^{(4k \log n)x} \\
&< \sum_{x=1}^{n} n^{3x} \left(\frac{1}{e}\right)^{(4 \log n)x} \\
&= \sum_{x=1}^{n} n^{3x} \left(\frac{1}{n}\right)^{4 \cdot x} \\
&\leq \sum_{x=1}^{n} \frac{1}{n} \\
&\leq 1
\end{aligned}$$