

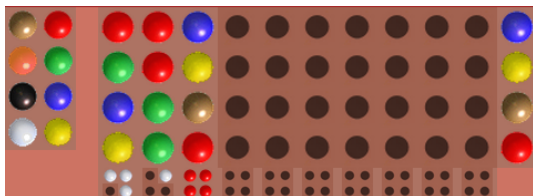
Mastermind

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Mastermind

- i. Codemaker vs. Codebreaker
- ii. Queries: Guess a vector from $\{1, 2, \dots, 6\}^4$
- iii. Response
 - i. Black (Red) hits
 - ii. White hits



Knuth Paper – 1976

- i. At most five turns needed

Minimax

For each possible guess

 For each possible response to that guess

 Check how many possible solutions remain

 Let *score* be max. number solutions remaining

Make guess with minimum score

Extensions

- i. Basic Extension: n spots, k colors
- ii. Repeats vs. no repeats
- iii. Non-adaptive vs. adaptive strategies

Entropy Method

Surprise Function: For an event x , we want

1. $S(x) = 0$ when $\mathbb{P}[x] = 1$
2. $S(x) = 1$ when $\mathbb{P}[x] = 1/2$
3. Decreasing as $\mathbb{P}[x]$ increases
4. $S(x \wedge y) = S(x) + S(y|x)$

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Let $S(x) = -\log_2(\mathbb{P}[x])$.

Entropy Method (cont'd)

Entropy is the expected surprise of a random variable.

Definition: Let X be a random variable with domain D .

$$H(X) = - \sum_{x \in D} \mathbb{P}[X = x] \cdot \log_2 (\mathbb{P}[X = x])$$

Probabilistic Method

Non-Adaptive Game: Set of queries $Q = \{q_1, q_2, \dots, q_s\}$.

$$\mathbb{P}[Q \text{ is a winning set of guesses}] > 0$$

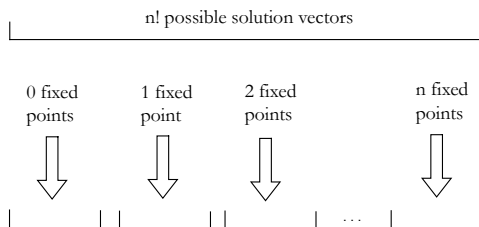


\exists a winning set of s guesses

Known Bounds

1. Lower Bounds: Try to beat $O(n)$.
2. Upper Bounds: Try to achieve/beat $O(n \log k)$

Bucket Method



- i. Uses more information than previous proofs, using “buckets.”
- ii. Bucket $i \iff i$ fixed points
- iii. Size of buckets:

$$\begin{aligned} |B_i| &= \binom{n}{i} \cdot D(n-i) \\ &= \binom{n}{i} \cdot (n-i)! \cdot \sum_{j=0}^{n-i} \frac{(-1)^j}{j!} \end{aligned}$$

Deriving $n \log \log n$ Upper Bound when $k = n$

[Doerr et. al., 2013]

- i. Split hidden vector into “coins” (subvectors).
- ii. Use coin weighing problem to eliminate colors.

Non-Adaptive Strategies

- i. Random Guessing
- ii. Evaluate probability of success