

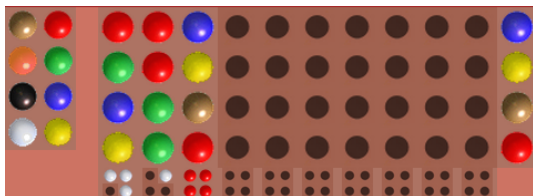
# Mastermind

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July 24, 2015

# Mastermind

- i. Codemaker vs. Codebreaker
- ii. Queries: Guess a vector from  $\{1, 2, \dots, 6\}^4$
- iii. Response
  - i. Black (Red) hits
  - ii. White hits



# Knuth Paper – 1976

- i. At most five turns needed

## Minimax

For each possible guess

    For each possible response to that guess

        Check how many possible solutions remain

        Let *score* be max. number solutions remaining

Make guess with minimum score

# Extensions

- i. Basic Extension:  $n$  spots,  $k$  colors
- ii. Repeats vs. no repeats
- iii. Non-adaptive vs. adaptive strategies

# Coin-Weighing Problem

[Grebinski & Kucherov, 2000], [Bshouty, 2009]

- i. Original Coin-Weighing algorithm by G&K,  
non-constructive (probabilistic method)
- ii. Refined polynomial-time algorithm [Bshouty]

[Doerr et. al., 2013]

- i. Split hidden vector into “coins” (subvectors).
- ii. “Weight” of each “coin” is # of black hits.
- iii. Use coin weighing algorithm to eliminate colors.

# Entropy Method

*Surprise Function:* For an event  $x$ , we want

1.  $S(x) = 0$  when  $\mathbb{P}[x] = 1$
2.  $S(x) = 1$  when  $\mathbb{P}[x] = 1/2$
3. Decreasing function of  $\mathbb{P}[x]$
4.  $S(x \wedge y) = S(x) + S(y|x)$  ( $= S(x) + S(y)$  if independent)

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$$\Rightarrow S(x) = -\log_2(\mathbb{P}[x]).$$

# Entropy Method (cont'd)

Entropy is the expected surprise of a random variable.

*Definition:* Let  $X$  be a random variable with domain  $D$ .

$$H(X) = \sum_{x \in D} \mathbb{P}[X = x] \cdot (-\log_2(\mathbb{P}[X = x]))$$



# Probabilistic Method

Non-Adaptive Game: Set of queries  $Q = \{q_1, q_2, \dots, q_s\}$ .

$$\mathbb{P}[Q \text{ is a winning set of guesses}] > 0$$



$\exists$  a winning set of  $s$  guesses