This proof will follow a similar path as [Grebinski and Kucherov (optimally...additive model)]. We want

$$\sum_{i=0}^{\frac{x}{2}} \Pr[\mathrm{Eq}(q, v_1) = i \wedge \mathrm{Eq}(q, v_2) = i] \leq \sum_{i=0}^{x} \Pr[\mathrm{Eq}(q, v_1) = i] \cdot \Pr[\mathrm{Eq}(q, v_2) = i]$$

We will show this in an appendix or something.

$$\begin{aligned}
&\sum_{i=0}^{x} \Pr[\mathrm{Eq}(q, v_1) = i] \cdot \Pr[\mathrm{Eq}(q, v_2) = i] \\
&\leq \sum_{i=0}^{x} \mathrm{P}_{\max}(\mathrm{Eq}(q, v_1)) \cdot \Pr[\mathrm{Eq}(q, v_2) = i] \\
&= \mathrm{P}_{\max}(\mathrm{Eq}(q, v_1)) \cdot \sum_{i=0}^{x} \Pr[\mathrm{Eq}(q, v_2) = i] \\
&= \mathrm{P}_{\max}(\mathrm{Eq}(q, v_1))
\end{aligned}$$

$\mathrm{P}_{\max}$ will always be bucket 0 unless $x = n$, in which case it's bucket 1. This is proved by taking a ratio of consecutive terms to show the sequence is decreasing for all other values of #of hits. [insert actual equation here]

$$\Pr[\mathrm{Eq}(q, v_1) = 0] = \left(1 - \frac{1}{n}\right)^{x}$$

$$\Pr[\mathrm{Eq}(q, v_1) = 1 | x = n] = \left(1 - \frac{1}{n}\right)^{n-1}$$

$$\text{So we can write } \mathrm{P}_{\max} \leq \left(1 - \frac{1}{n}\right)^{\min(x, n-1)}$$

So the sum of the probability that two vectors have the same response on all s questions is:

$$\sum_{x=2}^{n} (\text{\# of reduced pairs that disagree in } x \text{ spots})(\text{Probability this pair is in the same bucket})$$

$$
\begin{aligned}
&= \sum_{x=1}^{n} \binom{n}{x} n^x (n-1)^x \left(1 - \frac{1}{n}\right)^{s \cdot \min(x, n-1)} \\
&\leq \sum_{x=1}^{n} n^{3x} \left(1 - \frac{1}{n}\right)^{s \cdot \min(x, n-1)} \\
&= \sum_{x=1}^{n} n^{3x} \left(1 - \frac{1}{n}\right)^{(4n \log n) \min(x, n-1)} \\
&< \sum_{x=1}^{n} n^{3x} \left(\frac{1}{e}\right)^{(4 \log n) \min(x, n-1)} \\
&= \sum_{x=1}^{n} n^{3x} \left(\frac{1}{n}\right)^{4 \cdot \min(x, n-1)} \\
&\leq \sum_{x=1}^{n} \frac{1}{n} \\
&< 1
\end{aligned}
$$