Lecture 1

Review of basic material

1. Introduction

An electric circuit is a closed path around which an electrical current may flow.

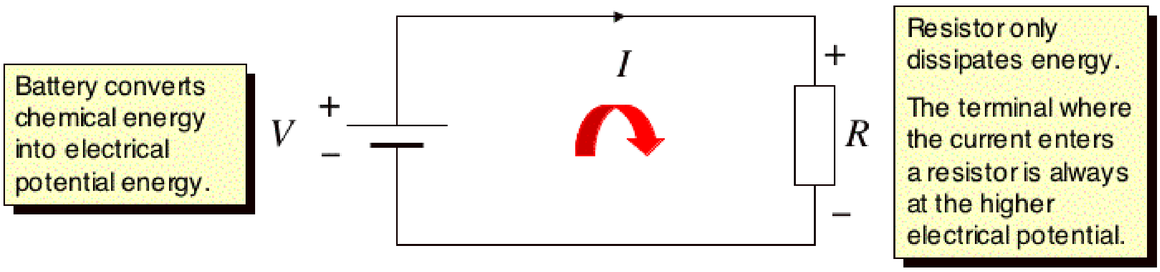


Figure 1.1: A simple electric circuit.

A circuit must contain one (or more) active device that delivers energy (e.g. a battery) and passive devices like resistors that dissipate energy (e.g. producing light or heat). Conventionally, current flows from points of high electrical potential (the positive terminal of a battery) to points of low electrical potential (the negative terminal). Note that this is opposite to the direction that electrons would flow.

Although we can speak of points with high or low electrical potential, in practice we measure only potential differences. It is sometimes convenient to designate a point in the circuit to have zero electrical potential. Usually we choose this point to be the negative terminal of the battery.

2. Definitions and units

- **Charge (coulomb, C)**
  The unit of electrical charge is the coulomb. Charge is quantised, that is, it comes in discrete amounts. The charge of an electron is -1.602×10^{-19} C.
- **Current (ampere, A)**
  Electric current is defined as the rate at which charge flows passed a given point in a circuit. Algebraically, we write

$$I = \frac{\Delta Q}{\Delta Q}$$

  where ΔQ = electric charge (coulombs) and Δt = time (seconds). One coulomb corresponds to the charge transferred when a current of 1 ampere (or amp) flows for 1 second. It is easy to see that this corresponds to a lot of electrons! Hence, the fact that charge is quantised does not usually matter.

- **Potential difference (volt, V)**
  Potential difference is the work done when transferring charge from one point in a circuit to another. Algebraically,
  $$V=\frac{\Delta W}{\Delta Q}$$
  where ΔW = work done (joules) and ΔQ = electric charge (coulombs). The potential difference is 1 volt when 1 joule of energy is required to transfer 1 coulomb of charge.
- **Resistance (ohm, Ω)**
  Resistance is defined as the ratio of the voltage across a circuit element to the current flowing through it.
  $$R=\frac{V}{I}$$
  For an important class of materials, the resistance is independent of the voltage or the current. That is, the resistance is constant, so that the voltage is directly proportional to the current. This is a statement of **Ohm's law**.

  $$V=IR, \quad R=constant$$

  Notice that there is a "$y = mx + c$" or *linear* relationship between the voltage and the current in an **ohmic** conductor ($c = 0$). We can define a linear circuit as one in which Ohm's law is obeyed. It is important to realise that not all conductors are ohmic, and indeed non-linear behaviour is desirable in some types of circuit. However, in this course we limit ourselves to the study of linear circuits.

  Before we leave this point, it is instructive to look at a plot of current against voltage for different types of conductor (resistor, lamp, diode).

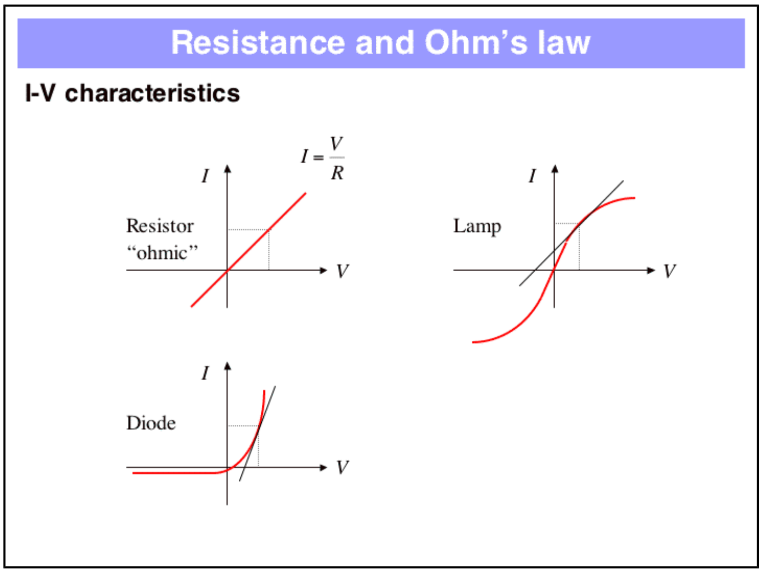


Figure 1.2: I-V characteristics of a resistor, lamp and diode.

Only the resistor has a constant slope (which because I've plotted I against V is given by one over the resistance). It is easy to see that the resistance of the lamp and diode changes as a function of potential difference across them (the gradient of the slope (black line) changes as a function of V or I). As the lamp heats up, its resistance increases, whereas the diode acts like a valve, only allowing current to flow easily in one direction.

- **Power (Watt, W)**

Power is defined as the rate of doing work. Since the work done in part of an electric circuit is defined as the potential difference multiplied by the charge transferred, $\Delta W = V{\cdot}\Delta Q$ (see Potential difference above),

$$P=\frac{\Delta W}{\Delta t}=V\cdot\frac{\Delta Q}{\Delta t}=VI$$

The power dissipated in a resistor may be determined by substituting for V or I using Ohm's law such that,

$$P=VI=I^2R=\frac{V^2}{R}$$

3. Conservation of charge and electrical potential energy

In any physical system it is often helpful to identify quantities that are conserved, that is quantities that are unchanging. The analysis of direct current (d.c.) circuits is based upon two important conservation laws. The first is that charge (or current) is conserved. The second is that the electrical potential energy supplied to a charge by any active devices, such as a battery, equals the energy dissipated by that charge in any resistors. That is, the sum of all the potential rises and drops around a circuit is equal to zero.

Here we are anticipating Kirchhoff's laws (after Gustav Kirchhoff, 1824-1887), which some of you may have come across before. We will state Kirchhoff's laws formally in a later exercise. What I want you to appreciate here are the physical principles that are involved and to see that they are reasonable.

Let's explore the consequences of this in the simple series circuit below.

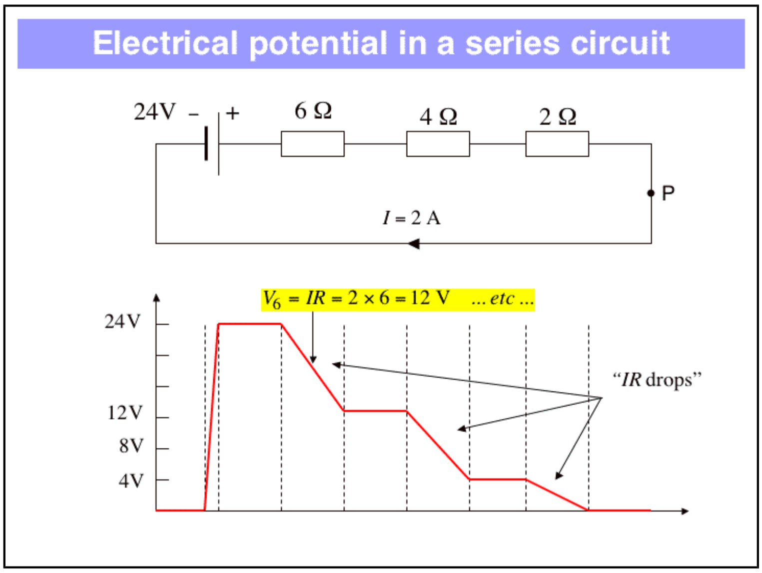


Figure 1.3: A graph of electrical potential energy versus position in the circuit.

The conservation of charge means that the current (or charge) flowing into any point in the circuit (e.g. point P) is equal to the current (or charge) flowing out. In a resistive circuit, charge is not allowed to simply build up at an arbitrary point. This means that the current must be the same at all points in this simple series circuit.

When charge passes through the battery, its electrical potential energy is raised. (In figure 1.3, we assume this is a positive charge passing from left to right through the battery, in the direction of conventional current flow, and that the negative terminal of the battery is at zero volts.) As the charge flows through the circuit it gives up its energy to the resistors in such a way, that when the charge returns to the negative terminal of the battery it has neither gained nor lost any net energy.

By analogy, if you climbed a hill via one route and then descended via another, starting and ending at the same location, you would have the same gravitational potential energy at the end of the walk as you did at the start.

Given that the current is the same all points in the circuit, we can use Ohm's law to determine the potential drop across each of the resistors as shown in figure 1.3. It is clear to see that the potential rise experienced by a hypothetical positive charge as it passes through the battery is equal to the sum of the "IR drops" suffered as it passes through the resistors.

4. Combinations of resistors

The concept of *equivalent circuits* is an extremely useful aid in the analysis of complicated networks. Here we will use it to derive a result you probably already know: how to combine resistors in series and parallel.

4.1 Resistors in series

Circuit (a) in figure 1.4 shows 3 resistors in series with a voltage source, or battery. Let circuit (b) be an equivalent circuit containing just one resistor. "Equivalent" means that the same current flows from the battery, so the circuit looks the same as far as the battery is concerned. We can now use this to determine what the equivalent resistance, R_{eq}, must be.

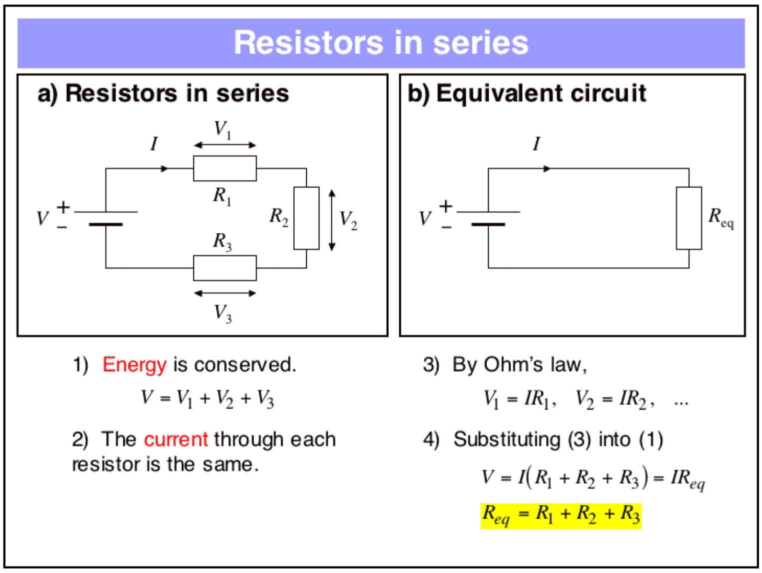


Figure 1.4: Resistors in series.

Step 1) in circuit (a) we know that electrical potential energy gained in the battery is equal to the electrical potential energy lost in the resistors. In terms of electrical potential (V = energy/charge Joules/coulomb), the potential rise across the battery must be equal to the sum of the potential drops across the resistors.

Step 2) in a series circuit we know that the same current flows through each resistor, since charge is conserved at very point in the circuit.

Step 3) we can use Ohm's law to find the voltage across each resistor in terms of the common current.

Step 4) Substituting for V_1, V_2 and V_3 in the equation given in step 1), we can factorise out the common current I. Now compare this situation with circuit (b), which is an equivalent circuit. This shows us that the equivalent resistance in (b) is equal to the sum of the resistances in (a). This would be true for any number of series resistors, so we may write

$$R_{eq}=R_1+R_2+R_3+L$$

4.2 Resistors in parallel

In Figure 1.5 in circuit (a) shows 3 resistors in parallel with a voltage source or battery. Let circuit (b) be an equivalent circuit containing just one resistor. As before, "equivalent" means that the same current flows from the battery.

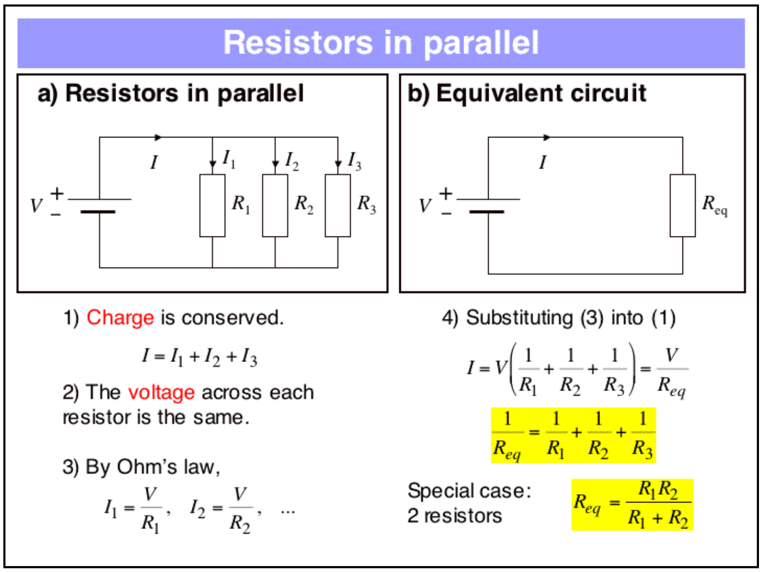


Figure 1.5: Resistors in parallel.

Step 1) in this case it is the current (or charge) that is conserved. In this context, charge conservation means that when the current is split between the three resistors in circuit a), the sum of the currents through each of the resistors individually, must be equal to the current supplied by the battery.

If this statement isn't completely obvious to you, consider the following. If we were studying a plumbing "circuit", the flow rate from the boiler (battery) should equal the sum of the flow rates in each of the radiators (resistors) if they were connected in parallel. If not, the flow rate back into the boiler would be less than the flow rate out, which would mean that water was accumulating

somewhere in the system, assuming no leaks. This is clearly nonsense! Following the plumbing analogy a step further you can hopefully see that what gets used up, or dissipated, as water flows around the plumbing circuit is thermal energy, not water molecules. Likewise in an electric circuit, it is not current (charge) that is used up, but electrical potential energy. In our case, if charged was allowed to accumulate in the circuit it would eventually stop the flow of current completely (see Capacitors later in the course).

Step 2) in a parallel circuit we know that the potential difference (or voltage) across each resistor is the same (since this is equal to the potential difference across the battery).

How do we know this? The lines that represent wires in a circuit diagram are assumed to have no resistance. If that is the case, no potential difference exists between the ends of the wire. Therefore, charge entering the top of R_1 has the same electrical potential energy as when it left the battery. Similarly for R_2 and R_3 it is the same. It is the amount of charge which flows through the three resistors that is different, not the amount of energy each charge gives up in the resistor.

Step 3) we can use Ohm's law to find the current through each resistor in terms of the common voltage.

Step 4) Substituting for I_1, I_2 and I_3 in the equation given in step 1), we can factorise out the common voltage V. Now compare this situation with circuit (b), which is an equivalent circuit. This shows us how to relate the equivalent resistance in (b) to the parallel combination of resistances in (a). Again, this would be true for any number of series resistors, so we may write

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + L$$

Note that in the special case of two resistors, you can write the equivalent resistance directly as the product over the sum of the two individual resistances. (Try to remember this, as I use it all the time.)

5. The voltage divider and the current splitter

When analysing circuits, we often wish to reduce them to a simpler equivalent circuit, where we can use Ohm's law to find the voltage across a particular component, or the current through it. We are usually left with one of two topologies, which I call the voltage divider and the current splitter. Figure 1.6 shows examples of each.

Note that circuit b) introduces a current source. We will do more on voltage and current sources later in the course. All you need to know for now is that an ideal current source produces a constant current. The potential difference across a current source depends upon the load resistance. By comparison, an ideal voltage source produces a constant potential difference and it is the current that may be varied by changing the load.

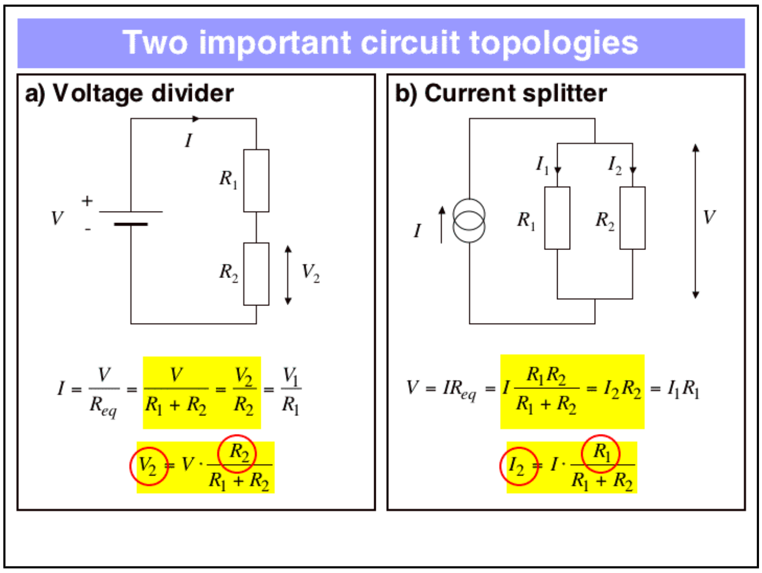


Figure 1.6: The voltage divider and the current splitter.

The problem can be expressed as follows:

In a **voltage divider**, you have a **known voltage** across two **series** resistors and you want to know the voltage across one of them. In a **current splitter**, you have a **known current** that is split between two **parallel** resistors and you want to know the current through one of them.

In the voltage divider case, we can write down an expression for the current from the voltage across both resistors, or the voltage across each resistor individually. If we consider the voltage across resistor R_2 (see yellow box) we find that V_2 is determined by the ratio $R_2/(R_1+R_2)$ multiplied by the total voltage.

In the current splitter case, we can write down an expression for the voltage across the parallel combination from the current through the combined resistance, or from the current through either resistor individually. If we consider the current through R_2 (see yellow box) we find that the current I_2 is determined by the ratio $R_1/(R_1+R_2)$ multiplied by the total current.

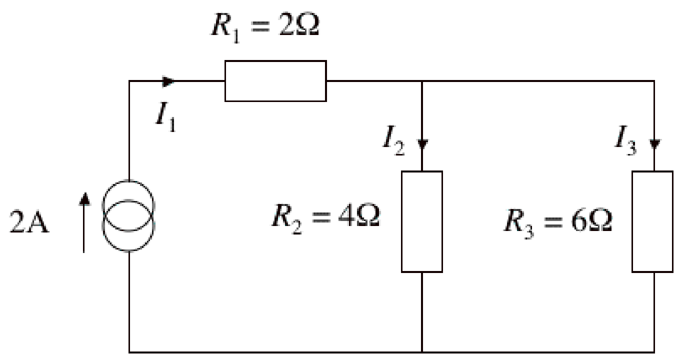
Since we encounter this problem so often when simplifying circuits, it is worth remembering the result and applying it directly to problems (see the worked problem below).

Voltage divider: The voltage across one of two resistors in series is determined by the ratio of its resistance to the sum of the two resistances.

Current splitter: The current in one branch of a parallel circuit is determined by the ratio of the resistance in the opposite branch to the sum of the two resistances.

6. Worked problem

Find the current in each of the resistors in the circuit below.



By inspection, $I_1 = 2$ A, since R_1 is in series with the current source.

I_1 is then split between R_2 and R_3. Using the current splitter rule, I_2 is given by

$$I_2 = I_1 \frac{R_3}{R_2 + R_3} = 2\frac{6}{4+6} = 1.2A$$

Since the current from the source is conserved, the remainder of the current must flow through R_3. Therefore,

$$I_3 = I_1 - I_2 = 0.8A$$

Notice that you only have to use the current splitter rule once. Now that the current through each resistor is known, you can find the voltage across each resistor using Ohm's law.

7. Try it for yourself

Repeat the above problem, substituting an 8.8 V voltage source for the current source. This time, find the voltage across each of the components. (Hint: combine R_2 and R_3 to form a voltage divider with R_1). Convince yourself that these circuits are in fact equivalent. (Answer: $V_1 = 4$ V, $V_2 = V_3 = 4.8$ V)

8. Further problems

Attempt problems 79 and 81 in Tipler, Chapter 25 (Answers in back of book).

9. Further reading

Tipler 25-1: Current and the motion of charges.
Tipler 25-2: Resistance and Ohm's law.
Tipler 25-4: Combinations of resistors.
Note: Tipler doesn't cover the voltage divider or current splitter, but they can be found in any electric circuits textbook. (e.g. Ray Powell's Introduction to Electric Circuits, p17 and 20).