University of Birmingham
School of Mathematics

Vectors, Geometry and Linear Algebra
VGLA

(Summative) Problem Sheet 4 (Semester 1)

All the questions on this sheet contribute to your grade in this module. You should upload your solutions to this problem sheet on the VGLA Canvas page. The deadline is **5pm on 11th December 2024**.

Remember that you should always carefully write out your solutions giving full explanations (not just stating the answer). Also, submissions should consist of a single pdf file.

Q1. Suppose that $z_1 = 1 + i$.

(i) Calculate z_1^8 in both modulus-argument form and exponential form giving the principal value of the argument.

(ii) Find all the fifth roots of z_1. Present your answers in exponential form giving the principal value of the argument.

Q2. Consider the polynomial $p(z) = z^3 - z^2 + 9z - 9$.

(i) Determine the roots of $p(z)$.

(ii) Write $p(z)$ as a product of real linear and real irreducible quadratic factors.

Q3. The circle of radius r in the xy-plane centred at $(a, b) \in \mathbb{R}^2$ is described by the equation $(x - a)^2 + (y - b)^2 = r^2$.

Describe the solution set to the equation $|z - 1| = 4|z - 3|$ (where $z \in \mathbb{C}$) as a geometrically defined subset of the Argand diagram.

Q4. Is the following system of linear equations homogeneous? Justify your answer.

$$\begin{aligned} 2x_1 - 7(x_4 + 1) - 3x_2 &= 2(x_3 + 3) - 13 \\ x_1 + x_2 + x_3 + x_4 &= 0 \\ (x_1 + 1) - 2(x_2 - 2) + x_3 &= 5 \end{aligned}$$

Q5. We define the following matrices:

$$\mathbf{A} = \begin{pmatrix} 1 & -1 & 2 \\ 1 & 3 & -1 \\ -1 & 1 & 4 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} -2 & 1 & 0 \\ 0 & 1 & 2 \\ 4 & -1 & 2 \end{pmatrix} \quad \mathbf{C} = \begin{pmatrix} 6 & 1 \\ 2 & 0 \\ -1 & 1 \end{pmatrix}.$$

For each of the following matrix calculations, either do the calculation or explain why it is not possible to do the calculation.

(i) $\mathbf{A} + \mathbf{B}$;

(ii) $\mathbf{B} \cdot \mathbf{A}$;

(iii) $\mathbf{C} \cdot \mathbf{A}$;

(iv) $(\mathbf{B} \cdot 2\mathbf{C}) + \frac{1}{2}\mathbf{C}$.