Electromagnetism

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Lecture 18

Mutual Inductance

Week 9

Last Lecture

- Devices designed to have a self-inductance are called ***Inductors.***
- ***Definition of self-inductance:***

$$N\Phi_m = LI \qquad \varepsilon = -L\frac{dI}{dt}$$

- ***Energy Stored in an inductor:***

$$U_L = \frac{1}{2}LI^2$$

- ***Energy Density in a Magnetic Field*** $u_B = \frac{1}{2}\frac{B^2}{\mu_0}$

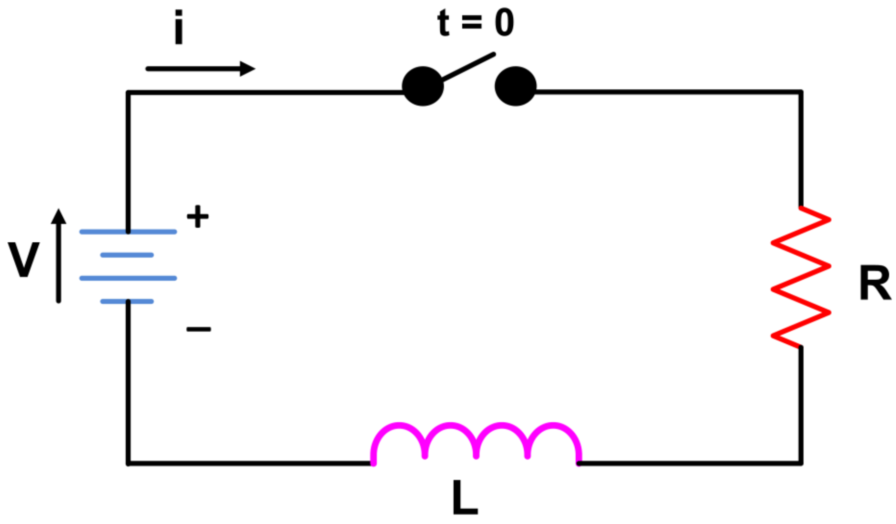
This Lecture

- Inductor in a circuit
- Quiz Time – inductance of a solenoid
- Mutual Inductance
- Applications of mutual inductance

- Magnetic materials

Inductor in a Circuit

- Consider the following circuit:

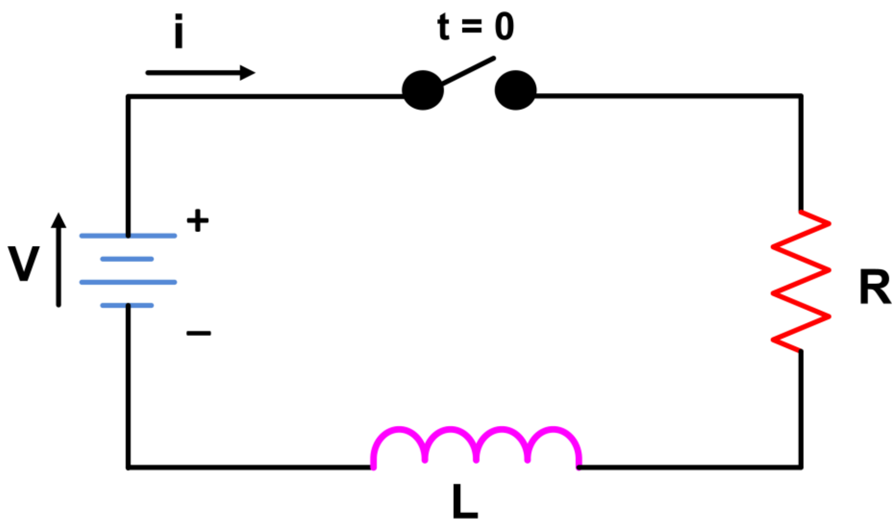


At $t = 0$ the switch is closed and an increasing current $I(t)$ begins to flow.

This causes an emf in the inductor that opposes the driving voltage, V. This back emf appears as a voltage drop across the inductor. Thus $V_L + V_R = V$

Inductor in a Circuit

- $V_L + V_R = V$

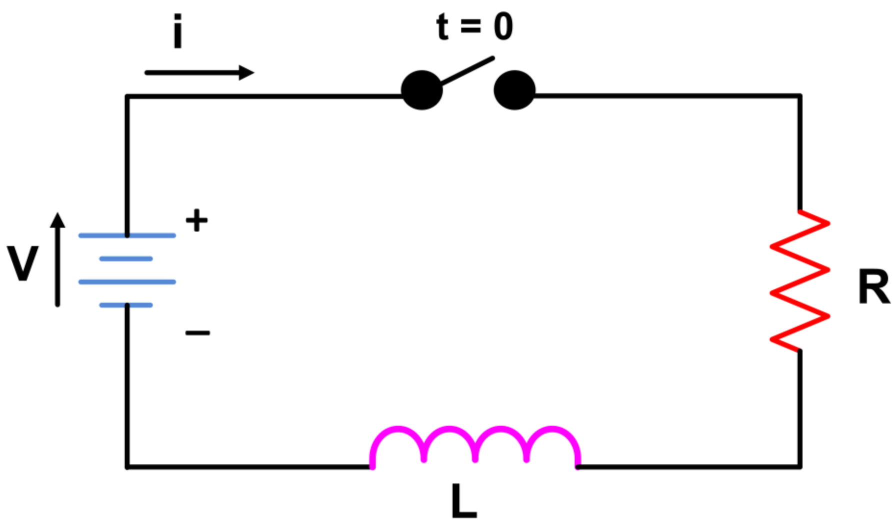


I.e

$$IR + L\frac{dI}{dt} = V$$

Let's use visualizer to solve for *I(t)*

Inductor in a Circuit



Current doesn't go to steady state value, I_0 immediately due to inductance.

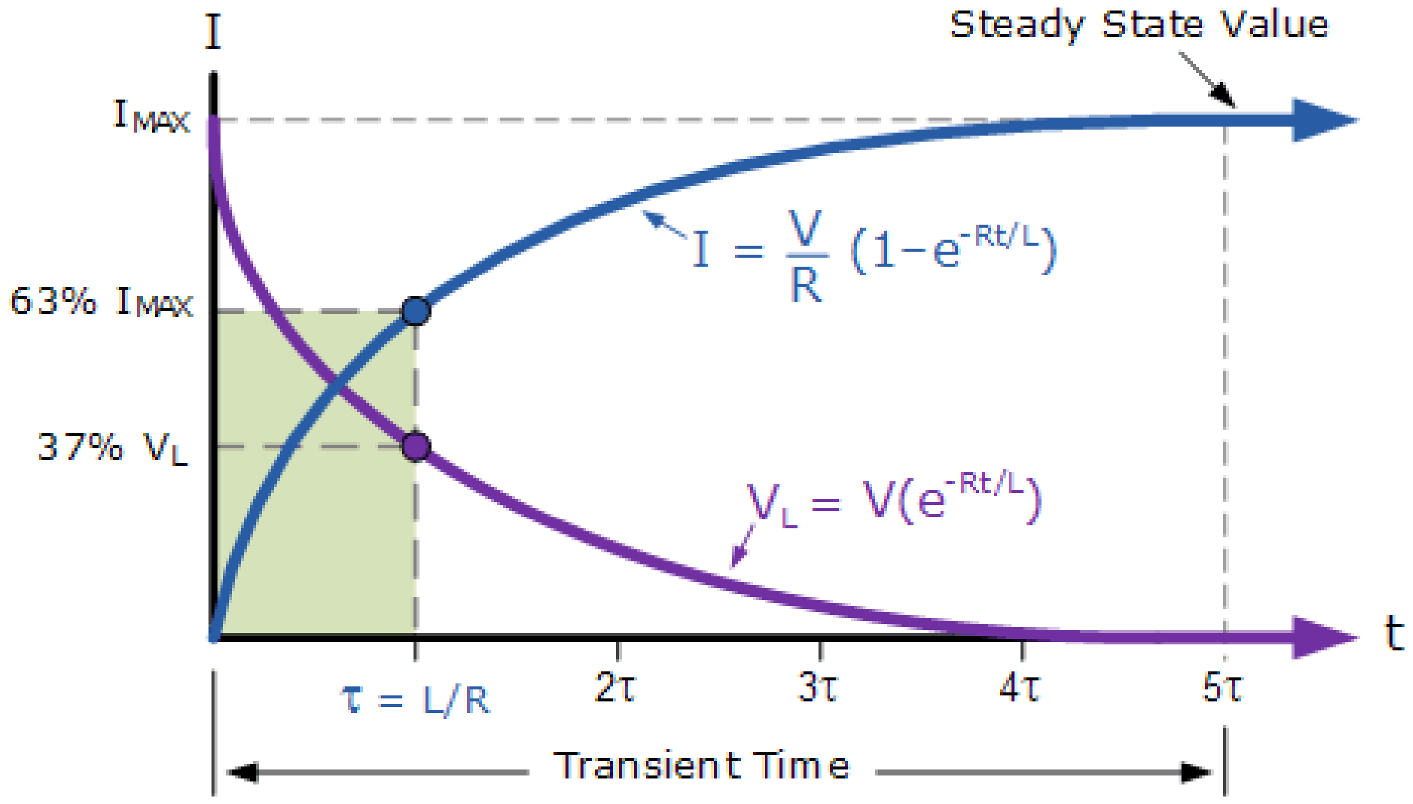
$$I(t) = I_0\left(1 - e^{-\frac{t}{\tau}}\right)$$

where $\tau = L/R$

When switch is opened from stead state: $I(t) = I_0 e^{-\frac{t}{\tau}}$

Inductor in a Circuit

Current in RL circuit after Switch is closed



Example 18.1

How would the self-inductance of a solenoid be changed if

1. the same length of wire were wound onto a cylinder of the same diameter but twice the length?
2. twice as much wire were wound onto the same cylinder?
3. the same length of wire were wound onto a cylinder of the same length but twice the diameter?

Answers

- $L = \mu_0 n^2 l \pi R^2$

1. [the same length of wire were wound onto a cylinder of the same diameter but twice the length?]

Since the diameter does not change, the number of turns and the area A remain constant. However, n^2 is diminished by a factor of 4 and l is increased by a factor of 2.

Thus L is reduced by a factor of 2

Answers

- $L = \mu_0 n^2 l \pi R^2$

2. [twice as much wire were wound onto the same cylinder?]

Using twice as much wire and making no other change, n^2 is increased by a factor of 4.

Hence, L is increased by a factor of 4

Answers

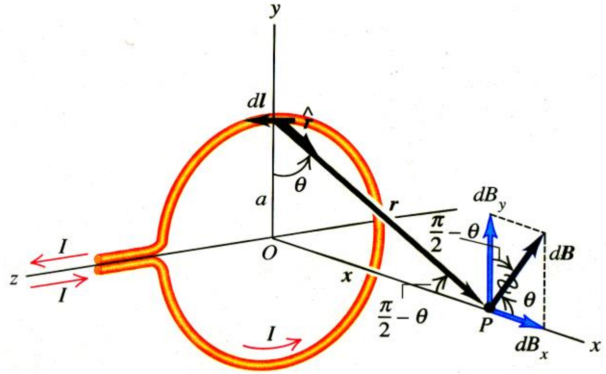
- $L = \mu_0 n^2 l \pi R^2 = \mu_0 n^2 l A$

3. [the same length of wire were wound onto a cylinder of the same length but twice the diameter?]

With twice the diameter, n^2 is reduced by a factor of 4, but A is increased by the same factor; L **is unchanged.**

Inductance

- A current loop produces a magnetic field

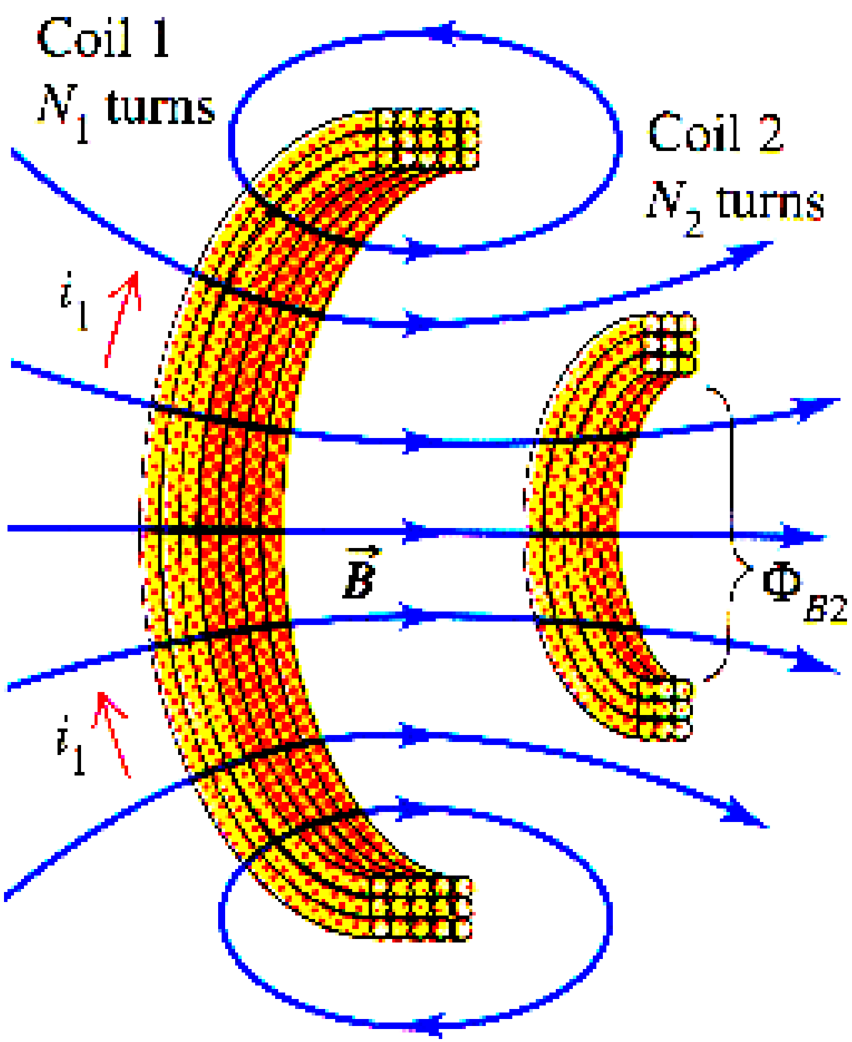


$$B_x = \frac{\mu_0 I\, a^2}{2(x^2+a^2)^{3/2}}$$

- But a changing magnetic flux produces an induced voltage in a conducting loop.

$$\varepsilon = -\frac{d\Phi_m}{dt}$$

Mutual Inductance



- A changing current in loop 1 causes a changing flux in loop 2 inducing a voltage
- $\varepsilon_2 = -N_2 \dfrac{d\Phi_2}{dt}$
- $\Phi_2 \propto B \propto i_1$

Mutual Inductance

- The mutual inductance M_{21} is defined by the equation:
- $N_2\Phi_2 = M_{21}i_1$
- The mutual inductance M_{21} may also be defined by the equation:
- $\varepsilon_2 = -M_{21}\dfrac{di_1}{dt}$
- As $\varepsilon_2 = -N_2\dfrac{d\Phi_2}{dt}$

Mutual Inductance

- It can be shown that the same value is obtained for M if one considers the flux threading the first loop when a current flows through the second loop:

- $M = \frac{N_2\Phi_2}{i_1} = \frac{N_1\Phi_1}{i_2}$ **Mutual Inductance**

- $\varepsilon_2 = -M\frac{di_1}{dt}$ & $\varepsilon_1 = -M\frac{di_2}{dt}$ **Mutually induced voltages**

- SI Unit of mutual inductance is the Henry (H)

Mutual Inductance

The induced emf, $\varepsilon_2 = -M\frac{di_1}{dt}$ has the following features:

- The induced emf opposes the magnetic flux change
- The induced emf increases if the currents changes very fast
- The induced emf depends on M, which depends only the geometry of the two coils and not the current.
- For a few simple cases, M can be calculated, but usually it is just measured.

Example 18.2

Two coils have mutual inductance of 3×10^{-4} H. The current in the first coil increases at a uniform rate of 800 A/s.

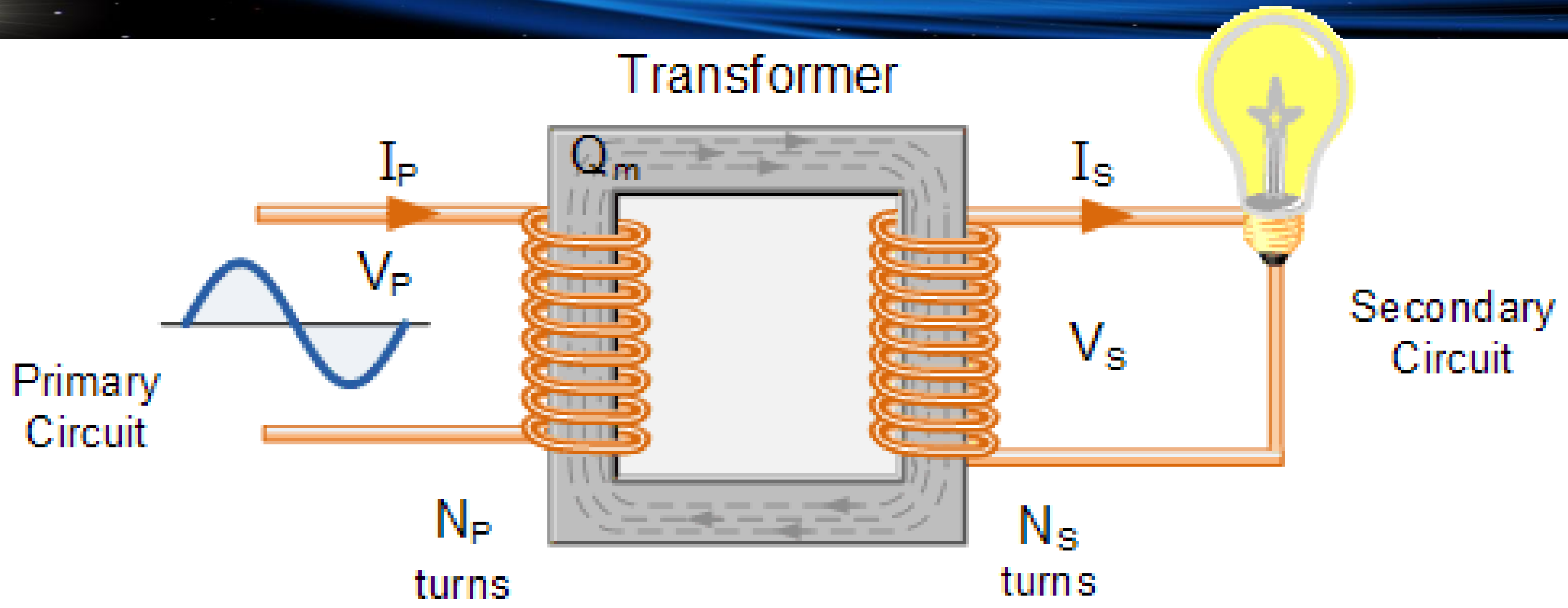
1. What is the magnitude of the induced emf in the second coil? Is it constant?

$$\varepsilon_2 = -M\frac{di_1}{dt} = -3 \times 10^{-4} \times 800 = \mathbf{-0.24}\boldsymbol{V}$$

(Yes, constant)

2. suppose that the current is instead in the 2nd coil, what is the magnitude of the induced emf in the 1st coil? $\varepsilon_1 = -M\frac{di_2}{dt} = -3 \times 10^{-4} \times 800 = \mathbf{-0.24}\boldsymbol{V}$

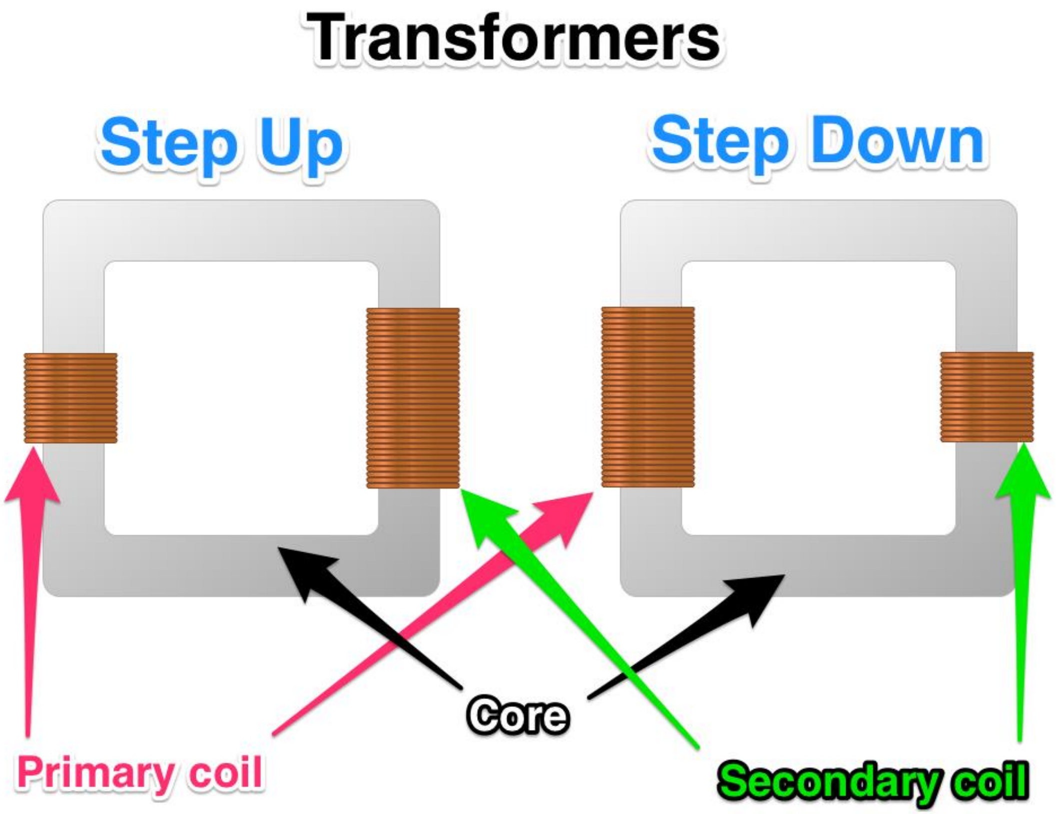
Application - Transformer



AC current in in primary coil induces a magnetic flux in the iron core. Assuming no flux leakage, the emf per turn is the same for primary and secondary coils. Hence

$$\frac{V_p(t)}{N_p} = \frac{V_s(t)}{N_s} \qquad \text{or} \qquad V_{s,\,rms} = \frac{N_s}{N_p} V_{p,\,rms}$$

Transformers



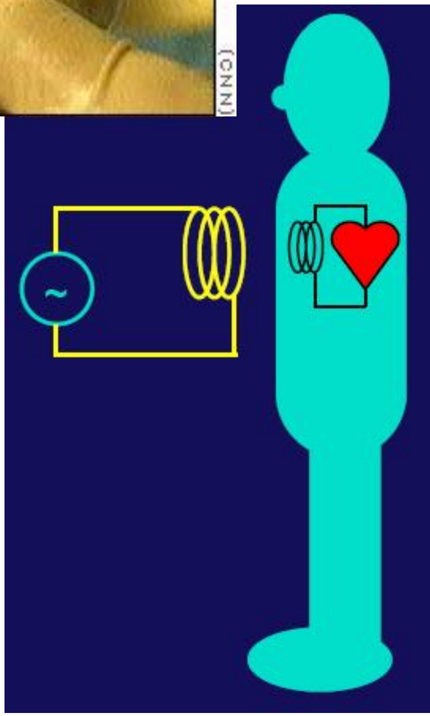
- Used to increase (step up) or decrease (step down) AC voltages.

Application 2 – Airport Metal Detectors

- Pulsed current $\rightarrow$ pulsed magnetic field $\rightarrow$ Induces emf in metal
- Ferromagnetic metals "draw in" more B $\rightarrow$ larger mutual inductance $\rightarrow$ larger emf
- Emf $\rightarrow$ current (how much, how long it lasts, depends on the resistivity of the material)
- Decaying current produces decaying magnetic field $\rightarrow$ induces current in receiver coils
- Magnitude & duration of signal depends on the composition and geometry of the metal object

Application 3

- **Pacemakers**
  - It's not easy to change the battery!
  - Instead, use an external AC supply.



- Alternating current:

$\rightarrow$ Alternating B-field

$\rightarrow$ alternating flux Φ_m inside "wearer"

$\rightarrow$ induces AC current to charge up pacemaker

Summary

- Current in RL circuit when switched on

$$I(t) = I_0\left(1 - e^{-\frac{t}{\tau}}\right)$$

- Mutual inductance from two coils:

$$M = \frac{N_2\Phi_2}{i_1} = \frac{N_1\Phi_1}{i_2}$$

$$\varepsilon_2 = -M\frac{di_1}{dt} \quad \& \quad \varepsilon_1 = -M\frac{di_2}{dt}$$

- AC Transformer $\dfrac{V_p(t)}{N_p} = \dfrac{V_s(t)}{N_s}$