Electromagnetism

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Lecture 8

Electric Dipoles in E-fields

Week 4

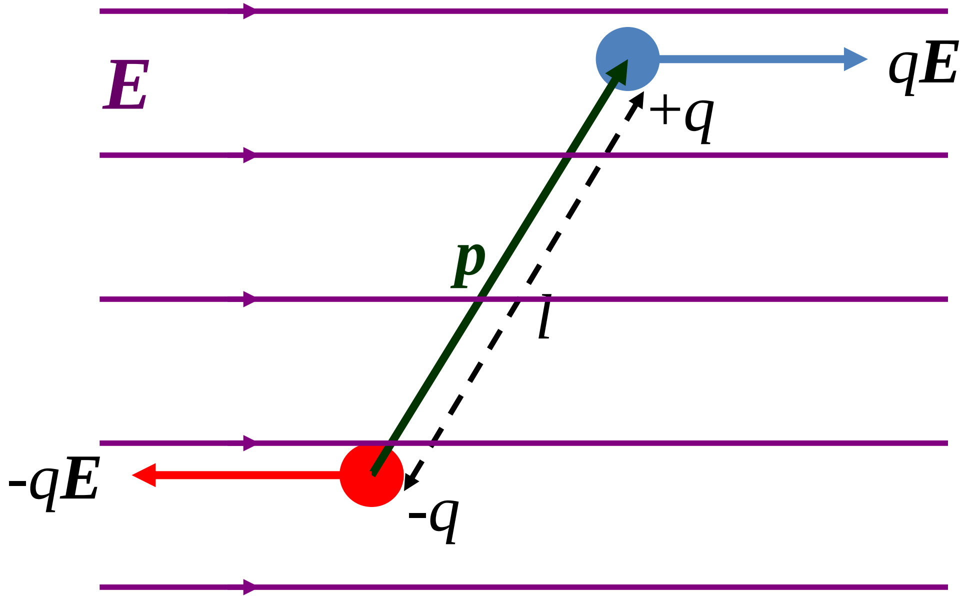
Last Lecture
Electrical Dipoles

- A dipole is two identical but opposite charges separated by a distance, say, $\underline{\boldsymbol{a}}$
- Define dipole moment as $\underline{\boldsymbol{p}} = q\underline{\boldsymbol{a}}$
- $V_p \approx \dfrac{p\cos\theta}{4\pi\varepsilon_0 r^2}$ (for $r >> a$)
- $\underline{\boldsymbol{E}} \approx \dfrac{2\,p\cos\theta}{4\pi\varepsilon_0 r^3}\underline{\hat{\boldsymbol{r}}} + \dfrac{p\sin\theta}{4\pi\varepsilon_0 r^3}\underline{\widehat{\boldsymbol{\theta}}}$ (for $r >> a$)
- (Note: if I defined the θ as the angle between $\underline{\boldsymbol{r}}$ and the *negative* charge $cos\theta \rightarrow -\cos\theta$ and $sin\theta \rightarrow -sin\theta$)

This Lecture

- Electric Dipoles in uniform E-fields
  - Torque
  - Potential energy
  - Work done
  - Example
  - Right-hand rule
  - examples

Electric Dipoles in Uniform Electric Fields



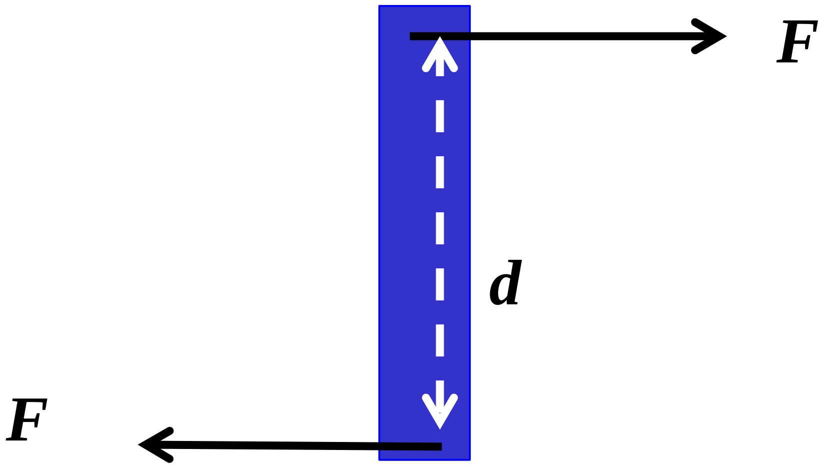
No Net Force

But Torque, $\underline{\tau}$ rotates the dipole clockwise

Two equal and opposite forces whose lines of action do not coincide constitute a (force) couple. The two forces always have a turning effect, called a torque.

Torque of a Couple

- The magnitude of the torque of a couple is:



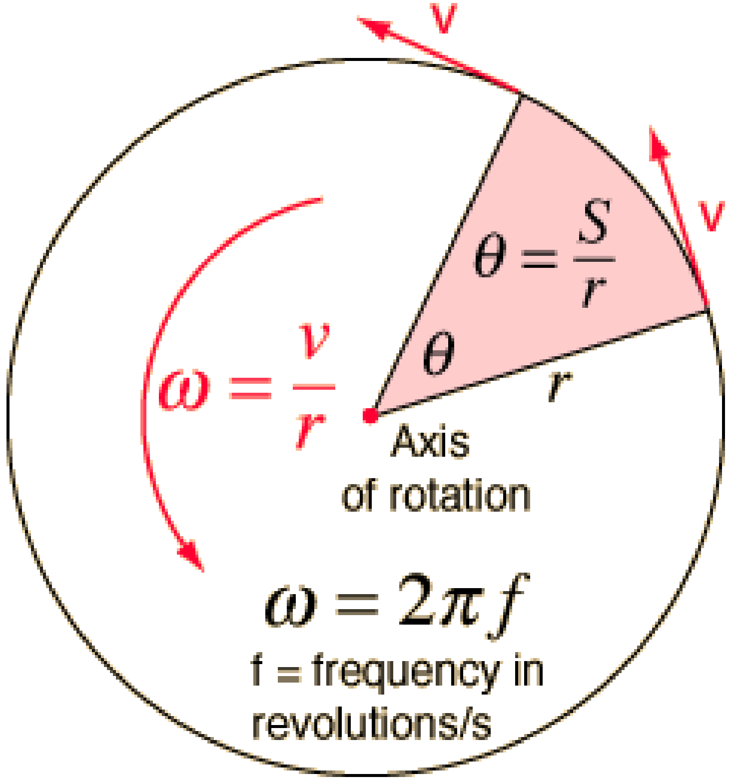
$$\boxed{|\underline{\boldsymbol{\tau}}| = Fd}$$

- torque = one force × perpendicular distance between forces

Aside: Circular Motion

A torque is to circular motion what a force is to linear motion

Angular Motion

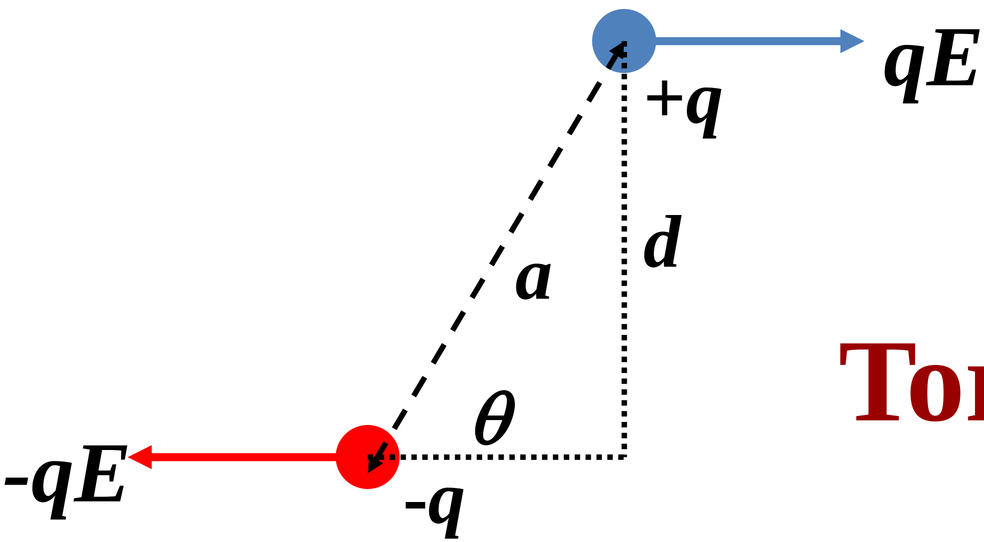


Linear vs Circular Motion

Distance, x	angle, θ (in radians of course)
Velocity, $v = \frac{dx}{dt} = \dot{x}$	angular velocity, $\omega = \frac{d\theta}{dt} = \dot{\theta}$
	($\omega = 2\pi/T$ where T is period)
Acceleration,	angular acceleration,
$a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \ddot{x}$	$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \ddot{\theta}$
Momentum, $\mathbf{p} = m\mathbf{v}$	angular momentum, $\mathbf{L} = \mathbf{r} \wedge \mathbf{p} = \mathbf{r} \wedge m\mathbf{v} = (mr^2)\,\boldsymbol{\omega}$ i.e. $\mathbf{L} = I\,\boldsymbol{\omega}$
	(I = moment of inertia)
Force, $\mathbf{F} = m\mathbf{a} = \frac{d\mathbf{p}}{dt}$	Torque, $\boldsymbol{\tau} = \mathbf{r} \wedge \mathbf{F} = I\,\boldsymbol{\alpha} = \frac{d\mathbf{L}}{dt}$
Kinetic energy $= \frac{1}{2}mv^2$	$= \frac{1}{2}I\omega^2$

See Classical Mechanics – Later this semester

Torque on Dipole



- The torque tends to align $\underline{p}$ and $\underline{E}$

Torque τ

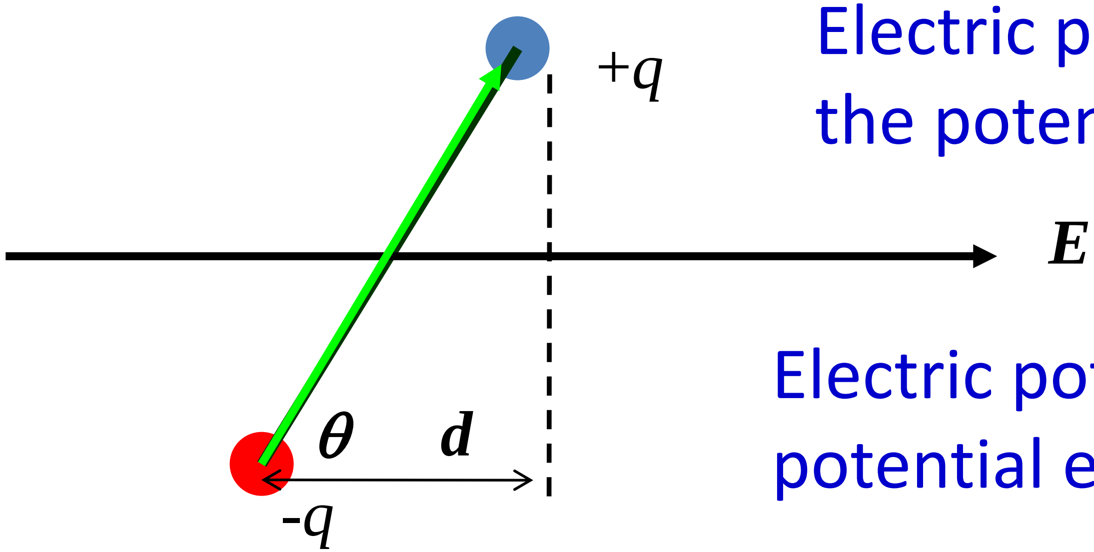
$$\tau = |\underline{\tau}| = Fd = qEd = qE\,a\sin\theta = qa\,E\sin\theta$$

$$= pE\sin\theta$$

Definition of Torque

- A torque is defined as the moment of a force
- Mathematically:
- $\underline{\boldsymbol{\tau}} = \underline{\boldsymbol{r}} \wedge \underline{\boldsymbol{F}}$
- In this case, torque on dipole is:
- $\underline{\boldsymbol{\tau}} = q\underline{\boldsymbol{a}} \wedge \underline{\boldsymbol{E}} = \underline{\boldsymbol{p}} \wedge \underline{\boldsymbol{E}}$

Potential Energy of Dipole in an E-field



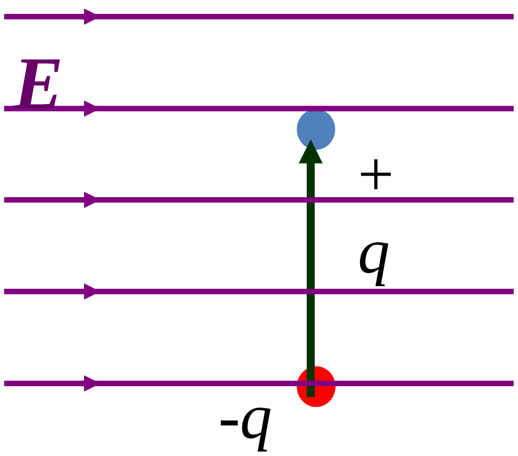
Electric potential at +q is V_+, the potential energy is qV_+

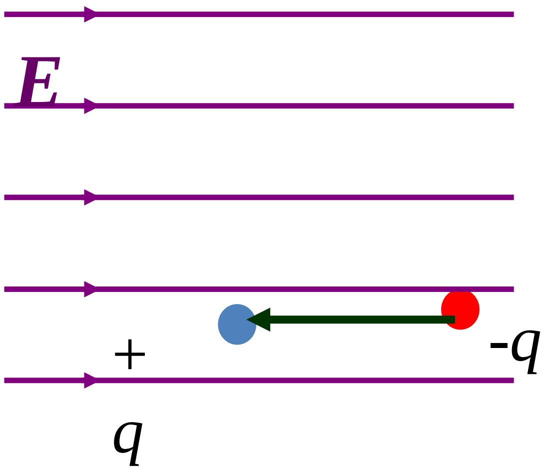
Electric potential at -q is V_-, the potential energy is $-qV_-$

- $U = q(V_+ - V_-) = -q\int_0^d \underline{\boldsymbol{E}} \cdot d\underline{\boldsymbol{x}} = -qEd = -qE\,acos\theta = -qa\,E\,cos\theta = -pE\,cos\theta.$
- i.e. $$U = -\underline{\boldsymbol{p}} \cdot \underline{\boldsymbol{E}}$$

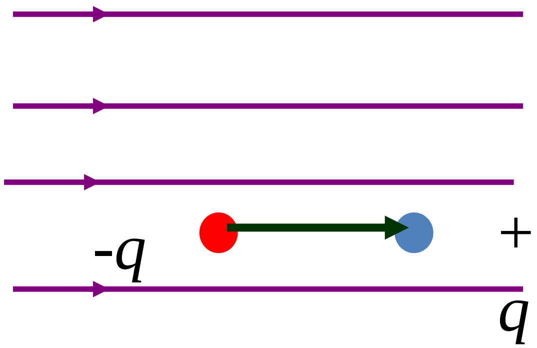
$$U = -\underline{p}\cdot\underline{E} = -pE\cos\theta$$

$\theta = \pi/2$





$\theta = \pi$



$\theta = 0$

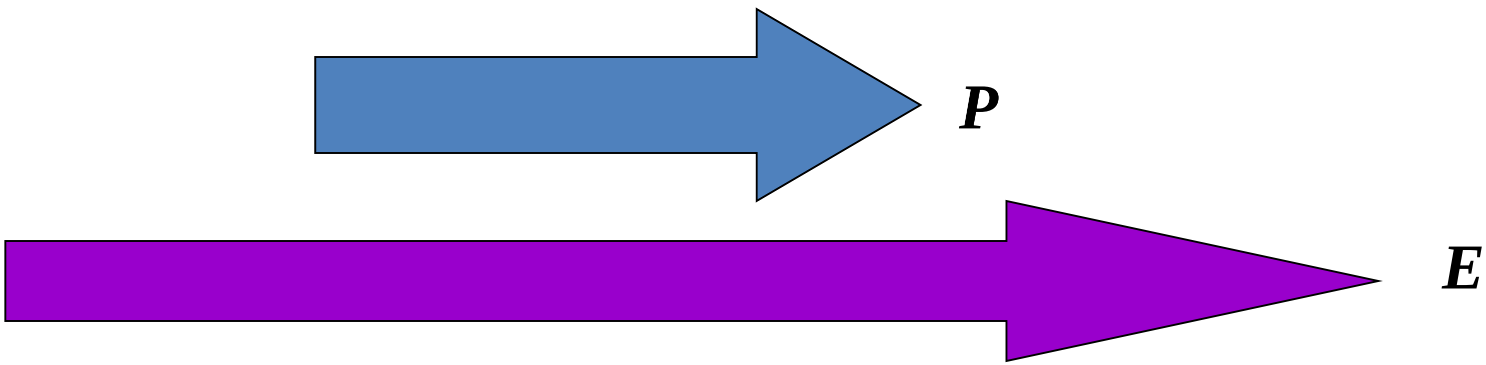
Potential Energy of Dipole

Thus the P.E. of an electric dipole in an E-field is:

$$U = -\underline{\boldsymbol{p}} \cdot \underline{\boldsymbol{E}} = -pE\cos\theta$$

Minimum at $\theta = 0$, maximum at $\theta = \pi$, and zero at $\theta = \pi/2$ (zero potential energy is not the minimum energy)

Example



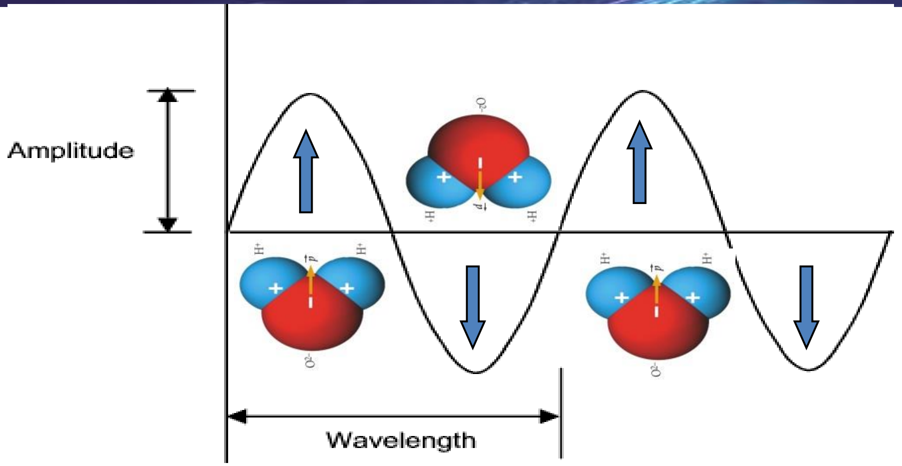
The potential energy of this dipole in E is

$$U = -\underline{p} \cdot \underline{E} = -pE$$

What is the torque on the dipole for the above configuration?

$$\tau = 0$$

Microwave Oven

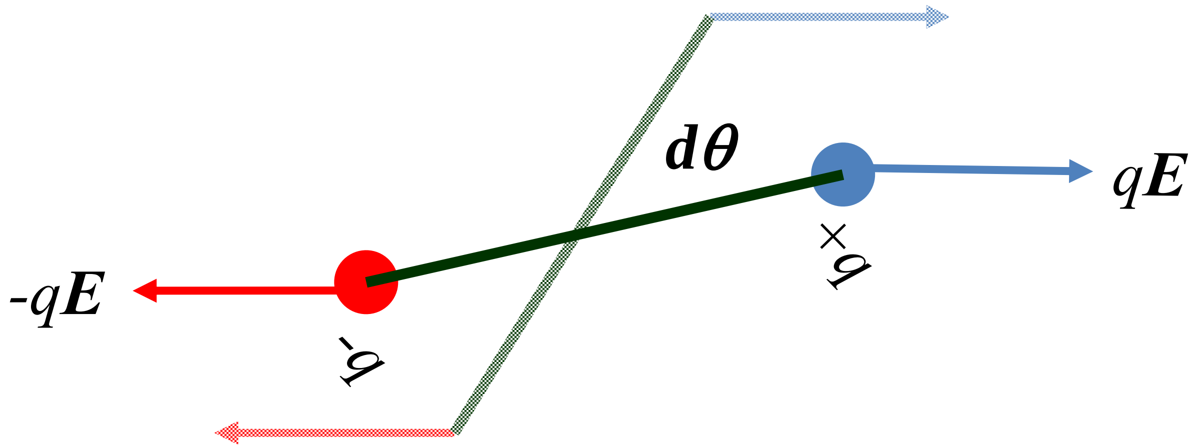


- Dipole of water molecules align with E-field of micro-waves.
- Alternating microwaves cause the water molecules to oscillate in phase
- i.e. they gain energy -> Heat
- https://www.youtube.com/watch?v=0OwYwvyYvx8

Summary of Dipole in an E-field

- An electric dipole in an electric field experiences a torque:
- $\underline{\boldsymbol{\tau}} = q\underline{\boldsymbol{a}} \wedge \underline{\boldsymbol{E}} = \underline{\boldsymbol{p}} \wedge \underline{\boldsymbol{E}}$
- The potential energy for an electric dipole in an electric field $\underline{E}$ depends on the orientation of the dipole moment $\underline{p}$ with respect to the field:
- $\boldsymbol{U} = -\underline{\boldsymbol{p}} \cdot \underline{\boldsymbol{E}}$

Work Done by Torque

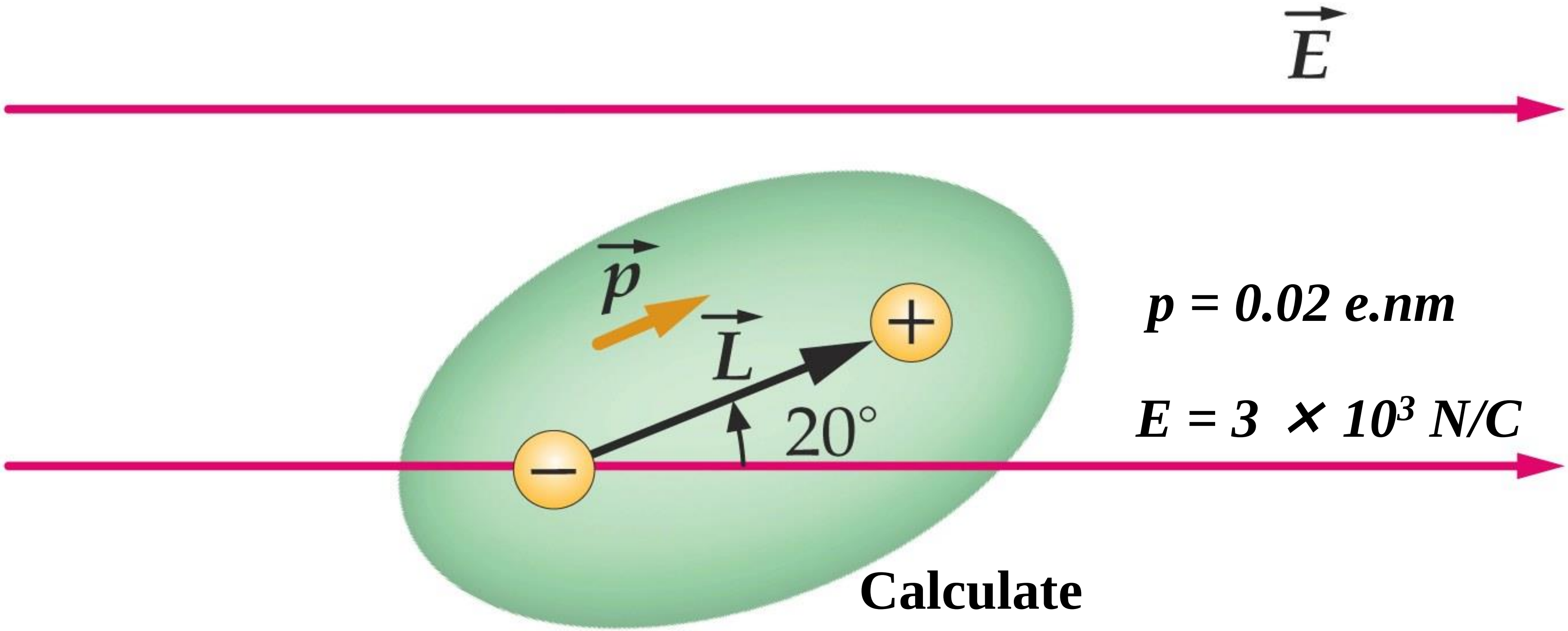


- Work done by $\underline{\tau}$ during an infinitesimal displacement δθ: $\delta W = \tau\, \delta\theta$
- The torque is in the direction of decreasing θ
- $\tau = -pE\, sin\theta$ hence $\delta W = -pE\, sin\theta\, \delta\theta$

Work Done by Torque

- Displacement from θ_1 to θ_2
- $W = \int dW = \int_{\theta_1}^{\theta_2} (-pE\ sin\theta)\ d\theta$
- Therefore $W = pE\ cos\theta_2 - pE\ cos\theta_1$
- Work (done by E-field) = - change in P.E.
- $W = -(U_2 - U_1) = (U_1 - U_2)$

Example



$p = 0.02$ *e.nm*

$E = 3 \times 10^3$ *N/C*

Calculate

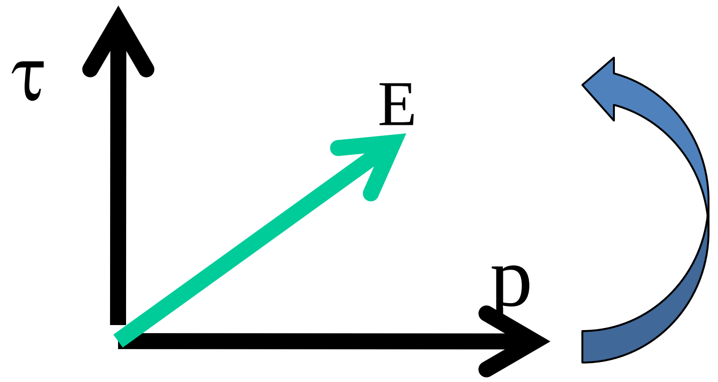
(a) the magnitude of the torque

(b) The potential energy

Let’s do it on the visualizer

Direction of Torque

- Torque is a vector and it is perpendicular to both $\underline{\boldsymbol{p}}$ and $\underline{\boldsymbol{E}}$.
- $\underline{\boldsymbol{\tau}} = \underline{\boldsymbol{p}} \wedge \underline{\boldsymbol{E}} = \underline{\hat{\boldsymbol{\tau}}}\, pE \, sin\theta$
- The direction of $\underline{\boldsymbol{\tau}}$ is:



Direction of Torque

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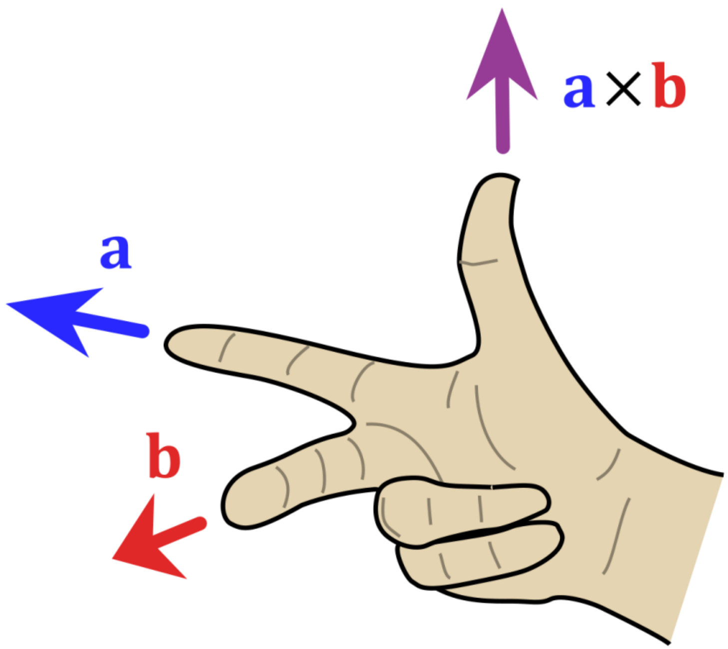
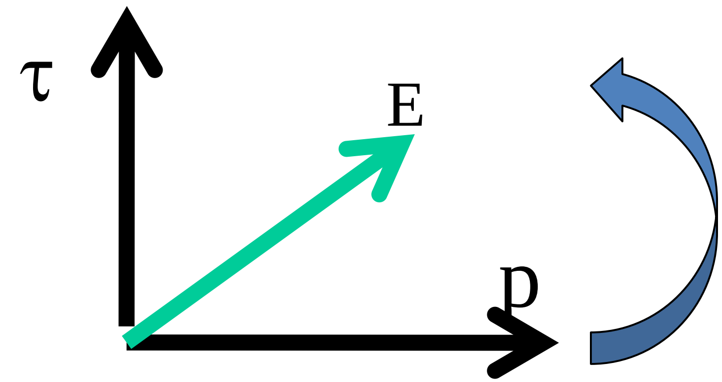


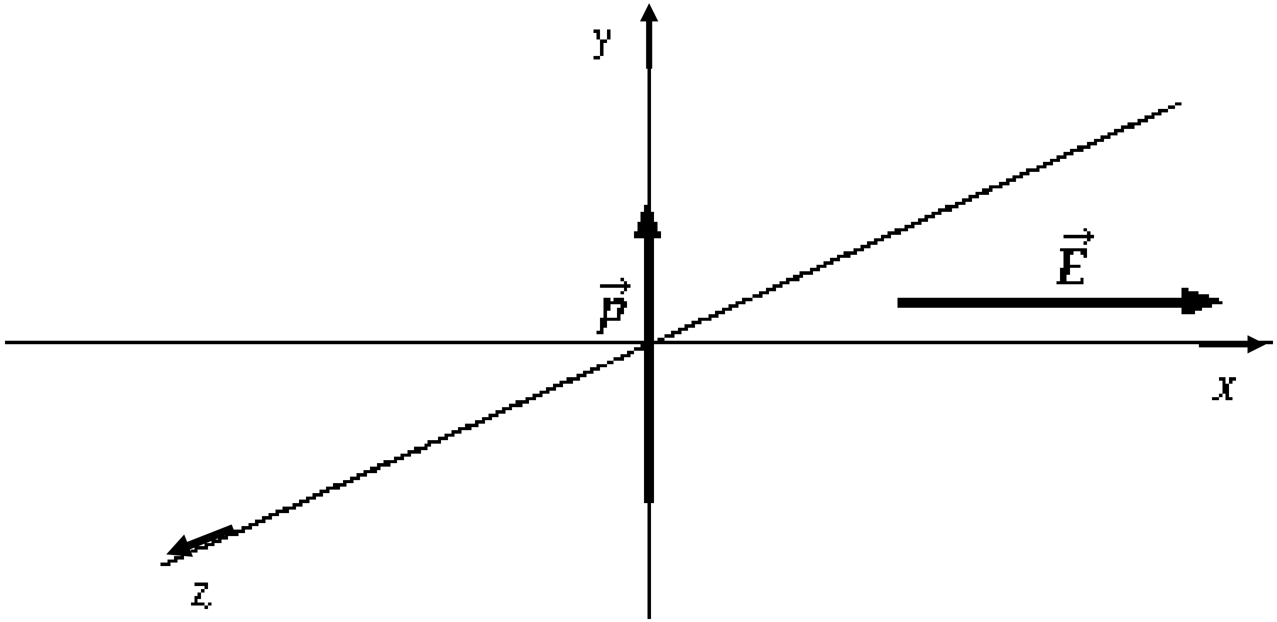


Use Right-Hand Rule

In this case

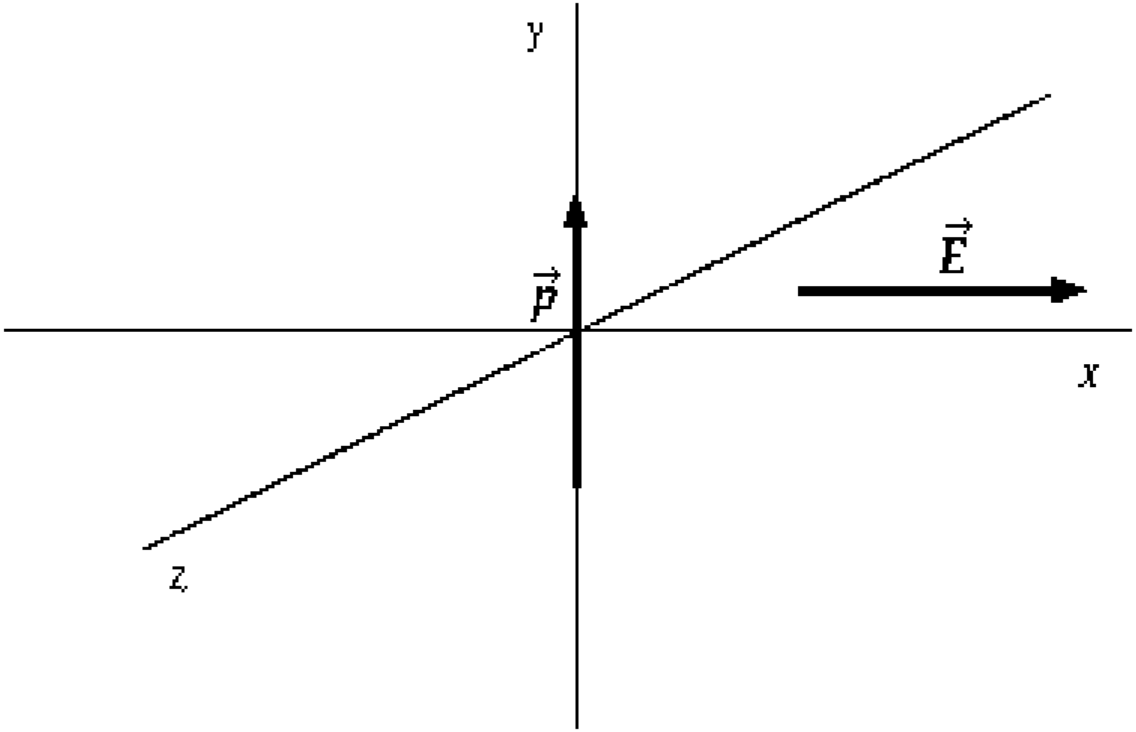
$\underline{\boldsymbol{a}}$ *is* $\underline{\boldsymbol{p}}$ *and* $\underline{\boldsymbol{b}}$ *is* $\underline{\boldsymbol{E}}$

Quiz Time



An electric dipole of moment p is placed in a uniform external electric field. The dipole moment vector is in the positive y direction. The external electric field vector is in the positive x direction. When the dipole is aligned as shown in the diagram, **the net torque is in the**

A)positive x direction. B)positive y direction. C)negative x direction. D)positive z direction. E)negative z direction.



An electric dipole of moment p is placed in a uniform external electric field as shown in the diagram. The dipole moment vector is in the positive y direction. The external electric field vector is in the positive x direction. If the dipole is to have minimum potential energy, **p should be in the**

A)positive x direction. B)negative x direction. C)positive y direction.

D)negative y direction. E)positive z direction.

Importance

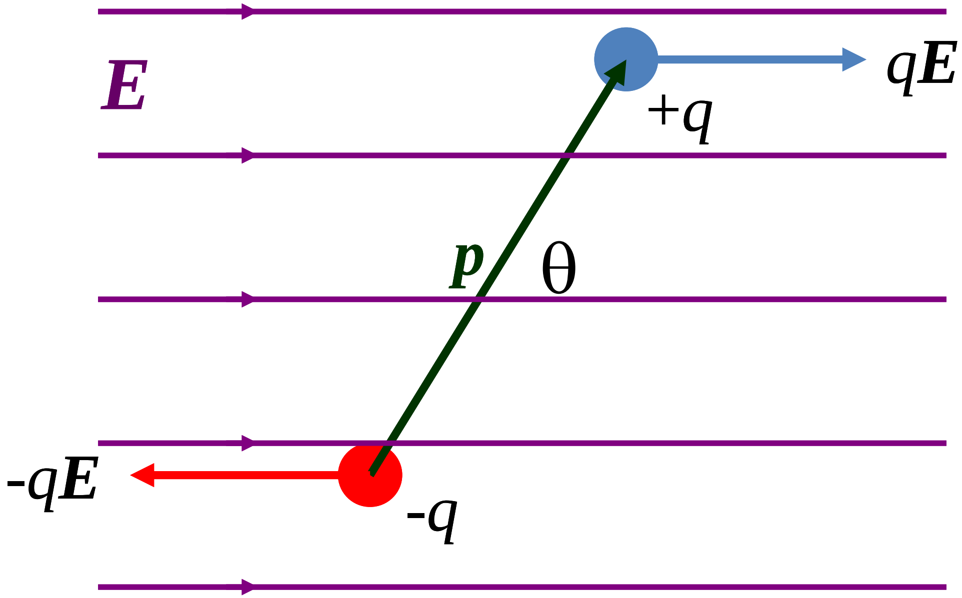
- Production and reception of radio and TV signals
- Interaction of molecules with EM radiation:

  - molecular spectroscopy
    - trace analysis
    - astrophysics
    - molecular physics

Dipoles - Summary

- Define dipole moment as $\underline{\boldsymbol{p}} = q\underline{\boldsymbol{a}}$
- $V_p \approx \dfrac{p \cos\theta}{4\pi\varepsilon_0 r^2}$ (for r >> a)
- $\underline{\boldsymbol{E}} \approx \dfrac{2\,p\cos\theta}{4\pi\varepsilon_0 r^3}\underline{\hat{\boldsymbol{r}}} + \dfrac{p\sin\theta}{4\pi\varepsilon_0 r^3}\underline{\widehat{\boldsymbol{\theta}}}$ (for r >> a)
- For Dipole in external uniform E-field
- $\underline{\boldsymbol{\tau}} = q\underline{\boldsymbol{a}} \wedge \underline{\boldsymbol{E}} = \underline{\boldsymbol{p}} \wedge \underline{\boldsymbol{E}}$
- $\boldsymbol{U} = -\underline{\boldsymbol{p}} \cdot \underline{\boldsymbol{E}}$

Dipole Example Problem



Consider a dipole in a uniform E-field $\underline{\boldsymbol{E}} = E\,\underline{\boldsymbol{i}}$ with a dipole moment vector $\underline{\boldsymbol{p}}$ making an angle θ with the x-axis.

Q1: what is the vector expression for $\underline{\boldsymbol{p}}$?

Q2: Find a vector expression for the torque on the dipole

Q3: What type of motion does the dipole undergo if free to move?

Next Installment

- Capacitance
  - How to calculate capacitance
  - Energy stored in a capacitor
  - Energy density of electric field
- Dielectric Materials
  - How dielectrics modify electric fields
  - Relative permittivity / Dielectric constant