CHAOS

1. A forced damped pendulum has an equation

$$\frac{d^2\theta}{dt^2} + 6\frac{d\theta}{dt} + 25\sin\theta = R\cos 4t \quad .$$

When $R \ll 1$, find the approximate solution for the attractor and transients. **[6]**
Using the two choices

$$p \equiv \frac{d\theta}{dt} \qquad P \equiv \frac{d\theta}{dt} + 3\theta$$

find the fundamental equations and propose which of these is more useful for the attractor and which is more useful for the transients; with an explanation for your choice. **[3]**
On what scale is the error? **[1]**

2. Consider the map

$$x_{n+1} = rx_n(1-x_n)^2$$

where $r > 0$ is a control parameter. Find all the possible 1-cycles and establish for which range of control parameter they are stable. **[4]**
Find all the possible 2-cycles and establish for which range of control parameter they are stable. **[5]**
How do you think this sequence of cycles continues? **[1]**

3. Consider the map

$$x_{n+1} = rx_n(1-x_n)^2 \quad .$$

where r is a control parameter. Employ the transformation

$$x_n = \frac{4}{3}\left[\sin\frac{\pi y_n}{2}\right]^2$$

to rewrite the map as

$$\left[\sin\frac{\pi y_{n+1}}{2}\right]^2 = \left[\sin\frac{3\pi y_n}{2}\right]^2$$

at a particular value of r that you should determine. **[5]**
On the assumption that $y_n \in [0,1]$, find the map that constitutes $y_{n+1} = M[y_n]$ and depict this map. **[4]**
Find all the possible 1-cycles of both maps and for which range of control parameter they are stable. **[1]**

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4. Consider the map defined by

$$x_{n+1} = M[x_n]$$

where

$$\begin{aligned} M[x] =& 3x & x \in \left[0, \frac{1}{3}\right] \\ =& 2 - 3x & x \in \left[\frac{1}{3}, \frac{2}{3}\right] \\ =& 3x - 2 & x \in \left[\frac{2}{3}, 1\right] \end{aligned}$$

Employ Base 3 to determine a useful representation of this map. **[5]**
Find all the possible 1-cycles and 2-cycles of this map using Base 3. **[3]**
Check your answers directly. **[2]**

5. A mass feels a potential

$$V(\mathbf{x}) = \frac{1}{2}\left[k_1 x_1^2 + k_2 x_2^2\right] - mgx_2$$

Find the equations of motion and determine a fundamental equation for the system. **[4]**
Solve for the motion in general and show that the system is integrable. **[4]**
Employ a Poincare section with x_1=0 and find the effective map. When does this map have n-cycles as a solution and when is it ergodic on the Poincare surface? **[2]**

6. A dynamical system is described by a fundamental equation

$$\frac{dx_1}{dt} = -x_1 + x_2 \qquad \frac{dx_2}{dt} = -x_1 - x_2 + 4x_1x_2 + 2x_1^2 - 2x_1^3 - 2x_2^3$$

Find the fixed points and determine the local trajectories in the vicinity of these fixed points. **[7]**
Depict the phase space portrait. **[3]**

Any calculator

CHAOS

8.

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11.

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