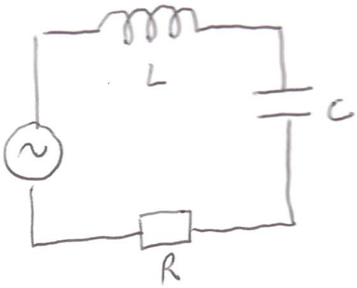
From last week : LRC circuit



$$\boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

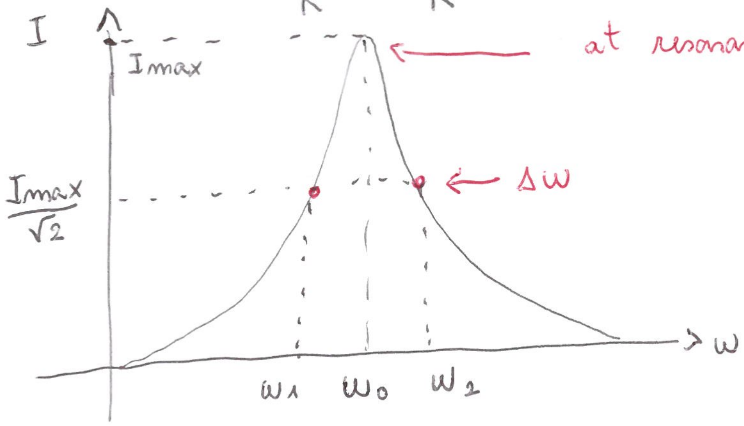
RESONANCE

$\mathrm{Im}(Z) = 0 \longrightarrow X_L = X_C$

$$I = \frac{V}{|Z|} = \frac{V}{R\sqrt{1+Q^2\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^2}} = \frac{I_{max}}{\sqrt{1+Q^2\left(\frac{\omega}{\omega_0}-\frac{\omega_0}{\omega}\right)^2}}$$

Q = quality factor $= \frac{X_L}{R} = \frac{X_C}{R}$

$I_{max} = \frac{V}{R}$



at resonance
- Z is min
- I is max

$$Q = \frac{\omega_0}{\omega_2 - \omega_1} = \frac{\omega_0}{\Delta\omega}$$

$$V, V_{rms} \Big\} \quad V_{rms} = \frac{V}{\sqrt{2}}$$

$$V_C = I X_C = I \frac{1}{\omega C} = \underset{\downarrow \; I}{\frac{V}{Z}} \frac{1}{\omega C} \underset{\uparrow}{=} \frac{V}{Z} \frac{Q \omega_0 R}{C} = VQ \frac{\omega_0 R}{\omega Z}$$

$$Q = \frac{1}{\omega_0 C R} \, , \quad \frac{1}{C} = Q \omega_0 R$$

$$V_L = I X_L = I \omega L = \frac{V}{Z} \omega L = Q \frac{\omega R}{\omega_0 Z} V$$

At resonance $\quad V_C = VQ \frac{\omega_0 \cancel{R}}{\cancel{\omega Z}} = QV$

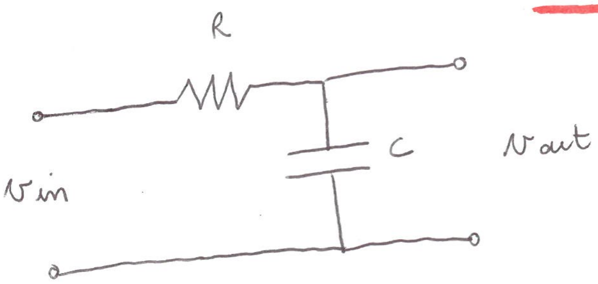
$\omega = \omega_0$

$Z = R$

$$V_L = Q \frac{\omega R}{\omega_0 Z} V = QV$$

$$\boxed{V_C = V_L = QV} \qquad Q >> 1$$

RC filter : law pass



$$\frac{v_{out}}{v_{in}} = \frac{Z_c}{Z_c + Z_R} = \frac{-j/\omega C}{-j/\omega C + R}$$

$$= \frac{1/(j\omega C)}{1/(j\omega C) + R} = \frac{1}{1 + j\omega RC} = \frac{1 - j\omega RC}{1 + \omega^2 R^2 C^2}$$

$$\left|\frac{v_{out}}{v_{in}}\right| = \sqrt{\frac{1}{[1+\omega^2R^2C^2]^2} + \frac{\omega^2R^2C^2}{[1+\omega^2R^2C^2]^2}} =$$

$$= \sqrt{\frac{\cancel{1+\omega^2R^2C^2}}{(1+\omega^2R^2C^2)^{\cancel{2}}}} = \frac{1}{\sqrt{1+\omega^2R^2C^2}} =$$

$$= \frac{1}{\sqrt{1+(\omega RC)^2}} =$$

$$= \frac{1}{\sqrt{1+(\omega/\omega_0)^2}}$$

amplitude of the ratio $\frac{v_{out}}{v_{in}}$

$\boxed{\omega_0 = 1/(RC)}$

$$\tan\phi = \frac{Im\,(v_{out}/v_{in})}{Re\,(v_{out}/v_{in})} = -\frac{\omega RC}{\cancel{1+\omega^2R^2C^2}} \cdot (\cancel{1+\omega^2R^2C^2})$$

$$= -\omega RC$$

$$\phi = -\arctan(\omega RC) = -\tan^{-1}(\omega RC)$$

in the sense of inverse function

relative phase between v_{out} and v_{in}