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Any Calculator

THE UNIVERSITY OF BIRMINGHAM

Degree of B.Sc./M.Sci. with Honours

Programmes in the School of Mathematics and Statistics

First examination

Programmes including Mathematics

First examination

Degree of M.Eng. with Honours

Mathematical Engineering

First examination

0611235/0611240

MSM1C: COMPUTATIONAL AND APPLIED MATHEMATICS

May/June, 2004

2 hours

Full marks may be obtained with complete answers to ALL questions in Section A (worth a total of 50 marks) and TWO (out of THREE) questions from Section B (worth 25 marks each). Only the best TWO answers from Section B will be credited. Calculators may be used in this examination but must not be used to store text. Calculators with the ability to store text should have their memories deleted prior to the start of the examination.

Turn over

SECTION A

1. (a) What are the correct S.I. units of Vt where V is volume and t is time? [6]

2. A projectile is fired from ground level with speed $100ms^{-1}$ at an angle of $\pi/4$ radians to the horizontal. Neglecting air resistance and assuming the landscape over which the projectile travels is flat, find the total horizontal distance travelled by the projectile before landing. Take $g = 10ms^{-2}$. [10]

3. Write down the unit vectors in polar coordinates \mathbf{e}_r and \mathbf{e}_θ in terms of the Cartesian unit vectors \mathbf{i} and \mathbf{j} , where the polar coordinates are (r, θ) . If θ depends on time t , write

$$\frac{d\mathbf{e}_\theta}{dt}$$

in terms of \mathbf{e}_r . [10]

SECTION B

4. A rain drop slides down a pane of glass. The speed of the drop's progress U depends on the volume of the droplet V , the kinematic viscosity of water ν , the surface tension of water σ , the density of water ρ , the acceleration due to gravity g and the angle that the water's surface makes with the surface of the glass θ .

(a) What are the dimensions of U , V , ρ , g and θ ? [5]

5. (a) What is Hooke's law? [1]

(b) A particle of mass m is attached by a horizontal spring to a fixed point. The particle is set in motion along the line of the spring. The particle slides across a table with negligible friction. If a is the natural length of the spring, the spring constant is k , and x is the displacement of the particle from some fixed point, then use Newton's second law to determine a differential equation for x . [4]

(c) A mechanical vibration is described by the differential equation

$$\frac{d^2x}{dt^2} + \alpha \frac{dx}{dt} + \omega^2 x = 0$$

where x is a displacement, t is time, and α and ω are constants. Let $x = 1$ and $dx/dt = 0$ at $t = 0$.

(i) If $\alpha = 0$ then determine an expression for x in terms of t . Describe briefly the nature of this oscillation. [10]

(ii) If $\alpha = 1$ and $\omega = 1$ then determine an expression for x in terms of t . Describe briefly the nature of this oscillation. [10]