Continuous Assessment

1. The energy of an oscillator is

$$E = \frac{1}{2}\left[\frac{dx}{dt}\right]^2 + \frac{1}{8}\left(x^2 - 1\right)^2$$

and is conserved. Determine the equations of motion. **[5]**

2. Find the two types of small-scale oscillations and provide the lowest order representation for the trajectory of these oscillations. **[5]**

3. For the oscillations centred around $x=1$ show that approximately, with $x=1+X$

$$\frac{d^2X}{dt^2} + X = -\frac{3}{4}R^2\left[1 + \cos 2(t - t_0)\right] - \frac{1}{8}R^3\left[\cos 3(t - t_0) + 3\cos(t - t_0)\right]$$ **[5]**

4. Solve this approximation to provide a supposedly more accurate representation for the trajectory. **[5]**

5. Explain why the trajectory might also be described by

$$\frac{d^2X}{dt^2} + X = -\frac{3}{4}R^2\left[1 + \cos 2\omega(t - t_0)\right] - \frac{1}{8}R^3\cos 3\omega(t - t_0) - \frac{3}{8}R^2X$$

with an appropriate choice of ω that you should choose. Solve this new equation and compare the new solution to the previous, suggesting which one is physically more appropriate. **[5]**