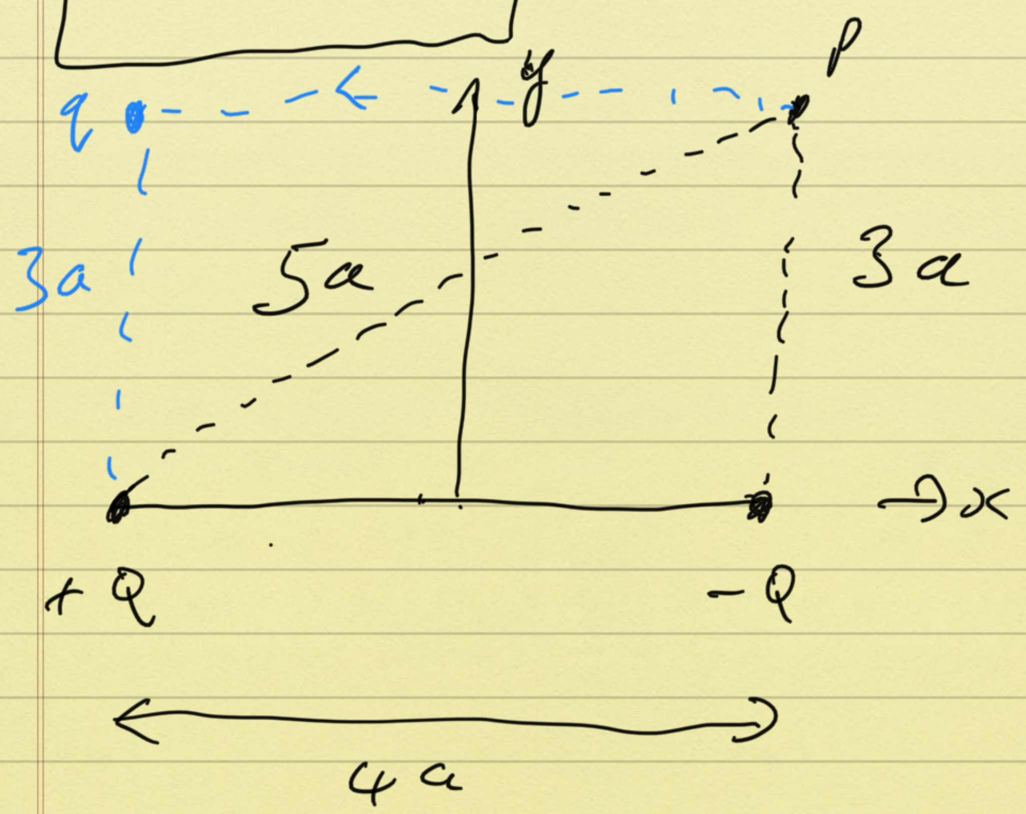
# Lecture 16 - revision for weeks 1-6

$\boxed{\text{Ex } 16 \cdot 1}$




$$V_P(+Q) = \frac{Q}{4\pi\varepsilon_0 (5a)}$$

$$V_P(-Q) = \frac{-Q}{4\pi\varepsilon_0 (3a)}$$

$$\therefore V_P = V_P(+Q) + V_P(-Q) = \frac{Q}{4\pi\varepsilon_0}\left\{\frac{1}{5a} - \frac{1}{3a}\right)$$

$$= \underline{\underline{\frac{-Q}{4\pi\varepsilon_0 a}\left(\frac{2}{15}\right)}}$$

$$W.D. = q\,\Delta V = q(V_f - V_i)$$

$$V_f(-2a, 3a) =$$

$$\frac{Q}{4\pi\varepsilon_0 (3a)} - \frac{Q}{4\pi\varepsilon_0 (5a)}$$

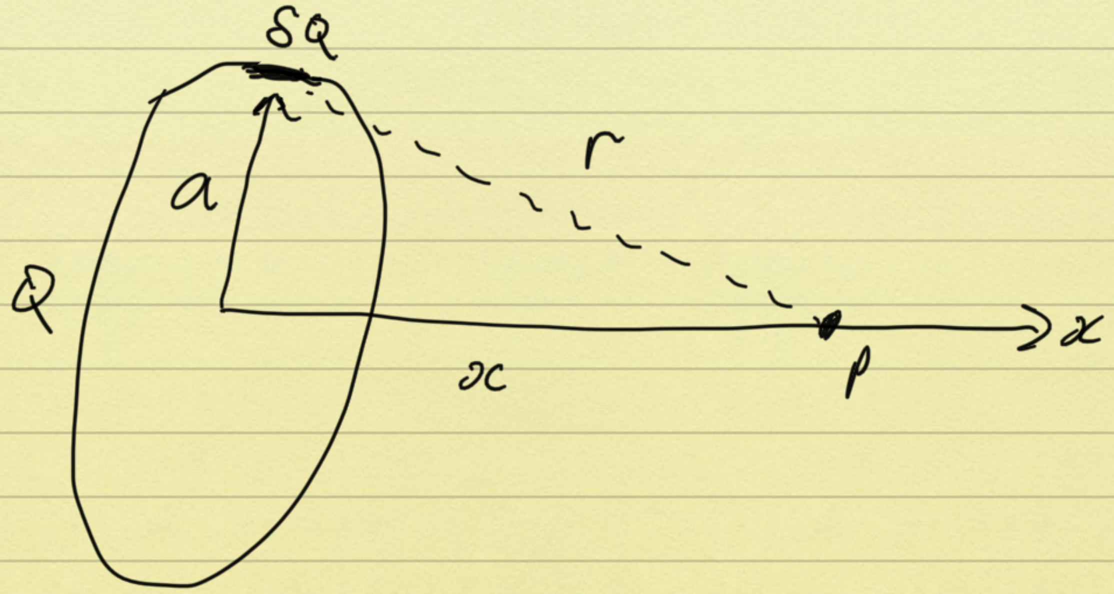
$$= \frac{Q}{4\pi\varepsilon_0 a}\left(\frac{2}{15}\right)$$

$$W.D. = q(V_f - V_i)$$

$$= \frac{qQ}{4\pi\varepsilon_0 a}\left(\frac{4}{15}\right)$$

$$= \frac{qQ}{15\pi\varepsilon_0 a}$$

$\boxed{\text{Ex 16-2}}$




(i)

$$\delta V = \frac{\delta Q}{4\pi\varepsilon_0 r} = \frac{\delta Q}{4\pi\varepsilon_0 (a^2+x^2)^{1/2}}$$

$$\therefore V = \frac{1}{4\pi\varepsilon_0 (a^2+x^2)^{1/2}} \oint dQ$$

$$= \frac{Q}{4\pi\varepsilon_0 (a^2+x^2)^{1/2}}$$

(ii) $$\underline{E} = -\nabla V = -\frac{\partial V}{\partial x}\underline{i} - \frac{\partial V}{\partial y}\underline{j} - \frac{\partial V}{\partial z}\underline{k}$$

but no $y$ or $z$ in eq$^n$

$$= \frac{\partial V}{\partial y} = \frac{\partial V}{\partial z} = 0 \Rightarrow \underline{E_y = E_z = 0}$$

$$\underline{E} = -\frac{d}{dx}\left(\frac{Q}{4\pi\varepsilon_0 (a^2+x^2)^{1/2}}\right)\hat{\underline{x}}$$

$$= -\frac{Q}{4\pi\varepsilon_0}\,\frac{-x}{(a^2+x^2)^{3/2}}\,\hat{\underline{x}}$$

$$\therefore\ \underline{E} = \frac{Q\,x}{4\pi\varepsilon_0 (a^2+x^2)^{3/2}}\,\hat{\underline{x}}$$

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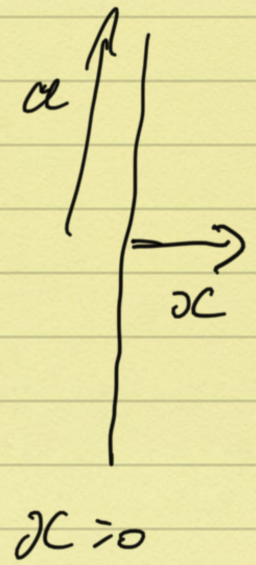
(iii)




$$x \ll a$$

$$\text{Force} = -e\underline{E}$$

$$\underline{F} = \frac{-e\,Q\,x}{4\pi\varepsilon_0 (a^2+x^2)^{3/2}}\,\hat{\underline{x}}$$

$$= \frac{-e\,Q\,x}{4\pi\varepsilon_0\, a^3 \left(1+\frac{x^2}{a^2}\right)^{3/2}}\,\hat{\underline{x}}$$

$x \ll a$ $$F \simeq \frac{-e\,Q\,x}{4\pi\varepsilon_0\, a^3}$$

$$F = m\frac{d^2x}{dt^2} = -\left(\frac{eQ}{4\pi\varepsilon_0 a^3}\right)x$$

$$\frac{d^2x}{dt^2} = -\left(\frac{eQ}{4\pi\varepsilon_0 m a^3}\right)x$$

Sol$^n$ $\sim$ $x(t) = x_0 \cos(\omega t)$

$$\Rightarrow \omega = 2\pi f = \left(\frac{eQ}{4\pi\varepsilon_0 m a^3}\right)^{1/2}$$

$$f = \frac{1}{2\pi}\left(\frac{eQ}{4\pi\varepsilon_0 m a^3}\right)^{1/2}$$

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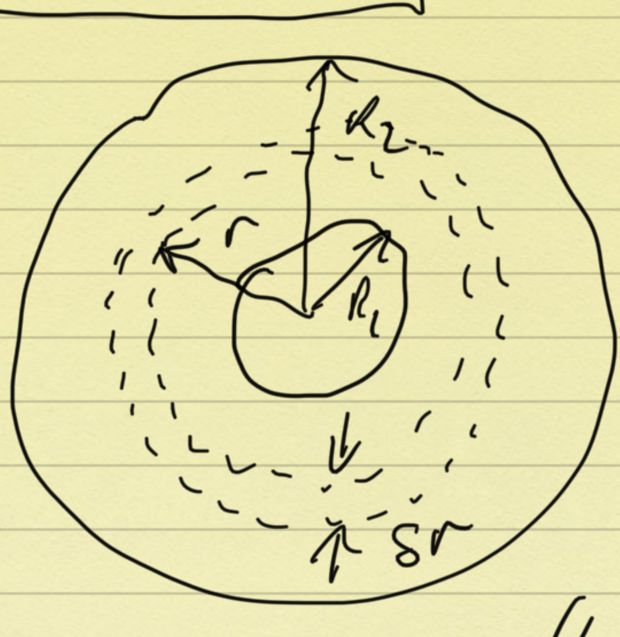
$\boxed{\text{Ex } 16.3}$ $\rho(r) = \rho_0 r$




shell at $r$ & thickness $\delta r$

$$\delta Q = \rho(r)\,\delta V$$

$$= \rho_0 r \cdot 2\pi r\,\delta r\,L$$

($L$ = some length)

(a)

$$\delta Q = 2\pi \rho_0 L r^2 \delta r$$

and per unit length ($\delta Q / L$)

$$\delta Q = 2\pi \rho_0 \; r^2 \delta r$$

$$\therefore Q = 2\pi \rho_0 \int_{R_1}^{R_2} r^2 \, dr$$

$$= 2\pi \rho_0 \left[ \frac{r^3}{3} \right]_{R_1}^{R_2}$$

$$= \frac{2\pi \rho_0}{3} \left( R_2^3 - R_1^3 \right)$$

(b)

(i) $r > R_2$

$$\int_S \underline{E} \cdot d\underline{S} = \frac{Q_{enc}}{\varepsilon_0}$$

By symmetry $\int_S \underline{E} \cdot d\underline{S} = \int_S E \, dS$

$$= E \int_S dS = E \cdot 2\pi r L$$

$$\underline{E} \cdot 2\pi r L = \frac{Q_{enc}}{\varepsilon_0} = \frac{Q\,(\text{per unit length})\,L}{\varepsilon_0}$$

$$E \cdot 2\pi r = \frac{2\pi \rho_0}{3\varepsilon_0}\left(R_2^{\,3} - R_1^{\,3}\right)$$

$$\underline{E} = \frac{\rho_0}{3\varepsilon_0}\left(R_2^{\,3} - R_1^{\,3}\right)\frac{1}{r} \quad \underline{\hat{r}}$$

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(b)

(ii) $R_1 < r < R_2$ : LHS $= E \cdot 2\pi r$

$Q_{enc}\,(R_1 < r < R_2)$

$$= \int_{R_1}^{r} \rho \, dV = 2\pi \rho_0 \int_{R_1}^{r} r^2 \, dr$$

$$= \frac{2\pi \rho_0}{3}\left(r^3 - R_1^{\,3}\right)$$

$$E \cdot 2\pi r = \frac{2\pi \rho_0}{3\varepsilon_0}\left(r^3 - R_1^{\,3}\right)$$

$$\underline{E} = \frac{\rho_0}{3\varepsilon_0}\left(r^2 - \frac{R_1^{\,3}}{r}\right)$$

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