UNIVERSITY OF BIRMINGHAM

School of Physics and Astronomy

DEGREE OF B.Sc. & M.Sci. WITH HONOURS

FIRST-YEAR EXAMINATION

03 19750

LC ELECTROMAGNETISM 1 / TEMPERATURE & MATTER / ELECTRIC CIRCUITS

SEMESTER 2 EXAMINATIONS 2022/23

Time Allowed: 3 hours

Answer five questions from Section 1 and three questions from Section 2.

Section 1 counts for 25% of the marks for the examination. Answer ***all five*** questions in this Section.
Section 2 consists of three questions and carries 75% of the marks.
Answer ***all three*** questions in this Section. Note that each question has two parts, of which only ***one part*** should be answered. If you answer both parts, credit will only be given for the best answer.

The approximate allocation of marks to each part of a question is shown in brackets **[ ]**.

PLEASE USE A SEPARATE ANSWER BOOK FOR SECTION 1 AND SECTION 2 QUESTIONS.

Calculators may be used in this examination but must not be used to store text. Calculators with the ability to store text should have their memories deleted prior to the start of the examination.

A formula sheet and a table of physical constants and units that may be required will be found at the end of this question paper.

SECTION 1

*Answer **all five** questions in this Section.*

1. A professor weighing 100 kg works out for an hour on an exercise bicycle, which registers a feeble 100 W power output.

   (a) Assuming that heat is generated in the professor's body at the same rate as work is done, and all the heat remains within their body, calculate the rise in the professor's temperature. **[2]**

   (b) Assuming, instead, that the professor's body temperature remains constant, and the heat loss occurs via perfect perspiration, calculate the change in the mass of the professor. **[2]**

   (c) How long would the professor have to work out to lose 30 kg? **[1]**

   [*The human body is mainly composed of water, which has heat capacity* 4190 J kg^{-1}K^{-1}*and a latent heat of vaporization of* 2260 kJ k^{-1}.]

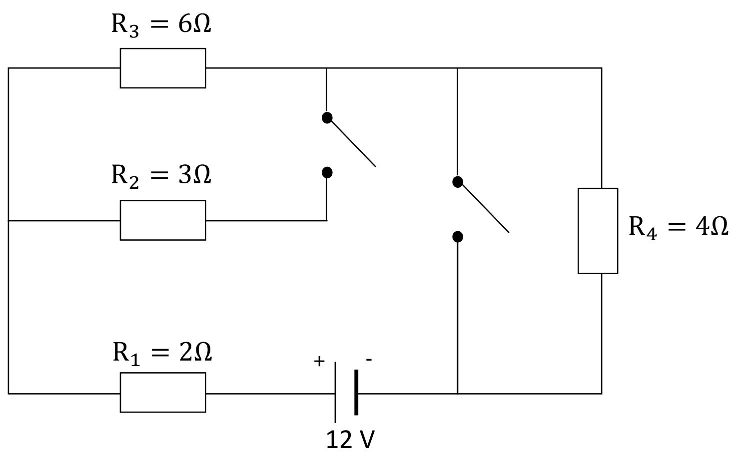
2. A bar of copper of length $\ell_{\text{c}}(T)$ and a bar of aluminium of length $\ell_{\text{a}}(T)$, maintain a temperature-independent length difference $\ell_{\text{c}}(T) - \ell_{\text{a}}(T) = 15$ cm. The coefficient of linear thermal expansion of copper is $\alpha_{\text{c}} = 1.7 \times 10^{-5}$ m K^{-1} and that of aluminium is $\alpha_{\text{a}} = 2.3 \times 10^{-5}$ m K^{-1}.

   Determine the lengths of the two bars, $\ell_{\text{c}}(T_0)$ and $\ell_{\text{a}}(T_0)$, at $T = T_0$. **[5]**

3. Point charges of $-2Q$ and Q, are fixed at (x, y) coordinates $(-3a, 0)$ and $(3a, 0)$ respectively. Derive an expression for the potential at point P at $(0, 4a)$. What is the work required to move a point charge, q from point P to $(0, 0)$? **[5]**

4. A conducting circular coil of radius a and N turns is rotating in a uniform magnetic field of magnitude, B, with angular frequency ω, around an axis through the centre and in the plane of the loop but perpendicular to $\mathbf{B}$. If the plane of the coil is initially perpendicular to the magnetic field (maximum flux through the coil), derive an expression for the $e.m.f$ induced on the coil. **[5]**

5. For the circuit shown below calculate the current through the resistance R_3:



(a) with both switches open;

(b) with both switches closed.

In which configuration (both switches open or both switches closed) is the dissipated power greater? **[5]**

SECTION 2

*Answer **all three** questions in this Section. Note that each question has two parts, of which only **one part** should be answered. If you answer both parts, credit will only be given for the best answer.*

6. **EITHER Part A**

Consider a thermodynamic cycle for **a single mole of an ideal gas** with four legs:

(i) Isothermal compression at temperature T_1 from volume V_1 to V_2.

(ii) Then, at fixed volume, a decrease in temperature from T_1 to T_2.

(iii) An isothermal expansion from V_2 to V_1 at temperature T_2.

(iv) Finally at fixed volume an increase of temperature from T_2 to T_1.

Then:

(a) Draw the cycle in the p-V plane indicating the four legs. **[5]**

(b) Write down the **two thermodynamic** defining features of an ideal gas. **[3]**

(c) Derive the work performed on the gas on each leg. **[3]**

(d) What is the total heat, $Q_{\text{in}}^{\text{tot}}$, carried into the gas on this cycle. **[3]**

(e) If we let $T_2 \to T_1$ what values are taken by: the total work, $W_{\text{on}}^{\text{tot}}$, performed on the gas; and heat into the gas, $Q_{\text{in}}^{\text{tot}}$? **[2]**

Consider a monatomic ideal gas in a cylindrical volume, V_0, subdivided into two cylindrical compartments, containing n_1 and n_2 moles respectively, by a circular partition of cross sectional area A and mass M. The partition moves along the axis of the cylinder without friction. The gas is in thermal contact with the container which is maintained at a constant temperature, T.

(f) What is the condition on the pressures in the compartments, p_1 and p_2, to ensure mechanical equilibrium of the partition? Determine the equilibrium volumes V_1 and V_2 in terms of V_0, n_1 and n_2. **[3]**

The partition is displaced from the equilibrium position isothermally by a small amount, x_0 ($x_0 A \ll V_1$ and V_2), along the axis of the cylinder and then released.

(g) Assuming the subsequent motion of the partition is sufficiently slow for the gas to be in equilibrium within each compartment and with the container, determine the equation of motion for the partition. (Gravity may be neglected.) **[4]**

(h) If the gas in the two chambers and the partition are now at a common temperature, T, and thermally isolated from the container, write down an expression for the total energy, E^{tot} and the modified First Law associated with E^{tot}. **[2]**

OR Part B

Consider an ideal gas of atoms of mass m. The three-dimensional Maxwell-Boltzmann distribution for the probability distribution, $p(v)$, for the atoms' speeds, $v \geq 0$, is

$$p(v)\mathrm{d}v = \mathcal{N}\, v^2 \exp\left(-\frac{mv^2}{2\mathrm{k_B}T}\right)\, \mathrm{d}v\ ,$$

where T is the temperature and $\mathcal{N}$ is the normalisation.

An average of a function of v, say $f(v)$, with respect to $p(v)$, is denoted by $\langle f(v)\rangle$.

(a) Sketch $p(v)$ indicating important aspects of the curve and labelling axes. **[6]**

(b) Show that the normalisation is: **[5]**

$$4\pi\left(\frac{m}{2\pi\mathrm{k_B}T}\right)^{3/2}\ .$$

(c) Derive the root-mean-square velocity, $\sqrt{\langle v^2\rangle}$, using $p(v)$. **[3]**

Now consider such an ideal gas of atoms, of mass m at temperature T, being released from the origin, $\mathbf{r} = (x, y, z) = (0, 0, 0)$, at time $t = 0$, with a Maxwell-Boltzmann distribution for their speeds.

(d) Write down the distance from the origin, d, at time t for an atom with velocity v. **[1]**

(e) Using the Maxwell-Boltzmann distribution, deduce the probability distribution, $\mathcal{P}(d, t)$, of the distance the particles have moved at time t. **[6]**

(f) Determine the maximum of the distribution, as a function of d at fixed t. **[2]**

(g) Provide a physical interpretation of the last result. **[2]**

7. **EITHER Part A**

(a) An infinitely long uniformly charged cylinder has charge density ρ and radius R.

i. Derive expressions for the magnitude of the electric field both inside and outside the cylinder. **[4]**

ii. Assuming the electrical potential at the axis ($r = 0$) is zero, derive expressions for the electrical potential both inside and outside the cylinder. **[6]**

iii. Derive an expression for the amount of work it takes to move a charge q from the centre of the cylinder to its edge. **[2]**

(b) An infinite solenoid of radius R has n turns per unit length and carries an alternating current of $I = I_0 \sin \omega t$.

i. Derive an expression for the magnitude of the magnetic field inside the solenoid as a function of time. (Assume the B-field is zero outside the solenoid). **[4]**

ii. Calculate the total magnetic energy per unit length inside the solenoid. **[4]**

iii. Derive an expression for the magnitude of the induced electric field inside the solenoid as functions of time. **[5]**

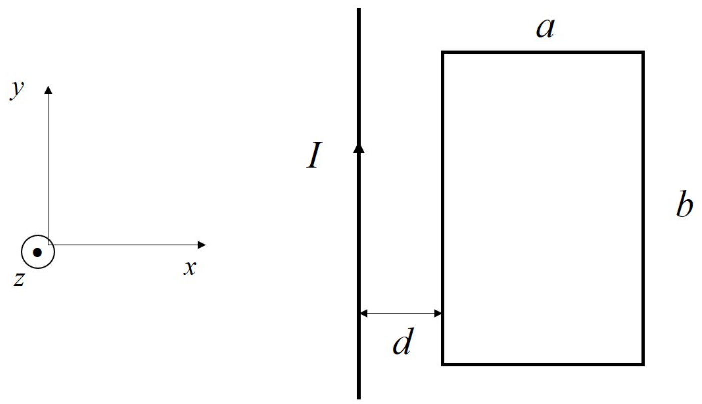
OR Part B

(a) A large parallel-plate capacitor, with circular plates separated by a distance d and of radius R, has a potential V between the plates. (Assume $R \gg d$ so edge effects can be ignored.)

i. Calculate the capacitance of the capacitor and the energy stored in it. **[5]**

ii. Calculate how much work is done by slowly increasing the separation between the plates from d to $2d$ (Note: the total charge remains constant but C and V vary). **[5]**

(b) An infinite straight wire carrying a current I lies in the same plane as a rectangular loop with sides a and b. The sides of length b are parallel to the wire at distances d and $d + a$ from it, as shown in the figure below.

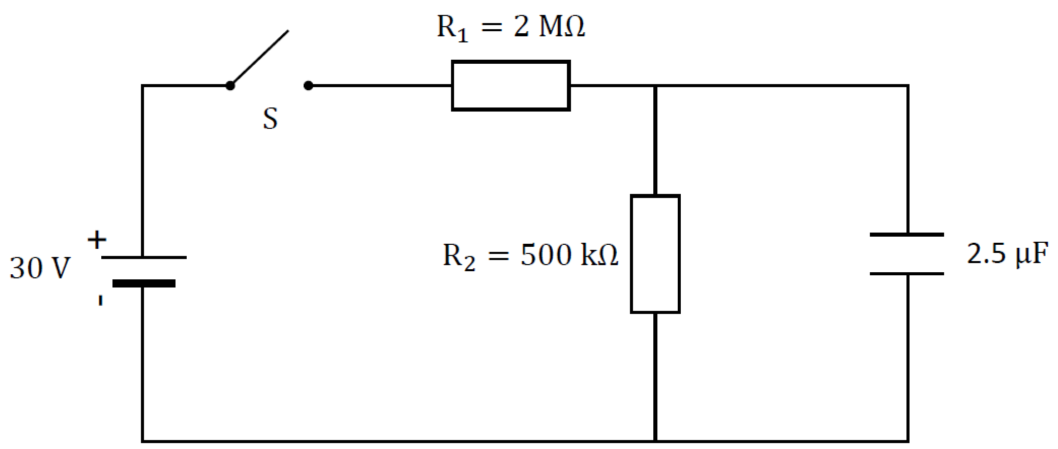


i. Derive an expression for the magnetic field outside the wire and hence the total magnetic flux inside the loop due to the current I in the long wire. **[9]**

ii. A current i is now passed around the rectangular loop in a clockwise direction. Calculate the direction and magnitude of the force on the loop caused by the current in the infinite wire. **[6]**

8. **EITHER Part A**

(a) In the circuit shown below,



answer the following questions. Note that before the switch is closed the capacitor is uncharged. [1 kΩ = 10^3 Ω; 1 MΩ = 10^6 Ω; 1 μF = 10^{-6} F].

i. What is the current through the resistance R_1 immediately after switch *S* is closed? **[2]**

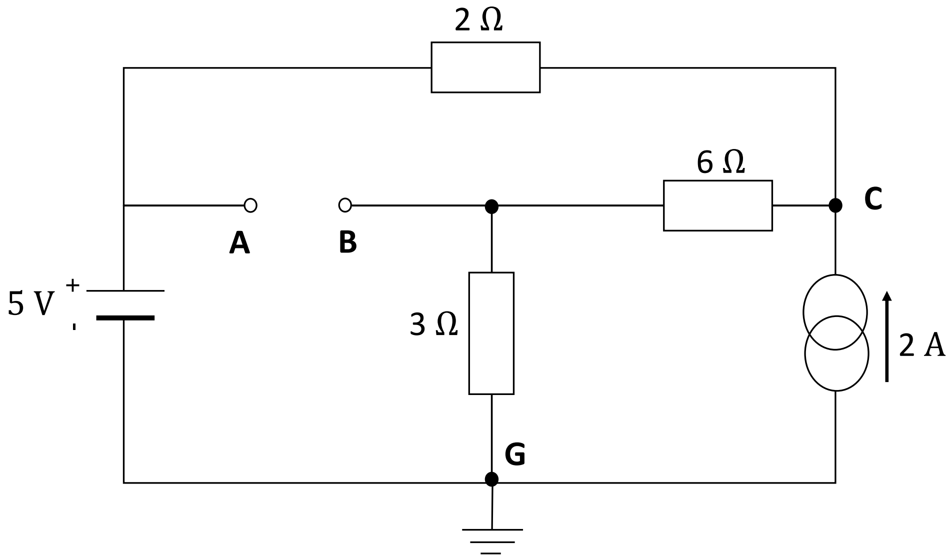
ii. What is the current though the resistance R_1 a long time after the switch *S* is closed? **[2]**

iii. What is the maximum voltage across the capacitor? **[2]**

iv. If the switch has been closed for a long time and is then opened, deduce an expression for the current through the R_2 resistor as a function of time. **[4]**

v. What is the energy dissipated in the R_2 resistor after the switch is opened as in part (iv)? **[2]**

(b) For the circuit shown in the figure below:



i. Using the superposition theorem show that the voltage at the point B is equal to $\left(\frac{21}{11}\right)$ V. **[2]**

ii. Show that the impedance between the points A and B is $\left(\frac{24}{11}\right)\ \Omega$. **[2]**

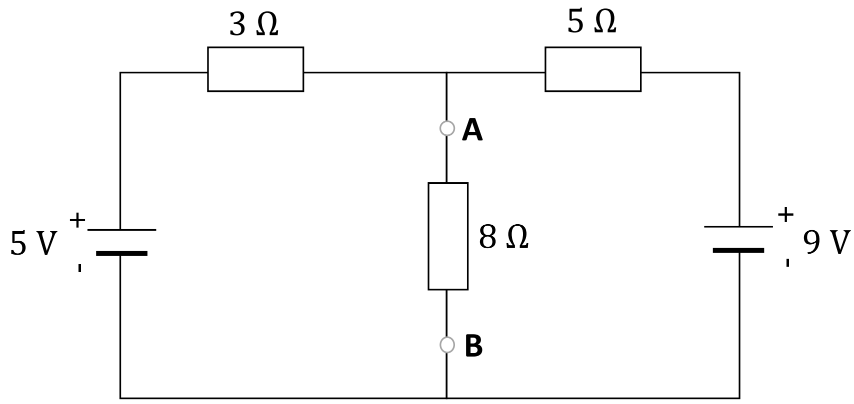
iii. Hence draw the Thevenin's equivalent circuit. **[3]**

iv. Hence find the current through a load resistor, $R_L = 1\ \Omega$, when it is connected between A and B. **[2]**

v. With the load resistor connected, calculate the voltage across the current source (V_{CG}). **[4]**

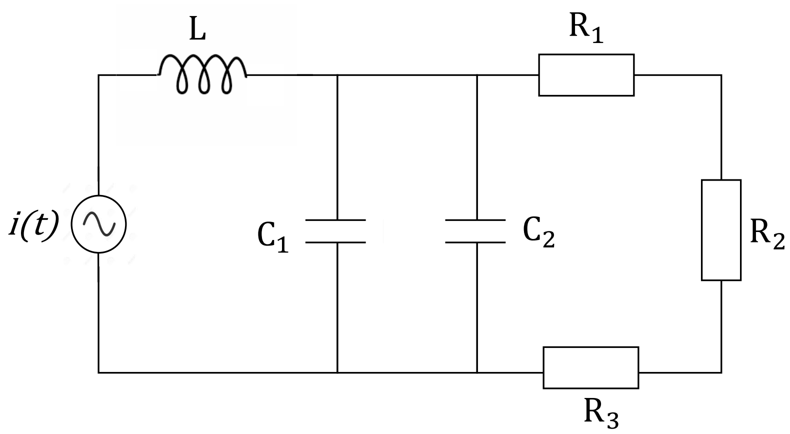
OR Part B

(a) Use Kirchhoff's laws to find the current in the 8 Ω resistor in the circuit shown below. **[3]**



(b) Repeat the above problem by first finding the Thevenin equivalent circuit between points A and B when the 8 Ω resistor is temporarily removed. **[7]**

(c) For the circuit shown below, the current is given by $i(t) = I\sin(\omega t)$.



i. In terms of the component values shown, find an expression for the total complex impedance of the LRC circuit. **[3]**

ii. Hence find an expression for the corresponding voltage $v(t)$. **[4]**

iii. Hence obtain an expression for the phase shift between $i(t)$ and $v(t)$. **[4]**

iv. Calculate the resonant frequency of the circuit. **[4]**

Formula Sheet
LC Electromagnetism 1 / Temperature & Matter / Electric Circuits

Useful Formulae for Electromagnetism 1

Force between two charges
$$\mathbf{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{\mathbf{r}}_{\mathbf{12}}$$

Coulomb's Law
$$\mathbf{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{\mathbf{r}}$$

Coulomb potential
$$V = \frac{Q}{4\pi\epsilon_0 r}$$

Gauss' Law
$$\int_S \mathbf{E} \cdot d\mathbf{S} = \frac{Q_{enc}}{\epsilon_0}$$

Field and potential relation
$$\mathbf{E} = -\nabla V$$

Electric Dipole
$$\mathbf{p} = q\mathbf{a}$$

Torque on Electric Dipole
$$\boldsymbol{\tau} = \mathbf{p} \wedge \mathbf{E}$$

Energy of Electric Dipole
$$U = -\mathbf{p} \cdot \mathbf{E}$$

Capacitance
$$C = \frac{Q}{V}$$

Stored energy in capacitor
$$U = \frac{1}{2}CV^2$$

Energy density of E-field
$$u_E = \frac{1}{2}\epsilon_0 E^2$$

Lorentz Force
$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \wedge \mathbf{B}\right)$$

Force on current length
$$\mathbf{F} = I\boldsymbol{\ell} \wedge \mathbf{B}$$

Magnetic dipole
$$\boldsymbol{\mu} = I\mathbf{A}$$

Torque on magnetic dipole
$$\boldsymbol{\tau} = \boldsymbol{\mu} \wedge \mathbf{B}$$

Energy of magnetic dipole
$$U = -\boldsymbol{\mu} \cdot \mathbf{B}$$

Biot-Savart Law
$$\delta\mathbf{B} = \frac{\mu_0 I}{4\pi}\frac{\delta\mathbf{l} \wedge \hat{\mathbf{r}}}{r^2}$$

Ampere's Law
$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

E.M.F
$$\epsilon = -N\frac{d\Phi_m}{dt}$$

Faraday's Law
$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_m}{dt}$$

Inductance
$$N\Phi_m = LI$$

Stored energy in inductor
$$U_L = \frac{1}{2}LI^2$$

Energy density of B-field
$$u_B = \frac{1}{2}\frac{B^2}{\mu_0}$$

Formula Sheet
LC Electromagnetism 1 / Temperature & Matter / Electric Circuits

Useful Formulae for Temperature & Matter

The first law of thermodynamics

$$\mathrm{d}U = \text{đ}Q - p\,\mathrm{d}V$$
$$\Delta U = Q_{\text{in}} + W_{\text{on}}$$

Ideal gas equation of state

$$pV = n\mathrm{R}T$$

Ideal gas adiabatic process

$$pV^{\gamma} = \text{constant}\,, \ \text{where}\ \gamma = C_p/C_V\ .$$

Heat Transfer

Rate of heat flow by conduction $\quad \dot{Q} = -\kappa A \dfrac{\partial T}{\partial x}$

Stefan-Boltzmann $\quad \dot{Q} = \sigma e A T^4$

Linear coefficient of thermal expansion, α

$$\ell(T) = \ell(T_0)[1 + \alpha(T - T_0)]\ .$$

Gamma function and Stirling's approximation

$$\Gamma(N+1) = N! = \int_0^{\infty} \mathrm{d}x\, x^N\,\mathrm{e}^{-x} \quad \text{and} \quad N! \approx \left(\frac{N}{\mathrm{e}}\right)^N \Leftrightarrow \ \ln N! \approx N(\ln N - 1)\,.$$

Gaussian integral

$$\int_{-\infty}^{\infty} \mathrm{e}^{-\alpha x^2}\mathrm{d}x = \sqrt{\frac{\pi}{\alpha}}\ .$$

Normalised one-dimensional Maxwell-Boltzmann distribution

$$p_{1\mathrm{d}}(v) = \sqrt{\frac{m}{2\pi \mathrm{k_B} T}}\exp\left(-\frac{mv^2}{2\mathrm{k_B}T}\right)\ .$$

Physical Constants and Units

Acceleration due to gravity	g	9.81 m s^{-2}
Gravitational constant	G	6.674×10^{-11} N m^2 kg^{-2}
Ice point	T_{ice}	273.15 K
Avogadro constant	N_A	6.022×10^{23} mol^{-1}
		[*N.B.* 1 mole $\equiv 1$ *gram-molecule*]
Gas constant	R	8.314 J K^{-1} mol^{-1}
Boltzmann constant	k, k_B	1.381×10^{-23} J K^{-1} $\equiv 8.62 \times 10^{-5}$ eV K^{-1}
Stefan constant	σ	5.670×10^{-8} W m^{-2} K^{-4}
Rydberg constant	R_∞	1.097×10^{7} m^{-1}
	$R_\infty hc$	13.606 eV
Planck constant	h	6.626×10^{-34} J s $\equiv 4.136 \times 10^{-15}$ eV s
$h/2\pi$	$\hbar$	1.055×10^{-34} J s $\equiv 6.582 \times 10^{-16}$ eV s
Speed of light *in vacuo*	c	2.998×10^{8} m s^{-1}
	$\hbar c$	197.3 MeV fm
Charge of proton	e	1.602×10^{-19} C
Mass of electron	m_e	9.109×10^{-31} kg
Rest energy of electron		0.511 MeV
Mass of proton	m_p	1.673×10^{-27} kg
Rest energy of proton		938.3 MeV
One atomic mass unit	u	1.66×10^{-27} kg
Atomic mass unit energy equivalent		931.5 MeV
Electric constant	ϵ_0	8.854×10^{-12} F m^{-1}
Magnetic constant	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Bohr magneton	μ_B	9.274×10^{-24} A m^2 (J T^{-1})
Nuclear magneton	μ_N	5.051×10^{-27} A m^2 (J T^{-1})
Fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	7.297×10^{-3}= 1/137.0
Compton wavelength of electron	$\lambda_c = h/m_e c$	2.426×10^{-12} m
Bohr radius	a_0	5.2918×10^{-11} m
angstrom	Å	10^{-10} m
barn	b	10^{-28} m^2
torr (mm Hg at 0 °C)	torr	133.32 Pa (N m^{-2})

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Do not complete the attendance slip, fill in the front of the answer book or turn over the question paper until you are told to do so.

Important Reminders

- Coats/outwear should be placed in the designated area.
- Unauthorised materials (e.g. notes or Tippex) must be placed in the designated area.
- Check that you do not have any unauthorised materials with you (e.g. in your pockets, pencil case).
- Mobile phones and smart watches **must** be switched off and placed in the designated area or under your desk. They must not be left on your person or in your pockets.
- You are not permitted to use a mobile phone as a clock. If you have difficulty seeing a clock, please alert an Invigilator.
- You are not permitted to have writing on your hand, arm or other body part.
- Check that you do not have writing on your hand, arm or other body part – if you do, you must inform an Invigilator immediately
- Alert an Invigilator immediately if you find any unauthorised item upon you during the examination.

Any students found with non-permitted items upon their person during the examination, or who fail to comply with Examination rules may be subject to Student Conduct procedures.