Electromagnetism I – Problem sheet 5 – Solutions

Problem 1.

(1) Each drop has capacitance: $C = 4\pi\epsilon_0 a$
As the spheres (rain drops) are far enough apart as to not affect each other, the total capacitance is just N times the capacitance of each i.e.

$$C_{total} = NC = 4\pi\epsilon_0 Na$$

[1 mark]

(2) If the total charge is Q, the charge on each raindrop is Q/N. Therefore the potential on each raindrop is:

$$V = \frac{Q/N}{4\pi\epsilon_0 a} = \frac{Q}{4\pi\epsilon_0 Na}$$

[1 mark]

(3) If the rain drops combine to form one big drop of radius R, say, the volume remains the same and hence:

$$\text{volume} = \frac{4\pi}{3}R^3 = N\frac{4\pi}{3}a^3 \implies R = aN^{1/3}$$

The capacitance of the single drop is hence:

$$C = 4\pi\epsilon_0 aN^{1/3}$$

[1 mark]

Note this is a factor $N^{2/3}$ smaller than the capacitance of the system of N rain drops.

Problem 2.

(1) Assume capacitance of body is roughly that of a conducting sphere with radius $r = 0.5m$ (Note: any value of r between $0.3m$ and $1.5m$ is ok)
So

$$C = 4\pi\epsilon_0 r = 4\pi \times 8.85 \times 10^{-12} \times 0.5 \approx 5.6 \times 10^{-11}F$$

[1 mark]

(Note value may vary between $3.3 \times 10^{-11}F$ and $1.7 \times 10^{-10}F$)

(2) Energy stored in a capacitor is:

$$U = \frac{1}{2}CV^2 = 0.5 \times 5.6 \times 10^{-11} \times \left(2 \times 10^3\right)^2 \approx 1.1 \times 10^{-4}J$$

[1 mark]

(Note value may vary between $6.5 \times 10^{-5}F$ and $3.3 \times 10^{-4}F$)

Problem 3.

(1) The energy stored in a capacitor is $\frac{1}{2}CV^2$ and the capacitance of a sphere is $C = 4\pi\epsilon_0 r$ Hence the initial and final energies are:

$$\text{initial } U_i = 0.5\,(4\pi\epsilon_0 R)\,\phi^2 = 2\pi\epsilon_0 R\phi^2$$

[1 mark]

$$\text{final } U_f = 0.5\,(4\pi\epsilon_0 \times 0)\,\phi^2 = 0$$

[1 mark]

(2) Relationship between charge and capacitance is $Q = CV$ Hence final charge on shell $(r = 0)$ is:

$$Q = CV = 0$$

As $C = 0$

[1 mark]

(3) The initial charge was Q, say, and the final charge was zero. Hence this charge has been transferred back to the battery through a potential difference of $-\phi$. So the work done by the battery is:

$$W_{by} = Q\,(-\phi) = -Q\phi = -\,(C_{iniital}\phi)\,\phi = -4\pi\epsilon_0 R\phi^2$$

The work done by the battery is negative which means work is actually done on the battery. I.e. the work done on the battery is:

$$W_{on} = 4\pi\epsilon_0 R\phi^2$$

[1 mark]

(Note, 0.5 marks deducted if sign is wrong).

(4) By conservation of energy, the work done by you is:

$$U_i + W_{you} + W_{by-battery} = U_f$$

$$W_{you} = U_f - U_i - W_{by-battery} = 0 - 2\pi\epsilon_0 R\phi^2 - \left(-4\pi\epsilon_0 R\phi^2\right) = 2\pi\epsilon_0 R\phi^2$$

[1 mark]

This work is positive i.e. work is done by you and you help to charge the battery.