L8

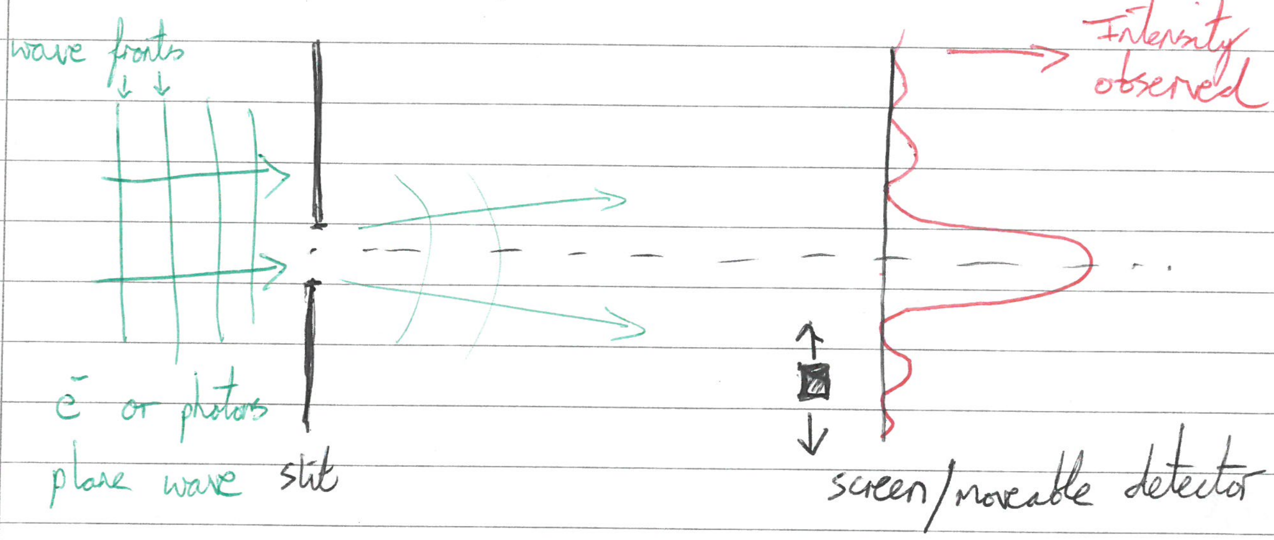
Wave - Particle Duality

We have seen particles do wave stuff.
And waves do particle stuff.

Seems to depend how we look?
- Wavey when in motion - interference etc
- Particulate when detected

° Single-slit (Fraunhofer) diffraction

(Can see both!)



Do this with either electrons or photons - both give interference diffraction pattern ⇒ superposition of waves.

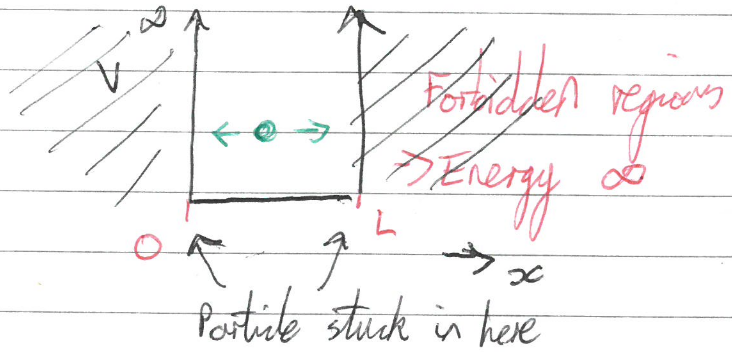
... But if we turn the source waaay down...
we will detect individual particles on our detector

↘ They arrive one-by-one but still build up that diffraction pattern!

- View intensity profile as probability distribution of any one particle impacting the
  - (we'll talk more on this next lecture)

- Particle in infinite potential well
  - → How does wave-like behaviour impact this?

Infinite potential well - particle in a box

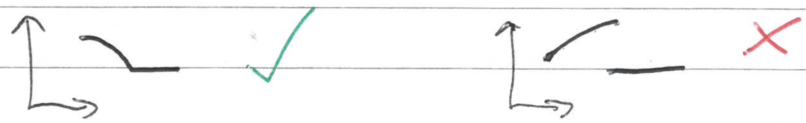


Classically - a squash court.

Quantum mechanically:

Rules: • Particle has finite energy
→ only has wave amplitude (exists) in $0 < x < L$
→ Amplitude must be 0 outside

- Wave function of particle must be continuous, no jumps

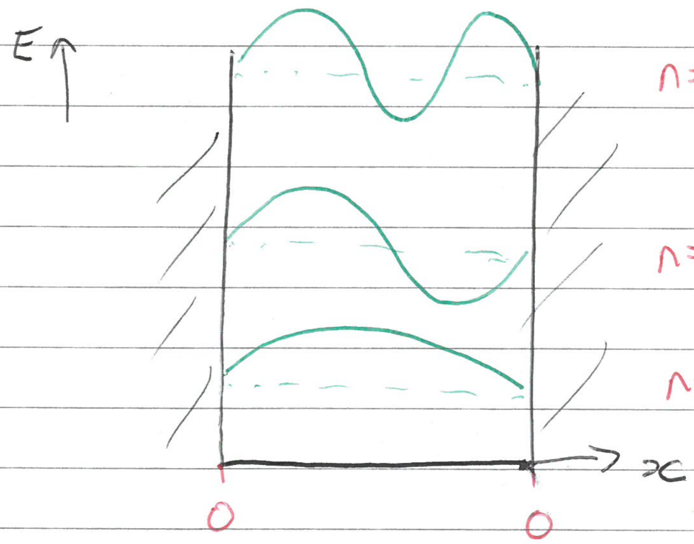


→ Wavefunction $\Psi(x) = 0$ at the boundary, $x=0$ and L

So we must have

$$\Psi(x) = A \sin \underbrace{\frac{n\pi x}{L}}_{= 2\pi \frac{x}{\lambda}} \qquad n = 1, 2, 3 \ldots$$

So $\lambda_n = \dfrac{2L}{n}$



$n=3, \quad \lambda_3 = \dfrac{3L}{2}$

$n=2, \quad \lambda_2 = L$

$n=1, \quad \lambda_1 = 2L$

(Note I drew bigger n as higher energy...)

Use de Broglie wavelength:

$$p = \frac{h}{\lambda} \rightarrow p_n = \frac{nh}{2L}$$

and $$E = \frac{p^2}{2M}$$

so... $$E = n^2 \frac{h^2}{8ML^2}$$

n.b. $E \propto n^2$ in ∞ 1D potential well.
Was $-\frac{1}{n^2}$ in the H-atom

- Energy is uniquely determined for each quantum state $\psi_n(x)$ in the well
- What about momentum?

Classically the electron is either moving ($\rightarrow$) or ($\leftarrow$). Magnitude is known, p, but not direction

For $n=1$, $$p_x = \pm \frac{h}{2L}$$

$$\Delta p_x = \frac{h}{2L}$$ uncertainty

$\langle p_x \rangle = 0$ Expectation / 'average'

And for x.. somewhere in the well, in the middle on average?

$$\Delta x = \frac{L}{2}$$

$$\langle x \rangle = \frac{L}{2}$$

What happens if we shrink the well?
Both p $(\propto \frac{1}{L})$ and E $(\propto \frac{1}{L^2})$ go up!
(This is <u>weird</u> in classical mechanics...)

So as we know better where the particle is, we know less about its momentum

(squash ball rattling back and forth really fast)

This is not a proof of, but a hint towards the...

- Heisenberg Uncertainty Principle

$$\boxed{\Delta x \, \Delta p_x \geqslant \frac{h}{4\pi}} \quad \left(\text{or } \frac{\hbar}{2}\right)$$

In QM there is a limit to the precision we can know 2 conjugate observables simultaneously.

(Fun note - can also be $\Delta E \Delta t$)

- But e.g. $\Delta p_x \; \Delta y$ has no restriction, these are independent.

In our example, $\Delta x \Delta p = \frac{L}{2} \frac{h}{2L} = \frac{h}{4} > \frac{h}{4\pi}$

- The single slit diffraction is a good example of this:

  If we measure x of a photon by passing through a slit, we lose p_x information and it 'spreads out' along x...

|| Uncertainty Principle

Narrower slit $\Rightarrow$ wider momentum spread $\Rightarrow$ wider diffraction peak.

- Conclusions:

  - Wave-particle duality. Diffraction but also quantised particles

- Wave-like particles mean quantisation of energy in a bound system
- $E \propto n^2$ in 1D ∞ well, $\frac{1}{n^2}$ in H atom
  - energy levels depend on the shape of the potential
- Heisenberg Uncertainty Principle