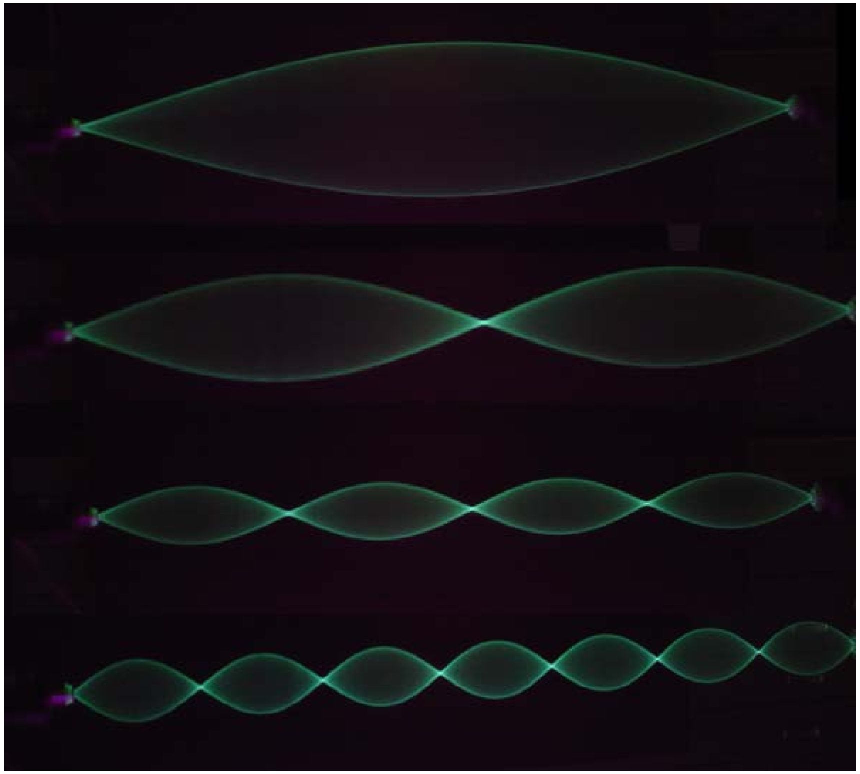


Wavelength, and thus frequency, is quantized! Energy is quantized.

Chladni Plates[**normal modes**]

Another interesting site if you have ten minutes to spare:
http://www.youtube.com/watch?v=67NPGP5A2EI

Chladni: German Physicist and Musician

Optics and Waves

Lecture 8

-Interference
-Beats
-Phase velocity and group velocity

Young and Freedman 16.6; 16.7;

Superposition

When waves overlap in the same region, the resulting wave is the algebraic sum of all waves- Interference

Interference of harmonic waves

consider two waves

$$y_1 = A\cos(kx - \omega t) \quad y_2 = A\cos(kx - \omega t + \delta) \quad \delta - \text{phase shift}$$

$$y = y_1 + y_2$$

$$y = A\cos\boxed{(kx - \omega t)} + A\cos\boxed{(kx - \omega t + \delta)}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$y = 2A\cos\left(\frac{\delta}{2}\right)\cos\left(kx - \omega t + \frac{\delta}{2}\right)$$

A travelling wave (not standing wave), amplitude depends on δ

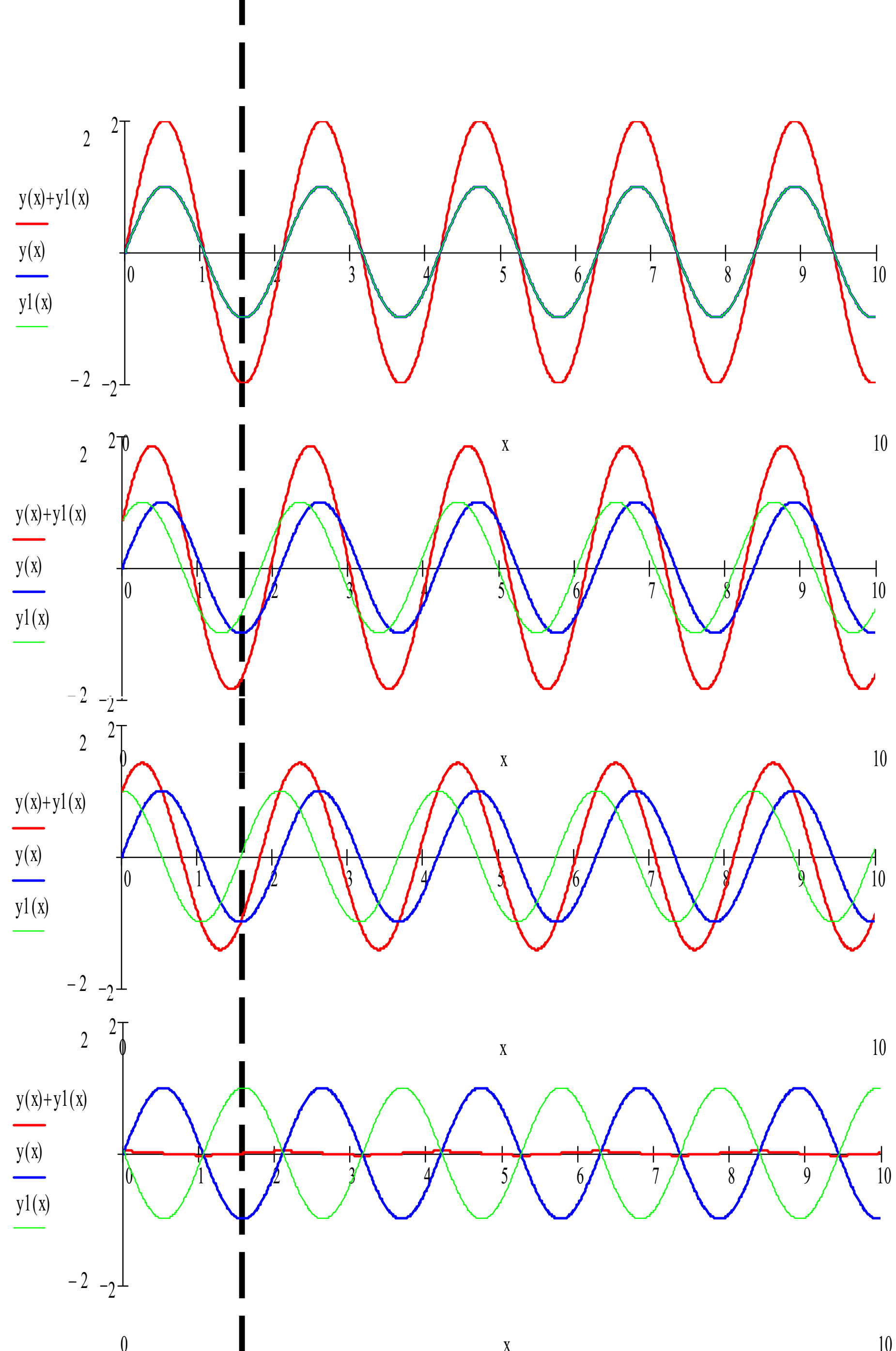
$$y = 2A\cos\left(\frac{\delta}{2}\right)\cos\left(kx - \omega t + \frac{\delta}{2}\right)$$

$$2\cos\left(\frac{\delta}{2}\right) = 2 \qquad \delta = 0$$

$$2\cos\left(\frac{\delta}{2}\right) = 1.85 \qquad \delta = \pi/4$$

$$2\cos\left(\frac{\delta}{2}\right) = 1.41 \qquad \delta = \pi/2$$

$$2\cos\left(\frac{\delta}{2}\right) = 0 \qquad \delta = \pi$$



Phase difference due to path difference

suppose two sound sources S_1 and S_2 oscillate in phase and emit harmonic waves of the same frequency and wavelength.
Now consider at the point P in space for which the path lengths differ.

If there is an integral number of wavelengths in path difference, then interference is constructive, if half num. of wavelengths then it is destructive.

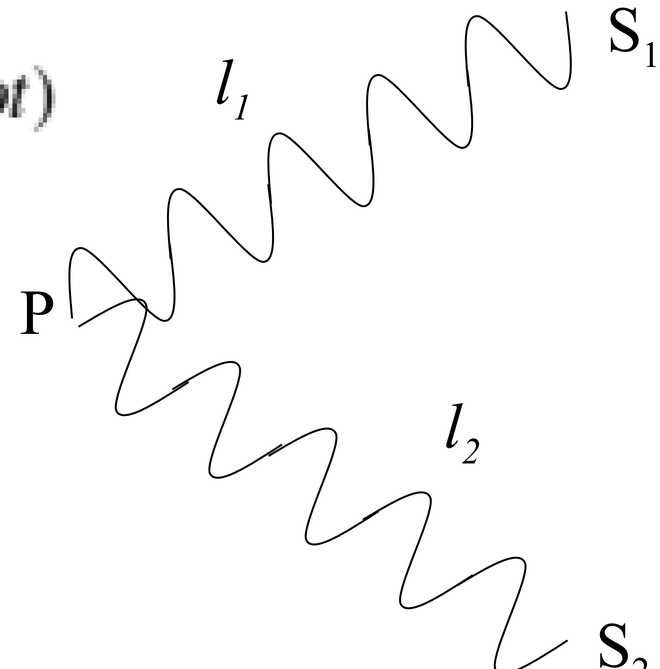
From S_1 $\quad y_1(l_1,t) = A\cos(kl_1 - \omega t)$

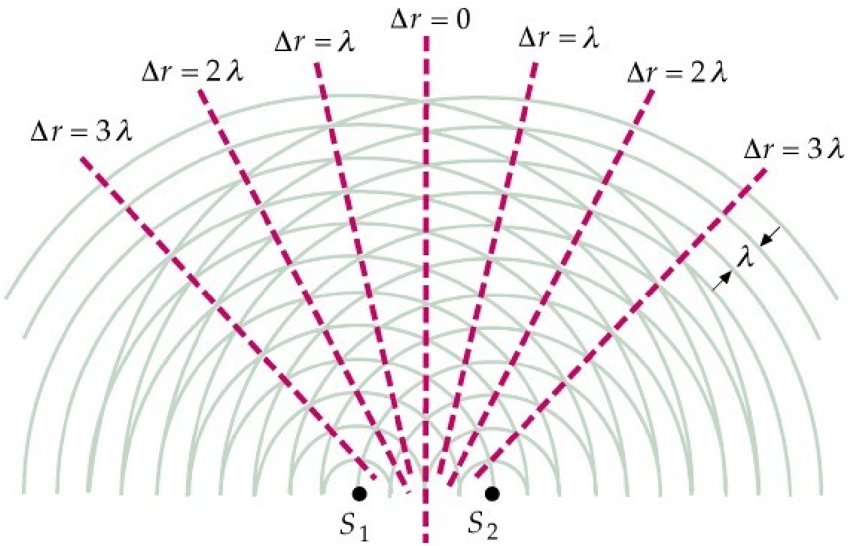
From S_2 $\quad y_2(l_2,t) = A\cos(kl_2 - \omega t)$

Hence the phase difference is

$$\delta = (kl_1 - \omega t) - (kl_2 - \omega t) = k(l_1 - l_2)$$

$$\delta = \frac{2\pi}{\lambda}\Delta l$$



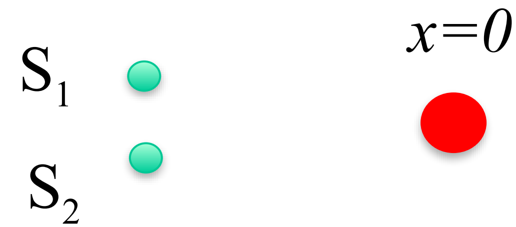


Water wavesinterference

Beats

Interference with sound waves with slightly **different** frequencies

Let's choose a fixed point (say *x*=0) so *kx* is 0



$$p_1 = A\cos(\omega_1 t)$$

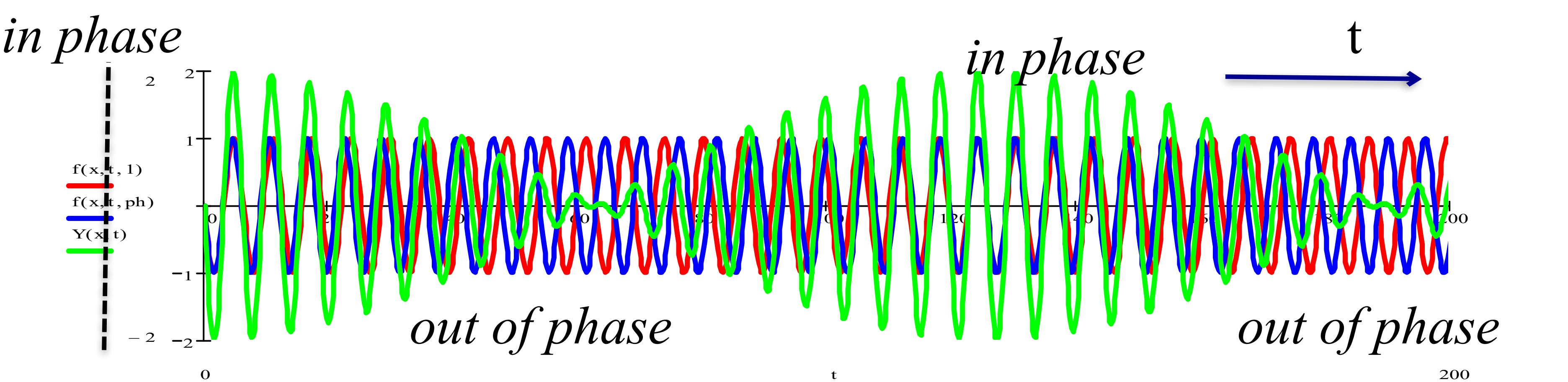
$$p_2 = A\cos(\omega_2 t)$$

$$p = p_1 + p_2 = A(\cos(\omega_1 t) + \cos(\omega_2 t))$$

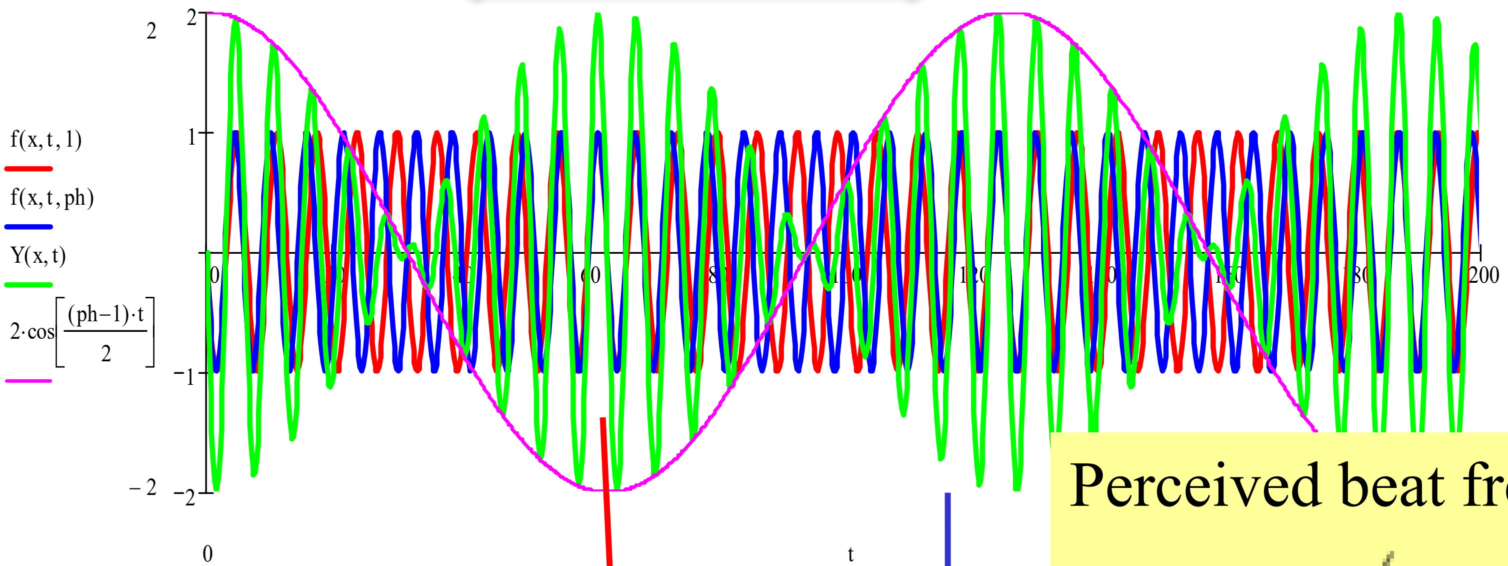
$$p = 2A\cos\left(\frac{\omega_1 - \omega_2}{2} t\right)\cos\left(\frac{\omega_1 + \omega_2}{2} t\right)$$

$$\omega_{av} = \frac{\omega_1 + \omega_2}{2} \qquad \Delta\omega = \frac{\omega_1 - \omega_2}{2}$$

$$p = \boxed{2A\cos(\Delta\omega t)}\cos(\omega_{av} t)$$



Beat period



$$p = \boxed{2A\cos\left(\frac{\omega_1 - \omega_2}{2}t\right)}\boxed{\cos\left(\frac{\omega_1 + \omega_2}{2}t\right)}$$

Perceived beat frequency:

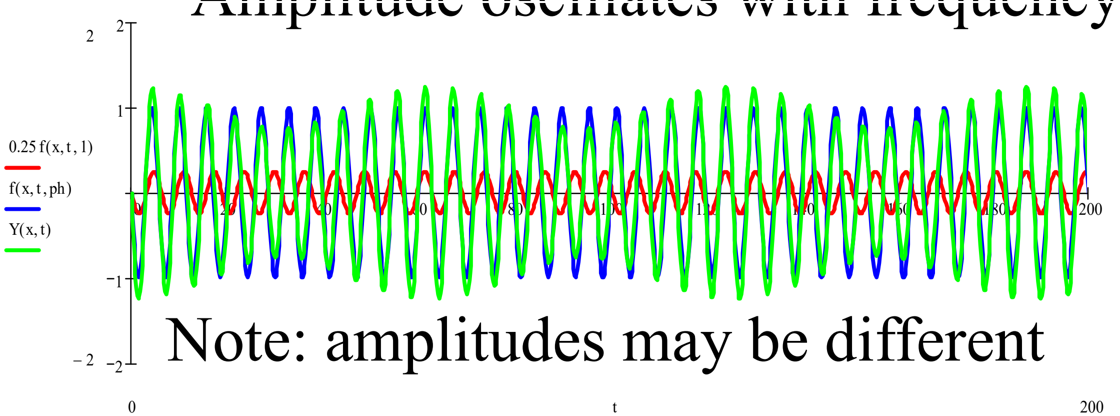
$$f_{beat} = 2\left(\frac{\omega_1 - \omega_2}{4\pi}\right)$$

$$= \frac{\omega_1}{2\pi} - \frac{\omega_2}{2\pi} = |f_1 - f_2|$$

Amplitude oscillates with frequency:

$$\omega = \left(\frac{\omega_1 - \omega_2}{2}\right) = 2\pi f$$

$$f = \left(\frac{\omega_1 - \omega_2}{4\pi}\right)$$



Note: amplitudes may be different

beat beat

Three tuning forks of frequencies 100Hz,101Hz and 102Hz are sounded together. How many beats will be heard in one second? If 2 forks are sounded together then the number of beats heard will be equal to the difference of their frequencies. But I don't know how to calculate the beat frequency if 3 forks are sounded together.

If you sound the 100 together with the 102, you will get a beat frequency of 2 beats/sec on a frequency of 101 Hz. When the 101 Hz is sounded there are no further beats added. So I would say, 2 beats. It is a little more complicated to analyse it by sounding the 100 and 101 first and then adding the 102. But if you pair 100 and 101, you get a 100.5 frequency and with 101 and 102 you get 101.5, each with one beat/sec. These two then beat down to 101 with another beat/sec. The beat frequency ends up 2/sec

Now put the space and time components together!

$$y_1 = A\cos(k_1 x - \omega_1 t) \qquad y_2 = A\cos(k_2 x - \omega_2 t)$$

$$y = y_1 + y_2$$

Remember:

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha-\beta}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$

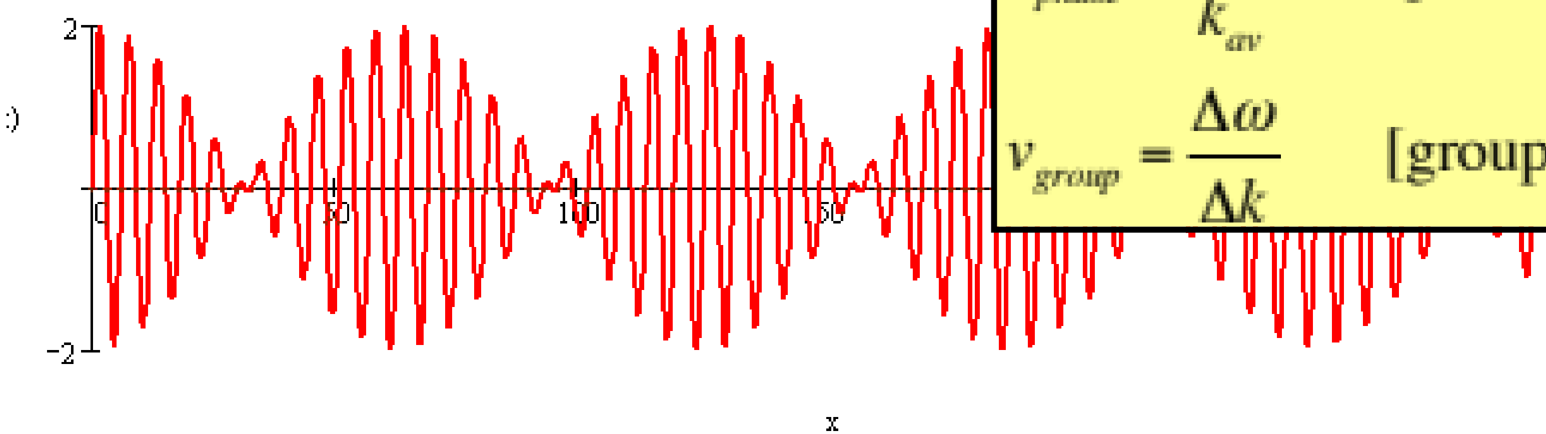
$$y = A\cos(\underbrace{k_1 x - \omega_1 t}_{\boxed{\alpha}}) + A\cos(\underbrace{k_2 x - \omega_2 t}_{\boxed{\beta}})$$

$$y = 2A\cos\left(\frac{k_1 - k_2}{2}x - \frac{\omega_1 - \omega_2}{2}t\right)\cos\left(\frac{k_1 + k_2}{2}x - \frac{\omega_1 + \omega_2}{2}t\right)$$

$$y = 2A\underbrace{\cos\left(\Delta k.x - \Delta\omega.t\right)}_{group}\underbrace{\cos\left(k_{av}x - \omega_{av}t\right)}_{carrier}$$

$$v_{phase} = \frac{\omega_{av}}{k_{av}} \qquad \text{[phase velocity]}$$

$$v_{group} = \frac{\Delta\omega}{\Delta k} \qquad \text{[group velocity]}$$



http://en.wikipedia.org/wiki/Phase_velocity

$$y = 2A\underbrace{\cos\left(\Delta k.x - \Delta\omega.t\right)}_{group}\underbrace{\cos\left(k_{av}.x - \omega_{av}t\right)}_{carrier}$$

Radio signal https://en.wikipedia.org/wiki/AM_broadcasting

$$v_{phase} = \frac{\omega_{av}}{k_{av}} \qquad \text{[phase velocity]}$$

$$v_{group} = \frac{\Delta\omega}{\Delta k} \qquad \text{[group velocity]}$$

Which of the two velocities is higher?

If wave speed is independent of frequency $v = \frac{\omega_1}{k_1}, \quad v = \frac{\omega_2}{k_2}$

$$v_{phase} = \frac{\omega_1 + \omega_2}{k_1 + k_2} = \frac{v(k_1 + k_2)}{k_1 + k_2} = v$$

$$v_{Group} = \frac{\omega_1 - \omega_2}{k_1 - k_2} = \frac{v(k_1 - k_2)}{k_1 - k_2} = v$$

In general, wave speed depends on frequency $v_{phase} \neq v_{Group}$

V_{phase} can be greater or smaller than v_{Group}

If wave speed depends on frequency, the medium is called dispersive.

Does sound speed in air depends on frequency?

Does speed of light in glass depends on frequency?

http://www.youtube.com/watch?v=tlM9vq-bepA

If you are really into the physics of group and phase velocities: visit:

http://resource.isvr.soton.ac.uk/spcg/tutorial/tutorial/Tutorial_files/Web-further-dispersive.htm

Coherence

Two sources that are in phase or have a constant phase difference, are said to be *coherent*.

Addition of coherent waves

(constant phase difference – hence can add amplitudes)

Intensity: $I = |A|^2 = (A_1 + A_2 + A_3 + ...)^2$

Addition of incoherent waves

(phase changes randomly – hence add intensities)

Intensity: $I = I_1 + I_2 + I_3 + ... = A_1{}^2 + A_2{}^2 + A_3{}^2 + ...$

Intensity of sound waves, decibel scale

Intensity and Loudness, the Decibel scale

A logarithmic scale has been adopted to measure intensity levels.

$$\beta = 10\log\left(\frac{I}{I_0}\right)$$

I is the intensity of the source

I_0 is a reference level defined as 10^{-12} W/m^2 (approx. threshold of hearing)

At threshold of hearing

$$\beta = 10\log\left(\frac{10^{-12}}{10^{-12}}\right) = 10\log(1) = 0\,dB$$

0 dB does not mean zero sound intensity!

Pain threshold: I= 1 Wm^{-2} (or 10 Wm^{-2})

$$\beta = 10\log\left(\frac{1}{10^{-12}}\right) = 10\log(10^{12}) = 120\,dB$$

A decibel is 1/10 of a *bel*. (Alexander Graham Bell)

How to calculate the intensity I?

$$I = \frac{power}{Area} = \frac{energy}{time \times Area} = \frac{energy \times length}{time \times volume}$$

$$= \frac{energy}{volume} \times \frac{length}{time} = \frac{energy}{volume} \times wave\ speed$$

How much is the displacement on the eardrum?

10^{-5} m for 120 dB, and 10^{-11} m for 0 dB.
10^{-11} m is smaller than an atom!

Threshold of pain
Plane takeoff
130
110
Underground Train
90
Factory Noise
70
60
Classroom Noise
50
40
Library
Whisper
30
20
Rustling Leaves
10
Breathing
dB
0
Threshold of Hearing

Doppler effect (for sound)

If a wave source and receiver are moving relative to each other, the frequency detected by the receiver is different from that emitted by the source.

Two scenarios (1) moving source (2) moving observer

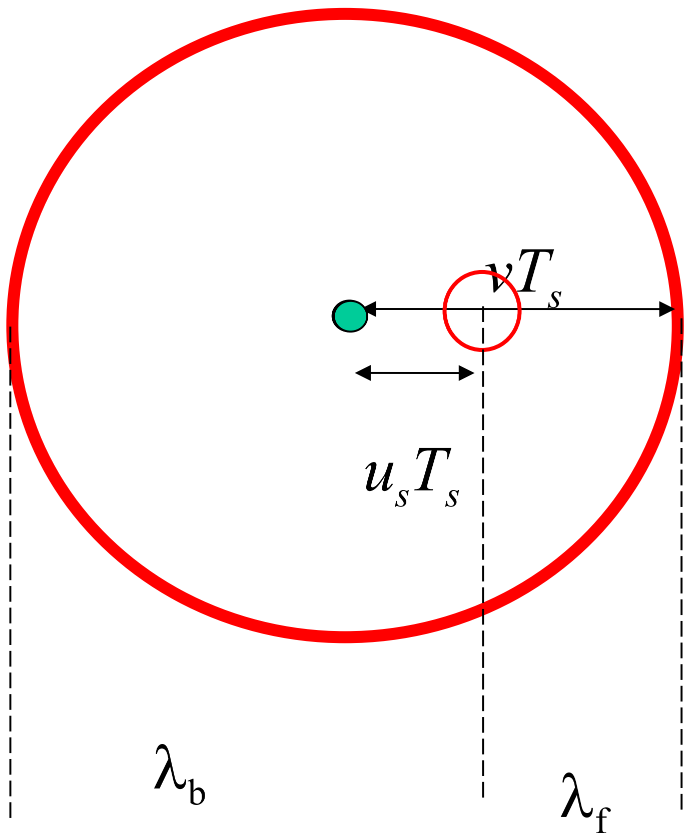
Source moving Doppler

1) moving source (receiver at rest)

Source has frequency: f_s (period $T_s=1/f_s$)

speed of source: u_s

velocity of waves in medium: v



Forwards

$$\lambda_f = \underbrace{\frac{v}{f_s}}_{\lambda} - \frac{u_s}{f_s} = \frac{v-u_s}{f_s}$$

observed frequency

$$f_f = \frac{v}{\lambda_f} = \frac{v}{v-u_s} f_s$$

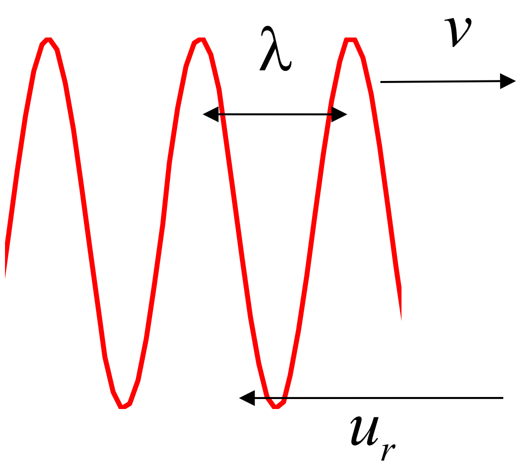
Backwards

$$\lambda_b = \underbrace{\frac{v}{f_s}}_{\lambda} + \frac{u_s}{f_s} = \frac{v+u_s}{f_s}$$

observed frequency

$$f_b = \frac{v}{\lambda_b} = \frac{v}{v+u_s} f_s$$

Case 2) receiver moving:
If moving towards source of the wave, encounters more wave crests per second and thus higher frequency.
Speed of receiver: u_r. To the receiver, wave with wavelength λ appears to travel at $v+u_r$



$$v + u_r = \lambda f_r$$

$$f_r = \frac{v+u_r}{\lambda} = \frac{v+u_r}{\dfrac{v}{f_s}} = \frac{v+u_r}{v} f_s$$

If receiver moves away from source $$f_r = \frac{v-u_r}{v} f_s$$

Source moving $f_r = \dfrac{v}{v \pm u_s} f_s$ (− towards, + away)

Receiver moving $f_r = \dfrac{v \pm u_r}{v} f_s$ (+ towards, − away)

Both moving $f_r = \dfrac{v \pm u_r}{v \pm u_s} f_s$

Moving towards, f increase; away, f decrease.

Doppler for light (relativity):
Doppler shift in freq. depends on if source or receiver is moving relative to the medium.
For light, for which there is no medium (propagates in vac), absolute motion cannot be detected; only relative motion of source and receiver can be determined.

$$f_r = f_s\sqrt{\frac{c \pm u}{c \mp u}} = f_s\left(1 \pm \frac{u}{c}\right)^{1/2}\left(1 \mp \frac{u}{c}\right)^{-1/2}$$

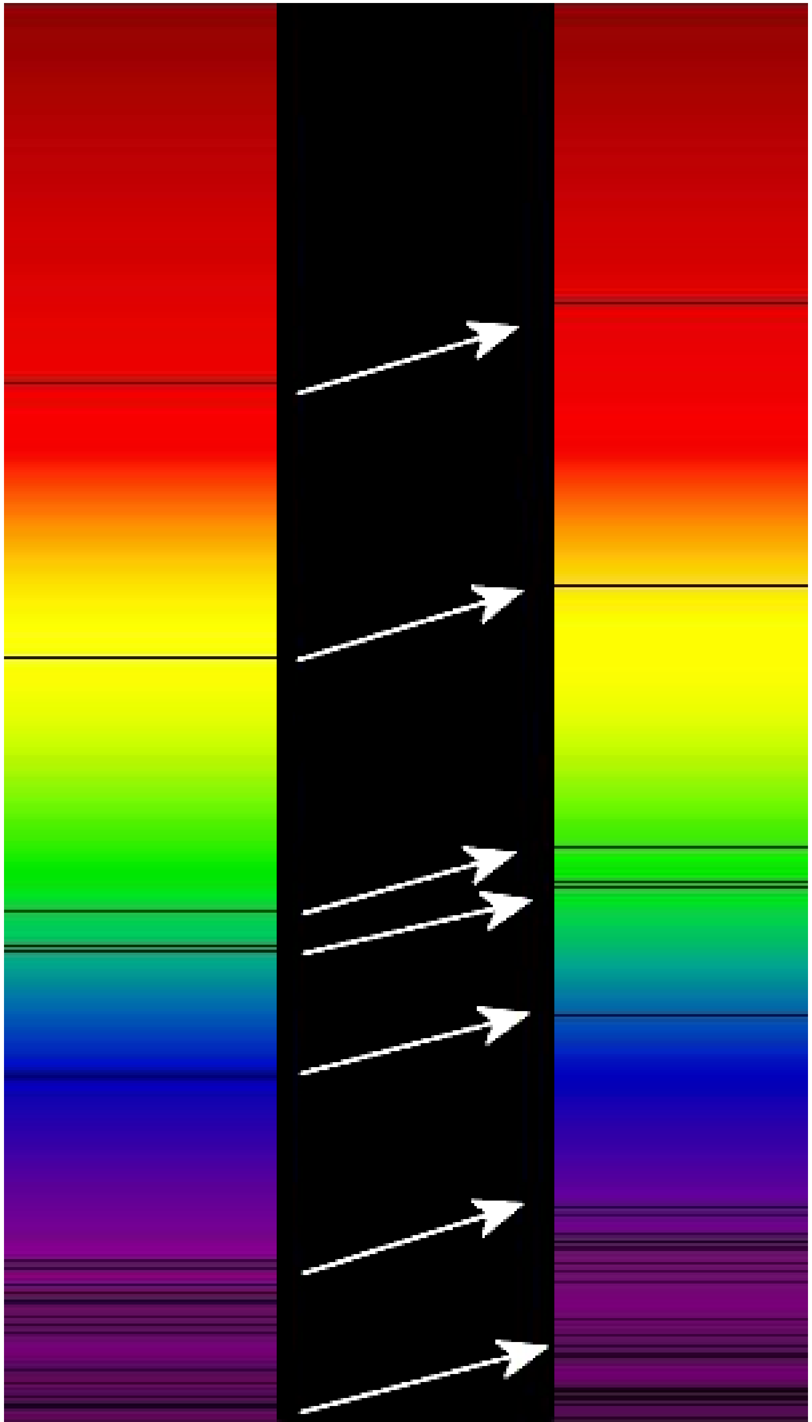
$$= f_s\left(1 \pm \frac{1}{2}\frac{u}{c}\right)\left(1 \pm \frac{1}{2}\frac{u}{c}\right) = f_s\left(1 \pm \frac{u}{c}\right)$$

ignoring terms in $\left(\dfrac{u}{c}\right)^2$

i.e.

$$\frac{\Delta f}{f_s} = \pm\frac{u}{c}$$

Austrian physicist Christian Doppler, who proposed it in 1842 in Prague.



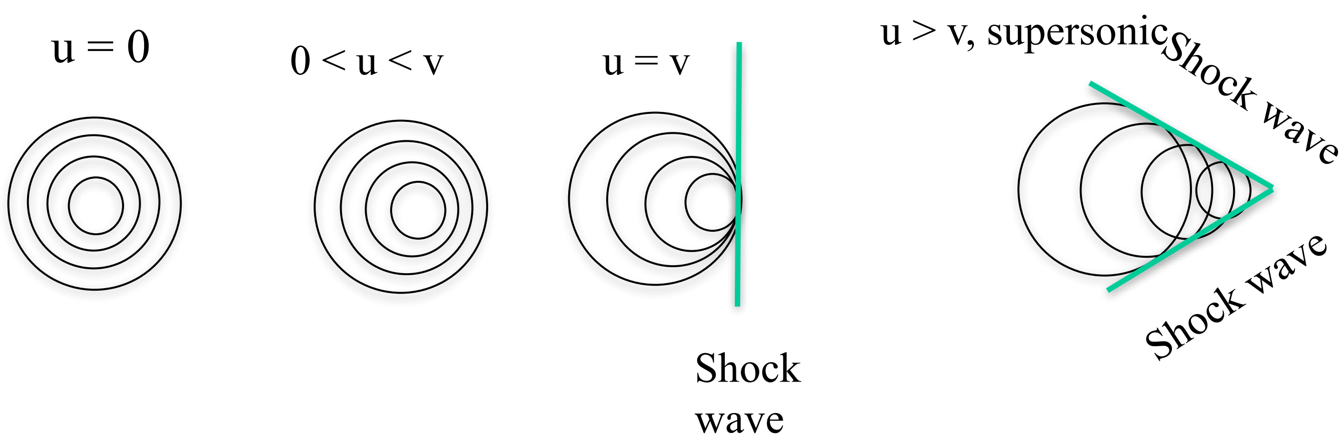
http://en.wikipedia.org/wiki/File:Redshift.png

Shock waves:

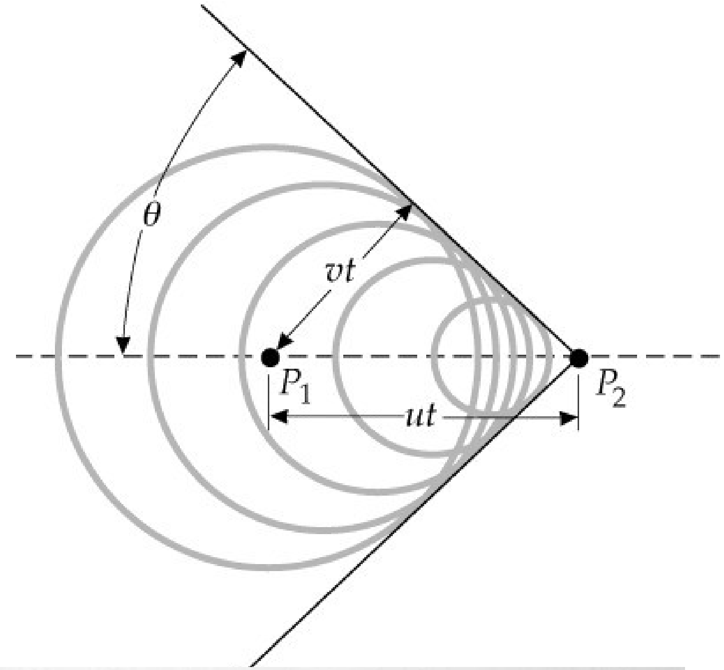
is what happens when the velocity of the source is faster than the velocity of waves in the medium

Speed boat phenomenon

v: Velocity of sound
u: velocity of source



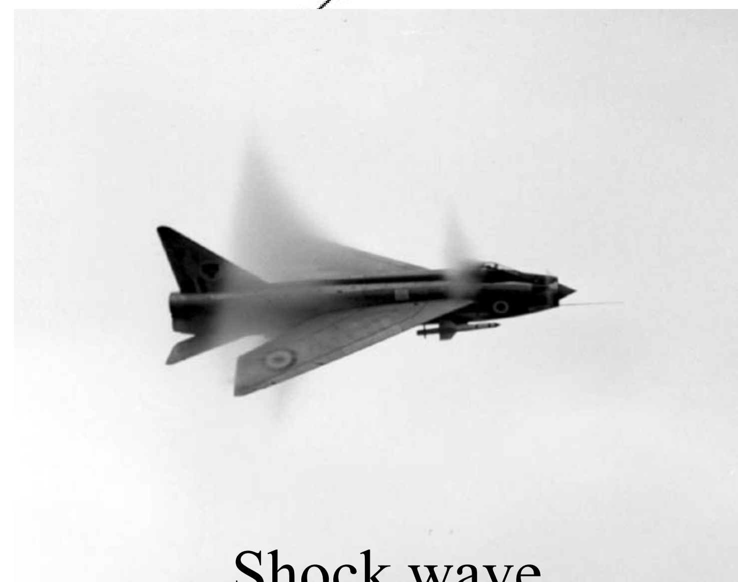
$$\sin\theta = \frac{vt}{ut} = \frac{v}{u}$$



Mach cone angle: θ

Mach number=u/v

Along the path of the plane, the sound waves produced later are heard first.



Shock wave

Sonic boom[sonic boom]