5 Probability revision

The problems are roughly in order of difficulty.

Problem 2.1 **More complicated outcomes**
Find the probability that a single throw of a die will give

1. a number less than 3,
2. an even number.

Problem 2.2 **Many balls**
In a box there are 2 white, 3 black and 4 red balls. If a ball is picked at random, what is the probability that it is (i) black, (ii) not red ?

Problem 2.3 **Family probabilities**
Given a family of 2 children and an equal probability of a given child being a boy or a girl,

1. what is the probability that at least one is a girl?
2. Given that at least 1 is a girl, what is the probability that both are girls?
3. given that they are both girls, what is the probability that an expected third child will be a boy?

Problem 2.4 **An irritating queue**
10 people stand in a queue. They are randomly chosen, one after the other, to come forward and leave the queue. What is the probability of a particular person being chosen on the third call?

Problem 2.5 **Monkeys, typewriters and Shakespeare**
It is often said that a million monkeys typing randomly on typewriters for millions of years will eventually manage to write all the works of Shakespeare. The following problem addresses the question of just how true this assertion is. – Consider a typical typewriter with 44 keys (including the space bar) and assume the monkeys only type in lower case. Consider 10^{20} monkeys, ($\sim 2 \times 10^{10} \times$ the number of people on earth) each typing at 10 characters/ sec. Show that the probability of them correctly typing Shakespeare's Hamlet (105 characters) in the age of the universe ($\sim 10^{18}$ sec.) is approximately $10^{-164,306}$.

Problem 2.6 **Penalty shoot-out**
In a penalty shoot-out in a football match, each penalty taker has a probability of 3/4 of scoring. If 5 penalties are taken, what is the probability that:

1. Exactly 2 goals are scored.

2. At least one goal is scored.

Problem 2.7 **Bill and Ben, gambling men**
Bill and Ben agree to settle an argument by throwing a pair of dice; the first to throw a double (i.e. the same number on both dice) wins. Bill throws first, then Ben, then Bill again, and so on until there is a winner. What is the probability that Bill wins?

Problem 2.8 **Combinatorics**
Three atoms each have 3 energy levels, 0, ϵ and 2ϵ : how many ways are there of getting a total energy of 3ϵ?