Resonance

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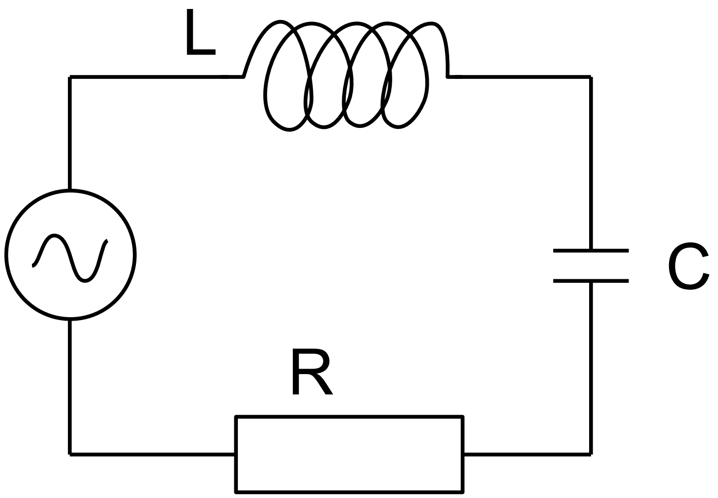
We know this condition is met for resistors.
This implies that at resonance, the impedance, Z, must be purely resistive (real) and that the reactance must cancel.



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$$Z =$$

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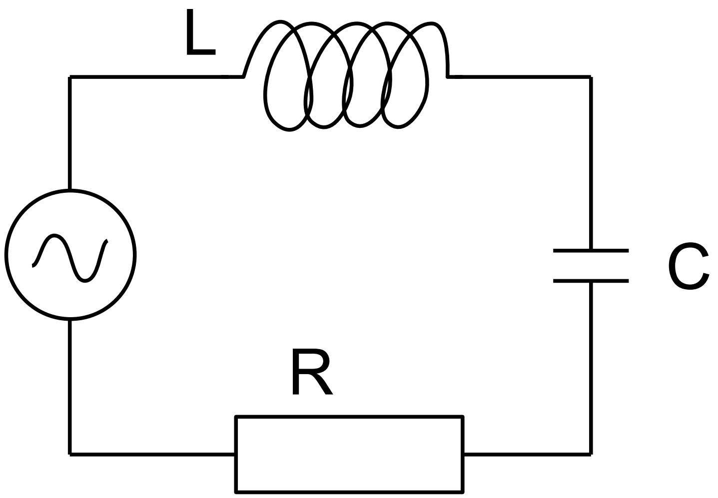

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$$Z = R + j\omega L + \frac{1}{j\omega C}$$

$$\Rightarrow Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

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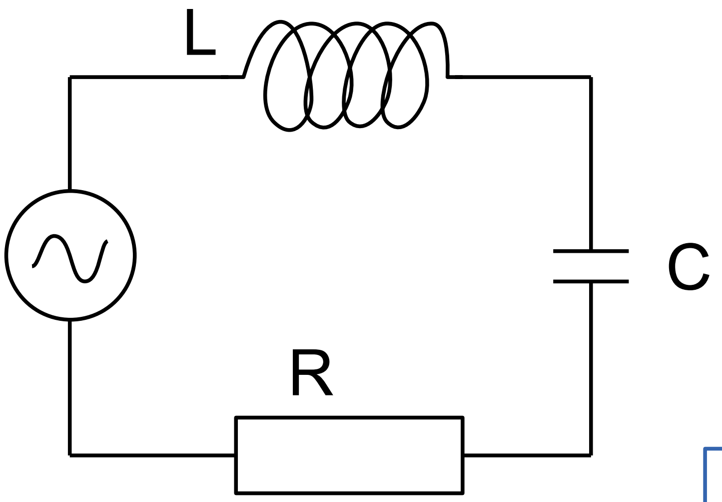

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$$\boxed{\Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}}$$

ω_0 is the resonant frequency.

Resonance 2

1) In the limit of very low frequency ($\omega \to 0$), the reactance of the capacitor dominates and $Z \to \infty$.

2) In the limit of very high frequency ($\omega \to \infty$), the reactance of the inductor dominates and $Z \to \infty$.

3) When the frequency is such that the inductive reactance equals the capacitive reactance and $Z = R$. At this frequency the impedance is at a minimum.

Resonance 3

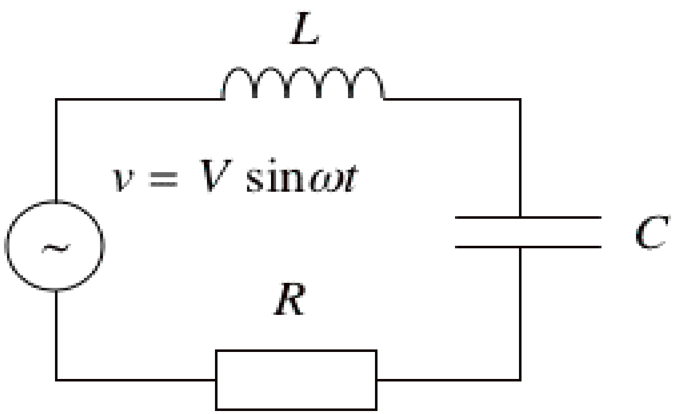
Notice that although the circuit behaves as if it is purely resistive at resonance (current and voltage are in phase) there is still a voltage across the capacitor and the inductor (equal in magnitude, but 180 degrees out of phase with each other). Energy is therefore being transferred back and forth between the capacitor and the inductor. During one half-cycle the capacitor receives energy from the inductor, during the next half-cycle the inductor receives energy from the capacitor. As we shall see, the stored energy is a maximum when the circuit is at resonance.

The ratio of energy stored oscillating between the capacitor and inductor becomes much larger than the energy dissipated in the resistor. This ratio is called the Q-factor (quality factor).

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{I^2 \chi_L}{I^2 R} = \frac{\chi_L}{R} \text{ and, using } \chi_L = 2\pi f_0 L = \omega_0 L,\ Q = \frac{\omega_0 L}{R}$$

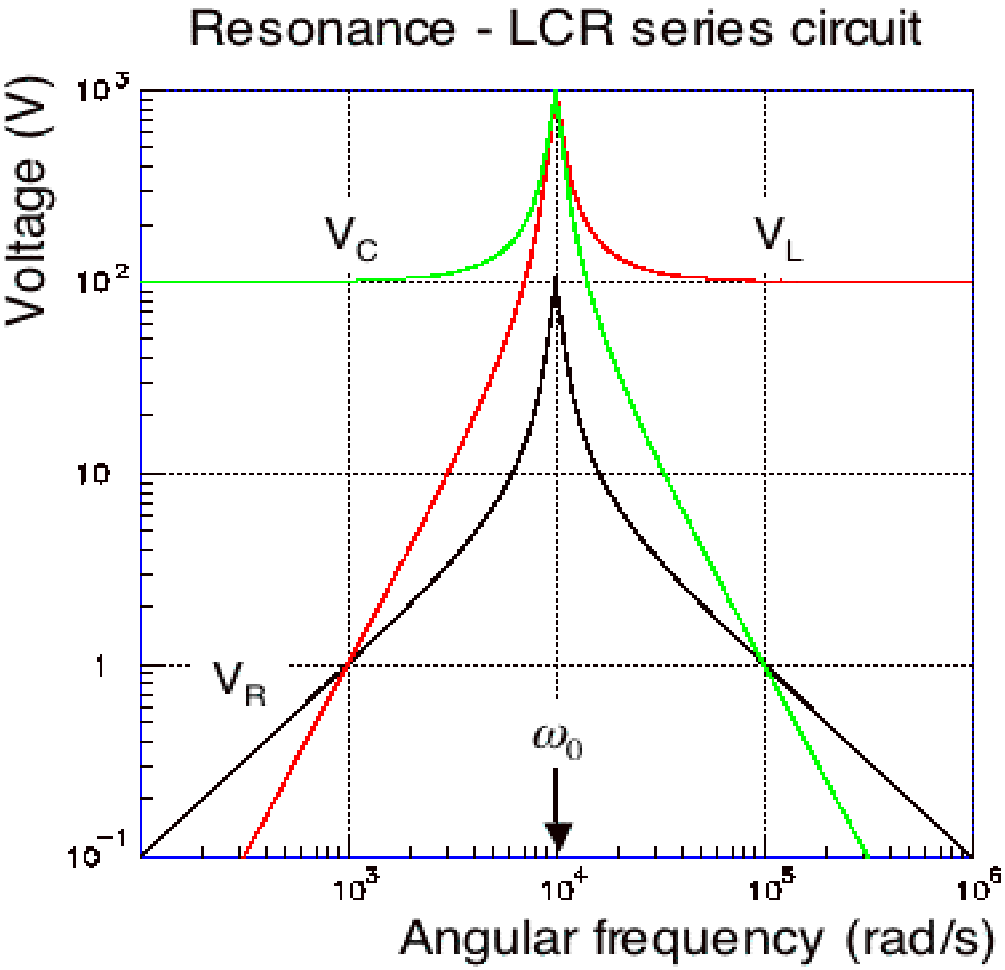
LCR series resonance

- **Frequency dependence**



$V = 100$ V
$L = 0.01$ H, $C = 1$ μF, $R = 10$ Ω
$Q = 10$

At resonance $V_C = V_L$ but they are 180° out-of-phase with each other. They exceed the applied voltage by $Q = 10$.



Try it yourself: LCR parallel resonance

Answer: $\Rightarrow \omega_0 = \dfrac{1}{\sqrt{LC}}$