Recap from last time

"Any substance in equilibrium at a temperature T has an average energy of $\frac{k_B T}{2}$ for each degree of freedom"

A degree of freedom is defined as any term in the expression for the energy of the system which contains a squared position or velocity component…

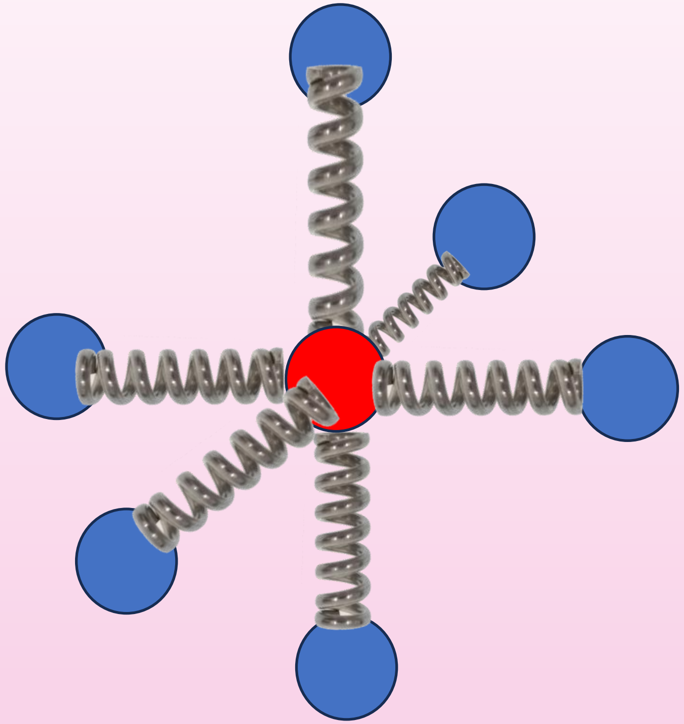
For our ideal gas in 1D, we have $E(v_x) = \frac{1}{2} m v_x^2 = \frac{1}{2} k_B T$ $\quad C_V = \frac{1}{2} R$

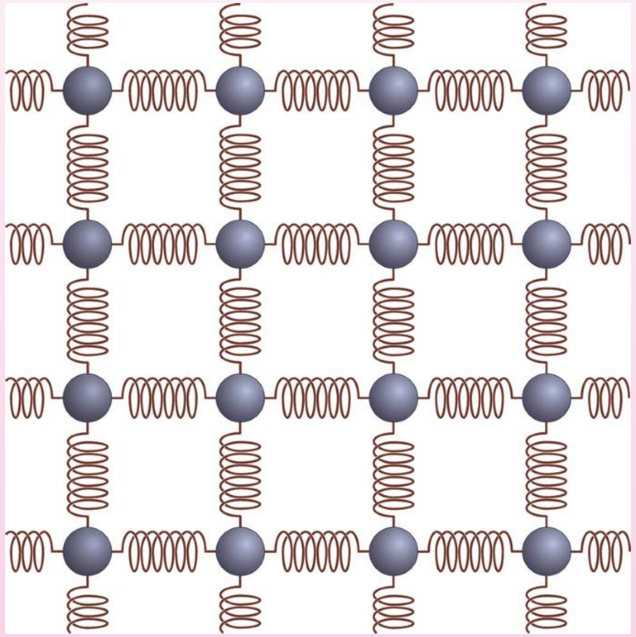
In 3D: $E(v_x, v_y, v_z) = \frac{1}{2} m v_x^2 + \frac{1}{2} m v_y^2 + \frac{1}{2} m v_z^2 = \frac{3}{2} k_B T$ $\quad C_V = \frac{3}{2} R$

No gravity (mgh) term because the position term is linear

Recap from last time

Each atom (mass m) is fixed, connected to its neighbours by interatomic forces (which we can treat like springs with spring constant K)





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Each spring has vibrational energy

$$E = \frac{1}{2}mv_x^2 + \frac{1}{2}Kx^2 = k_B T$$

Each atom connected to 6 springs (but each spring is shared by 2 atoms…)

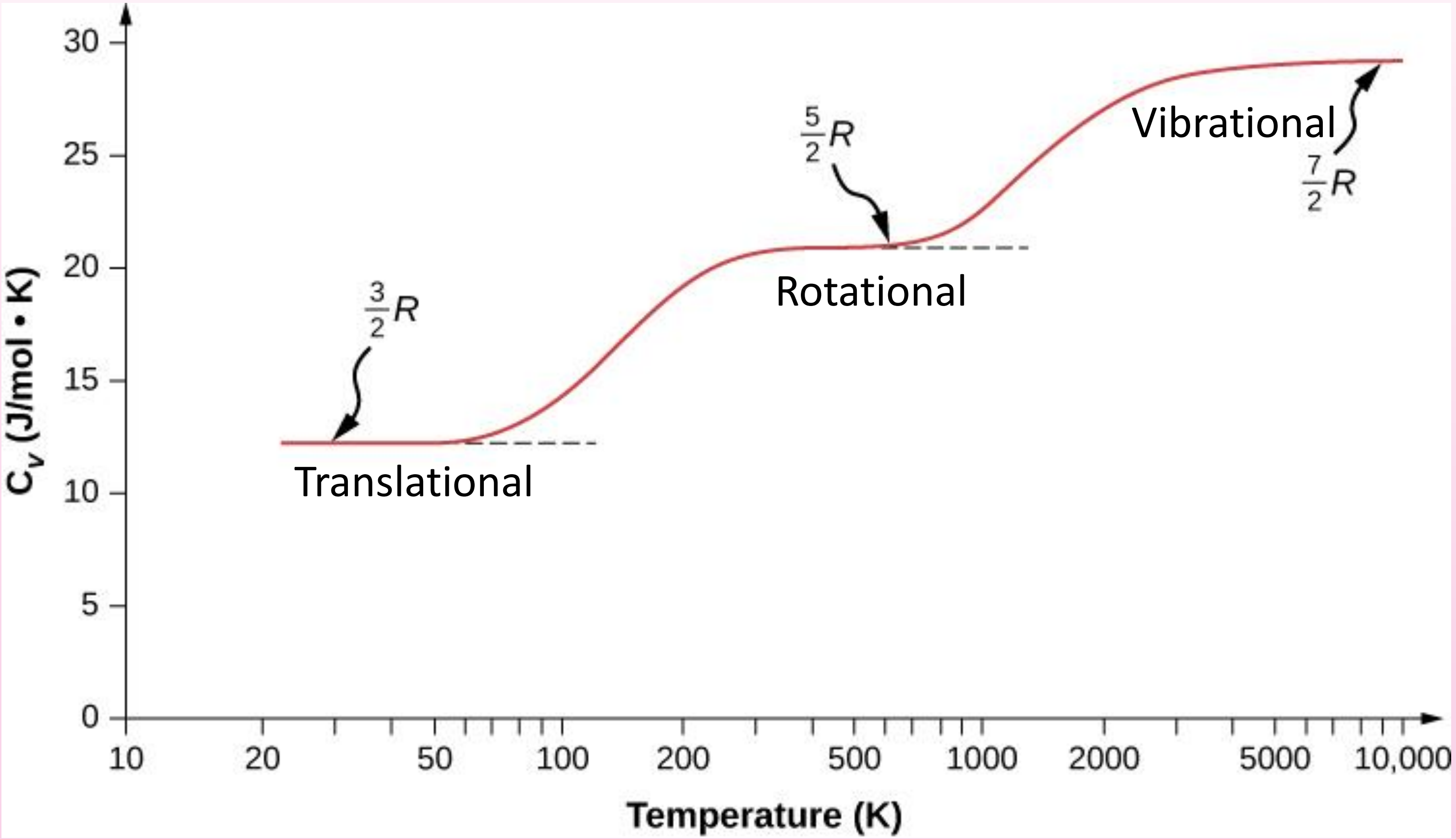
$$E = 3k_B T \Rightarrow c_V = 3R$$

Recap from last time

Degrees of freedom can be considered fully "excited" if

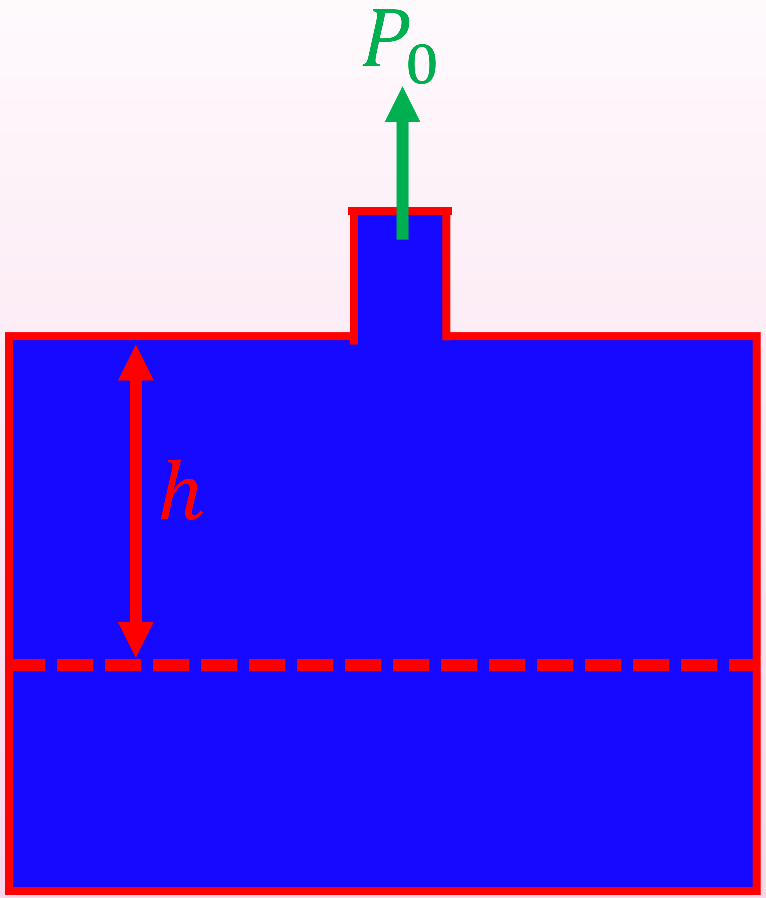
$$k_B T \gg E_1 - E_0$$

If $k_B T \ll E_1 - E_0$, the degree of freedom is said to be "frozen out"



Fluid dynamics

Say I have no applied force, and the pressure at the surface of the liquid is given by P_0… what is the pressure, P, at some arbitrary depth h?



$$P = P_0 + \rho g h$$

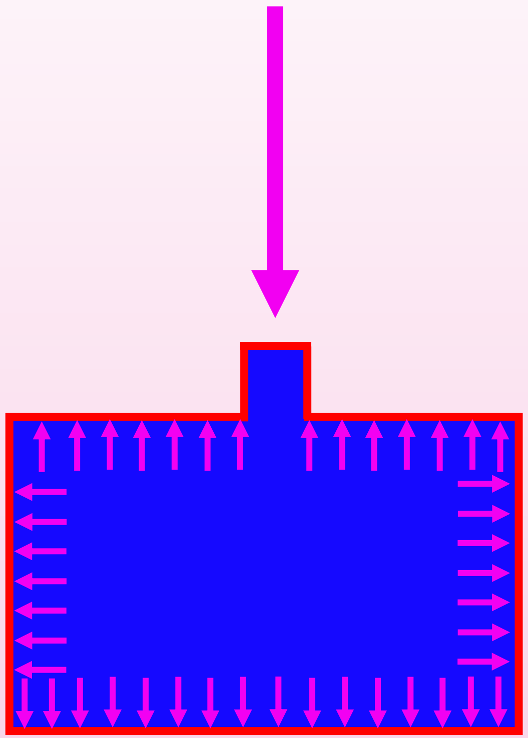
Needs units of pressure, so force/area

$[kg][m]^{-3}$ $[m][s]^{-2}$ $[m] \rightarrow [kg][m][s]^{-2}/[m]^2$

Only true for a fluid of constant density ρ!

Fluid dynamics

If I apply a force to the liquid shown in this container here, what will happen?



Pressure in the liquid will increase, so applied force to the walls of the container increases!

Pascal's law: If pressure is applied to an enclosed fluid, the pressure is applied equally (and undiminished) to every part of the fluid and the walls of enclosure.

Hydraulic presses

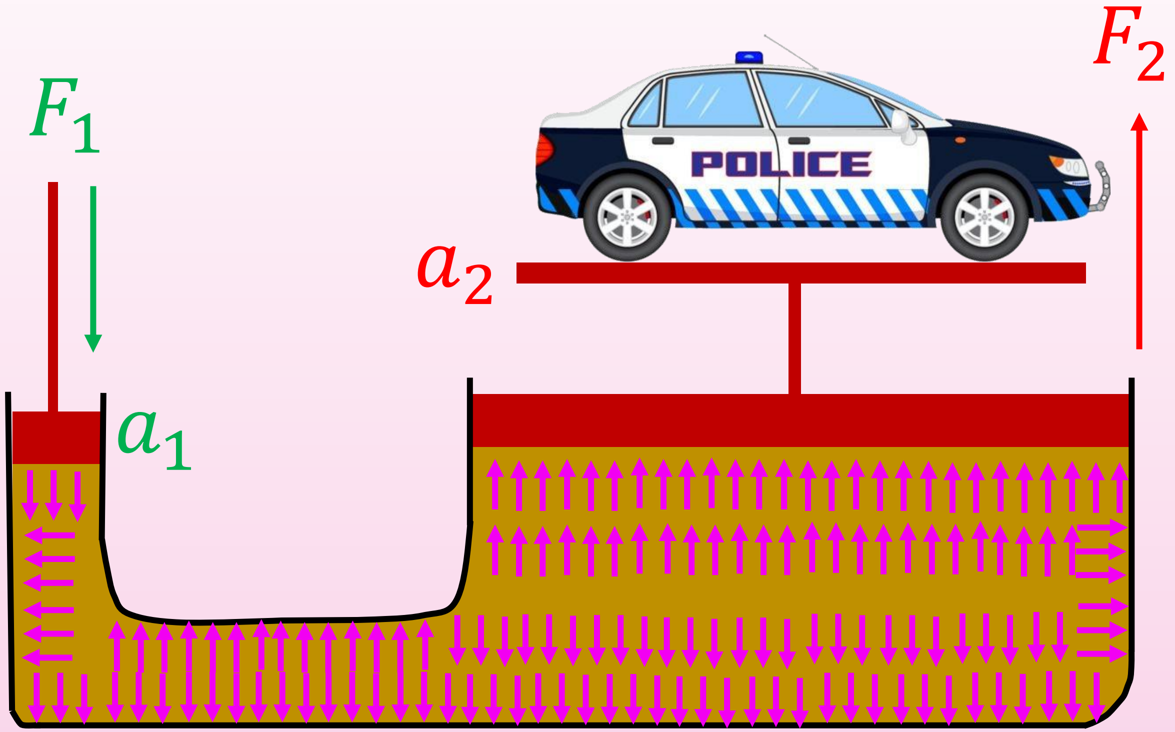
If a force F_1 is applied to a liquid through some relatively small area, a_1, then the pressure determined by these quantities must be applied at all points to the liquid

$$P = F_1/a_1$$

Therefore

$$P = F_2/a_2$$

$$F_2 = \frac{F_1 a_2}{a_1}$$



Q: incompressible fluid, with cross-sectional area a_1 of 0.25 m^2. On the other side, the cross sectional area is 2.5 m^2. What force is required to lift a 1 tonne car?

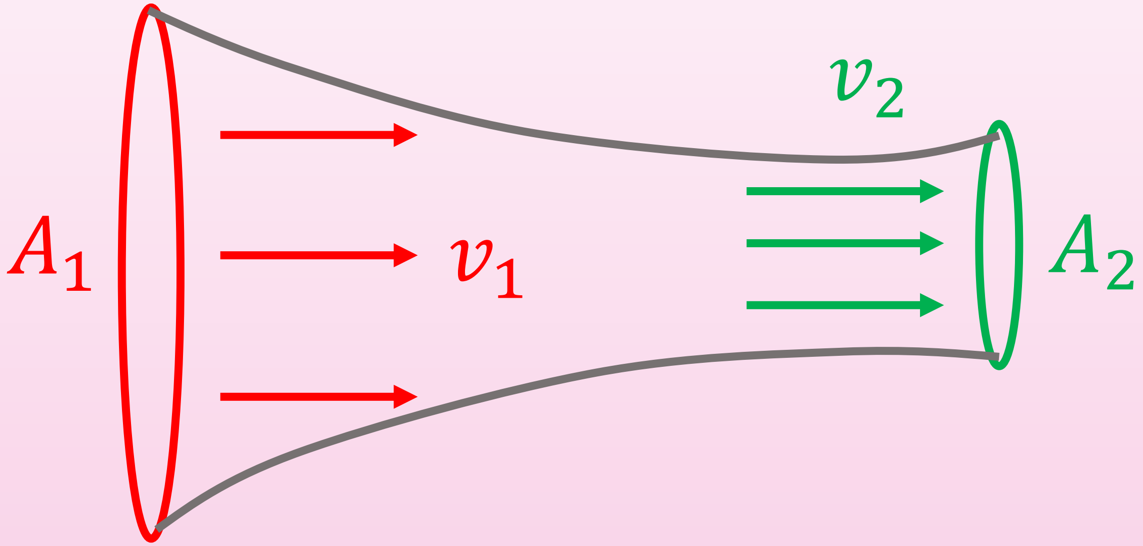
A: 980 N (100 kg of mass equivalent)

Fluid flowing through a pipe

In the steady state (unchanged with respect to time), the amount of water that passes a given point should always be the same at any time…

Speed of the flow depends on the area of the pipe, such that

$$A_1 v_1 = A_2 v_2$$



For a change in speed, we require an acceleration and hence a force

This force comes from the neighbouring water, but how does this affect pressure?

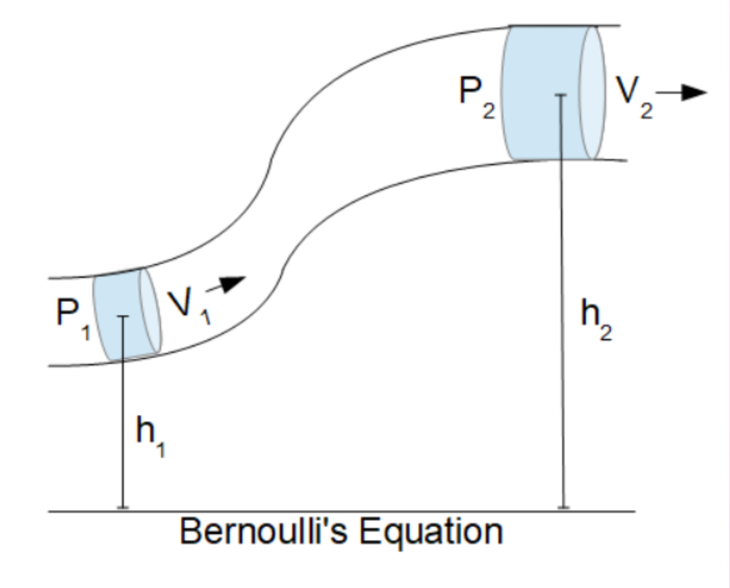
Bernoulli’s equation

Before, we had $$P = P_0 + \rho g h$$

To include the velocity, we need the pressure from the kinetic energy (think back to our gravitational pressure term)

$$\text{Constant} = P + \rho g h + \frac{1}{2}\rho v^2$$

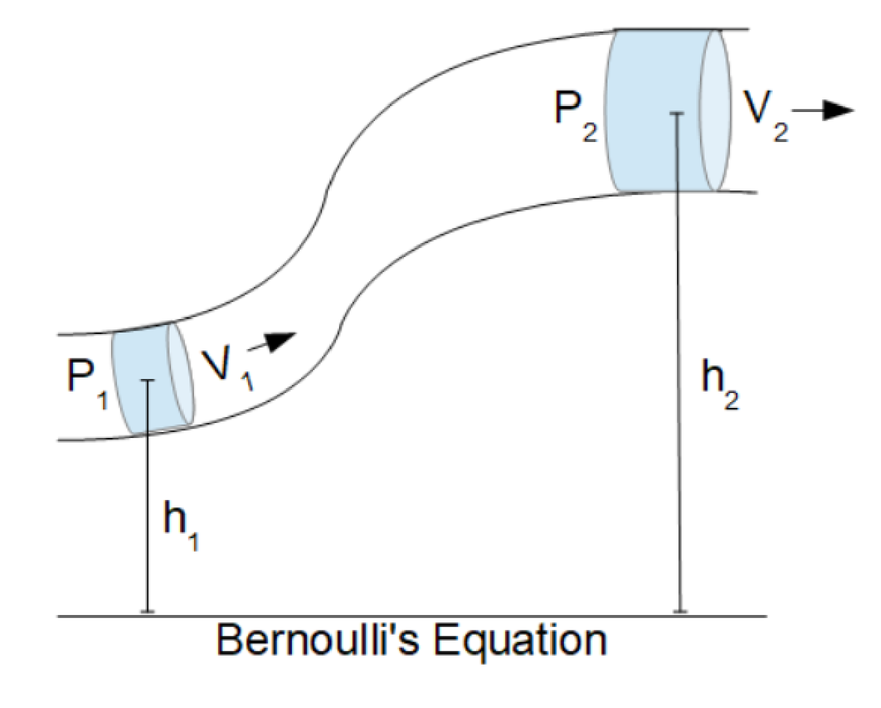
Static pressure | Hydrostatic pressure | Dynamic pressure



Bernoulli's equation example

$$\text{Constant} = P + \rho g h + \frac{1}{2}\rho v^2$$

Water enters the pipe at a speed of 1.5 ms^{-1} and pressure 4×10^5 Pa through a pipe of diameter 2 cm. It then travels through a pipe of diameter 1 cm to a tap in a bathroom 5 metres above.



What speed does the water flow out when the tap is opened?
What is the pressure at the tap?