University of Birmingham
School of Mathematics

RA Real Analysis Autumn 2024

Problem Sheet 4 - summative assessment
issued Week 8

Questions

(SUM) **Q1**. Find the derivatives of the following functions according to the definition, where they exist.

(a) $f(x) = x^3$.

(b) $f(x) = \cos x$.

Please note that only the definition of derivative can be used, and one should not assume these functions are differentiable a priori. In particular, one should not use L'Hospital's rule to evaluate the limits involved. You can use properties of limits and notable limits discussed in lectures (lecture notes).

(SUM) **Q10**. Find the derivatives of the following functions. Detailed computation is needed.

(a) $f(x) = 2x^3 + 5x^2 - 1$,

(b) $f(x) = x \ln x$,

(c) $f(x) = \ln(x^2 + 1)$,

(d) $f(x) = \dfrac{(x-1)(x-2)}{x^{\frac{1}{3}}}$,

(e) $f(x) = (x \sin x)^x$,

(f) $f(x) = \cos(\sin x)$.

(SUM) **Q11**. Determine the types of the following indeterminate form ($\frac{0}{0}, \frac{\infty}{\infty}, 0\cdot\infty, \infty-\infty, 0^0, \infty^0, 1^\infty$), and use L'Hospital's rule to find their limits where applicable:

(a) $\lim\limits_{x\to\frac{1}{2}} \dfrac{6x^2+5x-4}{4x^2+16x-9}$

(b) $\lim\limits_{x\to\infty} \dfrac{\ln x}{\sqrt{x}}$

(c) $\lim\limits_{x\to 1} \dfrac{x^8-1}{x^5-1}$

(d) $\lim\limits_{x\to 0^+} (1+\sin 3x)^{1/x}$