UNIVERSITY OF BIRMINGHAM

School of Physics and Astronomy

DEGREE OF B.Sc. & M.Sci. WITH HONOURS

FIRST YEAR EXAMINATION

03 19749

LC SPECIAL RELATIVITY/PROBABILITY AND RANDOM PROCESSES

SUMMER EXAMINATIONS 2018

Time Allowed: 1 hour 30 minutes

Answer Section 1 and two questions from Section 2.

Section 1 counts for 40% of the marks for the examination.
Full marks for this Section can be obtained by correctly answering ***four*** questions.
You may attempt more questions, but marks in excess of 40% will be disregarded.

Section 2 consists of three questions and carries 60% of the marks.
Answer ***two*** questions from this Section. If you answer more than two questions, credit will only be given for the best two answers.

The approximate allocation of marks to each part of a question is shown in brackets **[ ]**.

All symbols have their usual meanings.

Calculators may be used in this examination but must not be used to store text.
Calculators with the ability to store text should have their memories deleted prior to the start of the examination.

A table of physical constants and units that may be required will be found at the end of this question paper.

SECTION 1

*Full marks for this section can be obtained by correctly answering **four** questions. You may attempt as many questions as you wish, but any marks in excess of 40% will be disregarded.*

1. In an inertial frame Σ two events are separated by a time interval of 3 s and occur at distance 8×10^8 m away from each other.

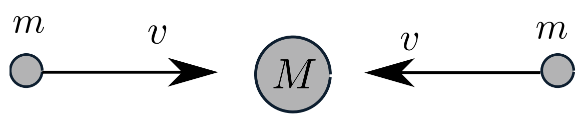
   (a) Can the earlier event be the cause for the later one? **[3]**

   (b) Calculate the square of space-time invariant interval Δs^2 between the events and find the proper time between the events. **[4]**

   (c) Find the velocity (relative to Σ) of the frame Σ' in which the events happen in the same space location. **[3]**

2. A relativistic brick is thrown at angle 45° with velocity $u' = c/\sqrt{2}$ inside a space ship which moves with velocity $3c/5$ in the positive x-direction with respect to the earth. Find the tangent of the angle of the brick's velocity with respect to x-axis in the earth's inertial frame. **[10]**

3. Two particles each having rest mass m are approaching each other with velocity v (in some inertial frame). After colliding they form a bigger particle with rest mass M.



   (a) Write down the equation for conserved energy and momentum and also write down the rest mass M of the resulting particle in terms of m and v. **[3]**

   (b) Transforming into frame Σ' where one of the colliding particles is at rest, find the velocity v' of the other particle. **[3]**

   (c) State the energy conservation equation in the frame Σ' in terms of m, v and v'. By using the results of part (b), verify that energy is conserved in the moving frame Σ'. **[4]**

4. A spy went to a pub where he has 1/8 chance to get drunk. When the spy is drunk he is ten times more likely to reveal state secrets than when he is sober. If

the state secrets were found to be leaked by the spy calculate the chance he was under influence of alcohol. **[10]**

5. Consider the generating function

$$g(z) = \left(\frac{1+z}{2}\right)^{17} = \sum_{n=0}^{\infty} p_n z^n$$

of a discrete probability distribution p_n.

(a) Show that p_n is normalised and calculate the probabilities for $n = 0$ and for $n = 18$. **[5]**

(b) Write down the definitions for the mean μ and the variance σ^2 of the distribution p_n and calculate them using the generating function above. **[5]**

6. A one-dimensional quantum mechanical *wavefunction* is

$$\psi(x) = C\left(1 - \frac{x^2}{b^2}\right) ,$$

where the particle is restricted to the region $x \in [-b, b]$ and C is the (real) normalisation constant. Find the normalisation constant C and calculate the uncertainty in position $\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$ of the particle. **[10]**

SECTION 2

Answer **two** *questions from this Section. If you answer more than two questions, credit will only be given for the best two answers.*

7. (a) Give a definition of *proper time*. **[3]**

   (b) Twin A flies with a constant velocity $V = 3c/5$ away from twin B who stays on Earth. According to the twin A's clock he travels for 10 years before deciding to return back to Earth. What is the distance (in light years, 1 light year$=$ $c\times$year) he travelled measured in the Earth's frame of reference? **[4]**

   (c) Draw the world line of the twin A on a Minkowski diagram with axes x and t of the Earth reference frame Σ. Indicate the event just before the U-turn and draw the line $x = ct$. **[3]**

   (d) The journey started on the 14th birthday of the twins, so the decision to return comes on the 24th birthday of twin A (according to his own clock). Just before the U-turn, he receives a video call with congratulations from his Earth sibling (who is an expert in Special Relativity and sent his video message so it would reach the spaceship just in time). How old does the Earth brother appear on this video call? [*Hint: Use the space-time interval* $\Delta s^2 = 0$ *between events on the light-cone*]. **[10]**

   (e) Twin A decides to hurry up and rushes back to Earth with velocity $V' = 3c/4$. How old is each twin when they finally meet? [*Hint: use* $\sqrt{7} \simeq 2.6$]. **[10]**

8. (a) Define *probability density* $p(x)$ of a random variable $a < x < b$ and explain how to normalise it. **[4]**

(b) If the random variable x is mapped onto y using the monotonically increasing function $y(x)$ show the following relation between their probability densities,

$$P(y) = p(x)\frac{dx}{dy}\,, \qquad \text{for} \qquad y(a) < y < y(b)\,.$$

[5]

(c) A source produces relativistic particles of (rest) mass m with energy E distributed according to probability density

$$P(E) = \frac{A}{E^3}\,, \qquad E_0 < E\,.$$

State the minimal possible value of E_0 and find the constant A. **[5]**

(d) Calculate the mean energy $\bar{E}$ of the produced particles. **[3]**

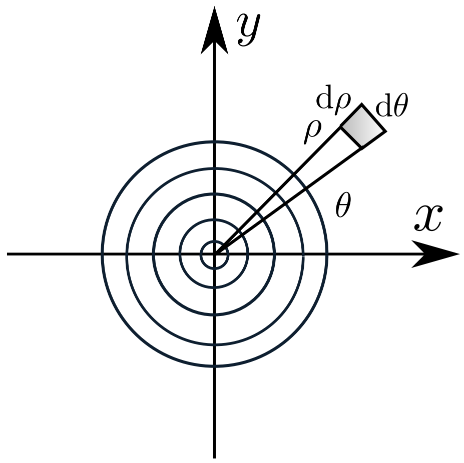
(e) State the relation $E(v)$ between the energy of a relativistic particle and its speed v. Particles are moving in the same direction and $v > 0$. Find the probability density $p(v)$. **[8]**

(f) Calculate the mean speed $\bar{v}$ and state whether $\bar{E} = E(\bar{v})$. **[5]**

9. An arrow is being shot at a target on a wall. The distribution of horizontal and vertical deviations x and y are identical normal distributions

$$p(x) = \sqrt{C}e^{-x^2/2\sigma^2}\,, \qquad p(y) = \sqrt{C}e^{-y^2/2\sigma^2}\,,$$

where C, σ are some constants.



(a) The vertical and horizontal deviations of the arrow are independent, calculate the joint probability distribution $p(x, y)$ of the coordinates of the arrow and state the probability for the arrow to hit a small area a situated near the point (x, y). **[3]**

(b) In terms of polar coordinates (ρ, θ)

$$x = \rho\cos\theta\,, \qquad y = \rho\sin\theta\,.$$

Calculate the area element subtended by small intervals $\mathrm{d}\rho, \mathrm{d}\theta$ around the point (ρ, θ) and state the probability density $P(\rho, \theta)$ in polar coordinates. **[9]**

(c) Normalise the probability density $P(\rho, \theta)$, *i.e.* find the value of C **[8]**

(d) Find the probability P_n to shoot the arrow in the annular region

$$\sigma\sqrt{n-1} < \rho < \sigma\sqrt{n}\,, \qquad n = 1, 2, \ldots.$$

and find the mean value $\langle n\rangle$ and the variance $\Delta n^2 = \langle n^2\rangle - \langle n\rangle^2$. [*Hint: you may want to use an appropriate generating function*] **[10]**

Physical Constants and Units

Acceleration due to gravity	g	9.81 m s^{-2}
Gravitational constant	G	6.674 $\times$ 10^{-11} N m^{2} kg^{-2}
Ice point	T_{ice}	273.15 K
Avogadro constant	N_A	6.022 $\times$ 10^{23} mol^{-1}
		[*N.B.* 1 mole $\equiv 1$ *gram-molecule*]
Gas constant	R	8.314 J K^{-1} mol^{-1}
Boltzmann constant	k, k_B	1.381 $\times$ 10^{-23} J K^{-1} $\equiv$ 8.62 $\times$ 10^{-5} eV K^{-1}
Stefan constant	σ	5.670 $\times$ 10^{-8} W m^{-2} K^{-4}
Rydberg constant	R_∞	1.097 $\times$ 10^{7} m^{-1}
	$R_\infty hc$	13.606 eV
Planck constant	h	6.626 $\times$ 10^{-34} J s $\equiv$ 4.136 $\times$ 10^{-15} eV s
$h/2\pi$	$\hbar$	1.055 $\times$ 10^{-34} J s $\equiv$ 6.582 $\times$ 10^{-16} eV s
Speed of light *in vacuo*	c	2.998 $\times$ 10^{8} m s^{-1}
	$\hbar c$	197.3 MeV fm
Charge of proton	e	1.602 $\times$ 10^{-19} C
Mass of electron	m_e	9.109 $\times$ 10^{-31} kg
Rest energy of electron		0.511 MeV
Mass of proton	m_p	1.673 $\times$ 10^{-27} kg
Rest energy of proton		938.3 MeV
One atomic mass unit	u	1.66 $\times$ 10^{-27} kg
Atomic mass unit energy equivalent		931.5 MeV
Electric constant	ϵ_0	8.854 $\times$ 10^{-12} F m^{-1}
Magnetic constant	μ_0	4π $\times$ 10^{-7} H m^{-1}
Bohr magneton	μ_B	9.274 $\times$ 10^{-24} A m^{2} (J T^{-1})
Nuclear magneton	μ_N	5.051 $\times$ 10^{-27} A m^{2} (J T^{-1})
Fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	7.297 $\times$ 10^{-3}= 1/137.0
Compton wavelength of electron	$\lambda_c = h/m_e c$	2.426 $\times$ 10^{-12} m
Bohr radius	a_0	5.2918 $\times$ 10^{-11} m
angstrom	Å	10^{-10} m
barn	b	10^{-28} m^{2}
torr (mm Hg at 0 °C)	torr	133.32 Pa (N m^{-2})

Do not complete the attendance slip, fill in the front of the answer book or turn over the question paper until you are told to do so

Important Reminders

- Coats/outwear should be placed in the designated area.
- Unauthorised materials (e.g. notes or Tippex) must be placed in the designated area.
- Check that you do not have any unauthorised materials with you (e.g. in your pockets, pencil case).
- Mobile phones and smart watches **must** be switched off and placed in the designated area or under your desk. They must not be left on your person or in your pockets.
- You are not permitted to use a mobile phone as a clock. If you have difficulty seeing a clock, please alert an Invigilator.
- You are not permitted to have writing on your hand, arm or other body part.
- Check that you do not have writing on your hand, arm or other body part – if you do, you must inform an Invigilator immediately
- Alert an Invigilator immediately if you find any unauthorised item upon you during the examination.

Any students found with non-permitted items upon their person during the examination, or who fail to comply with Examination rules may be subject to Student Conduct procedures.