? Any Calculator

THE UNIVERSITY OF BIRMINGHAM

Degree of B.Sc./M.Sci. with Honours

Programmes in the School of Mathematics and Statistics First examination

Programmes including Mathematics First examination

Degree of M.Eng. with Honours

Mathematical Engineering First examination

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MSM1C: COMPUTATIONAL AND APPLIED MATHEMATICS

May/June, 2001 2 hours

Full marks may be obtained with complete answers to ALL questions in Section A (worth a total of 50 marks) and TWO (out of THREE) questions from Section B (worth 25 marks each). Only the best TWO answers from Section B will be credited. Calculators may be used in this examination but must not be used to store text. Calculators with the ability to store text should have their memories deleted prior to the start of the examination.

Turn over

SECTION A

1. Consider the equation

$$\frac{d^2\rho}{dt^2} = \sqrt{\pi}\int \frac{m}{A^2}dv,$$

where ρ, t, m, A and v are density, time, mass, area and speed respectively.

(a) What are the dimensions of ρ, t, m, A and v? [4]

(b) Is the equation dimensionally homogeneous? Justify your answer. [3]

2. At time $t = 0$, a projectile of mass m is launched from the origin at an angle α to the horizontal with speed U.

(a) Let the position vector of the projectile be $\mathbf{r} = x\mathbf{i} + z\mathbf{k}$. Neglecting air resistance, use Newton's second law to show that

$$x = Ut\cos\alpha$$

and

$$z = -\frac{1}{2}gt^2 + Ut\sin\alpha.$$

[6]

Hence determine an expression for the speed v of the projectile at time t. [2]

(b) Hence determine an expression for the kinetic energy of the projectile in terms of the time t. [1]

3. Consider a star located at the origin of a coordinate system. A planet orbiting the star has polar coordinates (r, θ) where r is a constant. What is the shape of the orbit? If

$$\frac{d\theta}{dt} = \omega$$

where ω is constant, write down an expression for the speed of the planet in terms of r and ω. [4]

SECTION B

4. You may find the following formulae useful in this question:

$$\mathbf{e_r} = \cos\theta\mathbf{i} + \sin\theta\mathbf{j}$$

$$\mathbf{e}_\theta = -\sin\theta\mathbf{i} + \cos\theta\mathbf{j}.$$

Consider a particle with mass m and polar coordinates (r, θ), where r and θ are functions of time t. The particle has position vector $\mathbf{r} = r\mathbf{e_r}$.

(a) Determine expressions in terms of $\mathbf{e_r}$ and $\mathbf{e}_\theta$ for the velocity $\mathbf{v}$ and the acceleration $\mathbf{a}$ of the particle. [4]

(b) The particle experiences a central force $\mathbf{F} = -F(r)\,\mathbf{e_r}$. Write down Newton's second law for the motion of the particle. [1]

(c) Prove that $r^2\dot{\theta}$ is constant, where $\dot{\theta} = d\theta/dt$. [2]

(d) Prove that

$$\frac{d^2u}{d\theta^2} + u = \frac{F(1/u)}{mh^2u^2},$$

where $u = 1/r$ and $h = r^2\dot{\theta}$. [6]

(e) Determine the general solution to the differential equation in (d) when $F(r) = 1/r^2$. In this case, if the resulting orbit is circular, determine the radius of the orbit.

[4]