L5 # X-ray production and Diffraction

- How can we produce X-rays?

  → Just heat something to 1 MK ...

  → or... fire high-energy electrons (keV) into a metal target (in vacuum)

|| X-ray source

- What are X-rays? EM radiation / light, beyond UV

  0.1 keV → $\lambda = 10$ nm (soft)

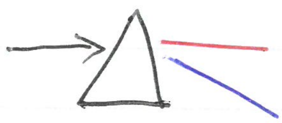
  ↓

  100 keV → $\lambda = 0.01$ nm (hard)

|| X-Rays

- How can we measure X-ray wavelengths?

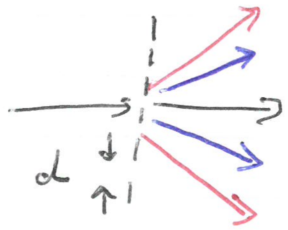
  → A Prism?



(turns λ into angle, measure angle)

... but $n \simeq 1$ for all materials at these frequencies, so this doesn't work

→ A grating?



... but we need $d \sim \lambda$

and $\lambda \sim 10^{-10}$ m, the size of an atom
- good luck making that grating!

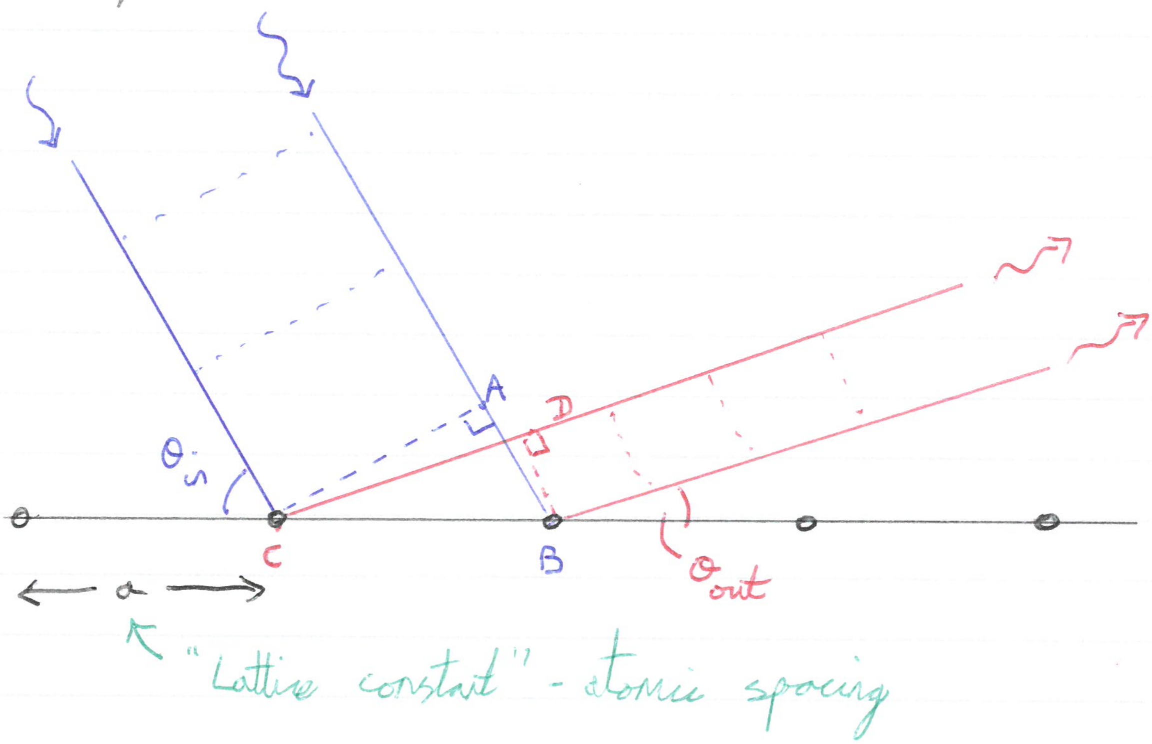
But! Nature has made these for us - crystalline materials.
Use the crystal lattice of a solid as a '3D grating' (Max von Laue, 1912)

|| Crystalline Materials

- X-ray diffraction from crystals
  - Treat the X-rays as EM waves
  - Each atom scatters independently
  - Atom absorbs X-ray then re-emits in <u>all</u> directions uniformly (at photon level, in a random direction)

→ Find the angles where constructive interference occurs. Zero intensity elsewhere.

Consider a single plane of equally spaced atoms:



The outgoing waves are in phase when the wavefronts line up

↳ when path lengths ~~AB~~ AB and CD are equal

$$a \cos\theta_{in} = a \cos\theta_{out}$$

$$\boxed{\theta_{out} = \theta_{in}}$$

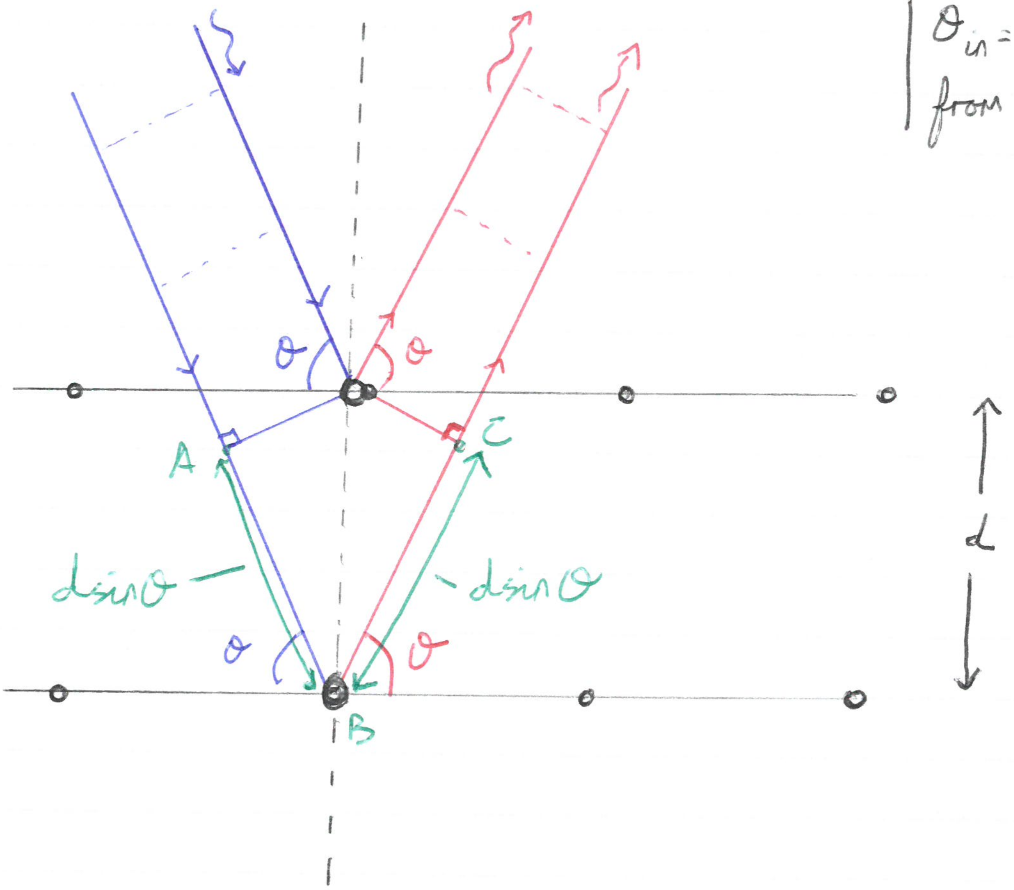
(higher orders possible too)

→ Independent of wavelength ("no dispersion") - not acting like a grating (yet)

N.b. → This is basically the same as a mirror

- Now consider criterion for constructive interference from two planes of atoms (and hence n planes)

$\theta_{in} = \theta_{out} = \theta$ from before

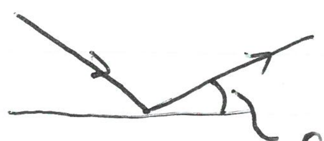


Ray that hits lower plane travels further.

For constructive interference, extra path length must be an integer number of wavelengths

$$AB + BC = \boxed{2d\sin\theta = n\lambda} \qquad n = 1, 2, 3\ldots$$

d – seperation of crystal planes



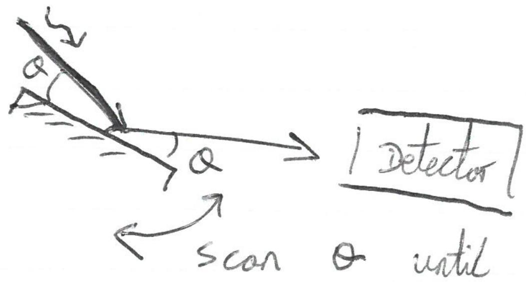
θ_{Bragg} – Bragg angle

Note! This is not the beam deflection angle

This condition gives different angles for different wavelengths

Lecture 3 foreshadowed this! Remember the Compton scattering setup had a "Bragg Analyser"?

|| Compton Scattering Setup



scan θ until detection $\Rightarrow$ know λ

Note θ is there twice - have to move detector too. Total angle beam deflected is then 2θ, which is the standard value plotted

So this lets us measure (or select!) an X-ray wavelength.

Alternatively, fire ~~at~~ a single known λ at an unknown crystal to find its atomic spacings. Scan 2θ and find angles where $n\lambda = 2d_i \sin\theta$ for various d_i

X-Rays for Crystallography

X-Ray crystallography - work out the structure of materials

- Conclusions
  - 2 Bragg conditions (Father and son, 1915 Nobel Prize)

$$\boxed{\begin{aligned} n\lambda &= 2d\sin\theta \\ \theta_{out} &= \theta_{in} \end{aligned}}$$

→ can use crystals to measure/select X-ray wavelength
"wavelength dispersive X-ray spectroscopy"