1Mech — Mechanics

Mechanics exercises 2 (weeks 3 and 4)

This sheet's assessed questions are number 2 and 4. To receive full marks your solution needs to be clear and well explained.

1. A cannon fires a shell from ground level, with velocity $\mathbf{V}$ at an angle α to the horizontal. Find the path the shell takes, and deduce the horizontal range (i.e. how far the shell travels before it hits the ground). What angle should you fire at, if you want the shell to travel the furthest distance possible, assuming the initial speed doesn't change?

   [NB. You should use vector and differential equation methods to solve this, not the SUVAT equations. If you don't know what the SUVAT equations are, please ignore this comment :)]

2. **Assessed**

   [Hint: Don't panic - this question looks a lot harder than it is initially, you should be able to get started OK! Follow the instructions in the question.]

   The force on a particle (mass m, charge e with position vector $\mathbf{r}$) moving under the influence of a constant magnetic field $\mathbf{B}$ is $\mathbf{F} = e\dot{\mathbf{r}} \times \mathbf{B}$. This is the **only force** acting on the particle, which starts at $\mathbf{0}$ with velocity $\mathbf{V}$ initially.

   (a) Show that

   $$m\ddot{\mathbf{r}} \;=\; e\dot{\mathbf{r}} \times \mathbf{B},$$

   and give appropriate initial conditions.

   (b) By integrating once, show that

   $$m\dot{\mathbf{r}} \;=\; e\mathbf{r} \times \mathbf{B} + m\mathbf{V}.$$

   (c) If $\mathbf{B} = (m/e, 0, 0)$ in Cartesians, use the definition of the vector product to write down an expression for $\mathbf{r} \times \mathbf{B}$ where $\mathbf{r} = (x, y, z)$.

   (d) We will now find the position of the particle in terms of $x(t)$, $y(t)$, $z(t)$, for an initial velocity $\mathbf{V} = (V_1, 0, V_3)$.

      i. By considering the $\mathbf{i}$ component of the governing equation, solve to find an expression for $x(t)$.

      ii. By combining your y and z equations (i.e. the $\mathbf{j}$ and $\mathbf{k}$ components) to form a single second order ODE for $y(t)$, find $y(t)$ and hence $z(t)$.

   (e) What shape is the particle path?

3. (a) Starting from expressions for the unit vectors $\mathbf{e}_r$, $\mathbf{e}_\theta$ in the r and θ directions, where r and θ are polar coordinates, derive the radial and transverse components of acceleration.

(b) If a particle of mass m moves under a central force of the form $\mathbf{F} = F(r)\mathbf{e}_r$, prove that $r^2\dot{\theta} = h$ is constant, and find the governing equation for the particle path in terms of $u = 1/r$ and θ.

4. **Assessed**

Find the value of the constant $h = r^2\dot{\theta}$, and suitable initial conditions for $u(\theta) = 1/r$, $du/d\theta$ for the following particles under the action of a central force.

(a) The particle is initially at $r = a$, moving with radial velocity v and transverse (also known as angular) velocity $a\omega$.

(b) The particle is initially at $r = b$, moving away from the origin with speed V in a direction which makes an angle $\pi/3$ with the outward pointing radial vector.

(c) The particle is initially at $r = c$, moving with speed w in a direction making an angle $\pi/4$ with the **inward** pointing radial vector.