? Any Calculator

THE UNIVERSITY OF BIRMINGHAM

????????Degree of B.Sc./M.Sci. with Honours???????????

Programmes in the School of Mathematics and Statistics First examination

Programmes including Mathematics First examination

????????Degree of M.Eng. with Honours???????????

Mathematical Engineering First examination

0611235/0611240

MSM1C: COMPUTATIONAL AND APPLIED MATHEMATICS

May/June, 2005 2 hours

Full marks may be obtained with complete answers to ALL questions in Section A (worth a total of 50 marks) and TWO (out of THREE) questions from Section B (worth 25 marks each). Only the best TWO answers from Section B will be credited. Calculators may be used in this examination but must not be used to store text. Calculators with the ability to store text should have their memories deleted prior to the start of the examination.

Turn over

SECTION A

1. (a) Let the symbols A, V, u, t, m, ρ represent area, volume, speed, time, mass and density. What are the dimensions of A, V, u, t, m, ρ. [3]

(b) In fluid dynamics Bernoulli's Law says that

$$B = \frac{p}{\rho} + \frac{1}{2}u^2 + gz,$$

where B is Bernoulli's constant, p is pressure, ρ is density, u is speed, g is the constant acceleration of gravity and z is vertical displacement. Given that Bernoulli's Law is dimensionally correct, what are the dimensions of pressure p and Bernoulli's constant B. [4]

2. A projectile with mass m is fired at an angle θ to the horizontal with speed V.

(a) Neglecting air resistance, and assuming that the acceleration of gravity is constant, use Newton's Second Law of Motion to determine the location of the projectile as a function of time. [5]

(b) Calculate the speed of the projectile as a function of time. [2]

(c) Show that the sum of the projectile's kinetic energy and gravitational potential energy is constant. [3]

3. A particle is located at position $\mathbf{r}$ with respect to the origin of a 2-dimensional co-ordinate system. If $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors in a (x, y) Cartesian co-ordinate system and $\mathbf{e}_r$ and $\mathbf{e}_\theta$ are the unit vectors in a (r, θ) polar coordinate system:

(a) Determine expressions for $\mathbf{e}_r$ and $\mathbf{e}_\theta$ in terms of $\mathbf{i}$ and $\mathbf{j}$. [4]

(b) Derive an expression for the velocity $\frac{d\mathbf{r}}{dt}$ of the particle in polar coordinates. [6]

SECTION B

4. A particle of mass m is attached to a spring with a Hooke's constant of k. Suppose that the particle is on a horizontal frictionless surface and x measures the distance of the particle away from the equilibrium length of the spring.

(a) Use Hooke's law to determine the force acting on the particle due to the spring. [3]

(b) Write down Newton's second law for the particle and hence determine the location of the particle as a function of time, if at time $t = 0$, $x = x_o$ and $\frac{dx}{dt} = 0$. [7]

(c) Show that the sum of the particles kinetic energy and potential energy is constant. [5]

(d) If we now assume that the particle is subject to a frictional resistance force equal to $-b\frac{dx}{dt}$, determine the location of the particle as a function of time, where once again at time $t = 0$, $x = x_o$ and $\frac{dx}{dt} = 0$. The constants k and b are related by $k = \frac{3b^2}{16m}$. In this case, what is $\lim_{t\to\infty} x$? [10]

5. In polar coordinates Newton's second law for a particle of mass m can be written as

$$\mathbf{F} = F_r\mathbf{e}_r + F_\theta\mathbf{e}_\theta = m\left(\left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right]\mathbf{e}_r + \left[2\frac{dr}{dt}\frac{d\theta}{dt} + r\frac{d^2\theta}{dt^2}\right]\mathbf{e}_\theta\right).$$

Assume that the particle is subject to a central force with $F_r = -g(r)$ (for some function g) and $F_\theta = 0$.

(a) Show that the quantity

$$h = r^2\frac{d\theta}{dt}$$

is constant and hence or otherwise show that the position vector of the particle sweeps out equal areas about the origin in equal times. [8]

(b) If $u = 1/r$ show that the radial component of Newton's second law becomes

$$\frac{d^2u}{d\theta^2} + u = \frac{g\,(1/u)}{mh^2u^2},$$

where h is the constant identified above. [10]

(c) Determine the general form of the orbit $r(\theta)$ of the particle if $g(r) = \gamma/r^2$ where γ is a dimensional constant. What are the dimensions of γ? [5]

(d) Under what condition are the orbits calculated above bounded? [2]

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