UNIVERSITY OF BIRMINGHAM

School of Physics and Astronomy

DEGREE OF B.Sc. & M.Sci. WITH HONOURS

FIRST-YEAR EXAMINATION

03 19750

LC ELECTROMAGNETISM 1 / TEMPERATURE & MATTER / ELECTRIC CIRCUITS

SUMMER EXAMINATIONS 2019

Time Allowed: 3 hours

Answer Section 1 and three questions from Section 2.

Section 1 counts for 25% of the marks for the examination.
Full marks for this Section can be obtained by correctly answering ***five*** questions.
You may attempt more questions, but marks in excess of 25% will be disregarded.

Section 2 consists of three questions and carries 75% of the marks.
Answer ***all three*** questions from this Section. Note that each question has two parts, of which only ***one part*** should be answered. If you answer both parts, credit will only be given for the best answer.

The approximate allocation of marks to each part
of a question is shown in brackets **[ ]**.

PLEASE USE A SEPARATE ANSWER BOOK FOR SECTION 1 AND SECTION 2 QUESTIONS.

Calculators may be used in this examination but must not be used to store text.
Calculators with the ability to store text should have their memories deleted prior to the start of the examination.

A table of physical constants and units that may be required
will be found at the end of this question paper.

SECTION 1

Full marks for this section can be obtained by correctly answering ***five*** *questions. You may attempt as many questions as you wish, but any marks in excess of 25% will be disregarded.*

1. State the zeroth and first laws of thermodynamics. **[5]**

2. An atom is part of a gas in thermal equilibrium at a temperature of 1800 K and has three possible states. Two of them are degenerate and have an energy which is larger than the third by 0.12 eV. Calculate the probability of finding the atom in the lowest energy state. **[5]**

3. Air in the cylinder of an engine at a temperature of 290 K is compressed adiabatically to one fifth of its original volume.

   (a) Sketch this process on a diagram of pressure versus volume.

   (b) Calculate the final temperature of the air, taking care to outline any assumptions that you make.

   [5]

4. The normalised Maxwell-Boltzmann distribution of the speeds v of molecules of mass m in a gas at temperature T that are constrained to move in only two dimensions is given by

$$P(v) = \left(\frac{m}{2\pi k_\mathrm{B} T}\right) 2\pi v \, \exp\left(\frac{-mv^2}{2k_\mathrm{B}T}\right)$$

   (a) Explain the origin of the $2\pi v$ term in this expression.

   (b) Derive an expression for the most probable molecular speed.

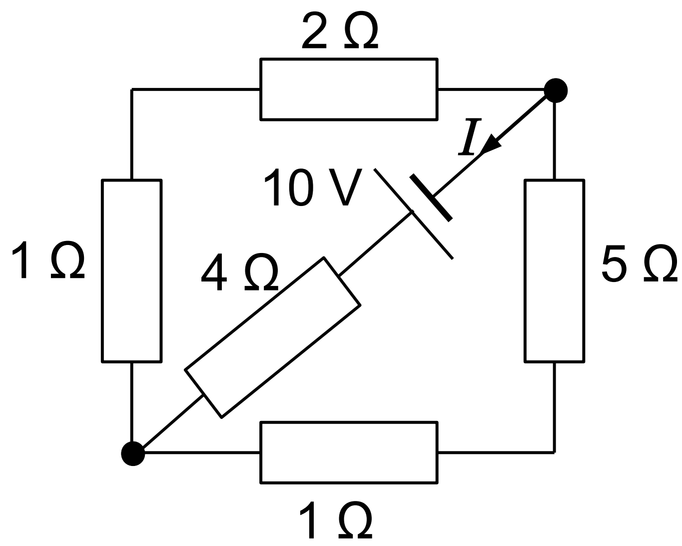
   [5]

5. A long, uniform, conducting wire of radius a has a current density $J = J_0 r/a$, where J_0 is a constant and r is the radial distance from the centre of the wire. Derive an expression for the magnetic field inside and outside the wire. **[5]**

6. Point charges of Q and $-2Q$ are fixed at (x, y) coordinates $(-a, 0)$ and $(a, 0)$, respectively. Derive an expression for the work needed to move a point charge of $-q$ from the point $(0, a)$ to the origin $(0, 0)$. **[5]**

7. A conducting circular loop of radius a and resistance R is placed in a uniform magnetic field $\underline{B}$ and rotates with an angular velocity ω around an axis perpendicular to $\underline{B}$. At $t = 0$ the loop is in a plane perpendicular to the magnetic field.
   Derive an expression for the induced *emf* in the loop.
   Give an expression for the induced current in the loop as a function of time. **[5]**

8. Find the current, I, for the circuit shown below. Hence, calculate the power dissipated. **[5]**



SECTION 2

*Answer **all three** questions from this Section. Note that each question has two parts, of which only **one part** should be answered. If you answer both parts, credit will only be given for the best answer.*

9. **EITHER Part A**

(a) Show that the height h, that a liquid of density ρ climbs inside a capillary of radius r, is given by

$$h = \frac{2\gamma \cos\alpha}{\rho g r}$$

where γ is the surface tension, g is the acceleration due to gravity and α is the contact angle between the liquid surface and capillary wall. **[7]**

(b) A monatomic liquid of element 'X' has a density of 0.51 g cm^{-3}. In a capillary of diameter 5.6 mm, the liquid is observed to climb 3.5 cm above the level of the liquid outside the capillary. The liquid contacts the capillary at an angle of 52°. Calculate the surface tension of the liquid. **[2]**

(c) Show that the latent heat of vaporisation L *per kilogram* of a liquid may be related to its surface tension and density by

$$L = \frac{2\gamma}{\rho\, a_0}$$

where a_0 is the typical distance between the centres of neighbouring atoms in the liquid. **[9]**

(d) The latent heat of vaporisation of liquid 'X' is 5.5×10^6 J kg^{-1}.
Calculate the molar mass of the liquid and hence identify element 'X'. **[4]**

(e) Estimate the specific heat capacity of solid 'X' at high temperatures, taking care to explain your reasoning. **[3]**

OR Part B

One mole of a monatomic ideal gas is prepared at a temperature T_c in an initial volume V_0.
The gas is expanded at constant temperature $T = T_c$ to a volume $4V_0$.
It is then heated at constant volume $V = 4V_0$ to temperature T_h.
Next it is compressed at constant temperature $T = T_h$ to its original volume V_0.
Finally it is cooled at constant volume $V = V_0$ to its original temperature T_c.

[You may assume throughout this question that all changes are reversible.]

(a) Sketch these changes on a diagram of pressure versus volume. **[4]**

(b) Determine the work done, heat transferred and change in internal energy in each change, writing your answers in terms of T_c, T_h and R, where R is the universal gas constant.
For each change, state whether work is done *on* or *by* the gas and whether heat is transferred *to* or *from* the gas. **[10]**

(c) The cycle is used as a refrigerator i.e. to remove heat from a cold bath at temperature T_c. Calculate the coefficient of performance of this refrigerator.

[The *coefficient of performance* is defined as the heat removed from the cold bath divided by the net work done on the gas in each cycle.
Hint: here the heat removed from the cold bath is that in the first constant-temperature step.]

[2]

(d) The cycle is used, with helium as the working gas, to cool a sphere to cryogenic temperatures, as shown in the diagram on the next page.

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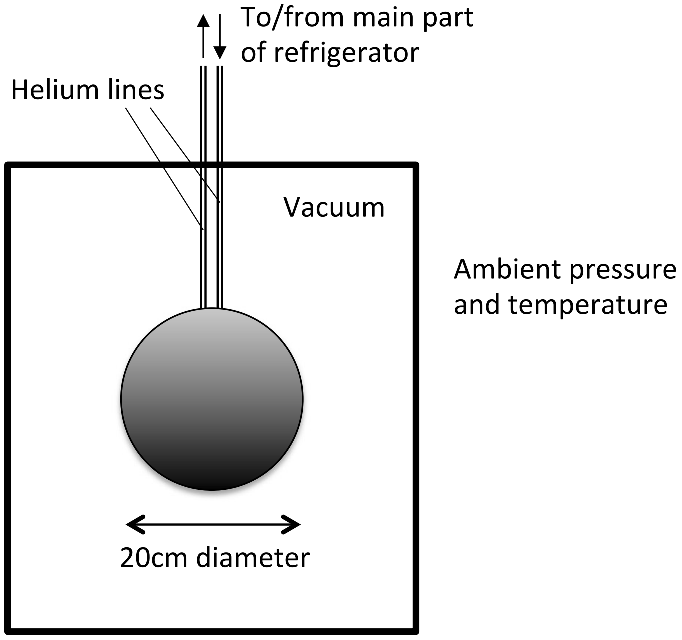
(d) *continued...*



The refrigerator uses 300 W of input power to cool the sphere.
Determine a value for the minimum temperature that the sphere will reach.

[You may neglect unwanted heat transfer through the helium lines.]

[5]

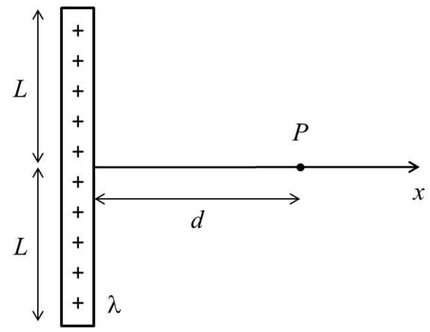
The coefficient of performance of the Carnot cycle working as a refrigerator is $T_c/(T_h - T_c)$.

(e) Explain why $T_c/(T_h - T_c)$ should be the maximum coefficient of performance that any refrigerator can attain. **[4]**

10. **EITHER Part A**

(a) State Coulomb's law and Gauss' law, briefly explaining the symbols in the equations. Write down an equation for the relationship between the electric field and the electric potential. **[5]**

(b) A thin wire of length $2L$ has a uniform charge density, λ, along its length. The wire is placed along the z-axis between $z = -L$ and $z = +L$ as shown in the diagram below.



Calculate the magnitude and direction of the electric field at point P, at a distance d from the wire, as shown in the diagram above.
You may find the follow integral useful:

$$\int_0^b \frac{du}{(a^2+u^2)^{3/2}} = \frac{b}{a^2(a^2+b^2)^{1/2}}$$

[5]

Show that the electric field at point P is independent of L when $d \ll L$. **[2]**

(c) The wire in part (b) is now replaced by an infinitely long wire, but still with a uniform charge density λ. Use Gauss' law, or otherwise, to show that the electric field at point P is now given by:

$$E_x = \frac{\lambda}{2\pi\epsilon_0 d}.$$

[4]

What is the change in the electrical potential if point P is now moved away from the infinite wire to $x = 2d$? **[4]**

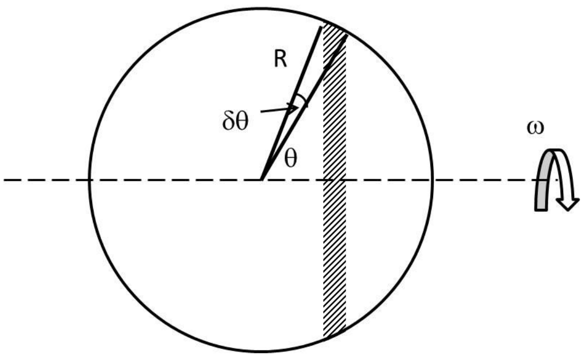
(d) A non-relativistic particle travels parallel to the infinite wire with speed v, at a constant distance d, and experiences a magnetic field. Derive an expression for the magnetic field experienced by the moving particle. **[5]**

OR Part B

(a) Write down the equation describing the Biot-Savart law for the magnetic field due to a current element. Give an explanation of all the symbols involved in the equation. **[4]**

(b) A hollow spherical shell of radius R, with a uniform surface charge density σ, is rotating about its diameter, along the x-axis, with angular velocity ω. By considering a thin ring of charge as illustrated in the diagram below, show that this represents a current element δI given by:

$$\delta I = \sigma 2\pi R^2 \delta\theta \sin\theta \frac{\omega}{2\pi}.$$



[5]

(c) Use the expression above and the Biot-Savart law, to give an expression for the magnetic field, δB, at the centre of the sphere resulting from this current element. **[9]**

Hence, determine the total magnetic field at the centre of the sphere.
You may assume the following definite integral: $\int_0^\pi \sin^3\theta d\theta = \frac{4}{3}$. **[2]**

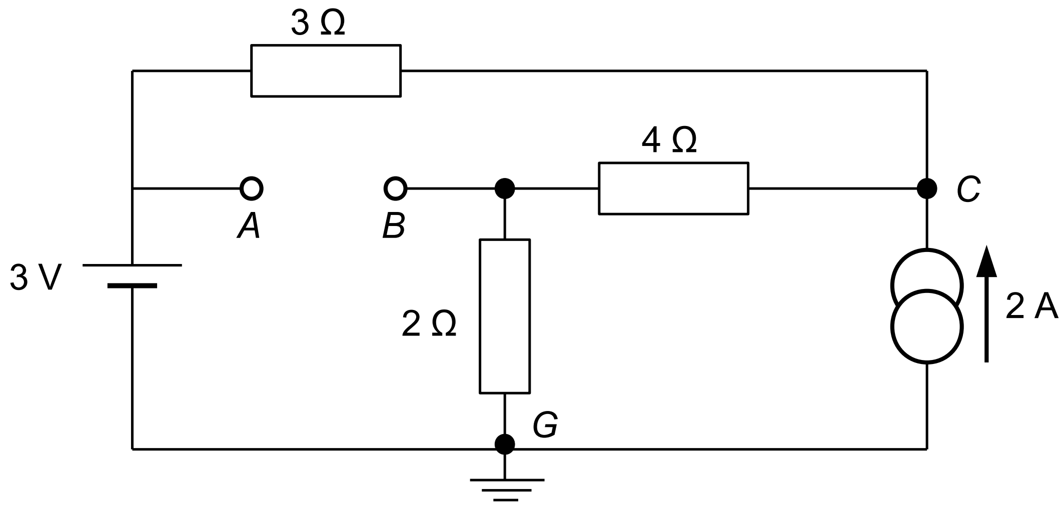
(d) Write down expressions for the electric field inside and outside this sphere as a function of the distance r, from the centre of the sphere. **[5]**

11. **EITHER Part A**

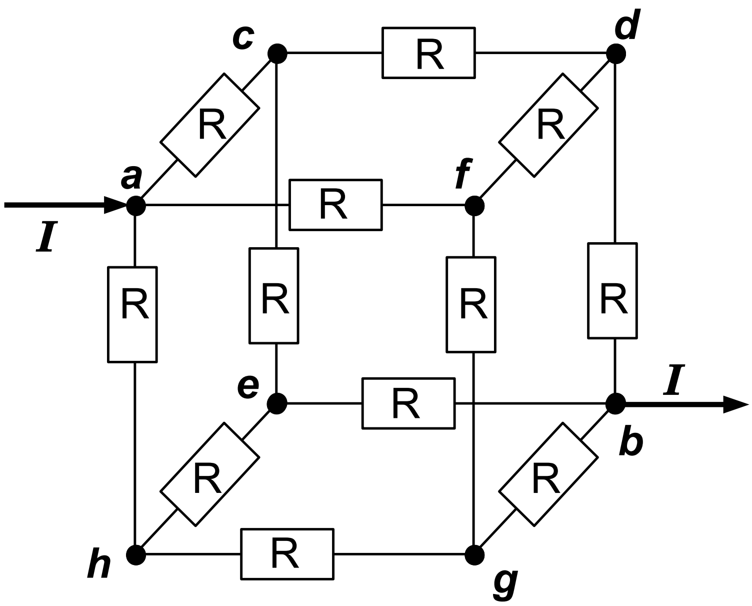
(a) For the circuit shown in the figure below, find the Thévenin equivalent circuit.

i. Hence, find the current through a load resistor $R_L = 1\ \Omega$ connected across $A - B$, *i.e.* I_{A-B}. **[13]**

ii. Using the current, I_L, from part (i.), with the load resistor connected, calculate the voltage across the current source (V_{CG}). **[7]**

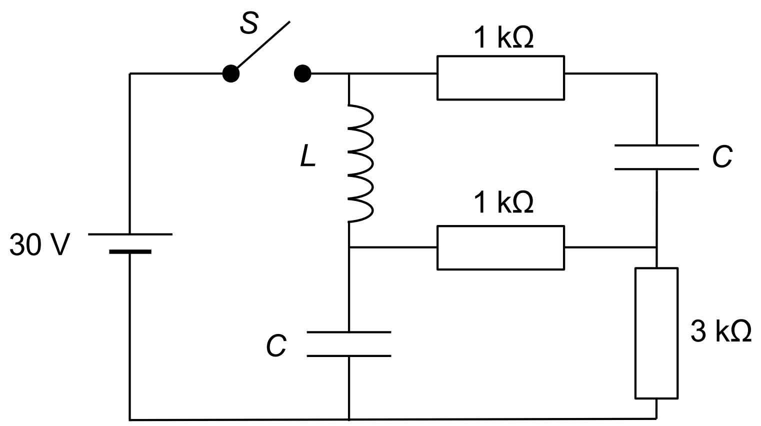


(b) For the circuit shown below, derive an expression for the potential difference between a and b in terms of the current, I, and resistances, R. **[5]**

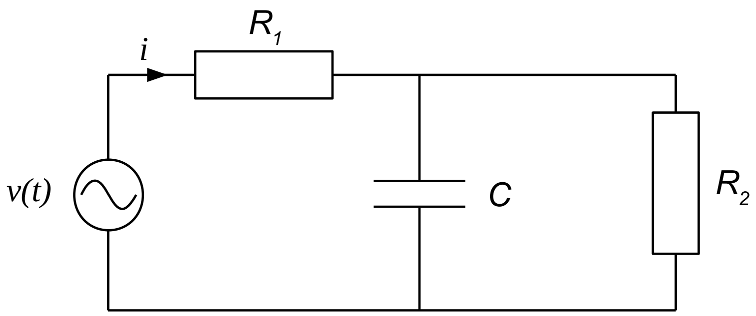


OR Part B

(a) In the circuit shown below, find the initial and final values of the voltage across the 3 kΩ resistor after the switch, S, is closed. Find the initial and final values of the power dissipated in the network. **[10]**



(b) The circuit below shows a resistor $R_1 = 4\ \Omega$ connected to a parallel combination of a capacitor $C = 8\ \mu F$ and a second resistor $R_2 = 20\ \Omega$. The source has a 100 V peak amplitude and operates at 500 Hz.



Express the impedance of the circuit as a complex number $z = a + jb$. Hence find: **[5]**

i. the magnitude of the impedance, **[2]**

ii. the magnitude of the current drawn from the source, **[2]**

iii. the phase angle between the source voltage and the current, **[2]**

iv. the power delivered by the source. Comment on whether the voltage leads the current, or vice versa. Explain your answer. **[4]**

Physical Constants and Units

Acceleration due to gravity	g	9.81 m s^{-2}
Gravitational constant	G	6.674×10^{-11} N m^2 kg^{-2}
Ice point	T_{ice}	273.15 K
Avogadro constant	N_A	6.022×10^{23} mol^{-1}
		[*N.B.* 1 mole $\equiv 1$ *gram-molecule*]
Gas constant	R	8.314 J K^{-1} mol^{-1}
Boltzmann constant	k, k_B	1.381×10^{-23} J K^{-1} $\equiv 8.62 \times 10^{-5}$ eV K^{-1}
Stefan constant	σ	5.670×10^{-8} W m^{-2} K^{-4}
Rydberg constant	R_∞	1.097×10^{7} m^{-1}
	$R_\infty hc$	13.606 eV
Planck constant	h	6.626×10^{-34} J s $\equiv 4.136 \times 10^{-15}$ eV s
$h/2\pi$	$\hbar$	1.055×10^{-34} J s $\equiv 6.582 \times 10^{-16}$ eV s
Speed of light *in vacuo*	c	2.998×10^{8} m s^{-1}
	$\hbar c$	197.3 MeV fm
Charge of proton	e	1.602×10^{-19} C
Mass of electron	m_e	9.109×10^{-31} kg
Rest energy of electron		0.511 MeV
Mass of proton	m_p	1.673×10^{-27} kg
Rest energy of proton		938.3 MeV
One atomic mass unit	u	1.66×10^{-27} kg
Atomic mass unit energy equivalent		931.5 MeV
Electric constant	ϵ_0	8.854×10^{-12} F m^{-1}
Magnetic constant	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Bohr magneton	μ_B	9.274×10^{-24} A m^2 (J T^{-1})
Nuclear magneton	μ_N	5.051×10^{-27} A m^2 (J T^{-1})
Fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	7.297×10^{-3}= 1/137.0
Compton wavelength of electron	$\lambda_c = h/m_e c$	2.426×10^{-12} m
Bohr radius	a_0	5.2918×10^{-11} m
angstrom	Å	10^{-10} m
barn	b	10^{-28} m^2
torr (mm Hg at 0 °C)	torr	133.32 Pa (N m^{-2})

Do not complete the attendance slip, fill in the front of the answer book or turn over the question paper until you are told to do so

Important Reminders

- Coats/outwear should be placed in the designated area.
- Unauthorised materials (e.g. notes or Tippex) must be placed in the designated area.
- Check that you do not have any unauthorised materials with you (e.g. in your pockets, pencil case).
- Mobile phones and smart watches **must** be switched off and placed in the designated area or under your desk. They must not be left on your person or in your pockets.
- You are not permitted to use a mobile phone as a clock. If you have difficulty seeing a clock, please alert an Invigilator.
- You are not permitted to have writing on your hand, arm or other body part.
- Check that you do not have writing on your hand, arm or other body part – if you do, you must inform an Invigilator immediately
- Alert an Invigilator immediately if you find any unauthorised item upon you during the examination.

Any students found with non-permitted items upon their person during the examination, or who fail to comply with Examination rules may be subject to Student Conduct procedures.