Live Session _ Week 06: Worked problem - CAPACITORS

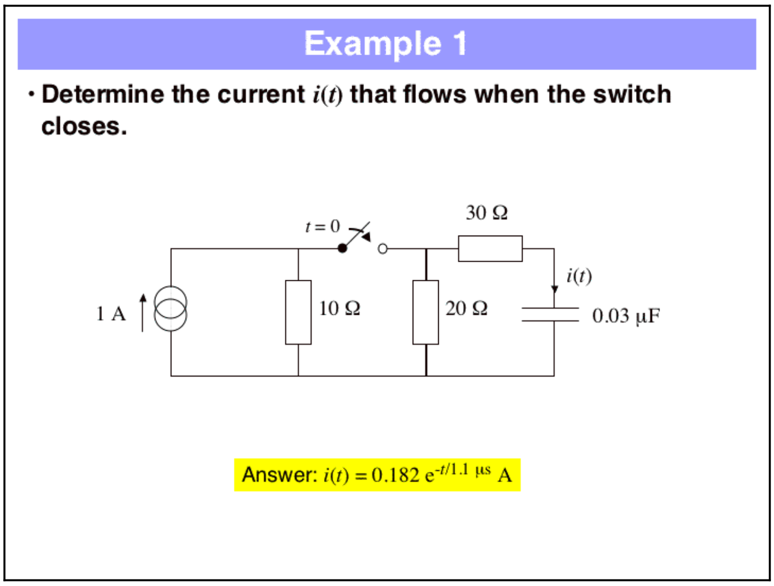

Figure 1: Capacitor charging example.

This problem requires us to find an expression for the time dependent current that flows "through" the capacitor. We will apply the shorthand method shown in Figure 1 and apply our knowledge of how capacitors respond to instantaneous changes.

We assume that the capacitor is initially uncharged. This is reasonable if the switch has been open for a long time before it is closed. When the switch is closed, the capacitor behaves like a short-circuit. So, the initial circuit has the current source in parallel with the 10 Ω resistor, which is in parallel with the 20 Ω resistor, which is in parallel with the 30 Ω resistor. (It may help if you redraw the circuit.)

To find the initial current, combine the 20 Ω and 30 Ω parallel combination. This gives an equivalent resistance of $20 \times 30 / (20+30) = 12$ Ω.

The current through the switch is found by the current splitter rule to be $1\times10(10+12)=0.455$ A.

The current through the 30 Ω resistor is found by applying the current splitter rule again to give $0.455\times20/(20+30)=0.182$ A. This is in the initial capacitor current.

The final current will be zero, since the charge accumulating on the capacitor will eventually block the current. The capacitor behaves like an open-circuit.

The big question is, what is the time constant? Well, we know it is given by the product, RC, but what is R? To answer this question, we need to visualise the circuit in Thévenin form. (This is another reason why Thévenin's theorem is useful.) To do this, remove the capacitor to give you two terminals. (The terminals are the ends of wires that are left when the capacitor is removed, as shown in figure 2a.)

After switching off the current source, what is the resistance between those terminals?

Replacing the current source with an open-circuit we are left with the 10 Ω and 20 Ω resistor in parallel (equivalent resistance = 10×20/(10+20)=6.67Ω). This combined resistance is in series with the 30Ω resistor. So, the Thévenin equivalent resistance is 36.67Ω. Putting the capacitor back into the circuit, the time-constant is evidently $R_{TH}C$ = 36.67 × 0.03 μF = 1.1 μs (see figure 2b). **Note that** we do not actually need to determine the Thévenin equivalent voltage source.

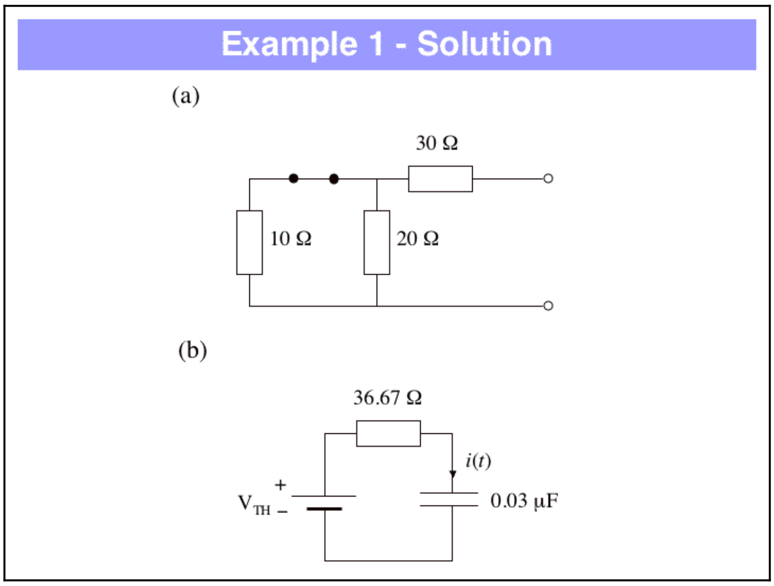

Figure 2: Example 1 – Solution.
a) Determining the Thévenin equivalent resistance of the network having removed the current source and the capacitor. b) The Thévenin equivalent circuit having replaced the capacitor.

Inserting our answers into the shorthand equation we obtain:

$$i(t)=i_f+(i_0-i_f)e^{-\frac{t}{RC}} \Rightarrow i(t)=0.182\,e^{-t/1.1\text{ms}}\,A$$

When multiple resistors are involved, you need to determine Thévenin's equivalent circuit in order to find the appropriate resistance to use in the time-constant. This reduces the problem to a simple series circuit as shown in Figure 2b.

6. The time constant

After a time equal to one time constant, the current in the previous example will have dropped to e^{-1} = *0.368 (or 38.8%) of its initial value. After a time equal to two time constants the current will have dropped to* e^{-2} = *0.135 (or 13.5%) of its initial value, and so on. After a time equal to five time constants the current will have dropped to less than 1% of its initial value. A useful rule of thumb therefore, is that after a sudden change, the circuit reaches a steady state after a time equal to five time constants.*

Try it for yourself

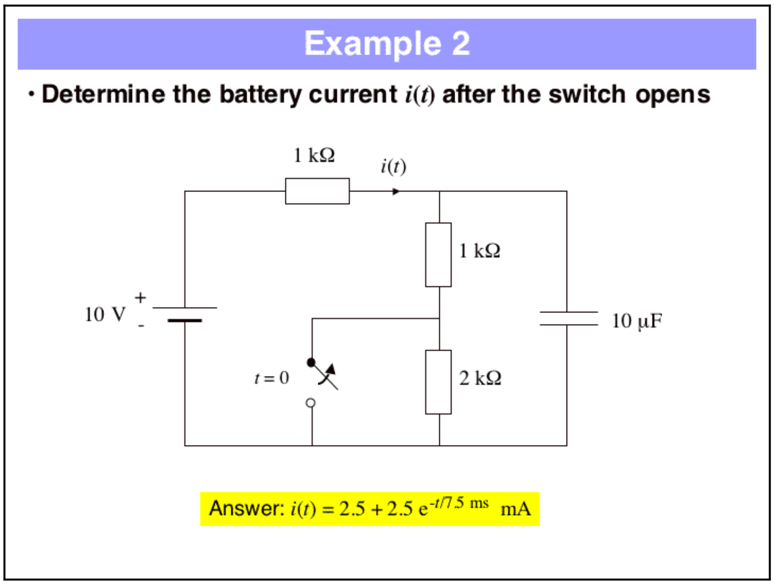

Figure 3: Example 2.

In this problem you are required to find the battery current as a function of time after the switch opens. The method you should use is similar to the preceding worked example, but you need to think carefully about what happens to the capacitor when the switch opens and what this means for the rest of the circuit. Since this is a bit tricky, I'll explain what happens and leave the numerical part to you. First of all, note that when the switch is closed, it shorts-out the 2 kΩ resistor. Secondly, note that there is a potential difference across the ends of the capacitor before the switch opens, since there is a current flowing down through the second 1 kΩ resistor. This means that the capacitor has already stored some charge. Since the voltage across the capacitor cannot change instantaneously when the switch opens, opening the switch initially has no effect on the battery current. To convince yourself of this, think about the voltage that is developed across the 1 kΩ resistor at the top of the circuit diagram, since this determines the battery current. (Answer: 5 V is developed across the top resistor when the switch opens, so the initial current is 5 mA. The final current is 2.5 mA. With the switch open, the Thévenin equivalent resistance seen by the capacitor is 750 Ω.)

In this example it would be wrong to replace the capacitor with a short-circuit when trying to determine the initial current. That's because the capacitor is initially charged, so we can't say there is no voltage across it, as would be the case for a short-circuit. Hopefully, you can see that the important point here is that the capacitor is initially unaffected by changes elsewhere in the circuit. However, it is true to say that capacitors always behave like open-circuits a long time after a switch is thrown, when acted upon by constant (that is d.c.) sources.

Further problems

Attempt problems 121 and 123 in Tipler, Chapter 25.