Problem 02

The circuit below shows a resistor $R_1 = 4\Omega$ connected to a parallel combination of a capacitor $C = 8\ \mu F$ and a second resistor $R_2 = 25\Omega$. The source has a 80 V peak amplitude and operates at 1000 Hz.

(i) Express the impedance of the circuit as a complex number $z = a + jb$.

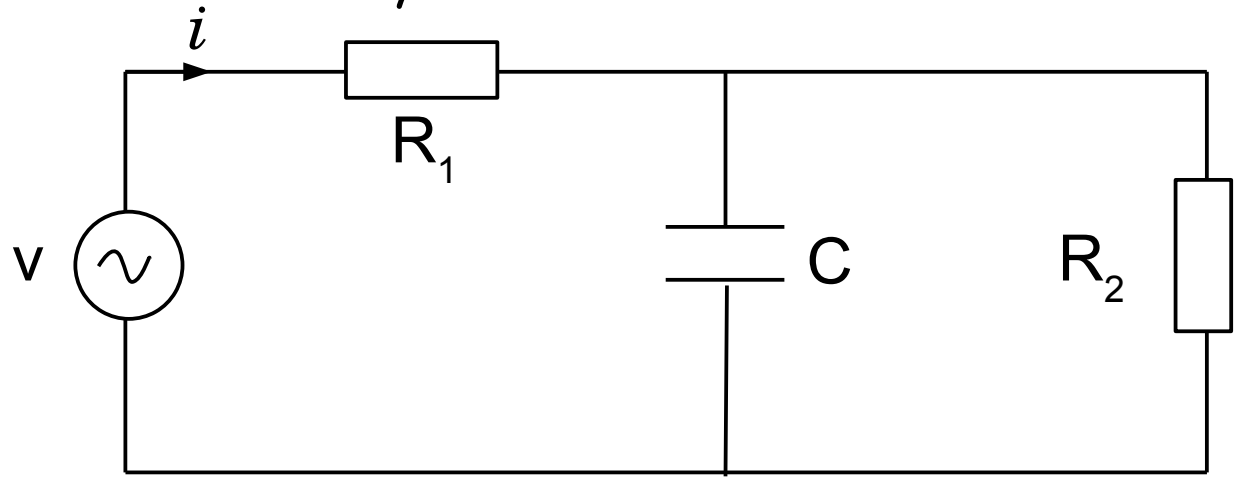
Hence find

(ii) the magnitude of the impedance,

(iii) the magnitude of the current drawn from the source,

(iv) the phase angle between the source voltage and the current,

(v) the power delivered by the source.



Solution 02 – #1

(i) In complex notation, the impedance of the **parallel** combination of C and R_2 is given by:

$$Z = \frac{Z_C Z_{R_2}}{Z_{R_2} + Z_C} = \frac{R_2 \times \frac{1}{j\omega C}}{R_2 + \frac{1}{j\omega C}} = \frac{R_2}{1 + j\omega C R_2}$$

Multiply the top and bottom of the second term by $(1 - j\omega C R_2)$ and separate the real and imaginary parts.

$$Z = \frac{R_2}{1 + (\omega C R_2)^2} - \frac{j\omega C R_2^2}{1 + (\omega C R_2)^2}$$

At f = 1000 Hz, ω = 6283.2 rad/s, Z = 9.69 – j12.18.
The total impedance is: Z = 4 + (9.69 – j12.18) = 13.69 – j12.18.

Solution 02 – #2

(ii) the magnitude of the impedance, is given by the real part squared plus the imaginary part squared, square rooted:

$$\mathbf{Z = 18.32\Omega.}$$

(iii) the magnitude of the current drawn from the source is the peak amplitude of the voltage divided by the impedance:

$$I = V_0/Z = 80/18.32 = \mathbf{4.37\ A}.$$

Note that the rms current would be $I_{rms} = V_{rms}/Z$, where $V_{rms} = \frac{V_0}{\sqrt{(2)}}$.

$I_{rms} = 80/(1.41*18.32) = \mathbf{3.09\ A}$. ($V_{rms} = \mathbf{56.6}$ V.)

(iv) the phase angle between the source voltage and the current can be found from the inverse tan of the ratio of imaginary and real parts of the impedance:

$$\varphi = \tan^{-1}(-12.18/13.69) = -41.7°.$$

The minus sign means the voltage lags the current.

(v) the power delivered by the source, P, is given by:

$$P = I_{rms} \times V_{rms} \times \cos(\varphi) = 3.09 \times 56.6 \times \cos(-41.7) = \mathbf{130.6\ W}.$$