UNIVERSITY OF BIRMINGHAM

School of Physics and Astronomy

DEGREE OF B.Sc. & M.Sci. WITH HONOURS

FIRST-YEAR EXAMINATION

03 19718

LC QUANTUM MECHANICS / OPTICS & WAVES

SUMMER EXAMINATIONS 2018

Time Allowed: 2 hours

Answer Section 1 and two questions from Section 2.

Section 1 counts for 28% of the marks for the examination.
Full marks for this Section can be obtained by correctly answering ***four*** questions.
You may attempt more questions, but marks in excess of 28% will be disregarded.

Section 2 consists of two questions and carries 72% of the marks.
Answer ***both*** questions from this Section. Note that each question has two parts, of which only ***one part*** should be answered. If you answer both parts, credit will only be given for the best answer.

The approximate allocation of marks to each part of a question is shown in brackets **[ ]**.

PLEASE USE A SEPARATE ANSWER BOOK FOR SECTION 1 AND SECTION 2 QUESTIONS.

Calculators may be used in this examination but must not be used to store text. Calculators with the ability to store text should have their memories deleted prior to the start of the examination.

A table of physical constants and units that may be required will be found at the end of this question paper.

SECTION 1

*Full marks for this section can be obtained by correctly answering **four** questions. You may attempt as many questions as you wish, but any marks in excess of 28% will be disregarded.*

1. When ultraviolet light with a wavelength of 400 nm falls on a certain metal surface, the maximum kinetic energy of emitted photoelectrons is 1.10 eV. What is the maximum kinetic energy of the photoelectrons when light of wavelength 300 nm falls on the same surface? **[5]**

2. A 15 cm long violin string is fixed at both ends and oscillates in its $n = 1$ mode. The speed of waves on the string is 250 ms^{-1}, and the speed of sound in air is 348 ms^{-1}. What are (i) the frequency and (ii) the wavelength of the emitted sound wave? What is the frequency of the sound perceived by a bird flying at 10 ms^{-1} toward the violin? **[5]**

3. The wave function of a free particle moving in the negative x-direction is

$$\Psi(x, t) = Ae^{-i(kx+\omega t)}.$$

Show that the wave function is an eigenfunction of the momentum operator, $\hat{p}_x$, and identify the corresponding eigenvalue. Explain the physical significance of the terms *eigenfunction* and *eigenvalue* in quantum mechanics. **[5]**

4. In a double-slit interference experiment, two slits of negligible width are separated by spacing d. The distance from the slits to the viewing screen is L with $L \gg d$. Treating the two slits as two sources of identical in-phase waves of wavelength λ, with the help of a sketch show that, for small angles, the m^{th} bright fringe occurs at position:

$$y_m = \frac{m\lambda L}{d}, \qquad m = 0, \pm 1, \pm 2, ...$$

[5]

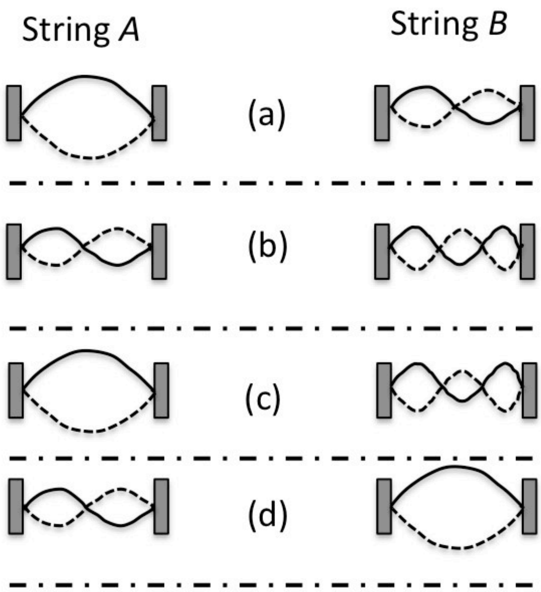
5. According to Planck's hypothesis, the average energy of cavity modes in the blackbody spectrum is wavelength dependent and given by

$$\langle E(\lambda)\rangle = \frac{hc/\lambda}{(\exp(hc/\lambda k_B T) - 1)}.$$

Evaluate $\langle E(\lambda)\rangle$ in the limits $\lambda \to +\infty$ and $\lambda \to 0$. Briefly explain how this behaviour solved the *ultraviolet catastrophe* problem in classical physics.
[*You may assume the approximation* $\exp(x) \approx 1 + x$ *for small* x.] **[5]**

6. Strings A and B have identical lengths (L) and linear densities (μ), but string B is under greater tension (T_B) than string A (T_A). Write down expressions for the standing wave frequencies for the two strings.

The figure below shows four situations, (a) through (d), in which standing wave patterns exist on the two strings. In which situation is there the possibility that strings A and B are oscillating at the same frequency?



[5]

SECTION 2

*Answer **both** questions from this Section. Note that each question has two parts, of which only **one part** should be answered. If you answer both parts, credit will only be given for the best answer.*

7. **EITHER (Part A)**

(a) A molybdenum X-ray tube has an operating voltage of 25 keV. Sketch the expected X-ray emission spectrum as a function of wavelength. Label the key features of the spectrum. Where possible, calculate numerical values for the associated wavelengths of those features.
[*Molybdenum has X-ray absorption edges at* 20 keV, 2.6 keV *and* 0.4 keV.] **[6]**

(b) A collimated beam of X-rays from a molybdenum X-ray tube is incident upon a crystal with a lattice spacing of 0.35 nm. What is the smallest angle at which you would orientate the crystal with respect to the photon beam direction to preferentially reflect molybdenum K_α X-rays? **[4]**

(c) The beam of molybdenum K_α X-rays prepared in part (b) is to be used to investigate photon scattering from a graphite rod. An X-ray detector is to be used to measure the energy of photons scattered through 90 degrees by the rod. Draw a sketch to show the possible arrangement of X-ray source, collimator, crystal, rod and detector, carefully indicating all relevant angles. **[4]**

(d) Photons are observed at two characteristic energies. The lower energy is measured to be at 16.8 $\pm$ 0.1 keV. Show that this result is compatible with Compton scattering of molybdenum K_α X-rays off electrons in the graphite rod. [*You may use without proof the Compton shift equation.*] **[6]**

(e) Calculate the energy of the scattered electrons and their emission angle with respect to the direction of the incoming molybdenum K_α X-ray photons.
[*You may assume that the electron was initially at rest.*] **[5]**

7. **OR (Part B)**

(a) The time-independent Schrödinger equation (TISE) in one dimension is

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x).$$

Explain the physical significance of each of the terms in the equation above. Briefly describe the assumptions upon which the Schrödinger equation is based. **[6]**

(b) Briefly explain the physical significance of the square modulus of the wave function, $|\psi\,(x)\,|^2$? **[2]**

(c) A particle of mass, m, and energy, E, moving along the positive x-direction is incident upon a potential step at $x = 0$. The potential function is $V(x) = 0$ for $x < 0$ (region 1) and $V(x) = V_0$ for $x \geq 0$ (region 2), where $V_0 > 0$ is a positive constant. The general solution of the TISE in region 1 is

$$\psi_1(x) = A\exp(ik_1x) + B\exp(-ik_1x).$$

i. Find an expression for the wavenumber, k_1, in terms of m and E. **[2]**

ii. Write down an expression for the solution of the TISE in region 2 when $E < V_0$. Where possible, express any new symbols that you introduce in terms of m, E and V_0. **[3]**

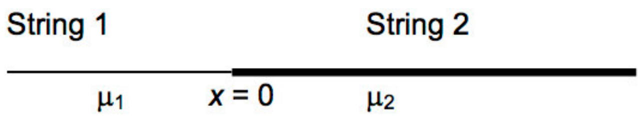
iii. By applying appropriate boundary conditions at $x = 0$, show that $|A|^2 = |B|^2$ when $E < V_0$. Explain the physical significance of this result. **[5]**

iv. Now, consider the situation where $E > V_0$. Write down a new expression for the solution of the TISE in region 2? Where possible, express any new symbols that you introduce in terms of m, E and V_0. **[3]**

v. Show that $B \neq 0$ in region 1 when $E > V_0$ and provide a physical interpretation of this result. How does this result differ from the classical expectation? **[4]**

8. **EITHER (Part A)**

(a) Two semi-infinite strings of different mass per unit length, μ_1 and μ_2, are joined at the position $x = 0$. The tension in the strings is T.



A wave, given by $y = Acos\left(k_1 x - \omega t\right)$, travels along string 1 from left to right. Show that the amplitudes of the reflected and transmitted waves, C and B, respectively, are given by:

$$B = \frac{2k_1}{k_1 + k_2} A \qquad \text{and} \qquad C = \frac{k_1 - k_2}{k_1 + k_2} A,$$

where k_1 and k_2 are the wave-numbers for strings 1 and 2, respectively. State the boundary conditions that you used for the derivation. **[8]**

(b) For $\mu_2 = 0$, the resultant wave on string 1 can be obtained by superposition of the incident and the reflected waves. Express the resultant wave in terms of A, k_1 and ω. Where is the first node from $x = 0$? **[7]**

(c) A string of length L is under constant tension T. The density of the string increases gradually from μ_1 to μ_2 according to the following expression:

$$\mu\left(x\right) = \mu_1 + \frac{\mu_2 - \mu_1}{L} x.$$

A single wave pulse is generated at $x = 0$ and it travels to $x = L$. Find an expression for the time taken for the pulse to arrive at $x = L$ in terms of L, T, μ_1 and μ_2. **[10]**

8. **OR (Part B)**

(a) An object 0.5 cm tall is placed 16 cm to the left of the vertex of a concave spherical mirror having a radius of curvature of 25 cm. Draw a principal-ray diagram showing the formation of the image. Determine the position, size, orientation, and nature (real or virtual) of the image. **[5]**

(b) A tank whose bottom is a mirror is filled with water to a depth of 30 cm. A small fish floats motionless 8 cm under the surface of the water and is viewed directly from above. The refractive index of water is 1.33.

i. What is the apparent depth of the fish?

ii. The fish has an image behind the mirrored bottom. What is the apparent depth of this image? **[8]**

(c) Two converging lenses are placed 40 cm apart. They have the same optical axis. The lens on the left has focal length 9 cm and the other lens has focal length 7 cm. An object 10 cm high is placed 15 cm to the left of the left lens. Find the position, size, and orientation of the image produced by the two lenses combined. **[12]**

Physical Constants and Units

Acceleration due to gravity	g	9.81 m s^{-2}
Gravitational constant	G	6.674×10^{-11} N m^2 kg^{-2}
Ice point	T_{ice}	273.15 K
Avogadro constant	N_A	6.022×10^{23} mol^{-1}
		[*N.B.* 1 mole $\equiv 1$ *gram-molecule*]
Gas constant	R	8.314 J K^{-1} mol^{-1}
Boltzmann constant	k, k_B	1.381×10^{-23} J K^{-1} $\equiv 8.62 \times 10^{-5}$ eV K^{-1}
Stefan constant	σ	5.670×10^{-8} W m^{-2} K^{-4}
Rydberg constant	R_∞	1.097×10^{7} m^{-1}
	$R_\infty hc$	13.606 eV
Planck constant	h	6.626×10^{-34} J s $\equiv 4.136 \times 10^{-15}$ eV s
$h/2\pi$	$\hbar$	1.055×10^{-34} J s $\equiv 6.582 \times 10^{-16}$ eV s
Speed of light *in vacuo*	c	2.998×10^{8} m s^{-1}
	$\hbar c$	197.3 MeV fm
Charge of proton	e	1.602×10^{-19} C
Mass of electron	m_e	9.109×10^{-31} kg
Rest energy of electron		0.511 MeV
Mass of proton	m_p	1.673×10^{-27} kg
Rest energy of proton		938.3 MeV
One atomic mass unit	u	1.66×10^{-27} kg
Atomic mass unit energy equivalent		931.5 MeV
Electric constant	ϵ_0	8.854×10^{-12} F m^{-1}
Magnetic constant	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Bohr magneton	μ_B	9.274×10^{-24} A m^2 (J T^{-1})
Nuclear magneton	μ_N	5.051×10^{-27} A m^2 (J T^{-1})
Fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	7.297×10^{-3}= 1/137.0
Compton wavelength of electron	$\lambda_c = h/m_e c$	2.426×10^{-12} m
Bohr radius	a_0	5.2918×10^{-11} m
angstrom	Å	10^{-10} m
barn	b	10^{-28} m^2
torr (mm Hg at 0 °C)	torr	133.32 Pa (N m^{-2})

Do not complete the attendance slip, fill in the front of the answer book or turn over the question paper until you are told to do so

Important Reminders

- Coats/outwear should be placed in the designated area.
- Unauthorised materials (e.g. notes or Tippex) must be placed in the designated area.
- Check that you do not have any unauthorised materials with you (e.g. in your pockets, pencil case).
- Mobile phones and smart watches **must** be switched off and placed in the designated area or under your desk. They must not be left on your person or in your pockets.
- You are not permitted to use a mobile phone as a clock. If you have difficulty seeing a clock, please alert an Invigilator.
- You are not permitted to have writing on your hand, arm or other body part.
- Check that you do not have writing on your hand, arm or other body part – if you do, you must inform an Invigilator immediately
- Alert an Invigilator immediately if you find any unauthorised item upon you during the examination.

Any students found with non-permitted items upon their person during the examination, or who fail to comply with Examination rules may be subject to Student Conduct procedures.