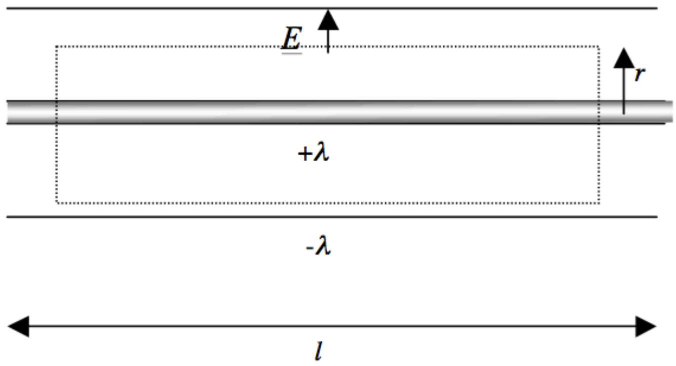
Electromagnetism I – Answers problem sheet 4



1. Applying Gauss's law to the closed cylindrical Gaussian surface as shown, no flux through end faces, E is constant over curved face and always perpendicular to the surface, one has:

$$\int_S \underline{\mathbf{E}} \cdot d\underline{\mathbf{S}} = \frac{Q}{\epsilon_0}$$

$$2\pi r l E = \frac{\lambda l}{\epsilon_0} \Rightarrow E = \frac{\lambda}{2\pi\epsilon_0 r}$$

The electric field is pointing in the direction shown in the figure. Students are expected to express the result in vector form or explicitly state the direction in which the field is pointing.

[2]

2. If the cylindrical Gaussian surface has now radius greater than r_2 the enclosed charge is zero. The total area of the surface is finite and therefore $\vec{E}$ outside the tube is zero everywhere. **[1]**

3. We apply Gauss's law again with the charge density on the outer tube which is twice the initial value:

   (a) For $r_1 < r < r_2$ nothing changes, and the result is as above; **[1]**

   (b) For $r > r_2$ the net charge is now different from zero and is $-\lambda$. The field is therefore

   $$\vec{E} = -\frac{\lambda}{2\pi\epsilon_0 r}\hat{r}\,.$$

   Students should express the result either in vector form, or if simply E is reported they should state that the field is pointing towards the centre of the rod/tube. **[1]**

4. We use

$$V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{r}$$

$$V_b - V_a = -\frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_b}{r_a}\right)$$

Computing it between the tube and the rod the result is:

$$V(r_2) - V(r_1) = -\frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_2}{r_1}\right)$$

[2]

5. If the potential is set to zero at infinity, $V(r \to \infty) = 0$, then $V(r = r_2) = 0$, because the field is zero outside the tube; Hence:

$$V(r_2) - V(r) = 0 - V(r) = -\frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_2}{r}\right)$$

and one obtains:

$$V(r_1) = \frac{\lambda}{2\pi\epsilon_0} \ln\left(\frac{r_2}{r_1}\right)$$

[1]

6. Sketch is shown below. Students should show that the field is zero inside the rod and outside the tube, the descrease of the field and of the potential between r_1 and r_2, with potential constant and non-zero inside the rod. **[2]**