L7

Matter Waves

We have seen light waves behaving as particles - photons have energy and momentum related to their 'wavey' frequency and wavelength

- De Broglie suggested matter / particles should have wave-like properties

De Broglie wavelength $\lambda = \frac{h}{p} = \frac{h}{mv}$

de Broglie Wavelength

→ Big momentum means small wavelength

- Wavelength of Paul Hollywood?

  Mass of 250 kg, can run at 43 mph = 20 ms^{-1}

  $\rightarrow p = 5{,}000 \ kg\,ms^{-1}$

  $$\lambda = \frac{6.6 \times 10^{-34}}{5000} m = 10^{-37} m$$

  → forget about it, tiny

- Wavelength of an electron given 54 V:

  KE = 54 eV

→ First, check – is this energy relativistic? Or can we use trusty Newton, $p = mv$?

$$m_e c^2 = (9\times10^{-31}\ kg)(3\times10^{8}\ ms^{-1})^2$$
$$= 8\times10^{-14}\ J$$
$$= \frac{8\times10^{-14}}{1.6\times10^{-19}}\ eV = 500{,}000\ eV$$

$\therefore \quad m_e c^2 \gg KE \rightarrow$ classical mechanics is fine

(Could also do $\frac{1}{2}mv^2$ and check $v \ll c$)

→ ok, now:

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m} \rightarrow p = \sqrt{2mE}$$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2meV}}$$

accelerating voltage

electron charge

$$\lambda = \frac{6.624\times10^{-34}\ (Js)}{\sqrt{2\times 9.109\times10^{-31}\ (kg) \times 1.6\times10^{-19}\ (C) \times 54\ (V=\frac{J}{C})}}$$

$$\lambda = \underline{\underline{1.67 \times 10^{-10}\ m}} = 1.67\ \mathring{A}$$

(Do the units work?

$$\frac{J s}{\sqrt{kg\ c\ \frac{J}{c}}} = \sqrt{\frac{J}{kg}}\ s$$

|| $J = kg(ms^{-1})^2$

$$= \sqrt{\frac{kg}{kg}}\ ms^{-1}s$$

$$= m$$ ✓)

- Experimental verification - Davisson and Germer experiment

Fired electrons into the surface of nickel (at 54V)

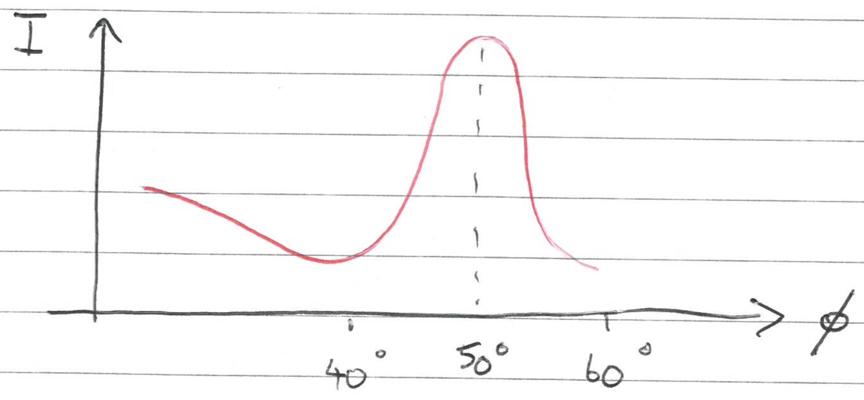
|| Davisson and Germer Experiment

Measure electron intensity in scattered beam vs angle

|| Davisson and Germer Experiment - Results

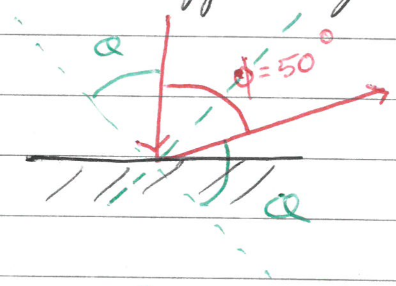
There are peaks in the data, just like we saw in X-ray crystallography!

Result: detector current vs scattering angle:



$\Rightarrow$ Interpret peak at $\phi = 50°$ as being due to electron diffraction from Ni crystal lattice

- Note $\Rightarrow$ this 'scattering angle' ϕ is not our θ Bragg angle...



$2\theta + 50° = 180°$

$\theta = 65°$

$d = 0.91\,Å$ for Ni (Handout on Canvas for super-enthusiasts!)

Bragg condition: $2d\sin\theta = n\lambda$

Assume $n=1$ (this is the first peak we've seen)

$\lambda = 2 \times 0.91 \times 10^{-10} \times \sin(65°)$

$\underline{\underline{\lambda = 1.65 \times 10^{-10}\,m}}$ $\rightarrow$ which we predicted earlier!

- Also - G.P. Thomson experiment

  - 'Powder diffraction' - grind up a crystalline sample into many small crystallites
    ↳ Will be arranged at random angles - all possible angles at once
    ↳ Some crystallites always satisfy the Bragg conditions

|| G.P Thomson Experiment

  - There is a rotational symmetry → rings
  - If we use electrons and X-rays of the same energy we can get the same pattern from the same target

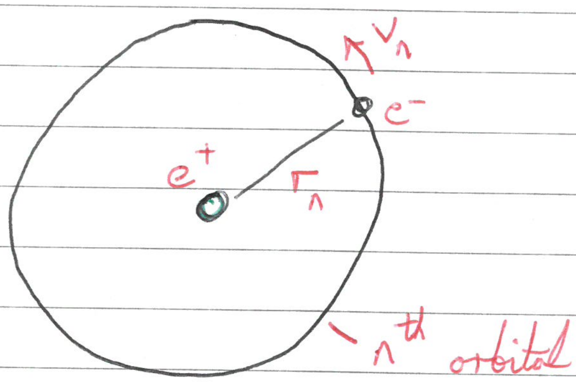
- What is the 'amplitude' of matter waves?
  ↳ with light this is clear - E and B fields

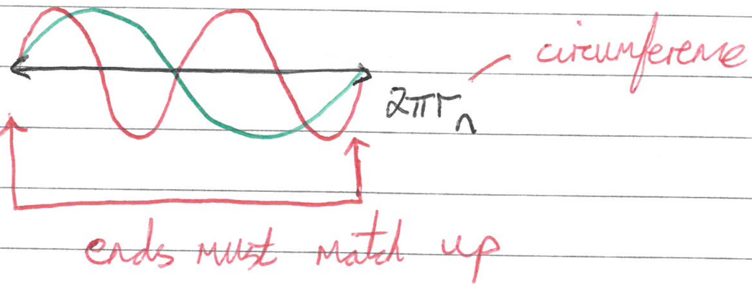
  For matter... ???
  In Lecture 9 we will see this amplitude is <u>complex</u> and not directly observable...

- Looking again at the Bohr model - electron waves

we want to get to: $E_n = \frac{-13.6\ eV}{n^2}$

- Starting point - electron wave must 'fit' in a circular orbit
- Electrons in potential well $V(r) = \frac{-e^2}{4\pi\varepsilon_0 r}$





$$2\pi r_n = n\lambda_n = n\frac{h}{p_n} = \underbrace{\frac{nh}{mv_n}}_{\text{note: non-relativistic}} \quad (1)$$

$n = 1, 2, 3 .$

And: $$F = ma = \frac{mv^2}{r}$$ (circular motion)

$$\frac{e^2}{4\pi\varepsilon_0 r_n^2} = \frac{m v_n^2}{r_n}$$

$$\frac{e^2}{4\pi\varepsilon_0 r_n} = m v_n^2 \qquad (2)$$

2 eqns. Exercise: eliminate v_n, solve for r_n to show:

$$\boxed{r_n = n^2 a_0} \qquad a_0 = \frac{\varepsilon_0 h^2}{\pi m e^2}$$

$$= 5.2918 \times 10^{-11}\ \mathrm{m}$$

'Bohr Radius'

OK, we know orbit radii, now find energy.

Back to (1): $$2\pi r_n = n\frac{h}{p_n}$$

sub in our r_n result

$$p_n = \frac{nh}{2\pi r_n} = \frac{h}{2\pi n a_0}$$

Kinetic Energy: $$T_n = \frac{p_n^2}{2m} = \frac{h^2}{8\pi^2 m n^2 a_0^2}$$

$$= \frac{me^4}{8\varepsilon_0^2 h^2 n^2} = +13.6\ \mathrm{eV}$$

Potential Energy:

$$U_n = \frac{-e^2}{4\pi\varepsilon_0 r_n} = \frac{-e^2}{4\pi\varepsilon_0 n^2 a_0}$$

$$= \frac{-me^4}{4\varepsilon_0^2 h^2 n^2} = -\frac{27.2}{n^2}\ \text{eV}$$

($-2 \times KE \Rightarrow$ Virial Theorem!)

Total energy:

$$E_n = T_n + U_n = -\frac{13.6}{n^2}\ \text{eV}$$

$\Rightarrow$ Bohr model result! QED

- Conclusions:

  - Matter has a de Broglie wavelength - wavey
  - Electron diffraction - particles found to interfere
  - Wave-like electron gives Bohr model result (and gives meaning to it)