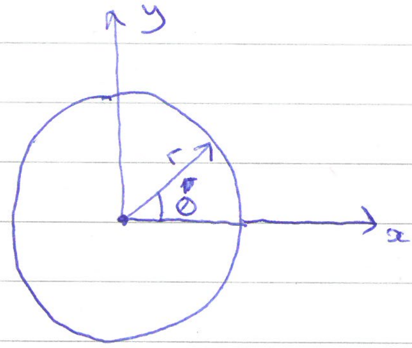
Proof $\int_{-\infty}^{\infty} e^{-ax^2}\,dx = \sqrt{\frac{\pi}{a}}$

$$I = \int_{-\infty}^{\infty} e^{-ax^2}\,dx = \int_{-\infty}^{\infty} e^{-ay^2}\,dy$$

$$I^2 = \int_{-\infty}^{\infty} e^{-ax^2}\,dx \int_{-\infty}^{\infty} e^{-ay^2}\,dy = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-a(x^2+y^2)}\,dxd$$



~~dx~~ $-\infty \le x \le \infty$

$-\infty \le y \le \infty$

$0 \le r \le \infty$

$0 \le \theta \le 2\pi$

$r^2 = x^2 + y^2$

(polar co-ordinate transform)

$dx\,dy = r\,dr\,d\theta$ (see lecture for explanati

$u = \cancel{2}ar^2$

$\frac{du}{dr} = 2ar$

$\frac{du}{2a} = r\,dr$

$$I^2 = \int_0^{2\pi} d\theta \int_0^{\infty} dr\, e^{-ar^2}\, r$$

$$I^2 = \int_0^{2\pi} d\theta \int_0^{\infty} e^{-u}\,\frac{du}{2a}$$

$[2\pi - 0] = 2\pi$

$$I^2 = \frac{2\pi}{2a}\left[-e^{-u}\right]_0^{\infty}$$

$$I^2 = \frac{\pi}{a}\left[0 - -1\right] = \frac{\pi}{a}$$

$$\boxed{\therefore I = \sqrt{\frac{\pi}{a}} = \int_{-\infty}^{\infty} e^{-ax^2}\,dx}$$