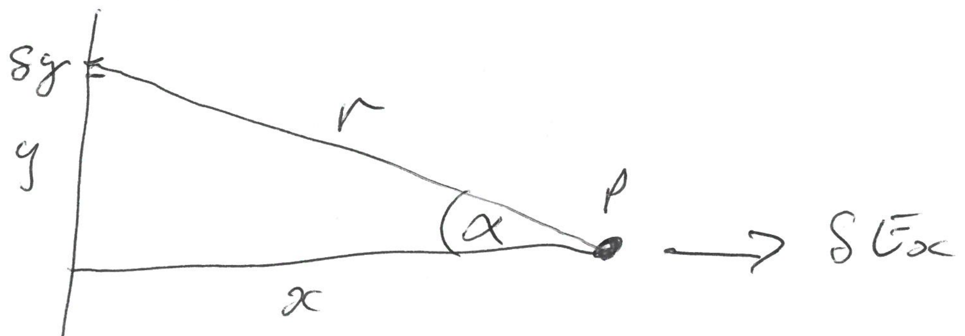
EM1 Lect 2

Example 2.1

E-field from thin line of charge λ per unit length. [length 2L]



$$\delta E_x = \frac{\delta Q}{4\pi\varepsilon_0 r^2} \cos\alpha$$

$$\delta Q = \lambda \delta y \qquad \cos\alpha = \frac{x}{r}$$

$$r^2 = y^2 + x^2$$

$$\therefore \delta E_x = \frac{\lambda \delta y}{4\pi\varepsilon_0} \frac{x}{[y^2 + x^2]^{3/2}}$$

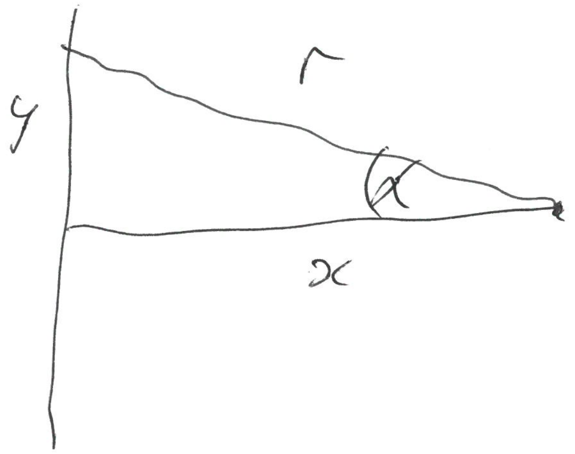
$$E_x = \frac{\lambda x}{4\pi\varepsilon_0} \int_{-L}^{L} \frac{dy}{[y^2 + x^2]^{3/2}}$$

Use standard integral (which would be given)

Example 2.1b

what about infinite line of charge?

As before: $\delta E_x = \dfrac{\lambda \delta y}{4\pi\varepsilon_0 r^2} \cos\alpha$



$$\cos\alpha = \frac{x}{r}$$

$$\Rightarrow r = \frac{x}{\cos\alpha}$$

$$\tan\alpha = \frac{y}{x} \Rightarrow y = x\tan\alpha$$

$$\therefore \delta y = \frac{x\,\delta\alpha}{\cos^2\alpha}$$

$$\left(\text{as } \frac{d(\tan\alpha)}{d\alpha} = \frac{1}{\cos^2\alpha} \right)$$

$$\delta E_x = \frac{\lambda}{4\pi\varepsilon_0} \underbrace{\left(\frac{x\,\delta\alpha}{\cos^2\alpha}\right)}_{\delta y} \underbrace{\left(\frac{\cos^2\alpha}{x^2}\right)}_{\frac{1}{r^2}} \cos\alpha$$

$$\delta E_x = \frac{\lambda}{4\pi\varepsilon_0 x} \cos\alpha \, \delta\alpha$$

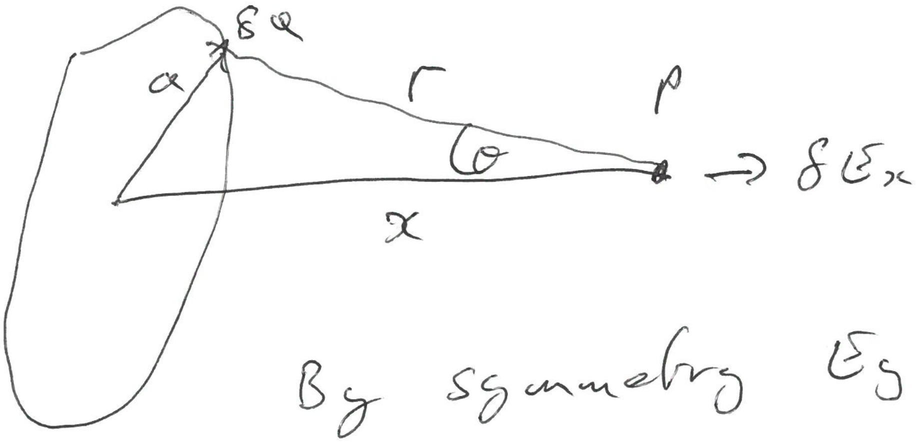
$$E_x = \frac{\lambda}{4\pi\varepsilon_0 x} \int_{-\pi/2}^{\pi/2} \cos\alpha \, d\alpha = \frac{\lambda}{2\pi\varepsilon_0 x} \int_0^{\pi/2} \cos\alpha \, d\alpha$$

$$E_x = \frac{\lambda}{2\pi\varepsilon_0 x}$$

Also ~~t~~ approx true for finite line if $x \ll L$.

Example 2.2

charged ring. Total charge Q



By symmetry $E_y = 0$

$$\delta E_x = \frac{\delta Q}{4\pi\varepsilon_0}\,\frac{1}{r^2}\cos\theta$$

$$\cos\theta = \frac{x}{r} \qquad r = \sqrt{a^2 + x^2}$$

$$\delta E_x = \frac{\delta Q}{4\pi\varepsilon_0}\,\frac{x}{\left[a^2 + x^2\right]^{3/2}}$$

So $$E_x = \frac{x}{4\pi\varepsilon_0}\,\frac{1}{\left[a^2+x^2\right]^{3/2}}\oint dQ$$

$$E_x = \frac{xQ}{4\pi\varepsilon_0}\,\frac{1}{\left[a^2+x^2\right]^{3/2}}$$