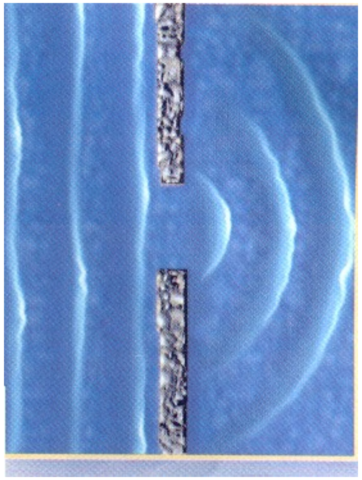
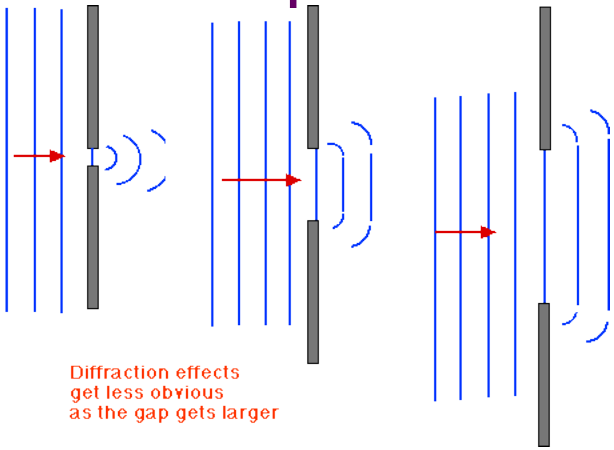
Lecture 19 Interference
Y&F Chapt 35

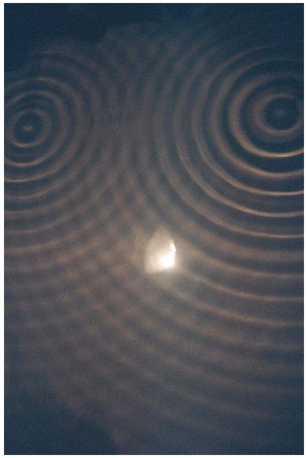


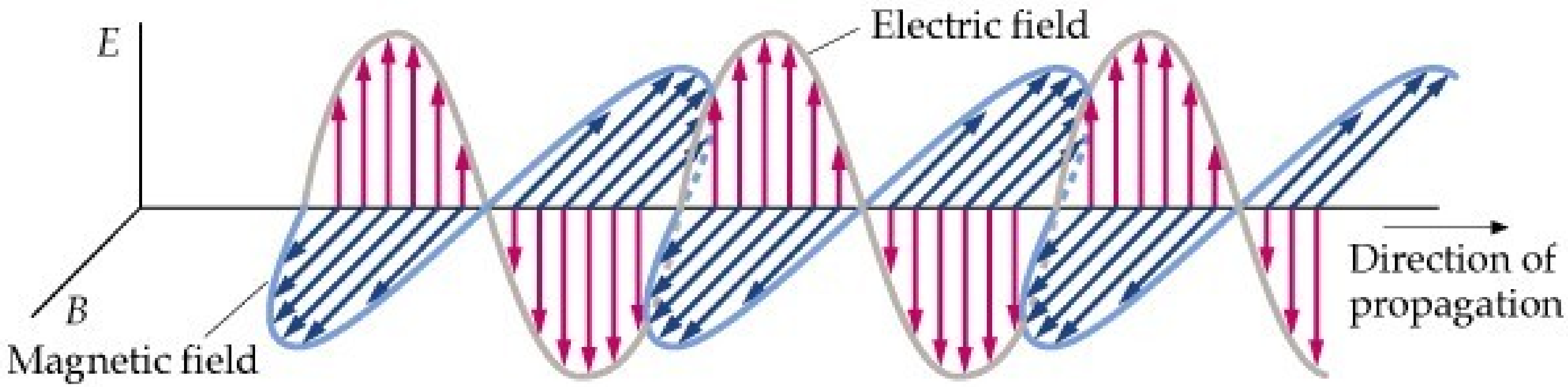


Geometric/ray optics. Lens/mirror are large.

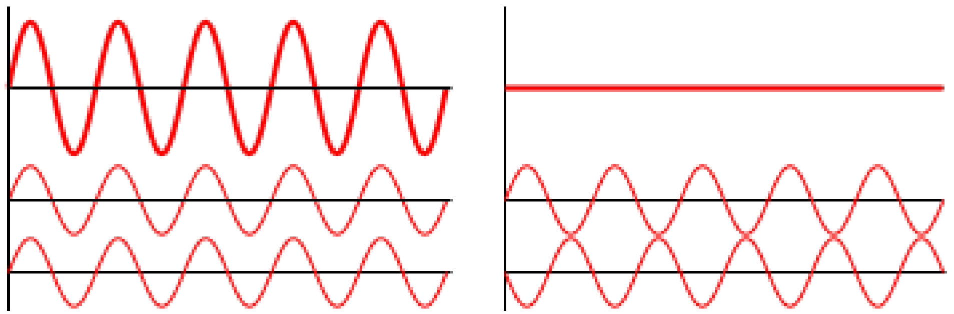
Diffraction: Waves `bending' around corners. (This happens with all waves, sound waves and EM waves etc.)

Interference: Waves interacting with one another. Standing waves are formed from interference.





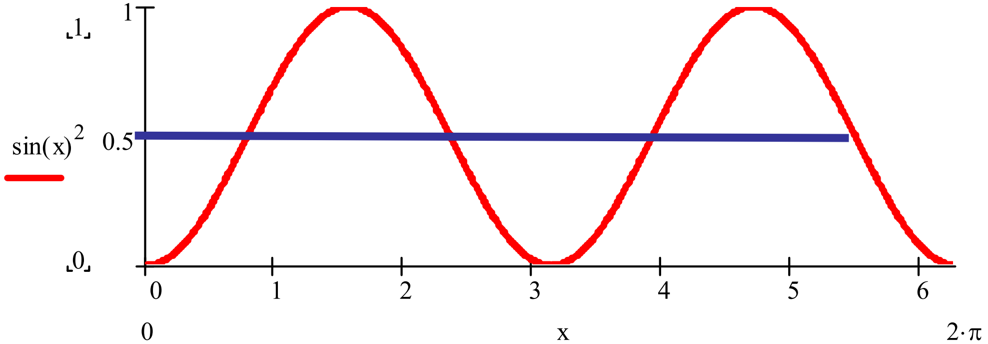
Visible light: Wave with a periodic changing electric/magnetic field.



Constructive interference

Destructive interference

For the superposition of two harmonic waves of the same frequency and wavelength, the resultant wave is also harmonic, but amplitude depends on phase difference –we better prove it



Why do we square the amplitude?

$$y_1 = A_1 \sin(kx - \omega t + \varphi_1)$$

$$y_2 = A_2 \sin(kx - \omega t + \varphi_2)$$

$$y = y_1 + y_2$$

$$y^2 = y_1^2 + y_2^2 + 2y_1y_2$$

$$I_1 = \left(y_1^2\right)_{average} = \frac{A_1^2}{2} \qquad I_2 = \left(y_2^2\right)_{average} = \frac{A_2^2}{2}$$

3rd term

$$2y_1y_2 = 2A_1A_2\sin(kx-\omega t+\varphi_1)\sin(kx-\omega t+\varphi_2)$$

$$\sin(kx-\omega t+\varphi_1) = \sin(kx+\varphi_1-\omega t)$$

Use : $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta,$

and $\alpha = kx+\varphi_1$

$\beta = \omega t,$ we get:

$$\sin(kx-\omega t+\varphi_1) = \sin(kx+\varphi_1-\omega t)$$

$$= \left[\sin(kx+\varphi_1)\cos(\omega t) - \cos(kx+\varphi_1)\sin(\omega t)\right]$$

By doing the same for the term involving φ_2, we get:

$$\text{3rd term} = 2A_1A_2\left[\sin(kx+\varphi_1)\cos(\omega t) - \cos(kx+\varphi_1)\sin(\omega t)\right]$$

$$\times\left[\sin(kx+\varphi_2)\cos(\omega t) - \cos(kx+\varphi_2)\sin(\omega t)\right]$$

$$= 2A_1A_2\left[\sin(kx+\varphi_1)\sin(kx+\varphi_2)\cos^2(\omega t)\right]+$$

$$2A_1A_2\left[\cos(kx+\varphi_1)\cos(kx+\varphi_2)\sin^2(\omega t)\right]-$$

$$2A_1A_2\left[\sin(kx+\varphi_1)\cos(kx+\varphi_2)+\cos(kx+\varphi_1)\sin(kx+\varphi_2)\right]\cos(\omega t)\sin(\omega t)$$

Time average

$$\left\langle \cos^2(\omega t)\right\rangle_T = \left\langle \sin^2(\omega t)\right\rangle_T = \frac{1}{2}$$

$$\left\langle \left[\sin(kx+\varphi_1)\cos(kx+\varphi_2)+\cos(kx+\varphi_1)\sin(kx+\varphi_2)\right]\cos(\omega t)\sin(\omega t)\right\rangle_T = 0$$

$$\left\langle Third\ term\right\rangle_T = 2A_1A_2\frac{1}{2}\left[\sin(kx+\varphi_1)\sin(kx+\varphi_2)\right]+$$

$$2A_1A_2\frac{1}{2}\left[\cos(kx+\varphi_1)\cos(kx+\varphi_2)\right]$$

$$= A_1A_2\left[\sin(kx+\varphi_1)\sin(kx+\varphi_2)+\cos(kx+\varphi_1)\cos(kx+\varphi_2)\right]$$

use $\cos\alpha\cos\beta+\sin\alpha\sin\beta=\cos(\alpha\text{-}\beta)$,

and $kx+\varphi_2=\alpha;\quad kx+\varphi_2=\beta$

$$\langle Third\, term \rangle_T = 2A_1A_2\left[\cos(kx+\varphi_1)-(kx+\varphi_2)\right]$$

$$= A_1A_2\left[\cos(\varphi_1-\varphi_2)\right]$$

$$= 2(I_1I_2)^{1/2}\cos\delta$$

$$I = \frac{\left(A_1^2 + A_2^2\right)}{2} + A_1A_2\cos\delta$$

δ : phase difference between the two waves.

$$I = \frac{\left(A_1^2 + A_2^2\right)}{2} + A_1 A_2 \cos\delta = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

The maximum intensity is observed for

δ=0,2π, 4π,

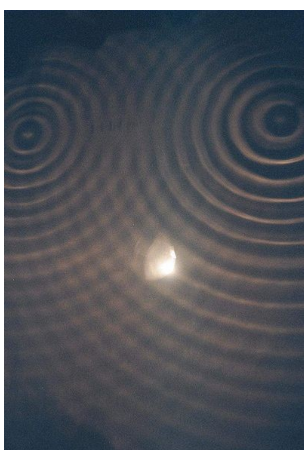
$$I_{\max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

If $I_1 = I_2 = I_0$, Then

$$I_{\max} = I_0 + I_0 + 2I_0 = 4I_0$$

The minimum intensity is observed for

δ=π, 3π....



$$I_{\min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$$

If $I_1 = I_2 = I_0$, Then

$$I_{\min} = I_0 + I_0 - 2I_0 = 0$$

If more than two waves:

$$y_1 = A_1 \sin(kx - \omega t + \varphi_1)$$

$$y_2 = A_2 \sin(kx - \omega t + \varphi_2)$$

$$\cdots\cdots$$

$$y_n = A_n \sin(kx - \omega t + \varphi_n)$$

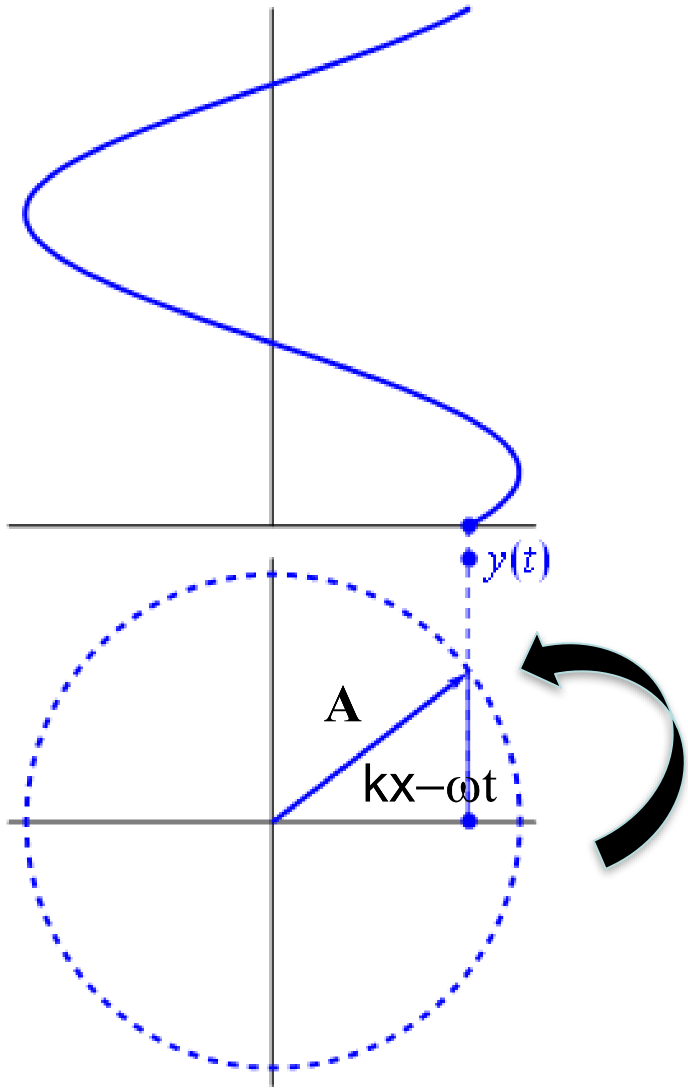
$$y = y_1 + y_2 + \ldots\ldots + y_n$$

$$y^2 =$$

Not difficult to do, but rather tedious, the alternative and much easier way is to use phasors

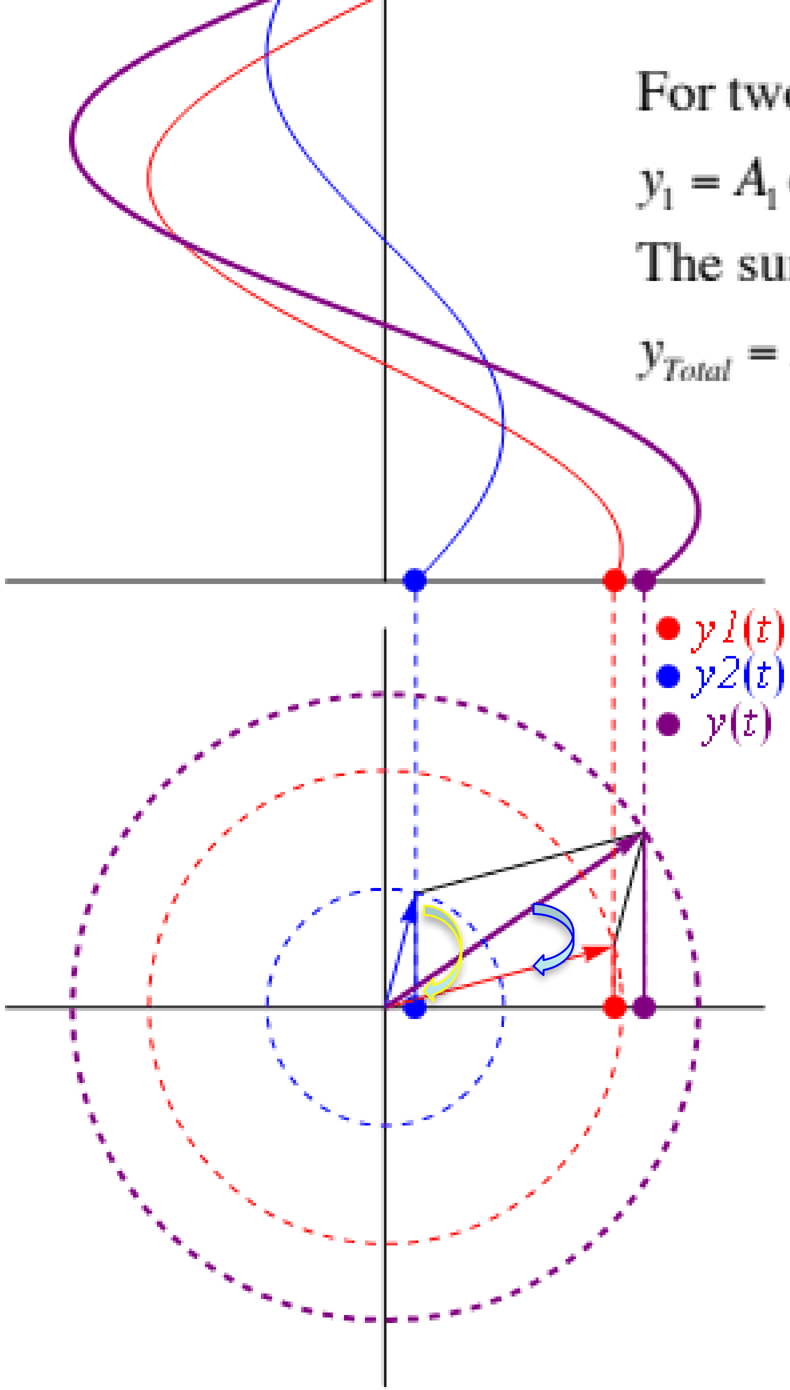
Phasor diagram

The wave $y = A\cos(kx - \omega t)$ can be represened by a rotating vector.



The rotating vector has a magnitude A, It rotates with angular frequency ω. The projection of this vector on the horizontal axis gives displacement y.

phasor



For two waves:

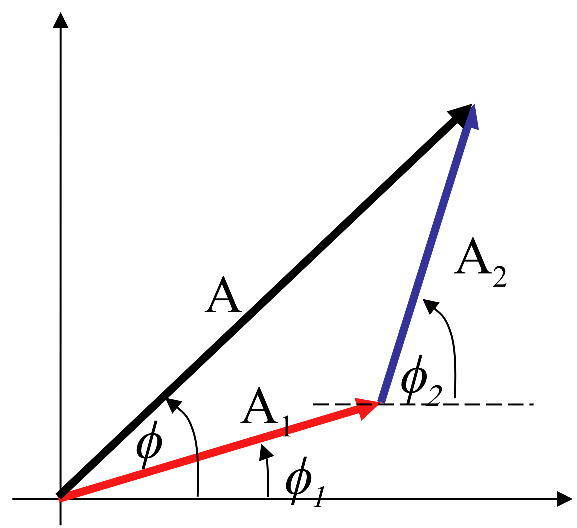
$y_1 = A_1 \cos(kx - \omega t)$ and $y_2 = A_2 \cos(kx - \omega t + \phi)$.

The sum of these two waves is:

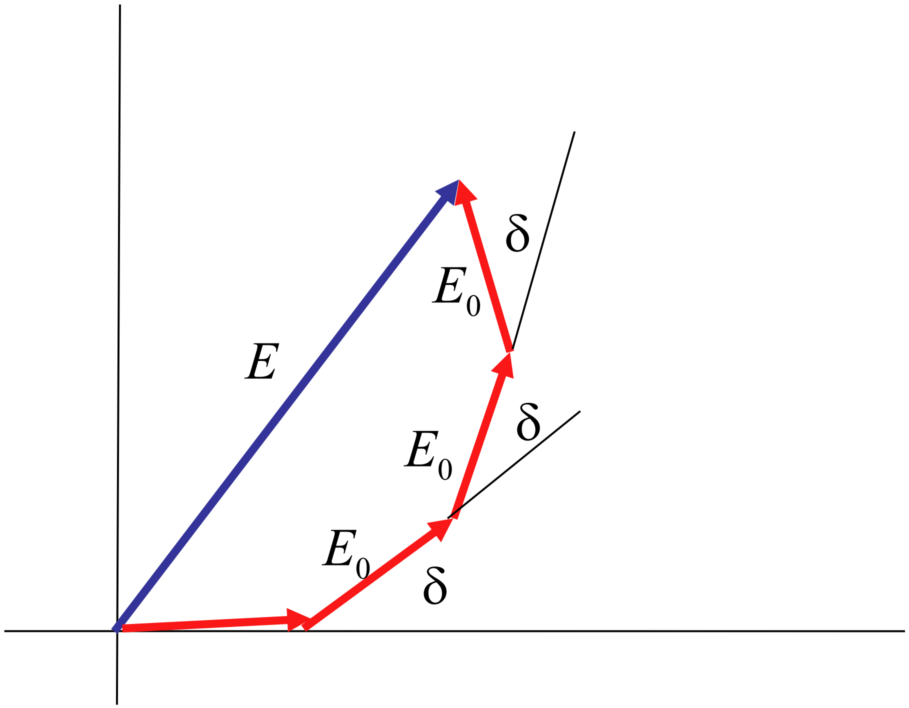
$y_{Total} = A_{Total} \cos(kx - \omega t + \beta)$

y_2 leads y_1 by ϕ, y_{Total} leads y_1 by β

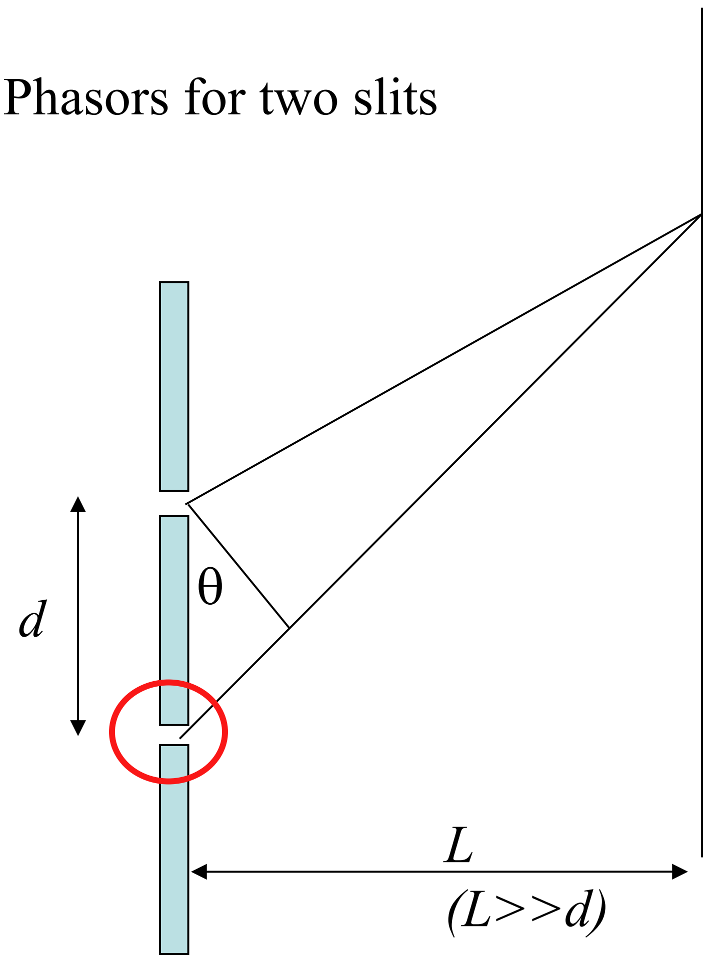
Instead of finding the projections of vectors $\mathbf{A_1}$ and $\mathbf{A_2}$, we add the vectors first, then find the projection of $\mathbf{A_{total}}$.



For many waves: (for simplicity, we consider waves of the same amplitude)



Phasors for two slits



$$\Delta = d\sin\theta$$

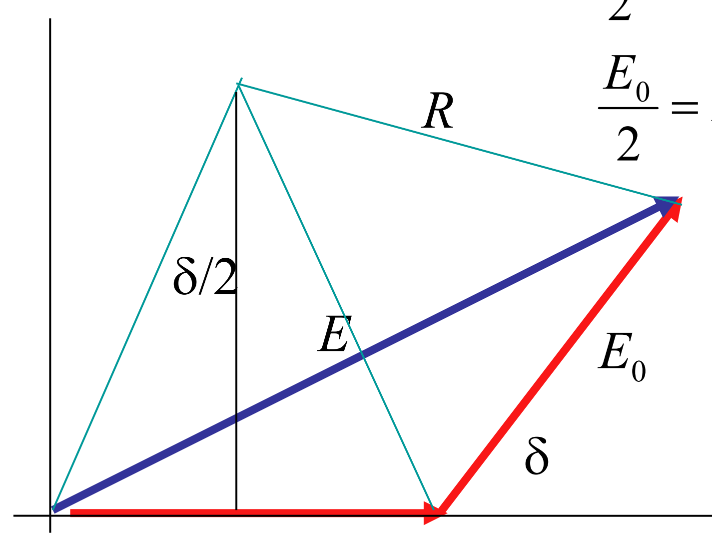
Phase diff

$$\delta = \frac{2\pi}{\lambda}\Delta = \frac{2\pi}{\lambda} d\sin\theta$$

Waves received at screen is sum of all the secondary waves from the slits – which all have the same intensity.

$$\frac{E}{2} = R\sin(\delta)$$

$$\frac{E_0}{2} = R\sin\left(\frac{\delta}{2}\right)$$

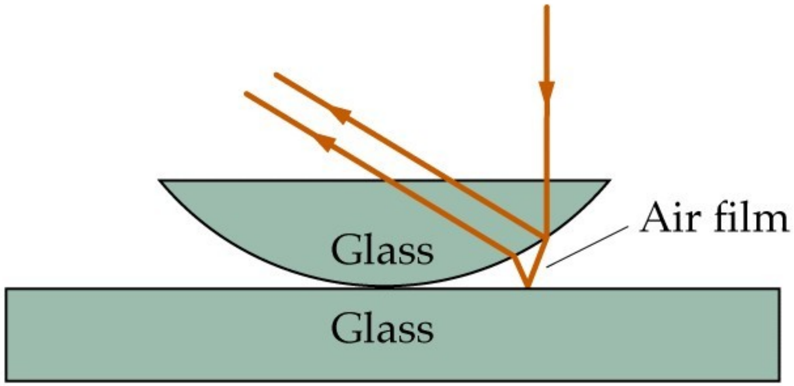


$$I = E^2 = (2R)^2 \sin^2(\delta)$$

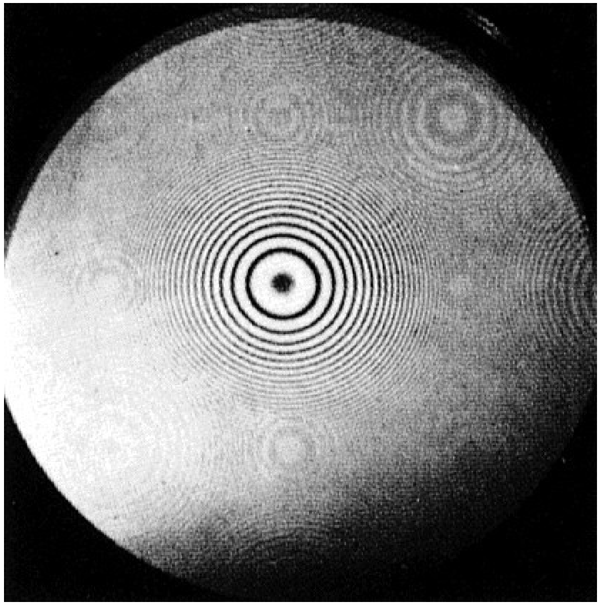
$$I = E_0{}^2 \frac{\sin^2(\delta)}{\sin^2\left(\frac{\delta}{2}\right)} = I_0 \frac{\sin^2(\delta)}{\sin^2\left(\frac{\delta}{2}\right)}$$

$$I = I_0 \frac{\sin^2(\delta)}{\sin^2\left(\frac{\delta}{2}\right)} = I_0 \frac{4\sin^2\left(\frac{\delta}{2}\right)\cos^2\left(\frac{\delta}{2}\right)}{\sin^2\left(\frac{\delta}{2}\right)} = 4 I_0 \cos^2\left(\frac{\delta}{2}\right)$$

Newton’s Rings



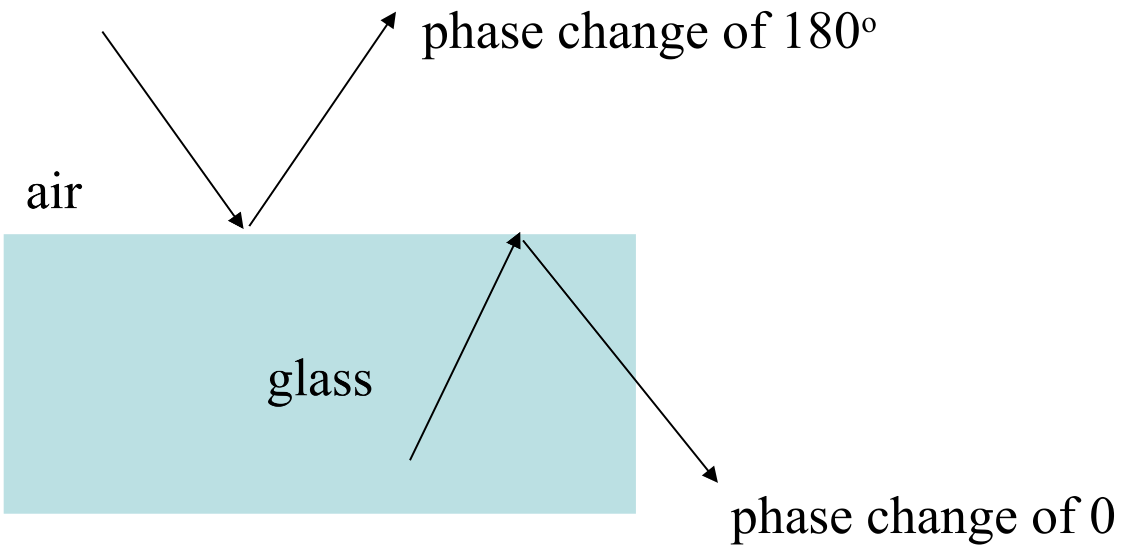
(b)



Reflection

low n $\rightarrow$ high n: inverted (phase change of 180°)

high n $\rightarrow$ low n: not inverted (phase change of 0)



For waves reflected and transmitted at a boundary

$$B = \frac{2k_1}{k_1 + k_2} A, \quad C = \frac{k_1 - k_2}{k_1 + k_2} A$$

B is the amplitude of the transmitted wave,

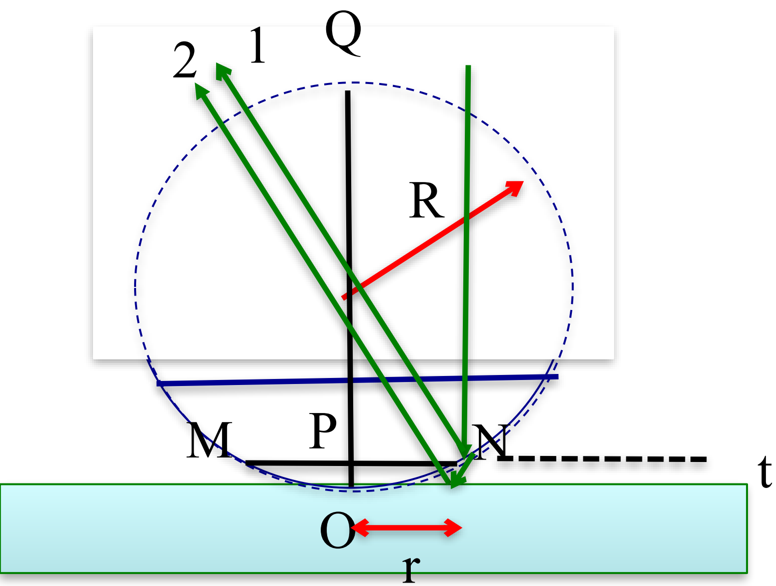
C is the amplitude of the refelcted wave

$$k = \frac{\omega}{v} = \frac{\omega}{\frac{c}{n}} = \frac{\omega n}{c},$$

C is the velocity of light in vacuum, n is the refractive index

$$k_1 = \frac{\omega}{v_1} = \frac{\omega}{\frac{c}{n_1}} = \frac{\omega n_1}{c} \qquad k_2 = \frac{\omega}{v_2} = \frac{\omega}{\frac{c}{n_2}} = \frac{\omega n_2}{c},$$

$$B = \frac{2n_1}{n_1 + n_2} A, \quad C = \frac{n_1 - n_2}{n_1 + n_2} A$$



Optical path for ray 2 is longer by 2t

$$(MP)\times(PN)=(QP)\times(PO)$$

$$r^2=(2R-t)t=2Rt-t^2$$

$$=2Rt, \text{ for } t<<R$$

Thus,

$$t=\frac{r^2}{2R}$$

For a bright ring with radius r,

$$2t+\frac{\lambda}{2}=n\lambda, \quad n=1,2,3....$$

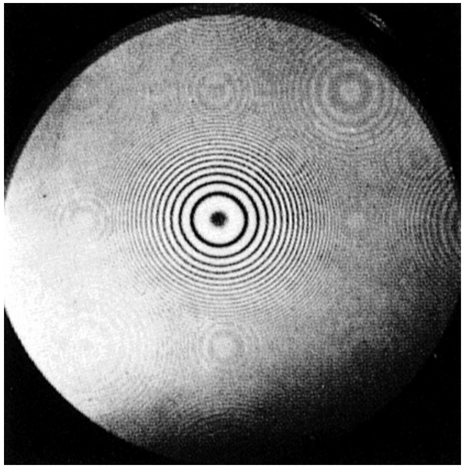
$$2t=\frac{2n-1}{2}\lambda$$

$$r=\sqrt{\frac{2n-1}{2}\lambda R}$$

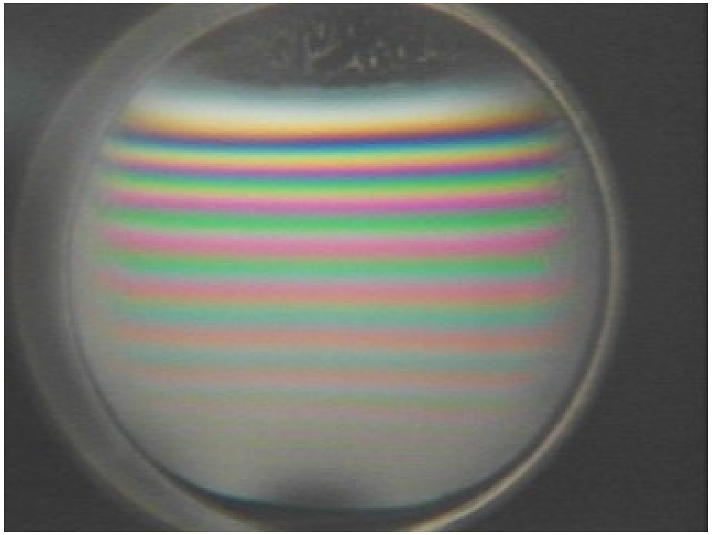
For a dark ring:

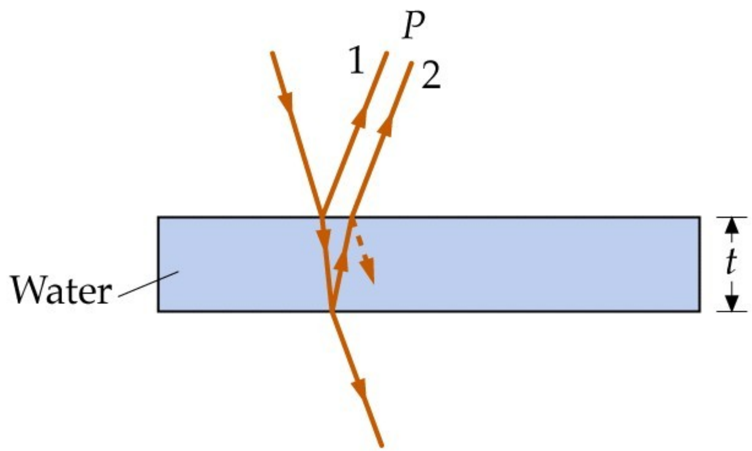
$$2t=n\lambda, \quad n=0,1,2,3,...$$

Why is it dark in the centre?



Interference in dielectric films





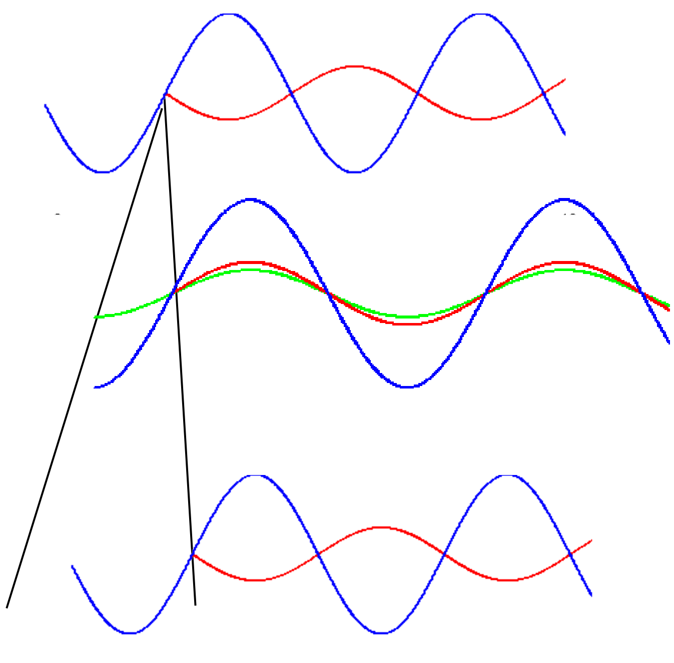
Ray 1 is inverted (phase change of 180°).
Ray 2 no phase change.
Rays 1 and 2 interfere.

Film thickness

destr.

constr.

destr.



Phase diff = $2\pi . d/\lambda + \pi$ $\quad$ d=2t
note λ is wavelength in medium.
Phase changes differs for blue and red