UNIVERSITY OF BIRMINGHAM

School of Physics and Astronomy

DEGREE OF B.Sc. & M.Sci. WITH HONOURS

FIRST-YEAR EXAMINATION

03 19718

LC QUANTUM MECHANICS / OPTICS & WAVES

SUMMER EXAMINATIONS 2019

Time Allowed: 2 hours

Answer Section 1 and two questions from Section 2.

Section 1 counts for 28% of the marks for the examination.
Full marks for this Section can be obtained by correctly answering ***four*** questions.
You may attempt more questions, but marks in excess of 28% will be disregarded.

Section 2 consists of two questions and carries 72% of the marks.
Answer ***both*** questions from this Section. Note that each question has two parts, of which only ***one part*** should be answered. If you answer both parts, credit will only be given for the best answer.

The approximate allocation of marks to each part of a question is shown in brackets **[ ]**.

PLEASE USE A SEPARATE ANSWER BOOK FOR SECTION 1 AND SECTION 2 QUESTIONS.

Calculators may be used in this examination but must not be used to store text. Calculators with the ability to store text should have their memories deleted prior to the start of the examination.

A table of physical constants and units that may be required will be found at the end of this question paper.

SECTION 1

*Full marks for this section can be obtained by correctly answering **four** questions. You may attempt as many questions as you wish, but any marks in excess of 28% will be disregarded.*

1. Give one reason why the photoelectric effect cannot be explained in terms of the classical wave picture of light.
   Electrons with a maximum kinetic energy of $1.7\ \mathrm{eV}$ are emitted when a metal is illuminated by light of frequency $1.0 \times 10^{15}\ \mathrm{Hz}$. Calculate the longest wavelength of light for which photoelectrons can be ejected from the metal. **[5]**

2. Light travelling through air is incident on a piece of optical glass. Explain what happens, and why, when the light is reflected at the angle called *Brewster's angle*. Derive an expression for Brewster's angle in terms of the refractive index of glass (you may assume that the refractive index of air, $n_{air} = 1$). What is Brewster's angle for optical glass with a refractive index of 1.4? **[5]**

3. When electrons in He^+ ions (helium atoms with one electron removed) are excited to the $n = 3$ level, photons are emitted with wavelengths $1.67 \times 10^{-7}\ \mathrm{m}$, $2.57 \times 10^{-8}\ \mathrm{m}$ and $3.05 \times 10^{-8}\ \mathrm{m}$. Estimate the ionisation energy of the electron in the ground state of He^+. **[5]**

4. Use the method of phasors to derive the intensity variation of the diffraction pattern for five slits of negligible width, each separated by a distance d. **[5]**

5. A particle confined to a one-dimensional potential well between $0 \leq x \leq L$ is described by the normalised wave function

$$\Psi(x,t) = \sqrt{\frac{2}{L}} \sin\left(\frac{\pi x}{L}\right) \exp(-i\omega t) .$$

Find the probability of finding the particle in the region $0 \leq x \leq \frac{L}{4}$.
How does this result differ from the classical expectation? **[5]**

6. A police radar gun operating at a frequency of $30\ \mathrm{GHz}$ is used to detect the speed of an approaching car. If a beat frequency between the transmitted waves and those reflected from the car of $2\ \mathrm{kHz}$ is observed, at what speed was the car moving in units of $\mathrm{km/h}$?
[You may use the fact that the Doppler shift for $v \ll c$ is given by $\Delta f / f_0 = v/c$.] **[5]**

SECTION 2

*Answer **both** questions from this Section. Note that each question has two parts, of which only **one part** should be answered. If you answer both parts, credit will only be given for the best answer.*

7. **EITHER (Part A)**

(a) Briefly describe the key features of the Davisson and Germer experiment and explain how it provided evidence for the wave nature of electrons. **[5]**

(b) When $100\ \mathrm{eV}$ electrons are incident upon a crystal, peaks in scattered intensity are observed as the crystal is rotated. If the largest angle between the incident and scattered beams at which a peak is observed is 126 degrees, calculate the angles at which all further peaks in the scattered intensity will be observed. **[9]**

(c) What energy of X-rays would give rise to peaks in reflected intensity at the same angles as those found in part (b)? **[2]**

(d) Use the Bohr model of the atom to obtain an expression for the approximate energy of K_α X-rays as a function of atomic number, Z. State any assumptions that you make. Hence show that the energy of K_α X-rays from copper ($Z = 29$) is approximately $8\ \mathrm{keV}$. **[6]**

(e) The X-ray absorption spectrum of copper has L absorption edges clustered around $950\ \mathrm{eV}$. If a sample of copper is bombarded with electrons, approximately what is the minimum electron energy required to observe K_α X-rays? Explain whether or not you would also expect to observe K_β X-rays. **[3]**

7. **OR (Part B)**

(a) The time-independent Schrödinger equation (TISE) in one dimension is

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x).$$

Using the TISE as an example, explain what is meant by the term *eigenvalue equation* in quantum mechanics. Identify the operators in the TISE and explain what they represent physically. **[5]**

(b) A particle of mass, m, is confined in an infinite one-dimensional potential well in the region $0 \leq x \leq L$. The non-zero potential inside the well is $V(x) = V_0$. Show that the wave function

$$\psi(x) = Ae^{ikx} + Be^{-ikx}$$

is a solution to the TISE in this case and find an expression for the wavenumber, k, in terms of E, V_0 and m. **[4]**

(c) State and explain the boundary condition that the wave function must satisfy at the boundaries of the well. Hence show that $A = -B$ and show that the allowed values of k in this problem are $k = \frac{n\pi}{L}$, where n is a positive non-zero integer. **[5]**

(d) Use the answer to part (c) to show that expectation value for the momentum in any state is equal to zero. (Hint: You do not need to normalise the wave function.) **[7]**

(e) Explain why the momentum of the particle inside the well cannot be equal to zero. Explain how your answer to this question is consistent with the expectation value for the momentum of the particle from part (d). **[4]**

8. **EITHER (Part A)**

(a) A wave is given by

$$y(x,t) = 0.01 \sin(3x + 6t),$$

where you may assume SI units for all parameters.
What is its amplitude and speed? In which direction does it travel? **[3]**

(b) A wave travels on a string of mass per unit length μ held under tension T. By considering a small element of length dx, show that the associated kinetic and potential energies are given by

$$\Delta \mathrm{KE} = \frac{1}{2}\,\mu\,dx\left(\frac{\partial y}{\partial t}\right)^2,$$

and

$$\Delta \mathrm{U} = \frac{1}{2}\,T\,dx\,\left(\frac{\partial y}{\partial x}\right)^2.$$

[Note that $(1+x)^n = 1 + nx + n(n-1)x^2/2! + \ldots$]

For the wave in part (a) deduce the kinetic and potential energies associated with an element dx, in terms of x, t, T and μ. **[10]**

(c) For the wave in part (a) deduce the average power transmitted along a string of mass per unit length $\mu = 0.02\ \mathrm{kg/m}$, held under a tension of $0.02\ \mathrm{N}$. **[6]**

(d) The wave passes from the string with mass per unit length of $\mu = 0.02\,\mathrm{kg/m}$ to a connected string of $\mu = 0.04\ \mathrm{kg/m}$. What are the amplitudes of the reflected and transmitted waves? **[6]**

8. **OR (Part B)**

(a) i. For a sphere with radius r and refractive index n_2 embedded in a medium with a refractive index n_1 the propagation of light from the medium into the sphere can be described by

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{r}.$$

Define the terms s and s' and give the sign conventions for s, s' and r. **[5]**

ii. Show that for a thin spherical lens in air (assume $n_{air} = 1$) the above equation becomes

$$\frac{1}{s} + \frac{1}{s'} = (n_l - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right),$$

where r_1 and r_2 are the radii of curvature of the two surfaces and n_l is the refractive index of the lens. **[8]**

(b) Two thin lenses are placed $20\ \mathrm{cm}$ apart. They both have a focal length of $10\ \mathrm{cm}$. An object $2\ \mathrm{mm}$ tall is placed $50\ \mathrm{cm}$ in front of the first lens. Where is the image produced by the second lens formed? What is the size of this image? **[7]**

(c) A thin film of oil (refractive index = 1.4) floats on the surface of water (refractive index = 1.3). Light is reflected from the film at normal incidence. The reflected light is observed to be red in colour ($\lambda = 650\ \mathrm{nm}$). What is the minimum thickness of the film? **[5]**

Physical Constants and Units

Acceleration due to gravity	g	$9.81\,\mathrm{m\,s^{-2}}$
Gravitational constant	G	$6.674 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$
Ice point	T_{ice}	$273.15\,\mathrm{K}$
Avogadro constant	N_A	$6.022 \times 10^{23}\,\mathrm{mol^{-1}}$
		[*N.B.* 1 mole $\equiv 1$ *gram-molecule*]
Gas constant	R	$8.314\,\mathrm{J\,K^{-1}\,mol^{-1}}$
Boltzmann constant	k, k_B	$1.381 \times 10^{-23}\,\mathrm{J\,K^{-1}} \equiv 8.62 \times 10^{-5}\,\mathrm{eV\,K^{-1}}$
Stefan constant	σ	$5.670 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$
Rydberg constant	R_∞	$1.097 \times 10^{7}\,\mathrm{m^{-1}}$
	$R_\infty hc$	$13.606\,\mathrm{eV}$
Planck constant	h	$6.626 \times 10^{-34}\,\mathrm{J\,s} \equiv 4.136 \times 10^{-15}\,\mathrm{eV\,s}$
$h/2\pi$	$\hbar$	$1.055 \times 10^{-34}\,\mathrm{J\,s} \equiv 6.582 \times 10^{-16}\,\mathrm{eV\,s}$
Speed of light *in vacuo*	c	$2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$
	$\hbar c$	$197.3\,\mathrm{MeV\,fm}$
Charge of proton	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Mass of electron	m_e	$9.109 \times 10^{-31}\,\mathrm{kg}$
Rest energy of electron		$0.511\,\mathrm{MeV}$
Mass of proton	m_p	$1.673 \times 10^{-27}\,\mathrm{kg}$
Rest energy of proton		$938.3\,\mathrm{MeV}$
One atomic mass unit	u	$1.66 \times 10^{-27}\,\mathrm{kg}$
Atomic mass unit energy equivalent		$931.5\,\mathrm{MeV}$
Electric constant	ϵ_0	$8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Magnetic constant	μ_0	$4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$
Bohr magneton	μ_B	$9.274 \times 10^{-24}\,\mathrm{A\,m^2}$ ($\mathrm{J\,T^{-1}}$)
Nuclear magneton	μ_N	$5.051 \times 10^{-27}\,\mathrm{A\,m^2}$ ($\mathrm{J\,T^{-1}}$)
Fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	$7.297 \times 10^{-3} = 1/137.0$
Compton wavelength of electron	$\lambda_c = h/m_e c$	$2.426 \times 10^{-12}\,\mathrm{m}$
Bohr radius	a_0	$5.2918 \times 10^{-11}\,\mathrm{m}$
angstrom	Å	$10^{-10}\,\mathrm{m}$
barn	b	$10^{-28}\,\mathrm{m^2}$
torr (mm Hg at 0 °C)	torr	$133.32\,\mathrm{Pa}$ ($\mathrm{N\,m^{-2}}$)

Do not complete the attendance slip, fill in the front of the answer book or turn over the question paper until you are told to do so

Important Reminders

- Coats/outwear should be placed in the designated area.
- Unauthorised materials (e.g. notes or Tippex) must be placed in the designated area.
- Check that you do not have any unauthorised materials with you (e.g. in your pockets, pencil case).
- Mobile phones and smart watches **must** be switched off and placed in the designated area or under your desk. They must not be left on your person or in your pockets.
- You are not permitted to use a mobile phone as a clock. If you have difficulty seeing a clock, please alert an Invigilator.
- You are not permitted to have writing on your hand, arm or other body part.
- Check that you do not have writing on your hand, arm or other body part – if you do, you must inform an Invigilator immediately
- Alert an Invigilator immediately if you find any unauthorised item upon you during the examination.

Any students found with non-permitted items upon their person during the examination, or who fail to comply with Examination rules may be subject to Student Conduct procedures.