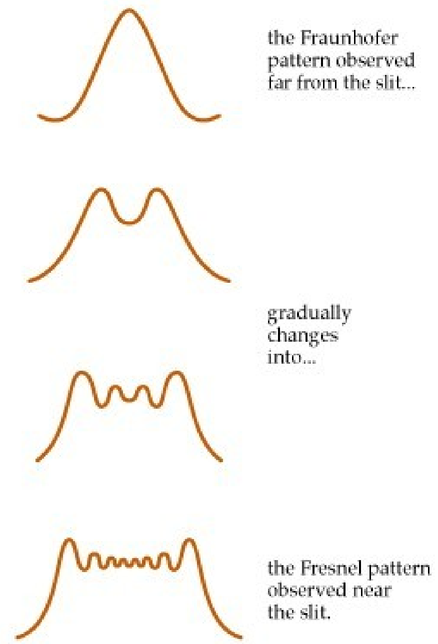
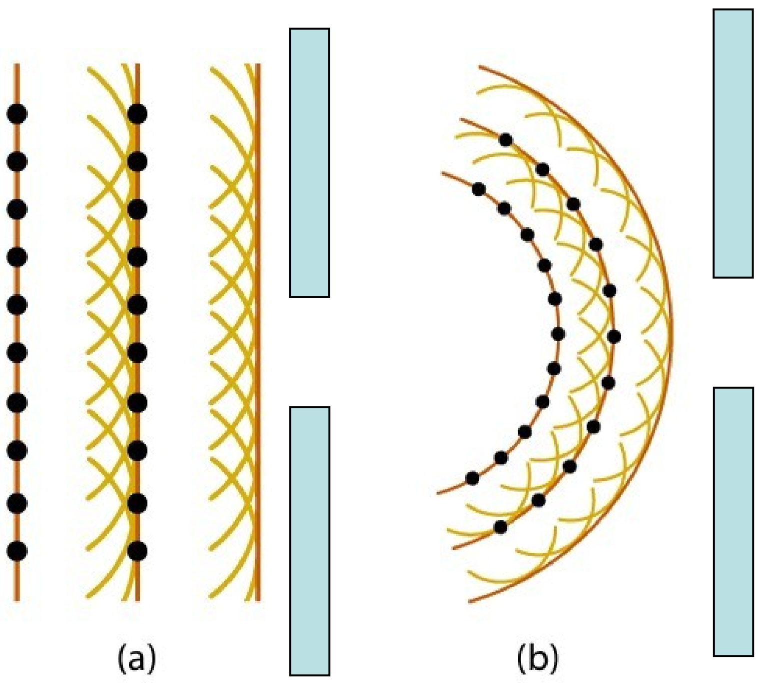
Fraunhofer and Fresnel Diffraction

Fraunhofer – far field
when incoming and outgoing waves can be considered planar or parallel

Fresnel – near field
when the slit and source are at finite distances from each other



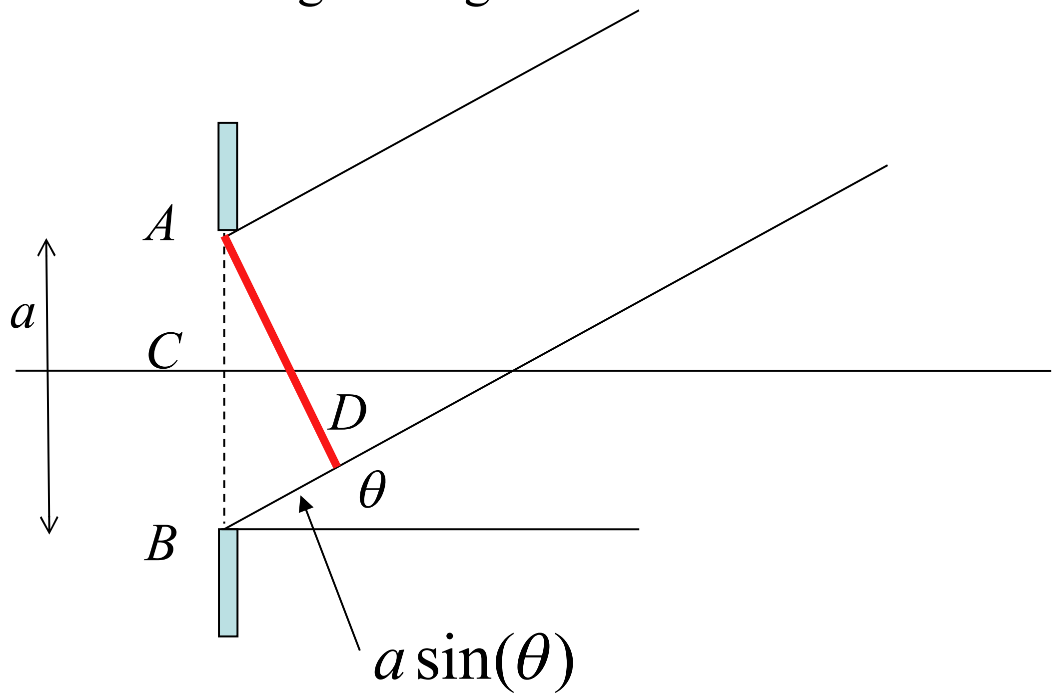


Fraunhofer if: $$d > \frac{a^2}{\lambda}$$

d is the smaller of source and screen distance from slit, and a is slit size

Here we consider only Fraunhofer as it is simpler!

Diffraction through a single slit



Consider wavefront to be composed of a large number of emitters *A* to *B* *(Huygens)*

Total phase difference between wavelets from *A* and *B* is:

$$\phi = 2\pi \frac{a\sin(\theta)}{\lambda}$$

Phase diff of each source is: $\dfrac{\phi}{N} = \dfrac{2\pi}{N}\dfrac{a\sin(\theta)}{\lambda}$

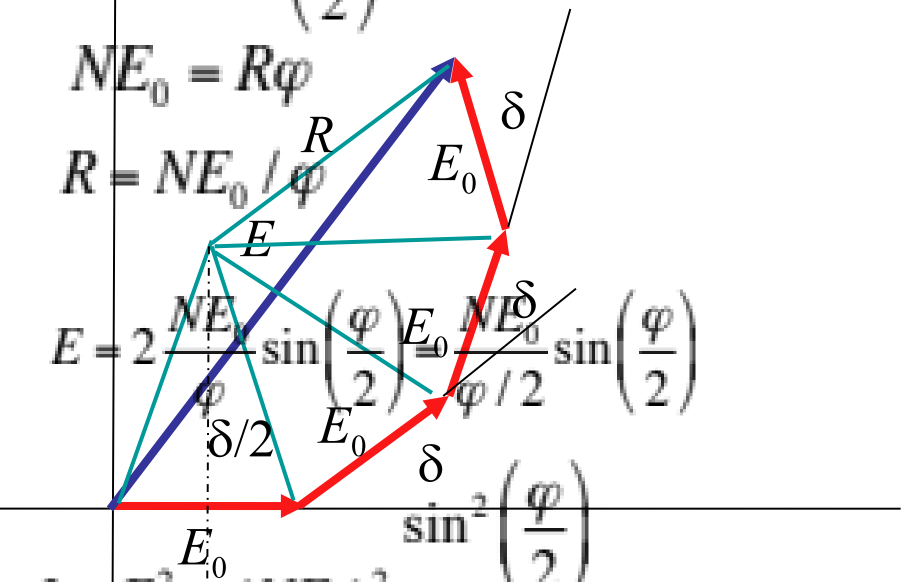
4 slits

$$E = 2R\sin\left(\frac{\varphi}{2}\right)$$

$$NE_0 = R\varphi$$

$$R = NE_0 / \varphi$$

$$E = 2\frac{NE_0}{\varphi}\sin\left(\frac{\varphi}{2}\right) = \frac{NE_0}{\varphi/2}\sin\left(\frac{\varphi}{2}\right)$$

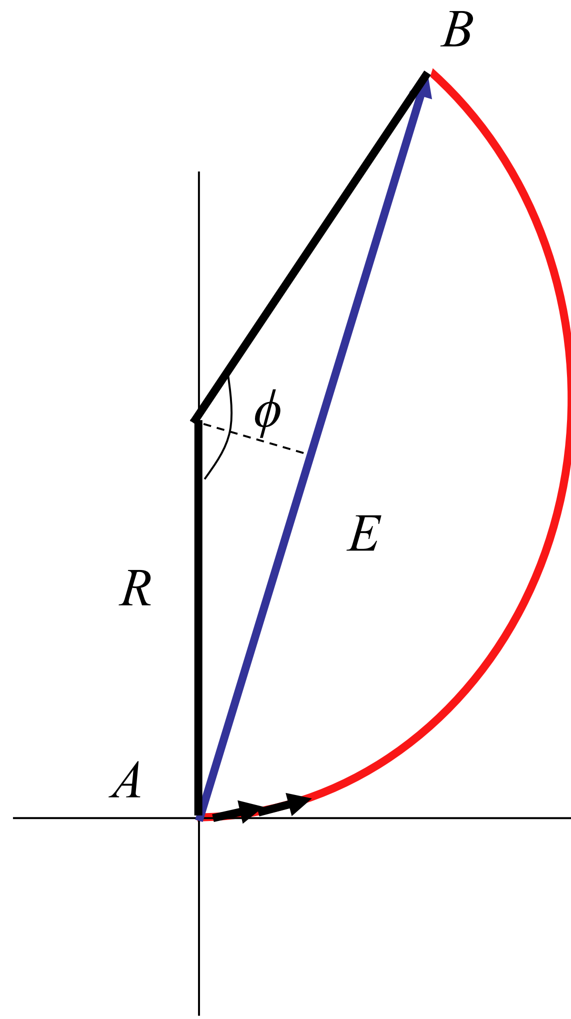


$$I \propto E^2 = (NE_0)^2 \frac{\sin^2\left(\frac{\varphi}{2}\right)}{(\varphi/2)^2}$$

$$\varphi = 0, E = NE_{0},$$

$$I(0) = N^2 E_0^2,$$

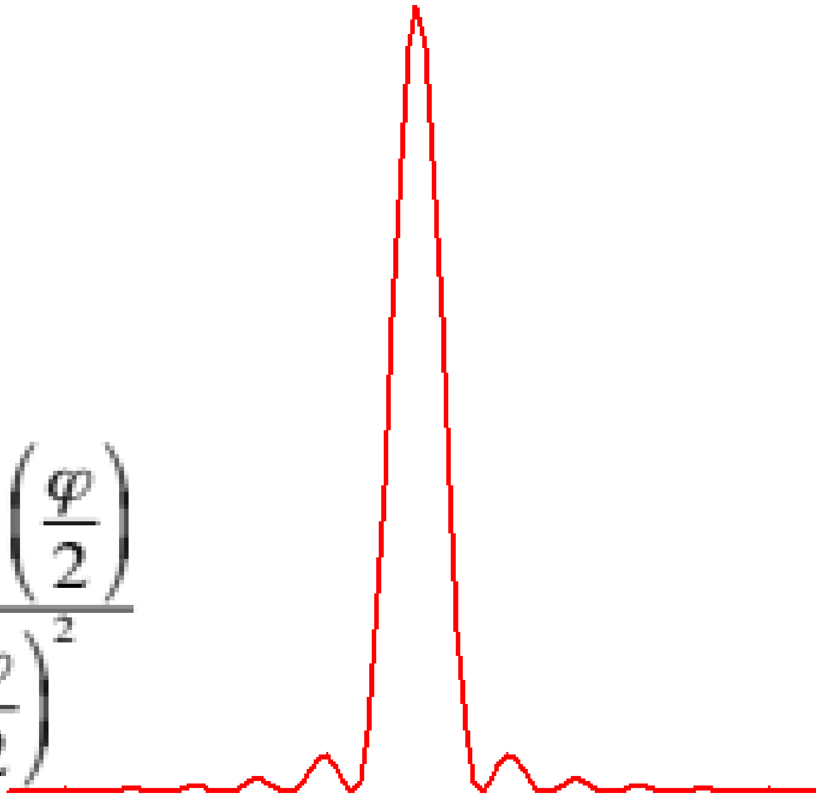
this is the intensity for $\varphi = 0$, i.e. $\theta = 0$



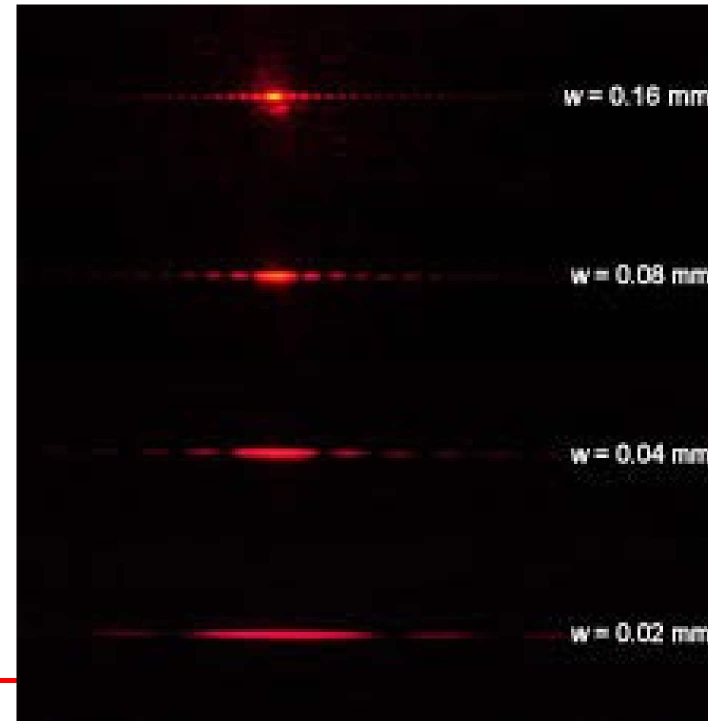
$$\phi = 2\pi\,\frac{a\sin(\theta)}{\lambda}$$

$$\frac{I}{I(0)} = \frac{E^2}{N^2 E_0^2} = \frac{\sin^2\left(\frac{\varphi}{2}\right)}{\left(\frac{\varphi}{2}\right)^2}$$

$$\phi = 2\pi \frac{a \sin(\theta)}{\lambda}$$



$$I = I_0 \frac{\sin^2\left(\frac{\varphi}{2}\right)}{\left(\frac{\varphi}{2}\right)^2}$$



I_0=I(0), the intensity at zero angle

Compute location of max and minima

I is a max/min when dI/dϕ=0

$$I = I_0 \frac{\sin^2\left(\frac{\varphi}{2}\right)}{\left(\frac{\varphi}{2}\right)^2} = uv, \quad \sin^2\left(\frac{\varphi}{2}\right) = u, \quad \frac{1}{\left(\frac{\varphi}{2}\right)^2} = v$$

$$\frac{dI}{d\varphi} = \frac{2\sin\left(\frac{\varphi}{2}\right)\cos\left(\frac{\varphi}{2}\right)\frac{1}{2}}{\left(\frac{\varphi}{2}\right)^2} + \frac{\sin^2\left(\frac{\varphi}{2}\right)(-2)\frac{1}{2}}{\left(\frac{\varphi}{2}\right)^3} = 0$$

$$\frac{\sin\left(\frac{\varphi}{2}\right)\cos\left(\frac{\varphi}{2}\right)}{\left(\frac{\varphi}{2}\right)^2} = \frac{\sin^2\left(\frac{\varphi}{2}\right)}{\left(\frac{\varphi}{2}\right)^3}$$

$$\sin\left(\frac{\varphi}{2}\right)\left(\frac{\varphi}{2}\cos\left(\frac{\varphi}{2}\right) - \sin\left(\frac{\varphi}{2}\right)\right) = 0$$

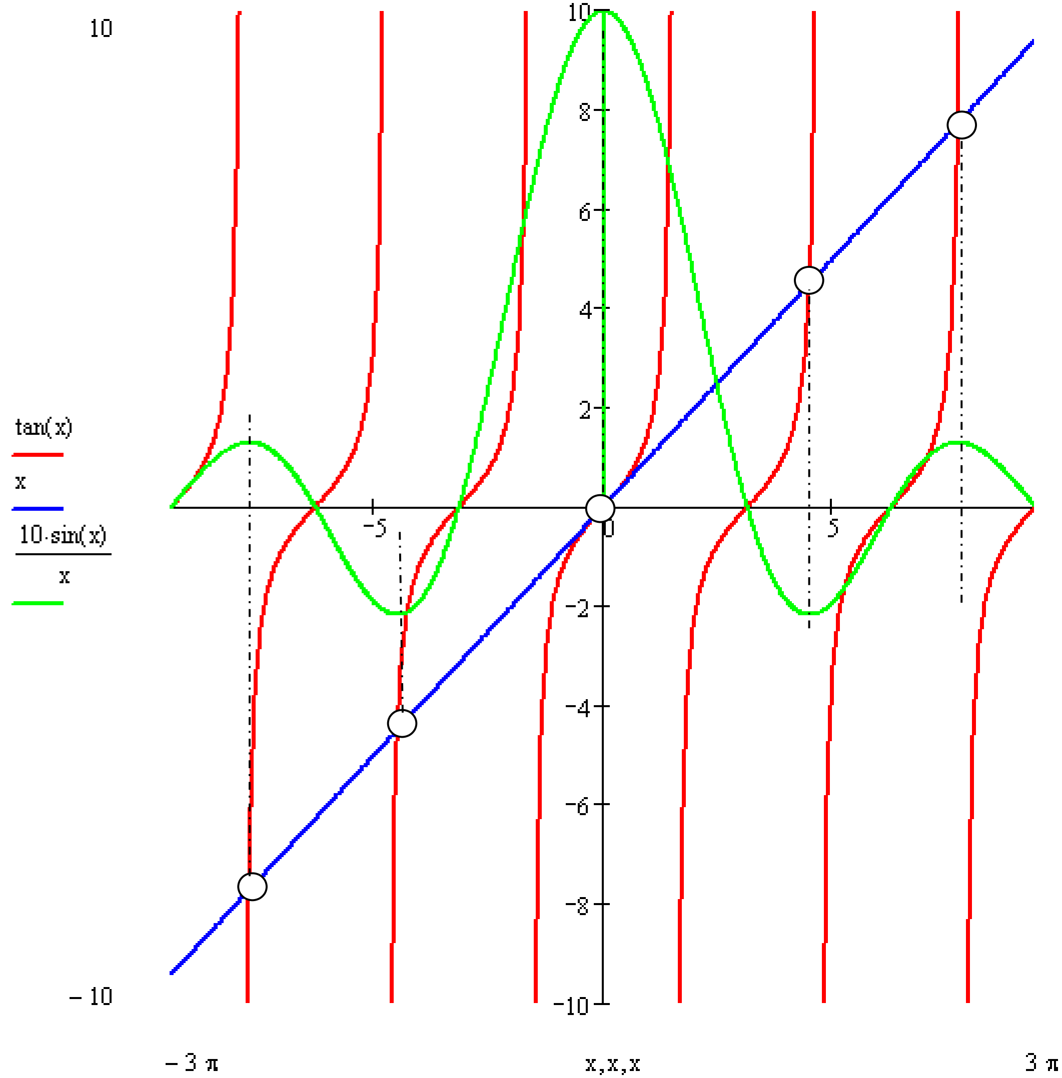
min

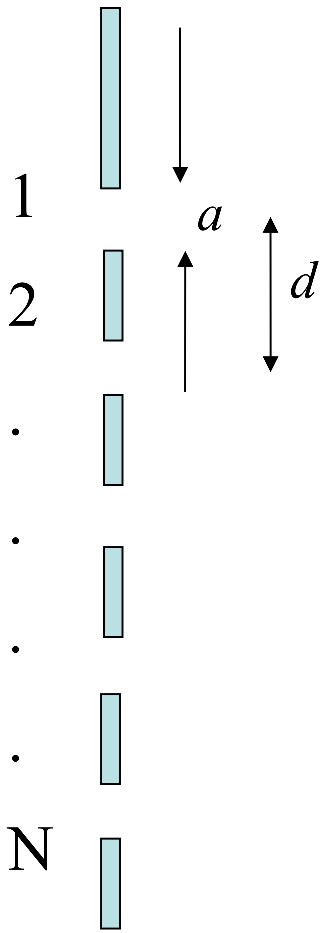
$$\sin\left(\frac{\varphi}{2}\right)=0, \quad \text{excluding } \varphi=0$$

max

$$\left(\frac{\varphi}{2}\cos\left(\frac{\varphi}{2}\right)-\sin\left(\frac{\varphi}{2}\right)\right)=0$$

$$\frac{\varphi}{2}=\tan\left(\frac{\varphi}{2}\right)$$





$$I_{\text{single}} = I_0 \frac{\sin^2\left(\frac{\varphi}{2}\right)}{\left(\frac{\varphi}{2}\right)^2}$$

Single slit $\phi = 2\pi \frac{a\sin(\theta)}{\lambda}$

$$I = I_{\text{single}} \frac{\sin^2\left(N\frac{\delta}{2}\right)}{\sin^2\left(\frac{\delta}{2}\right)}$$

N-slits $\delta = \frac{2\pi}{\lambda} d \sin\theta$

Combining both the response of the diffraction patterns

$$I = I_0 \frac{\sin^2\left(\frac{N\delta}{2}\right)}{\sin^2\left(\frac{\delta}{2}\right)^2} \frac{\sin^2\left(\frac{\phi}{2}\right)}{\left(\frac{\phi}{2}\right)^2}$$

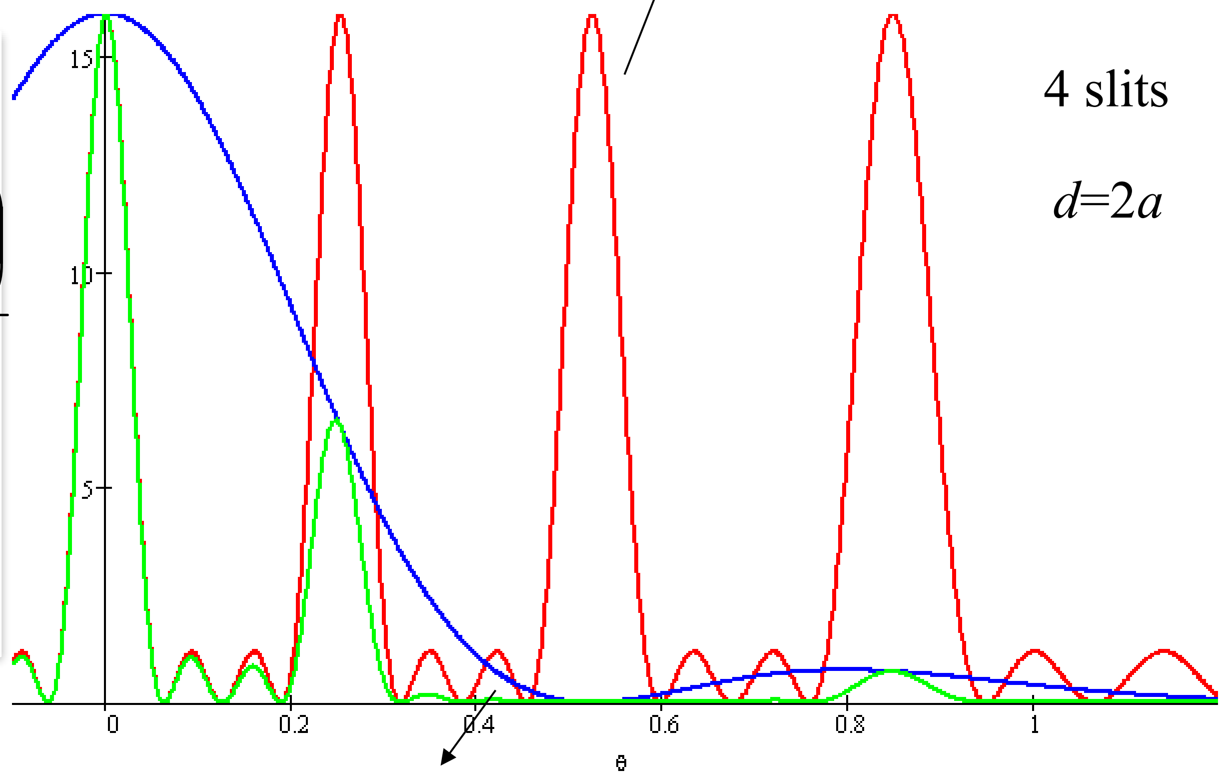
Where are the maxima and the minima?

Max when: $\delta = 2m\pi, \quad \frac{d\sin(\theta)}{\lambda} = m$

$\sin(\theta) = m\lambda / d$

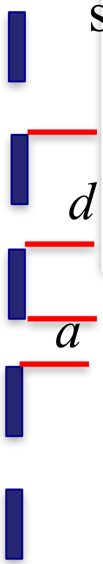
single slit

$$I = I_0 \frac{\sin^2\left(\frac{N\delta}{2}\right)}{\sin^2\left(\frac{\delta}{2}\right)^2} \frac{\sin^2\left(\frac{\phi}{2}\right)}{\left(\frac{\phi}{2}\right)^2}$$



4 slits

d=2*a*



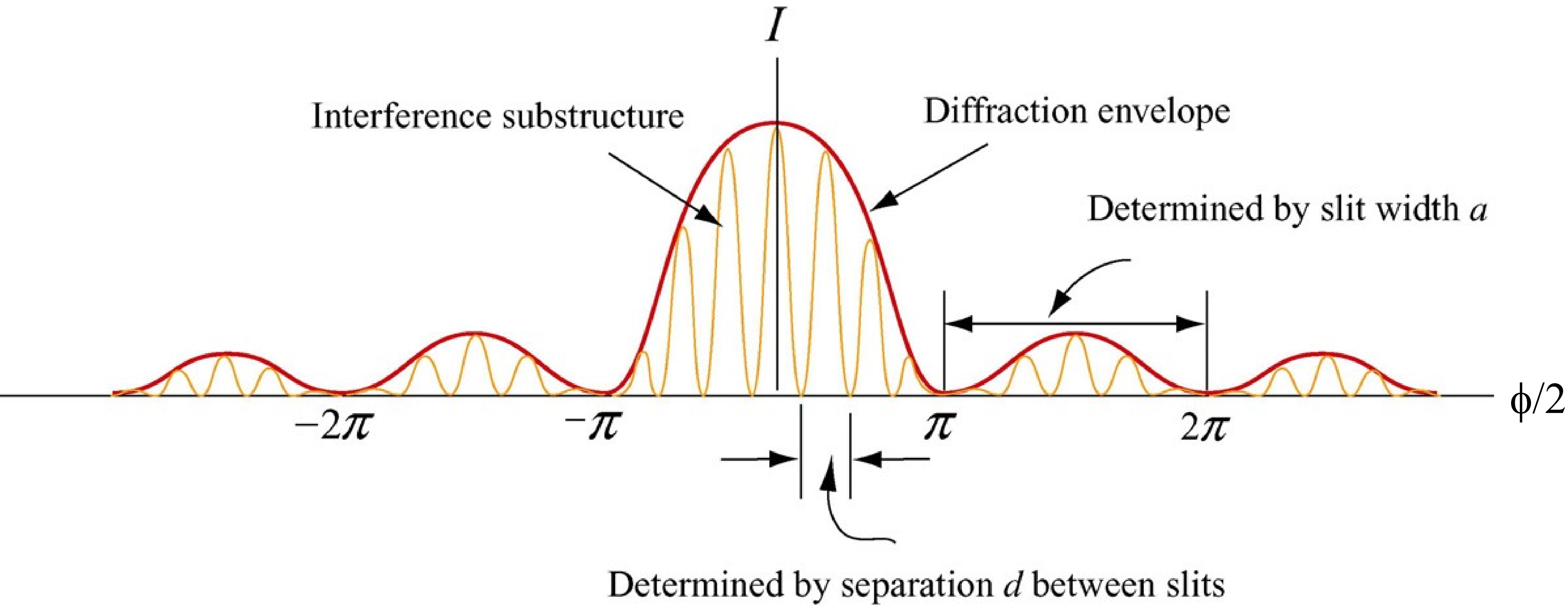
Has min when

$\sin\left(\frac{\varphi}{2}\right) = 0$

$\frac{a\sin(\theta)}{\lambda} = 1,2,3,4,...$

i.e. $\frac{\varphi}{2} = \pi, 2\pi, 3\pi,..$ $\sin(\theta) = \lambda / a,\ 2\lambda / a,\ 3\lambda / a,...$

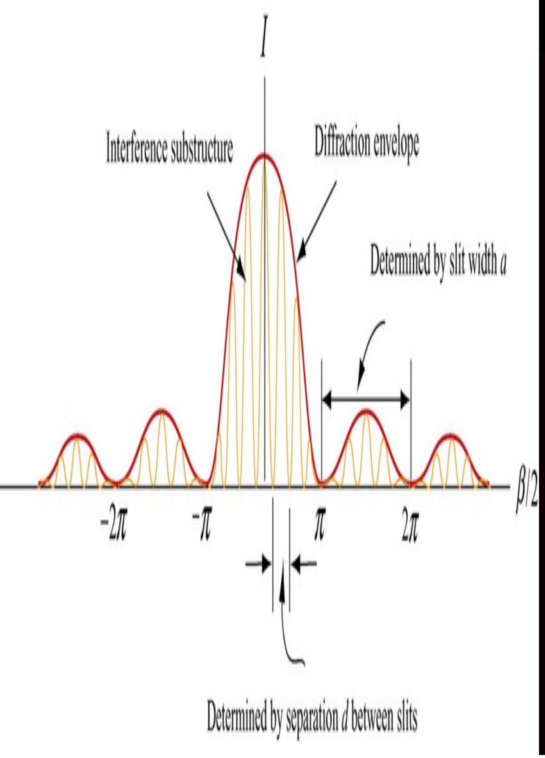
Can get missing orders in the diffraction pattern

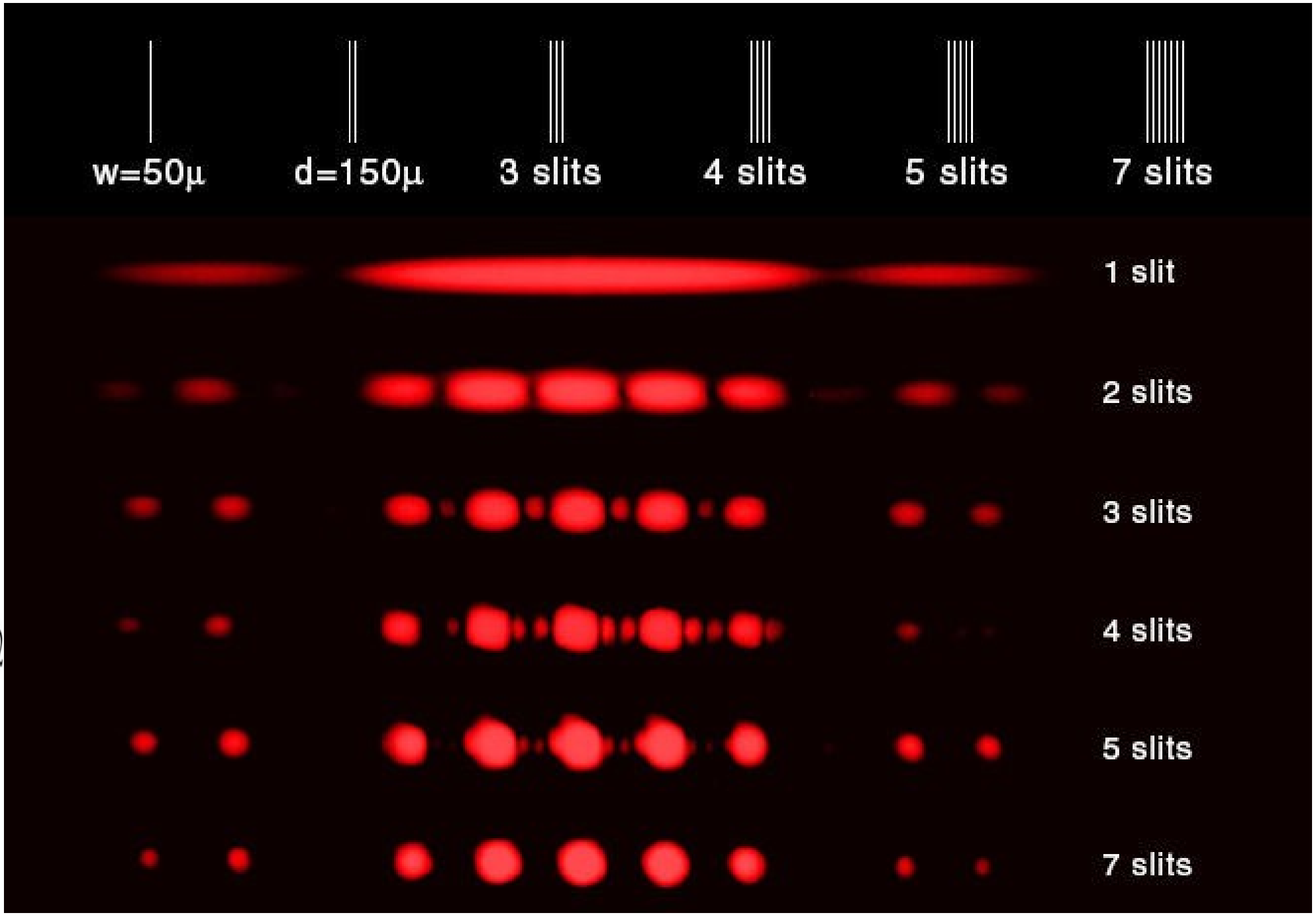


Double-slit interference with diffraction.

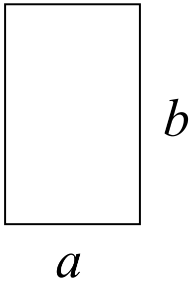
DiffractionDiff

particle size





Two dimensional diffraction (rectangular slit):

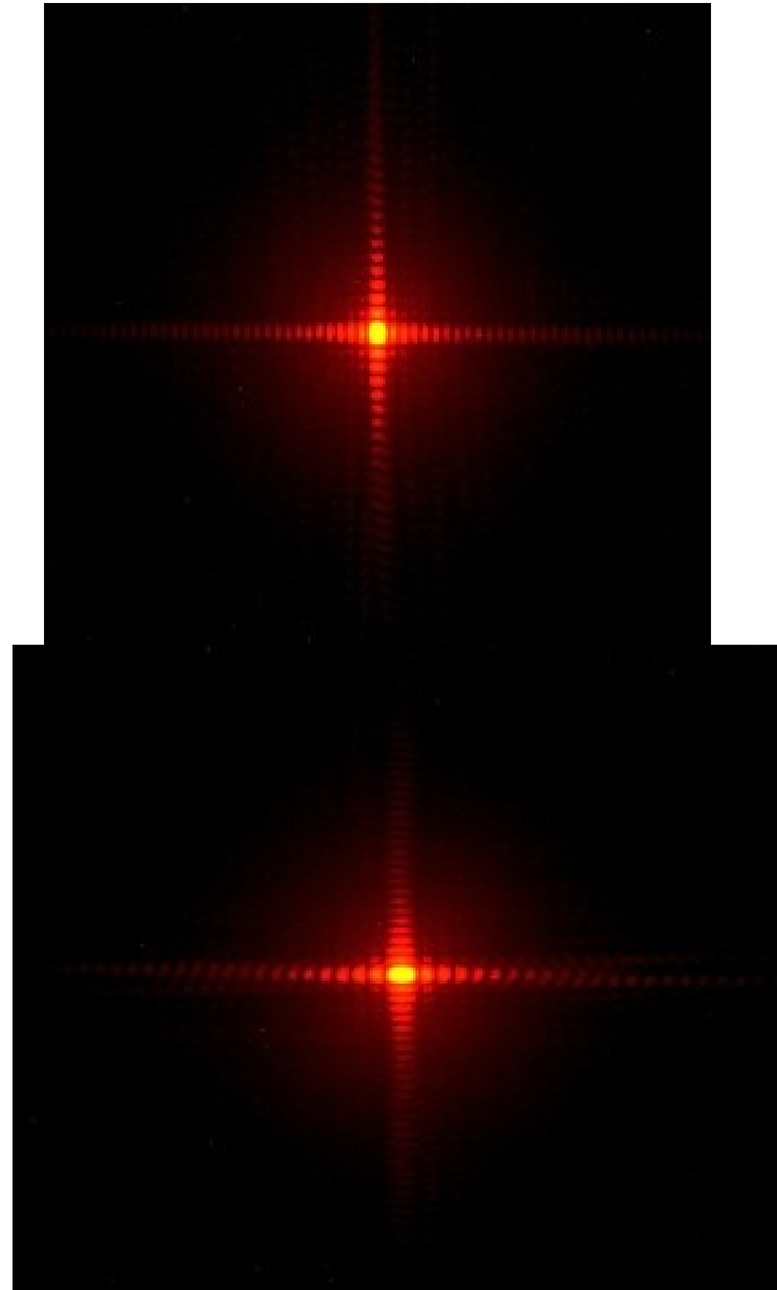


Rectangular slit

$$\alpha = 2\pi \frac{a \sin(\theta)}{\lambda}$$

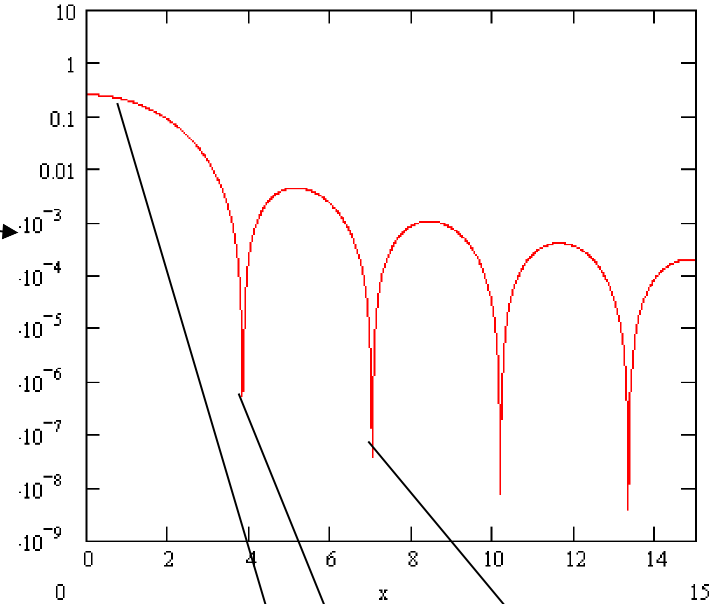
$$\beta = 2\pi \frac{b \sin(\theta')}{\lambda}$$

$$I = I_0 \frac{\sin^2\left(\frac{\alpha}{2}\right)}{\left(\frac{\alpha}{2}\right)^2} \frac{\sin^2\left(\frac{\beta}{2}\right)}{\left(\frac{\beta}{2}\right)^2}$$



The circular aperture

$$I = I_0 \left(\frac{2J_1\left(\frac{2\pi}{\lambda} a \sin\theta\right)}{\frac{2\pi}{\lambda} a \sin\theta} \right)^2$$

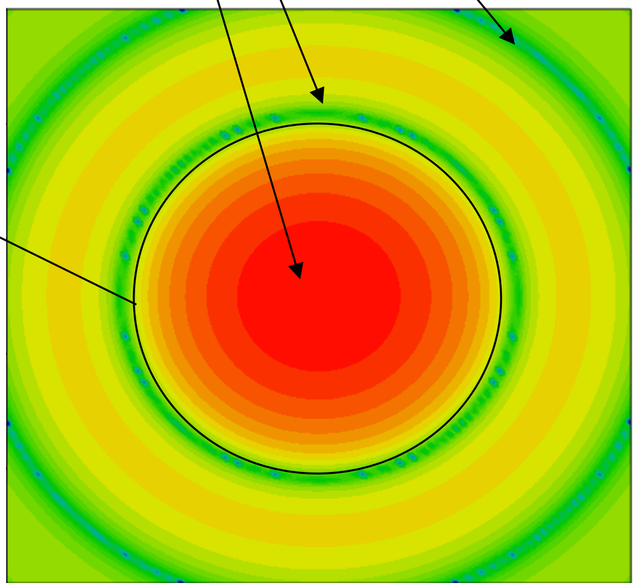


note:
log scale

J_1 is a first order Bessel function

$J_1(x)=0$ at x=0, 3.83, 7.018

Airy disc

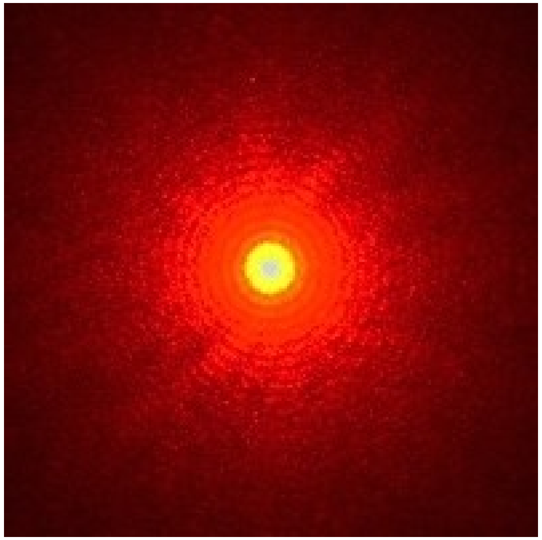


first dark ring

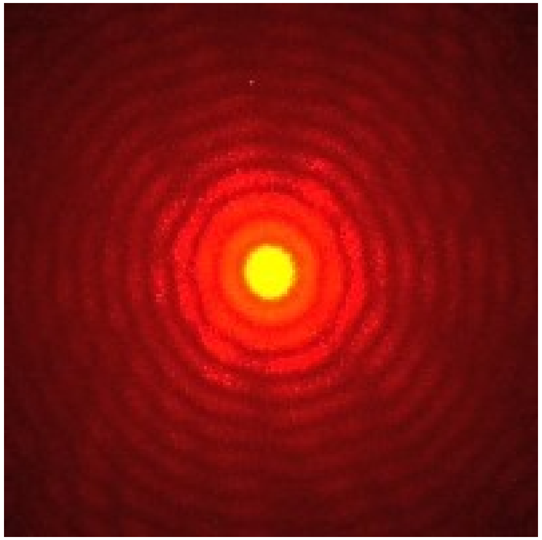
$$\frac{2\pi}{\lambda} a \sin\theta = 3.83$$

$$\sin\theta = \frac{3.83}{\pi} \frac{\lambda}{2a} = 1.22 \frac{\lambda}{D}$$

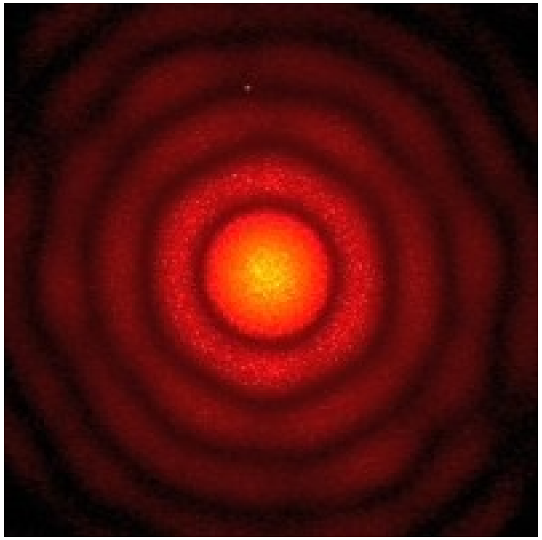
$D = 2a$ (diameter of aperture)



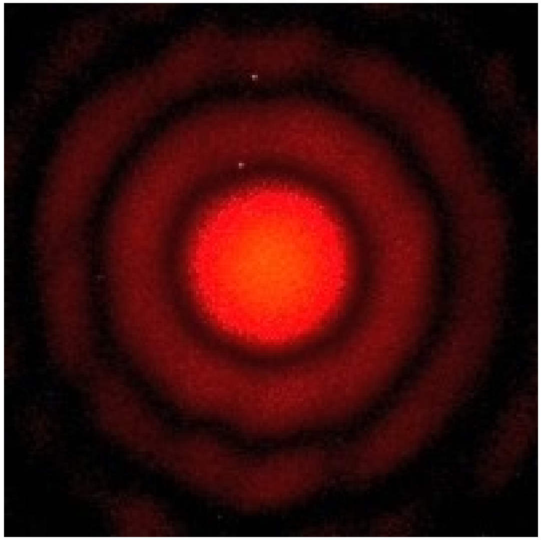
0.48 mm diameter



0.24 mm diameter



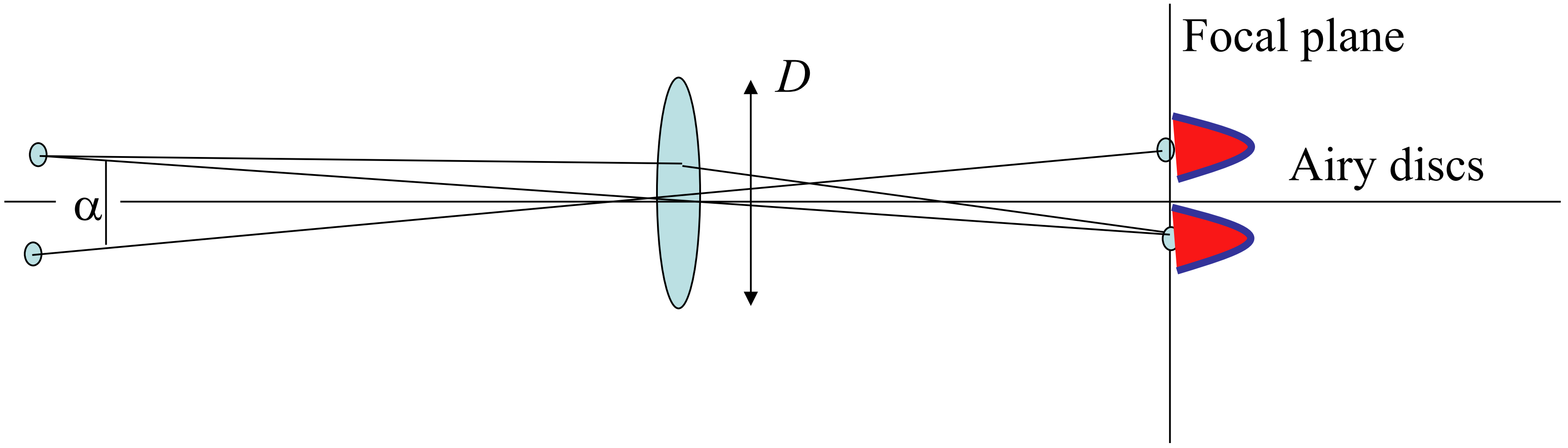
0.12 mm diameter



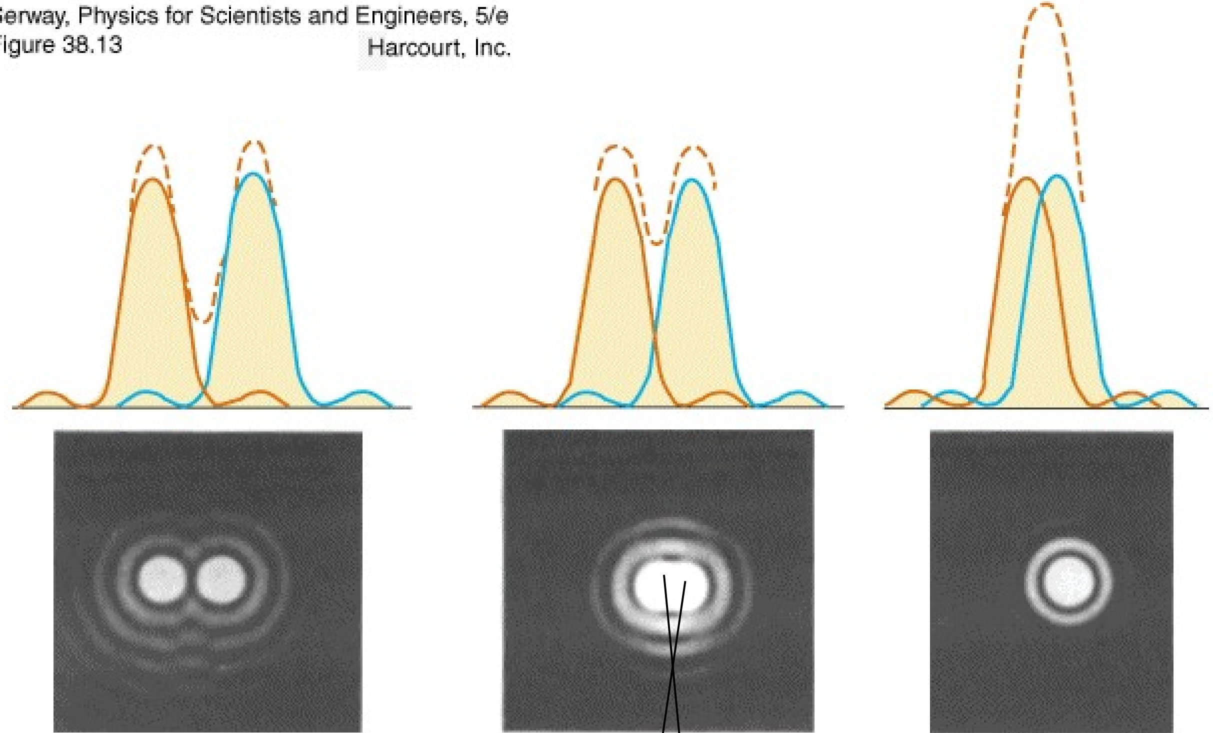
0.08 mm diameter

Resolution of Imaging Systems

Lens forms an image which can be regarded as an Airy pattern (circular aperture).
When the Airy discs overlap too much, the two objects cannot be resolved.



Serway, Physics for Scientists and Engineers, 5/e
Figure 38.13 Harcourt, Inc.



(a) (b) (c)

Clearly resolved just resolved (Raleigh) Not resolved

To improve resolution:
Use light with small λ;
Use large aperture!

α

The Raleigh criterion: Centre of one disc occurs at the first minimum of the other.

$$\alpha = 1.22\frac{\lambda}{D}$$

for small angles

Resolution of the human eye

$$\theta = 1.22\frac{\lambda}{D}$$

λ~ 500 nm, D=5 mm, (Size of pupil)

$$\theta = 1.22\frac{\lambda}{D} \sim 0.0001 \text{ rad} \sim 0.005 \text{ degrees}$$

Cannot resolve two objects separated by <1mm at 10 m distance.

Interference and diffraction explained without using any mathematics, and in an entertaining way:

Diffraction interference patterns with phasor diagrams - YouTube