? Any Calculator

THE UNIVERSITY OF BIRMINGHAM

????????Degree of B.Sc./M.Sci. with Honours???????????

Programmes in the School of Mathematics and Statistics First examination

Programmes including Mathematics First examination

????????Degree of M.Eng. with Honours???????????

Mathematical Engineering First examination

0611235/0611240

MSM1C: COMPUTATIONAL AND APPLIED MATHEMATICS

May/June, 2006 2 hours

Full marks may be obtained with complete answers to ALL questions in Section A (worth a total of 50 marks) and TWO (out of THREE) questions from Section B (worth 25 marks each). Only the best TWO answers from Section B will be credited. Calculators may be used in this examination but must not be used to store text. Calculators with the ability to store text should have their memories deleted prior to the start of the examination.

Turn over

SECTION A

1. A particle of mass m has Cartesian coordinates (x, y, z) at time t given by

$$x = Ut\cos\alpha,$$

$$y = 0$$

and

$$z = -\frac{1}{2}gt^2 + Ut\sin\alpha.$$

(a) Determine expressions for the velocity $\mathbf{v}$ and the acceleration $\mathbf{a}$ of the particle. [4]

(b) Determine the kinetic energy of the particle. [2]

(c) Determine the net force acting on the particle. [2]

(d) Briefly describe what this set of equations represents. [2]

2. A particle is located at position $\mathbf{r}$ with respect to the origin of a 2-dimensional co-ordinate system. If $\mathbf{i}$ and $\mathbf{j}$ are the unit vectors in a (x, y) Cartesian co-ordinate system and $\mathbf{e}_r$ and $\mathbf{e}_\theta$ are the unit vectors in a (r, θ) polar coordinate system given by

$$\mathbf{e}_r = \cos\theta\mathbf{i} + \sin\theta\mathbf{j}$$

and

$$\mathbf{e}_\theta = -\sin\theta\mathbf{i} + \cos\theta\mathbf{j}$$

then determine an expression for the velocity $\frac{d\mathbf{r}}{dt}$ of the particle in polar coordinates. [3]

If a particle's location in Cartesian co-ordinates is $(x, y) = (1, t)$, what is the particle's velocity in polar co-ordinates [3]

SECTION B

3. (a) What is Hooke's law? [2]

(b) A particle of mass m is attached to a spring which has a spring constant k. The natural length of the spring is a, and the only force that acts on the particle is due to the spring. If x measures the displacement of the particle from some fixed point along the line of the spring, then use Newton's second law to determine a differential equation for x. [3]

(c) Show that the sum of the potential and kinetic energy of the particle in part (b) is constant. [5]

(d) A mechanical vibration is described by

$$\frac{d^2x}{dt^2} + \alpha\frac{dx}{dt} + \omega^2 x = f(t)$$

where x measures displacement, t is time, α and ω are dimensional constants which characterize the system, and $f(t)$ is some time dependent function which represents an external forcing. At time $t = 0$, assume that $x = 1$ and $\frac{dx}{dt} = 0$.

If $\alpha = 0$ and $f(t) = 0$ then determine an expression for x in terms of t. [4]

Describe briefly the nature of the oscillation. [1]

If $\alpha = 1$, $\omega = 1$ and $f(t) = 0$ then determine an expression for x in terms of t. [4]

Describe briefly the nature of the oscillation. [1]

If $\alpha = 0$, $\omega = 1$ and $f(t) = \cos(t)$ then determine an expression for x in terms of t. [4]

Describe briefly the nature of the oscillation. [1]

4. In polar coordinates Newton's second law for a particle of mass m can be written as

$$\mathbf{F} = F_r\mathbf{e}_r + F_\theta\mathbf{e}_\theta = m\left(\left[\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2\right]\mathbf{e}_r + \left[2\frac{dr}{dt}\frac{d\theta}{dt} + r\frac{d^2\theta}{dt^2}\right]\mathbf{e}_\theta\right).$$

Assume that the particle is subject to a central force with $F_r = g(r)$ (for some function g) and $F_\theta = 0$.

(a) Show that the quantity

$$h = r^2\frac{d\theta}{dt}$$

is constant and hence or otherwise show that the position vector of the particle sweeps out equal areas about the origin in equal times. [8]

(b) If $u = 1/r$ show that the radial component of Newton's second law becomes

$$\frac{d^2u}{d\theta^2} + u = -\frac{g\,(1/u)}{mh^2u^2},$$

where h is the constant identified above. [10]

(c) Assume that $g(r) = -\gamma/r^2 + \lambda/r^3$ where γ and λ are positive dimensional constants. What are the dimensions of γ and λ? Determine the orbit of the particle. Under what conditions are the orbits closed? [7]

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