Optics and Waves

Lectures 14-15

Spherical Mirrors
Y&F, Chapter 34,
34.2 Geometric optics/ Ray optics

Spherical Mirrors:

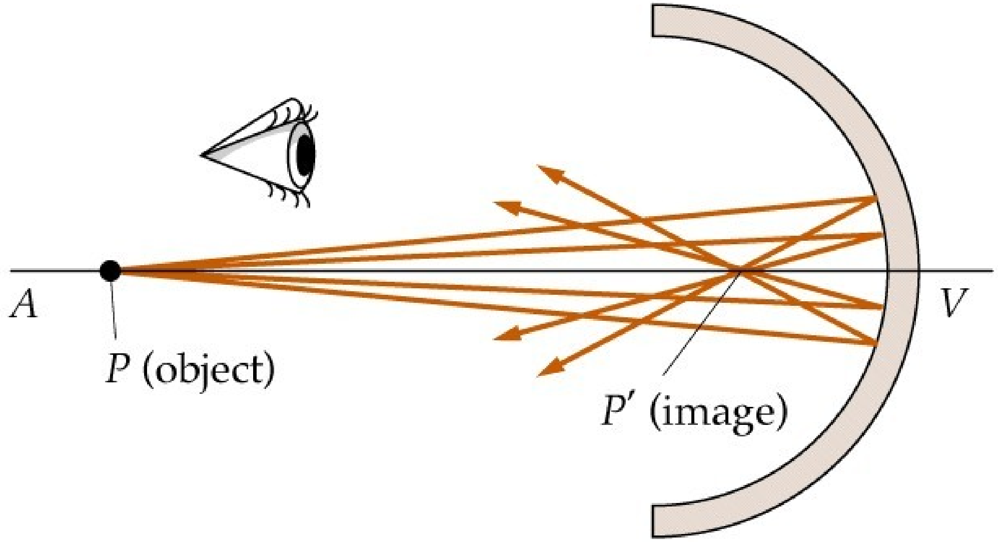
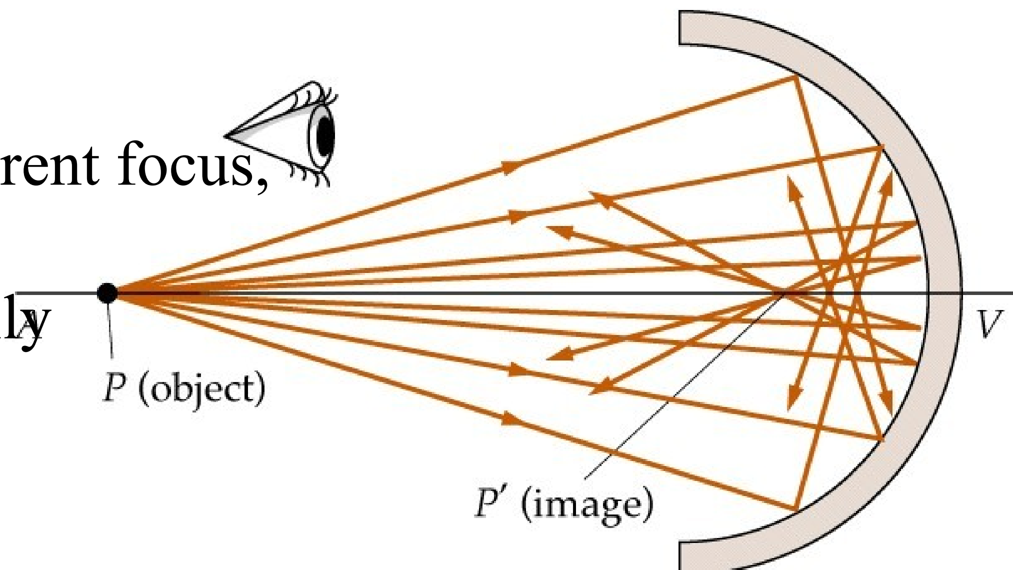


image is real –
light rays pass through it
and image could be projected
on a screen – not true for
virtual images.



Spherical aberrations

- non axial rays come to a different focus,

and thus image is blurred.
The non-paraxial rays are usually
removed using an aperture.

- Paraxial ray: a ray makes a small angle to the optical axis.

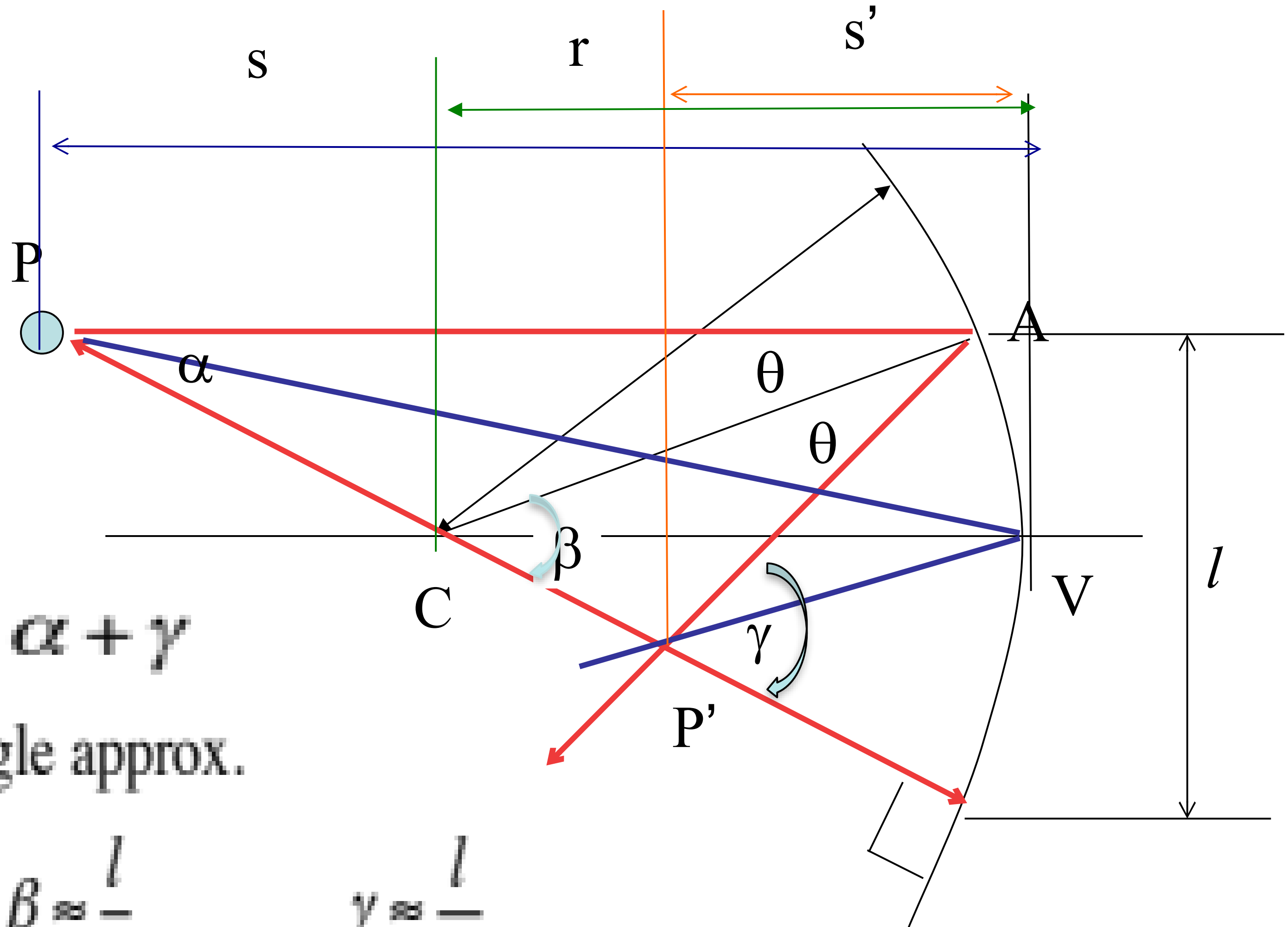
Objectives:

1. Use simple geometry to derive the mirror equation

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f} \qquad \text{(The mirror equation)}$$

2. Learn to use the mirror equation to find images

3. Use ray-diagrams to locate images.



$$\beta = \alpha + \theta$$

$$\gamma = \alpha + 2\theta$$

$$i.e. \quad 2\beta = \alpha + \gamma$$

using small angle approx.

$$\alpha \approx \frac{l}{s} \qquad \beta \approx \frac{l}{r} \qquad \gamma \approx \frac{l}{s'}$$

$i.e.$

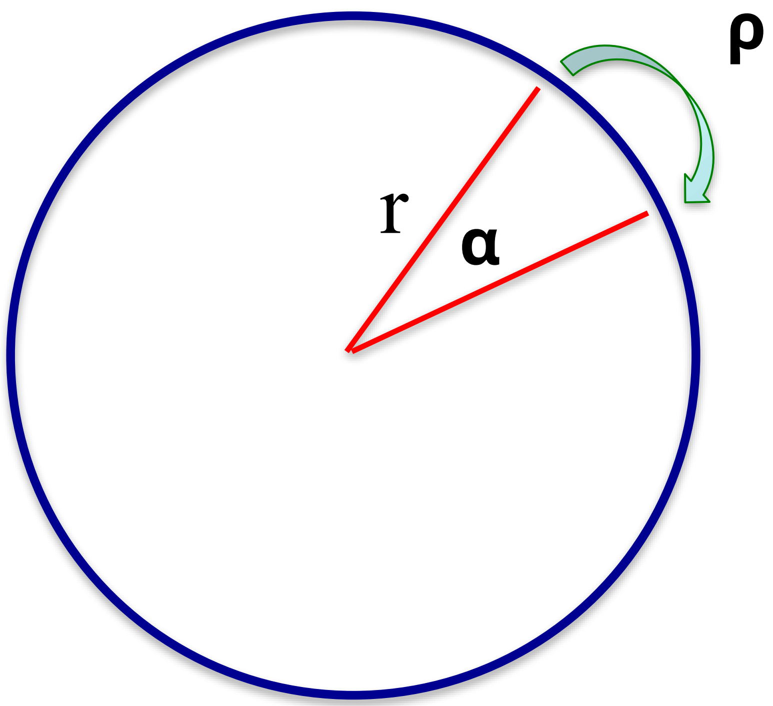
$$\frac{2}{r} = \frac{1}{s} + \frac{1}{s'}$$

V: Vertex of the mirror

CV: Optic axis

All distances measured from V

$$\alpha = \frac{\rho}{r}$$



$$\frac{1}{s}+\frac{1}{s'}=\frac{2}{r}=\frac{1}{r/2}$$

Introduce a new parameter f, $f=\frac{r}{2}$

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \quad \text{(The mirror equation)}$$

For plane mirror, r=∞, hence f=∞, and s= –s'

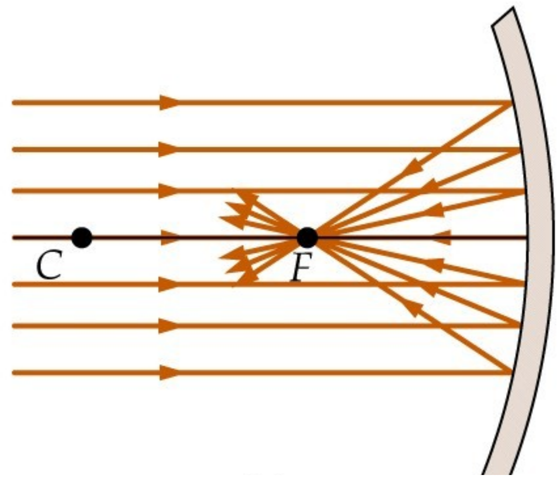
f is the focal length of the mirror

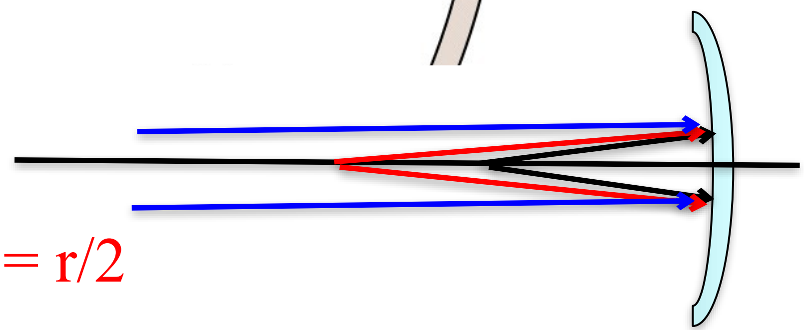
When parallel rays incident onto the mirror, they are focused at the focal point which is at distance f from the vertex.

$$\frac{2}{r} = \frac{1}{s} + \frac{1}{s'}$$

so as $s \rightarrow \infty$

$$s' \rightarrow \frac{r}{2}$$





Focal length of a spherical mirror: f = r/2
f takes the same sign as r.

Sign for r. When the centre of curvature C is on the same side as the outgoing light, the radius of curvature is positive.

F, the focal point. When parallel rays incident onto mirror, the rays are brought to focus at F. If a point object is placed at F, all reflected ray are parallel to the optic axis.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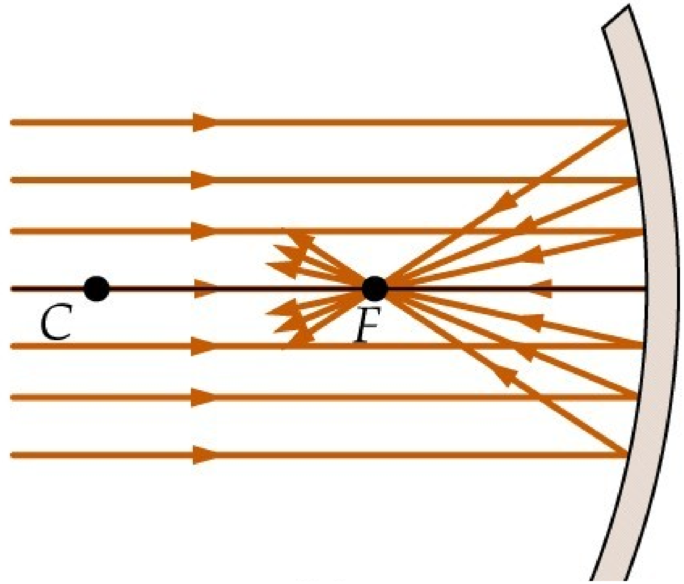
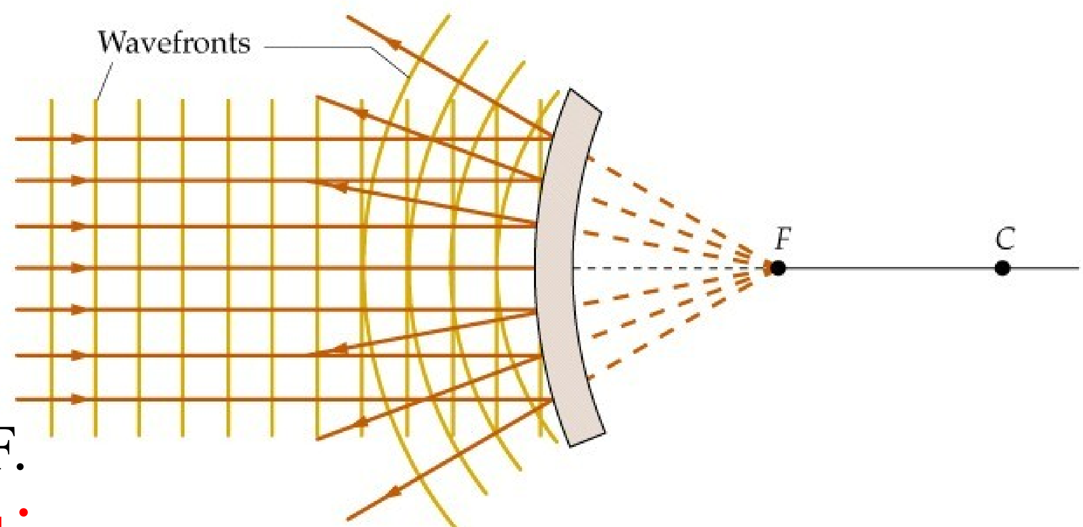
Concave mirror

- parallel rays strike the mirror and are focused at F at a distance r/2



Convex mirror



- the outgoing wavefronts appear to emanate from F behind the mirror.

Rays appear to diverge from F.

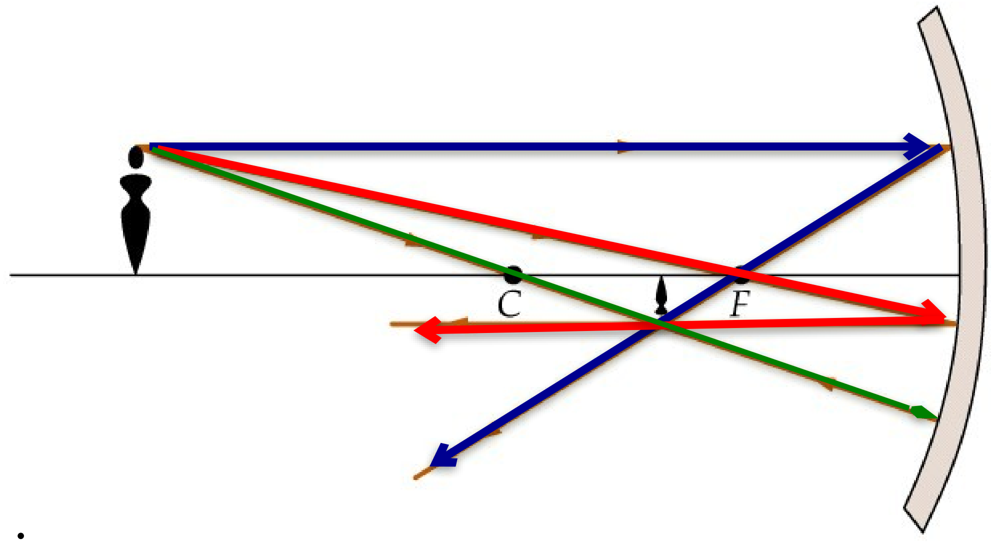
r is negative, f is negative

The mirror equation is applicable to both Concave and convex mirrors.

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f} \quad \text{(The mirror equation)}$$

Different signs for r and hence f.

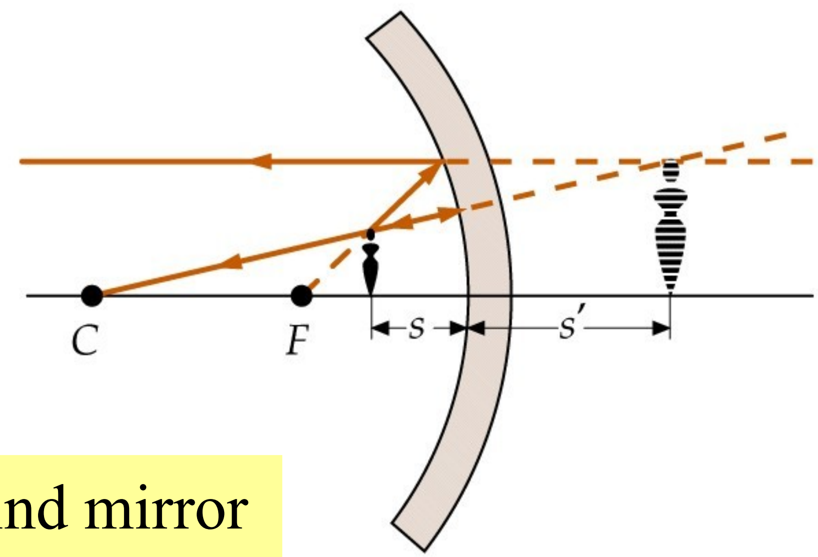
Ray diagrams



Draw 3 rays

1. parallel
2. focal
3. radial

Note. if s < f image is behind mirror and virtual and need a different construction

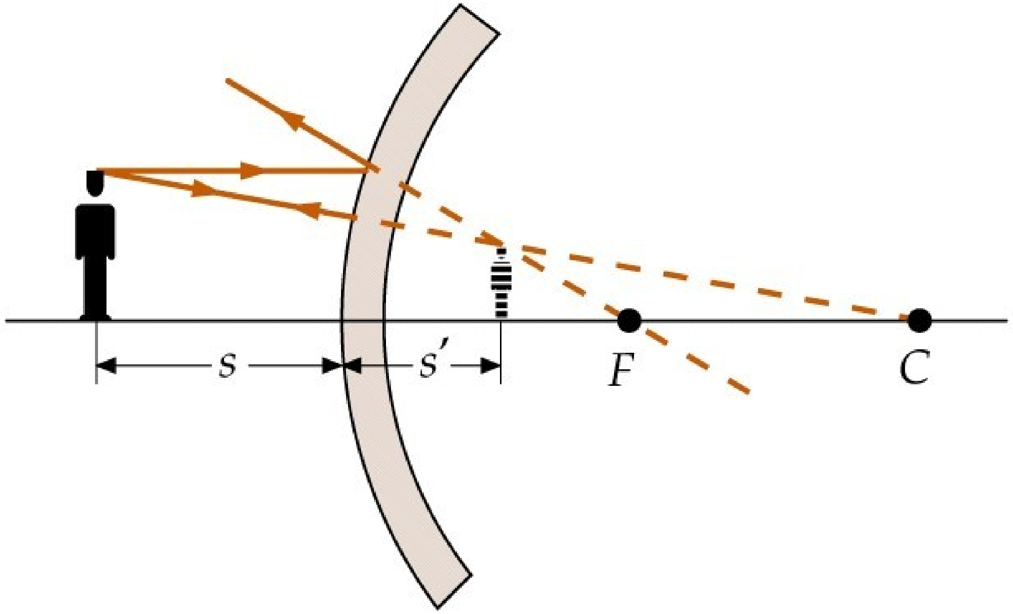


Convention for $\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$

s is +ve/-ve if object is in front/behind mirror
s’ is +ve/-ve if image is in front/behind mirror
f/r +ve/-ve if mirror is concave/convex

For a concave mirror, depending on the object distance,
The image can be real and inverted,
or virtual and erect. (Cosmetic mirror)

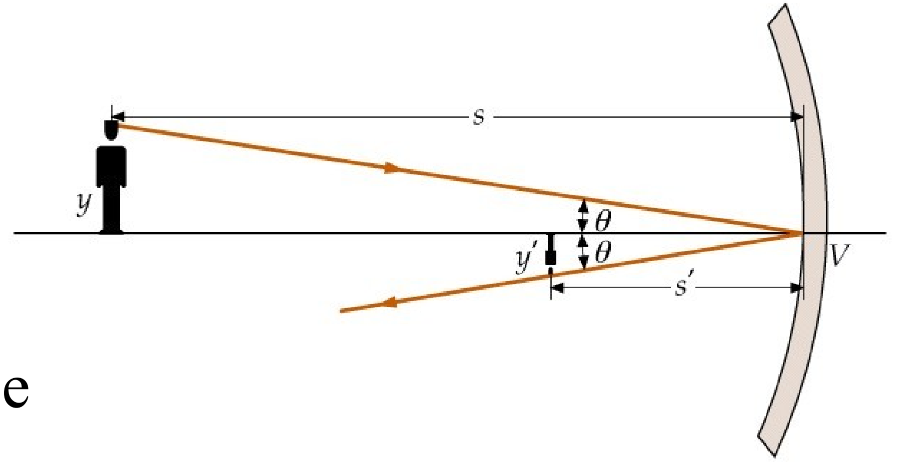
Ray diagram for convex mirror



Magnification:

$$\tan\theta = \frac{y}{s} \qquad \tan\theta = \frac{-y'}{s'}$$

$$m = \frac{y'}{y} = -\frac{s'}{s}$$



negative magnification hence image is inverted.

Principle rays for concave mirror: Y&F P1124.

1) Ray parallel to axis reflects through the focal point.
2) Ray through focal point reflects parallel to axis.
3) Ray through centre reflects along its original path.
4) Ray to vertex reflects symmetrically around optic axis.

Principle rays for convex mirror: Y&F P1124.

1) Reflected parallel ray appears to come from focal point.
2) Ray towards focal point reflects parallel to axis.
3) Ray towards centre reflects along its original path.
4) Ray to vertex reflects symmetrically around optic axis.

Better do an example or two!

Concave mirror, 40 cm radius of curvature, object 1cm high, placed 100 cm from the mirror (beyond C) – where is the image and what is the magnification ?

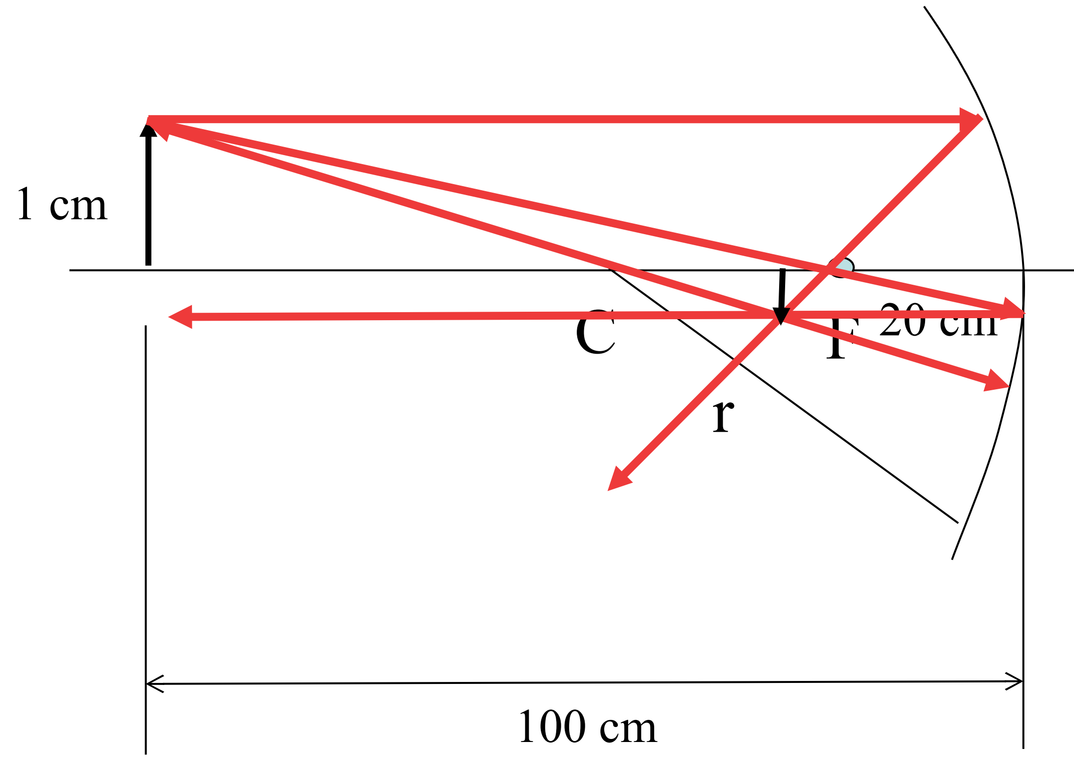
$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

$$\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{1}{20}-\frac{1}{100}=\frac{4}{100}$$

$$s'=25cm$$

$$m=-\frac{s'}{s}=-\frac{25}{100}=-0.25$$

Thus image is 0.25 cm high and inverted



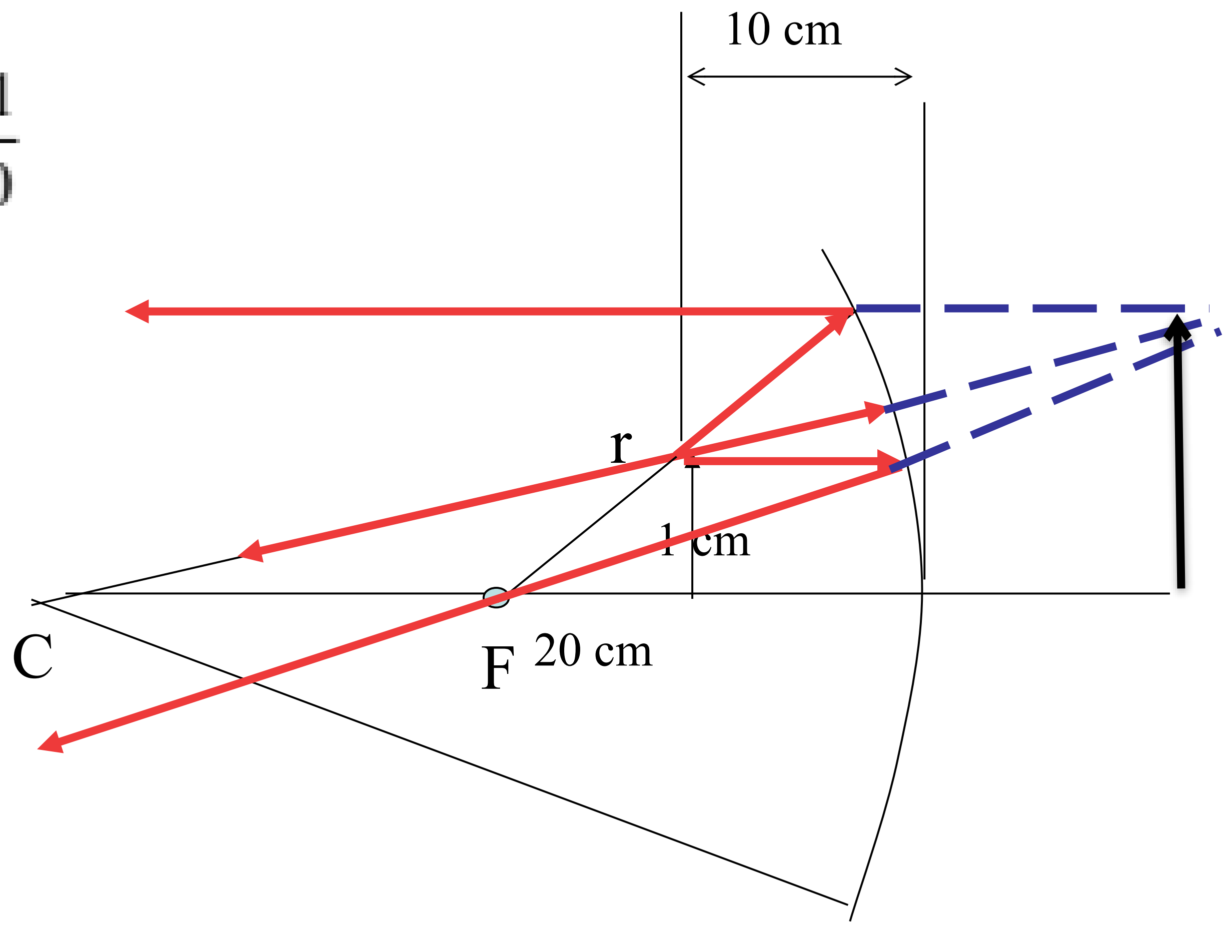
Concave mirror, 40 cm radius of curvature, object 1cm high, placed 10 cm from the mirror (inside F) – where is the image and what is the magnification

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

$$\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{1}{20}-\frac{1}{10}=\frac{-1}{20}$$

$$s'=-20cm$$

$$m=-\frac{s'}{s}=-\frac{-20}{10}=2$$



Thus image is 2 cm high and erect and virtual

Concave mirror, 40 cm radius of curvature, object 1cm high, Placed 30 cm from the mirror (between C and F) – where is the image and what is the magnification?

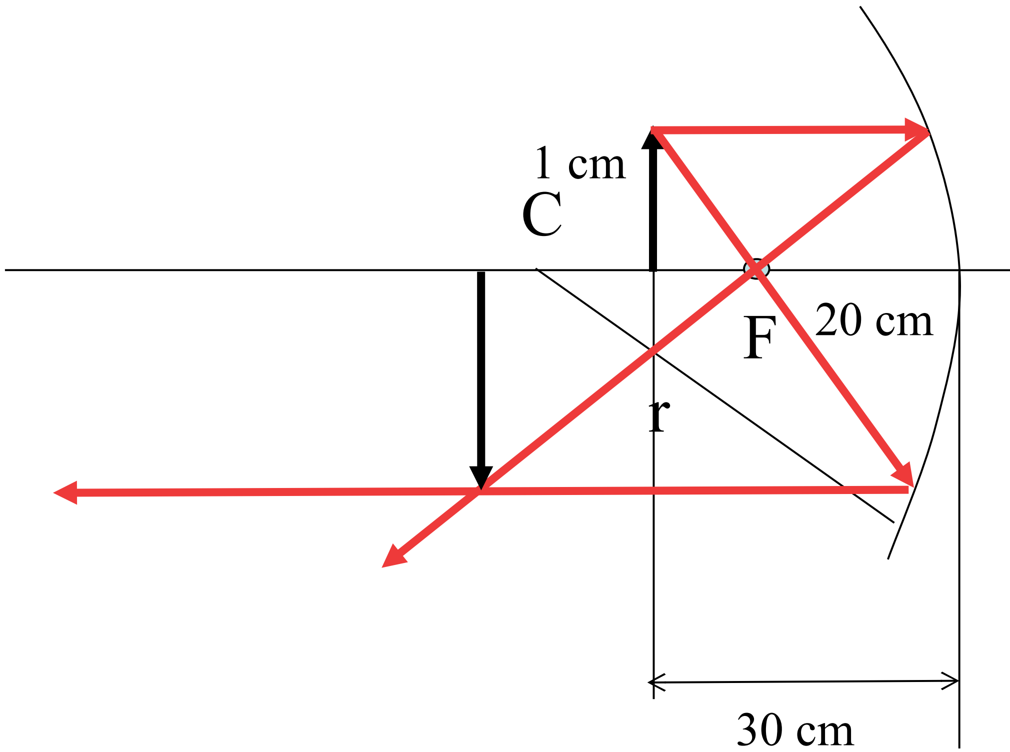
$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

$$\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=\frac{1}{20}-\frac{1}{30}=\frac{1}{60}$$

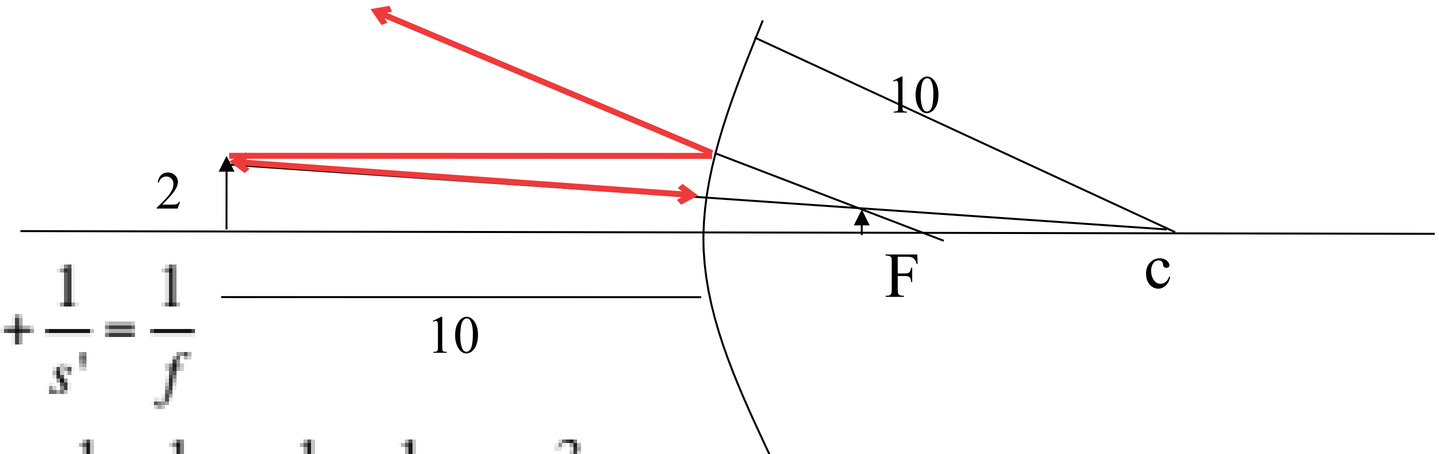
$$s'=60\,cm$$

$$m=-\frac{s'}{s}=-\frac{60}{30}=-2$$

Thus image is 2 cm high, real and inverted



An object is 2 cm high and 10 cm from a convex mirror with a radius of curvature 10 cm. (a) locate the image (b) find the image height



$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

$$\frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=-\frac{1}{5}-\frac{1}{10}=-\frac{3}{10}=-0.3$$

$$s'=-3.3\text{ cm}$$

$$m=-\frac{s'}{s}=-\frac{-3.3}{10}=0.33$$

Mirror type	Object location	Image location	Image type	Orientation	f	Sign r	m
Plane	Anywhere						
Concave	Inside F						
	Outside F						
Convex	Anywhere						