Quantum Mechanics 1 – Problem 10

A particle of mass *m* is trapped in an infinite one-dimensional potential well of width *L*, where the potential inside the well is zero and the origin is chosen to be at the *centre* of the well. The particle is in a state described by the wave function

$$\Psi(x,t) = A\cos(kx)\, e^{-i\omega t}.$$

a) Show that the wave function is an eigenfunction of the kinetic energy operator. [2 marks]

b) Show that the wave function is <u>not</u> an eigenfunction of the momentum operator $\hat{p}_x$. [2 marks]

c) In quantum mechanics, the "average" value expected for an observable, corresponding to the measurement of many particles prepared in the same quantum state, is known as an *expectation value*. Find the *expectation value* of the *x*-component of momentum, which is given by

$$\langle p_x \rangle = \int_{-\infty}^{+\infty} \Psi^* \, (\hat{p}_x \Psi) dx.$$

(Hint: because I have specified that the origin is at the centre of the well, you can solve the integral by inspection. Clearly state your reasoning.)

[2 marks]

d) Explain how these results are consistent with each other.

[4 marks]