1Mech — Mechanics

Mechanics exercises 1 (weeks 1 and 2)

Please submit your answers to questions 3 and 4 to get feedback. To get full marks your solution must be clearly presented and explained.

1. In this question ρ is density, t is time, $\mathbf{v}$ is velocity, x is position, v is speed, m is mass, a is acceleration, V is volume, A is area and g is acceleration due to gravity.

   (a) What are the dimensions of the following expressions?

   i. $\frac{d\rho}{dt}$

   ii. $\dfrac{d^2\mathbf{v}}{dx^2}$

   (b) Are these equations dimensionally correct? Make sure you justify your answer.

   i. $vma = \frac{dV}{dt}$

   ii. $\int \rho \, \mathrm{d}t = \dfrac{m}{Av} + \dfrac{\sqrt{2}mt}{V}$

   iii. $A^{1/2}g = v^2 + \dfrac{mg}{\rho A}$

2. If a particle moves along a path $\mathbf{r} = Vt\mathbf{i} + (h - gt^2/2)\,\mathbf{j}$, what is its velocity and acceleration? By eliminating t, what path does the particle travel along in space? What shape is this?

3. Suppose a charged particle moving under the influence of a magnetic field follows a helical path, at constant rate $\dot{\theta} = \omega$. The position vector is given by

$$\mathbf{r} \;=\; a\cos(\theta(t))\,\mathbf{i} + a\sin(\theta(t))\,\mathbf{j} + b\theta(t)\,\mathbf{k},$$

   where a and b are constants. Calculate the velocity and acceleration of the particle. Hence determine the speed which the particle moves along the helical path, and the magnitude of the acceleration. Which direction does the acceleration point in (in terms of the helix geometry)?

4. A particle of mass m is moving on a straight line under the action of a force of the form $F = F_0 e^{-\lambda t} - F_1$, where F_0, F_1 and λ are positive constants. The particle passes the point x_0 with velocity v_0 at time $t = 0$. Find the displacement and the velocity of the particle at time t. Describe the motion in words - what happens for large time?

5. A particle of mass m is attached to a spring with spring constant k, which is fixed at the opposite end. The mass is subject to an additional force $F_0 \cos\Omega t$ directed away from the fixed end of the spring, for constant Ω.

(a) Show that the equation of motion is

$$\ddot{x} + \omega^2 x = \frac{F_0}{m} \cos \Omega t,$$

where x gives the displacement from the equilibrium position (when the spring is neither stretched nor compressed), and $\omega^2 = k/m$.

(b) Find the displacement as a function of time of a particle subject to the forcing described, given that initially the particle is stationary at $x = 0$, and assuming that $\omega \neq \Omega$.

(c) **Optional extension:** What happens when $\omega = \Omega$?