Video material Week 03 NOTES

1. Introduction

In this week's video material we will look at source characteristics and the consequences of internal resistance. This will enable us to make a very general statement about all linear circuits, which will expand upon the idea of equivalent circuits introduced to you in the second week.

2. Ideal source characteristics

In figure 2.1 are shown the I-V characteristics of an ideal voltage source and an ideal current source.

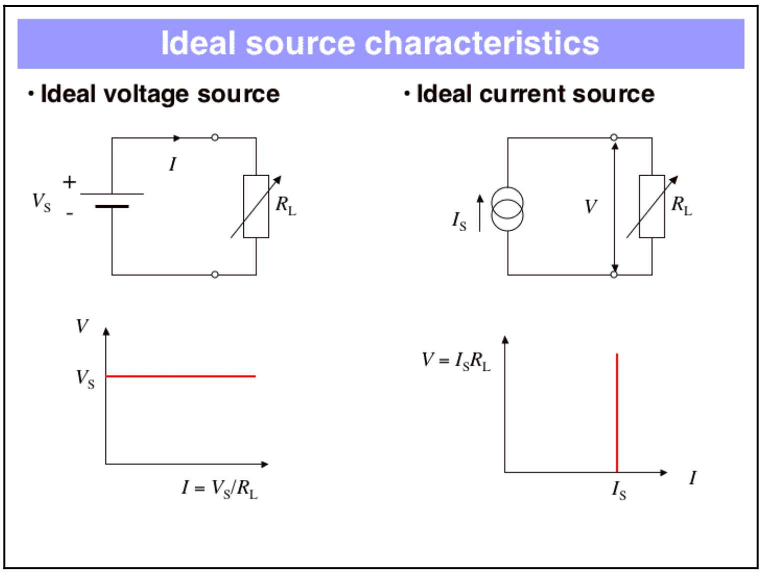


Figure 2.1: The I-V characteristics of ideal voltage and current sources

An **ideal** voltage source maintains a constant potential difference (V_S) across its terminals regardless of the value of the load resistance. The current that is drawn from the voltage source depends on the load.

An **ideal** current source maintains a constant current (I_S) flowing through it. In this case, it is the potential difference across the current source that depends on the load.

Whilst these ideal energy sources are useful abstractions, in practice there is a problem…

- *What happens to the current from the voltage source when the load resistance is zero?* This is equivalent to connecting a wire between the terminals (indicated by the small open circles), creating a short-circuit.

- What happens to the voltage appearing across the current source when the load is infinite? This is equivalent to removing the load completely, leaving an open-circuit.

The answer is that the current from the battery becomes infinite and the voltage across the current source becomes infinite. *Clearly, this is impossible!* An infinite current (or voltage) would require infinite energy. In practice, real sources do not behave this way because they have internal resistance.

3. Real (practical) source characteristics.

Figure 2.2, shows a practical voltage and current source. This is indicated by the elements shown in the shaded area inside the dashed box.

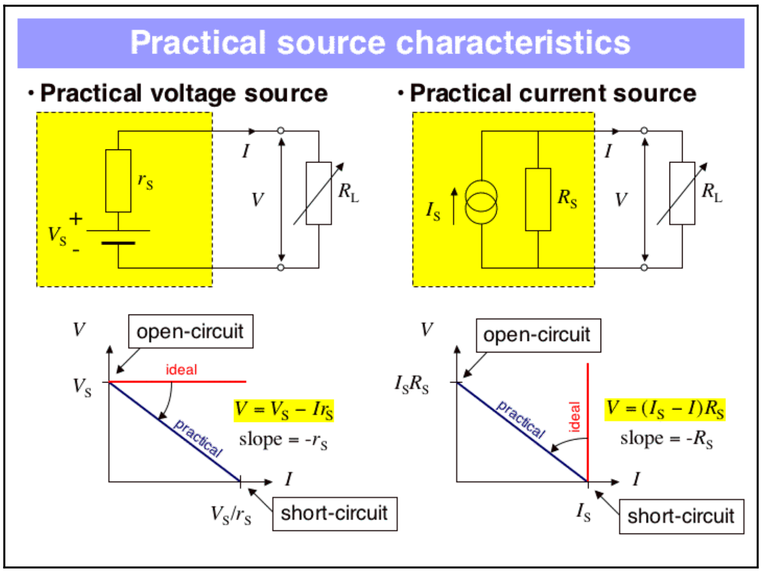


Figure 2.2: The I-V characteristics of a practical voltage and current sources.

The **practical voltage source** is represented by a **small** internal resistance, r_S, in **series** with an ideal voltage source. (The subscript "S" indicates that this is the resistance of the source). Let's see what happens when we attach different loads. When the load is infinite (open-circuit between the terminals), no current flows. A high resistance voltmeter would measure V_S across the terminals as before. However, when the load is finite a current flows, thus, there is a potential drop across the internal resistance, r_S. The remainder of the voltage ($V = V_S - Ir_S$) is dropped across the load (remembering that the potential rise across the battery, must equal the sum of the potential drops across the two resistors.) Effectively, the terminal voltage of the battery drops, becoming smaller the larger the current, or the smaller the load.

Now let's look at the problem of short-circuiting the voltage source, by replacing the load with a zero resistance wire. The potential gained across the battery is now equal to the potential dropped across the internal resistance. The maximum current is $I_{MAX}=V_S/r_S$, by Ohm's law.

> ***Test your understanding.*** *Is it really necessary to replace the load with the wire? What would happen if I place a wire between the terminals keeping the load in place, so that the wire and the load resistance are connected in parallel? What is the combined resistance of the wire and the load? For the answer, go to the end of this lecture notes.*

The I-V characteristics of the practical voltage source is therefore given by the blue line in figure 2.2. Notice that there is a linear relationship between the voltage across the load and the current through the load, the slope being given by $-r_S$.

The **practical current source** is represented by a **large** internal resistance, R_S, in **parallel** with an ideal current source. When the load is zero (short-circuit between the terminals) all the current flows through the short-circuit. (See "Test your understanding" above.) A zero resistance ammeter would therefore measure a current I_S from the source. When the load is non-zero and finite, current I flows through the load and the remainder ($I_S - I$) flows through the internal resistance. The voltage that appears across the load is IR_L, which is the same as the voltage that appears across the internal resistance, $V = (I_S - I)R_S$. The current in the load drops as the load resistance increases.

When the load is infinite, the practical current source is open-circuit. Now, all the current flows through the internal resistance. The maximum voltage that appears across the current source is $V_{MAX} = I_S R_S$.

Notice that the I-V characteristics of the practical current source, shown in figure 2.2, is similar to the practical voltage source. Once again, there is a linear relationship between the voltage across the load and the current through the load, the slope this time being given by $-R_S$.

4. Maximum power transfer theorem

From figure 2.2, it should be apparent that to be close to ideal, the internal resistance of a voltage source should be as small as possible (and the internal resistance of a current source should be as large as possible). The presence of internal source resistance has an implication for the power that is dissipated in a load. Figure 2.3, shows the practical voltage source and its I-V characteristics again.

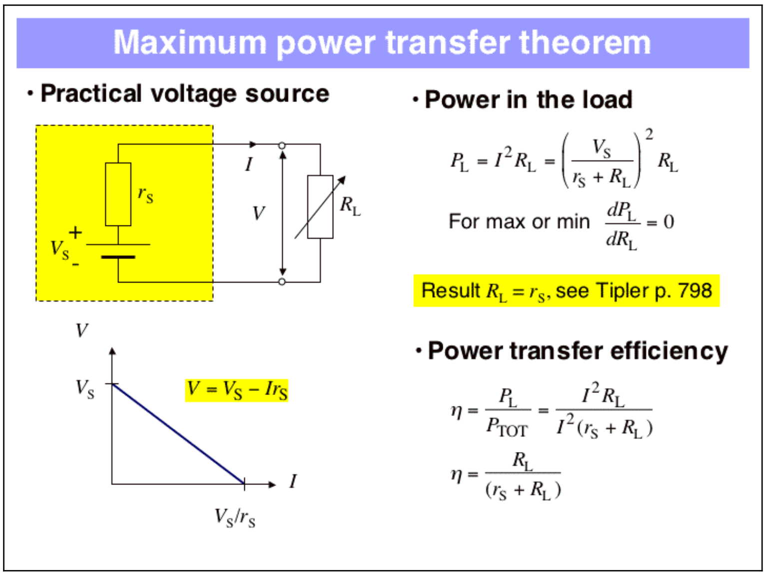


Figure 2.3: The effect of source internal resistance on power delivered to a load.

The power in the load can calculated using $P_L = I^2R_L$, where the current in this simple series circuit is found by taking the ratio of the source voltage to the total resistance. We now know P_L as a function of R_L. (V_S and r_S being constant.) To find the maxima (or minima) of a function with respect to a given variable, you differentiate the function with respect to that variable and set the result to zero.

This introduces the concept of **"impedance matching"**, which is an issue when connecting different pieces of electrical equipment together in the laboratory. Impedance and resistance are synonymous in d.c. circuits, although these terms have slightly different meanings in a.c. circuits, as we will see later in this course.

Having derived the condition for maximum power to be transferred to a load, it is instructive also to also look at the power transfer efficiency. It should be fairly apparent that if the load resistance matches the source resistance, half of the source voltage is dropped across each of them. This means that only half of the total power is dissipated in the load, the other half goes to heating up the battery! This is clearly rather wasteful. We can define the power transfer efficiency as the ratio of the power dissipated in the load to the total power (see figure 2.3).

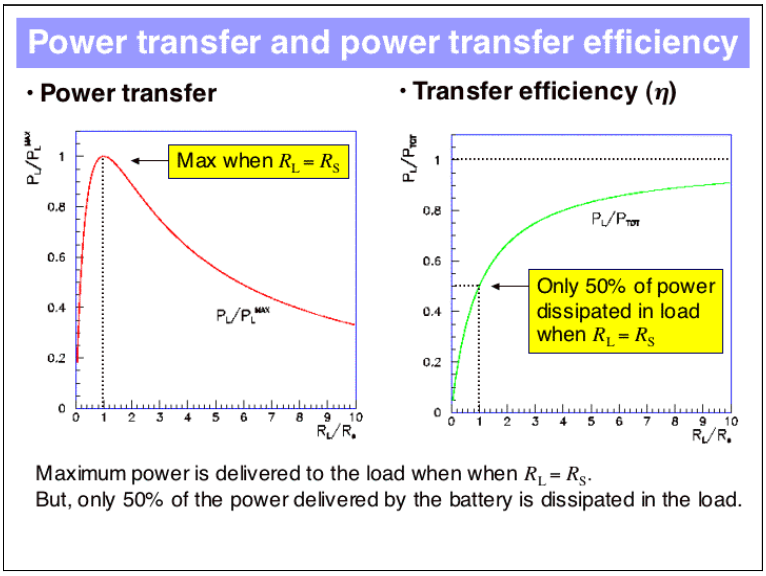


Figure 2.4: Power transfer and power transfer efficiency.

Figure 2.4, shows the power transfer and power transfer efficiency. The power in the load has been divided by the maximum value of the power transfer (when $R_L = r_S$), so that the curve peaks at one on the vertical axis. From the right-hand plot, you can see how by increasing R_L we get a larger fraction of the source power dissipated in the load. However, the left-hand plot, says that the source power steadily decreases $R_L > r_S$. The reason for this is that current decreases as R_L increases and as we have seen, the power in a resistor depends on the square of the current.

5. Equivalent circuits. Thévenin's theorem

We now turn to the main topic of this exercise. Let's start off with a statement of Thévenin's theorem, which is given in figure 2.5.

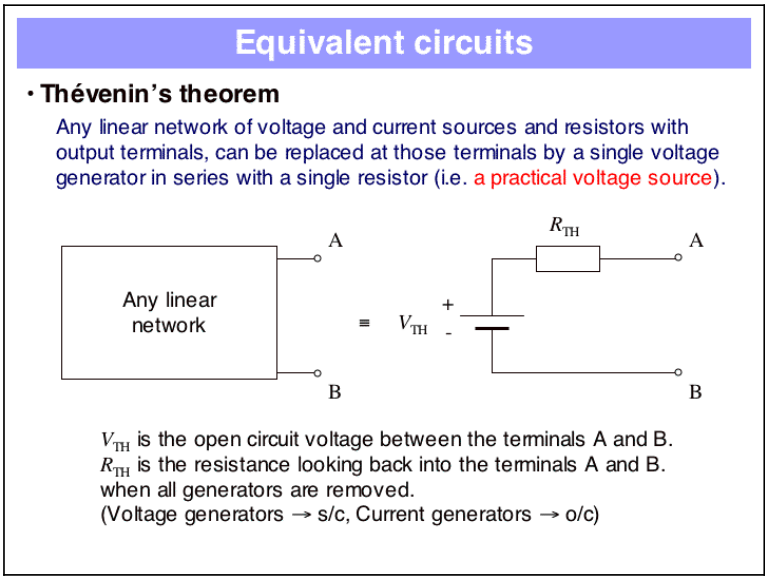


Figure 2.5: A statement of Thévenin's theorem
and the procedure for finding Thévenin's equivalent circuit.

Let that sink in for a moment. Thévenin's theorem states that for any circuit, no matter how complicated, as long as it is a linear circuit (elements within obey Ohm's law), there exists an equivalent circuit that comprises an ideal voltage source and a resistor connected in series. This is nothing more than a practical voltage source, as we have seen. That's astounding!

Thévenin's theorem is actually quite hard to prove for the general case, but we do have a specific case where we can see that this is true. Look back at figure 2.2 for a moment and ask yourself is there any way of knowing whether the source inside the box is a practical voltage source or a practical current source, from the point of view of the external circuit (i.e. the load)?

The short answer is no! All you could do is change the load and measure the voltage across it and the current flowing through it. You would find that there exists a linear relationship between the two quantities, but there is no way of deciding which type of source it might be. (You might think that the slope of the blue line might tell you something, but you could not be sure that you didn't have a voltage source with an unusually large internal resistance, or a current source with an unusually small internal resistance.) In this case, and Thévenin's theorem tells us every other case involving a linear circuit, both situations are equally well described by a practical voltage source.

Some of you may be thinking, wait a minute, what is special about the practical voltage source? The fact that the external circuit can't distinguish between a practical voltage source and a practical current source means that I could equally describe any linear circuit with a practical current source. And you'd be absolutely right! That is Norton's theorem. Thévenin came up with his theorem in 1883. Norton restated it for the current source in 1926.

OK, that's a neat trick, but is it useful? The answer is yes! Generally, we are only interested in what is happening in part of a circuit. On these occasions it simplifies things greatly if we can reduce the rest of the circuit to something more manageable. For example, you want to test the effects of different loads on a circuit, or you are trying to decide how to match the impedance of your load with the rest of your circuit.

I am sometimes asked why we don't study really complicated circuits in this course. The answer is that unless you are an electronic engineer, you are generally not interested in circuits at the individual component level. If you want to study what a circuit can do for you, take the simplest equivalent circuit that describes the one you've got. For linear circuits, where you can identify a pair of output terminals, this is nothing more than a practical voltage source. Now we will see what Thévenin's theorem can do for us.

6. Thevenin procedure.
Example 1

In this example we see how to use Thévenin's theorem in practice. The problem is to find the current in the 15 Ω resistor, which can be interpreted as the load for the remainder of the circuit. The analysis procedure is summarised in figure 2.6.

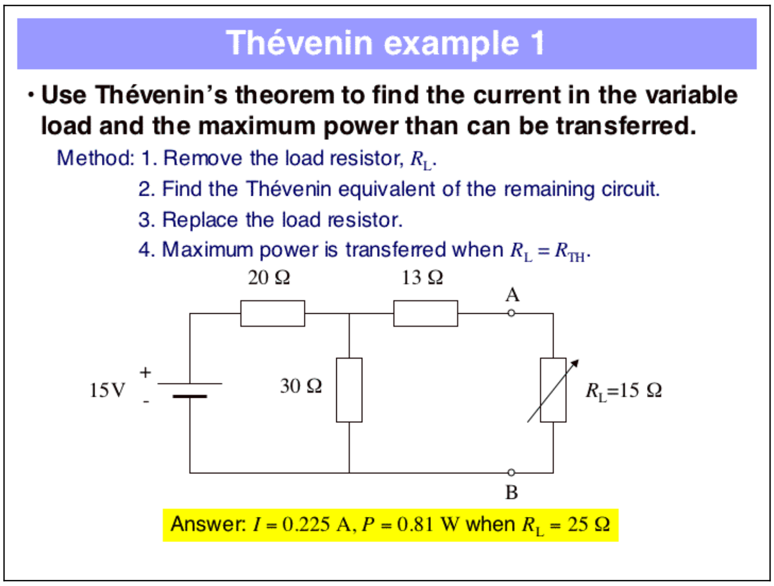


Figure 2.6: A worked example.

Thévenin's theorem refers to terminals. These may be notional, rather than physical parts of the circuit. We create a pair of terminals when we break the circuit, by removing the load. These are shown by A and B above. Now we are required to find the practical voltage source that replaces everything to the left of AB.

The first thing to notice is that when the load is removed (leaving an open-circuit), the 13 Ω resistor is only connected via one end. Therefore no current flows through that resistor. Already this is quite a simplification, as we are left with only the 20 Ω and 30 Ω resistors connected in series with the battery. These two resistors form a potential divider (see exercise 1), so we can determine that 9 V is the potential difference across the 30 Ω resistor. The terminals A and B are connected across this resistor. Since no current flows through the 13 Ω resistor when the load is removed, the open-circuit voltage V_{AB} = 9 V. This is precisely Thévenin's equivalent voltage source, $V_{TH} \equiv V_{AB}$.

We now need to determine Thévenin's equivalent resistance to put in series with this voltage source. This is done, as Figure 2.5 suggests, by removing the voltage source. Voltage sources are removed by replacing them with short-circuits (or a plain piece of wire if you prefer). A current source would be removed by replacing it with an open circuit. The following section attempts to offer an explanation.

6.1 Removing sources

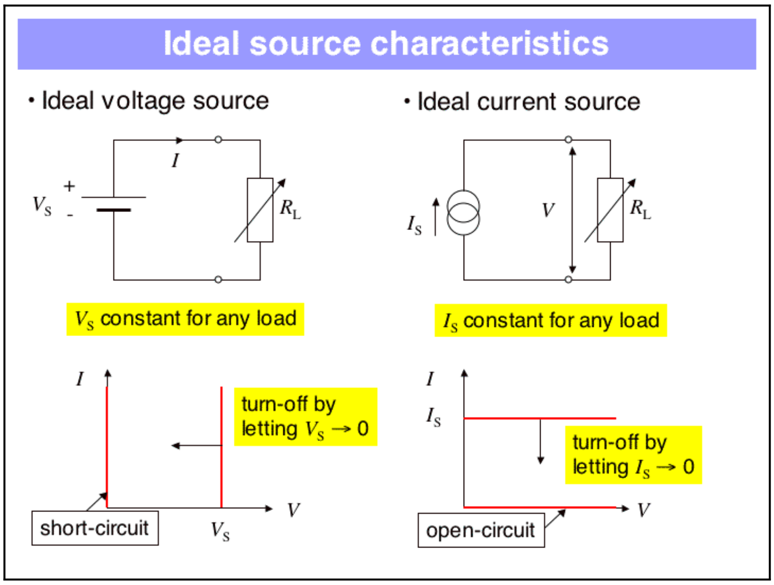


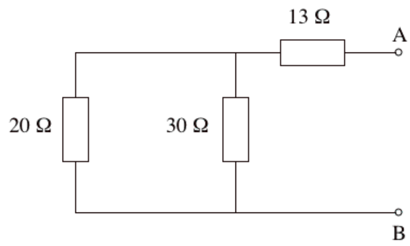
Figure 2.7: The effect of removing sources from a circuit.

Figure 2.7, reminds us of the I-V characteristics of ideal energy sources. Take the voltage source for example. What does removing the source really mean? We can think of it as turning down the magnitude of the voltage source. In this case the vertical red line would move to the left until, when $V_S = 0$, we are left with the red line directed along the vertical axis. This describes a part of the circuit, which can pass any current, but no potential difference is seen across its ends. This is exactly what we expect for a piece of wire, or a short-circuit.

If we think of turning off the current source in the same way, we are left with the red line along the horizontal axis. This describes part of a circuit through which no current flows, but across which any value of potential difference may be found. This is exactly what we'd expect for a gap in the circuit, or an open-circuit.

This is worth remembering since it comes up more than once during this course.

Back to the problem. We remove the voltage source by replacing it with a short-circuit and then ask, what single resistor could we place between A and B that would be equivalent to the network we have got. I've redrawn the network below, with the source removed.



I've also moved the 20 Ω resistor. Convince yourself that I haven't altered the circuit in any way by doing this. The procedure for finding the equivalent resistance is always to start from the end **furthest** from the terminals. Combine the 20 Ω and 30 Ω resistors, which are in parallel, to find a single equivalent resistor (12 Ω), which is in series with the 13 Ω resistor. Since resistors in series add, the total resistance between A and B is R_{TH} = 25 Ω.

Thévenin's equivalent of the circuit to the left of AB, shown in figure 2.6, is a 9 V source in series with a 25 Ω resistor. Now we can reconnect the load resistor and calculate the current that would flow. The maximum power transfer takes place when $R_L = R_{TH}$. I leave it as an exercise for you to calculate these quantities. (The answers are given in figure 2.6.)

Test your understanding (question on p.3):

Answer: *The result of placing the wire across the load (that is, in parallel with the load) would have been the same as replacing it with the wire. Intuitively, what we have done is provided an alternative path for the current to flow back to the battery, which has zero resistance. It is infinitely easier for current to flow down the wire, so no current flows through the load resistor. This is exactly what the phrase "shorted-out" means. It is like the shorted component isn't there at all. Algebraically, you can show this by applying the current splitter rule to this example.*

$$I_{wire} = \frac{R_L}{R_L + R_{wire}} I \qquad \text{where} \qquad R_{wire} = 0$$

The current in the wire is given by the ratio of the resistance in the other branch (the load) to the sum of the two resistances (which is this case is the load plus zero) multiplied by the current.