Quantum Mechanics 1 – Solution 5

a) The wavelength of the X-rays is obtained from $E = \frac{hc}{\lambda}$.

$$\lambda = \frac{hc}{E} = \frac{4.14 \times 10^{-15} \times 3 \times 10^{8}}{3.55 \times 10^{3}} = 3.50 \times 10^{-10}\ \mathrm{m} = 350\ \mathrm{pm}.$$

Note that I have used the value of Planck's constant in units of eV s, therefore the energy of the X-ray is given in eV. [1 mark]

To find the distance between the crystal planes, use the Bragg equation and, for the first interference maximum, set $n = 1$:

$$2d \sin\theta = n\lambda$$

[1 mark]

$$d = \frac{n\lambda}{2\sin\theta} = \frac{1 \times 350}{2 \times \sin(18^{o})} = 566\ \mathrm{pm}.$$

[2 marks]

b) To calculate the other angles, use

$$\sin\theta = \frac{n\lambda}{2d} = n \times \frac{350\ \mathrm{pm}}{2 \times 566\ \mathrm{pm}} = n \times 0.309$$

[1 mark]

Hence:

$n = 2$ gives $\sin\theta = 0.618$ and so $\theta = 0.667$ radians or 38.2^{o}

$n = 3$ gives $\sin\theta = 0.928$ and so $\theta = 1.19$ radians or 68.1^{o}

Values of $n > 3$ give values of $\sin\theta$ greater than unity. Hence there may only be two further interference maxima. Condition is that $\sin\theta \le 1$.

[3 marks]

c) Using the same formula, we now require that $\sin\theta \le 1$ for $n = 2$. This then gives the condition

$$\sin\theta = \frac{n\lambda}{2d} \le 1$$

$$\frac{2\lambda}{2d} = \frac{\lambda}{d} \le 1$$

$$\lambda \le 566\ \mathrm{pm}$$

[2 marks]