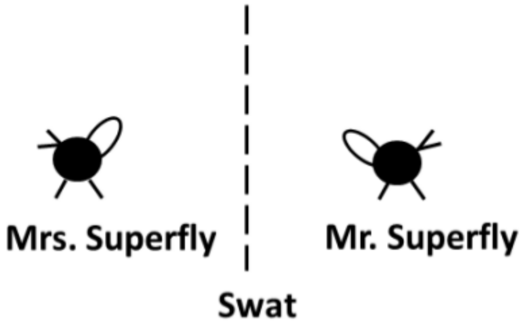
You may assume the standard Lorentz Transformations between the inertial frame Σ and another inertial frame Σ' moving with velocity v in the x-direction. At $t = t' = 0$ the axes of Σ and Σ' coincide. This arrangement is called *standard configuration.*

$$\begin{array}{rcl} t' &=& \gamma(v)(t - vx/c^2) \\ x' &=& \gamma(v)(x - vt) \\ y' &=& y \\ z' &=& z \end{array} \quad \text{and} \quad \begin{array}{rcl} t &=& \gamma(v)(t' + vx'/c^2) \\ x &=& \gamma(v)(x' + vt') \\ y &=& y' \\ z &=& z' \end{array} \quad \text{with } \gamma(v) = \frac{1}{\sqrt{1 - v^2/c^2}}$$

You should take $c \simeq 3 \times 10^8\, m\, s^{-1}$ in any numerical calculations.

1. (a) Mr. and Mrs. Superfly retreat in opposite directions from a flyswat with speeds $3c/4$ relative to the flyswat.



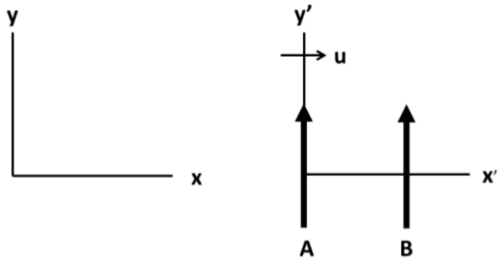
What speed does Mrs. Superfly have relative to Mr. Superfly?

(b) Having escaped the flyswat, Mr. and Mrs Superfly enjoy a well deserved rest together by the side of a railway track. To their horror, they realise that baby Superfly is trapped inside the Birmingham to Wolverhampton express train which, as is well known, travels with a speed $3c/5$ relative to the track. The guard in the train notices that Baby Superfly crawls vertically up a window with a speed $c/2$ relative to the window. What do Mr. and Mrs. Superfly decide is Baby Superfly's speed relative to the track and in what direction do they judge Baby Superfly to be travelling?

2. Two frames of reference Σ and Σ' are in the usual standard configuration with respect to each other. Their origins coincide at $t = t' = 0$ and Σ' moves to the right along the common $x - x'$ axis. An object moves, in the frame of reference Σ', with a velocity V_y' parallel to the y' axis.

(a) Show that an observer in Σ judges the object to have velocity components (V_x, V_y) where $V_x = u$ and $V_y = V_y'/\gamma(u)$.
Show that an observer in Σ judges the object to have velocity components (V_x, V_y) where $V_x = u$ and $V_y = V_y'/\gamma(u)$.

(b) The diagram shows two beetles A and B. In the frame of reference Σ' which has a velocity u along the $x - x'$ axis relative to the frame Σ, the beetles move perpendicular to the x'−axis and parallel to the y' axis of Σ' with a velocity V_0 and for convenience beetle A has elected to move along the y' axis of Σ' as shown.



The line AB joining the beetles remains parallel to the x' axis and has a length L_0 in the frame of reference Σ'. The beetles A and B each cross the x' axis at a time $t' = 0$ and remain separated in Σ' by a horizontal distance L_0 along the x' axis so that the line AB joining the beetles is, and remains, parallel to the x' axis.

What space and time coordinates does an observer in the frame Σ ascribe to the two events consisting of $A's$ crossing the x' axis of Σ' at $t' = 0$ and $B's$ crossing the x' axis of Σ' at $t' = 0$? Show that these events are not simultaneous in Σ and find the time difference between these events as judged by an observer in Σ.

(c) What angle does the line AB joining the two beetles make with the horizontal x axis?

3. It is well known that the speed limit for a rocket in space is $2 \times 10^7\,\mathrm{km\,hr^{-1}}$. A rocket travelling to a remote planet is piloted by a physicist and is arrested for driving through a set of red light in space. At the subsequent trial the physicist claims that the car was going so fast that the red light appeared green to the pilot. "Plea accepted" said the judge, "but I fine you £1 for each kilometer per hour by which you exceeded the speed limit." If the wavelength of green light is 530 nm and that of red light is 630 nm, calculate the fine.

4. The H_α line has a wavelength $656.3\,nm$ when measured by an observer who is at rest with respect to the hydrogen atom. What is the Doppler shift of this line from a star which moves directly away from the Earth with a velocity $300\,\mathrm{km\,s^{-1}}$?

5. A force F acts on a particle with rest mass m_0 and which moves along the $x-$axis. The work done by the force in displacing the particle through a distance dx is $Fdx = Fudt$ where $u = dx/dt$ is the velocity of the particle. This work done by the force is the change, dE, in the energy of the particle so that $dE/dt = Fu$.

(a) Show that use of the Newtonian relation between $F$and the non relativistc momentum of the particle leads to the expression

$$\Delta E = \Delta\left(\frac{m_0 u^2}{2}\right) = \text{ change in the non relativistic kinetic energy}$$

(b) If $\gamma(u) = 1/(1 - u^2/c^2)$, show that

$$\frac{d\gamma(u)}{dt} = \frac{u\dot{u}}{c^2}\gamma^3(u)$$

(c) Show that if the relativistic momentum $p = m(u)u$ is used in the expression $F = dp/dt$, then the change in the energy of the particle is given by

$$\Delta E = \Delta(m(u)c^2)$$

where $m(u) = \gamma(u)m_0$. This calculation is yet another argument for writing $E = mc^2$.

6. ★ A pion has rest mass 139.58MeV and a mean-lifetime of 2.5×10^{-8} s whereas a muon has rest mass 105.65MeV and a mean-lifetime of 2.2×10^{-6} s. A pion usually decays into a muon and a neutrino. You may assume that a neutrino is massless and lives forever.

   Assuming that the pion is initially at rest, find the velocity of the muon and the mean-distance that the muons travel in the pion rest-frame.

7. The deuteron is the nucleus of "heavy hydrogen" and is made up of a neutron and a proton bound together. The deuteron has a mass $m_d = 3.3433 \times 10^{-27}\,kg$. An unbound proton has a mass $m_p = 1.6724 \times 10^{-27}\,kg$ and an unbound neutron has a mass $m_n = 1.6747 \times 10^{-27}\,kg$. When a neutron and a proton fuse to form a deuteron an energy ΔE, equivalent to the mass difference $(m_n + m_p - m_d)$ is released as heat. Calculate this energy release in both $Joules$ and MeV. Estimate *very roughly* the energy that would be released in forming $1\,g$ of deuterium by fusing together an appropriate number of neutrons and protons. How long would a $10\,MW$ power station need to run to produce this quantity of energy?

8. (a) A particle with rest mass m_0 has a velocity u. For what value of u/c does the momentum of the particle differ by 1% from its non-relativistic value m_0u. Is the correct relativistic value greater or less than that obtained from the non-relativistic result?

   (b) In a chemical reaction in which $1\,kg$ of hydrogen combines with $8\,kg$ of oxygen to give $9\,kg$ of water about $10^8\,J$ of heat are released. To what mass is this equivalent? How does the fractional change in mass in this chemical reaction compare with the fractional mass change in a nuclear fusion process such as that described in the previous question?

1. (a) Mr. Superfly is on the left and Mrs. Superfly on the right. In the middle is the flyswat. Each retreats from the flyswat with velocity $3c/4$ relative to the flyswat. Care is needed here because we have three frames of reference - that of Mr. Superfly, that of the swat and finally the frame of Mars Superfly. The Lorentz Transformations deal with pairs of frames of reference in uniform motion with respect to each other. In the frame in which Mr. Superfly is at rest, the swat moves to the right with velocity $3c/4$. However in the frame of reference in which the swat is at rest, Mrs. Superfly has a velocity $3c/4$ to the right of the swat. In order to calculate the velocity of Mrs. Superfly relative to Mr. Superfly we thus need to use the relativistic composition law for velocities in which we think of Mr. Superfly as being at rest, the swat as having a velocity $u_1 = 3c/4$ relative to the Mr. Superfly, and Mrs. Superfly as having a velocity $u_2 = 3c/4$ relative to the swat. Mrs. Superfly's velocity V relative to Mr. Superfly is then given by using the relativistic composition law with both $u_1 = 3c/4$ and $u_2 = 3c/3$.

$$V = \frac{(u_1 + u_2)}{(1 + u_1 u_2/c^2)} = \frac{3c/4 + 3c/4}{1 + (3c/4)\,(3c/4)/c^2} = \frac{3c/2}{1 + 9/16} = \frac{3c}{2} \times \frac{16}{25} = \frac{24}{25}c.$$

Thus $V < c$ as expected. It is **tempting but wrong** to focus on the swat and say that the relative velocity of the two flies must me $3c/4 + 3c/4 = 3c/2$ ($> c$)!

(b) Let Σ be the frame of reference of the track in which Mr. and Mrs. Superfly are at rest after their exertions and let Σ' be the frame of reference of the train in which the train is at rest. In the frame of reference Σ' of the train, Baby Superfly has only a vertical component of velocity $V_y' = c/2$ as he crawls up the window. However the train has a velocity $u = 3c/5$ relative to the track. We know that if the components of the velocity of Baby Superfly are (V_x', V_y') relative to the train (i.e. in Σ') then relative the the track Baby Superfly will have a velocity with components (V_x, V_y) with

$$V_x = \frac{u + V_x'}{1 + uV_x'/c^2} \text{ and } V_y = \frac{1}{\gamma(u)} \frac{V_y'}{1 + uV_x'/c^2}$$

where u is the speed in the common $x - x'$ direction of Σ' relative to Σ. In this problem Baby Superfly moves vertically up the window of the train and so $V_x' = 0$. Thus in this problem

$$V_x = u \text{ and } V_y = \frac{V_y'}{\gamma(u)}$$

Since $u = 3c/5$, $\gamma(u) = 1/\sqrt{1 - (3c/5)^2} = 5/4$ and $V_y' = c/2$ we find that

$$V_x = \frac{3c}{5} \text{ and } V_y = \frac{c/2}{5/4} = \frac{2c}{5}.$$

Thus relative to the track, Baby Superfly has a *speed* $\sqrt{V_x^2 + V_y^2} = (c/5)\sqrt{3^2 + 2^2} = c\sqrt{13}/5$ and crawls in a direction making and angle θ to the horizontal given by $\tan\theta = V_y/V_x = 2/3 \to \theta = \tan^{-1}(2/3) \simeq 33.7^0$ or $0.57\,\mathrm{rad}$.

2. (a) Σ' moves to the right along the $x - x'$ axis relative to Σ. $V'_x = dx'/dt'$, but $dx' = \gamma(u)(dx - udt)$ and $dt' = \gamma(u)\left(dt - udx/c^2\right)$ and hence

$$V'_x = \frac{dx'}{dt'} = \frac{\gamma(u)(dx - udt)}{\gamma(u)\left(dt - udx/c^2\right)} = \frac{dx/dt - u}{1 - u(dx/dt)/c^2} = \frac{V_x - u}{1 - V_x/c^2}$$
$$V'_y = \frac{dy'}{dt'} = \frac{dy}{\gamma(u)\left(dt - udx/c^2\right)} = \frac{dy/dt}{\gamma(u)\left(1 - u(dx/dt)/c^2\right)} = \frac{V_y}{\gamma(u)\left(1 - V_x/c^2\right)}.$$

In Σ the object has a velocity with components $V_x = 0$ and V_y and so

$$V'_x = -u \text{ and } V'_y = \frac{V_y}{\gamma(u).}$$

The first expression simply simply tells us that Σ retreats from Σ' in the negative direction with speed u.

(b) The two frames are in standard configuration in which the origins coincide at $t = t' = 0$ and so if A, crosses the $x-$axis at $x = 0$ and $t = 0$, then in Σ', A will cross the x' axis at $x' = 0$ and $t' = 0$. B will then cross the $x - x'-$axis at $x' = L_0$ and at $t' = 0$ since we are told the line AB remains parallel to the $x-$axis. The Lorentz Transformation equations will give the coordinates in Σ of the event that B crosses the $x - x'$ axis.

$$x = \gamma(u)(x' + ut') = \gamma(u)(L_0 + u \times 0) = \gamma(u)L_0 \text{ and } t = \gamma(u)(0 + uL_0/c^2).$$

Thus in the frame of reference Σ, B crosses the common $x - x'$ axis at a time $t = u\gamma(u)L_0/c^2$ i.e. in Σ, B will cross the $x - x'$ axis **after** A crosses this axis by an amount $u\gamma(u)L_0/c^2$ seconds i.e. the events of A and of B crossing the $x - x'$ axis are not simultaneous in Σ'. The velocity in Σ of the line joining AB is $\left(V'_x, V'_y\right) = (0, V_0)$, and so its velocity components in Σ are $(V_x, V_y) = (u, V_0/\gamma(u))$. We have shown that an observer in Σ judges A to cross the axis at $t' = 0$, and B to cross the axis at a time later than this by $u\gamma(u)L_0/c^2$. In this time interval an observer in Σ judges B to move **vertically parallel to the y' axis** through a distance

$$V_y \times u\gamma(u)L_0/c^2 = \frac{V_0}{\gamma(u)} \times \frac{u\gamma(u)L_0}{c^2} = \frac{uV_0L_0}{c^2}$$

and horizontally through a distance

$$V_x \times \left(u\gamma(u)L_0/c^2\right) = (u) \times \left(u\gamma(u)L_0/c^2\right) = \gamma(u)\frac{u^2}{c^2}L_0.$$

(c) To obtain the horizontal separation of A and B, we must note the position of A and of B in the same frame in which the measurement is being made. A crosses the axis at $t = 0$ but B crosses the axis at the later time $t = u\gamma(u)L_0/c^2$ and at a position $x = \gamma(u)L_0$. However in Σ, B has a horizontal velocity component $V_x = u$, and so in the time interval $u\gamma(u)L_0/c^2$ its horizontal position changes by $V_x \times \left(u\gamma(u)L_0/c^2\right) = u^2\gamma(u)L_0/c^2$. Thus we need to measure the horizontal separation of A and B at the same time in Σ

$$\gamma(u)L_0 - \gamma(u)\frac{u^2}{c^2}L_0 = \gamma(u)L_0\left(1 - \frac{u^2}{c^2}\right) = L_0\frac{(1 - u^2/c^2)}{\sqrt{(1 - u^2/c^2)}} = L_0\sqrt{(1 - u^2/c^2)} = \frac{L_0}{\gamma(u)}.$$

At a time $t = t' = 0$, in Σ, the line AB will make an angle θ with the common $x - x'$ axis given by

$$\tan\theta' = \frac{uV_0L_0/c^2}{L_0/\gamma(u)} = \gamma(u)\frac{V_0u}{c^2}.$$

3. Choose $\lambda_0 = 630\,\mathrm{nm}$ to be the wavelength of the red light and $\lambda = 530\,\mathrm{nm}$ to be the wavelength of the green light then

$$\begin{aligned}
\frac{\lambda}{\lambda_0} &= \frac{530}{630} = \sqrt{\frac{1-u/c}{1+u/c}} \to \left(\frac{530}{630}\right)^2 = \frac{1-u/c}{1+u/c} \simeq .708 \to \frac{u}{c} = 0.171 \\
u &\simeq 0.171 \times 3 \times 10^8\,\mathrm{ms}^{-1} \simeq 0.51 \times 10^8\,\mathrm{ms}^{-1} = 5.1 \times 10^7\,\mathrm{ms}^{-1} = 5.1 \times 10^4\,\mathrm{kms}^{-1} \\
&= 5.1 \times 10^4 \times 3600\,\mathrm{km\,hr}^{-1} = 18.4 \times 10^7\mathrm{km\,hr}^{-1}
\end{aligned}$$

The speed limit is $2 \times 10^7\,\mathrm{km\,hr}^{-1}$ and so the driver exceeded the speed limit by $\left(18.4 \times 10^7 - 2 \times 10^7\right)\mathrm{km\,hr}^{-1} = 16.4 \times 10^7\mathrm{km\,hr}^{-1}$ and so the pilot of the rocket pays a fine of £1.64×10^7. That will teach her to speed!

4. We have a relativistic Doppler Shift with source and observer retreating and so

$$\nu = \nu_0\left(\frac{1-u/c}{1+u/c}\right)^{1/2}.$$

Since $c = \nu\lambda = \nu_0\lambda_0$ we have

$$\lambda = \lambda_0\left(\frac{1+u/c}{1-u/c}\right)^{1/2}.$$

Since $u = 300\,\mathrm{km\,s}^{-1} = 300 \times 10^3\mathrm{m\,s}^{-1} = 3 \times 10^5\mathrm{m\,s}^{-1}$ then taking $c = 3 \times 10^8\mathrm{m\,s}^{-1}$ we see that $u/c = 10^{-3}$ and so

$$\lambda = \lambda_0\left(\frac{1+10^{-3}}{1-10^{-3}}\right)^{1/2} = \lambda_0\left(\frac{1.001}{.999}\right)^{1/2} \simeq 1.001\lambda_0.$$

The shift in wavelength is $\lambda - \lambda_0 = 0.001\lambda_0 \simeq 0.656\,\mathrm{nm}$.

5. (a) The energy change of the non relativistic particle is the work done on if by the force F

$$\begin{aligned}
dE &= Fdx = F\frac{dx}{dt}dt = Fudt \text{ where } u = dx/dt \\
\frac{dE}{dt} &= Fu.
\end{aligned}$$

By Newton's second law for a non relativistic particle

$$F = \frac{dp}{dt} = \frac{dm_0u}{dt} = m_0\frac{du}{dt} \to \frac{dE}{dt} = m_0\frac{du}{dt}u.$$

But

$$\begin{aligned}
u\frac{du}{dt} &= \frac{1}{2}\frac{d}{dt}(u^2) \to \frac{dE}{dt} = \frac{m_0}{2}\frac{du^2}{dt} = \frac{d}{dt}\left(\frac{1}{2}m_0u^2\right) \\
&\to E = \frac{1}{2}m_0u^2 + \text{const}
\end{aligned}$$

The change ΔE in E when the velocity changes from u_1 to u_2 is thus

$$\Delta E = \frac{1}{2}m_0 u_2^2 - \frac{1}{2}m_0 u_1^2$$

as required.

(b) The relativistic calculation is a little more complicated. we use the equation $dE/dt = Fu$ to define E. However p is now the relativistic momentum given by $p = m(u)u = m_0\gamma(u)u$ (rather than just $m_0 u$).

$$\begin{aligned}
F &= \frac{dp}{dt} = m_0\frac{d}{dt}(\gamma(u)u) = m_0\left(u\frac{d\gamma(u)}{dt} + \gamma(u)\frac{du}{dt}\right)\\
\frac{d\gamma(u)}{dt} &= \frac{d\gamma(u)}{du}\times\frac{du}{dt} = \frac{du}{dt}\left(\frac{d(1-u^2/c^2)^{-1/2}}{du}\right) = \dot{u}\left(-\frac{1}{2}\times(-\frac{2u}{c^2}\right)(1-u^2/c^2)^{-3/2}\\
&= \frac{\dot{u}u}{c^2}\frac{1}{(1-u^2/c^2)^{3/2}} = \frac{\dot{u}u}{c^2}\gamma^3(u)\\
F &= m_0\left(u\times\frac{\dot{u}u}{c^2}\gamma^3(u) + \gamma(u)\frac{du}{dt}\right) = m_0\left(u\times\frac{\dot{u}u}{c^2}\gamma^3(u) + \gamma(u)\dot{u}\right)\\
&= m_0\dot{u}\left(\frac{u^2}{c^2}\gamma^3(u) + \gamma(u)\right)
\end{aligned}$$

Thus

$$\begin{aligned}
Fu &= m_0 u\dot{u}\left(\frac{u^2}{c^2}\gamma^3(u) + \gamma(u)\right) = m_0 u\dot{u}\gamma^3(u)\left(\frac{u^2}{c^2} + \frac{1}{\gamma^2(u)}\right)\\
&= m_0 u\dot{u}\gamma^3(u)\left(\frac{u^2}{c^2} + 1 - \frac{u^2}{c^2}\right) = m_0 u\dot{u}\gamma^3(u).
\end{aligned}$$

But

$$\begin{aligned}
\frac{d}{dt}m(u)c^2 &= c^2 m_0\frac{d\gamma(u)}{dt} = c^2 m_0\frac{d\gamma(u)}{du}\times\frac{du}{dt} = c^2 m_0\dot{u}\frac{d(1-u^2/c^2)^{-1/2}}{du}\\
&= c^2 m_0\dot{u}\times\left(-\frac{1}{2}\right)\left(-\frac{2u}{c^2}\right)(1-u^2/c^2)^{-3/2} = m_0\dot{u}u\gamma^3(u)
\end{aligned}$$

Hence

$$Fu = m_0 u\dot{u}\gamma^3(u) = \frac{d}{dt}m(u)c^2.$$

But $Fu = dE/dt$ and so

$$Fu = \frac{dE}{dt} = \frac{d}{dt}m(u)c^2.$$

Integrating this gives that a change $\Delta m(u)$ in th mass give s rise to a change

$$\Delta E = \Delta(m(u)c^2) = c^2\Delta(m(u))$$

so that a change $\Delta m(u)$ in the mass gives rise to a change $c^2\Delta m(u) = \Delta E$ in the mechanical energy of the particle.

6. We start with the pion at rest, so its energy and momentum are

$$E_i = m_\pi c^2 \,, \qquad P_i = 0$$

After the decay the produced muon and neutrino are moving away from each other along the line which we assume to coincide with x-axis. The muon moves with velocity V in the positive x-direction and the neutrino moves with momentum p in the negative x-direction.

The energy and momentum of the muon are

$$E_\mu = \frac{m_\mu c^2}{\sqrt{1-\frac{V^2}{c^2}}} \,, \qquad P_\mu = \frac{m_\mu V}{\sqrt{1-\frac{V^2}{c^2}}}$$

and those of the neutrino are

$$E_\nu = cp \,, \qquad P_\nu = -p \,.$$

The conservation of energy/mass gives

$$E_i = m_\pi c^2 = E_f = E_\mu + E_\nu = \frac{m_\mu c^2}{\sqrt{1-\frac{V^2}{c^2}}} + cp$$

and the conservation of momentum gives

$$P_i = 0 = P_f = P_\mu + P_\nu = \frac{m_\mu V}{\sqrt{1-\frac{V^2}{c^2}}} - p$$

Substituting p from the last equation into the energy conservation gives

$$m_\pi = \frac{m_\mu}{\sqrt{1-\frac{V^2}{c^2}}} + \frac{m_\mu V/c}{\sqrt{1-\frac{V^2}{c^2}}} = m_\mu \sqrt{\frac{1+V/c}{1-V/c}} \,,$$

hence

$$\frac{m_\pi^2}{m_\mu^2} = \frac{1+V/c}{1-V/c} \,, \qquad V/c = \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 + m_\mu^2} \simeq 0.27 \,.$$

If the life-time of the muon in its rest-frame is τ, then as a result of time dilation its mean life-time in the frame where it moves with velocity V will be $\tau' = \gamma(V)\tau$ and the mena distance traveled is

$$L = V\tau' = \frac{0.27 c\tau}{\sqrt{1-(0.27)^2}} \simeq 185\text{m} \,.$$

7. Let m_p,m_n and m_d be the masses of the proton,neutron and deuteron respectively. Then

$$\begin{aligned} m_p + m_n &= (1.6724 + 1.6747) \times 10^{-27}\,\text{kg and } m_d = 3.3433 \times 10^{-27}\,\text{kg. Thus} \\ (m_p + m_n - m_d) &= 0.0038 \times 10^{-27}\,\text{kg} = 3.8 \times 10^{-30}\,\text{kg.} \end{aligned}$$

Thus the energy released is

$$\Delta E = (m_p + m_n - m_d)\, c^2 \simeq (3.8 \times 10^{-30} \; \times 9 \times 10^{16})\,\mathrm{J} \simeq 3.4 \times 10^{-13}\mathrm{J}$$

where we have taken $c \simeq 3 \times 10^8\,\mathrm{ms}^{-1}$. But $1\mathrm{eV} \equiv 1.6 \times 10^{-19}\,\mathrm{J}$ and so the energy released in eV is

$$\Delta E \simeq \frac{3.4 \times 10^{-13}}{1.6 \times 10^{-19}}\,\mathrm{eV} \simeq 2.1 \times 10^6\,\mathrm{eV} \simeq 2.1\,\mathrm{MeV}.$$

The rest mass of the electron is $\simeq 0.5\,\mathrm{MeV}$ and so by comparison $\Delta E \sim 4$ electron masses.

The deuteron contains one neutron and one proton and so 2 g of deuterium contains $\sim 6 \times 10^{23}$ deuterons and 1g of deuterium contains roughly 3×10^{23} deuterons. Forming each deuteron by fission of a proton and a neutron releases about 2 MeV of energy and so the energy released in forming $1g$ of deuterium by fission is roughly $\left(3 \times 10^{23}\right) \times 2 \sim 6 \times 10^{23}\,\mathrm{MeV} \sim 6 \times 10^{29}\,\mathrm{eV} \sim 6 \times 10^{29} \times 1.6 \times 10^{-19}\,\mathrm{J} \sim 9.6 \times 10^{10}\,\mathrm{J}$. By comparison, a 10 MW power station produces $10 \times 10^6\,\mathrm{Js}^{-1}$ and so the time needed to produce $9.6 \times 10^{10}\,\mathrm{J}$ of energy is roughly

$$\frac{9.6 \times 10^{10}\,\mathrm{J}}{10 \times 10^6\,\mathrm{Js}^{-1}} \sim 9.6 \times 10^3\,s \sim 2.7\,\mathrm{hr}.$$

8. (a) The relativistic momentum associated with a particle of rest mass m_0 and velocity u is $p_{rel} = m_0\gamma(u)u$ and the non relativistic momentum is $p_{nonrel} = m_0 u$. We require that

$$\frac{p_{rel} - p_{nonrel}}{p_{nonrel}} = \frac{1}{100} = \frac{\gamma(u) - 1}{1} = \gamma(u) - 1 \rightarrow \gamma(u) = 1 + \frac{1}{100} = 1.01$$

Hence

$$\begin{aligned}\gamma^2(u) &= \frac{1}{1 - u^2/c^2} = 1.01^2 = 1.0201 \rightarrow 1 - u^2/c^2 = \frac{1}{1.0201} \rightarrow u^2/c^2 = 1 - \frac{1}{1.0201} \\ &= 0.0197 \rightarrow u/c = 0.14.\end{aligned}$$

Since $\gamma(u) > 1$ and $p_{rel} = \gamma(u)p_{nonrel}$ then $p_{rel} > p_{nonrel}$.

(b) Since $\Delta E = 10^8 J$ of energy are released, the mass equivalent of this is $\Delta E/c^2 \simeq 10^8/\left(3 \times 10^8\right)^2\,\mathrm{kg} = 0.111 \times 10^{-8}\mathrm{kg} = 1.11 \times 10^{-9}\mathrm{kg}$.
(N.B. $10^8 J \equiv 10^8/\left(1.6 \times 10^{-19}\right)\,\mathrm{eV} = 6.25 \times 10^{26}\,\mathrm{eV}$). The fractional mass change in the chemical reaction in which 9 kg of water are produced is thus

$$\frac{1.11 \times 10^{-9}\mathrm{kg}}{9\,\mathrm{kg}} = 0.123 \times 10^{-9} = 1.23 \times 10^{-10}$$

and the percentage change is approximately $1.2 \times 10^{-8}\,\%$. By comparison, the fractional change in mass in the nuclear fission reaction is

$$\frac{(m_p + m_n - m_d)}{m_d} = \frac{3.8 \times 10^{-30}\,\mathrm{kg}}{3.3433 \times 10^{-27}\,\mathrm{kg}} = 1.13 \times 10^{-3}$$

and so the percentage change in mass in the nuclear reaction is

$$1.13 \times 10^{-3} \times 10^{2} \sim 0.11\,\%.$$

Hence whereas the percentage mass change in the nuclear reaction is $1.1 \times 10^{-1}\%$ whereas in a chemical reaction the percentage change in mass is $1.2 \times 10^{-8}\,\%$ i.e. the nuclear reaction produces a measurable mass change which is seven orders of magnitude larger than the unmeasurably small percentage change which occurs in a chemical reaction.