UNIVERSITY OF BIRMINGHAM

School of Physics and Astronomy

DEGREE OF B.Sc. & M.Sci. WITH HONOURS

FIRST-YEAR EXAMINATION

03 19750

LC ELECTROMAGNETISM 1 / TEMPERATURE & MATTER / ELECTRIC CIRCUITS

SUMMER EXAMINATION 2024

Time Allowed: 3 hours

Answer five questions from Section 1 and three questions from Section 2.

Section 1 counts for 40% of the marks for the examination. Answer ***all five*** questions in this Section.
Section 2 consists of three questions and carries 60% of the marks.
Answer ***all three*** questions in this Section. Note that each question has two parts, of which only ***one part*** should be answered. If you answer both parts, credit will only be given for the best answer.

The approximate allocation of marks to each part of a question is shown in brackets **[ ]**.

PLEASE USE A SEPARATE ANSWER BOOK FOR SECTION 1 AND SECTION 2 QUESTIONS.

A formula sheet and a table of physical constants and units that may be required will be found at the end of this question paper.

SECTION 1

*Answer **all five** questions in this Section.*

1. Define and explain the zeroth and first laws of thermodynamics. What is meant by the terms isothermal, isobaric, isochoric and adiabatic? **[8]**

2. 12 grams of ^{16}O gas (molar mass of 32 amu) is expanded from its original volume of 2 litres at a fixed temperature of 300 K. During this expansion, its pressure changes from 200 kPa to 100 kPa. Calculate

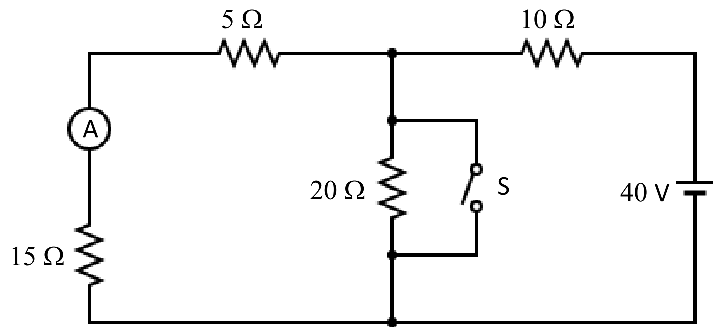
   (a) The work done by the gas; **[6]**

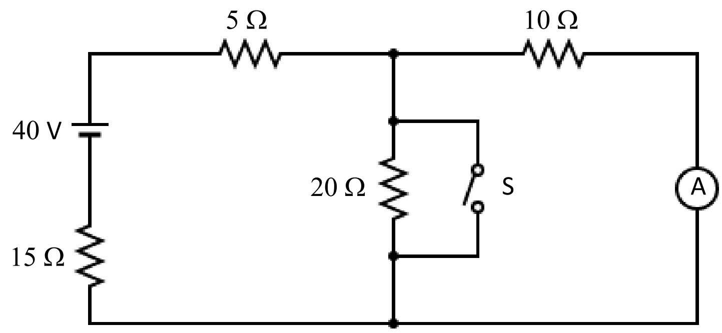
   (b) The heat added to the gas. **[2]**

3. Point charges of $-2Q$, Q, and q are fixed at (x, y) coordinates $(-a, 0)$, $(a, 0)$, and $(a, 2a)$ respectively. Derive the vector expression for the net electric force on the charge q. **[8]**

4. Derive an expression for the inductance of a solenoid of length l, radius R, and having n turns per unit length. (Hint: use Ampere's law and a sensible integration path to find the B-field and hence flux inside the solenoid for a given current. Then use the correct expression from the formula sheet.) **[8]**

5. Consider the two circuits shown below, where the symbol A represents an ammeter (an instrument to measure the current in a circuit).





(a) What is the assumption that is generally made for a good ammeter?

(b) With switches S open, calculate the readings at the ammeter in the two circuits and explain the comparison between the results.

(c) With switches S closed, calculate again the ammeter readings in the two circuits. Why are the currents different with respect to the previous case? Also, in which of these circuits with closed switches is the greatest power dissipated?

[8]

SECTION 2

*Answer **all three** questions in this Section. Note that each question has two parts, of which only **one part** should be answered. If you answer both parts, credit will only be given for the best answer.*

6. **EITHER Part A**

(a) Sketch the approximate form of the potential felt by an atom that is bound to another atom as a function of the interatomic distance. On this sketch, label both ϵ, the work done to separate the two atoms to infinity, and r_0, the equilibrium bond length between the two atoms. **[5]**

(b) Now, instead sketch the force described by this potential as a function of interatomic distance, indicating where this force is attractive and repulsive, with r_0 clearly labelled. **[3]**

(c) The general form of the Lennard-Jones potential is given by

$$V(r) = \frac{A}{r^{12}} - \frac{B}{r^6}.$$

Show that the generalised constants A and B can be related to ϵ and r_0, with $A = \epsilon r_0^{12}$ and $B = 2\epsilon r_0^6$. **[5]**

(d) A student puts some liquid argon, which has density ρ and surface tension γ, into a refrigerated tank with a small tube of radius r protruding vertically out of it. The liquid rises into the tube to a height h and forms a meniscus, with a contact angle θ between surface of the liquid and the wall of the tube. Derive an expression for the height the liquid reaches in the tube. You may assume the liquid undergoes an acceleration due to gravity of g m s^{-2}. **[7]**

OR Part B

(a) State the law of the equipartition of energy. **[3]**

(b) Use this law to predict and justify the molar heat capacity at constant volume, c_V, in terms of the gas constant R, for

i. a monatomic gas; **[2]**

ii. a diatomic gas; **[3]**

iii. a solid comprised of atoms in a lattice. **[3]**

(c) Oxygen gas (O_2) and chlorine gas (Cl_2) are experimentally found to have $c_V = 20.7$ J K^{-1} mol^{-1} and $c_V = 24.8$ J K^{-1} mol^{-1} respectively. What is the reason for this difference? **[3]**

(d) For a set of particles of mass m at a temperature T, the distribution of their speeds v is given by the normalised Maxwell-Boltzmann distribution

$$P(v) = 4\pi \left(\frac{m}{2\pi k_B T}\right)^{\frac{3}{2}} v^2 \exp\left(\frac{-mv^2}{2k_B T}\right)$$

i. Show that the most probable speed is $\sqrt{2k_B T/m}$. **[3]**

ii. Through use of the law of the equipartition of energy, justify that the root-mean square speed is given by $\sqrt{3k_B T/m}$. **[3]**

7. **EITHER Part A**

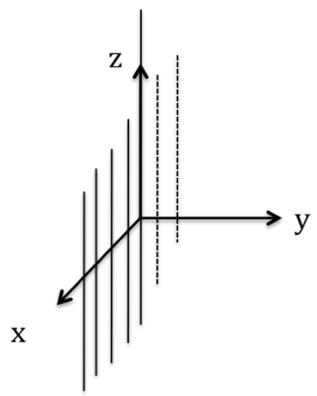
(a) An infinite long wire with a uniform charge density, λ, along its length is placed along the z-axis. By choosing a suitable Gaussian surface, use Gauss' Law to find an expression for the electric field at a distance r from this wire along the y-axis. **[5]**

(b) An infinite number of such infinite long wires, each identical to the one described in part (a), are placed in the x-z plane, as shown in the figure below. All the wires are parallel to the z-axis and equally spaced so that there are N wires per unit length along the x direction.
Use the superposition principle in combination with the result from (a) to derive an expression for the electric field at a point, with distance d from the origin, along the y-axis **[10]**

You may find the follow sum useful:

$$\sum_{n=-\infty}^{\infty} \frac{1}{a^2+n^2} = \frac{\pi \coth(\pi a)}{a},$$

where a is any constant.



(c) Show that when d is much greater than the wire spacing, your result from part (b) approximates to the electric field from an infinite charged plane with charge per unit area, σ. **[5]**

OR Part B

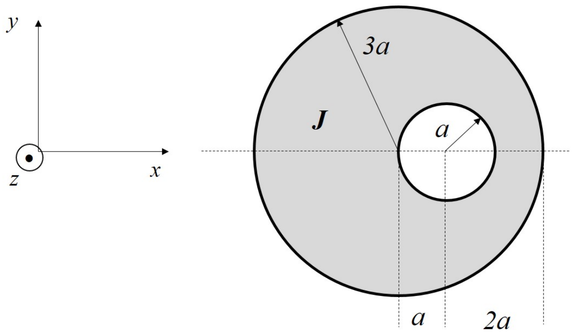
(a) A uniform current density, $\mathbf{J}$ flows through an infinitely long wire of radius R. Use Ampere's Law to derive expressions (in terms of J, R, r, and any constants) for the magnetic field:

i. outside the wire at a distance $r > R$; **[3]**

ii. inside the wire at distance $r < R$. **[4]**

(b) Describe (or make a sketch showing) the direction of the magnetic field outside the wire mentioned in part (a) above. **[3]**

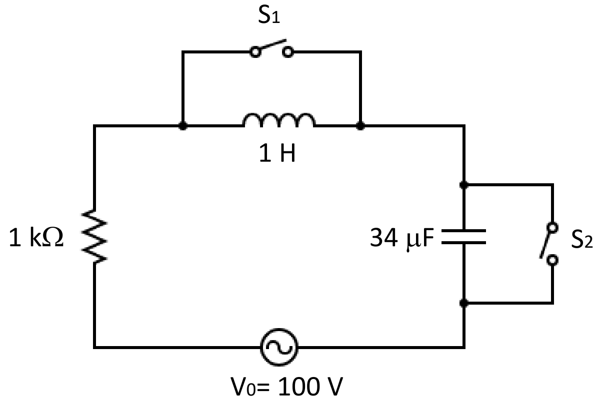
(c) The figure below shows the cross-section of an infinitely long conducting cylinder of radius $3a$ carrying a current density of J in the positive z direction (coming out of the page). Inside the cylinder is an infinitely long cylindrical hole of radius a, which is displaced so that its centre is at a distance a from the centre of the the conducting cylinder (obviously, no current flows inside the hole).



Derive a vector expression for the magnitude and direction of the magnetic field inside the hole.
(Hint: use the superposition principle and remember the magnetic field is perpendicular to both the current density and radial vector i.e. $\mathbf{B} \propto \mathbf{J} \wedge \mathbf{r}$.) **[10]**

8. **EITHER Part A**

In the circuit below consider an AC voltage source of peak value V_0=100 V, and frequency 60Hz.

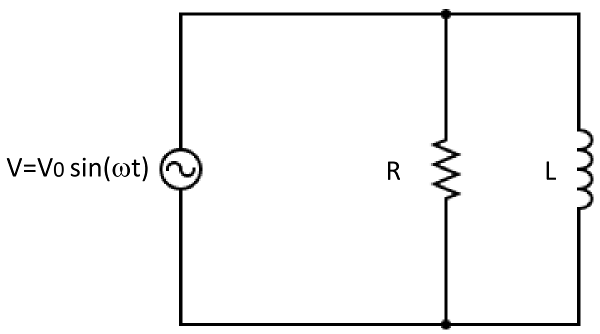


(a) Find an expression for the current I(t), and calculate the average power delivered to the circuit when both the switches are closed. **[5]**

(b) Open the switch S_1 and find an expression for the current as a function of time (neglect the transient). **[3]**

(c) Now open both switches S_1 and S_2, at which frequency the current is in phase with the voltage?
For this frequency, calculate the total impedance of the circuit.
Then calculate the maximum energy stored in the capacitor during the oscillations. **[8]**

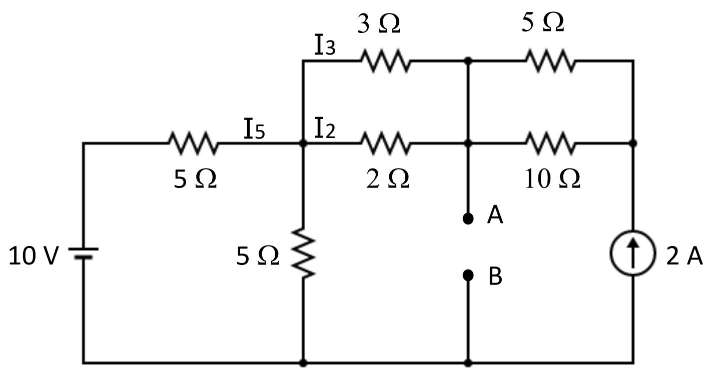
(d) Consider now the circuit below: obtain its series equivalent such that the



series circuit draws the same current and power at a given voltage and frequency ω. **[4]**

OR Part B

Consider the circuit below:



(a) Calculate the currents I_5, I_2, and I_3, as indicated in the figure, and specify their direction. **[5]**

(b) Calculate the voltage between terminals A and B, V_{AB}=V_A-V_B. **[5]**

(c) Draw the Thevenin's equivalent circuit between points A and B, showing the element values. **[5]**

(d) Add a 10 μF capacitor between A and B, and calculate the time constant of the resulting circuit. What does the time constant represent in the dynamics of the circuit? **[5]**

Formula Sheet
LC Electromagnetism 1 / Temperature & Matter / Electric Circuits

Useful Formulae for Electromagnetism 1

Force between two charges
$$\mathbf{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{\mathbf{r}}_{\mathbf{12}}$$

Coulomb's Law
$$\mathbf{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{\mathbf{r}}$$

Coulomb potential
$$V = \frac{Q}{4\pi\epsilon_0 r}$$

Gauss' Law
$$\int_S \mathbf{E} \cdot d\mathbf{S} = \frac{Q_{enc}}{\epsilon_0}$$

Field and potential relation
$$\mathbf{E} = -\nabla V$$

Electric Dipole
$$\mathbf{p} = q\mathbf{a}$$

Torque on Electric Dipole
$$\boldsymbol{\tau} = \mathbf{p} \wedge \mathbf{E}$$

Energy of Electric Dipole
$$U = -\mathbf{p} \cdot \mathbf{E}$$

Capacitance
$$C = \frac{Q}{V}$$

Stored energy in capacitor
$$U = \frac{1}{2}CV^2$$

Energy density of E-field
$$u_E = \frac{1}{2}\epsilon_0 E^2$$

Lorentz Force
$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \wedge \mathbf{B}\right)$$

Force on current length
$$\mathbf{F} = I\boldsymbol{\ell} \wedge \mathbf{B}$$

Magnetic dipole
$$\boldsymbol{\mu} = I\mathbf{A}$$

Torque on magnetic dipole
$$\boldsymbol{\tau} = \boldsymbol{\mu} \wedge \mathbf{B}$$

Energy of magnetic dipole
$$U = -\boldsymbol{\mu} \cdot \mathbf{B}$$

Biot-Savart Law
$$\delta\mathbf{B} = \frac{\mu_0 I}{4\pi}\frac{\delta\mathbf{l} \wedge \hat{\mathbf{r}}}{r^2}$$

Ampere's Law
$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

E.M.F
$$\epsilon = -N\frac{d\Phi_m}{dt}$$

Faraday's Law
$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_m}{dt}$$

Inductance
$$N\Phi_m = LI$$

Stored energy in inductor
$$U_L = \frac{1}{2}LI^2$$

Energy density of B-field
$$u_B = \frac{1}{2}\frac{B^2}{\mu_0}$$

Formula Sheet
LC Electromagnetism 1 / Temperature & Matter / Electric Circuits

Useful Formulae for Temperature & Matter

The first law of thermodynamics

$$\mathrm{d}U = đQ - p\,\mathrm{d}V$$
$$\Delta U = Q_{\text{in}} + W_{\text{on}}$$

Ideal gas equation of state

$$pV = n\mathrm{R}T$$

Ideal gas adiabatic process

$$pV^{\gamma} = \text{constant}\,,\ \text{where}\ \gamma = C_p/C_V\ .$$

Heat Transfer

Rate of heat flow by conduction $\dot{Q} = -\kappa A \dfrac{\partial T}{\partial x}$

Stefan-Boltzmann $\dot{Q} = \sigma e A T^4$

Linear coefficient of thermal expansion, $\boldsymbol{\alpha}$

$$\ell(T) = \ell(T_0)[1 + \alpha(T - T_0)]\ .$$

Gamma function and Stirling's approximation

$$\Gamma(N+1) = N! = \int_0^{\infty} \mathrm{d}x\, x^N\, \mathrm{e}^{-x} \quad \text{and} \quad N! \approx \left(\frac{N}{\mathrm{e}}\right)^N \Leftrightarrow\ \ln N! \approx N(\ln N - 1)\,.$$

Gaussian integral

$$\int_{-\infty}^{\infty} \mathrm{e}^{-\alpha x^2} \mathrm{d}x = \sqrt{\frac{\pi}{\alpha}}\,.$$

Normalised one-dimensional Maxwell-Boltzmann distribution

$$p_{\text{1d}}(v) = \sqrt{\frac{m}{2\pi \mathrm{k_B} T}} \exp\left(-\frac{mv^2}{2\mathrm{k_B}T}\right)\ .$$

Physical Constants and Units

Acceleration due to gravity	g	9.81 m s^{-2}
Gravitational constant	G	6.674×10^{-11} N m^2 kg^{-2}
Ice point	T_{ice}	273.15 K
Avogadro constant	N_A	6.022×10^{23} mol^{-1}
		[*N.B.* 1 mole $\equiv 1$ *gram-molecule*]
Gas constant	R	8.314 J K^{-1} mol^{-1}
Boltzmann constant	k, k_B	1.381×10^{-23} J K^{-1} $\equiv 8.62 \times 10^{-5}$ eV K^{-1}
Stefan constant	σ	5.670×10^{-8} W m^{-2} K^{-4}
Rydberg constant	R_∞	1.097×10^7 m^{-1}
	$R_\infty hc$	13.606 eV
Planck constant	h	6.626×10^{-34} J s $\equiv 4.136 \times 10^{-15}$ eV s
$h/2\pi$	$\hbar$	1.055×10^{-34} J s $\equiv 6.582 \times 10^{-16}$ eV s
Speed of light *in vacuo*	c	2.998×10^8 m s^{-1}
	$\hbar c$	197.3 MeV fm
Charge of proton	e	1.602×10^{-19} C
Mass of electron	m_e	9.109×10^{-31} kg
Rest energy of electron		0.511 MeV
Mass of proton	m_p	1.673×10^{-27} kg
Rest energy of proton		938.3 MeV
One atomic mass unit	u	1.66×10^{-27} kg
Atomic mass unit energy equivalent		931.5 MeV
Electric constant	ϵ_0	8.854×10^{-12} F m^{-1}
Magnetic constant	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Bohr magneton	μ_B	9.274×10^{-24} A m^2 (J T^{-1})
Nuclear magneton	μ_N	5.051×10^{-27} A m^2 (J T^{-1})
Fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	7.297×10^{-3}= 1/137.0
Compton wavelength of electron	$\lambda_c = h/m_e c$	2.426×10^{-12} m
Bohr radius	a_0	5.2918×10^{-11} m
angstrom	Å	10^{-10} m
barn	b	10^{-28} m^2
torr (mm Hg at 0 °C)	torr	133.32 Pa (N m^{-2})

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Do not complete the attendance slip, fill in the front of the answer book or turn over the question paper until you are told to do so.

Important Reminders

- Coats/outwear should be placed in the designated area.
- Unauthorised materials (e.g. notes or Tippex) must be placed in the designated area.
- Check that you do not have any unauthorised materials with you (e.g. in your pockets, pencil case).
- Mobile phones and smart watches **must** be switched off and placed in the designated area or under your desk. They must not be left on your person or in your pockets.
- You are not permitted to use a mobile phone as a clock. If you have difficulty seeing a clock, please alert an Invigilator.
- You are not permitted to have writing on your hand, arm or other body part.
- Check that you do not have writing on your hand, arm or other body part – if you do, you must inform an Invigilator immediately
- Alert an Invigilator immediately if you find any unauthorised item upon you during the examination.

Any students found with non-permitted items upon their person during the examination, or who fail to comply with Examination rules may be subject to Student Conduct procedures.