Electromagnetism

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Lecture 4c

Final part on Gauss’ Law

Week 2

Last Lecture

- Gauss's Law
  - Examples using Gauss's Law
  - E-fields in conductors
  - E-field between charged Conducting Plates

Lecture 4c Content

- Final Note on Gauss's Law
  - Choosing a Gaussian surface
- E-field between non-conducting plates

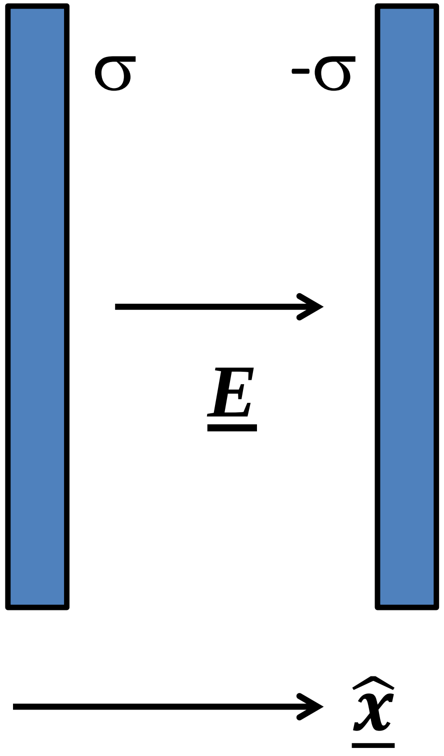
Gaussian Surface

- Use Gauss's Law in case where there is symmetry
  - E.g. sphere, infinite planes, infinite line charges
- Choose Gaussian surface such that the E-field is normal to the Gaussian surface such that:

$\underline{E} \cdot d\underline{S}$ = E dS and E is constant at surface i.e. $\int_S \underline{E} \cdot d\underline{S} = \int_S E\,dS = E \int_S ds$

- Hence $\int_S ds$ is just the area of the surface.

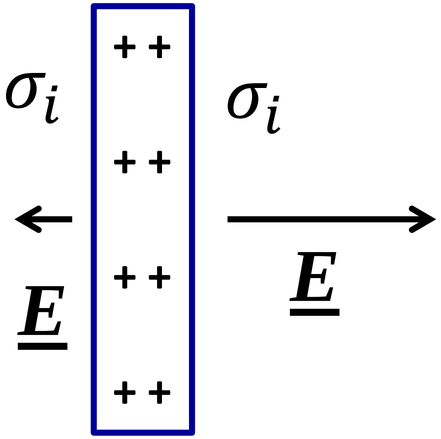
Non-conducting Charged Parallel Plates (infinite)



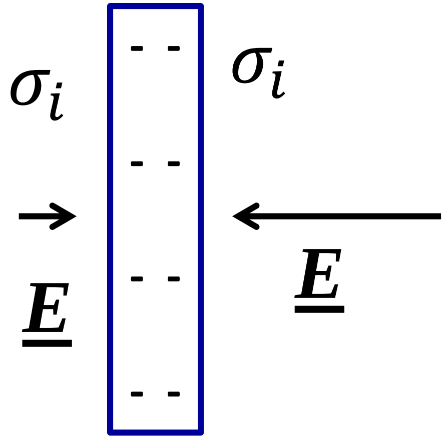
- Surface charge from LHS plate gives field: $\underline{E}_L = \dfrac{\sigma}{2\varepsilon_0}\hat{\underline{x}}$ *(see Lec. 3)*
- Surface charge from RHS plate gives field: $\underline{E}_R = \dfrac{-\sigma}{2\varepsilon_0}\hat{\underline{n}} = \dfrac{\sigma}{2\varepsilon_0}\hat{\underline{x}}$
  - Where $\hat{\underline{n}}$ is the unit vector normal to the surface.
- Superposition principle gives:
- $\underline{E} = \underline{E}_L + \underline{E}_R = \dfrac{\sigma}{\varepsilon_0}\hat{\underline{x}}$

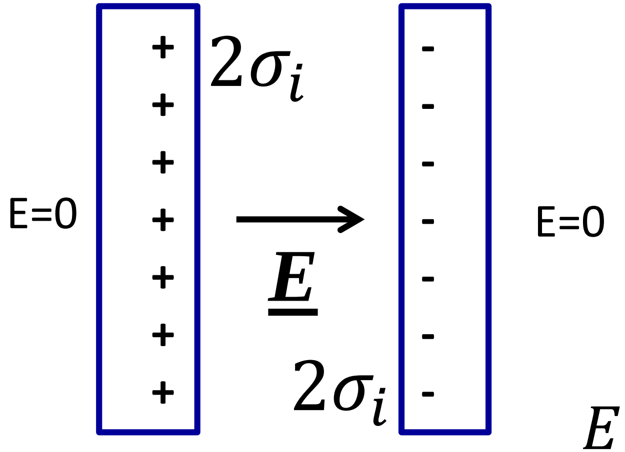
Note: Same result as for conducting parallel plates (last lecture) – WHY?

Conducting Charged Parallel Plates (infinite)



Consider two oppositely charged plates a long way apart. They have charges on each surface.



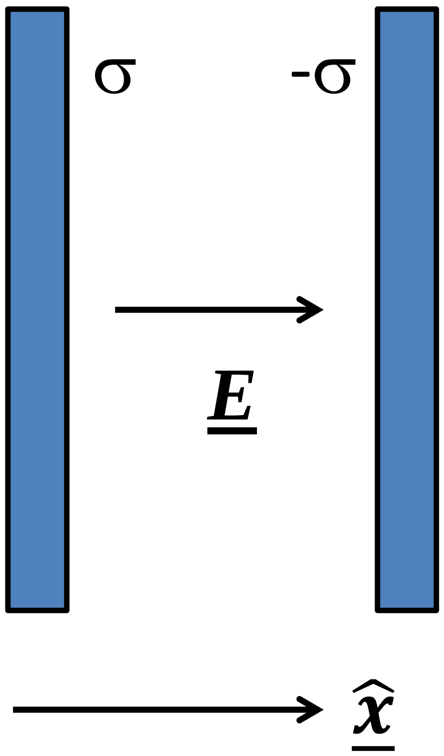


As the plates are moves close together, their E-fields attract the charges on each plates until all the charges move to the front faces, doubling the surface charge density. So add initial E-fields:

$$E = \frac{\sigma_i}{\varepsilon_0} + \frac{\sigma_i}{\varepsilon_0} = \frac{2\sigma_i}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}$$

Where σ is final charge density

Back to Non-conducting Charged Parallel Plates



- E-field in between plates

$$\underline{E} = \frac{\sigma}{2\varepsilon_0}\underline{\hat{\boldsymbol{x}}} + \frac{\sigma}{2\varepsilon_0}\underline{\hat{\boldsymbol{x}}} = \frac{\sigma}{\varepsilon_0}\underline{\hat{\boldsymbol{x}}}$$

- E-field to the left of plates:

$$\underline{E} = -\frac{\sigma}{2\varepsilon_0}\underline{\hat{\boldsymbol{x}}} + \frac{\sigma}{2\varepsilon_0}\underline{\hat{\boldsymbol{x}}} = 0$$

- E-field to the right of plates:

$$\underline{E} = \frac{\sigma}{2\varepsilon_0}\underline{\hat{\boldsymbol{x}}} - \frac{\sigma}{2\varepsilon_0}\underline{\hat{\boldsymbol{x}}} = 0$$

- E-field outside non-zero if plates have different charge densities. (see week 3 problem 1).