Year 1 Assessed Problems

Semester 1

Assessed Problems 3

SOLUTIONS TO BE SUBMITTED ON CANVAS BY

Wednesday 23rd October

at 17:00

Classical mechanics and Relativity
Problem Sheet 2

A stunt car leaves a jump at a height of 5.0 meters above ground level at an angle of 45 degrees upwards from the horizontal, traveling with a total velocity of 150 km per hour.

1. Find the car's distance from take-off point and speed 3.0 seconds after the jump.

   [5 marks]

2. How long after the jump will the car hit the ground, how far will it have travelled horizontally at that point, and what will its speed be?

   [5 marks]

Quantum Mechanics 1 – Problem 2

a) Show that the Boltzmann probability function

$$P(E) = exp(-E/k_B T)/k_B T$$

is properly normalized (i.e. show that $\int_0^\infty P(E)\, dE = 1$). [2 marks]

b) Show that the Boltzmann probability function results in an average energy

$$\bar{E} = \int_0^\infty E\, P(E)\, dE = k_B T.$$

Hint: to show this you will need to integrate by parts. Indeterminate limits of the form ∞/∞ can be evaluated using L'Hopital's rule:

$$\lim_{E\to\infty} \frac{f(E)}{g(E)} = \lim_{E\to\infty} \frac{f'(E)}{g'(E)}.$$

where $f'(E) = df(E)/dE$ and $g'(E) = dg(E)/dE$. [6 marks]

c) Blackbody radiation from the early Universe is estimated to be 2.73 K. Use Wien's displacement law to calculate the peak wavelength. To what part of the electromagnetic spectrum does radiation at this wavelength belong? [2 marks]

[Notes on calculus with (natural) exponential functions: to differentiate an exponential function, write down the exponential function and multiply it by the derivative of the power, remembering that $exp(ax)$ is just Euler's number raised the power ax, where a is a constant.

$$\frac{d}{dx} exp(ax) = a\, exp(ax)$$

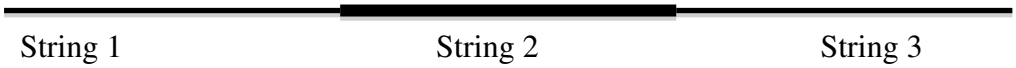
Likewise, to integrate an exponential function, write down the exponential function and divide it by the derivative of the power.

$$\int exp(ax)\, dx = \frac{exp(ax)}{a} + C$$

Here, I've added the constant of integration, C, as I have not specified any limits.]

Optics and Waves, Week 3

Consider the system of strings below, which is maintained at a fixed tension. All strings are 1.0 m long. Strings 1 and 3 have a mass of 10.0 g each and string 2 a mass 40.0 g. For a short time interval a wave pulse of amplitude 1.0 mm is generated on the left end and travels through strings 1, 2 to the end of 3.



String 1 String 2 String 3

a) What is the ratio of the wavelength of the waves on strings 1, 2 and 3? [4]
b) What is the amplitude of the wave pulse that arrives at the end of string 3? (Reflection at the end of String 3 not to be considered) [6]