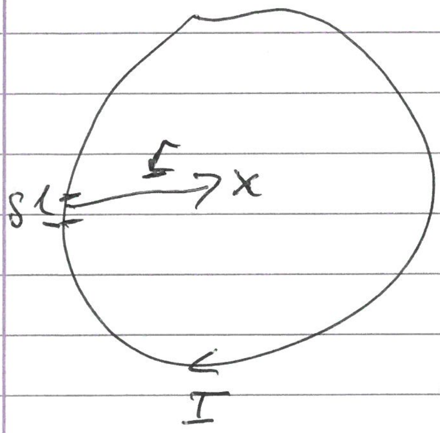
EM - lecture 13

Ex 13 - 1

B-field at centre of current loop.



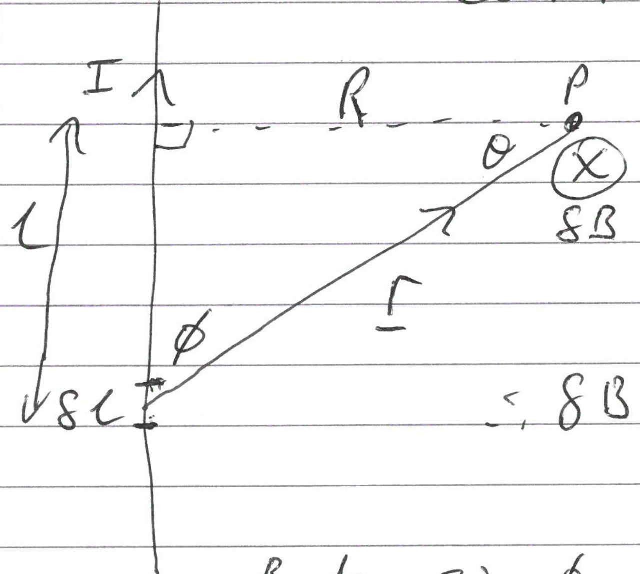
$$\delta \underline{B} = \frac{\mu_0}{4\pi} I \frac{\delta \underline{l} \wedge \hat{\underline{r}}}{r^2}$$

$$\therefore \delta B = \frac{\mu_0 I}{4\pi} \frac{\delta l}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi} \frac{1}{r^2} \oint dl = \frac{\mu_0 I}{4\pi} \frac{1}{r^2} \cdot 2\pi r$$

$$\therefore B = \frac{\mu_0 I}{2r}$$

Ex 13.2   B-field from line of current



$$\delta \underline{B} = \frac{\mu_0 I}{4\pi r^2}\, \delta \underline{l} \wedge \hat{\underline{r}}$$

$$\therefore \delta B = \frac{\mu_0 I}{4\pi r^2} \sin\phi \, \delta l \quad (1)$$

But $\sin\phi = \sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$ (2)

$$r = R/\cos\theta \quad (3)$$

$$\tan\theta = \frac{l}{R} \Rightarrow l = R\tan\theta \Rightarrow \frac{dl}{d\theta} = \frac{R}{\cos^2\theta}$$

$$\Rightarrow \delta l = \frac{R}{\cos^2\theta}\, \delta\theta \quad (4)$$

Plug (2), (3), & (4) into (1)

$$\Rightarrow \delta B = \frac{\mu_0 I}{4\pi} \cdot \frac{\cos^2\theta}{R^2} \cdot \cos\theta \cdot \frac{R\,\delta\theta}{\cos^2\theta}$$

$$= \frac{\mu_0 I}{4\pi} \frac{1}{R} \cos\theta \, \delta\theta$$

$$\therefore B = \frac{\mu_0 I}{4\pi R} \int_{\theta_1}^{\theta_2} \cos\theta \, d\theta \quad (2)$$

(Ex 13.2)

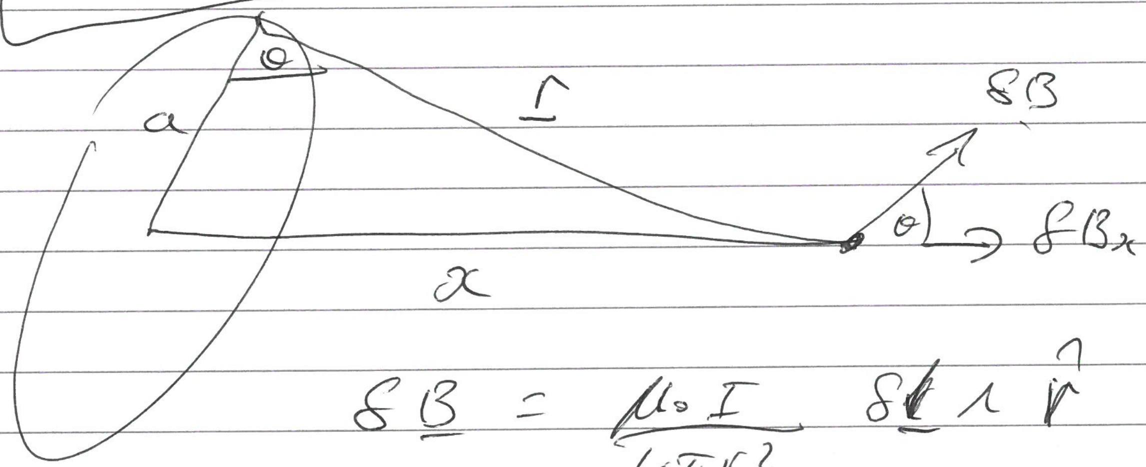
$$B = \frac{\mu_0 I}{4\pi R}\left(\sin\theta_2 - \sin\theta_1\right)$$

For infinite line of current:

$\theta_2 = \pi/2$, $\theta_1 = -\pi/2$

$$\therefore \boxed{B = \frac{\mu_0 I}{2\pi R}} \quad [\text{Eq } 13.2]$$

$\boxed{\text{Ex 13.3}}$



$$\delta \underline{B} = \frac{\mu_0 I}{4\pi r^2} \, \delta \underline{l} \wedge \hat{r}$$

$$\delta B_x = \frac{\mu_0 I}{4\pi r^2} \delta l \cos\theta \quad (1)$$

$$r^2 = a^2 + x^2 \quad (2) \quad \& \quad \cos\theta = \frac{a}{r}$$

$$\therefore \cos\theta = \frac{a}{(a^2+x^2)^{1/2}} \quad (3)$$

(2) & (3) in (1)

$$\delta B_x = \frac{\mu_0 I}{4\pi} \frac{a}{(a^2+x^2)^{3/2}} \delta l$$

$$B_x = \frac{\mu_0 I}{4\pi} \frac{a}{(a^2+x^2)^{3/2}} \oint dl$$

$2\pi a$

$$\therefore B_x = \frac{\mu_0 I}{2} \frac{a^2}{(a^2+x^2)^{3/2}}$$

(4)