Year 1 Assessed Problems

Semester 1

Problem Sheet 6

SOLUTIONS TO BE SUBMITTED ON CANVAS

DEADLINE

13th November 2024 @ 17:00hrs

3 Assessed – Limits and Differentiation 1

Problem 3.1 **Limits**
Determine the following limits, taking square roots to be positive:

$$\text{(i)} \quad \lim_{x\to 0^+} \frac{\sqrt{\sin 2x}}{\sqrt{x}} \; ; \; \text{(ii)} \quad \lim_{x\to 1} \frac{x^2-3x+2}{x^2-4x+3} \; .$$

The notation 0^+ indicates approaching $x = 0$ from the positive side.

Problem 3.2 **Derivatives**
Differentiate:

$$\text{(i)} \quad \sin(2x+3) \; ; \; \text{(ii)} \quad \frac{1}{(3x-1)^2} \; ; \text{(iii)} \quad \tan\left(\frac{x+1}{2}\right) \; .$$

probstat_ps5

October 31, 2024

1 Introduction to Probability and Statistics

Statistics - Problem Sheet 3

1.1 Question 1

Use linear regression to fit a straight line model ($M(x, \theta) = mx + c$) to the following data. The noise on the data is known to be drawn from a normal distirbution and the uncertainty on each data point is given.

x = [5.52, 3.73, 5.24, 4.20]

D = [23.6, 18.6, 21.9, 20.9]

σ_D = [1.0, 1.2, 2.6, 1.4]

Marks will be awarded in the following way:

i) State the likelihood function you are using (log likelihood is fine) **[1 mark]**

ii) Calculate the best-fit estimates, $\hat{m}$ and $\hat{c}$ and show your working **[3 marks]**

iii) Calculate the uncertainty on these estimates, $\sigma_{\hat{m}}$ and $\sigma_{\hat{c}}$ **[1 mark]**

iv) Estimate the covariance of the best fit parameters, $\mathrm{Cov}(\hat{m}, \hat{c})$. **[1 mark]**

v) Make a plot (hand drawn or computer generated is fine) of the data, together with the best-fit model **[2 marks]**

vi) Make a prediction, including uncertainty in $\hat{m}$ and $\hat{c}$, on the value of the model at the exact value $x = 2$. (Hint: use the variance formula including accounting for covariance) **[2 marks]**

[ ]:

A pion has rest mass 139.58MeV and a mean-lifetime of 2.5×10^{-8} s whereas a muon has rest mass 105.65MeV and a mean-lifetime of 2.2×10^{-6} s. A pion usually decays into a muon and a neutrino. You may assume that a neutrino is massless and lives forever.

Assuming that the pion is initially at rest, find the velocity of the muon and the mean-distance that the muons travel in the pion rest-frame.

[10]