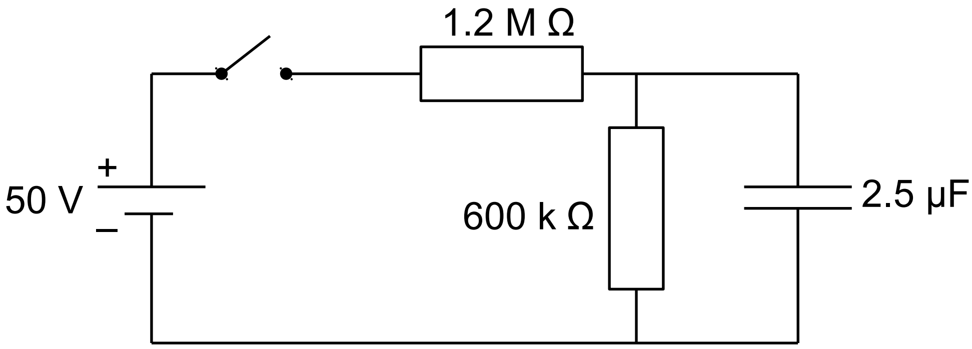
Electric circuits
Non-assessed Problem Sheet 5, Solution

1. In the circuit shown below:
    (a) What is the initial battery current immediately after the switch is closed?
    (b) What is the battery current a long time after the switch is closed?
    (c) What is the maximum voltage across the capacitor?
    (d) If the switch has been closed for a long time and is then opened, deduce an expression for the current through the 600 kΩ resistor as a function of time.
    (e) What is the energy dissipated in the 600 kΩ resistor after the switch is opened?
    (*Note* that 1 kΩ = 10^3 Ω; 1 MΩ = 10^6 Ω; 1 μF = 10^{-6} F.)



Solution

1. (a) Initially, the capacitor acts like a short-circuit, bypassing the 600 kΩ resistor.
    I = V/R = 50 V/ 1.2MΩ = 41.7 μA

    (b) A long time after the switch is closed, the capacitor is fully charged and acts like an open-circuit.
    I = V/R = 50 V/ (1.2 + 0.6)MΩ = 50 V/1.8MΩ = 27.8 μA

    (c) At this stage, the potential difference across the capacitor is
    50 V x 0.6/1.8 = 16.7 V, by the potential divider rule.

    (d) Total response = Final Value + (Initial Value – Final Value) $e^{-t/\tau}$
    The final value of the current will be zero, as the capacitor is discharging. The total response is therefore, $I = I_0 e^{-t/RC}$, where I_0 = V/R = 16.7 V / 600 kΩ = 27.8 μA and τ=RC= 1.5 s.

    (e) The energy dissipated is equal to the energy stored in the capacitor.
    E = 1/2 CV^2 = 0.5 (2.5 x 10^{-6} x $(16.7)^2$)= 3.5 x 10^{-4} Joules.