Electromagnetism

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Lecture 13
Magnetic Fields from Currents
Biot-Savart Law
Week 7

Last Lecture

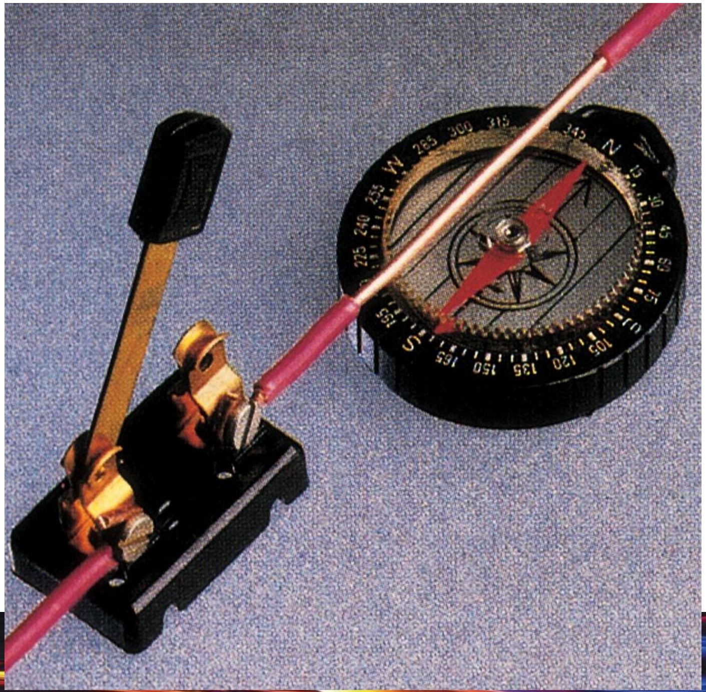
- Special cases of magnetic force
- Force on current carrying conductor
  - $\underline{\boldsymbol{F}} = I\,\underline{\boldsymbol{l}} \wedge \underline{\boldsymbol{B}}$
- Current Loops and Magnetic Dipoles
  - $\underline{\mu} = I\underline{A}$
- Torque on magnetic dipole in B-field
  - $\underline{\boldsymbol{\tau}} = \underline{\boldsymbol{\mu}} \wedge \underline{\boldsymbol{B}}$
- Potential energy of magnetic dipole in B-field
  - $U = -\underline{\boldsymbol{\mu}} \cdot \underline{\boldsymbol{B}}$

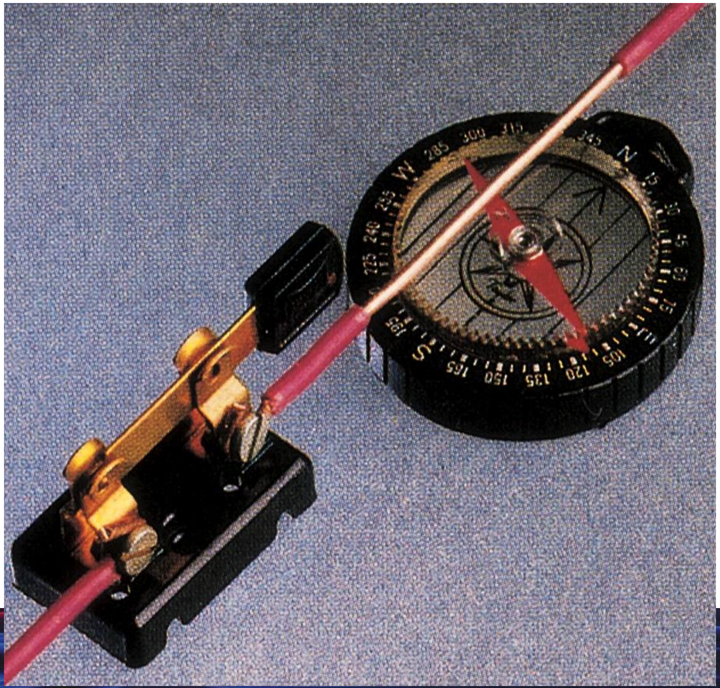
This Lecture

- Magnetic field from moving charge
- Magnetic field from current element
- **Biot-Savart Law**
  - B-Field at centre of current loop (magnetic dipole)
  - B-field from line of current
  - B-field from infinite line of current
  - B-field along axis of current loop (magnetic dipole)
  - B-field and E-field from dipoles

Magnetic Field From Current

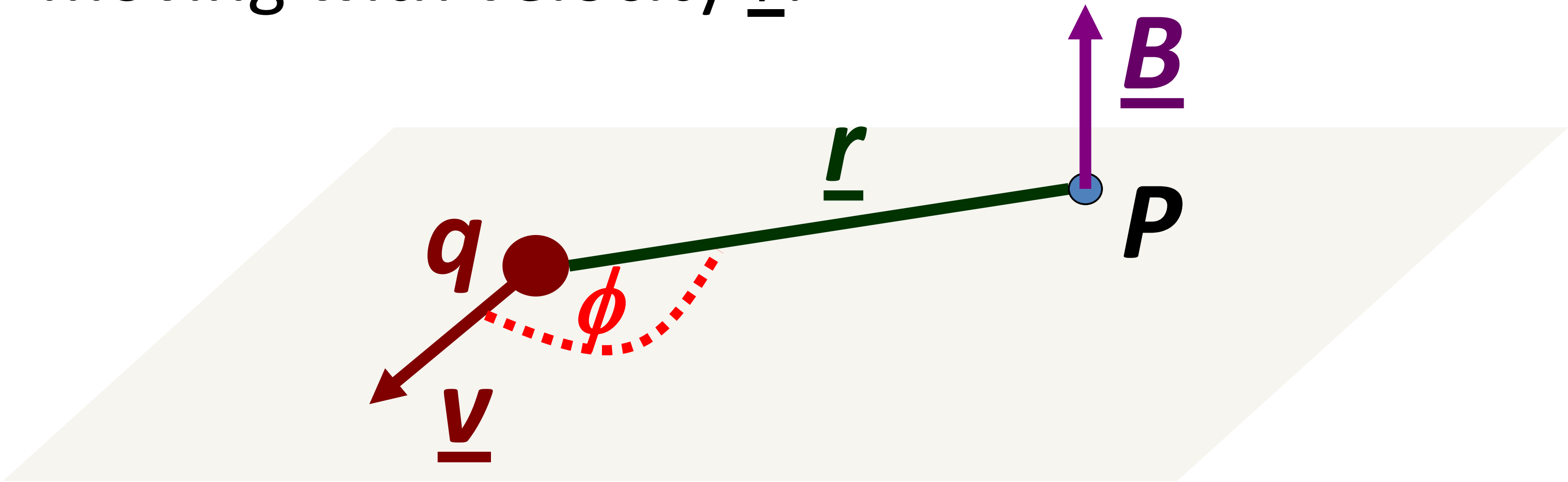
- **Hans Christian Oersted,** Danish Physicist (1777-1851)
- In 1820 Oersted demonstrated that a magnetic field exists near a current-carrying wire - first connection between electric and magnetic phenomena.





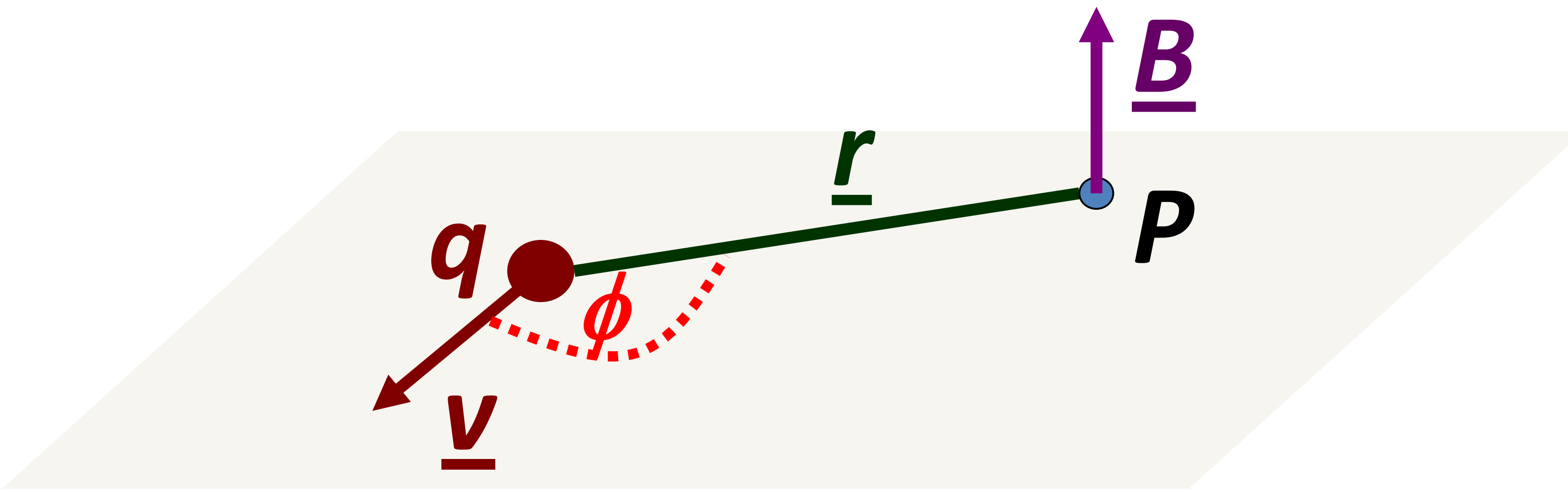
Magnetic Field from Moving Charge

- Consider a B-field at a point $\underline{\boldsymbol{r}}$ from a charge q moving with velocity $\underline{\boldsymbol{v}}$.



- $B \propto \dfrac{qv \sin \phi}{r^2}$ perpendicular to $\underline{\boldsymbol{v}}$ and $\underline{\boldsymbol{r}}$.

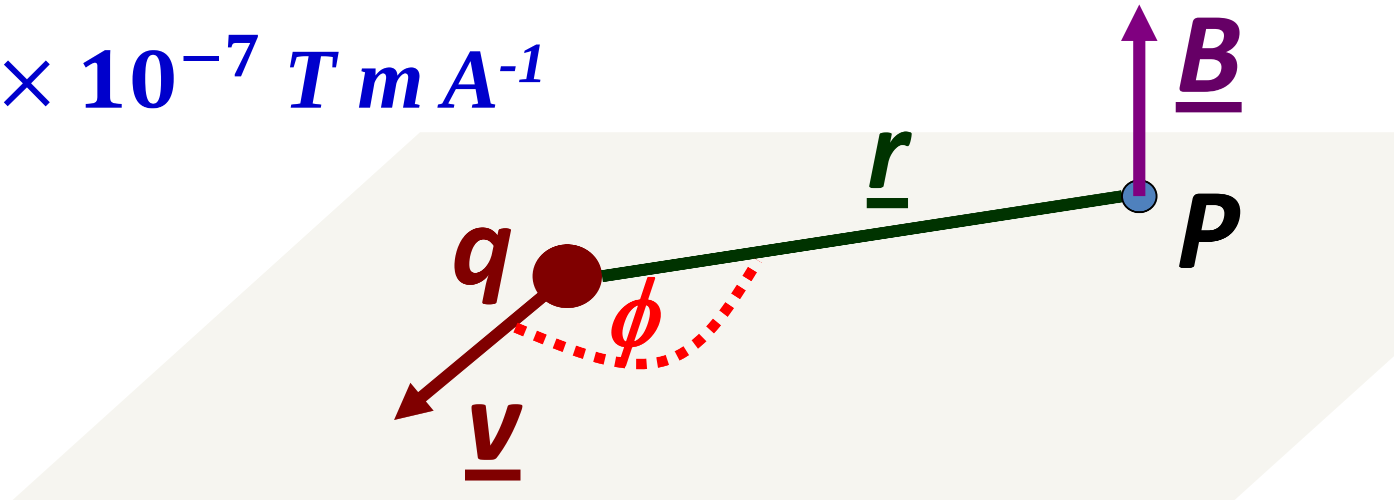
Magnetic Field from Moving Charge



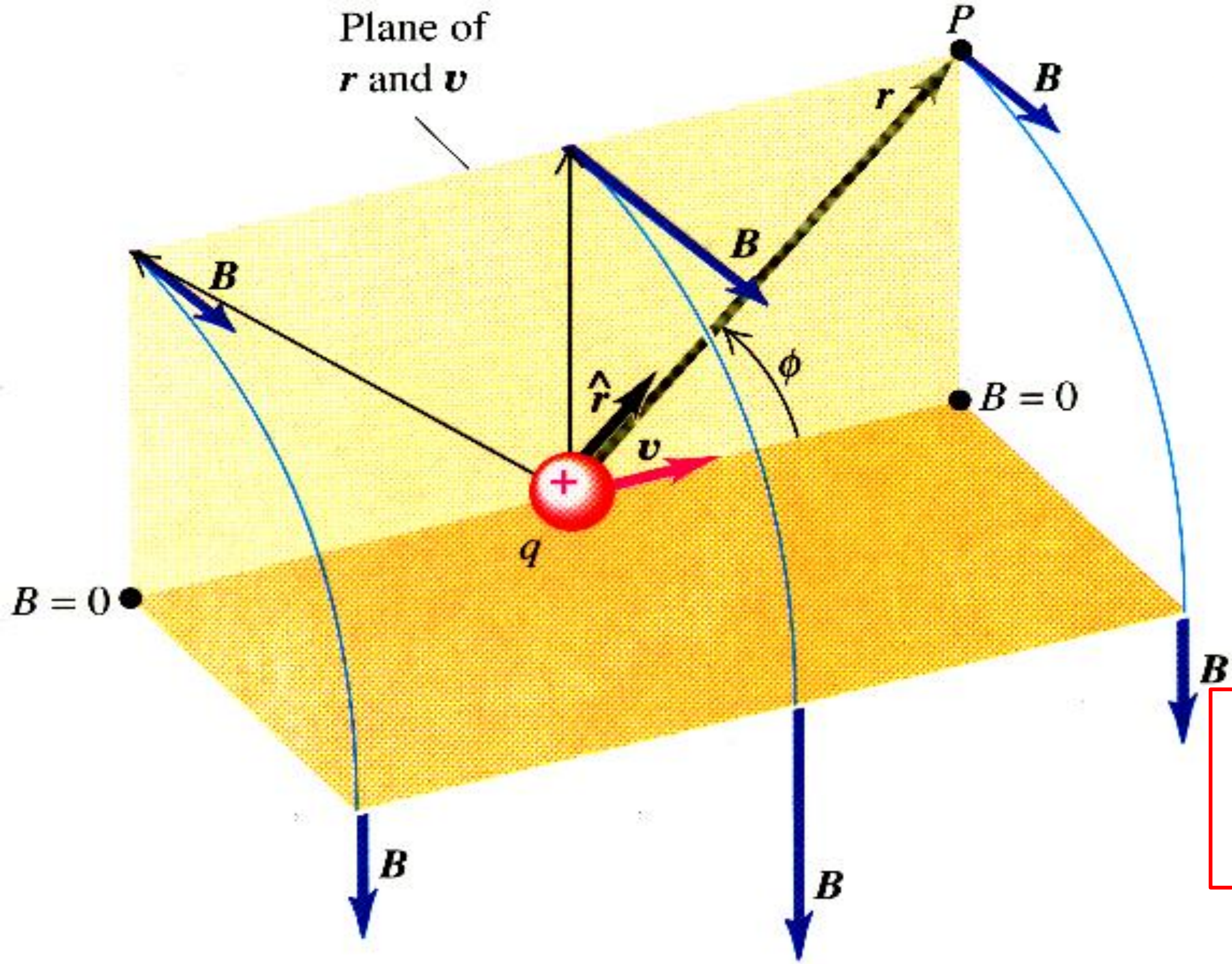
- In vector form: $\underline{\boldsymbol{B}} \propto \frac{q}{r^2} \underline{\boldsymbol{v}} \wedge \underline{\hat{\boldsymbol{r}}}$

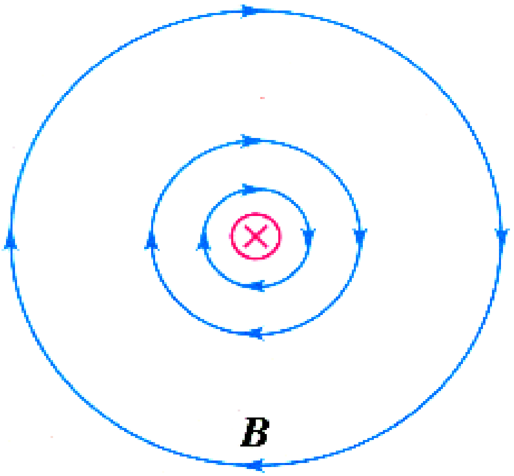
Magnetic Field from Moving Charge

- In SI units
- In vector form: $\underline{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{q}{r^2}\underline{\boldsymbol{v}}\wedge\underline{\hat{\boldsymbol{r}}}$
- μ_0 is called the **Permeability of free space**.
- $\boldsymbol{\mu_0 = 4\pi \times 10^{-7}\ T\ m\ A^{-1}}$



Magnetic Field from Moving Charge

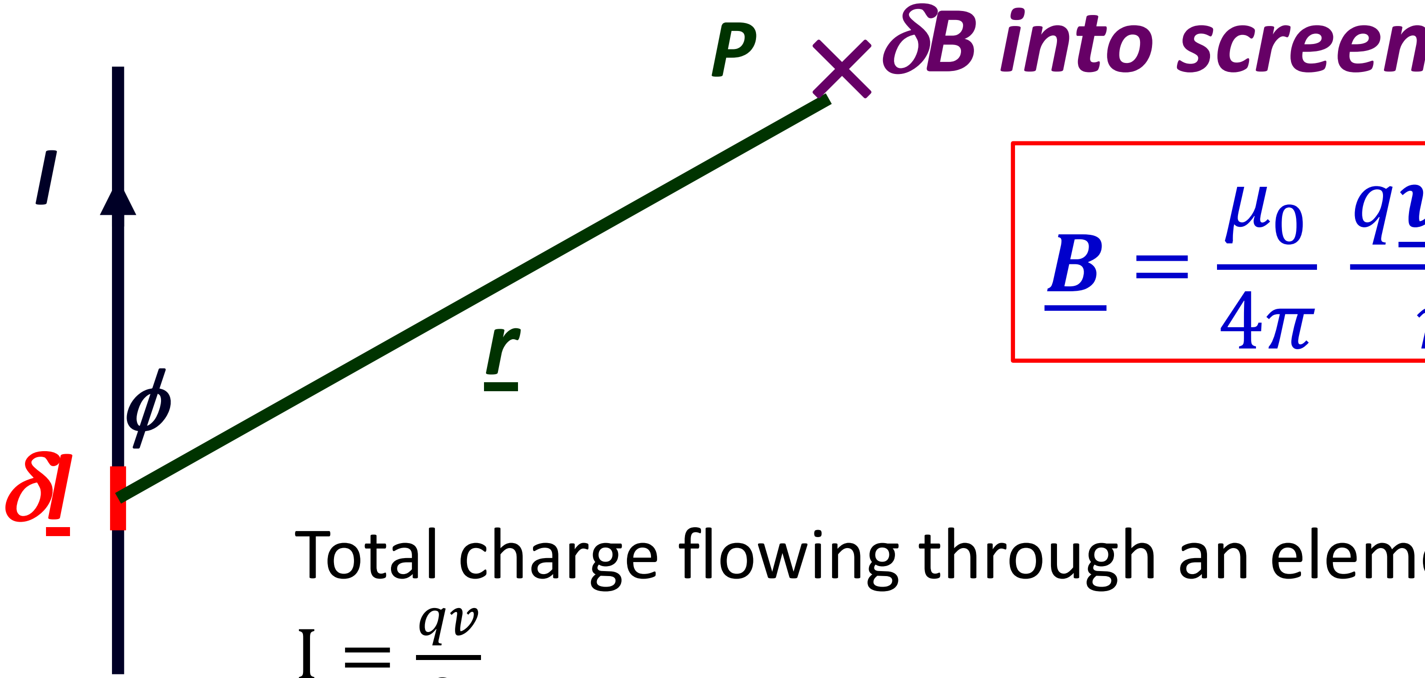




Direction of B same a screw

$$\underline{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\,\frac{q\underline{\boldsymbol{v}}\wedge\underline{\hat{\boldsymbol{r}}}}{r^2}$$

Magnetic Field due to Current Element



$$\underline{\boldsymbol{B}} = \frac{\mu_0}{4\pi} \frac{q\underline{\boldsymbol{v}} \wedge \underline{\hat{\boldsymbol{r}}}}{r^2}$$

Total charge flowing through an element δl is $\mathrm{I} = \frac{qv}{\delta l}$

So, $q\underline{\boldsymbol{v}} = \frac{qv}{\delta l}\delta \underline{\boldsymbol{l}} = I\delta \underline{\boldsymbol{l}}$ where $\delta \underline{\boldsymbol{l}}$ is defined to be in the same direction as the current.

Magnetic Field due to Current Element

- Plugging this in, gives us the Biot Savart Law:

Biot Savart Law

$$\boxed{\delta \underline{\boldsymbol{B}} = \frac{\mu_0}{4\pi} \frac{I \, \delta \underline{\mathbf{l}} \wedge \underline{\hat{\mathbf{r}}}}{r^2}}$$

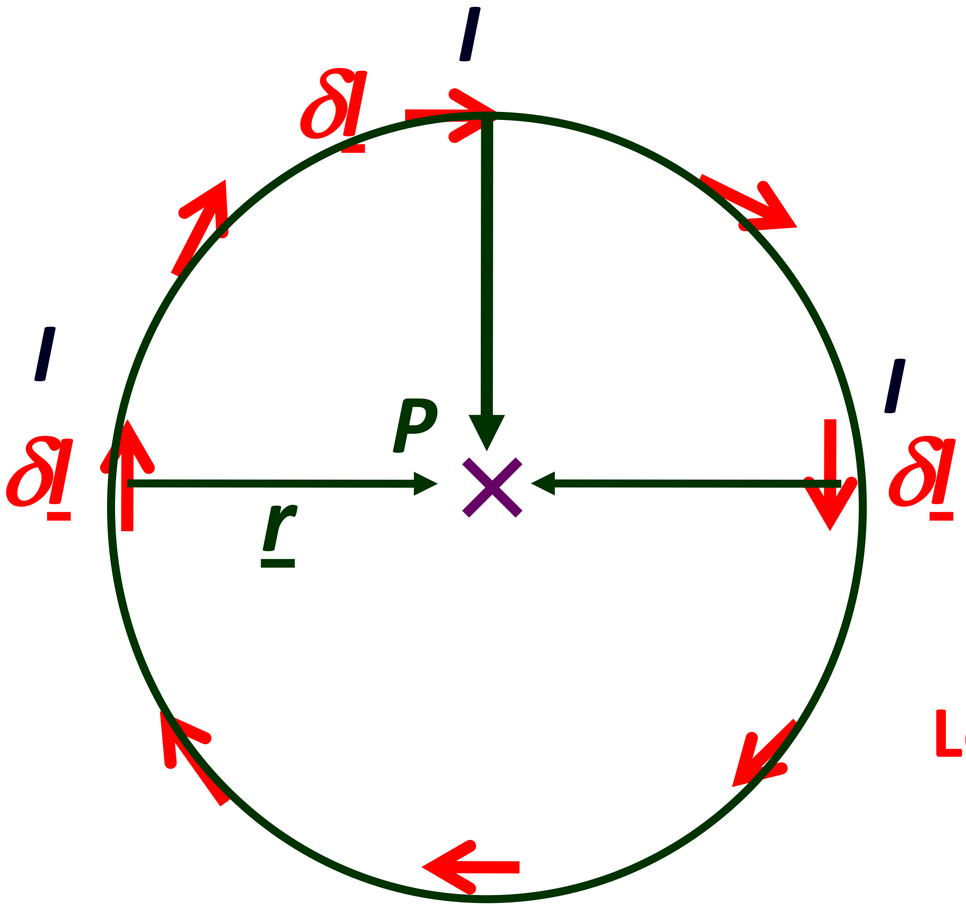
I'm sorry, this is just one of those few equations you need to learn.

The Magnetic Fields from Circuits

Procedure

- Write down $d\underline{B}$ in terms of a *single* variable
- Integrate between the limits applicable to the problem
- Be careful about the directions of the vector quantities
- Use symmetry to simplify the problem

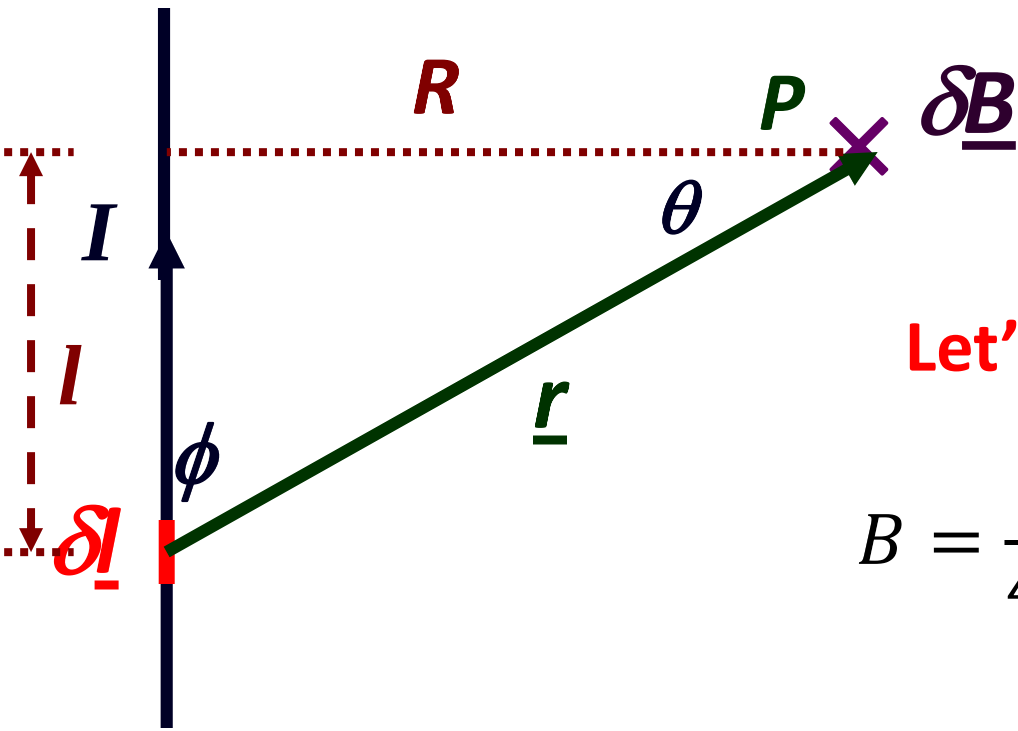
B-field in Centre of Current Loop (magnetic dipole)



$$\delta \underline{\boldsymbol{B}} = \frac{\mu_0}{4\pi} \frac{I\ \delta \underline{\boldsymbol{l}} \wedge \underline{\hat{\boldsymbol{r}}}}{r^2}$$

Let's do it on the visualizer

B-field from a Line of Current



$$\delta \underline{\boldsymbol{B}} = \frac{\mu_0}{4\pi} \frac{I\, \delta \underline{\boldsymbol{l}} \wedge \hat{\underline{\boldsymbol{r}}}}{r^2}$$

Let's do it on the visualizer

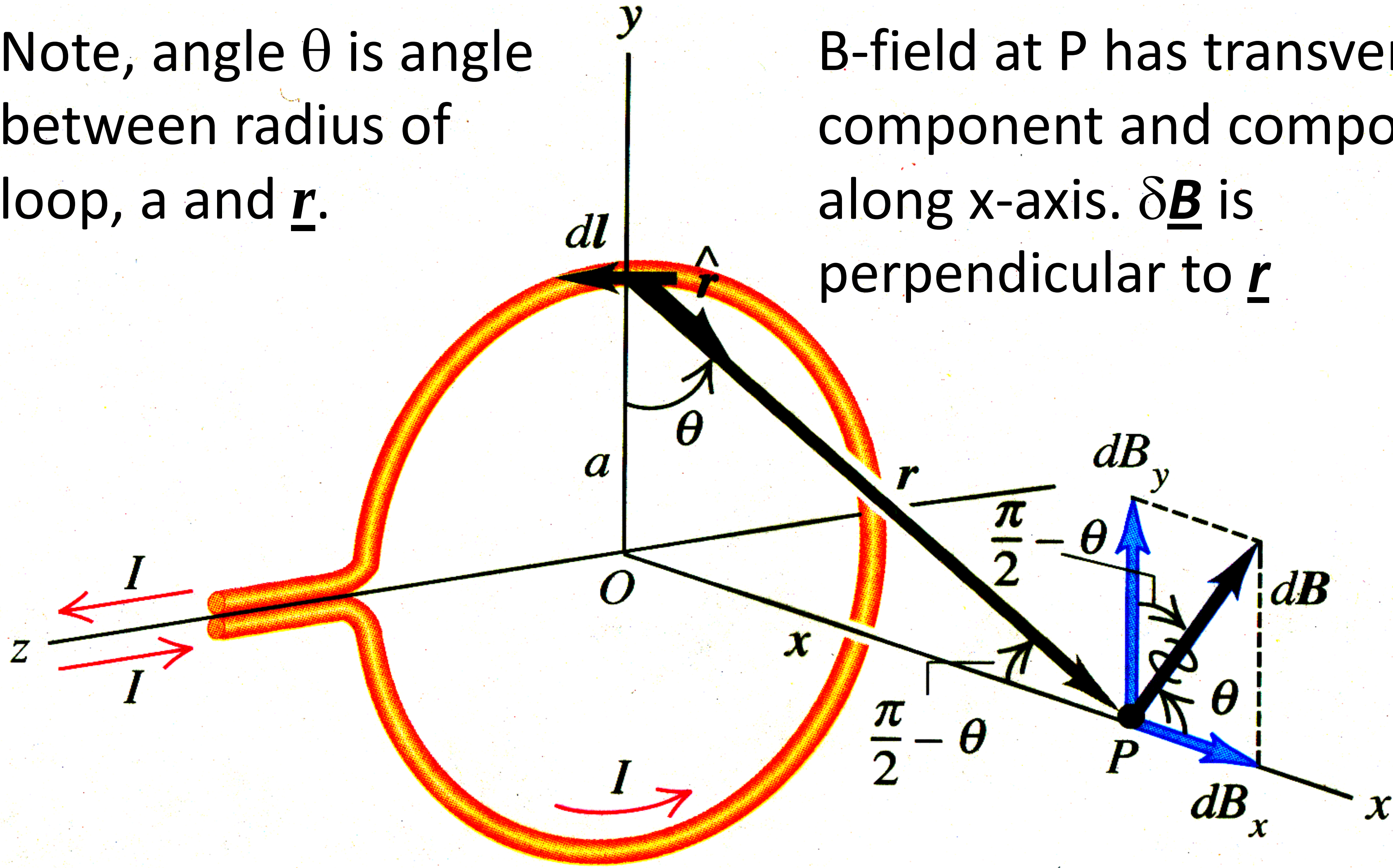
$$B = \frac{\mu_0 I}{4\pi R}(\sin\theta_2 - \sin\theta_1)$$

For infinite line of current $\boldsymbol{B} = \frac{\boldsymbol{\mu_0 I}}{\boldsymbol{2\pi R}}$

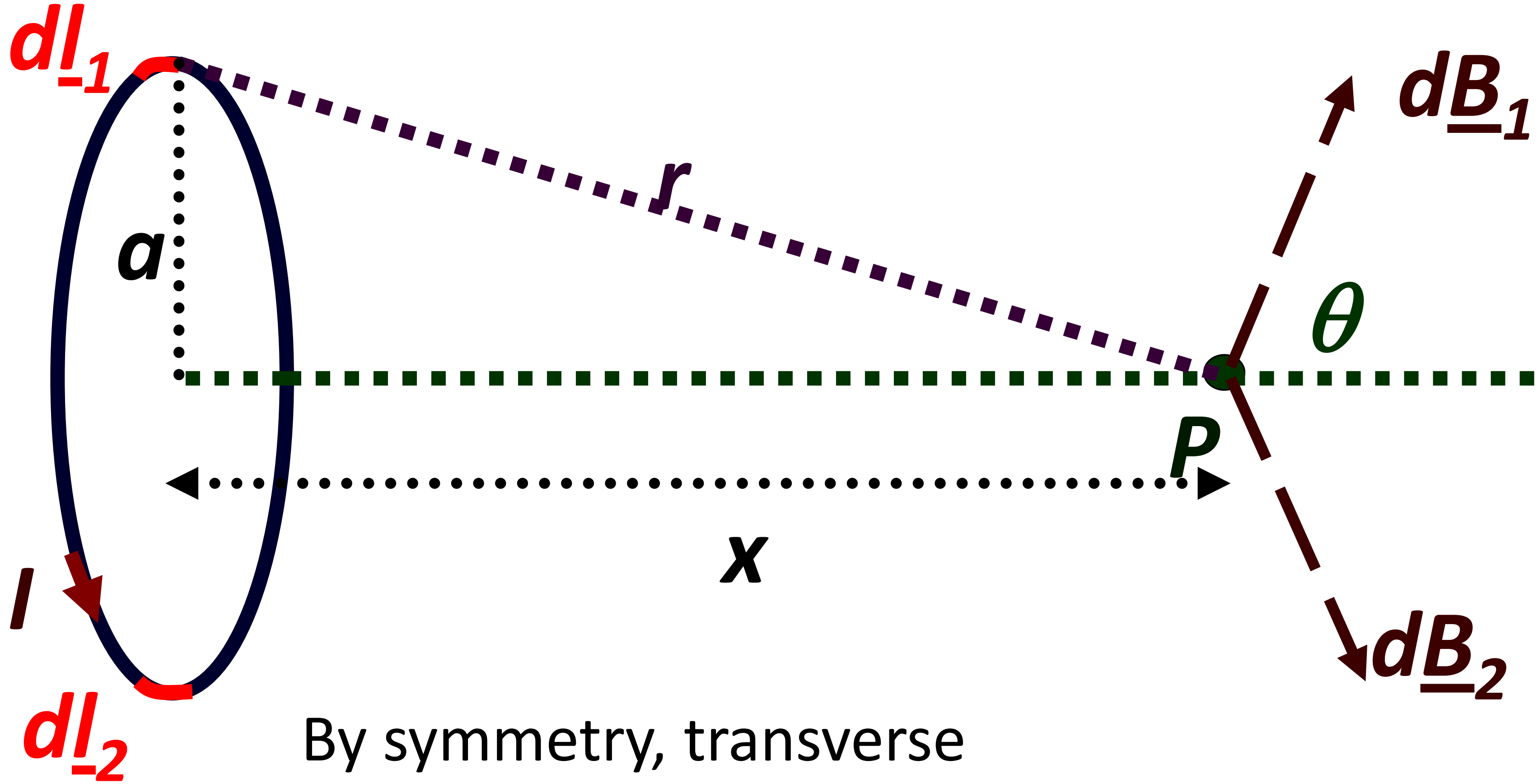
B at any Point on the Axis of a Single Current Loop

Note, angle θ is angle between radius of loop, a and ***r***.

B-field at P has transverse component and component along x-axis. δ***B*** is perpendicular to ***r***

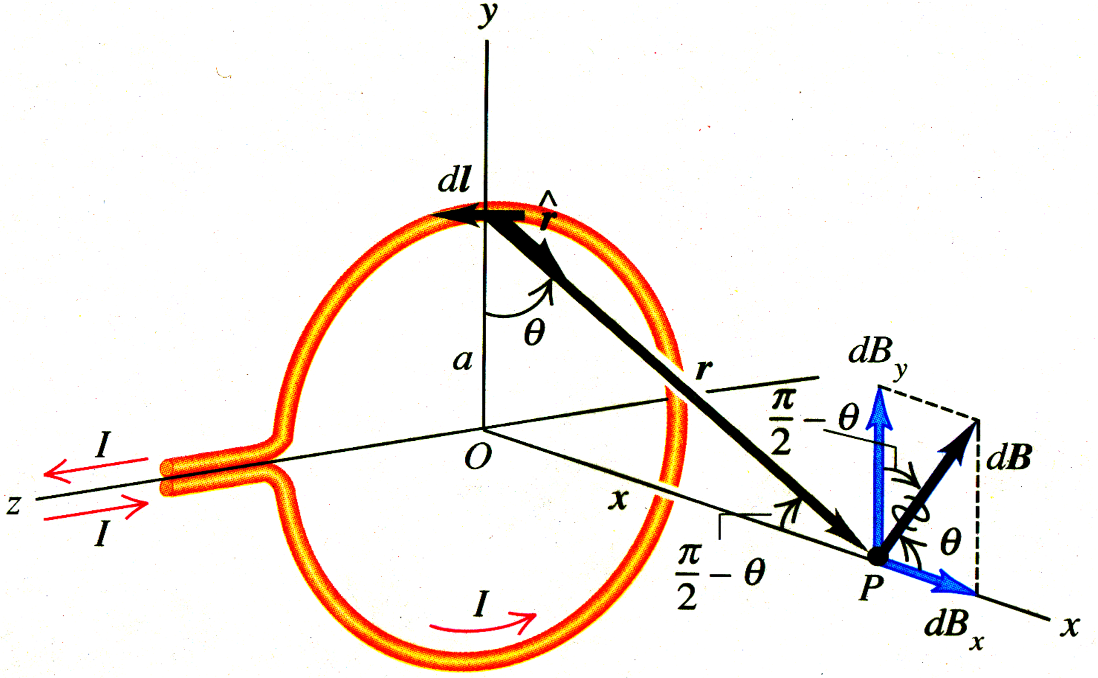


B at any Point on the Axis of a Single Current Loop



By symmetry, transverse components to the axis cancel.

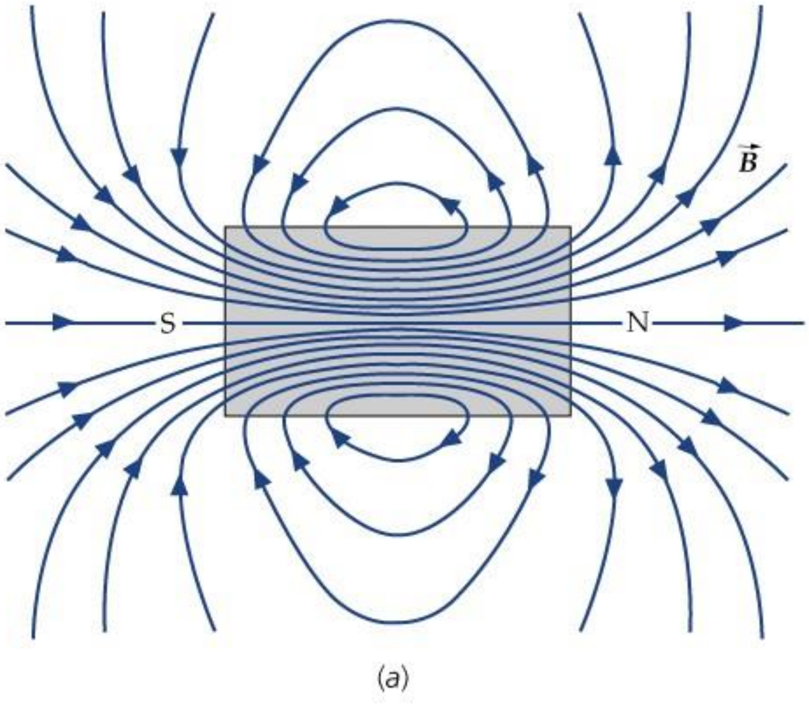
B at any Point on the Axis of a Single Current Loop



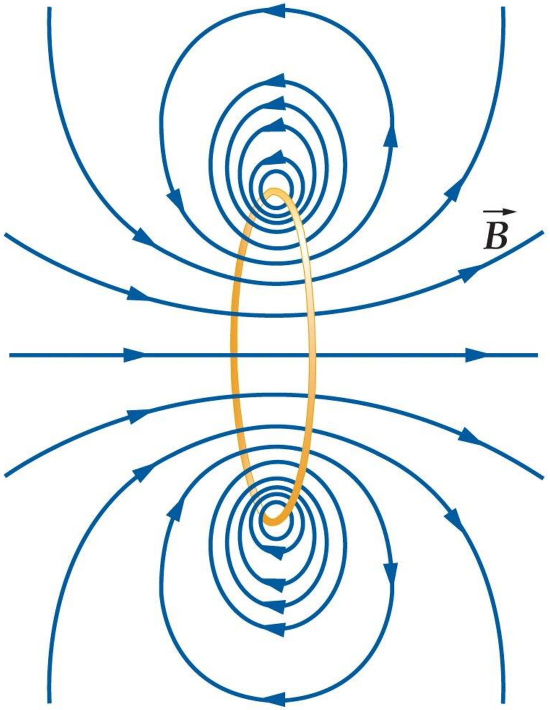
- Do problem on visualizer.

Result: $B_x = \dfrac{\mu_0 I\, a^2}{2(x^2+a^2)^{3/2}}$

B-field from Current Loop



B-field from bar magnet



B-field from current loop (magnetic Dipole)

Exercise Time

- **Obtain an expression for the magnetic field**
- **(i) At the centre of the loop**

Use previous result: $B_x = \frac{\mu_0 I\, a^2}{2(x^2+a^2)^{3/2}}$

Hence $B_0 = \frac{\mu_0 I}{2a}$ (as shown in Ex 13.1)

- **(ii) At *x* >> *a***

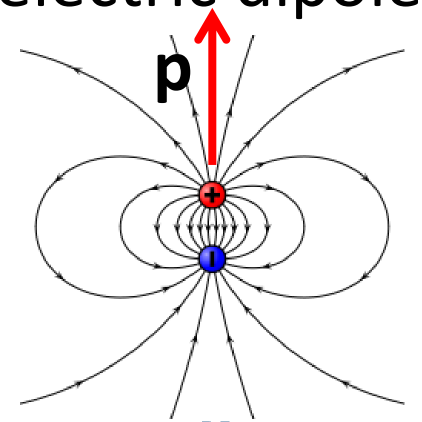
Here we see: $B_x \to \frac{\mu_0 I\, a^2}{2x^3}$ (x >> a)

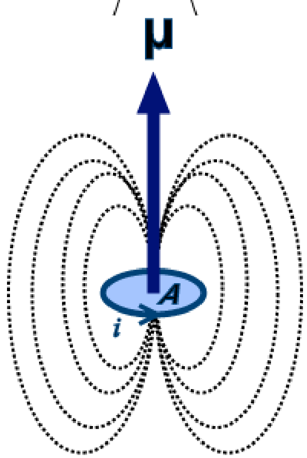
B-field from Magnetic Dipole (x >> a)

- For x >> a (usually the case for atoms)
- $B_x = \frac{\mu_0 I\, a^2}{2x^3}$
- But magnetic dipole moment
- $\mu = I \times area = I\, \pi a^2$ Hence
- $B_x = \frac{\mu_0}{4\pi}\frac{2\mu}{x^3}$ (don't get μ_0 and μ confused)

Magnetic and Electric Dipole Fields

- Compare B-field from magnetic dipole to E-field from electric dipole





Electric Dipole (Lecture 7)

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{2p}{x^3}$$

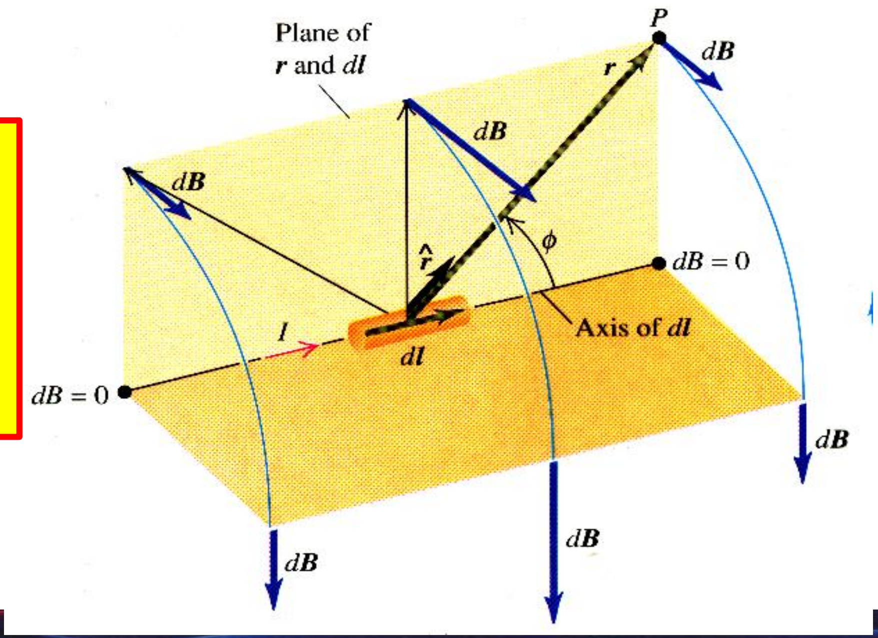
Magnetic Dipole

$$B_x = \frac{\mu_0}{4\pi}\frac{2\mu}{x^3}$$

Review – Biot-Savart Law

- The magnetic field set up by a current-carrying conductor can be found from the Biot-Savart law. This law asserts that the contribution $\delta\underline{\boldsymbol{B}}$ to the field set up by a current element $I\ \delta\underline{\boldsymbol{l}}$ at a point P, a distance $\underline{\boldsymbol{r}}$ from the current element, is:

$$\delta\underline{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{I\ \delta\underline{\boldsymbol{l}}\wedge\underline{\hat{\mathbf{r}}}}{r^2}$$



Next Lecture

- **Ampere's Law**
  - B-fields inside and outside current carrying wires
  - B-fields inside solenoids
  - B-field from Toroidal Solenoid
- Force between two long parallel currents