Electromagnetism

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Lecture 5

Electrical Potential

Week 3

Last Lecture

- **Gauss's Law**
  - **Examples using Gauss's Law**
  - **E-fields in conductors**
  - **E-field between charged conducting Plates**
  - **E-field between charged non-conducting Plates**

The Electric Potential, V

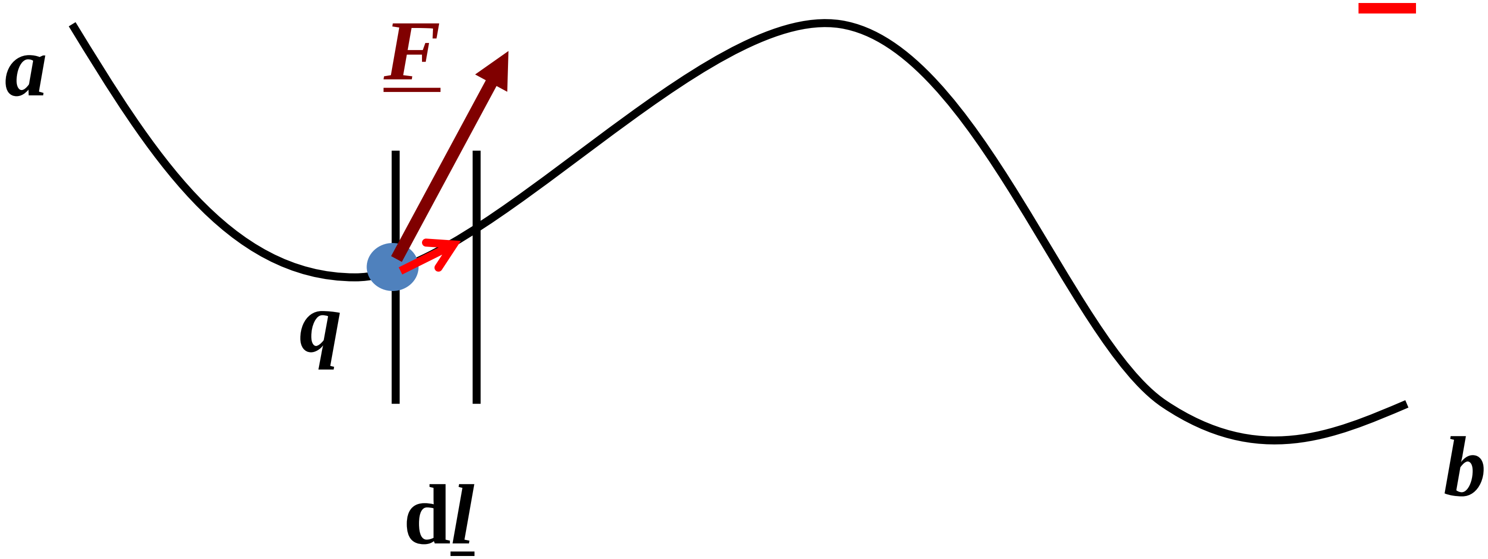
- This Lecture
  - Define electrical potential
  - Electrical potential energy
  - Relationship between potential and electric field
- Next lecture (Lecture 6)
  - We do examples calculating the electrical potential

Charged Particle in an *E-field*

From Classical Mechanics:

work done = force × distance moved in direction of force

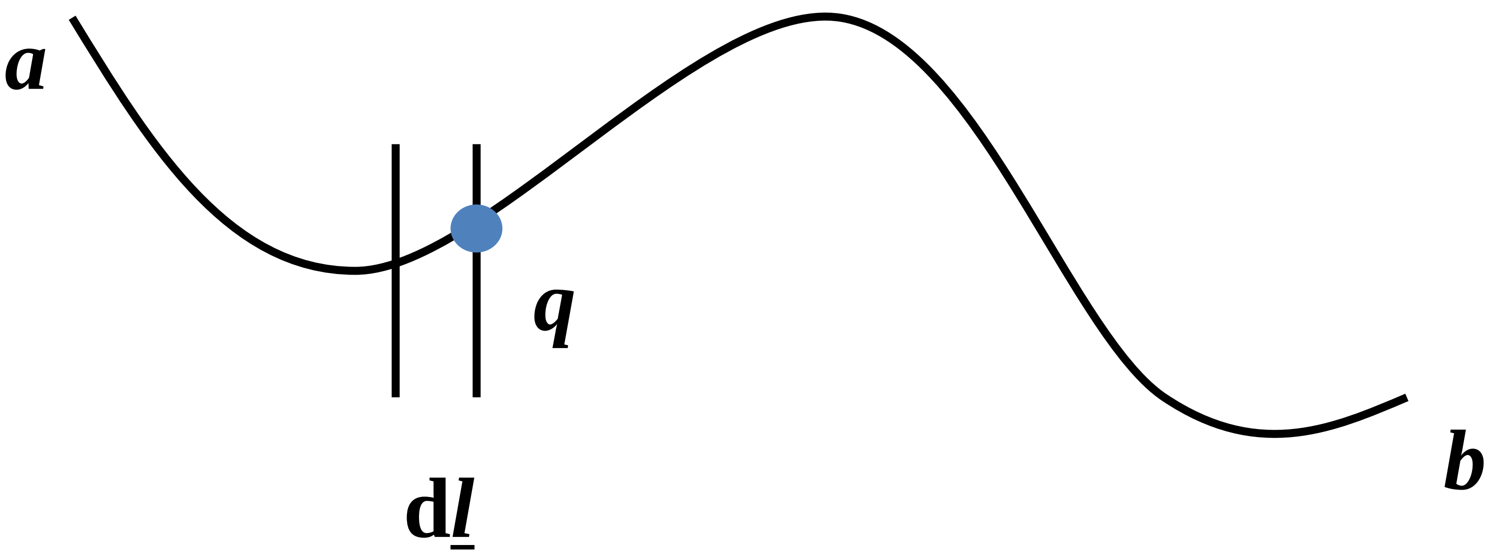
$$dW = \underline{F} \cdot d\underline{l}$$



Charged Particle in an $\underline{E}$-field

work done = force × distance moved in direction of force

$$\delta W = \underline{\boldsymbol{F}} \cdot d\underline{\boldsymbol{l}} = q\underline{\boldsymbol{E}} \cdot d\underline{\boldsymbol{l}}$$



Charged Particle in an <u>*E*</u>-*field*

- <u>*dl*</u> is an infinitesimal displacement along the particle's path.
- It is a vector. It can be at any angle with <u>E</u>.

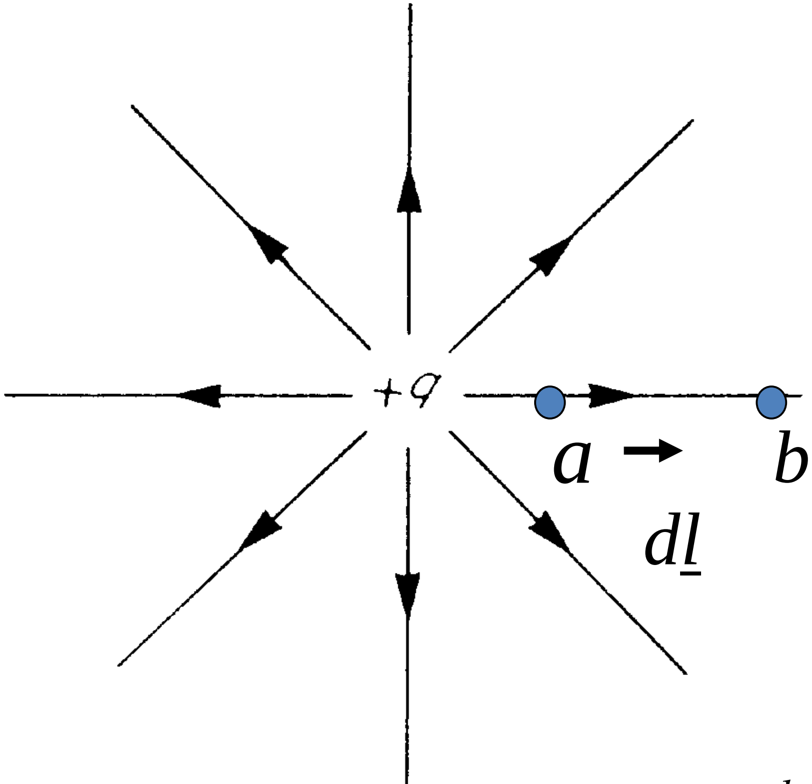
$$\delta W = q\underline{\boldsymbol{E}} \cdot d\underline{\boldsymbol{l}}$$

- This work done by an <u>*E*</u>-field represents a decrease in the electric potential energy $(\boldsymbol{\delta U = -\delta W})$

$$\delta U = -q\underline{\boldsymbol{E}} \cdot d\underline{\boldsymbol{l}}$$

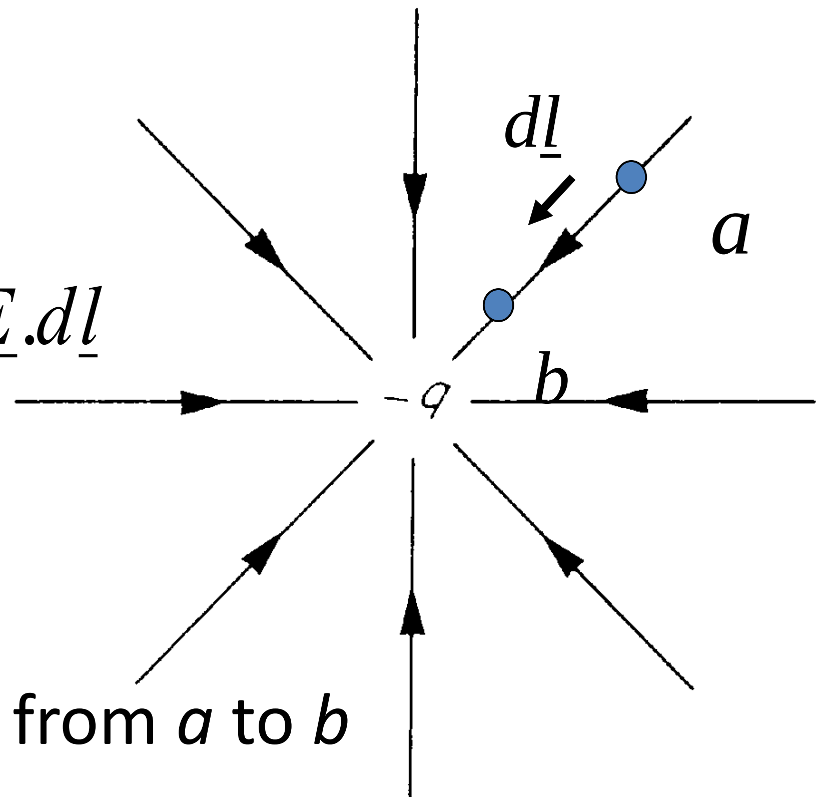
Electrical Potential Energy

- For +q, moving along E for dl distance, δU is negative. **Energy decrease**.
- For –q, moving along E for dl distance, δU is positive. **Energy increase**.
- If the $\underline{E}$-field varies as particle moves from a point *a* to *b* we need to integrate to find the total change in potential energy
- $U_b - U_a = -q \int_a^b \underline{E} \cdot d\underline{l}$



$$U_b - U_a = -q' \int_a^b \underline{E}.d\underline{l}$$

$$U_b < U_a$$



$$U_b - U_a = -q' \int_a^b \underline{E}.d\underline{l}$$

$$U_b < U_a$$

q’ is positive charge moving from *a* to *b*

Electrical Potential Energy

- Electric potential energy for a charged particle in an electric field depends on:

1) The property of the electric field.
2) The charge, both magnitude and sign.

- We want to describe the potential energy on a "per unit charge" basis.
  - Similar to: Electric field "E" describes the force per unit charge.

Definition of Electric Potential, *V*

- V is Potential Energy of the system per unit charge

$$V = \frac{U}{q} \qquad U_b - U_a = -q\int_a^b \underline{E}\cdot d\underline{l}$$

$$V_b - V_a = -\int_a^b \underline{E}\cdot d\underline{l}$$

It is a property of a point in an $\underline{E}$-field. It is a *scalar*. The unit of potential, *Joule Coulomb*$^{-1}$, is called a *Volt* (V).

Alessandro Volta (1745-1827)

The first electric battery

Potential Difference

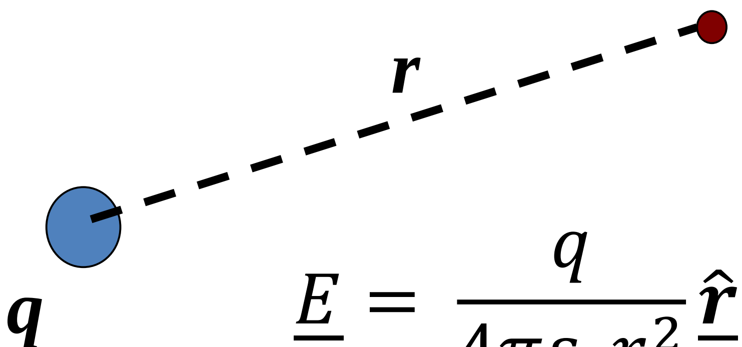
- How to calculate potential difference?

$$\boxed{V_b - V_a = -\int_a^b \underline{E} \cdot d\underline{l}}$$

- Use the above equation to calculate the potential difference, if you already know E. Choice of an integration path.

Electric Potential at a distance r from a point charge q

- Electric Potential at a distance r from a point charge q



$$\mathbf{V(\infty) - V(r)}$$

$$\underline{E} = \frac{q}{4\pi\varepsilon_0 r^2}\underline{\hat{\mathbf{r}}}$$

$$V_\infty - V_r = -\int_\infty^r \underline{E} \cdot d\underline{r} = \frac{q}{4\pi\varepsilon_0 r}$$

Coulomb Potential

- $V(r) = \frac{q}{4\pi\varepsilon_0 r}$
- Electrons in all atoms experience the Coulomb potential.
- q can be positive or negative, so is the potential.

Potential Energy

The *U* of a charge q_0 at a distance *r* from *q* is:

$$U(r) = V(r)q_0 = \frac{1}{4pe_0}\frac{qq_0}{r}$$

$$F = -\frac{dU(r)}{dr} = \frac{1}{4pe_0}\frac{qq_0}{r^2}$$

Electric Potential

The Electric potential at distance r away from a point charge is

$$V(r) = \frac{q}{4\pi\varepsilon_0 r}$$

For a negative charge:

$$V(r) = \frac{-q}{4\pi\varepsilon_0 r}$$

What if we have more than one charge?

Potential due to a Collection of Point Charges

$$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$$

r_i is the distance from the i^{th} charge, q_i, to the point at which V is being evaluated

Electrical Potential Different

The *electric potential difference* between two points is:

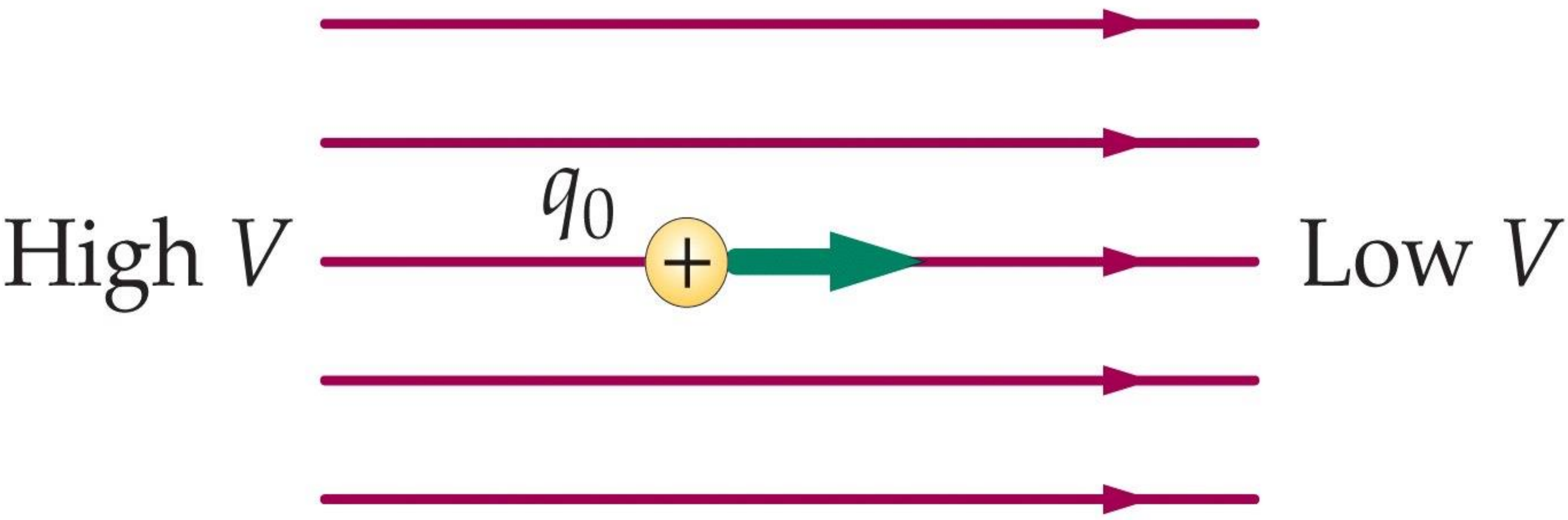
$$V_b - V_a = \frac{U_b - U_a}{q} = -\int_a^b \underline{E}.d\underline{l}$$

Most commonly used for electric fields

[Volt] = [N/C] [m] ➔[N/C]= **[V/m]**

E-field lines and Equipotential Surfaces

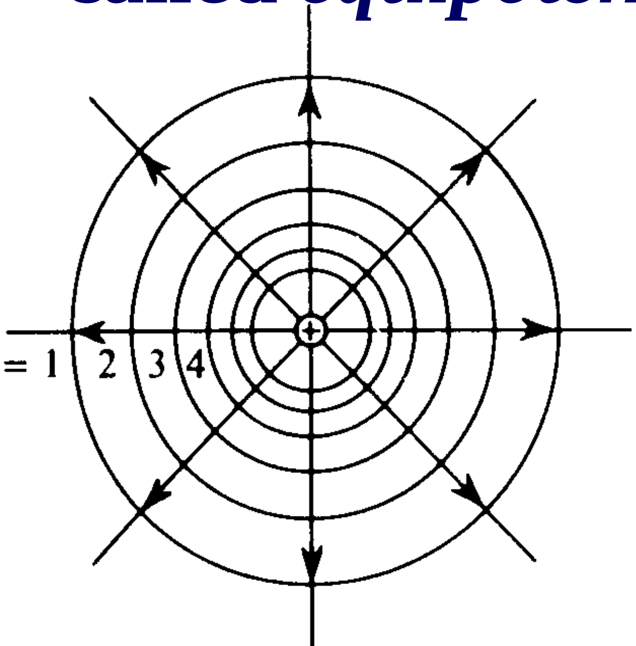
- An E-field line traces the path that a +ve test charge would follow under the action of electrostatic forces. If released the positive charge will accelerate in the direction of the electric field, from high V to low V.



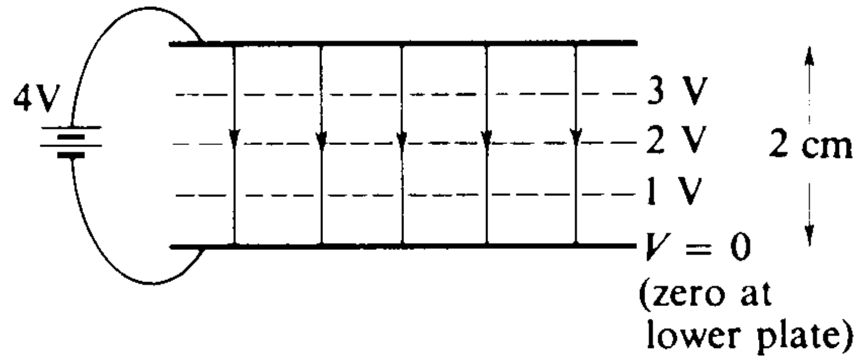
E-field lines and Equipotential Surfaces

- **Surfaces over which *V* is constant are called *equipotentials***

$$V_b - V_a = -\int_a^b \underline{E}.d\underline{l}$$



(*b*)



Note that lines of force are always perpendicular to the equipotentials

Calculation of E from the Electric Potential:

$$V = -\int \underline{E} \cdot d\underline{l}$$

- Electric field can be calculated from V, and there are cases where it is easier finding V first, because V is a scalar quantity

Finding $\underline{E}$ from V

In general

$$\underline{E} = -\left(\frac{\partial V}{\partial x}\underline{i} + \frac{\partial V}{\partial y}\underline{j} + \frac{\partial V}{\partial z}\underline{k} \right) = -\underline{\nabla} V$$

The negative of the *gradient* of the electric potential

In Plane Polar coordinates

$$\boxed{\underline{E} = -\left(\frac{\partial V}{\partial r}\,\underline{\hat{r}} + \frac{1}{r}\frac{\partial V}{\partial \theta}\,\underline{\hat{\theta}}\right)}$$

The Coulomb Potential

$$V(r) = \frac{q}{4\pi\varepsilon_0 r}$$

$$\underline{E} = -\left(\frac{\partial V}{\partial r}\underline{\hat{r}} + \frac{1}{r}\frac{\partial V}{\partial \theta}\underline{\hat{\theta}} \right)$$

V is a function of *r* only.

$$E(r) = -\frac{dV}{dr}\hat{r} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{r}$$

E-field is a Conservative Field

$\underline{E}$-field is a *conservative field* (electrostatics)

If the charge returns to its original position, *by any route*, *NO WORK IS DONE*

$$\oint \underline{E}.d\underline{l} = 0$$

i.e. the change in potential between two points is the same whichever path is taken

Summary

$$V = \frac{1}{4\pi\varepsilon_0}\sum_i \frac{q_i}{r_i}$$

$$\boxed{\underline{E} = -\underline{\nabla} V}$$

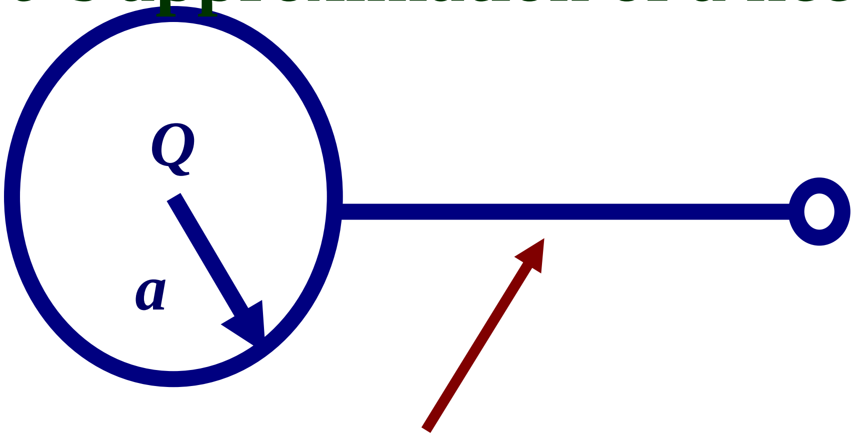
Application:

Field ion microscope - used to image atoms

Works by having a high electric field around a point of a needle.

How is this high electric field achieved?

Physicist's approximation of a needle is:



Radius b

Charge q

Long conducting wire

Electric potential of larger sphere:

$$V_a = \frac{Q}{4pe_0 a}$$

Electric potential of smaller sphere:

$$V_b = \frac{q}{4pe_0 b}$$

But potentials are *equal* as connected:

$$V_a = V_b$$

$$\backslash \quad \frac{Q}{q} = \frac{a}{b}$$

Compare electric fields at the surface of each sphere

$$E_a \big/ E_b = \frac{Q}{q}\frac{b^2}{a^2} = \frac{b}{a}$$

Smaller radius of curvature, the higher the E-field

High Voltage Power Lines

Losses are higher than normal in damp weather.
Why?

Charged water droplets on wire become elongated to a point because of repulsion. The resulting high *E*-field leads to ionisation and heating of the air (energy loss)

Results in TV and radio interference