VGLA: CONICS PRACTISE QUESTIONS

The following questions relate to Chapter 6 - Conics, and Appendix C - Conics: Optional Extra Content. Questions are ranked in difficulty from A (basic) to C (challenging).

(A) Question 1. Show that each of the following equations represents a single straight line.

(a) $x^2 + 4xy - 6x + 4y^2 - 12y + 9 = 0$.
(b) $x^2 - 4xy + 2x + 4y^2 - 4y + 1 = 0$.

(A) Question 2. Show that each of the following equations represents a pair of straight lines and find their point of intersection.

(a) $2x^2 - xy + x - y^2 + 2y - 1 = 0$.
(b) $2x^2 - xy + 5x - y^2 + y + 2 = 0$.

(A) Question 3. By completing the square (translation), eliminate the linear terms in the following equations, and hence, identify the conics.

(a) $3x^2 + 24x + 53 - 2y^2 + 4y = 0$;
(b) $2x^2 - 4x + 10 + 4y^2 + 16y = 0$;
(c) $y^2 - 2y - 11 - 6x = 0$.

(A) Question 4. Find equations, in x, y co-ordinates, for the given ellipses.

(a) Foci at $(0, \pm 2)$ semi-major axis (or equivalently, the distance from centre to vertex) 3;
(b) Foci at $(0, 1)$ and $(4, 1)$ and eccentricity $\frac{1}{2}$;

(A) Question 5. Sketch the following ellipses and find their centre and semi-axes.

(a) $9x^2 + 4y^2 - 18x + 8y = 23$;
(b) $2x^2 - 4x + 10 + 4y^2 + 16y = 0$.

(A) Question 6. Find the vertex, focus and directrix of the parabola with equation

(a) $y = -(1/6)x^2$;
(b) $2y^2 = -3x$; and
(c) $y^2 = 100x$.

(A) Question 7. Find the equation of the parabola with vertex $V(x, y) = (-4, 2)$ and directrix $y = 5$ and sketch it.

(A) Question 8. Find the equation of the tangent to the parabola with equation $y^2 = x$ at the point $P(x, y) = (16, -4)$.

(A) Question 9. Find the vertices and the foci of the hyperbola with given equation. Moreover, sketch the hyperbola, asymptotes and foci.

(a) $y^2/9 - x^2/4 = 1$;
(b) $x^2 - 2y^2 = 8$.

(A) Question 10. Find the equation of the tangent to the hyperbola with equation

$$\frac{x^2}{6} - \frac{y^2}{8} = 1,$$

at the point $P(x, y) = (3, 2)$.

(B) Question 11. By rotating the coordinate system in the following equations, eliminate the cross term xy. Hence identify the type of conics from the equations.

(a) $2x^2 + 2xy + 2y^2 - 5 = 0$;
(b) $-2x^2 + 2\sqrt{3}xy - 4 = 0$;
(c) $\frac{3}{4}x^2 + xy\frac{\sqrt{3}}{2} + \frac{y^2}{4} + 6x - 6y\sqrt{3} = 0$.

(B) Question 12. By a combination of rotation and translation, simplify the following equations so they are given by a standard equation for a conic. *You may find it helpful to use the double angle formula for* $\tan(2\theta)$.

(a) $34x^2 + 24xy - 40x + 41y^2 + 30y = 0$;
(b) $39x^2 - 96xy - 270x + 11y^2 + 140y = 0$.

(B) Question 13. Find the equations of the tangent and the normal lines to the given curves at the given value of x. *Note this means there may be two possible values of y and hence two tangent lines and two normal lines. Use implicit differentiation to solve this question.*

(a) $2x^2 + 2xy + 2y^2 - 5 = 0, \quad x = 0$;
(b) $34x^2 + 24xy - 40x + 41y^2 + 30y = 0, \quad x = 0$;
(c) $8x^2 + 12xy + 17y^2 - 20, \quad x = 1$.

(B) Question 14. If an ellipse is defined parametrically by

$$\begin{aligned} y &= 2\sqrt{2}\sin\theta \\ x &= 4\cos\theta - 1 \end{aligned}$$

show that the normal to the ellipse at $\theta = \frac{\pi}{3}$ intersects $(x, y) = (0, 0)$.

(B) Question 15. Find the equation of the parabola that satisfies the given conditions.

(a) Focus $F(3, 0)$ and directrix $x = -3$;
(b) Vertex $V(-2, 3)$ and directrix $y = 6$.

(B) Question 16. Find the vertex and focus of the given parabola and sketch it.

(a) $y = x^2 - 4x + 2$;
(b) $4x^2 + 40x + y + 106 = 0$.

(B) Question 17. Find the equation of the tangents to $y = (x - 1)^2 + 8$ that intersect the (0,0).

(B) Question 18. Find an equation for the hyperbola with centre at (0,0) and:

(a) has foci $F(\pm 5, 0)$ and vertices $V(\pm 3, 0)$;
(b) has vertices $V(\pm 4, 0)$ and intersects $P(8, 2)$.

(B) Question 19. *You should be able to make a suitable rotation and translation to the conic in standard form to derive your answer, after sketching the conic.*

(a) For the parabola with directrix $y = \frac{1}{\sqrt{3}}x$ and focus at $F\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$, sketch the vertex, axis of the parabola, focus and directrix on the xy-plane. Additionally, determine a quadratic equation in x and y that describes the parabola.
(b) For the ellipse foci $F_1(-\sqrt{2}, 0)$ and $F_2(0, -\sqrt{2})$ and eccentricity $e = \frac{1}{2}$, sketch the vertices, major axis, minor axis, foci and directrices on the xy-plane. Additionally, determine a quadratic equation in x and y that describes the ellipse.