

# Recap from last lecture

Latent heat of vaporisation,  $L$ , (J/kg): the amount of heat required to completely vaporise a liquid into a gas (at a constant temperature).

$$L = 10 U_0 / 2 \times \frac{\text{atoms}}{\text{kg}}$$

Q: How much energy do we need to remove 1 mole of (connected) atoms from the centre of the structure?

$$\text{Amount of substance (moles)} = \frac{\text{Number of particles}}{N_A}$$

$$E = \frac{n N_A \varepsilon}{2}$$

$$N_A = \frac{\text{atoms}}{\text{mol}}$$

# Recap from last lecture

Atoms/molecules at the surface of a liquid have fewer nearest neighbours and hence a smaller force acting upon them than those deep within the liquid

We thus need to do work to move atoms from inside of the liquid to the surface (increasing the surface area of the liquid)

$$\text{Surface tension, } \gamma = \left( \frac{n\varepsilon}{2} - \frac{n\varepsilon}{4} \right) \times N = \frac{n\varepsilon N}{4}$$

Work done per particle (deep in liquid – surface)      Number of particles per m<sup>2</sup>

# Recap from last lecture

Surface tension has units of energy over area, or force over length

The force due to surface tension acts parallel to the surface of the liquid, but perpendicular to the division we are trying to create by exerting a separate force on the liquid



# Recap from last lecture

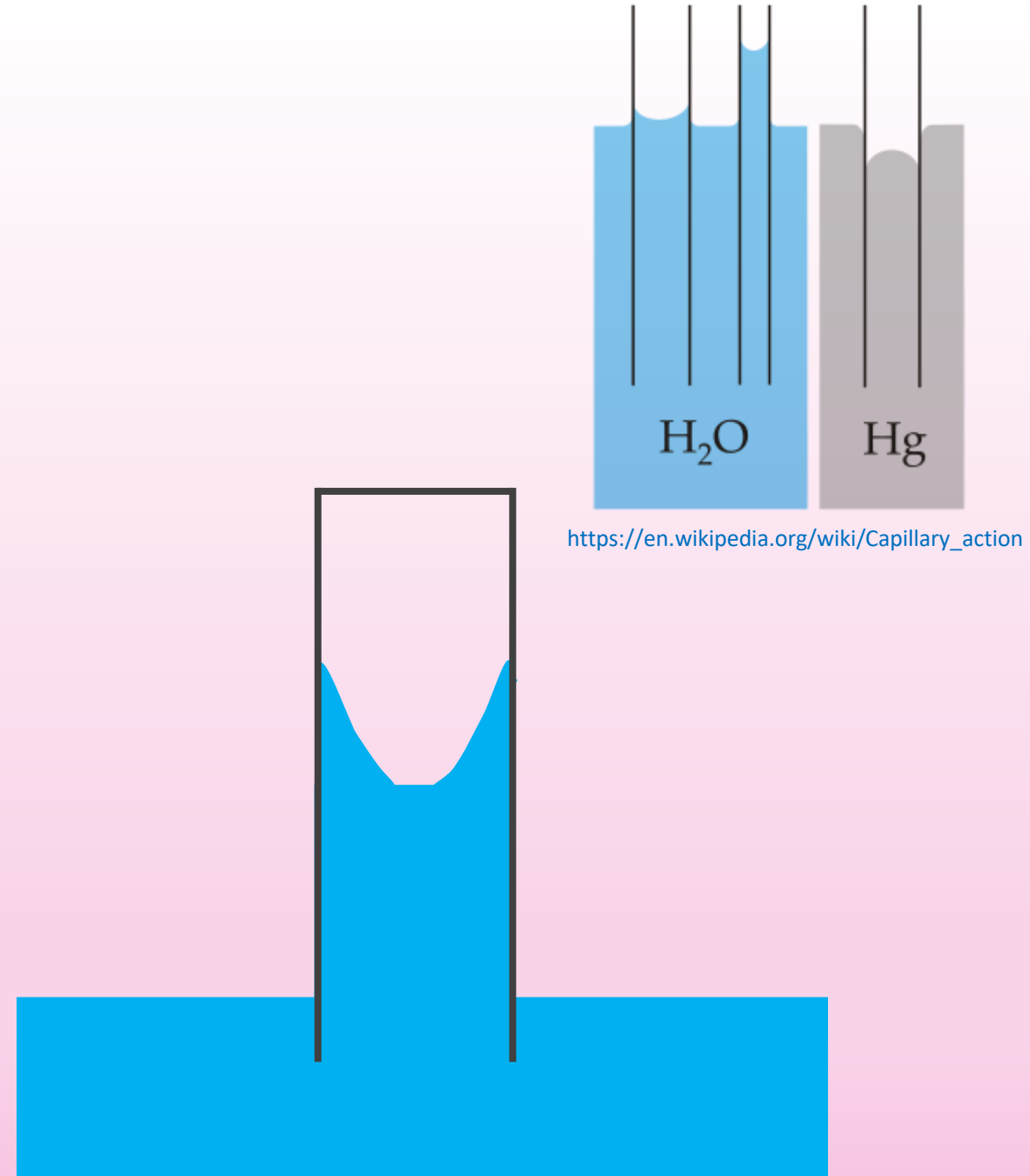
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# Capillary action

Open tube placed in contact with the surface of a liquid causes the liquid to rise up the tube



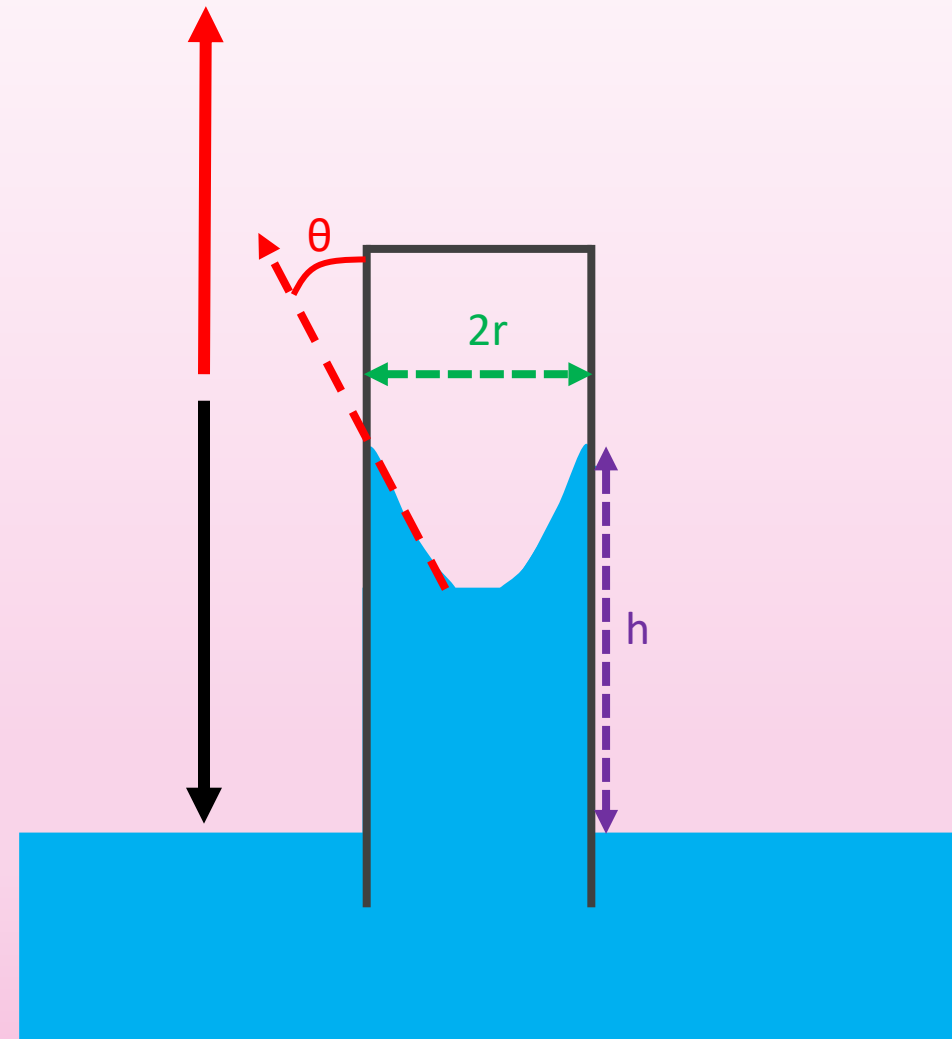
# Capillary action

Open tube placed in contact with the surface of a liquid causes the liquid to rise up the tube

Competing forces/potentials:

- 1) Gravity of the liquid
- 2) Surface tension – total surface energy is reduced as adhesion contributes to liquid in the tube

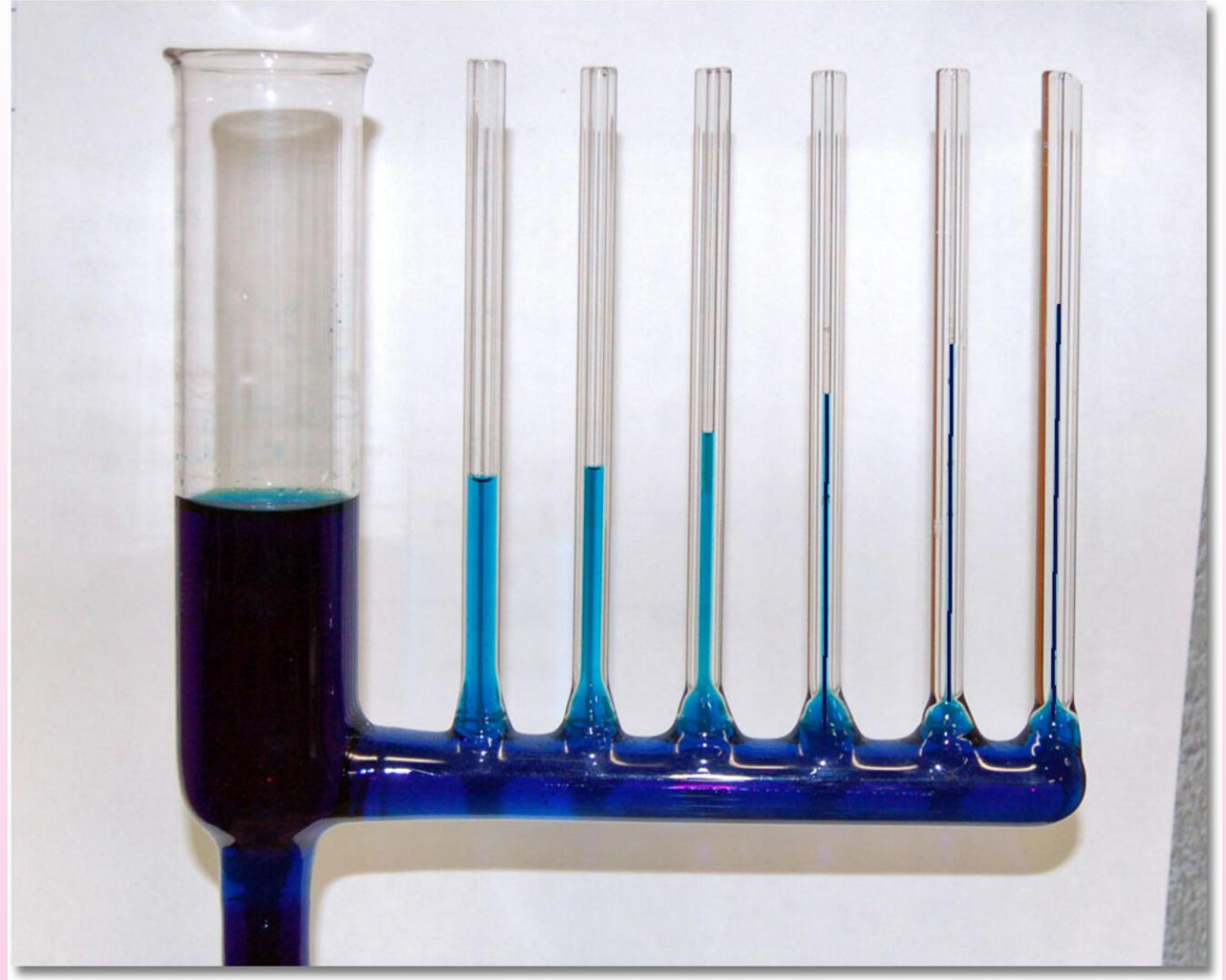
$$h = \frac{2\gamma \cos \theta}{\rho g r}$$







<https://www.zurich.com/en/media/magazine/2021/can-reforestation-uproot-climate-change>

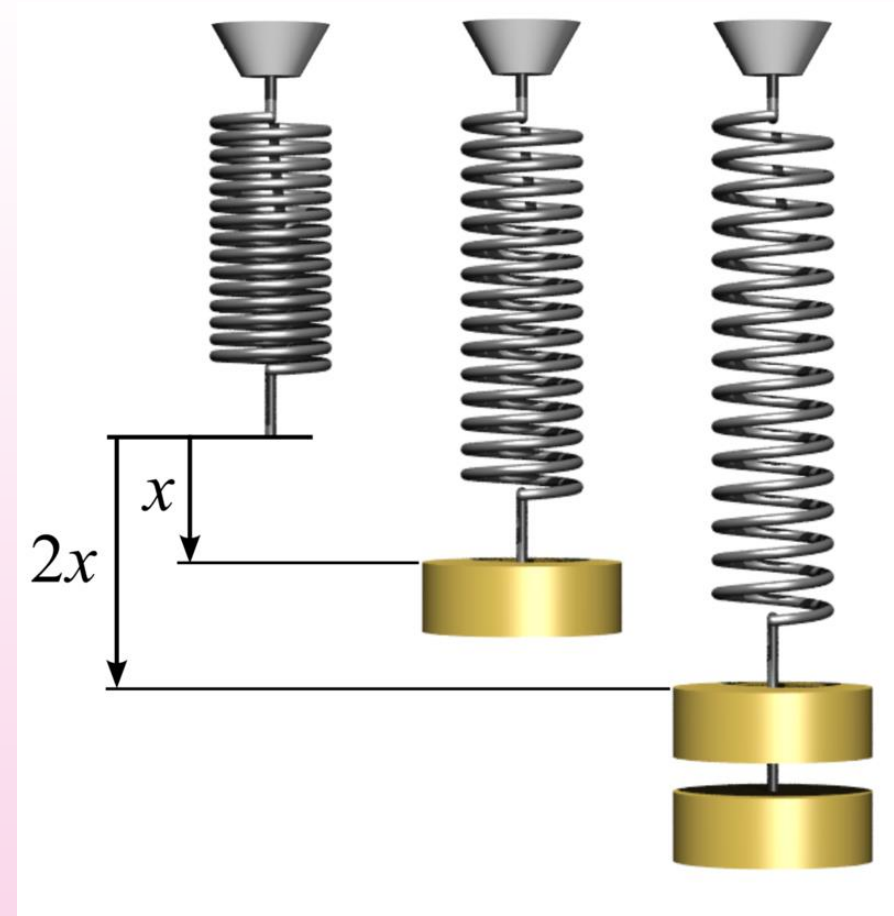


Credit: Dr. Keith Hayward  
<https://www.usgs.gov/media/images/narrower-tube-openings-allow-capillary-action-pull-water-higher>

# Refresher: Hooke's law

A spring with one end fixed is pulled at the other end by some force,  $F$ , and extended by a distance  $x$

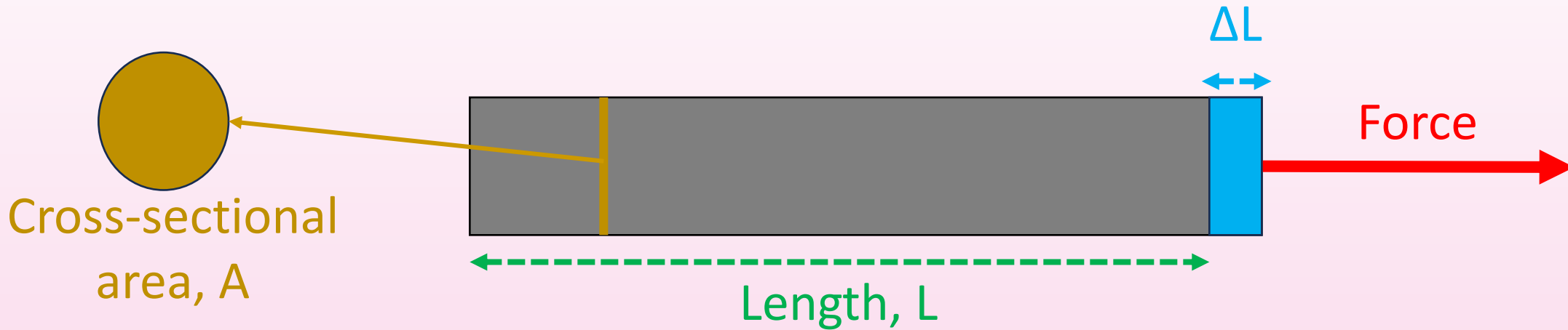
In the elastic region the extension is proportional to the applied force,  $F = kx$



<https://upload.wikimedia.org/wikipedia/commons/thumb/f/fc/Hookes-law-springs.png/800px-Hookes-law-springs.png>



# Some definitions

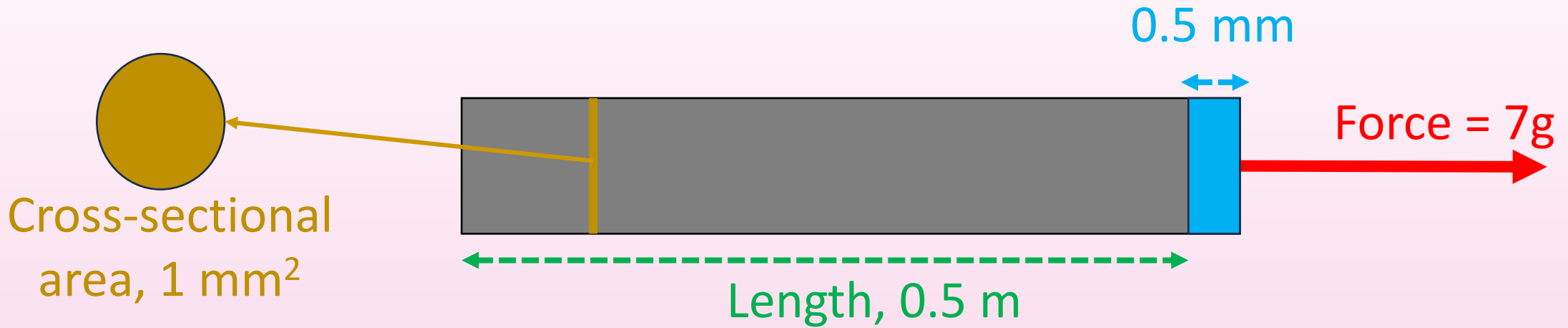


**Stress** is the force applied per unit area on a material [ $\text{kg m}^{-1} \text{s}^{-2}$ ]

**Strain** is the fractional change in length  $\frac{\Delta L}{L}$

Young's modulus describes the ratio of stress/strain for a linear extension

# Some definitions



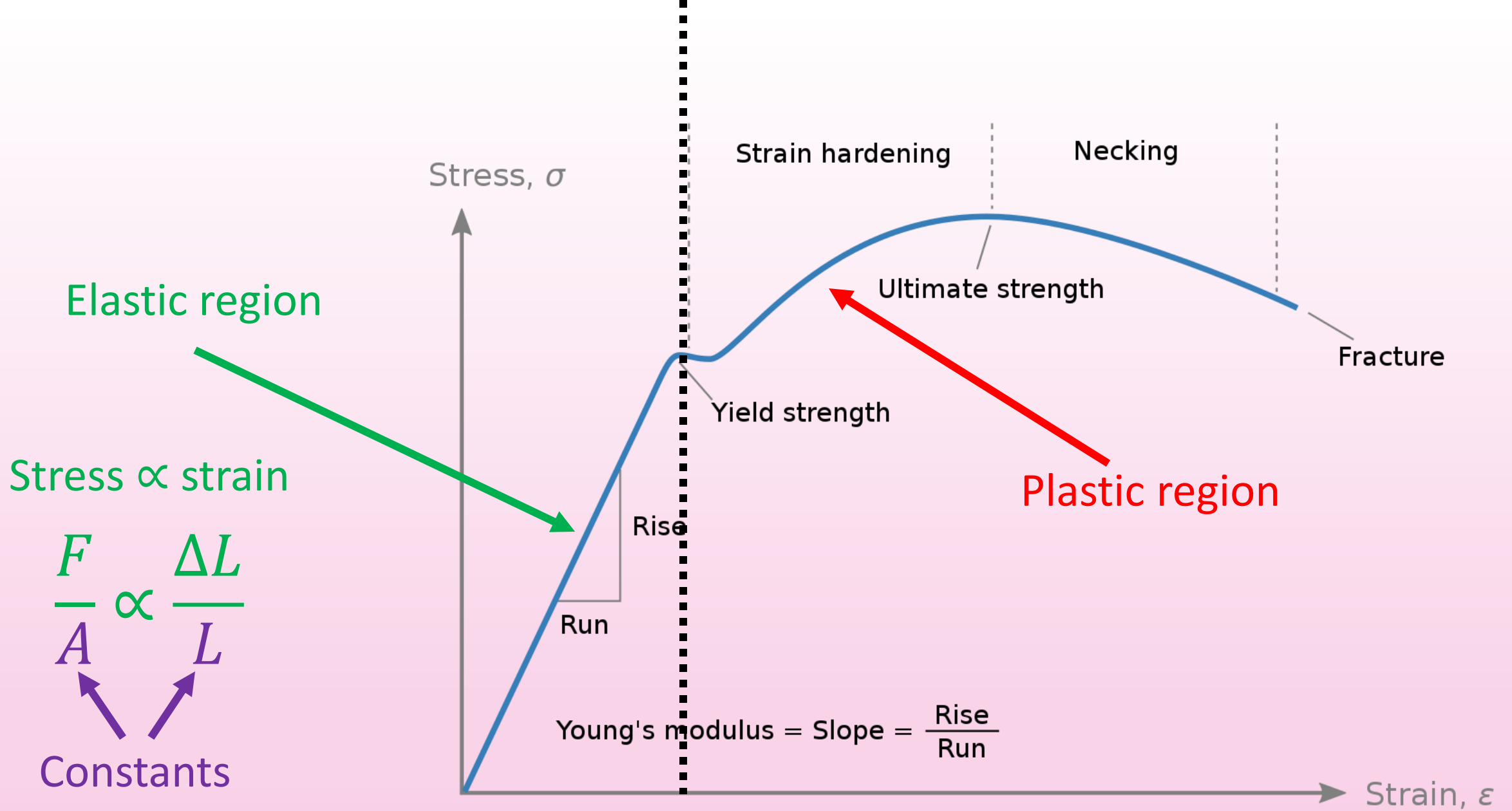
Q) For the system above, calculate

i) stress = force / area =  $(7 \times 10) \text{ kg m s}^{-2} / 1 \times 10^{-6} \text{ m}^2 = 7 \times 10^7 \text{ kg m}^{-1} \text{ s}^{-2}$

ii) strain = extension/length =  $5 \times 10^{-4} \text{ m} / 0.5 \text{ m} = 1 \times 10^{-3}$

iii)  $Y = \text{stress} / \text{strain} = 7 \times 10^7 / 1 \times 10^{-3} = 7 \times 10^{10} \text{ kg m}^{-1} \text{ s}^{-2} = 70 \text{ GPa}$

Units of pressure!



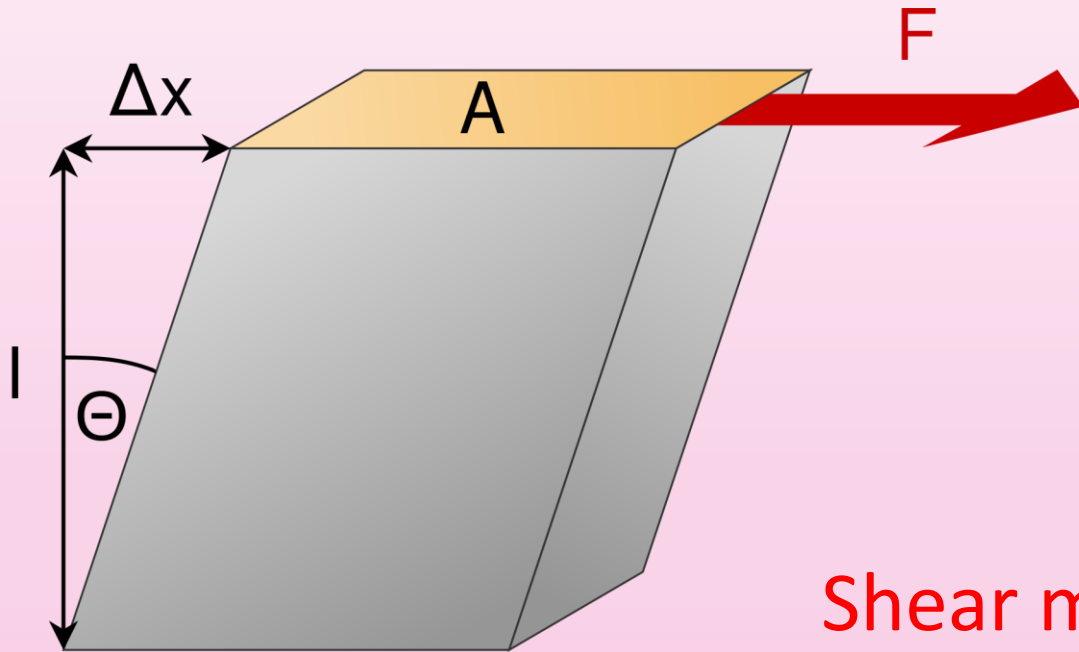
[https://upload.wikimedia.org/wikipedia/commons/thumb/c/c1/Stress\\_strain\\_ductile.svg/1280px-Stress\\_strain\\_ductile.svg.png](https://upload.wikimedia.org/wikipedia/commons/thumb/c/c1/Stress_strain_ductile.svg/1280px-Stress_strain_ductile.svg.png)

$$F = k\Delta L = kx$$

If force is removed, material returns to original length

# Other elastic moduli

Still defined as modulus = stress/strain



[https://en.wikipedia.org/wiki/Shear\\_modulus](https://en.wikipedia.org/wiki/Shear_modulus)

Rather than a force linear to length, we apply a **tangential force**

$$\text{Shear stress} = F/A$$

$$\text{Shear strain} = \frac{\Delta x}{l}$$

$$\text{Shear modulus} = (\text{shear stress})/(\text{shear strain})$$

Small for liquids and very small for gases

# Other elastic moduli

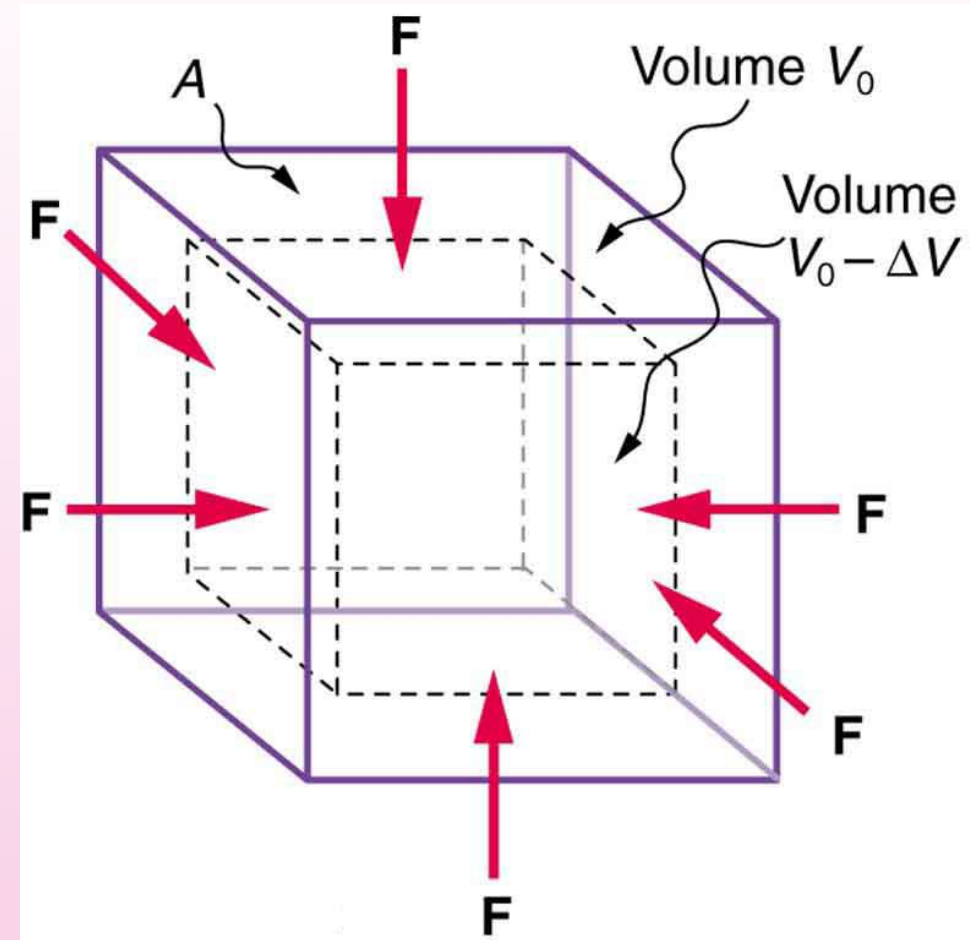
**Bulk modulus**, for objects deformed under a **uniform pressure** – particularly useful for liquids!

Bulk stress = force/area = pressure,  $P$

$$\text{Bulk strain} = \frac{\Delta V}{V_0}$$

$$\text{Bulk modulus} = -V \left( \frac{dP}{dV} \right)$$

Comparable for solids and liquids



[https://nigerianscholars.com/assets/uploads/2018/03/Figure\\_06\\_03\\_09a.jpg](https://nigerianscholars.com/assets/uploads/2018/03/Figure_06_03_09a.jpg)

Compressibility =  $1/(\text{Bulk modulus})$

# Elastic modulus data

Material	Young's modulus (GPa)	Shear modulus (GPa)	Bulk modulus (GPa)
Aluminium	70	30	70
Brass	91	36	61
Copper	110	42	140
Glass	55	23	37
Iron	190	70	100
Lead	16	5.6	7.7
Nickel	210	77	260
Steel	200	84	160
Tungsten	360	150	200

Young's modulus and Bulk modulus are usually fairly close,  
Shear modulus is slightly lower (often 2-3 times smaller)



# Summary

Learnt about capillary action in the context of cohesion and adhesion

Discussed the different kinds of deformation, elastic and plastic

Identified three different elastic moduli for describing forces of different kinds (linear -> Young's, tangential -> Shear, compression -> Bulk)