# Electromagnetism

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## Lecture 17

## Self Inductance

## Week 9

# Last Two Lectures

- **Magnetic Inductance**
  - Motion of conductor in B-field
  - Induced voltage (e.m.f)
  - Lenz's Law (polarity of induced voltage)
- Induced E-fields
- Faraday's Law

$$\varepsilon = -\frac{d\Phi_m}{dt}$$

$$\oint \underline{\boldsymbol{E}} \cdot d\underline{\boldsymbol{l}} = -\frac{d\Phi_m}{dt}$$

**[Last Lecture]()**

**Past Revision Questions**

# This Lecture

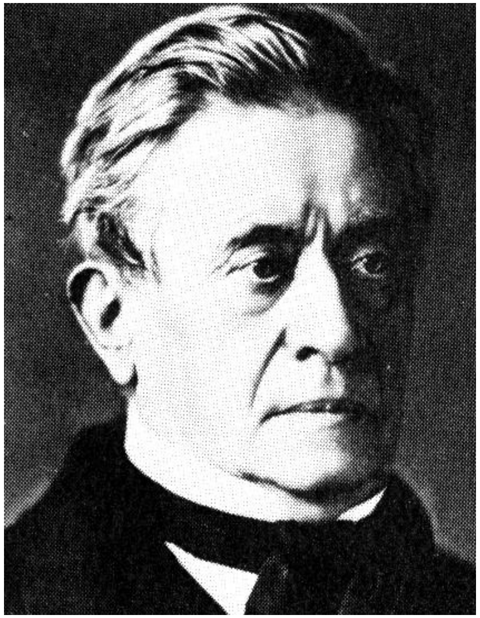
- Definition of Self-Inductance
- Calculation of Self-Inductance

- Energy Stored by an Inductor

- Energy Density of a magnetic field

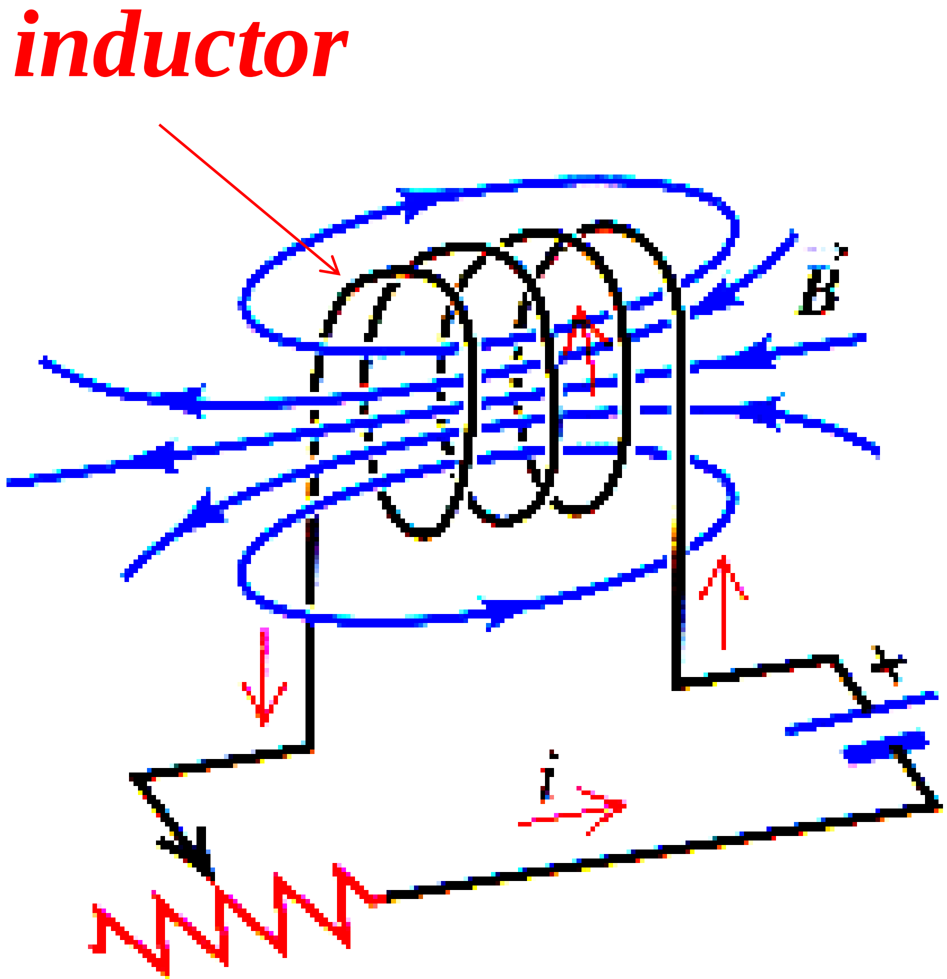
# Self-Inductance

- The phenomenon of self-inductance was discovered by Joseph Henry in 1832 (Princeton University).



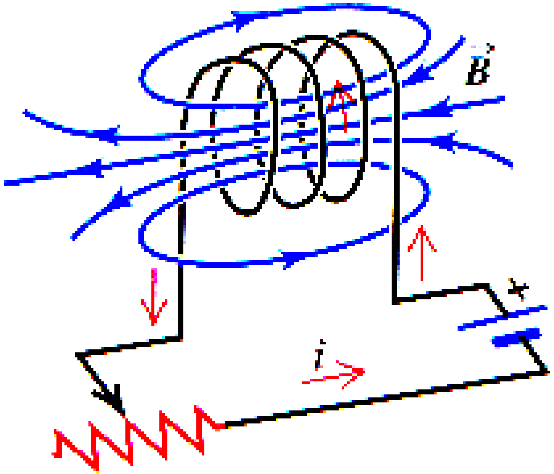
**Joseph Henry 1797-1878**

# Self-Inductance




- When current in the circuit changes, the flux changes also, and a self-induced voltage appears in the circuit.

# Self-Inductance




- $I$ constant => ε = 0.
- Note: ε is induced voltage <u>not</u> the voltage from the battery.
- For $I$ increasing or decreasing, ε not zero.

- ***We define, L as the self-inductance of the coil.***

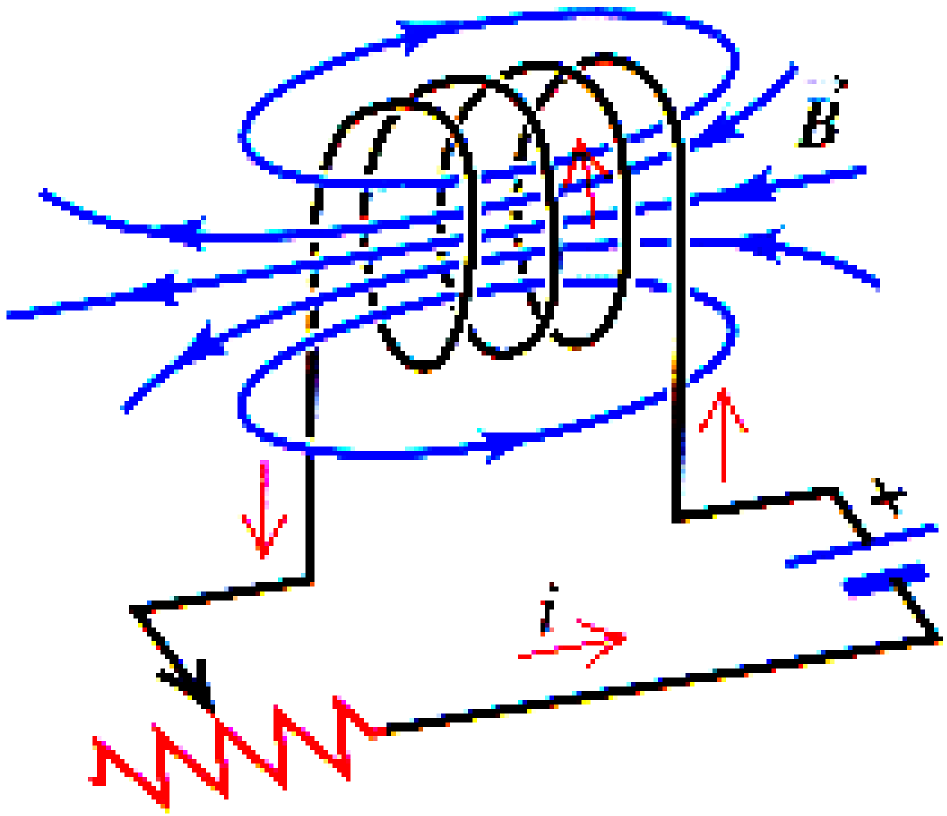
- $\boldsymbol{\varepsilon \propto -\frac{d\Phi_m}{dt} \propto -\frac{dB}{dt} \propto -\frac{dI}{dt} = -L\frac{dI}{dt}}$

"resistance" to the change of current

# Definition of Inductance

- Suppose a current $I$ in a coil of $N$ turns causes a flux $\Phi_m$ to thread each turn.
- $N\Phi_m \propto B \propto I$
- It's easier to measure current than flux so:
- Self Inductance is defined as:
- $N\Phi_m = LI$

# Self Inductance




- $N\Phi_m = LI$

- $L = \dfrac{N\Phi_m}{I}$

# Self Inductance

- From Faraday's Law of Induction

$$\varepsilon = -N\frac{d\Phi_m}{dt}$$

$$\varepsilon = -\frac{d(N\Phi_m)}{dt} = -\frac{d(LI)}{dt} = -L\frac{dI}{dt}$$

# Two Equivalent Definitions of L

$$N\Phi_m = LI$$

$$\varepsilon = -L\frac{dI}{dt}$$

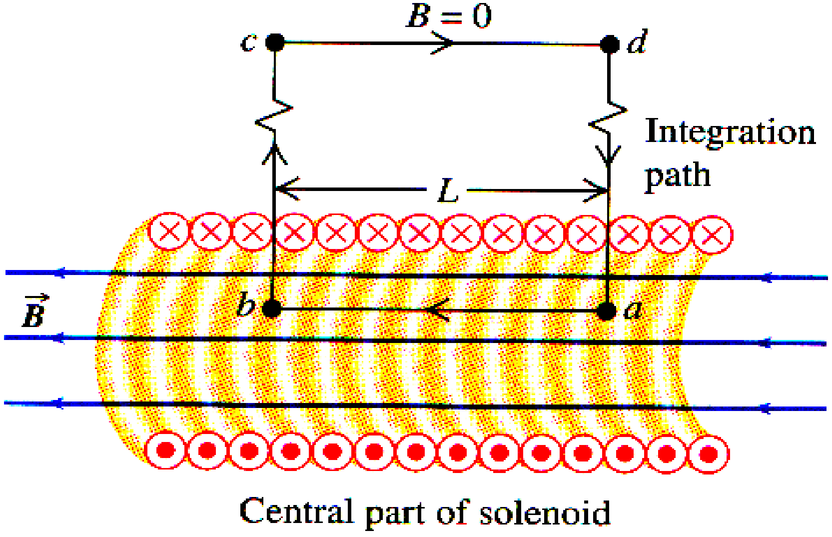
# SI Unit of inductance

$$\varepsilon = -L\frac{dI}{dt}$$

- SI unit for inductance is: V s $A^{-1}$
- This is called the **Henry (H)**

- If a current changing by 1A/s is to generate 1V, the inductance is 1H.

# Self-Inductance of a Solenoid

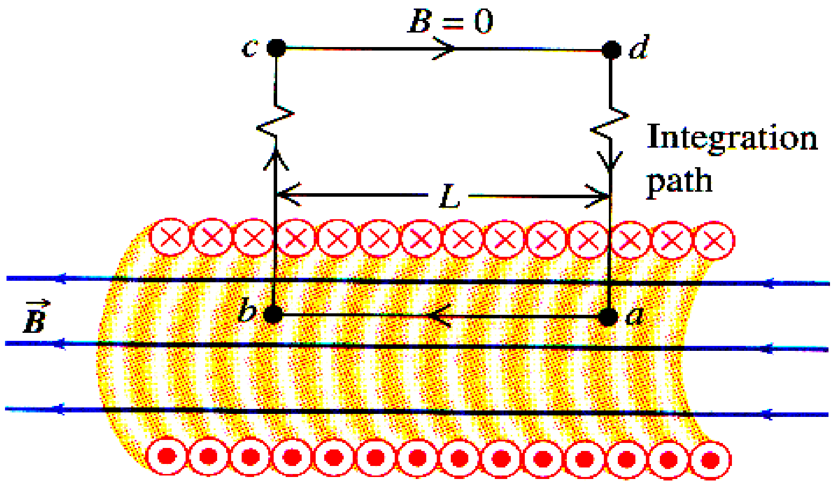



Definition of inductance:

$$L = \frac{N\Phi_m}{I}$$

- $n$ turns per unit length, radius $R$ and the length of the solenoid is $l$
- **Find inductance -> use visualizer**

# Self-Inductance of a Solenoid



Central part of solenoid

$$L = \mu_0 n^2 l \pi R^2$$

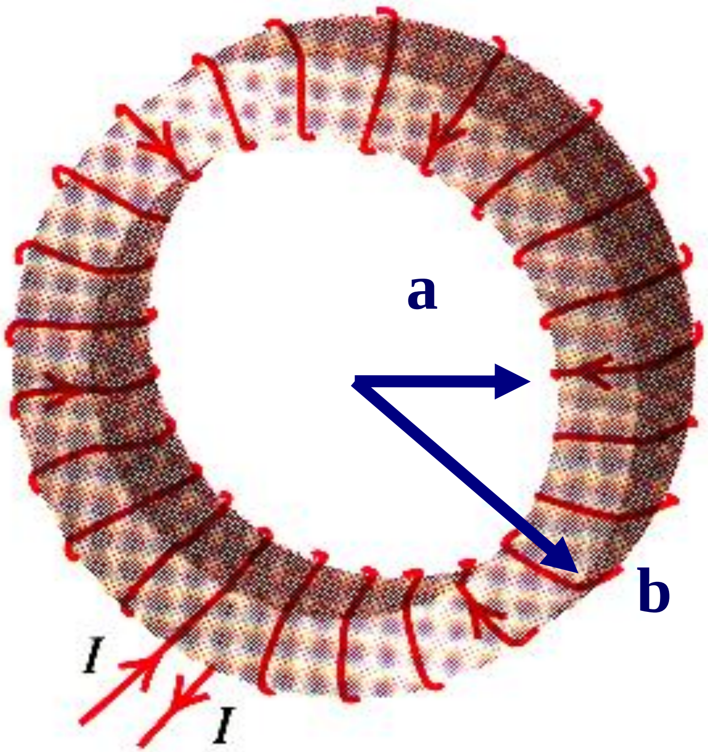
- The inductance does not depend on current or voltage, **it is a property of the coil**. (length, width, and number of turns per unit length)

# Example

- Find the inductance of a solenoid of length 10 cm, area 5 cm$^2$, and 100 turns.
- $n = \frac{100}{0.1} = 1000$ turns/metre
- $L = \mu_0 n^2 l \pi R^2 = 4\pi \times 10^{-7} \times 10^6 \times 0.1 \times 5 \times 10^{-4} = 6.27 \times 10^{-5}$ H.
- *At what rate must the current in the solenoid change to induce a voltage of 20 V?*
- *Answer: 3.18 x 10$^5$ A/s*

$$\varepsilon = -L\frac{dI}{dt}$$

# Self-Inductance of a Toroid Magnet

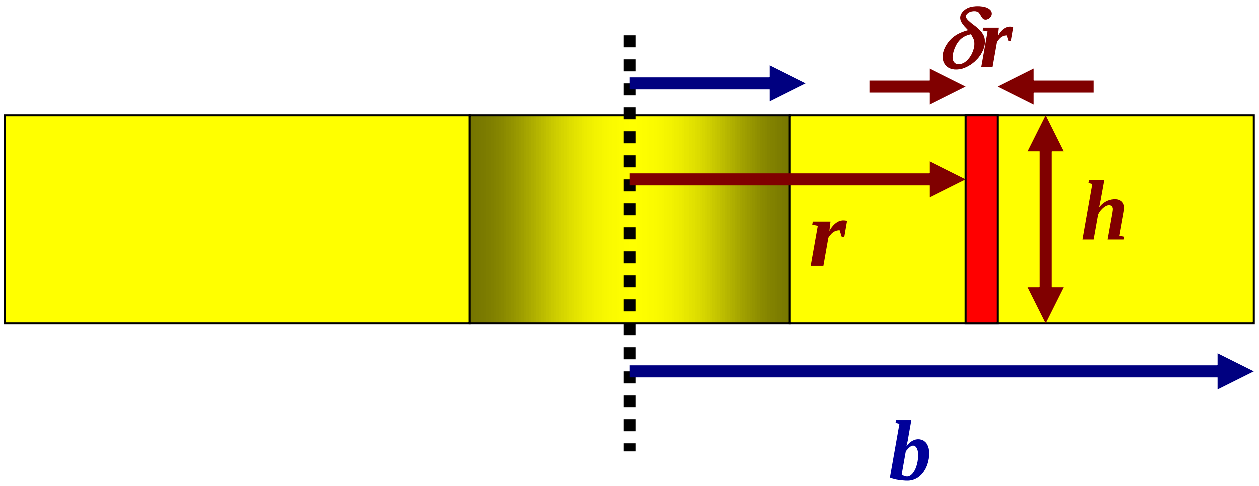



- From Lecture 14:
- Magnetic field inside yoke of toroid is:

$$B = \frac{\mu_0 NI}{2\pi r}$$

- Take a cross-sectional view of toriod.

# Self-Inductance of a Toroid Magnet




$$B = \frac{\mu_0 N I}{2\pi r}$$

Consider a thin band of radius $r$, width $h$, and thickness $\delta r$. The cross-sectional area of the thin band is $h\,\delta r$.

Let's do working on the visualizer.

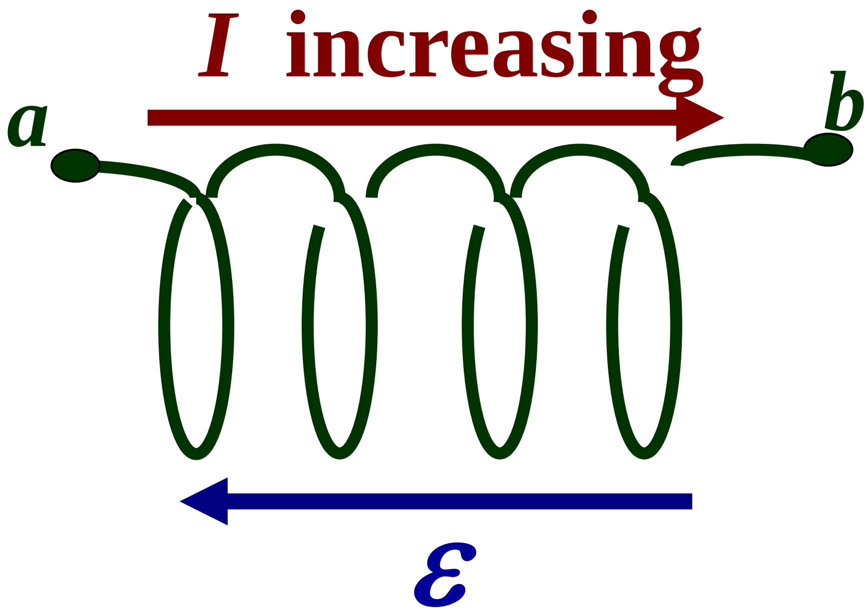
# Self-Inductance of a Toroid Magnet

- Result: $L = \frac{\mu_0 N^2 h}{2\pi} \ln \frac{b}{a}$

- Inductance – like capacitance – depends only on geometric factors

- From the worked examples it can be seen that: **$\mu_0$ also has units of H $m^{-1}$.**

# Energy Stored in an Inductor




- $\varepsilon = -L\dfrac{dI}{dt}$
- Power *delivered* to inductor:

$$P = \varepsilon I = LI\frac{dI}{dt}$$

The energy $\delta U$ supplied to the inductor during an infinitesimal time interval $\delta t$ is:

$$\delta U = P\,\delta t = L\,I\,\delta I$$

# Energy Stored in an Inductor

- $\delta U = P\ \delta t = L\ I\ \delta I$
- The total energy $U$ supplied while the current increases from zero to a final value $I$ is
- $U = L \int_0^I I\ dI = \frac{1}{2} L I^2$
- This is the energy stored in the magnetic field in the inductor.

# Energy Stored in Capacitor & Inductor

- Note: analogy between energy stored in a capacitor and in an inductor.

$$U_L = \frac{1}{2} L I^2$$

$$U_C = \frac{1}{2} \frac{Q^2}{C}$$

# Example: Energy Stored in a Solenoid

- $U = \frac{1}{2}LI^2 = \frac{1}{2}\mu_0 n^2 l\pi R^2\, I^2$
- Energy per unit volume (magnetic energy density)
- $u_B = \frac{U}{\pi R^2 l} = \frac{1}{2}\mu_0 n^2 I^2$
- But $B = \mu_0 n I$
- So $\mu_0 n^2 I^2 = \frac{B^2}{\mu_0}$

# Megnatic Energy Density in a Vacuum

$$\boxed{u_B = \frac{1}{2}\frac{B^2}{\mu_0}}$$

- The equation is true for all magnetic field configurations
- Compare with the energy density in an electric field:

$$u_E = \frac{1}{2}\varepsilon_0 E^2$$

# Summary
# Self Inductance, L

- When a current flow through a circuit, a magnetic field is produced.
- If that current changes (e.g. switching off, AC current etc.) the magnetic field will change.
- The changing magnetic field will induce a voltage, $\varepsilon$ and hence a current *opposing* the changing current.
- This is called self-inductance.

# Summary
# Self Inductance, L

- Devices designed to have a self-inductance are called ***Inductors.***
- ***Definition of inductance:***

$$N\Phi_m = LI \qquad \varepsilon = -L\frac{dI}{dt}$$

- ***Energy Stored in an inductor:***

$$U_L = \frac{1}{2}LI^2$$

- ***Energy Density in a Magnetic Field*** $u_B = \frac{1}{2}\frac{B^2}{\mu_0}$