Propagation of light. Michelson-Morley experiment

In the previous lecture we have derived the law of velocity transformation which states that in the moving frame the velocity $\boldsymbol{v}'$ is related to the velocity $\boldsymbol{v}$ in the stationary frame as

$$\boldsymbol{v}' = \boldsymbol{v} - \boldsymbol{V} \,,$$

where $\boldsymbol{V}$ is the velocity of the moving frame *relative* to the stationary frame.

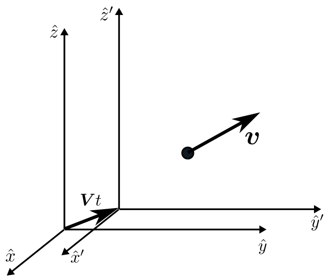

Figure 1: Trajectory and Galilean transformation into a moving frame.

The transformation of velocities can be applied to waves as well. The familiar example is the Doppler effect: the sound from a moving source moves with a different velocity and it affects its frequency. There is an important difference though: the sound waves propagate in a medium (air, water) thus there is a preferential frame, the one in which the medium is in rest. In this respect different frequency shifts are obtained depending whether the emitter or the observer move with respect to the medium.

What about another well-known wave phenomena, the light, which is an example of electromagnetic radiation? It was long believed that light waves, similarly to sound waves, are vibrations of a certain medium called "luminiferous aether". In this case one would be able to tell the difference in speed of light propagation if either the light source or a detector is moving. Of course, the effect will be tiny, as the speed of light $c \simeq 300000\text{km/s} = 3 \times 10^8\text{m/s}$ is huge compared to typical velocities we experience in everyday life. It is still ten thousand times bigger than the orbital speed of the earth, $v_\mathrm{e} = 30\text{km/s}$. Nevertheless, an American physicist Albert A. Michelson (1852-1931) devised an instrument, called Michelson interferometer which, in principle, would be able to detect tiny differences in the speed of light propagation.

A schematic depiction of Michelson interferometer is shown in Fig. 2. Light from a source S (a distant star, for example) falls on an inclined semitransparent plate (beam splitter) P and splits into two parts. One part travels on through the plate and strikes the mirror M_1. It retraces its path to the point where the beam was first split. A second light path is reflected from the beam splitter to a mirror M_2, goes back and then is recombined with the first beam arriving at P. Both beams travel together from P to a detecting device D.

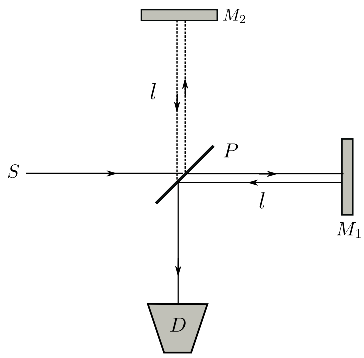

Figure 2: Michelson interferometer

This is an idealised picture: we neglect the thickness of the beam splitter P, losses and imperfections of the mirrors. We also assume that the lengths of both arms PM_1 and PM_2 of the interferometer are the same and equal to l. Nevertheless we will be able to get an idea of the measurement. It is based on the fact that if the interferometer moves with respect to the presumed aether the time intervals to travel along each path will be different. Let us assume that the device moves in the direction PM_1 velocity v with respect to hypothesised aether. Then the light of the first beam PM_1 would have a resultant velocity $c - v$ (relative to the interferometer) when travelling towards the mirror M_1 a resultant velocity $c + v$ on the return trip. Thus it takes it

$$t_1 = \frac{l}{c-v} + \frac{l}{c+v} = \frac{2lc}{c^2-v^2} = \frac{2l}{c}\frac{1}{1-v^2/c^2}$$

to travel to M_1 and back. The time t_2 for the light to travel along the second path. To calculate it let us place ourselves in the frame of aether and imagine the trajectory from the beam splitter to the mirror and back in this frame. It is shown in Fig. 3. It consists of two segments, each taking time $t_2/2$. In this time the mirror M_2 travels a distance $vt_2/2$ in horizontal direction, so from the Pythagoras theorem we have

$$\left(\frac{ct_2}{2}\right)^2 - \left(\frac{vt_2}{2}\right)^2 = l^2 \quad \Rightarrow \quad t_2 = \frac{2l}{c}\frac{1}{\sqrt{1-v^2/c^2}}\,,$$

which is clearly different from t_1.

As it was noted before, the ratio v/c is extremely small. Taking v to be the earth's orbital velocity $v/c \simeq 30/300000 = 10^{-4}$. This allows us to use the approximations (Taylor expansion to first order):

$$\frac{1}{1-v^2/c^2} \simeq 1 + \frac{v^2}{c^2}$$
$$\frac{1}{\sqrt{1-v^2/c^2}} \simeq 1 + \frac{1}{2}\frac{v^2}{c^2}\,,$$

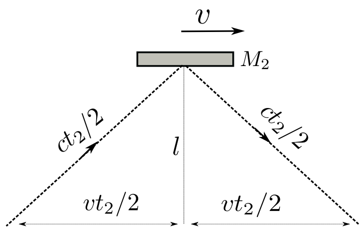

Figure 3: Trajectory of the second beam in the æther frame.

so that the time delay between the two light beams is

$$\Delta t = t_1 - t_2 = \frac{2l}{c}\left(\frac{1}{1-v^2/c^2} - \frac{1}{\sqrt{1-v^2/c^2}}\right) \simeq \frac{l}{c}\frac{v^2}{c^2}\,.$$

This time delay is extremely small: not only it takes $l/c \simeq 1\text{m}/3\cdot 10^8\text{m}\,\text{s}^{-1} \simeq 3.3\cdot 10^{-8}\text{s}$ for the light beam to travel a typical size (1m) of the interferometer, it is also gets multiplied by the square of the ratio v/c. There was no chance to measure such minuscule time intervals. However the principle of interferometric measurements is based on detecting the difference in *optical path* $\Delta l_{\text{opt}} = c\Delta t$ by comparing it to the *wavelength* λ of the light. What one measures is the relative shift of the interference pattern,

$$\delta = \frac{\Delta l_{\text{opt}}}{\lambda} = \frac{c\Delta t}{\lambda} = \frac{l}{\lambda}\frac{v^2}{c^2}\,.$$

For a typical wavelength $\lambda \simeq 500\text{nm} = 5\cdot 10^{-7}\text{m}$ the smallness of v^2/c^2 is compensated and $\delta \simeq 0.02$ can be detected.

The Michelson-Morley experiment was conducted in 1887. The interferometer was rotated so that the mirrors M_1 and M_2 change their position with respect to the direction of motion which, be the luminiferous aether a reality, would result in a detectable shift of the interference pattern. However it was concluded beyond any doubt that $\delta = 0$. This, in turn, ruled out the Galilean law of velocity transformation for light waves.

In the next lecture we will follow Einstein and modify Galilean transformation to accommodate for invariance of light speed observed in MM experiment. Thus we will formulate postulates of Special Relativity and find the new space-time (Lorentz) transformation.