Electromagnetism

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Lecture 12

Magnetic Force & Dipoles

Week 6

Last Lecture

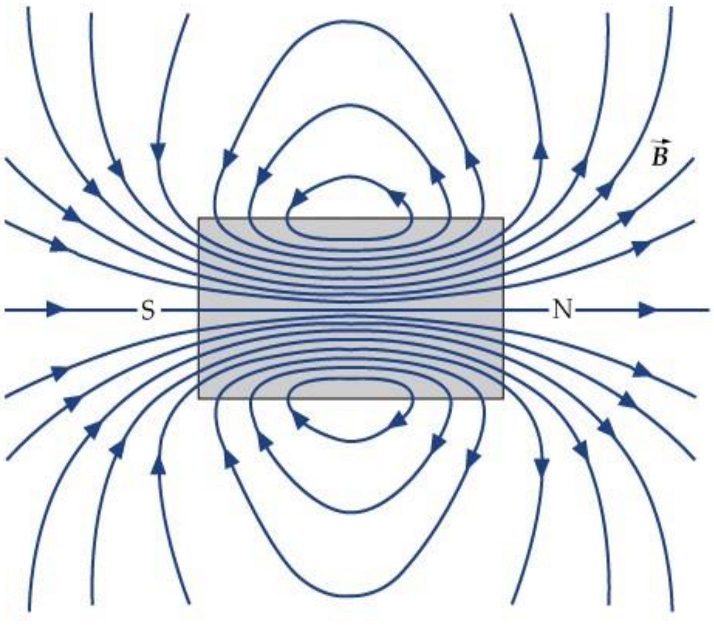
We started Part II – Magnetism

- Definition of Current
- Current Density
- Magnetic force on a moving charge
- The Lorentz Force
- Magnetic field lines

Gauss 's Law for Magnetism

- For E-fields, net electric flux: $\int_S \underline{E} \cdot d\underline{S} = \frac{Q_{enc}}{\varepsilon_0}$
- But there are no magnetic monopoles so for magnetic fields:
- Net magnetic flux:
- $\int_S \underline{B} \cdot d\underline{S} = 0$

(not much use for this course but does form Maxwell's 2nd equation)



(a)

Summary

- A *magnetic field* $\underline{B}$ is defined in terms of the force $\underline{F}_m$ acting on a test particle with charge q and moving through the field with velocity $\underline{v}$:

$$\boxed{\underline{\boldsymbol{F}}_m = q\underline{\boldsymbol{v}} \wedge \underline{\boldsymbol{B}}}$$

- The general case of both B-fields and E-fields is the Lorentz equation (Lorentz Force):

$$\boxed{\underline{\boldsymbol{F}} = q(\underline{\boldsymbol{E}} + \underline{\boldsymbol{v}} \wedge \underline{\boldsymbol{B}})}$$

This Lecture

- Special cases of magnetic force
- Force on current carrying conductor
- Current Loops and Magnetic Dipoles
  - Torque on magnetic dipole in B-field
  - Potential energy of magnetic dipole in B-field

Special Cases of Magnetic Force

- $\underline{\boldsymbol{F}}_m = q\underline{\boldsymbol{v}} \wedge \underline{\boldsymbol{B}}$
- Special cases:
- **v** parallel to **B** $\quad (\underline{\boldsymbol{F}}_m = q\underline{\boldsymbol{v}} \wedge \underline{\boldsymbol{B}}$ = 0)
- **v** perpendicular to **B**
- **v** makes an angle θ to **B**

Direction of B-field

The direction of B field in sketches (in to and out of page)

$\otimes$ $\odot$

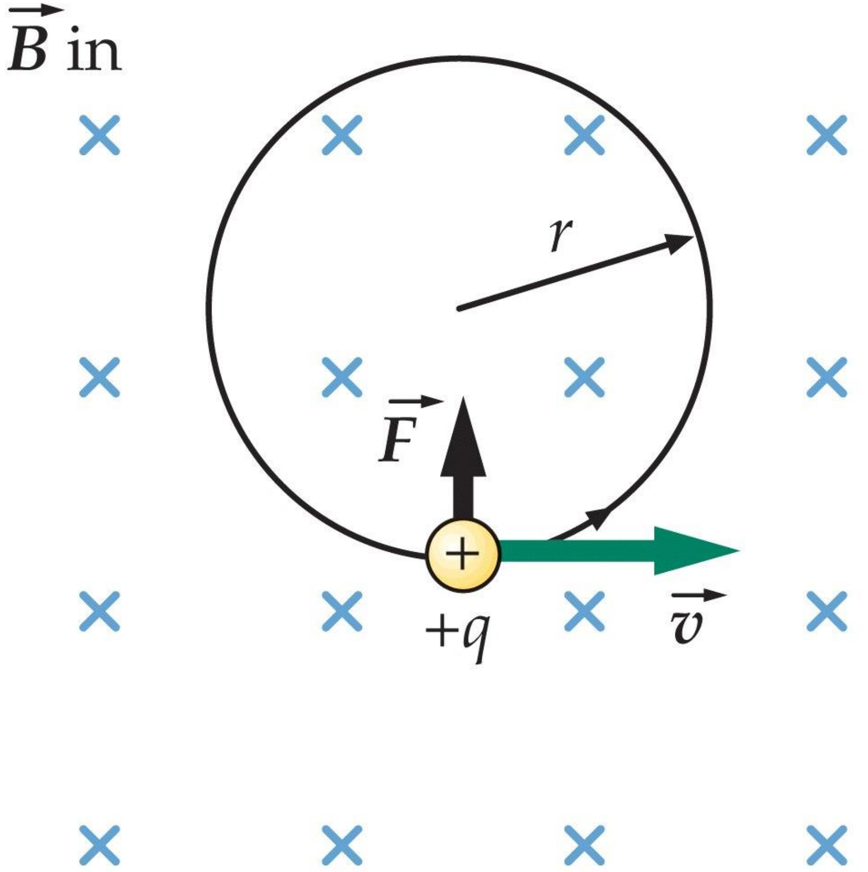
B-field going **<u>in to</u>** the page

B-field coming **<u>out</u>** of the page

i.e. like the back or front of a dart

$\underline{V}$ Perpendicular to $\underline{B}$

- $\underline{\boldsymbol{V}} \perp \underline{\boldsymbol{B}}$ $\quad$ ***F*** $\perp$ **to the plane containing *B* and *v***



$$|\underline{\boldsymbol{F}}_m| = |q\underline{\boldsymbol{v}}\wedge\underline{\boldsymbol{B}}| = \boldsymbol{qvB}$$

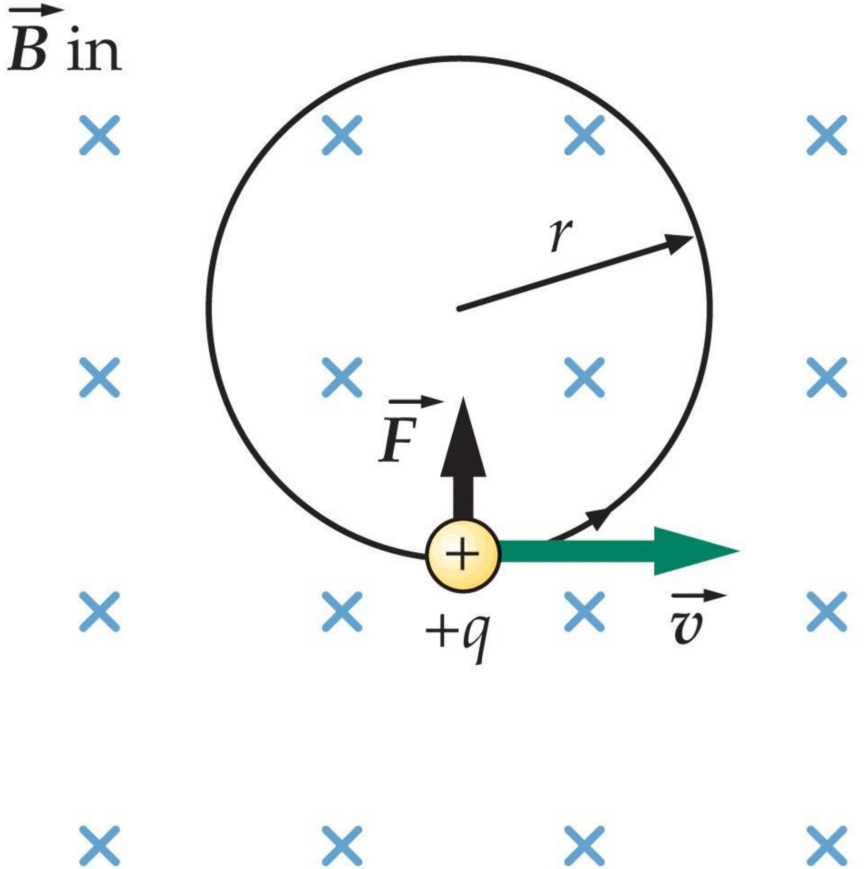
Gives circular motion
Equate forces:

$$qvB = \frac{mv^2}{r}$$

Use visualizer

$\underline{V}$ Perpendicular to $\underline{B}$

- $\underline{\boldsymbol{V}} \perp \underline{\boldsymbol{B}}$ **$\underline{F}$ ⊥ to the plane containing $\underline{B}$ and $\underline{v}$**



$$r = \frac{mv}{Bq} = \frac{p}{Bq}$$

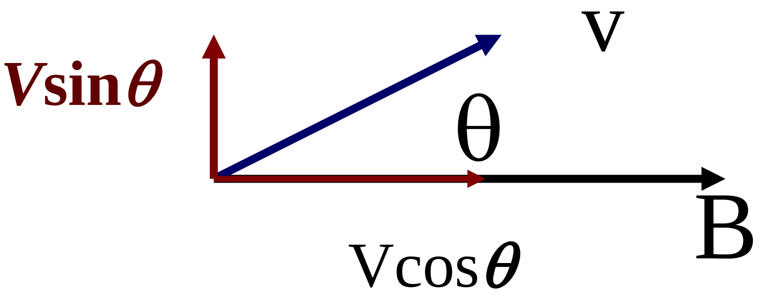
where p is momentum

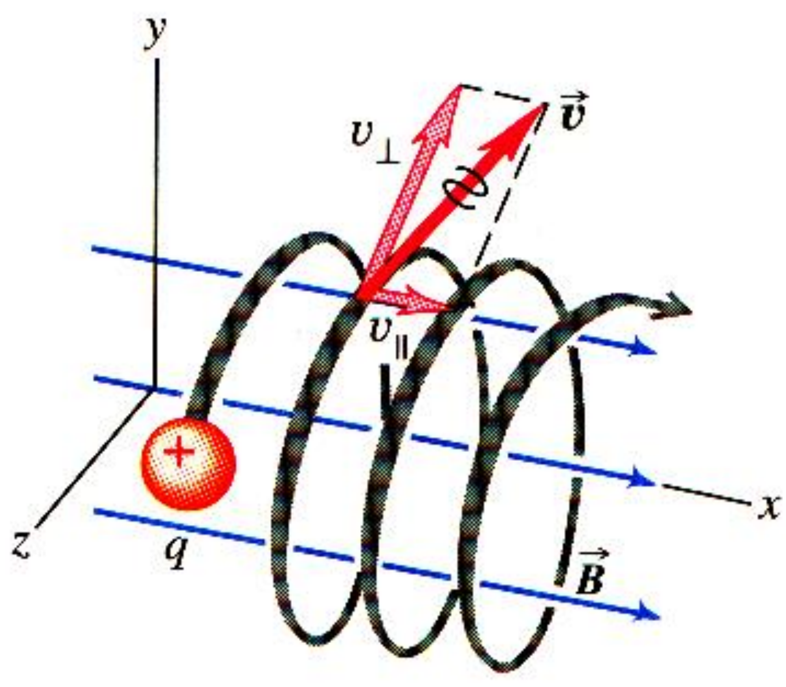
Also: $$\frac{Bq}{m} = \frac{v}{r} = \omega$$

So frequency of "orbital motion": $f = \frac{\omega}{2\pi} = \frac{Bq}{2\pi m}$

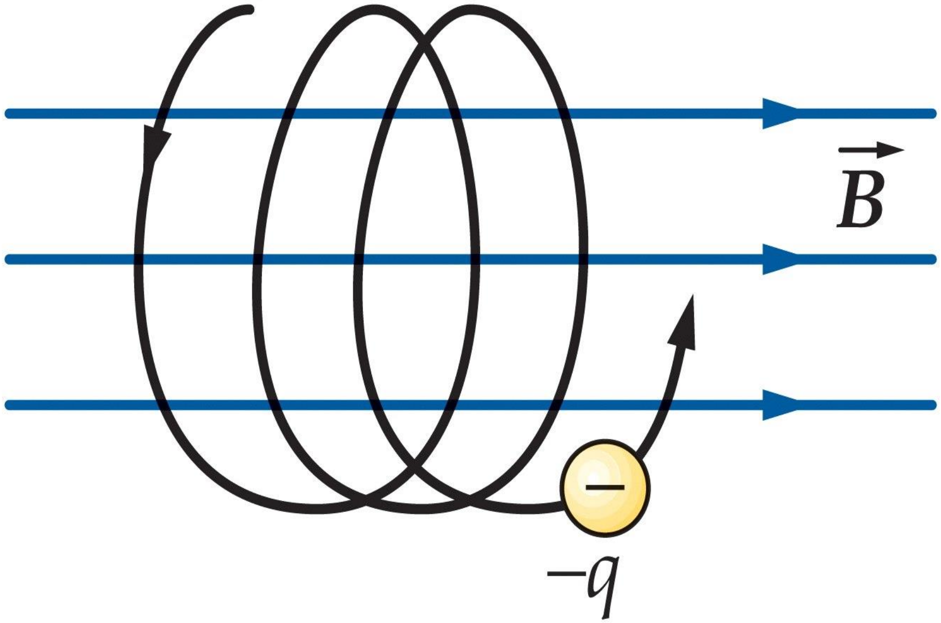
$\underline{v}$ makes an angle θ with $\underline{B}$

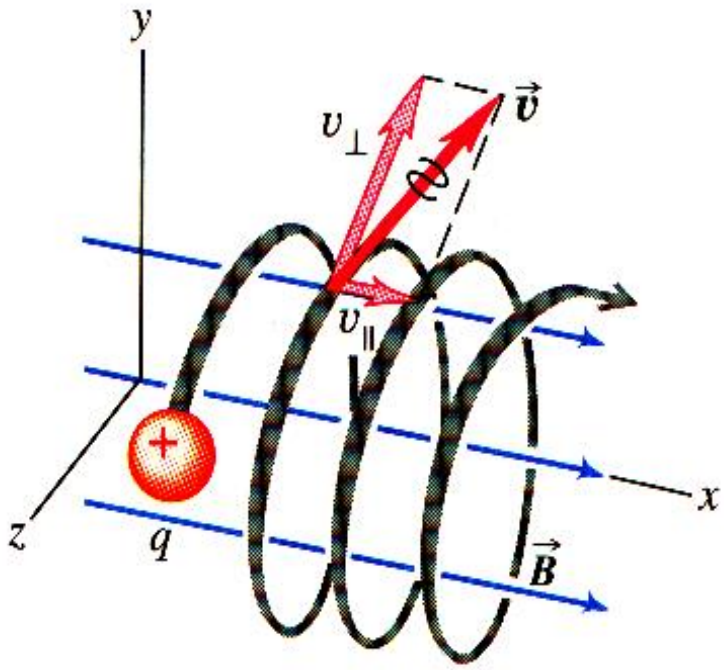
- a uniform circular motion (with "cyclotron angular frequency" ω = 2π/T) in which it has the speed V sinθ in a plane perpendicular to the direction of B.
- a steady speed of magnitude V cosθ along the direction of B
- **Helical Motion**





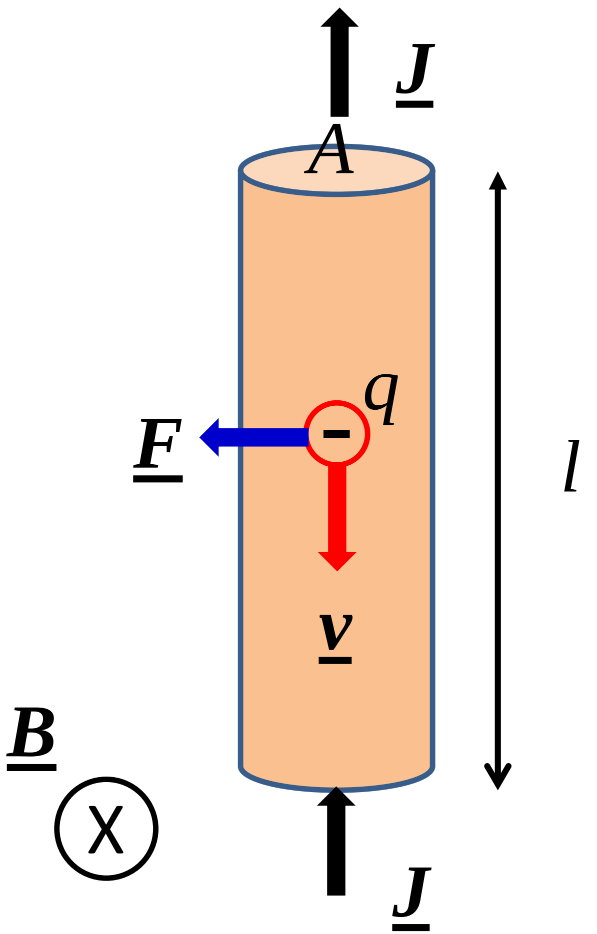
$\underline{v}$ makes an angle θ with $\underline{B}$





Looking along x direction, clockwise motion for –q, anticlockwise motion for +q

Force on Current carrying Conductor

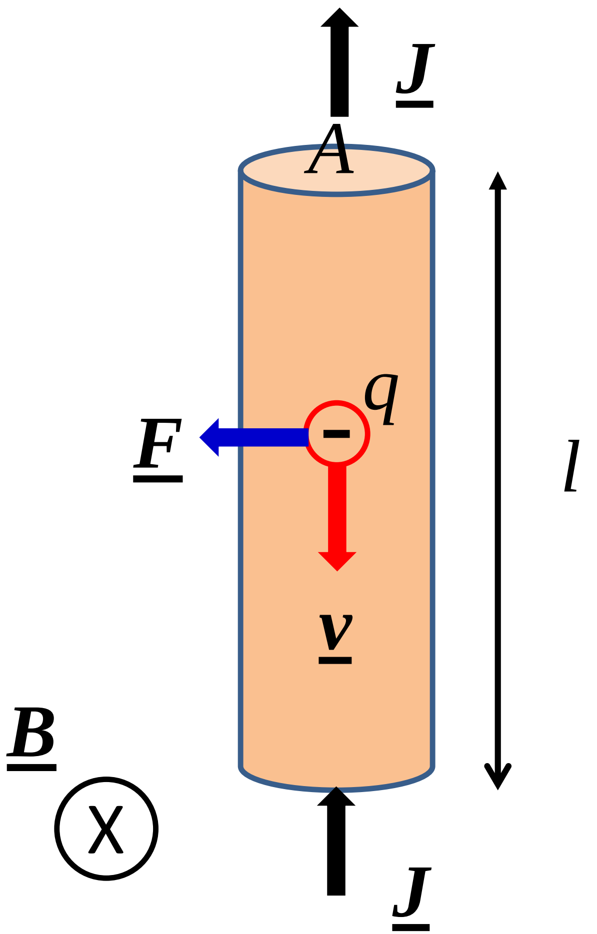


- Single charge $\underline{\boldsymbol{F}}_m = q\underline{\boldsymbol{v}}\wedge\underline{\boldsymbol{B}}$
- N, number of charge carriers in volume Al is $N = nAl$

where n is charge number density

- Total force $F = Nq\underline{\boldsymbol{v}}\wedge\underline{\boldsymbol{B}} = -nAle\underline{\boldsymbol{v}}\wedge\underline{\boldsymbol{B}}$
- But current: $\underline{I} = -nAe\underline{\boldsymbol{v}}$
- So: $\underline{\boldsymbol{F}} = l\,\underline{\boldsymbol{I}}\wedge\underline{\boldsymbol{B}}$ but by convention we define l as the vector I as a scalar.

Force on Current carrying Conductor



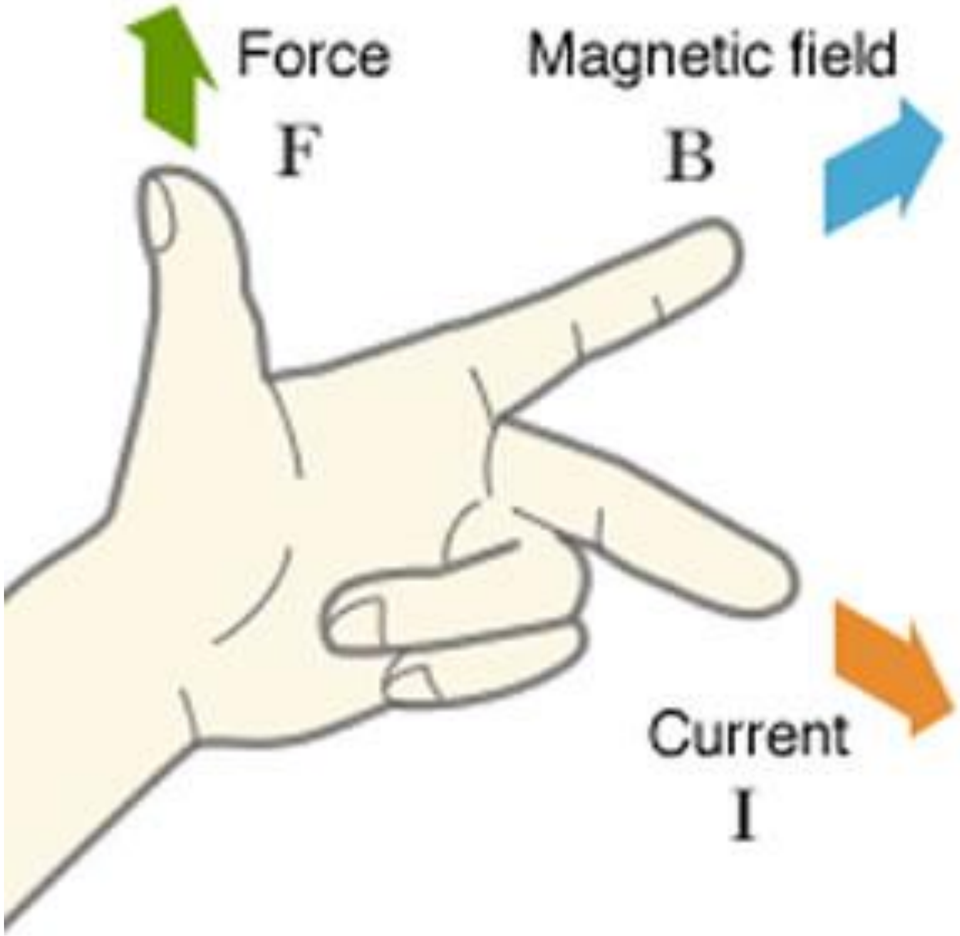
In general

$$\boxed{\underline{F} = I\,\underline{l} \wedge \underline{B}}$$

magnetic force on a straight wire segment.

The direction of $\underline{l}$ is defined as the direction of the current I

Left Hand Rule



FBI

Force on Non-Straight Conductor

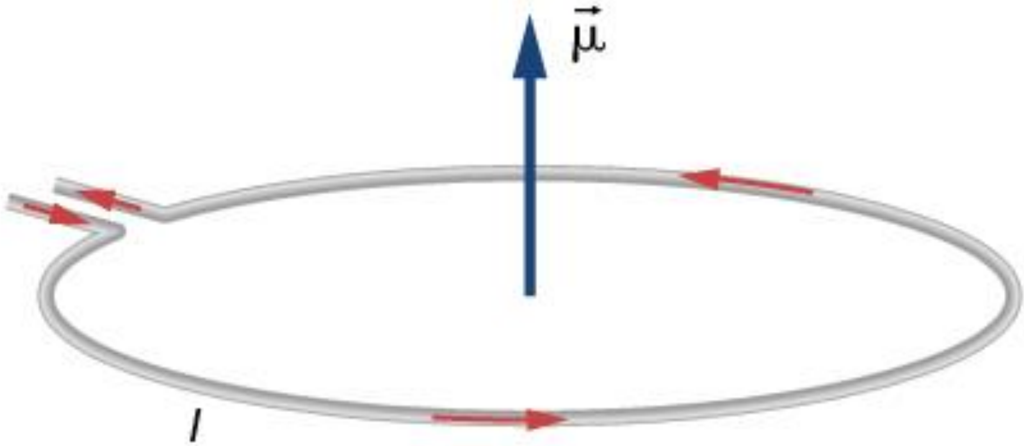
- If the conductor is not straight, consider individual segments and use:
- $\delta \underline{\boldsymbol{F}} = I\ \delta \underline{\boldsymbol{l}} \wedge \underline{\boldsymbol{B}}$
- magnetic force on an infinitesimal wire segment

- Total force:
- $\underline{F} = \int_a^b I\ d\underline{\boldsymbol{l}} \wedge \underline{\boldsymbol{B}}$

Current Loops
Magnetic Dipoles
Magnetic Dipole Moment

Magnetic Dipoles

A current loop is known as a Magnetic Dipole



(a) Current-carrying loop

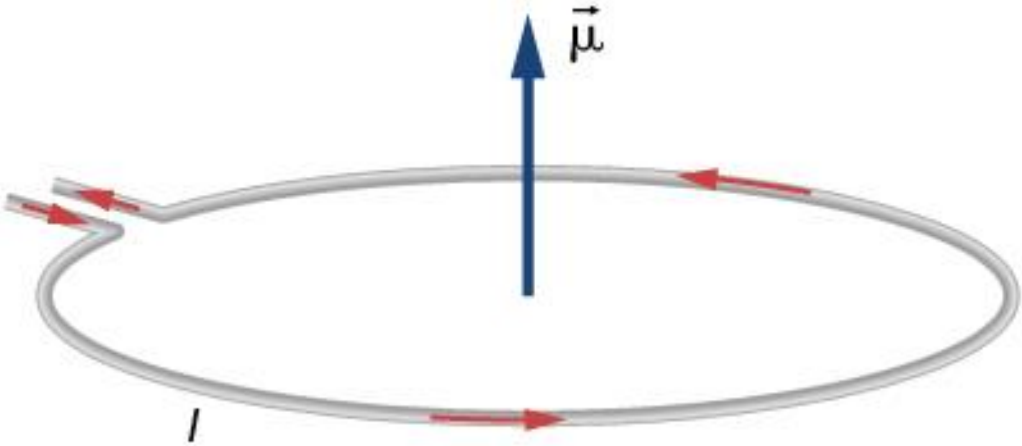
The magnitude of the of the **Magnetic Dipole Moment** is:

Current x Area of loop

$$\boxed{\mu = I \times A}$$

Magnetic Dipole Moment

The **Magnetic Dipole Moment** is a Vector



(a) Current-carrying loop

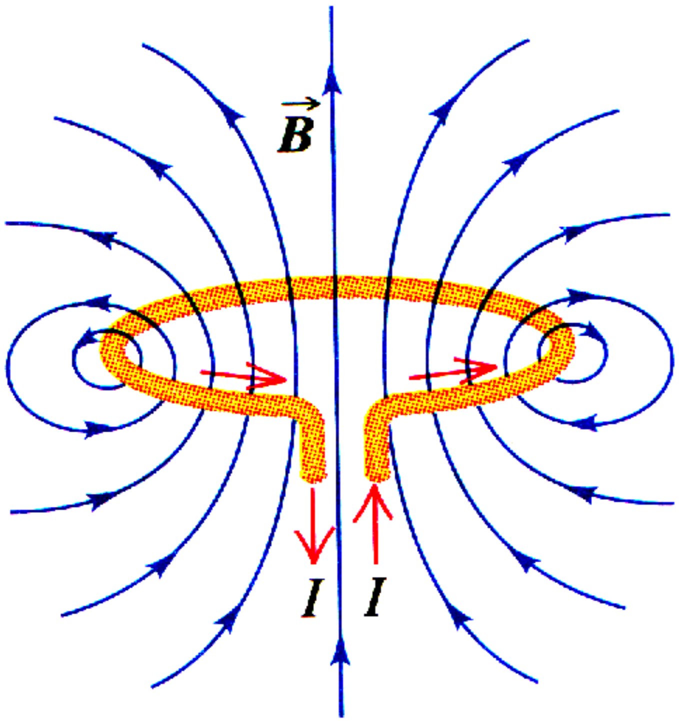
The area enclosed by the loop may be defined as a vector $\underline{\boldsymbol{A}} = A\underline{\hat{\boldsymbol{n}}}$ where $\underline{\hat{\boldsymbol{n}}}$ is a unit vector normal to the area

$$\boxed{\underline{\mu} = I\underline{A}}$$

Units are Am^2

Magnetic Dipole

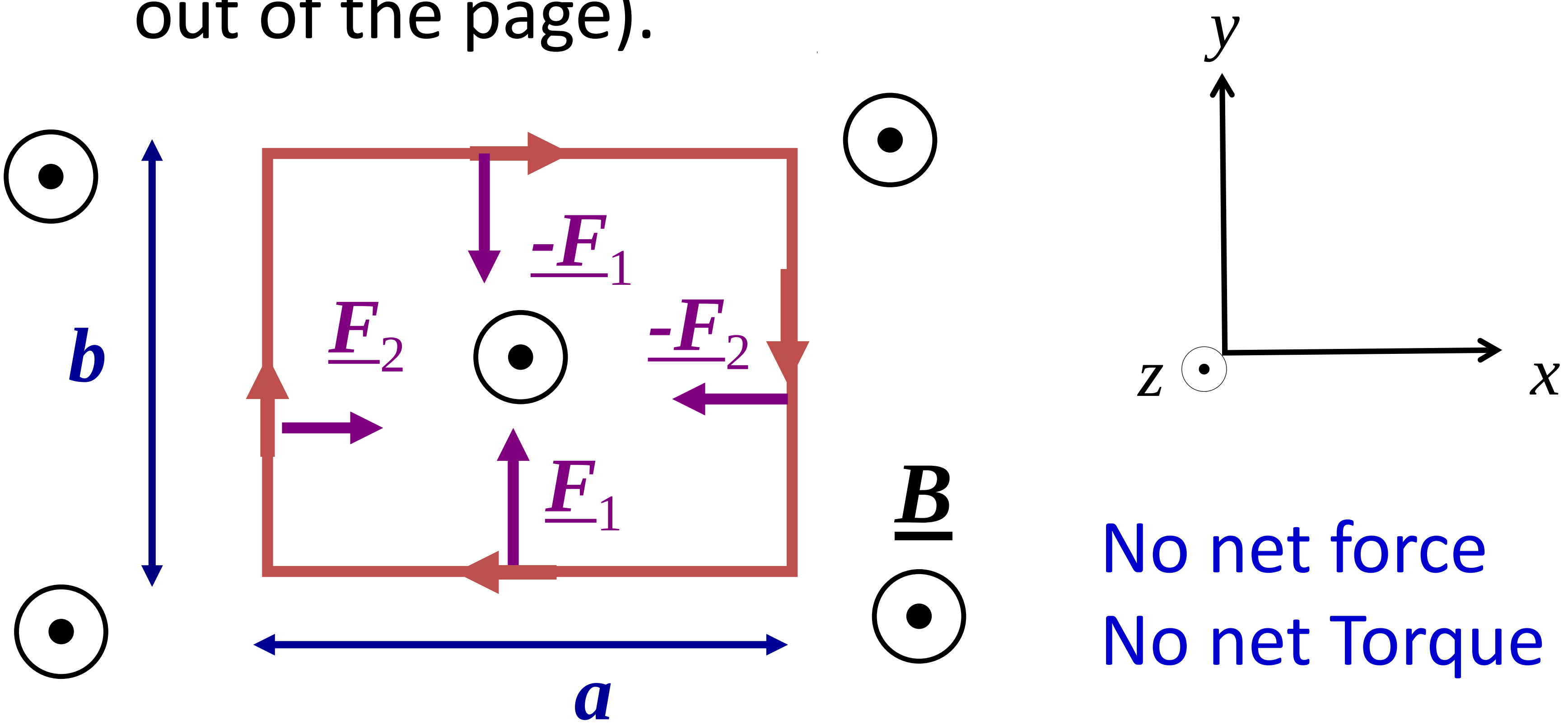
- A current loop produces a magnetic field, similar to that produced by a tiny bar magnet.



- We will cover B-fields produced by currents in the next lecture.

Current Loop in B-field

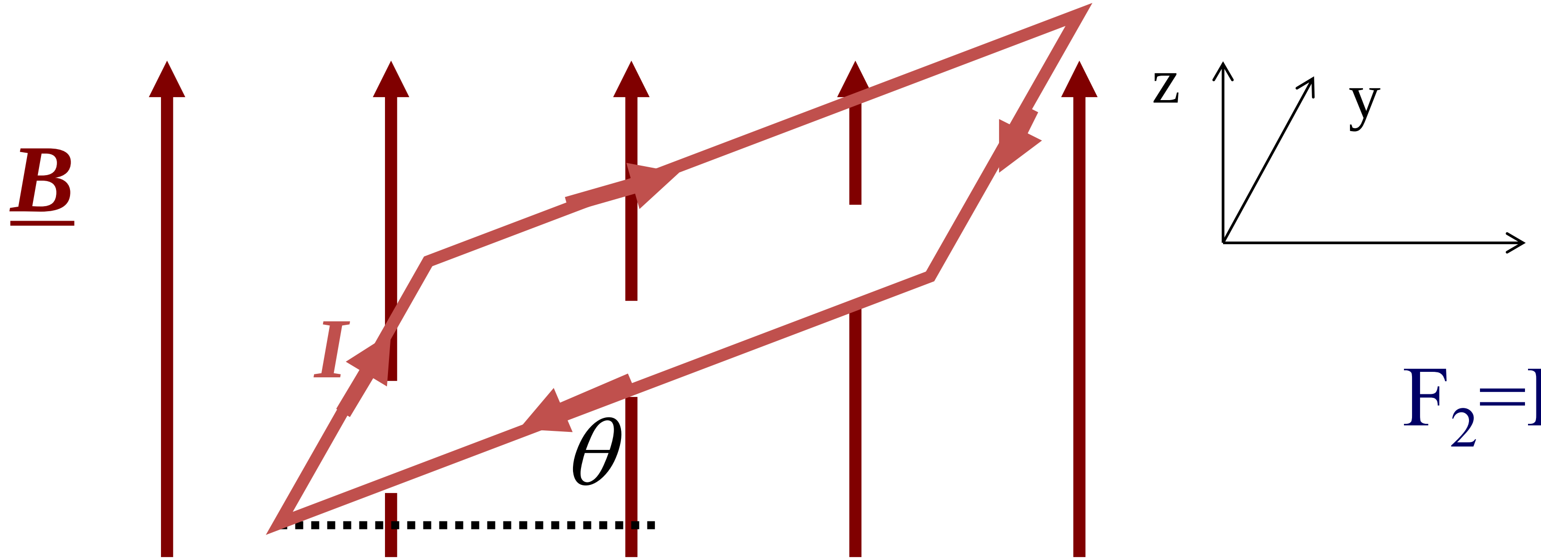
- Consider a current loop in a B-field (coming out of the page).



No net force
No net Torque

Torque on Current Loop

- Now consider loop at an angle to x-axis

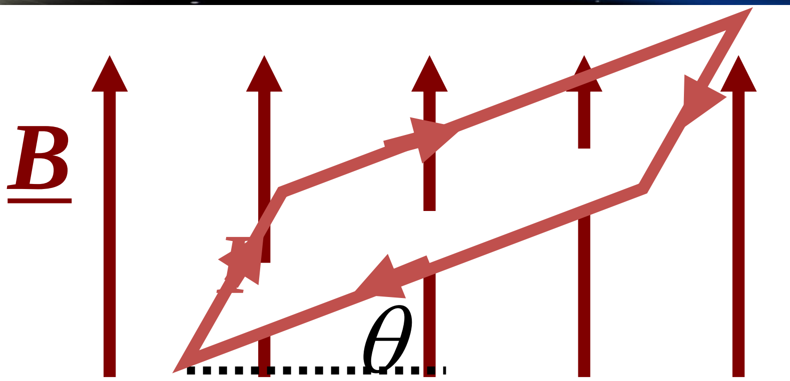


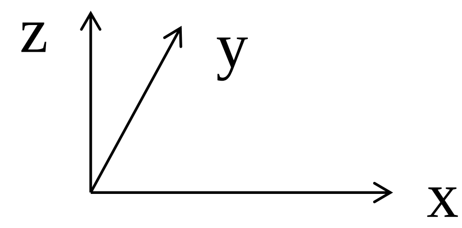
$F_2 = BIb$

b parallel to ***y***

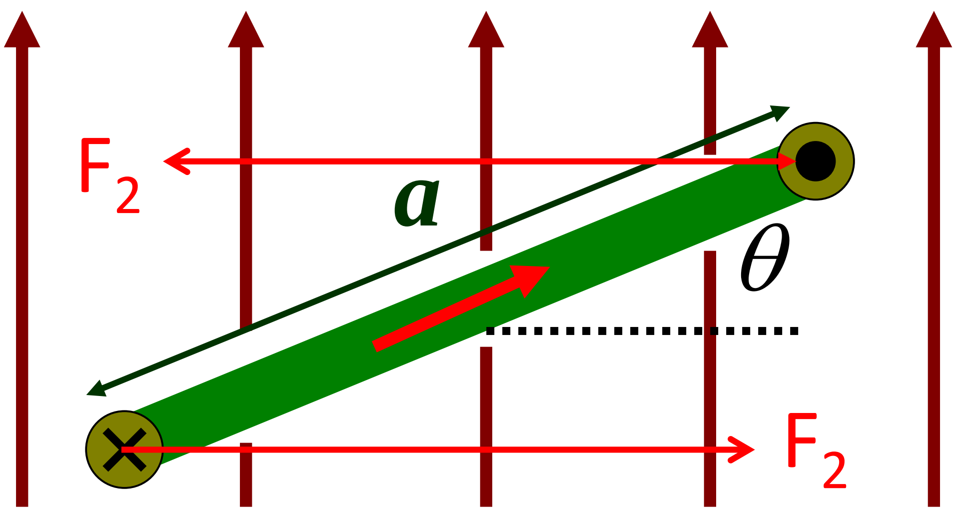
a makes an angle θ to ***x***

Torque on Current Loop





Cross-sectional view



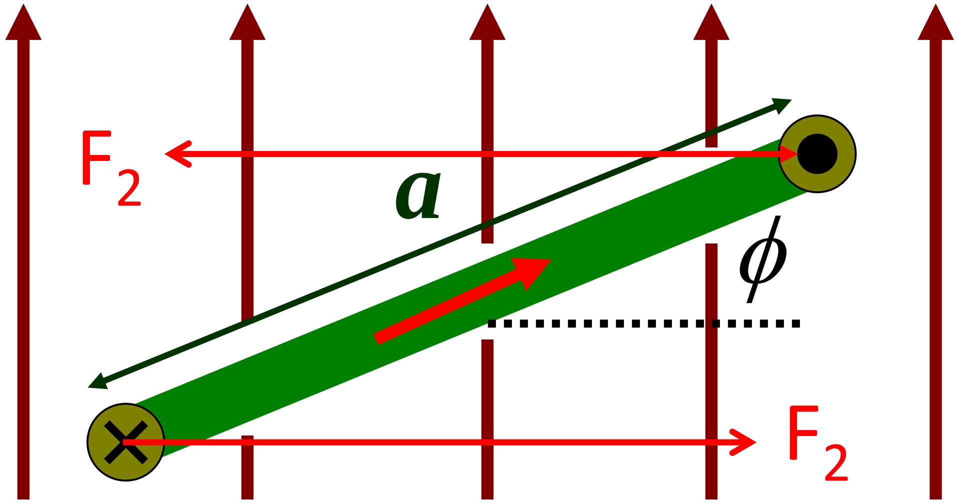
$F_2 = B\ I\ b$

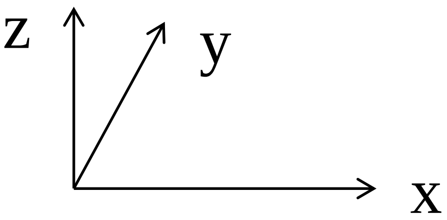
Torques: $\underline{\tau} = \underline{r} \wedge \underline{F}$

$\tau = a\ F_2 \sin\theta$

Torque on Current Loop

Cross-sectional view





$$F_2 = B\, I\, b$$

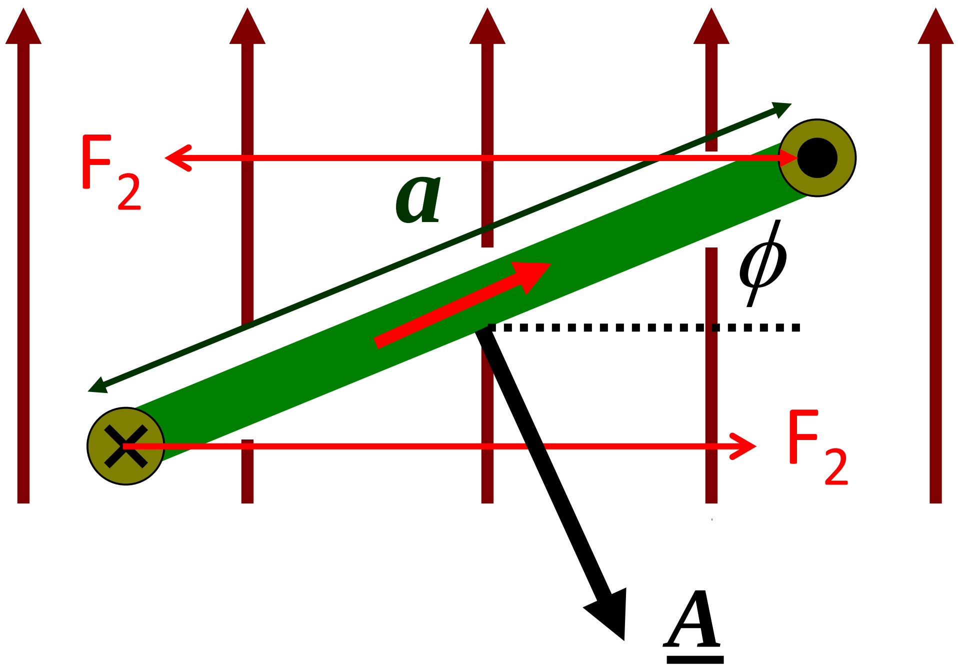
Torques: $\underline{\tau} = \underline{r} \wedge \underline{F}$

$$\tau = a\, F_2 \sin\theta$$

$$\text{So } \tau = a\,(B\, I\, b) \sin\theta = BI\, ab\, \sin\theta = BIA \sin\theta$$

Torque on Current Loop

Cross-sectional view



$$\tau = BIA\sin\theta$$

$$\boxed{\underline{\tau} = \mathrm{I}\,\underline{A}\wedge\underline{B}}$$

Torque on Current Loop

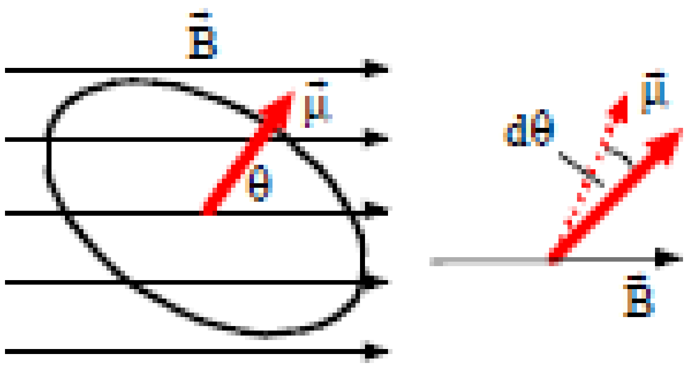
$$\underline{\tau} = \mathrm{I}\, \underline{A} \wedge \underline{B}$$

- True for loops of any shape
- But magnetic dipole moment
- So torque on any current loop

$$\underline{\mu} = I\, \underline{A}$$

$$\boxed{\underline{\tau} = \underline{\mu} \wedge \underline{B}}$$

Potential Energy of Magnetic Dipole in B-field

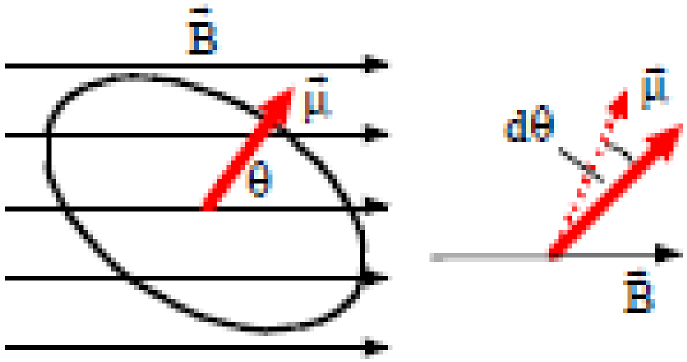


Work done in moving dipole by a small angle, δθ:

$\delta W = \tau\,\delta\theta$

- Torque is in the direction of decreasing θ so:
- $\delta W = -\ \mu B \sin\theta\ \ \delta\theta$
- work done is equal to *decrease* in potential energy: $\delta U = -\ \delta W = \mu B \sin\theta\ \ \delta\theta$

Potential Energy of Magnetic Dipole in B-field



$$\delta U = -\delta W = \mu B \sin\theta \;\delta\theta$$

$$U = \mu B \int \sin\theta \, d\theta$$

$$U = -\mu B \cos\theta + C$$

Define $U = 0$ when $\theta = \pi/2$

- So : $U = -\mu B \cos\theta$ i.e.

$$\boxed{U = -\underline{\mu} \cdot \underline{B}}$$

Comparison between Magnetic & Electric Dipoles

Electric Dipole

$$\underline{\boldsymbol{p}} = q\underline{\boldsymbol{a}}$$

$$\underline{\boldsymbol{\tau}} = \underline{\boldsymbol{p}} \wedge \underline{\boldsymbol{E}}$$

$$U = -\underline{\boldsymbol{p}} \cdot \underline{\boldsymbol{E}}$$

Magnetic Dipole

$$\underline{\boldsymbol{\mu}} = I\underline{\boldsymbol{A}}$$

$$\underline{\boldsymbol{\tau}} = \underline{\boldsymbol{\mu}} \wedge \underline{\boldsymbol{B}}$$

$$U = -\underline{\boldsymbol{\mu}} \cdot \underline{\boldsymbol{B}}$$

Example

- A square coil with sides equal to $20\ cm$ carries a current of $2A$. It lies on the $Z=0$ plane in a B-field = $(0.5\underline{\mathbf{i}} + 0.2\underline{\mathbf{k}})T$ with the current anti-clockwise when viewed from a point on the +ve z-axis. The coil has 5 turns of wire.

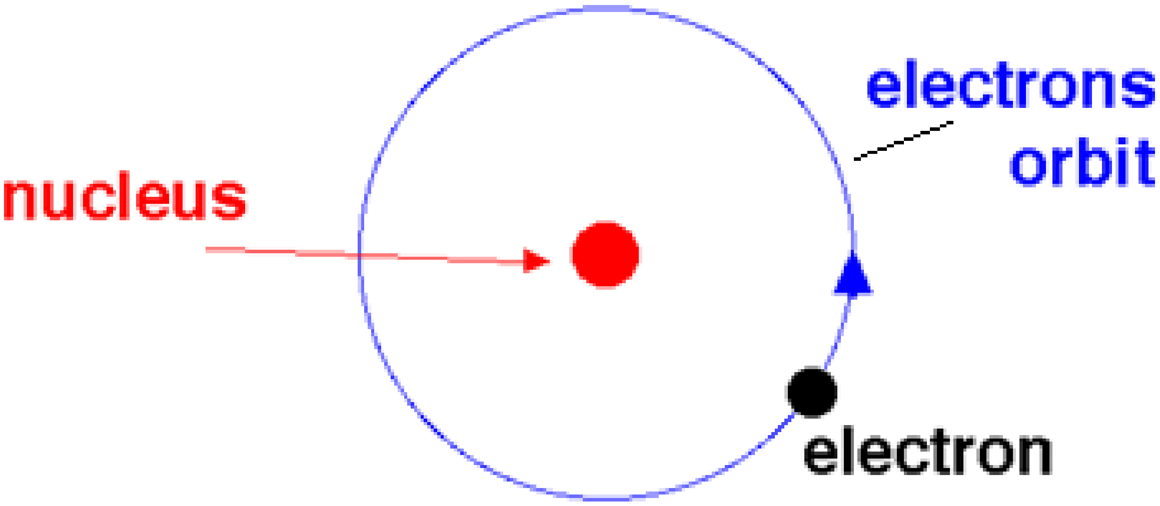
1. What is the magnetic moment of coil?
2. What is the torque
3. What is the potential energy

Time to use the visualizer

Dipoles in Nature

- Molecules behave like electric dipoles
- What in nature behaves like a magnetic dipole?

ATOMS



Summary

- Force on a length l of a current carrying conductor (where $\underline{\boldsymbol{l}}$ is defined to be in the dir^n of current flow)

$$\boxed{\underline{\boldsymbol{F}} = I\,\underline{\boldsymbol{l}} \wedge \underline{\boldsymbol{B}}}$$

- Magnetic Dipole moment: $\boxed{\underline{\mu} = I\,\underline{A}}$

- Torques on magnetic dipole: $\boxed{\underline{\boldsymbol{\tau}} = \underline{\boldsymbol{\mu}} \wedge \underline{\boldsymbol{B}}}$

- Potential energy of magnetic Dipole: $\boxed{U = -\underline{\boldsymbol{\mu}} \cdot \underline{\boldsymbol{B}}}$