3 Thermodynamics and Ideal gases

The problems are roughly in order of difficulty. The ones with ♣ are the hardest ones, which might only occur as a "sting in the tail" at the end of a long examination question.

Problem 3.1 **Mixing water**
100g of water and 100°C is mixed with 400g of water at 20°C. What is the final temperature of the mixture?

Problem 3.2 **Hot electronics**
Some electronic circuitry on a piece of silicon of mass 20 mg produces heat at a rate of 5mW. If there is no means of transporting heat out of the device, at what rate does the temperature of the silicon increase? [The specific heat capacity of silicon is 705 $\mathrm{Jkg^{-1}K^{-1}}$.]

Problem 3.3 **Expansion**
A gas is kept at constant pressure. If its temperature is changed from $50°$C to $100°$C, by what factor does its volume change?

Problem 3.4 **Isothermal expansion**
An ideal gas is initially at temperature $20°$C, pressure 200 kPa and has a volume of 4 litres. It undergoes an isothermal expansion until its pressure is reduced to 100 kPa. Find the work done by the gas and the heat added to the gas during expansion.

Problem 3.5 **Adiabatic expansion**
One mole of an ideal monatomic gas expands adiabatically from a pressure of $p_\mathrm{i} = 10$ atmospheres and a temperature of $T_\mathrm{i} = 0°$C to a pressure of $p_\mathrm{f} = 2$ atmospheres. Find the initial, V_i, and final, V_f, volumes, the final temperature, T_f, and the work done by the gas, W_by.

Problem 3.6 **A simple thermodynamic cycle**
Consider an ideal gas undergoing the following cycle. An isothermal compression, at temperature T_0, is made from $(p_\mathrm{i}, V_\mathrm{i})$ to V_f. Calculate p_f.

At constant volume, V_f, the gas cools with the pressure dropping from p_f to p_i. Finally the gas expands at constant pressure to V_i.

Calculate the work performed on the gas, W_on, and the heat input into the gas, Q_in.

Problem 3.7 **Bow tie cycle**
Consider the bow tie contour in Fig. (3.1).

We work with an ideal gas. The section $(p_1, V_1) \to (p_2, V_2)$ is adiabatic and $(p_3, V_2) \to (p_4, V_1)$ is isothermal, at temperature T_0 and $V_2 = V_1/2$. Note that $\gamma > 1$.

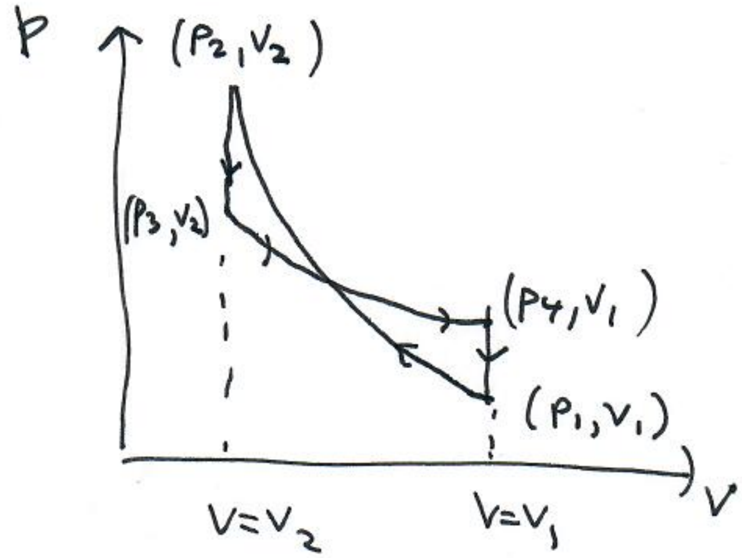

Figure 3.1: The bowtie cycle.

1. Derive p_2 in terms of p_1, n, the number of moles and V_1.
2. Derive p_4 and p_3 In terms of T_0 and V_1.
3. Determine the work done on the adiabatic and isothermal paths.
4. What is the work done on the other two sections?
5. If the whole cycle is to have zero net work, what value must T_0 have?

Problem 3.8 **The atmospheric lapse rate**

The atmosphere does not conduct heat very well – which is why convection often occurs. Can we use this observation to deduce how the temperature drops in the troposphere with altitude, or at least its stability?

If the atmosphere does not conduct well, any process where one displaces some air should be adiabatic.

Start from the First Law in the form:

$$mc_{\rm V}\ {\rm d}T + p\ {\rm d}V = 0$$

where m is the mass of the air being displaced, and $c_{\rm V}$ is the specific heat per unit mass. Assume the change is adiabatic and show that

$$c_{\sf p}\ {\rm d}T - \frac{1}{\rho}{\rm d}p = 0\ ,$$

where ρ is the density.

Write down the expression for the hydrostatic equilibrium we considered when discussing the isothermal atmosphere in differential form. involving p and the height h.

Hence show that the *dry adiabatic lapse rate*, $\Gamma_{\sf d}$, is

$$\Gamma_{\sf d} = \frac{{\rm d}T}{{\rm d}h} = -\frac{g}{c_{\sf p}}\ .$$

Given, for air, $c_\mathsf{p} \simeq 1$kJ kg^{-1}K^{-1}, derive the numerical value of Γ_d. The decrease in temperature is actually 6.5 K km^{-1}. What can you say about stability of such an atmosphere? Provide a physical argument. [Actually the answers are closer if one takes into account moisture in the air.]