2 Elasticity

The problems are roughly in order of difficulty. The ones with ♣ are the hardest ones, which might only occur as a "sting in the tail" at the end of a long examination question.

Problem 2.1 **Stretching**
When a mass of 600 kg is hung from a steel cable (density, $\rho_{\text{steel}} = 8 \times 10^3$ kgm^{-3}) with a cross sectional area of 0.20 cm^2, it stretches 0.05 cm beyond the no-load length of 5 m. Determine the stress, strain and Young's modulus of the steel in the cable.

Problem 2.2 **Lateral contraction**
Poisson's ratio for steel is $\nu_{\text{steel}} = 0.3$. By how much does the steel cable in problem 1 change laterally when it is stretched by the weight?

Problem 2.3 **Stretching and cooling**
Consider the same steel cable as in problem 1. The coefficient of (linear) thermal expansion of steel is $\alpha_{\text{steel}} = 1.3 \times 10^{-5}$ K^{-1}. How much would the cable need to be *cooled* to compensate for the extension due to the weight?

Problem 2.4 **Auxetic materials**
Consider the two-dimensional auxetic material with geometry defined in Fig. (2.1). All joints are freely hinged. As we vary θ, and wish to disallow the structure intersecting itself,

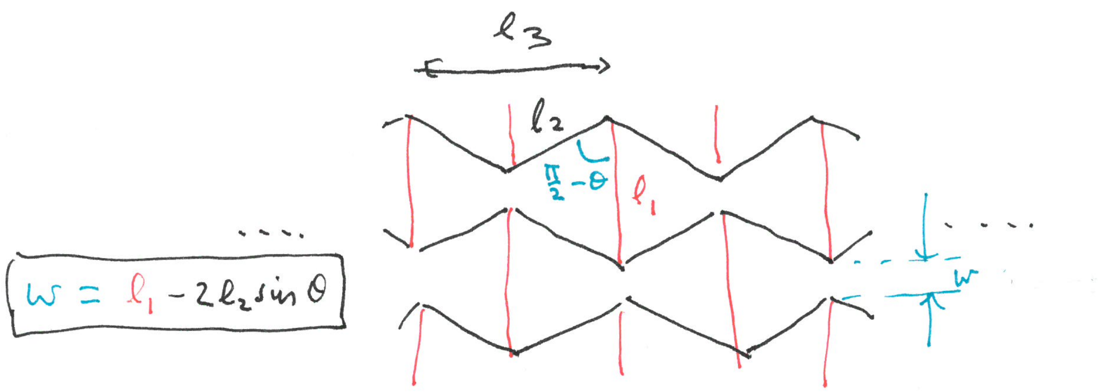

Figure 2.1: The notation for auxetic materials.

what is the inequality which ℓ_1 and ℓ_2 must obey?

What is the Poisson ratio for this material? (Use the same definition as in three-dimensions, i.e. $\nu = \delta w/\delta\ell$, where δw is the change in width, assumed to be a *contraction* in the definition, induced by the change in length $\delta\ell$.

Problem 2.5 **Elastic energy of a compressed rod**
Consider a rod of square cross section with cross sectional area $4t^2$. If the rod has an unstressed, or natural, length L_0, what force is required to compress the rod to $L_0 - d$, if the rod is composed of material with Young's modulus Y? What is the amount of elastic energy stored in the rod when it has been compressed by d?

♣*Problem 2.6* **Elastic energy of a bent rod and the Euler instability**
Consider the same uncompressed rod as in the last problem, but now bend the centre of the rod into an arc of a circle with radius a with angle θ. See Fig. (2.2).

Figure 2.2: The rod of thickness $2t$, bent through an angle θ.

What is the length of the outer side of the rod and the inner side of the rod, assuming the ends of the rod remain flat?

Considering the rod as a set of thin rectangular plates which have been stretched or compressed, determine the elastic energy of the rod.

If one now compares the energy of the compressed rod (in the last problem) with the elastic energy of the bent rod, at what applied force will the initially straight rod buckle as the elastic energy is lower than the compressed case? This is a variant of the *Euler instability*. The assumption of a circular arc is not quite right – so the estimate is a little too high.