UNIVERSITY OF BIRMINGHAM

School of Physics and Astronomy

DEGREE OF B.Sc. & M.Sci. WITH HONOURS

FIRST-YEAR EXAMINATION

03 19750

LC ELECTROMAGNETISM 1 / TEMPERATURE & MATTER / ELECTRIC CIRCUITS

SEMESTER 2 EXAMINATIONS 2021/22

Time Allowed: 3 hours

Answer five questions from Section 1 and three questions from Section 2.

Section 1 counts for 25% of the marks for the examination. Answer ***all five*** questions in this Section.
Section 2 consists of three questions and carries 75% of the marks.
Answer ***all three*** questions in this Section. Note that each question has two parts, of which only ***one part*** should be answered. If you answer both parts, credit will only be given for the best answer.

The approximate allocation of marks to each part of a question is shown in brackets **[ ]**.

PLEASE USE A SEPARATE ANSWER BOOK FOR SECTION 1 AND SECTION 2 QUESTIONS.

Calculators may be used in this examination but must not be used to store text. Calculators with the ability to store text should have their memories deleted prior to the start of the examination.

A formula sheet and a table of physical constants and units that may be required will be found at the end of this question paper.

SECTION 1

*Answer **all five** questions in this Section.*

1. A kettle, of negligible heat capacity, is filled with water at 20°C. If it takes 4 minutes for the water to reach boiling point, how much longer will it take for the kettle to boil dry?

   [Specific heat capacity of water = 4.2 kJ K^{-1} kg^{-1}. Latent heat of vaporisation of water = 2.26 MJ kg^{-1}.] **[5]**

2. A classical system contains particles which can exist in only three states: one, A, with energy $E = 0$, and two states, B and C, with the same energy $E = \epsilon$. The particles do not interact.

   If the system is at temperature T, what is the probability of a particle being observed:

   (i) with $E = 0$? **[2]**

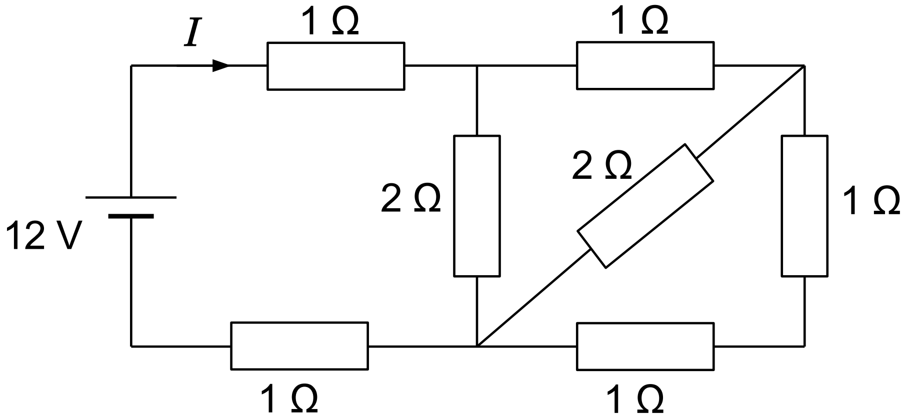
   (ii) with $E = \epsilon$? **[2]**

   (iii) as being in state C? **[1]**

3. Point charges of Q, $-2Q$, and q are fixed at (x, y) coordinates $(0, a)$, $(a, 0)$, and $(a, 2a)$ respectively. Derive the vector expression for the net electric force on the charge q. **[5]**

4. A non-relativistic particle of mass m and charge e is moving with velocity $\mathbf{v}$ in a region of uniform B-field and zero E-field. The direction of the B-field is perpendicular to the velocity of the particle.
   Derive an expression for the radius of the particle's path in terms of its momentum and the B-field. You may ignore gravity. **[5]**

5. Find the current, I, for the circuit shown below. **[5]**



SECTION 2

*Answer **all three** questions in this Section. Note that each question has two parts, of which only **one part** should be answered. If you answer both parts, credit will only be given for the best answer.*

6. **EITHER Part A**

A hypothetical diatomic molecule may be described by a potential :

$$V_0(r) = \frac{L^2}{r^2} - \frac{Z^2}{r} \, ,$$

where r is the inter-atomic separation, with L and Z constants.

(i) Sketch the potential $V_0(r)$, and indicate notable features. **[5]**

(ii) Find the equilibrium separation of the atoms, r_0, and the binding energy, ϵ_0. **[6]**

Now consider the molecule experiencing in addition a potential $V_1(r)$, such that the total potential is

$$V_{\text{tot}}(r) = V_0(r) + V_1(r) \quad \text{where} \quad V_1(r) = -\mathcal{E} r \, ,$$

and $\mathcal{E} > 0$ is a constant.

(iii) Sketch $V_{\text{tot}}(r)$ in the limits: (I) small $\mathcal{E}$; (II) large $\mathcal{E}$. **[4]**

(iv) Find the value of $\mathcal{E}$, $\mathcal{E}_{\text{c}}(L, Z)$, where for $\mathcal{E} \geq \mathcal{E}_{\text{c}}$, there is no minimum of $V_{\text{tot}}(r)$ for $r < \infty$. **[6]**

(v) Briefly describe a physical situation where $V_{\text{tot}}(r)$ might be appropriate. **[4]**

OR Part B

Consider n moles of an ideal gas initially at pressure p_1 and with volume V_1. All stages in the following cycle are performed sufficiently slowly that the gas remains in equilibrium. The ratio of the specific heats, C_p/C_V, is γ.

Compress the gas *adiabatically* until the volume is halved.

(i) What is the final pressure, p_2, and temperature T_2? **[3]**

Now, reduce the pressure to $p_3 < p_2$ at constant volume. Then expand the gas adiabatically to the original volume, at pressure p_4. Finally, increase the pressure, at constant volume, to p_1.

(ii) What is the pressure, p_4, in terms of p_3? **[3]**

(iii) What is the total work, $W_{\text{on}}^{\text{tot}}$, performed on the gas in the cycle? **[5]**

(iv) What is the total heat extracted from the gas, $Q_{\text{out}}^{\text{tot}}$? **[3]**

(v) What is the value of $W_{\text{on}}^{\text{tot}}$ if $p_2 - p_3 \to 0$ and $p_1 - p_4 \to 0$? **[1]**

Consider a monatomic ideal gas in a cylindrical volume, subdivided into two cylindrical compartments by a circular partition of cross sectional area A and mass M, which moves without friction. No heat or particle transfer occurs between the two compartments, or with the cylinder's exterior.

Initially, the partition divides the cylinder into compartments of equal volume, V_0, each containing an identical number of moles, n.

(vi) What is the condition on the pressures in the compartments, p_1 and p_2, to ensure mechanical equilibrium of the partition? **[1]**

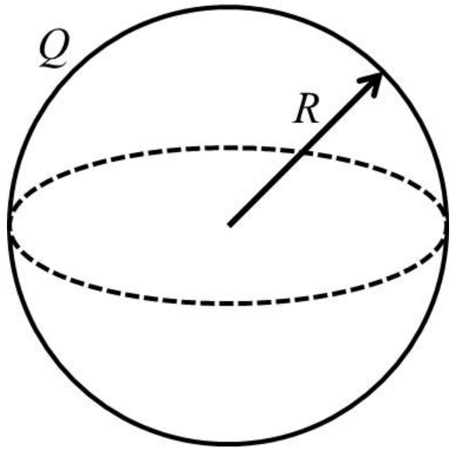
The partition is displaced adiabatically by a small amount, x_0 ($x_0 A \ll V_0$), along the axis of the cylinder and then released.

(vii) Assuming the subsequent motion of the partition is sufficiently slow for the gas in both compartments to be always in equilibrium, determine the equation of motion for the partition. (Gravity may be neglected.) **[7]**

(viii) Write down, and motivate, any criteria for the motion to be "sufficiently slow". **[2]**

7. **EITHER Part A**

(a) A spherical shell, of radius R, has a total charge Q distributed uniformly over its surface, as shown in the diagram below.



Derive expressions for the magnitude of the electric field inside and outside of the sphere. Hence derive expressions for the electric potential, as a function of the distance from the centre, r, inside and outside of the sphere, assuming the potential tends to zero as the distance tends to infinity. **[6]**

Derive expressions for the capacitance of this sphere and the energy stored. **[4]**

(b) A point charge, q, is located at a fixed distance a from the centre of the spherical shell, where $a > R$. The point charge does not affect the charge distribution on the sphere. What is the magnitude of the electric field at the centre of the sphere now? **[2]**

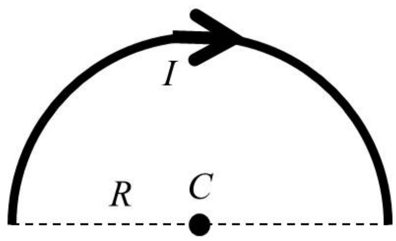
What is the magnitude of the force on the point charge? **[4]**

If the shell shrinks to half its radius, without losing any charge, what happens to the force on the point charge? Explain your reasoning. **[2]**

(c) In part (b), work must be done in order to shrink the spherical shell. The work done is balanced by a change in energy stored in the electric field. Using the general expression for the energy density, u_E, stored in an electric field, $\mathbf{E}$, derive an expression for the change in energy in the electric field when the sphere shrinks to half its radius. **[7]**

OR Part B

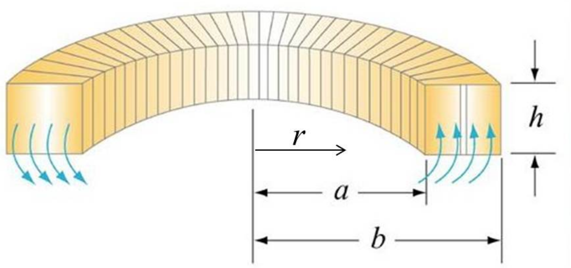
(a) The figure below shows a wire in the shape of a semi-circle with centre C and radius R. The wire carries a current I.



Derive an expression for the magnetic field at point C and state its direction.

[6]

(b) A toroid magnet, with a square cross-section, has an inner radius of a, an outer radius of $b = a+h$, and a total of N square windings, as shown below. (You may assume the relative permeability, $\mu_r = 1$ everywhere.)



Use Ampere's law to derive expressions for the magnetic field at a distance r from the centre of the toroid for
(i) $r < a$,
(ii) $a < r < b$, and
(iii) $r > b$
when a current I flows through the toroid. **[9]**

(c) Derive an expression for the total energy stored in the magnetic field of the toroid. **[10]**

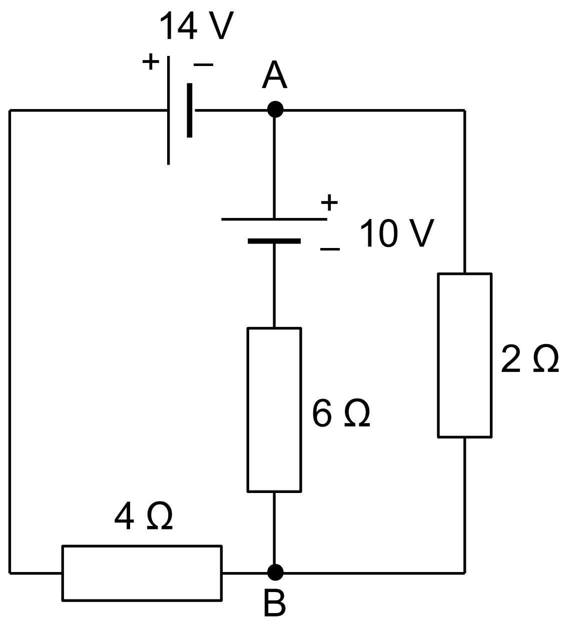
8. **EITHER Part A**

(a) In the circuit shown below, use the superposition theorem to find:

i. the current in each resistor; **[13]**

ii. the potential difference between points A and B; **[2]**

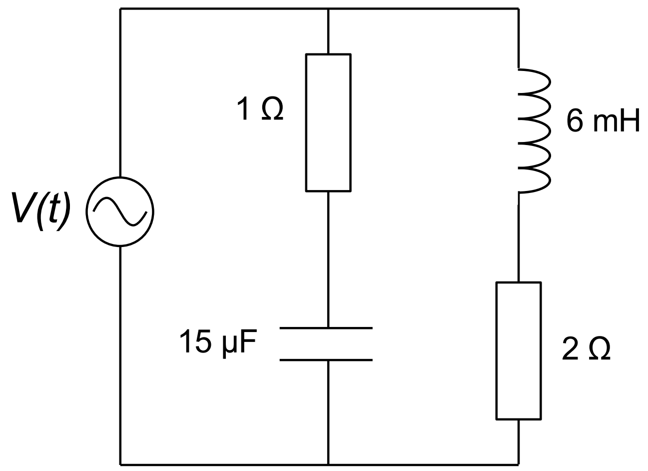
iii. the power supplied by each battery. **[2]**



(b) State Kirchhoff's laws and the conserved quantity from which each law is derived.
For the same circuit as shown in part (a), use Kichhoff's laws to find the current in each resistor. **[8]**

OR Part B

(a) In the circuit shown below, the voltage source has a peak amplitude of V_0=20 V and an angular frequency ω = 820.5 rad/s. Calculate all your answers at the frequency of the voltage source and to two decimal places.



i. Calculate the reactance of both the capacitor and the inductor. **[2]**

ii. Using complex notation, express the impedance of each branch of the circuit in the form $z = a + jb$, calculating the values for a and b. **[2]**

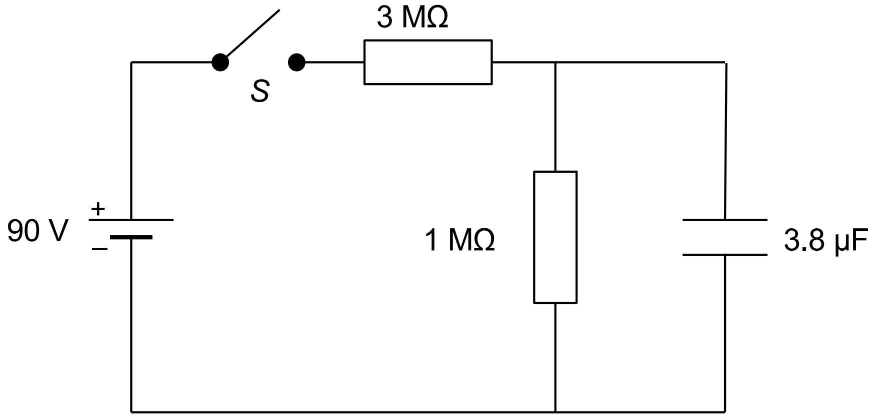
iii. Hence find the magnitude of the impedance in each branch and the phase relationship between the applied voltage and the current in each case. **[5]**

iv. Find a complex expression for the current in each branch of the circuit $V(t = 0) = 20\ V$. **[2]**

v. Hence find an expression for the total complex current. **[2]**

Continued over page...

(b) For the circuit shown below,



answer the following questions.

i. What is the initial battery current immediately after switch, *S*, is closed? **[2]**

ii. What is the battery current a long time after switch, *S*, is closed? **[2]**

iii. What is the maximum voltage across the capacitor? **[2]**

iv. If the switch has been closed for a long time and is then opened, deduce an expression for the current through the 1 MΩ resistor as a function of time. **[4]**

v. What is the total energy dissipated in the 1 MΩ resistor after the switch is opened as in part (iv)? **[2]**

Formula Sheet
LC Electromagnetism 1 / Temperature & Matter / Electric Circuits

Useful Formulae for Electromagnetism 1

Force between two charges
$$\mathbf{F} = \frac{q_1 q_2}{4\pi\epsilon_0 r^2}\hat{\mathbf{r}}_{\mathbf{12}}$$

Coulomb's Law
$$\mathbf{E} = \frac{Q}{4\pi\epsilon_0 r^2}\hat{\mathbf{r}}$$

Coulomb potential
$$V = \frac{Q}{4\pi\epsilon_0 r}$$

Gauss' Law
$$\int_S \mathbf{E} \cdot d\mathbf{S} = \frac{Q_{enc}}{\epsilon_0}$$

Field and potential relation
$$\mathbf{E} = -\nabla V$$

Electric Dipole
$$\mathbf{p} = q\mathbf{a}$$

Torque on Electric Dipole
$$\boldsymbol{\tau} = \mathbf{p} \wedge \mathbf{E}$$

Energy of Electric Dipole
$$U = -\mathbf{p} \cdot \mathbf{E}$$

Capacitance
$$C = \frac{Q}{V}$$

Stored energy in capacitor
$$U = \frac{1}{2}CV^2$$

Energy density of E-field
$$u_E = \frac{1}{2}\epsilon_0 E^2$$

Lorentz Force
$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \wedge \mathbf{B}\right)$$

Force on current length
$$\mathbf{F} = I\boldsymbol{\ell} \wedge \mathbf{B}$$

Magnetic dipole
$$\boldsymbol{\mu} = I\mathbf{A}$$

Torque on magnetic dipole
$$\boldsymbol{\tau} = \boldsymbol{\mu} \wedge \mathbf{B}$$

Energy of magnetic dipole
$$U = -\boldsymbol{\mu} \cdot \mathbf{B}$$

Biot Savart law
$$\delta\mathbf{B} = \frac{\mu_0 I}{4\pi}\frac{\delta\mathbf{l} \wedge \hat{\mathbf{r}}}{r^2}$$

Ampere's Law
$$\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I$$

E.M.F
$$\epsilon = -N\frac{d\Phi_m}{dt}$$

Faraday's Law
$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_m}{dt}$$

Inductance
$$N\Phi_m = LI$$

Stored energy in inductor
$$U_L = \frac{1}{2}LI^2$$

Energy denisty of B-field
$$u_B = \frac{1}{2}\frac{B^2}{\mu_0}$$

Formula Sheet
LC Electromagnetism 1 / Temperature & Matter / Electric Circuits

Useful Formulae for Temperature & Matter

The first law of thermodynamics

$$\mathrm{d}U = đQ - p\,\mathrm{d}V$$
$$\Delta U = Q_{\mathsf{in}} + W_{\mathsf{on}}$$

Ideal gas equation of state

$$pV = nRT$$

Ideal gas adiabatic process

$$pV^{\gamma} = \text{constant}\,,\ \text{where}\ \gamma = C_p/C_V\ .$$

Gamma function and Stirling's approximation

$$\Gamma(N+1) = N! = \int_0^{\infty} \mathrm{d}x\, x^N\, \mathrm{e}^{-x} \qquad \text{and} \quad N! \approx \left(\frac{N}{\mathrm{e}}\right)^N \Leftrightarrow\ \ln N! \approx N(\ln N - 1)\,.$$

Gaussian integral

$$\int_{-\infty}^{\infty} \mathrm{e}^{-\alpha x^2} \mathrm{d}x = \sqrt{\frac{\pi}{\alpha}}\ .$$

Normalised one-dimensional Maxwell-Boltzmann distribution

$$p_{\mathsf{1d}}(v) = \sqrt{\frac{m}{2\pi k_{\mathsf{B}} T}}\, \exp\left(-\frac{mv^2}{2k_{\mathsf{B}}T}\right)\ .$$

Physical Constants and Units

Acceleration due to gravity	g	$9.81\,\mathrm{m\,s^{-2}}$
Gravitational constant	G	$6.674 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$
Ice point	T_{ice}	$273.15\,\mathrm{K}$
Avogadro constant	N_A	$6.022 \times 10^{23}\,\mathrm{mol^{-1}}$
		[*N.B.* 1 mole $\equiv 1$ *gram-molecule*]
Gas constant	R	$8.314\,\mathrm{J\,K^{-1}\,mol^{-1}}$
Boltzmann constant	k, k_B	$1.381 \times 10^{-23}\,\mathrm{J\,K^{-1}} \equiv 8.62 \times 10^{-5}\,\mathrm{eV\,K^{-1}}$
Stefan constant	σ	$5.670 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$
Rydberg constant	R_∞	$1.097 \times 10^{7}\,\mathrm{m^{-1}}$
	$R_\infty hc$	$13.606\,\mathrm{eV}$
Planck constant	h	$6.626 \times 10^{-34}\,\mathrm{J\,s} \equiv 4.136 \times 10^{-15}\,\mathrm{eV\,s}$
$h/2\pi$	$\hbar$	$1.055 \times 10^{-34}\,\mathrm{J\,s} \equiv 6.582 \times 10^{-16}\,\mathrm{eV\,s}$
Speed of light *in vacuo*	c	$2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$
	$\hbar c$	$197.3\,\mathrm{MeV\,fm}$
Charge of proton	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Mass of electron	m_e	$9.109 \times 10^{-31}\,\mathrm{kg}$
Rest energy of electron		$0.511\,\mathrm{MeV}$
Mass of proton	m_p	$1.673 \times 10^{-27}\,\mathrm{kg}$
Rest energy of proton		$938.3\,\mathrm{MeV}$
One atomic mass unit	u	$1.66 \times 10^{-27}\,\mathrm{kg}$
Atomic mass unit energy equivalent		$931.5\,\mathrm{MeV}$
Electric constant	ϵ_0	$8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Magnetic constant	μ_0	$4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$
Bohr magneton	μ_B	$9.274 \times 10^{-24}\,\mathrm{A\,m^2}$ ($\mathrm{J\,T^{-1}}$)
Nuclear magneton	μ_N	$5.051 \times 10^{-27}\,\mathrm{A\,m^2}$ ($\mathrm{J\,T^{-1}}$)
Fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	$7.297 \times 10^{-3} = 1/137.0$
Compton wavelength of electron	$\lambda_c = h/m_e c$	$2.426 \times 10^{-12}\,\mathrm{m}$
Bohr radius	a_0	$5.2918 \times 10^{-11}\,\mathrm{m}$
angstrom	Å	$10^{-10}\,\mathrm{m}$
barn	b	$10^{-28}\,\mathrm{m^2}$
torr (mm Hg at 0 °C)	torr	$133.32\,\mathrm{Pa}$ ($\mathrm{N\,m^{-2}}$)

A22259 LC ELECTROMAGNETISM 1 / TEMPERATURE & MATTER / ELECTRIC CIRCUITS

Do not complete the attendance slip, fill in the front of the answer book or turn over the question paper until you are told to do so.

Important Reminders

- Coats/outwear should be placed in the designated area.
- Unauthorised materials (e.g. notes or Tippex) must be placed in the designated area.
- Check that you do not have any unauthorised materials with you (e.g. in your pockets, pencil case).
- Mobile phones and smart watches **must** be switched off and placed in the designated area or under your desk. They must not be left on your person or in your pockets.
- You are not permitted to use a mobile phone as a clock. If you have difficulty seeing a clock, please alert an Invigilator.
- You are not permitted to have writing on your hand, arm or other body part.
- Check that you do not have writing on your hand, arm or other body part – if you do, you must inform an Invigilator immediately
- Alert an Invigilator immediately if you find any unauthorised item upon you during the examination.

Any students found with non-permitted items upon their person during the examination, or who fail to comply with Examination rules may be subject to Student Conduct procedures.