Example Sheet 1: Classical Dynamics

1. An equation of motion is

$$\frac{d^2x}{dt^2} + x + x^3 = 0$$

find a fundamental representation to describe this system. Can you find a conserved energy?

2. An equation of motion is

$$\frac{d^2x}{dt^2} + \nu x^2 \frac{dx}{dt} + x = 0$$

find a fundamental representation to describe this system. Can you find a conserved energy? Can you find the attractor?

3. An equation of motion is

$$\frac{d^3x}{dt^3} + \nu \frac{d^2x}{dt^2} + \frac{dx}{dt} = 0$$

find a fundamental equation to describe the system. Solve for the general motion and deduce the attractor.

4. An equation of motion is

$$\frac{d^2x}{dt^2} + \nu \frac{dx}{dt} + x^3 - x = 0$$

find a fundamental representation to describe this system. Can you find the attractor?

5. The Van der Pol oscillator has the equation of motion

$$\frac{d^2x}{dt^2} + \lambda(x^2 - 1)\frac{dx}{dt} + x = 0$$

Employ the velocity as a dynamical variable to find a fundamental equation. Now employ

$$P = \frac{x^3}{3} - x + \frac{1}{\lambda}\frac{dx}{dt}$$

as a dynamical variable to find a fundamental representation. Explain why the first representation is useful when λ is small and the second representation is useful when λ is large.

6. A one-dimensional particle moves in a potential

$$V(x) = -\frac{1}{2}kx^2 + \frac{1}{4}Kx^4$$

Find the equation of motion and deduce a representation of the fundamental equation. Deduce the energy and create a phase space portrait for this system. Find the separatrices. What are the possible long-time limits?

7. A one-dimensional particle moves in a potential

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Find the equation of motion and deduce a representation of the fundamental equation. Deduce the energy and create a phase space portrait for this system. Find the separatrices. What are the possible long-time limits?

8. A sprung pendulum has l=0 and a spring constant k, find the equations of motion using Cartesian coordinates. Find the energy and demonstrate that there is a second conservation law. Solve the system completely and find the Poincare section associated with x=0. A second spring is added which also has l=0 but a spring constant K which acts only along the x-axis. Find the Poincare section associated with x=0.

9. A bob of mass m is suspended from two springs, with l=0 and spring constants k_1 and k_2 respectively, with fixed points on the same level separated by distance d. Show that the system is integrable. Find the equilibrium position and an appropriate Poincare section.

10. A pendulum of length a and mass m pivots about one end but has a spring of natural length l=0 and spring constant k attached to the other end. This spring is also attached to a second pivot at a horizontal distance d from the first pivot. Show that the system is equivalent to a single pendulum with parameters that you should find.

11. Consider a square billiard with specular reflection. Show that the system is integrable. Find a general trajectory using the idea of reflection and images. Use the points of reflection to set up a collection of Poincare sections. Find the low order n-cycles and establish when the system is ergodic on the Poincare sections.