Electromagnetism

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Lecture 11

Currents & Magnetic Force

Week 6

Last Lecture

- Potential energy of capacitor $U = \frac{1}{2} C V^2$
- Energy density of Electric field $u_E = \frac{1}{2} \varepsilon_0 E^2$
- **Dielectrics**
  - Polarisation, $\underline{\boldsymbol{P}} = N\underline{\boldsymbol{p}} = Nq\underline{\boldsymbol{a}}$
  - Electric susceptibility χ_E: $\underline{\boldsymbol{P}} = \chi_{\boldsymbol{E}} \boldsymbol{\varepsilon_0 E}$
  - Relativity permittivity $\varepsilon_r = 1 + \chi_{\boldsymbol{E}}$
  - Gauss's Law for dielectrics: $\int_S \underline{E} \cdot d\underline{S} = \frac{Q_{enc}}{\varepsilon_r \varepsilon_0}$

This Lecture

We start Part II – Magnetism

- Definition of Current
- Current Density
- Magnetic force on a moving charge
- The Lorentz Force
- Magnetic field lines

Greek Alphabet

	Alpha	beta	gamma	delta	epsilon	zeta	eta
Lower case	α	β	γ	δ	ε	ζ	η
Upper case	Α	Β	Γ	Δ	Ε	Ζ	Η

	theta	iota	kappa	lambda	mu	nu	xi	omicron	pi
	θ	ι	κ	λ	μ	ν	ξ	ο	π
	Θ	Ι	Κ	Λ	Μ	Ν	Ξ	Ο	Π

rho	Sigma	tau	upsilon	phi	chi	psi	omega
ρ	σ	τ	υ	ϕ	χ	ψ	ω
Ρ	Σ	Τ	Υ	Φ	Χ	Ψ	Ω

Definition of a Current

- Suppose a conductor carries a current, I
- Rate of flow of charge Q past a given cross-section is defined by:

$$I = \frac{dQ}{dt}$$

Andre Ampere

The SI unit of current is the *ampere*: one ampere is defined to be *coulomb per second*.

André Marie Ampére (1775-1836)

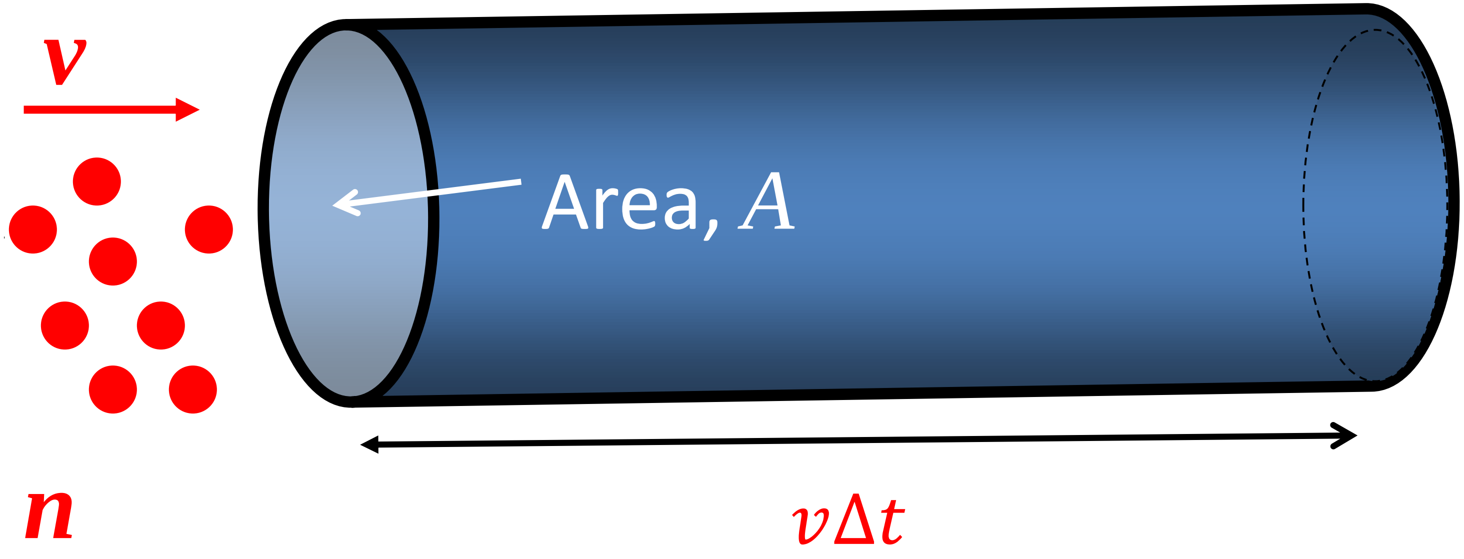
Nature of Current

- Experimentally, recognizable effects of current flow are:

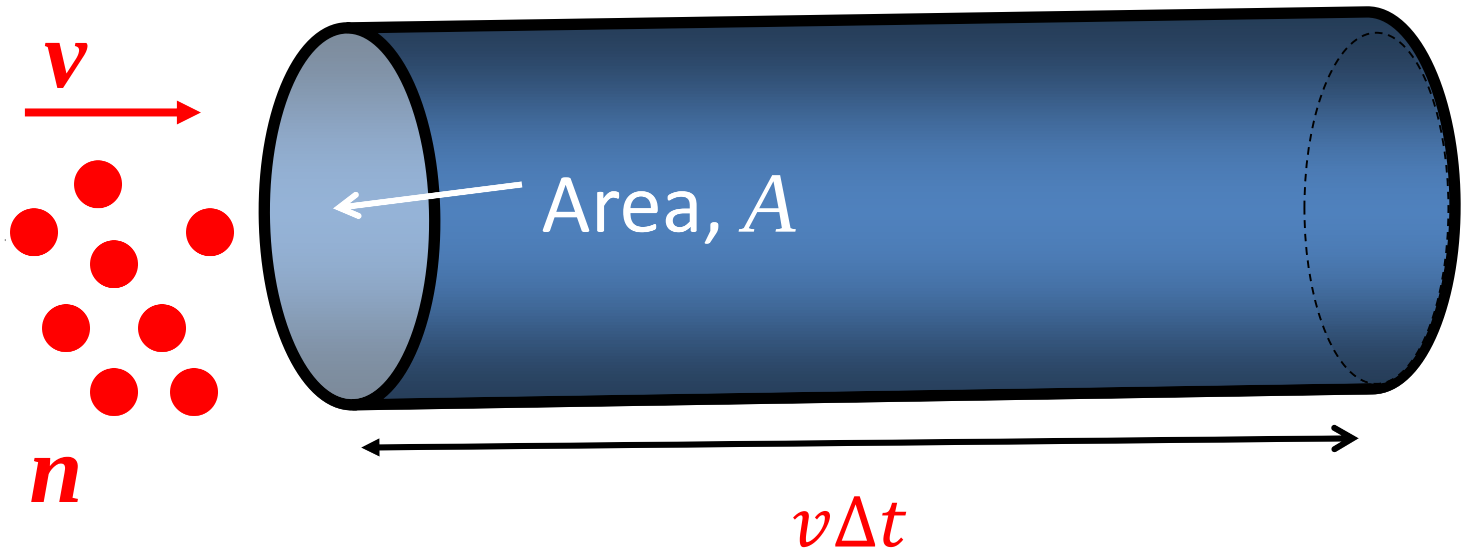
1. Heating
2. Magnetic fields

Current in a Conductor

- Consider a current flowing through a conductor: current is caused by electrons moving with average drift velocity (due to applied E-field), v
- In time Δt the volume swept out is $Av\Delta t$



Current in a Conductor



If there are n conducting electrons per unit volume: the total charge in this volume is: $\Delta Q = n(Av\Delta t)(-e)$

Therefore:

$$\underline{\boldsymbol{I}} = \frac{\Delta Q}{\Delta t} = -nAe\underline{\boldsymbol{v}}$$

Negative charge

Current Density

- The current per unit cross-section (area) is called the current density, J

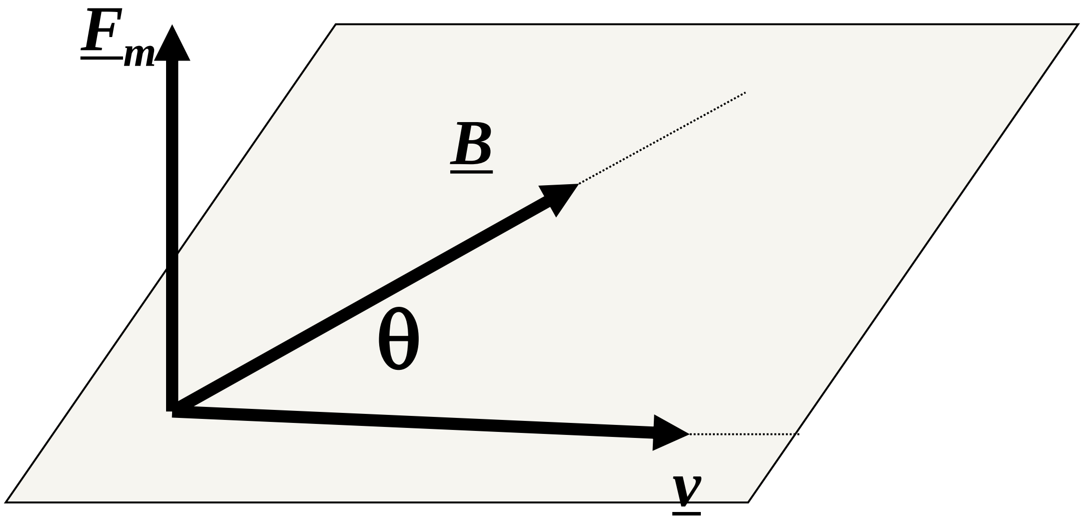
$$\underline{\boldsymbol{J}} = \frac{\underline{\boldsymbol{I}}}{A} = -ne\underline{\boldsymbol{v}}$$

Typically, the net drift velocity of conducting electrons when a current is flowing $v < 1\ mm/s$

Magnetic Force on Moving Charge

- A magnetic field exerts a force $\underline{\boldsymbol{F}}_m$ on any moving charge (or current) that is present in the field.
- A particle of charge $+q$ moving with velocity $\underline{\boldsymbol{v}}$ in a magnetic field $\underline{\boldsymbol{B}}$ experiences a force $\underline{\boldsymbol{F}}_m$.
- Experimentally:
  - $\underline{\boldsymbol{F}}_m$ is $\perp$ $\underline{\boldsymbol{v}}$ and $\underline{\boldsymbol{B}}$,
  - $\underline{\boldsymbol{F}}_m \propto \underline{\boldsymbol{v}}$, $\underline{\boldsymbol{F}}_m \propto \underline{\boldsymbol{B}}$,
  - $\underline{\boldsymbol{F}}_m \propto q$

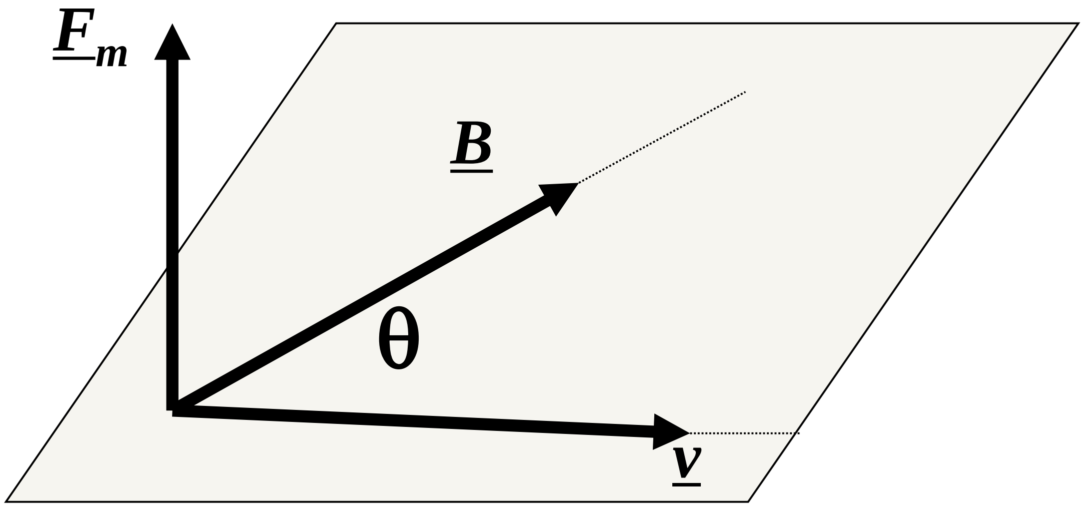
$$F_m = Bqv\sin\theta$$



Magnetic Force on Moving Charge

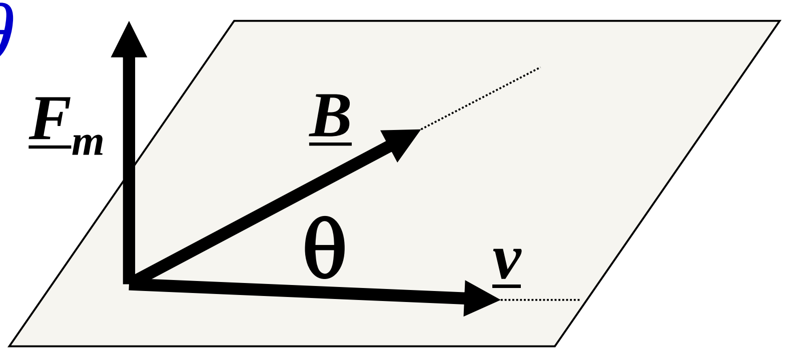
In vector form

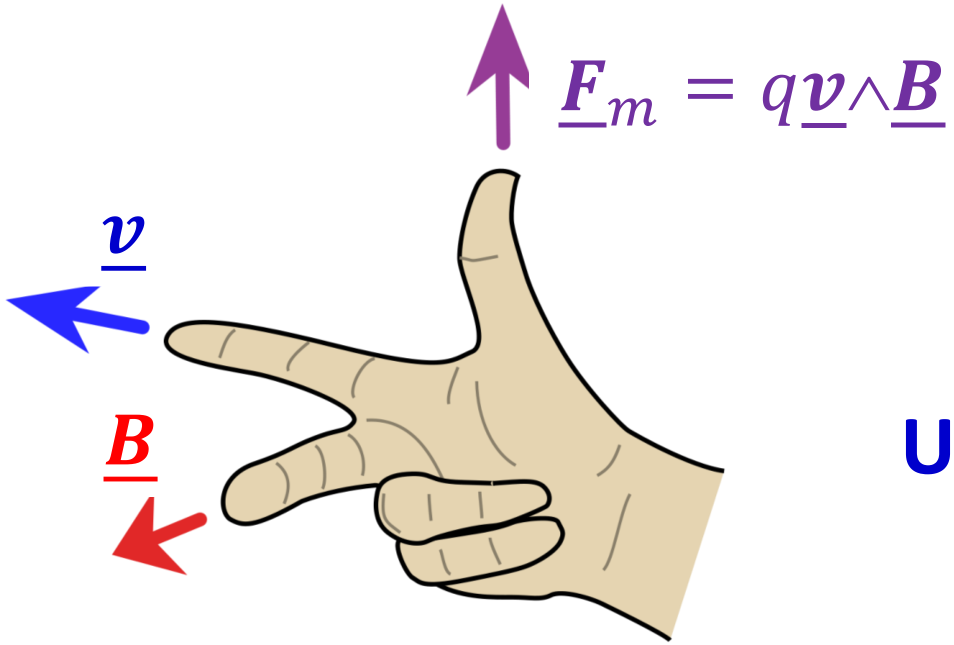
$$\boxed{\underline{\boldsymbol{F}}_m = q\underline{\boldsymbol{v}}\wedge\underline{\boldsymbol{B}}}$$



Direction of Magnetic Force

- $\underline{\boldsymbol{F}}_m = q\underline{\boldsymbol{v}} \wedge \underline{\boldsymbol{B}} = \underline{\hat{\boldsymbol{F}}}\, qvB \sin\theta$
- The direction of $\underline{\boldsymbol{F}}_{\boldsymbol{m}}$ is:





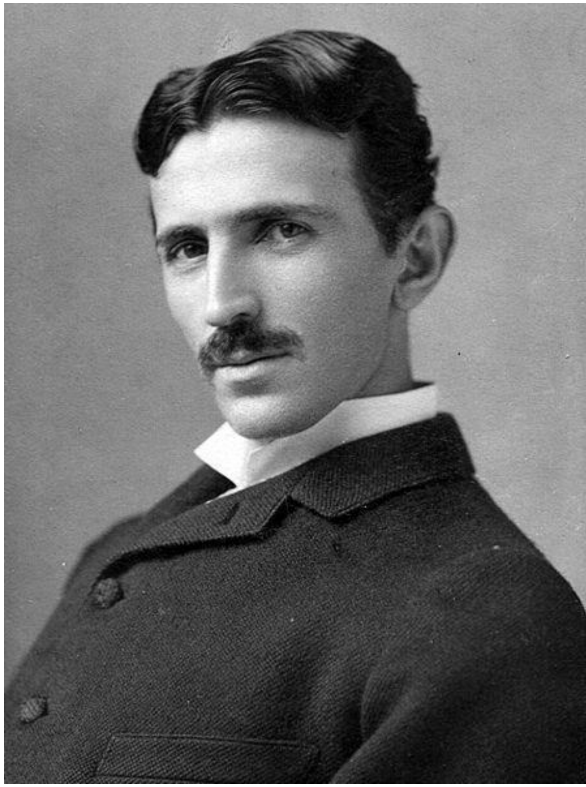
Use Right-Hand Rule

Tesla

- $\underline{\boldsymbol{F}}_m = q\underline{\boldsymbol{v}}\wedge\underline{\boldsymbol{B}}$

  N C ms^{-1}

- The unit of $\underline{\boldsymbol{B}}$ is $\mathrm{N\ C^{-1}\ m^{-1}\ s}$
- It’s given the name **Tesla (T)**
- in honour of Nikola Tesla (1856 – 1943)



- The unit of Gauss (1G = 10^{-4} T) is sometimes used but the S.I unit is Tesla.

Tesla

- If 1 C of charge moving at 1 m/s perpendicular to a magnetic field experiences a force of 1 Newton, the magnetic field is 1 Tesla.
- **Earth's magnetic field $\sim 5 \times 10^{-5}$ T (~ 0.5 Gauss)**
- **Poles of a large electromagnet ~ 2 T**
- **Surface of a neutron star $\sim 10^{8}$ T**
- **The magnetic field that can be created in Lab ~ 50 T.**

Earth's Magnetic Field

- The origin of the Earth's magnetic field is believed to be generated by electric currents in the conductive iron alloys of its core, created by convection currents due to heat escaping from the core although not completely understood yet.
- The North and South poles swap places on average every ~ 300,000 years.
- **The last time the poles swap was 780,000 years ago !**
  - **It take ~ 7000 years to switch**

Lorentz Force

- In regions where both $\underline{E}$ and $\underline{B}$ fields are present, the total force is the *vector sum* of the electric and magnetic forces:

$$\boxed{\underline{\boldsymbol{F}} = q(\underline{\boldsymbol{E}} + \underline{\boldsymbol{v}} \wedge \underline{\boldsymbol{B}})}$$

In direction of $\underline{\boldsymbol{E}}$

$\perp$ to $\underline{\boldsymbol{v}}$ and $\underline{\boldsymbol{B}}$

Lorentz Equation (you need to know this)

Example 11.1

- The Earth's magnetic field at a particular region is represented by
- $\underline{\boldsymbol{B}} = B\cos 70^{o}\underline{\boldsymbol{j}} - B\sin 70^{o}\underline{\boldsymbol{k}}$ (B = 5 x 10^{-5} T)
- A proton is moving in this magnetic field with a velocity
- $\underline{\boldsymbol{v}} = 10^{7}\,\underline{\boldsymbol{j}}\ m/s$
- **Obtain an expression for the direction and magnitude of magnetic force acting on the proton.**
- **Visualizer Time**

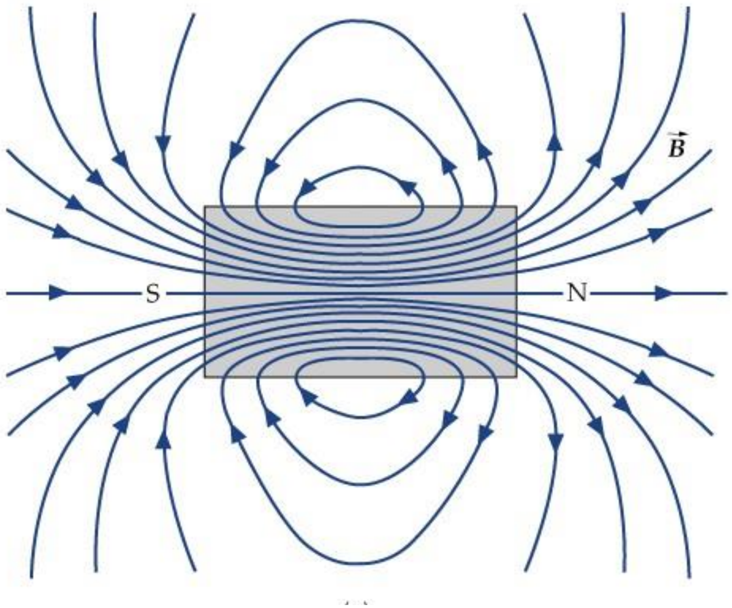
Exercise 11.2

- **True or False: The magnetic force does not accelerate a moving charged particle because the force is perpendicular to the velocity of the particle.**
- **FALSE (it does but not in the direction of travel)**
- **Ex 11.2:** What is the force acting on an electron with velocity $\underline{\boldsymbol{v}} = \left(2\underline{\boldsymbol{i}} - 3\underline{\boldsymbol{j}}\right) \times 10^6\ m/s$
- In a magnetic field:

$\underline{\boldsymbol{B}} = \left(0.8\underline{\boldsymbol{i}} + 0.6\underline{\boldsymbol{j}} - 0.4\underline{\boldsymbol{k}}\right) T$?

Let's do it on the visualizer

Magnetic Field Lines

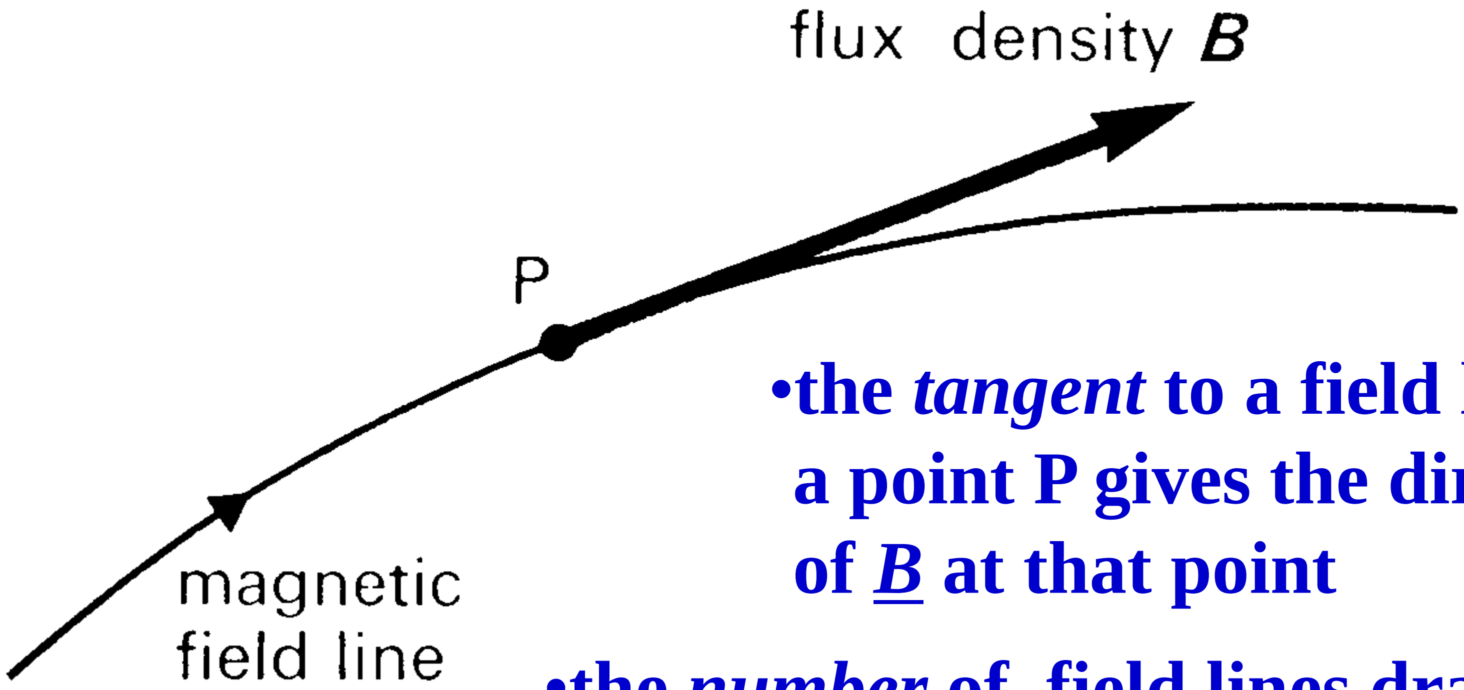


Current upper limit on magnetic monopoles per nucleon is < 10^{-29}

(~4×10^{28} nucleons in your body)

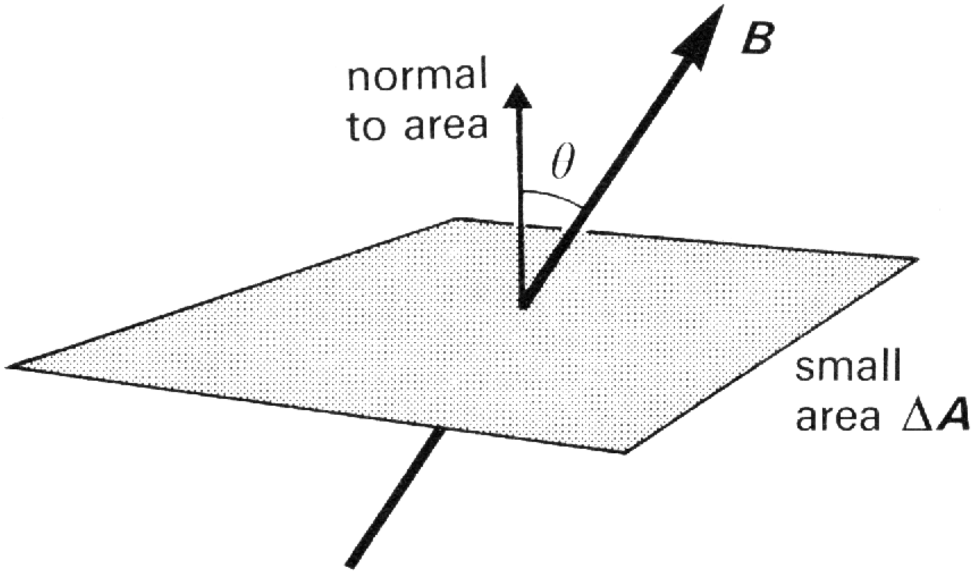
NOTE: unlike electric field lines, magnetic field lines are ALWAYS continuous (**no magnetic monopoles**) and they do not point in the direction of the force on the moving charge in a magnetic field.

Magnetic Field Lines



- **the *tangent* to a field line at a point P gives the direction of <u>*B*</u> at that point**
- **the *number* of field lines drawn per unit cross sectional area is proportional to the magnitude of <u>*B*</u>**

Magnetic Flux ϕ_B



The magnetic flux $\Delta\phi_B$ passing through the small area ΔA shown is defined by:

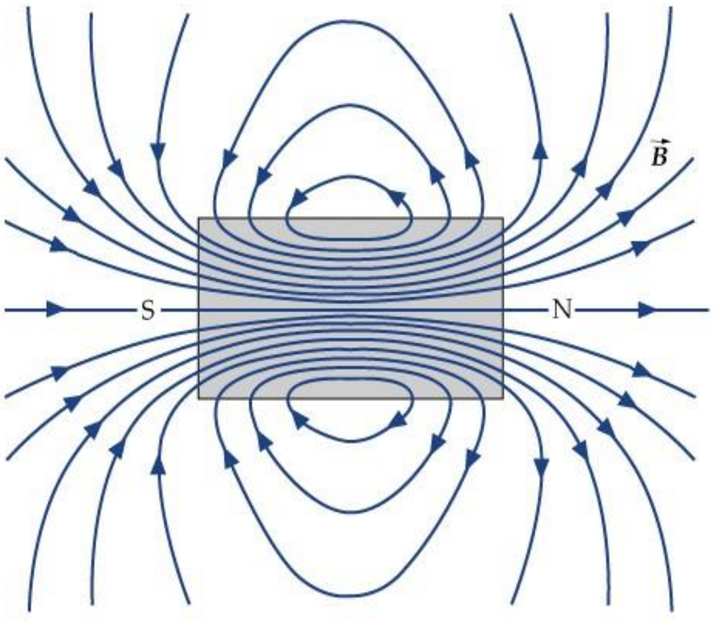
$$\Delta\emptyset_B = B\cos\theta \times \Delta A$$

$$\emptyset_B = \int_S \underline{B} \cdot d\underline{S}$$

Gauss 's Law for Magnetism

- For E-fields, net electric flux: $\int_S \underline{E} \cdot d\underline{S} = \frac{Q_{enc}}{\varepsilon_0}$
- But there are no magnetic monopoles so for magnetic fields:
- Net magnetic flux:
- $\int_S \underline{B} \cdot d\underline{S} = 0$

(not much use for this course but does form Maxwell's 2nd equation)



(a)

Summary

- A *magnetic field* $\underline{B}$ is defined in terms of the force $\underline{F}_m$ acting on a test particle with charge q and moving through the field with velocity $\underline{v}$:

$$\boxed{\underline{\boldsymbol{F}}_m = q\underline{\boldsymbol{v}} \wedge \underline{\boldsymbol{B}}}$$

- The general case of both B-fields and E-fields is the Lorentz equation (Lorentz Force):

$$\boxed{\underline{\boldsymbol{F}} = q(\underline{\boldsymbol{E}} + \underline{\boldsymbol{v}} \wedge \underline{\boldsymbol{B}})}$$