Video Material WEEK 04

Superposition theorem

1. Introduction

In Week 2 we saw how Kirchhoff's laws could be used to generate a set of simultaneous equations, which could be solved to find the unknown currents in a circuit. This method allowed us to analyse circuits that could not be simplified by combining series and parallel resistors, or circuits containing more than one source. This week, we will look at another useful technique for analysing circuits containing more than one source: the principle of superposition. In some cases (small networks containing two or three sources) it is often easier and quicker than setting up and solving a set of simultaneous equations. However, the drawback of this method is that *you* have to keep track of the direction of currents and the sign of potential differences, so there are pitfalls for the unwary.

2. Linearity

As we have previously noted, there is an important class of circuits for which Ohm's law is obeyed in all parts of the circuit. These are **linear circuits.** The term linear arises because there exists a linear relationship ($y = mx$) between the potential difference across a device obeying Ohm's law and the current flowing through it ($V = RI$).

Algebraically, if y is a function of x, such that $y = f(x) = mx$, it is quite straightforward to show that

$$f(kx) = kf(x)$$
$$f(x_1 + x_2) = f(x_1) + f(x_2)$$

where k is a constant. The first equation is almost self-evident and is a result of the proportionality between x and $f(x)$. The second equation is slightly less obvious. It says that if I have two sources, represented by x_1 and x_2, the result of them acting on a circuit is the same as if I had considered each source acting independently and summed the results. In other words, when we have a linear circuit with multiple sources, the voltage (or current) in any part of the circuit will be a superposition of the voltage (or current) that would have been obtained with each source acting alone.

3. Superposition

A formal statement of the principle of superposition can be seen in Figure 5.1. The key points are that
a) the circuit must be linear,
b) the sources must be independent of one another and
c) we must be able to turn off the other sources, so that we can determine the effect of each source acting alone.

Does this ring any bells?

We learned how to turn off sources when we studied Thévenin's theorem in Week 3. Before I continue... do you recall how to remove current and voltage sources from a circuit?

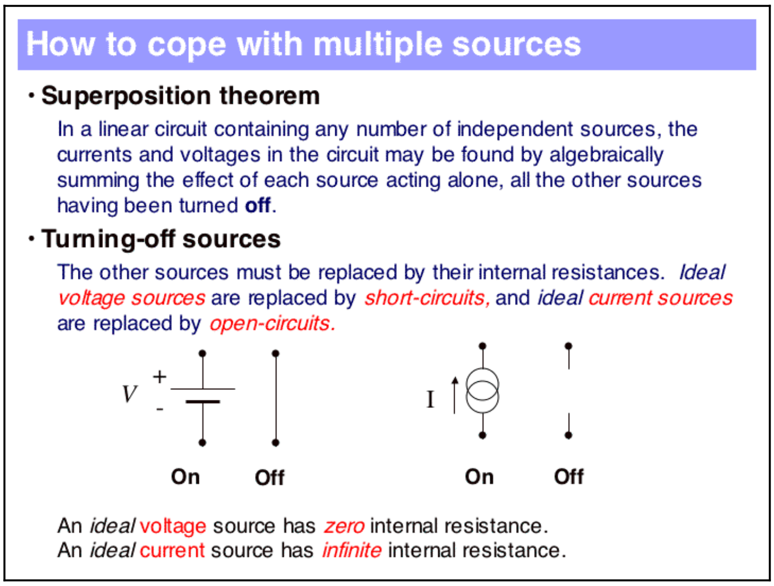

Figure 5.1: The principle of superposition.

Voltage sources are removed by replacing them with a short-circuit. Current sources are removed by replacing them with an open-circuit. Go back to Week 2 to remind yourself how we arrived at this conclusion. What we are really doing is replacing each source by its internal resistance. For ideal sources, the internal resistance of a voltage source is zero, for a current source, it is infinite.

To see how the superposition theorem works in practice, consider the following example.

4. Worked problem

Figure 5.2 shows a problem that we considered in the previous Lecture, so we already know what the answer should be!

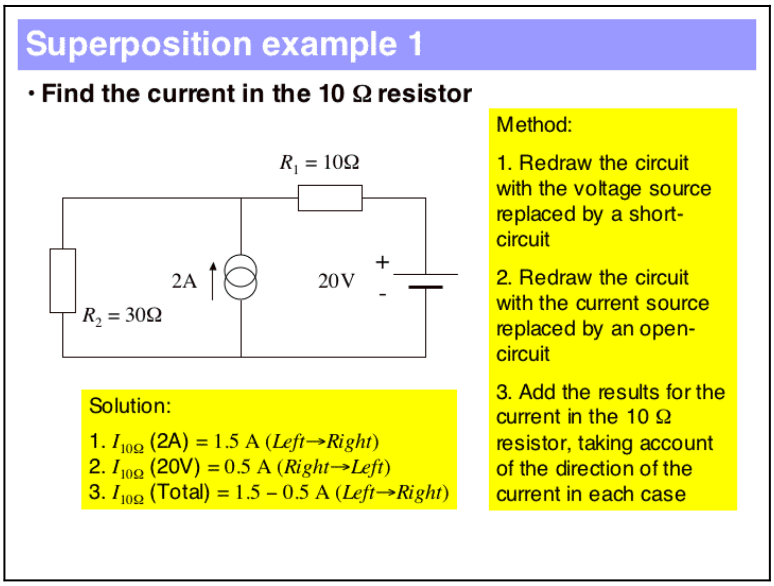

Figure 5.2: Superposition example 1.

To apply the superposition theorem, we must redraw the circuit considering each source in turn. This means that we have to turn off the other source, either replacing the voltage source with a short-circuit, or replacing the current sources with an open-circuit. This simplifies the resulting circuit considerably. In this example, I chose to consider the 2 A current source first and then the 20 V voltage source, although the order is unimportant. You can see the circuit redrawn for each source acting independently in Figure 5.3.

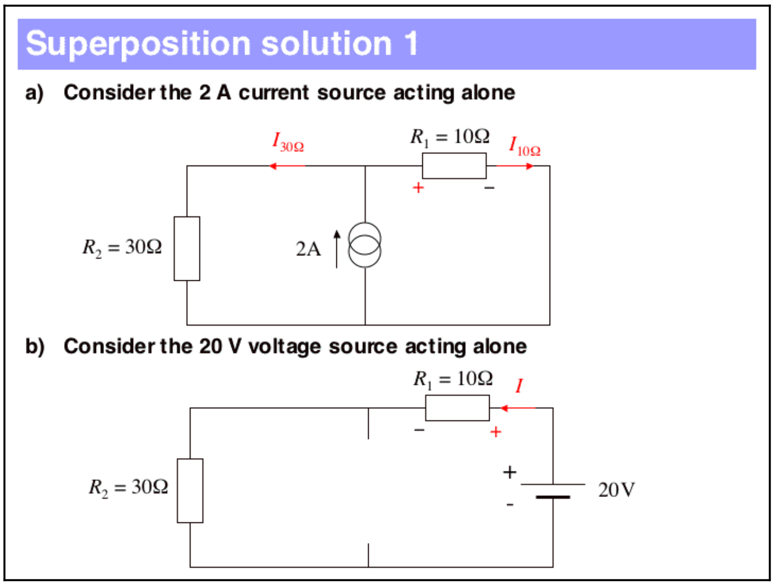

Figure 5.3: Example 1, redrawing the circuit to consider each source in turn.

The problem is now very straightforward. In the case of the current source acting alone, you should recognise the resulting circuit as a current splitter. If you need reminding about current splitters, look back at the material from Week 1. Using the current splitter rule, the current in the 10 Ω resistor is given by the ratio of the current in the opposite branch to the sum of the resistances in both branches, multiplied by the total current.

$$I_{10\Omega}=\frac{R_2}{R_1+R_2}\times I=\frac{30}{30+10}\times 2=1.5\,A$$

Notice that this gives you the magnitude of the current, but not its direction. To keep track of this information, you should also note down that this current flows from left-to-right through the 10 Ω resistor.

In the case of the voltage source acting alone, the circuit is even simpler. The current in a series circuit is found by taking the ratio of the applied voltage to the total resistance. Since the resistors are in series, the same current flows through each of them.

$$I=I_{10\Omega}=\frac{V}{R_1+R_2}=\frac{20}{30+10}=0.5\,A$$

Notice that this current flows from right-to-left, and therefore opposes the current from the current source. The total current through the 10 Ω resistor is therefore 1.5 – 0.5 = 1 A (left-to-right).

5. Try it for yourself

Here is an example for you to try. The problem is to find the potential difference across the 5 Ω resistor. In many respects this problem is similar to the one above. Consider each source in turn, removing the other sources in way we have described. Redraw the circuit with the other sources removed. This problem is quite straightforward if you can remember the expressions for a current splitter and a voltage divider.

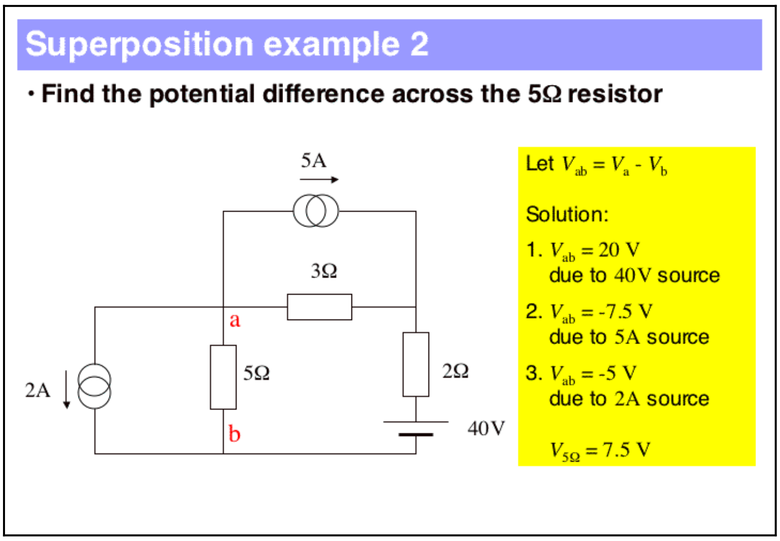

Figure 5.4: Superposition example 2.

The key point in this problem is to keep track of which end of the resistor is at the higher electrical potential. In the solution shown in Figure 5.4, I've define the potential difference to be the potential at a, minus the potential at b.

$$V_{ab} = V_a - V_b$$

This means that the potential difference is positive if a is at a higher electrical potential; negative if it is at a lower electrical potential. Remember that the end at which the current enters a resistor is at the higher electrical potential. Redraw the circuit for each source acting independently. This means that you will have to redraw the circuit 3 times. In the case of the two current sources you must apply the current splitter rule. Think carefully about which resistors are in series and which are in parallel. It may help you to draw the current directions on your circuit diagrams.

6. Further reading

Tipler does not cover the principle of superposition in electric circuits. You will find some further worked examples in Ray Powell's *Introduction to Electric Circuits* (p. 45) and Silvester's *Electric Circuits* (p. 124).