Electromagnetism I – Problem sheet 7

Problem 1.

The loop of wire in the figure below forms a right triangle and carries a current $I = 5$ A in the direction shown. The loop is in a uniform magnetic field that has magnitude $B = 3$ T and the same direction as the current in side PQ of the loop, see figure.

1. Find the force exerted by the magnetic field on each side of the triangle.

   For side PQ: current is parallel to B-field $\implies$ F = 0

   [1 mark]

   For side PR: current is perpendicular to B-field $F = Il_{PR}B = 5 \times 0.8 \times 3 = 12N$

   [1 mark]

   For side QR: $F_{QR} = Il_{QR}B \sin\alpha$ where α is the angle between QR and B.
   From the diagram $\tan\alpha = \frac{0.8}{0.6} \implies \alpha = 53.1^o$ hence
   $F_{QR} = Il_{QR}B \sin 53.1^o = 5 \times 1 \times 5 \times \sin 53.1^o = 12N$

   [1 mark]

2. The loop is pivoted about an axis that lies along side PR.

   (a) Use the forces calculated in part (1) to calculate the net torque on the loop.
   F_{PR} acts along the pivoted axis so no contribution to torque *i.e.* $\tau = \underline{\mathbf{r}} \wedge \underline{\mathbf{F}}$ and $\underline{\mathbf{r}} = 0$.
   There is no force from the side PQ so no contribution to the torque from here.
   The force acting on QR can be taken as acting on the centre of mass of QR, whose distance from the pivot hinge is $0.3m$. Hence, the Torque from QR is:

   $$\tau = F_{QR} = 12 \times 0.3 = 3.6Nm$$

   [1 mark]

   (b) Is the torque to rotate point Q into the plane of the figure or out of the plane?
   The torque tends to move point Q out of the plane

   [1 mark]

3. Use the equation $\vec{\tau} = I\vec{A} \times \vec{B}$ to find the torque on the loop.

   $$\tau = I\underline{\mathbf{A}} \wedge \underline{\mathbf{B}} = 5 \times (0.5 \times 0.8 \times 0.6) \times 3 = 3.6Nm$$

   Same as before, of course

   [1 mark]

Problem 2.

An interstellar dust grain of mass $m = 10^{-16}$ kg is (roughly) spherical with a radius of 3×10^{-7} m. It has acquired a negative charge such that its potential is -0.15 V.

1. How many extra electrons has it picked up?
   The potential is:

$$V = \frac{q}{4\pi\epsilon_0 R} \implies q = V\,(4\pi\epsilon_0 R) = -0.15 \times 4\pi \times 8.85 \times 10^{-12} \times 3 \times 10^{-7} = -5 \times 10^{-18} C$$

   But $e = -1.6 \times 10^{-19} C$ hence the number of electrons is:

$$N = \frac{q}{e} = \frac{5 \times 5 \times 10^{-}18}{1.6 \times 10^{-19}} \approx 31$$

[1 mark]

2. What is the strength of the electric field on its surface?
   The elctric field is:

$$E = \frac{q}{4\pi\epsilon_0 R^2} = \frac{5 \times 10^{-}18}{4\pi \times 8.85 \times 10^{-12} \times 9 \times 10^{-14}} = 5 \times 10^5 N/C$$

[1 mark]

3. The dust grain moves freely (with velocity much smaller than the speed of light) in a plane perpendicular to the interstellar magnetic field, which in that region has a strength of 3×10^{-10} T.

   (a) What is the "cyclotron frequency" associated to the dust grain motion?
   The charged particle undergoes circular motion in the magnetic field. Equating the forces, we have:

$$qvB = \frac{mv^2}{r} \implies \frac{v}{r} = \omega = \frac{qB}{m} = \frac{5 \times 10^{-18} \times 3 \times 10^{-10}}{10^{-16}} = 1.5 \times 10^{-11} s^{-1}$$

   So frequency $f = \omega/2pi = 2.4 \times 10{-}12 s^{-1}$ (or Hz)

[1 mark]

   (b) How many years will it take to complete a circular orbit?
   The period $T = 2\pi/\omega = 4.2 \times 10^{11} s$. Hence, number of years to complete an orbit is:

$$t = \frac{4.2 \times 10^{11}}{3.1 \times 10^7} \approx 1.3 \times 10^4 \text{ years.}$$

[1 mark]