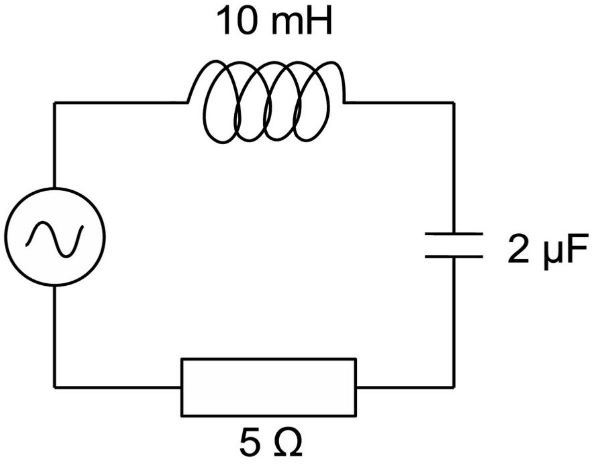
Non-assessed Week11 solution

1. Worked problem from Lecture 9.

A series LCR circuit with L = 10 mH, C = 2 μF and R = 5 Ω is driven by a generator with an amplitude of 100 V and variable angular frequency ω. Find **(a)** the resonant frequency ω_0 and **(b)** the rms current at resonance. When ω is 8000 rad/s find **(c)** the impedance Z, **(d)** the rms current.



The resonance condition is found by requiring the generator voltage to be in phase with the current. This occurs when the inductor reactance equals the capacitor reactance.

(a) $$\omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}} = 7071 \text{ rad}/s.$$

At resonance the impedance is purely resistive, Z = R. The current magnitude and rms current are given by

(b) $$I = \frac{V}{R} = \frac{100}{5} = 20 \text{ A} \Rightarrow I_{rms} = \frac{I}{\sqrt{2}} = 14.142 \text{ A}.$$

(c) To calculate the impedance, sum the imaginary parts for *L* and *C* and the real part from *R*,

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right) = R + j\left(\frac{\omega^2 LC - 1}{\omega C}\right)$$

The magnitude of the impedance is thus,

$$Z = \sqrt{R^2 + \left(\frac{\omega^2 LC - 1}{\omega C}\right)^2} = 18.2 \ \Omega.$$

The impedance is defined as the ratio of the voltage magnitude to the current magnitude (Z = V/I). This is just the expression of Ohm's law in a.c. circuits. Hence the rms current at this frequency is

(d) $$I_{rms} = \frac{1}{\sqrt{2}} \times \frac{V}{Z} = \frac{100}{\sqrt{2}\,18.2} = 3.885 \text{ A}.$$