Year 1 Assessed Problems

Semester 1

Assessed Problems 9

SOLUTIONS TO BE SUBMITTED ON CANVAS BY

Wednesday 4th December

at 17:00

Classical mechanics and Relativity
Problem Sheet 5

1. U^{238} uranium nucleus is radioactive. It spontaneously disintegrates into a small fragment that is ejected with a measured speed of 1.5×10^7 m/s, and a daughter nucleus that recoils with a measured speed of 2.56×10^5 m/s. What are the atomic masses of the ejected fragment and the daughter nucleus? [**2 marks**]

2. Consider dropping a ball of mass m from a height h onto the horizontal floor of a tall box standing on the Earth. Ignoring air resistance:

   (a) What is its speed v on hitting the ground? [**1 mark**]

   (b) Assume that after the ball hits the ground, it bounces up with the same speed v. By how much is its momentum changed by the bounce? [**1 mark**]

   (c) If the ball is in contact with the ground for a time δt, what is the average contact force it experiences during that time? [**1 mark**]

   (d) Imagine that, just before contact with the ball, the Earth was motionless. What is the speed of the Earth (whose mass is M_{Earth}) just after collision? If $v = 10$ m/s, $m = 10$ g, and $M_{\text{Earth}} = 6 \times 10^{24}$ kg, what is the numerical value of the post-collision speed of the Earth? (Note: you only need to use the numerical values for this part of the problem.) [**1 mark**]

   (e) How long does it take for the ball to reach its original height, measured from the moment it was dropped? [**1 mark**]

   (f) How many bounces are there per second? [**1 mark**]

   (g) What is the average force exerted by the ball on the floor over many bounces? [**1 mark**]

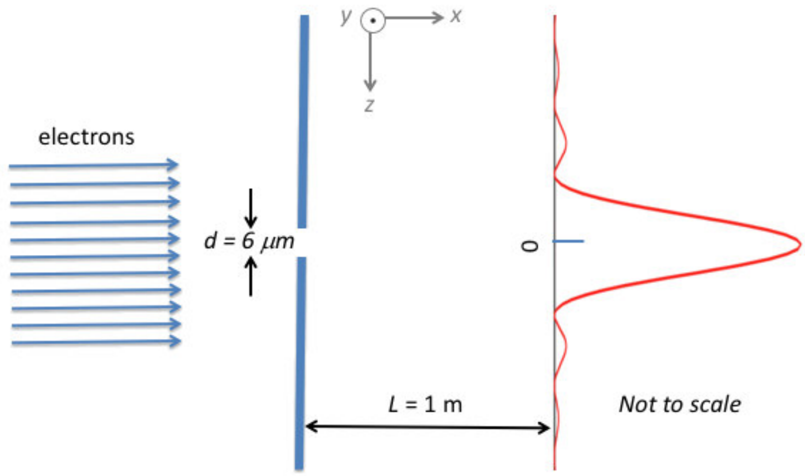
   (h) If an identical ball is dropped from a height $2h$, how is the average force exerted on the floor over many bounces changed? [**1 mark**]

Quantum Mechanics 1 – Problem 8

a) Estimate the minimum energy of a proton of mass 1.673 x 10^{-27} kg confined to an infinite one-dimensional potential well of width 10^{-14} m. Express your answer in MeV. [2 marks]

b) State Heisenberg's Uncertainty Principle for momentum and position and briefly explain its physical significance. [2 marks]

c) A beam of 50 eV electrons travelling in the +x direction passes through a slit that is parallel to the y-axis and 6 μm wide. A diffraction pattern is observed at a distance 1 m from the slit as illustrated in the figure below.



From classical wave theory, the angle of the first minimum in the diffraction pattern is given by $\theta = \sin^{-1}(\lambda/d)$, where λ is the de Broglie wavelength of the electrons and d is the width of the slit. Show that the observed width of the central maximum, measured by the distance between the two minima either side of z =0, does not violate Heisenberg's Uncertainty Principle. [6 marks]

[Hint: To answer the last part of this question, you need to estimate the minimum width of the image of the slit using Heisenberg's Uncertainty Principle. The idea is that passing the electrons through a slit in the z-direction introduces an uncertainty in the z-component of momentum after the slit. Use this momentum uncertainty to estimate the minimum width of the image of the slit and compare it with the width obtained using classical wave theory. Don't overlook the fact that in the absence of any other effect (quantum or classical) you'd expect the image to have the same width as the slit due to electrons that pass through the slit unperturbed.]

Optics and Waves (week 9)

(a) An object is 2 cm tall and 10 cm from a convex mirror with a radius of curvature 10 cm.

i) Draw the corresponding ray diagram.

ii) Where does the image lie?

iii) Find the image height.

iv) Is the image real or virtual, erect or inverted?

(b) A 2 cm-tall object is placed in front of a mirror. A 1 cm-tall erect image is formed behind the mirror, 150 cm from the object. Without numerical calculation, determine if the mirror is concave or convex, and then use the mirror equation to calculate the focal length of the mirror.