Electromagnetism I

Spring 2025

Professor D. Evans
Email: d.evans@bham.ac.uk
Office: Physics East 303

Start Panopto

Give out Number

Recommended textbook is Young and Freedman

To Contact Me

- Please feel free to contact me if you have questions, concerns, complaints, requests, etc.
  - I'm always happy to talk and answer emails
- **Don't use Canvas** or any other web-based media to contact me.
- Use email: d.evans@bham.ac.uk
- Salutation: Dear Professor Evans ….

Delivery of Course

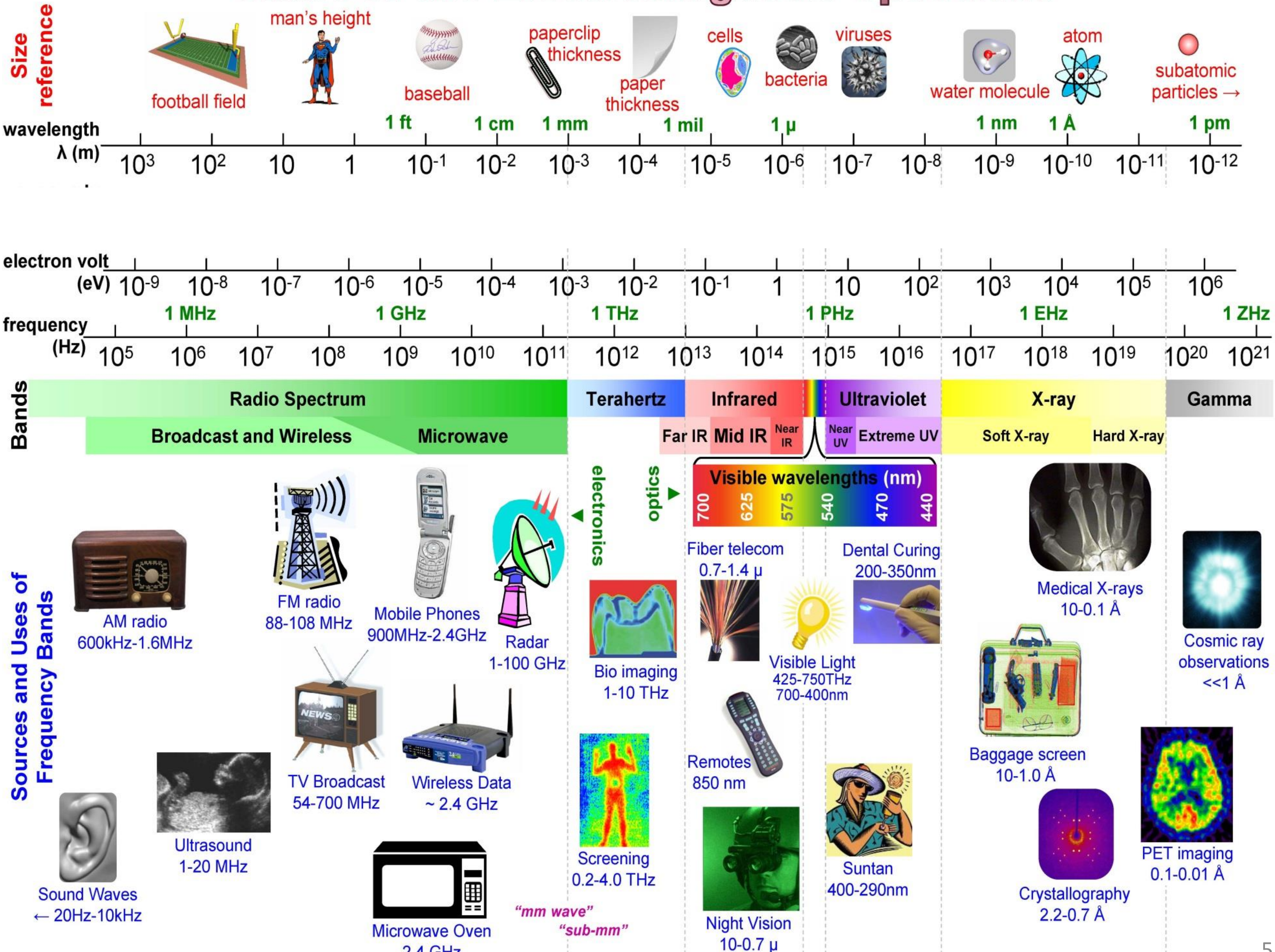
- PowerPoint
  - Mainly for background material
  - Deriving formulae etc.

- Hand-written on visualiser (or board if you prefer)
  - Doing worked examples
  - The things you really need to know

Why is EM Important?

- The whole modern World and the technology we take for granted is all based on our knowledge of electro-magnetism.
- It’s responsible for the properties of all the elements.
- It’s responsible for life itself.

Chart of the Electromagnetic Spectrum



Why is this course important?

Everyday materials are held together by electromagnetic forces

Optics can only be fully understood through electromagnetic theory

Technical applications – power generation, construction and development of electronics, telecommunications and computers…

EM II in year 2

Aim of Course

The aim of this course is to Lay Down the Foundations Leading to an Understanding and use of:

Maxwell's Laws of Electromagnetism

Maxwell's Equations

By around the 1860s, Maxwell had put together four equations which summarise all of the properties of electric and magnetic fields in a free space.

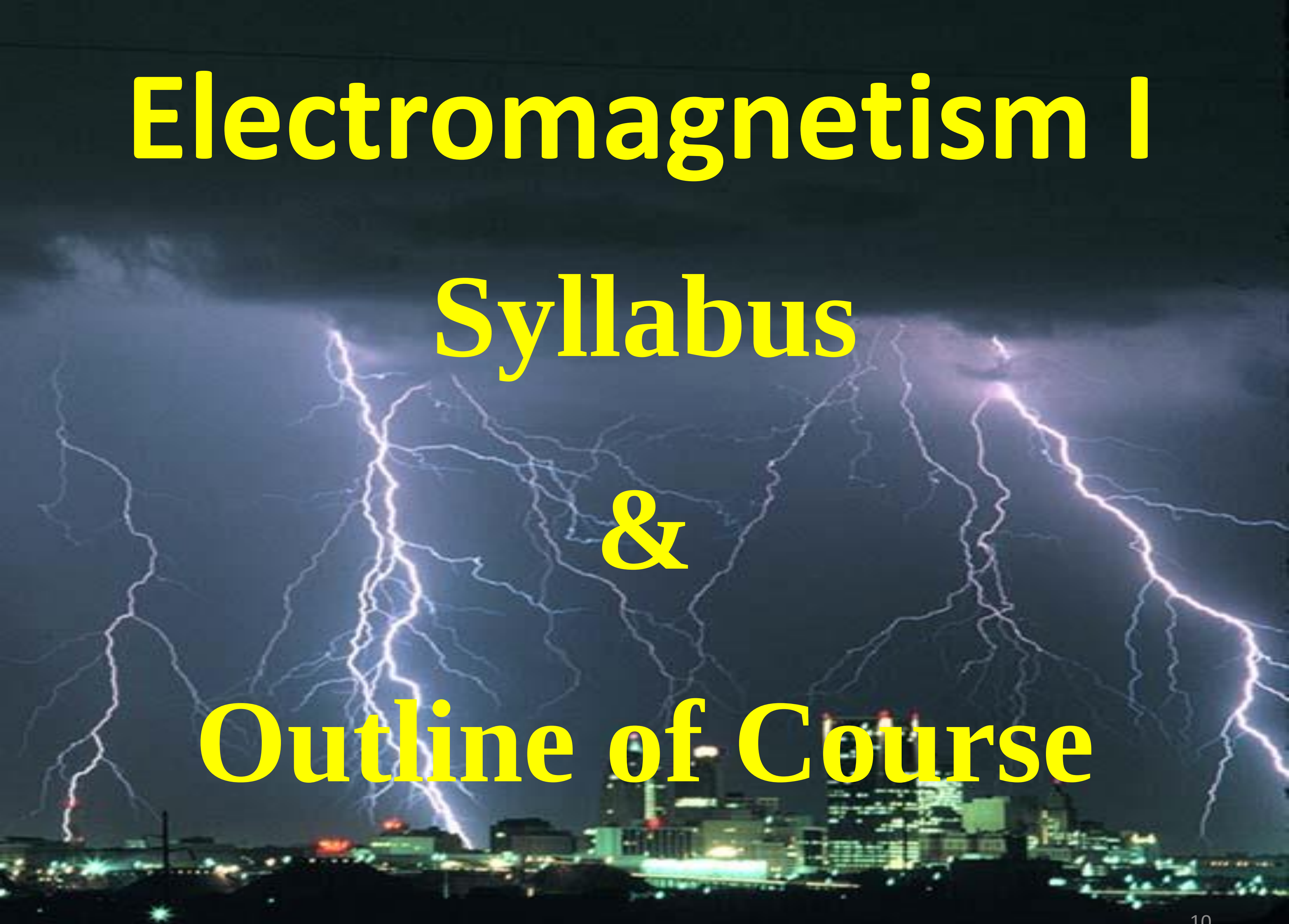
$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0} \qquad \nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \wedge \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \qquad \nabla \wedge \boldsymbol{B} = \mu_0 \left(\boldsymbol{J} + \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t} \right)$$

where $\nabla = \boldsymbol{i}\frac{\partial}{\partial x} + \boldsymbol{j}\frac{\partial}{\partial y} + \boldsymbol{k}\frac{\partial}{\partial z}$ (see EM2 lectures next year)

These equations show that electric and magnetic forces are related and are aspects of a single force - electromagnetism

- **We cannot fully understand Maxwell's equations until we can appreciate the meaning of the fields $\underline{E}$ and $\underline{B}$ and the sources of these fields, the charges and the currents** ***(hence the need for this course)***


Electromagnetism I
Syllabus
&
Outline of Course

Part I. Electric Fields

Electrostatics

- **CHARGE AND COULOMB’S LAW**
- **THE ELECTRIC FIELD**
- **GAUSS’S LAW**
- **ELECTRIC POTENTIAL**
- **CAPACITORS**

Part II. Magnetic Fields

- **Magnetic Fields**
- **Charged Particles in B Fields**
- **Electromagnetic Induction**
- **Magnetic Dipoles**

Lecture 1: Introduction to EM

- Introduction to electromagnetism (the bit we just had)
- Electric charge
- Force between charges
- The concept of the Electric Field (E-field)

Electric Charge

- **First discovery attributed to Thales (pronounced: "Thay leez")**
  - **(**Θαλῆς, *Thalēs*; c. 624 BC – c. 546 BC **)**
  - ***Elektron*** $(\eta\lambda\epsilon\kappa\tau\rho o\upsilon) \rightarrow$ ***amber***

- **Experiments by *Franklin, Coulomb* showed that:**
  - **there are *two types of charge*: *positive* and *negative***
  - **(Franklin named "*positive electricity*" to the charge on a glass rod rubbed with silk, and "*negative electricity*" to that on an ebonite rod rubbed with fur)**

- ***like charges repel, unlike charges attract***

Aside

Ebonite is one of the earliest forms of plastic. A hard, rigid and shiny resin, it was intended as an artificial substitute for ebony wood. It is often used in bowling balls, smoking pipe mouthpieces, fountain pen nib feeds, and high-quality saxophone and clarinet mouthpieces. It is also commonly used in physics classrooms to demonstrate static electricity .

Ebony wood is one of the most intensely black woods known. With its very high density, it is one of the very few woods that sink in water.

Unit of Charge

We now know, the elementary (smallest) unit of charge is the charge of an electron or proton (electrons negative charge, protons positive charge) i.e.

$$e = 1.602176565 \times 10^{-19}\ \mathrm{C}$$

$$\boxed{e = 1.6 \times 10^{-19}\ \mathrm{C}}$$ **Good enough for us**

(Yes, I know quarks have fractional electric charge (d, s, b have -1/3 e and u, c, t have +2/3 e) but we don't get free quarks

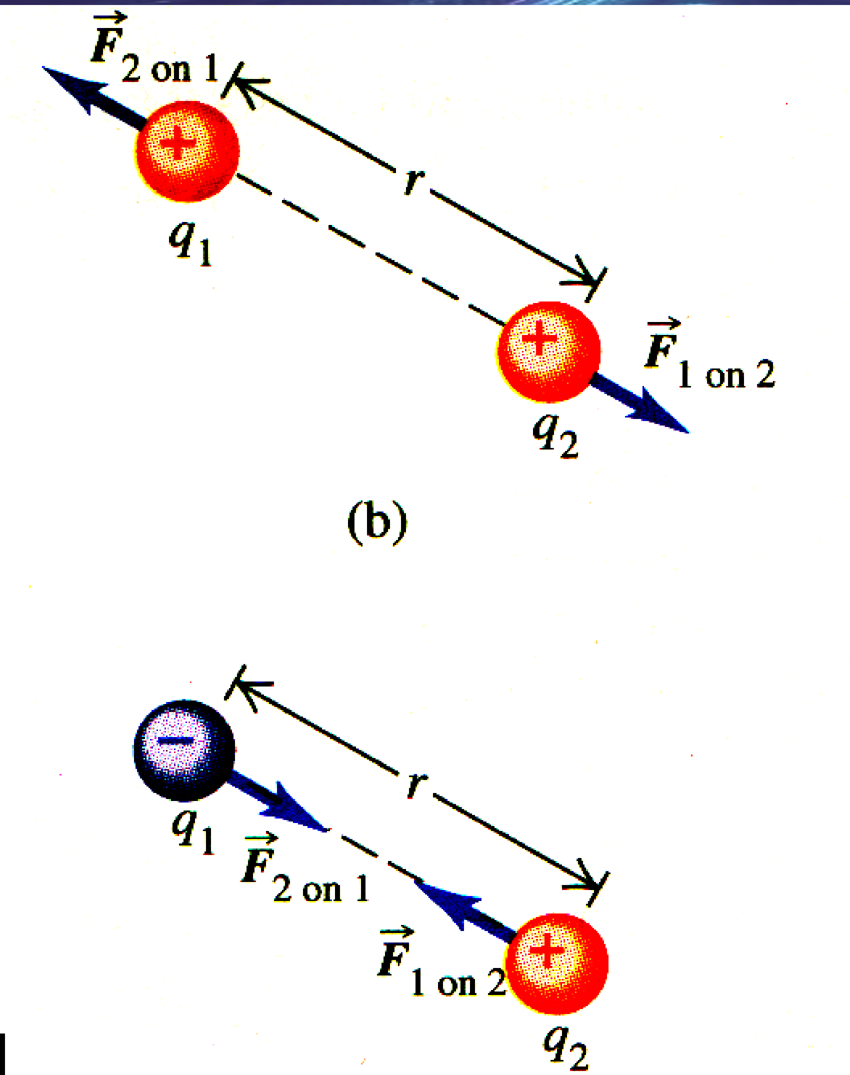
The unit is charge in S.I. units is the Coulomb.

Charge Conservation

Electrons **and** ***protons*** **are stable (proton decay > 10^{31} years). This means that the** ***net charge of an isolated system is constant*****. FURTHERMORE:**

$$\left|q_p\right| = \left|q_e\right| = e$$

Electro-static Force



Electric charges of the same sign repel and those of opposite sign attract.

The force between two charges is proportional to the product of both charges i.e.

$$F \propto q_1 q_2$$

(note a negative force means attractive and positive means repulsive in this context)

Force at a Distance

- So, just like the case of masses and gravity, an electric charge will exert a force on another charge at a distance – without actually coming into contact with it.
- So there must be something between the charges which mediates the force.
- (In my field of research the mediators of the electromagnetic force are virtual photons, electrons, and positrons – but quantum field theory is a little advanced for year 1).

The Electric Field

- In classical physics, we say a charge produces an electric field around it. Another charge (a test charge) would interact with this field and that causes the force.
- The electric field, E is define thus:

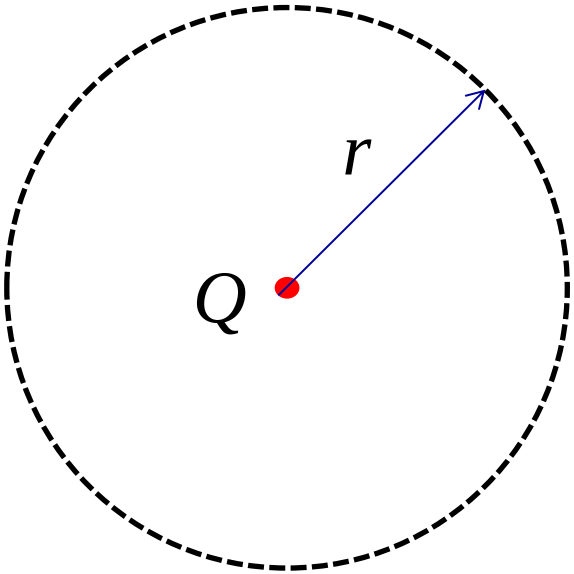
$$\underline{\boldsymbol{F}} = \underline{\boldsymbol{E}}\, q$$

where $\boldsymbol{F}$ is the force exerted on a test charge q by a charge (say Q) producing a field $\boldsymbol{E}$. E-field has units N C^{-1}.

$$\underline{|\boldsymbol{E}|} \propto Q \;\text{ as }\; |\underline{\boldsymbol{F}}| \propto Qq$$

E-Field from Point Charge, Q

Consider a point charge with an electric field spreading out from it in all directions.



As the distance r, from the charge Q, increases the field will be spread out over a larger area.

At a distance r from the charge, we can imagine a spherical surface where the E-field is spread over. As the surface area of a sphere of radius r is $4\pi r^2$ the strength of the field must decrease with distance as $4\pi r^2$.

Coulomb's Law

Hence $|\underline{\boldsymbol{E}}| \propto \frac{Q}{4\pi r^2}$

The (inverse) constant of proportionality, in a vacuum, is known as the permittivity of free space, ε_0.

In S.I. units: $\varepsilon_0 = 8.854 \times 10^{-12}$ $C^2m^{-2}N^{-1}$.

So:

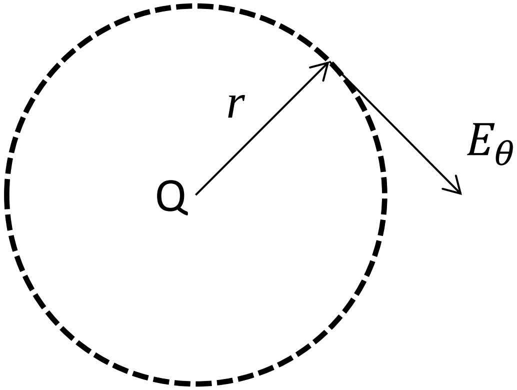
$$\boxed{E = |\underline{\boldsymbol{E}}| = \frac{Q}{4\pi\varepsilon_0 r^2}}$$

This is known as **Coulomb's Law**

(I assume you all know this already)

Direction of E-field (Force)

- Force is a vector and hence so is electric field
  - $\underline{\boldsymbol{F}} = q\ \underline{\boldsymbol{E}}$
- Consider E-field from a point charge Q at a distance r
  - Assume E has components E_r , E_θ and E_ϕ .



If E_θ was, say, in the clockwise direction from our view point, then from the other side of the charge, E_θ would be in the anti-clockwise direction but everything else would like exactly the same. The laws of Physics should be the same from all view points so $\boldsymbol{E_\theta}$ **must be zero and the same goes for** $\boldsymbol{E_\emptyset}$**.**

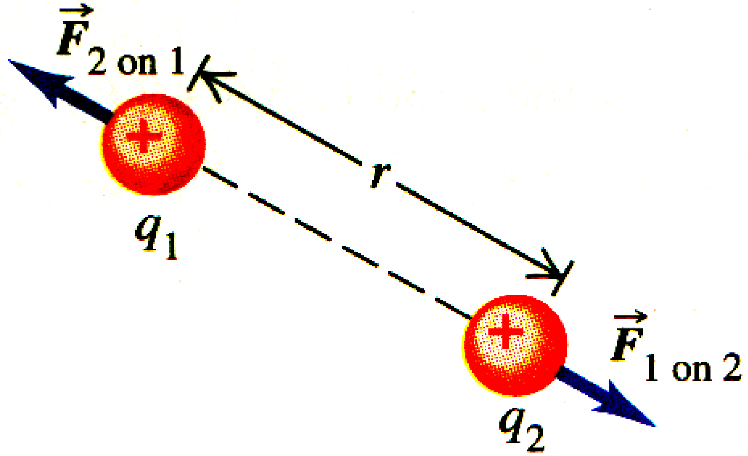
E-Field from Point Charge

- So E-field of a point charge is radial

i.e.

$$\underline{\boldsymbol{E}} = \frac{Q}{4\pi\varepsilon_0 r^2}\underline{\hat{\boldsymbol{r}}}$$

Electro-static Force



Force between two charges, q_1 and q_2:

Force on q_2 due to E-field from q_1 is $\underline{\mathbf{F}}_1 = \underline{\mathbf{E}}_1 q_2$

This equals force on q_1 due to E-field from q_2 : $\underline{\mathbf{F}}_2 = \underline{\mathbf{E}}_2 q_1$

I.e. force between two charges is:

$$\boxed{\underline{\boldsymbol{F}} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\underline{\hat{\boldsymbol{r}}}_{\mathbf{12}}}$$

In SI units where:

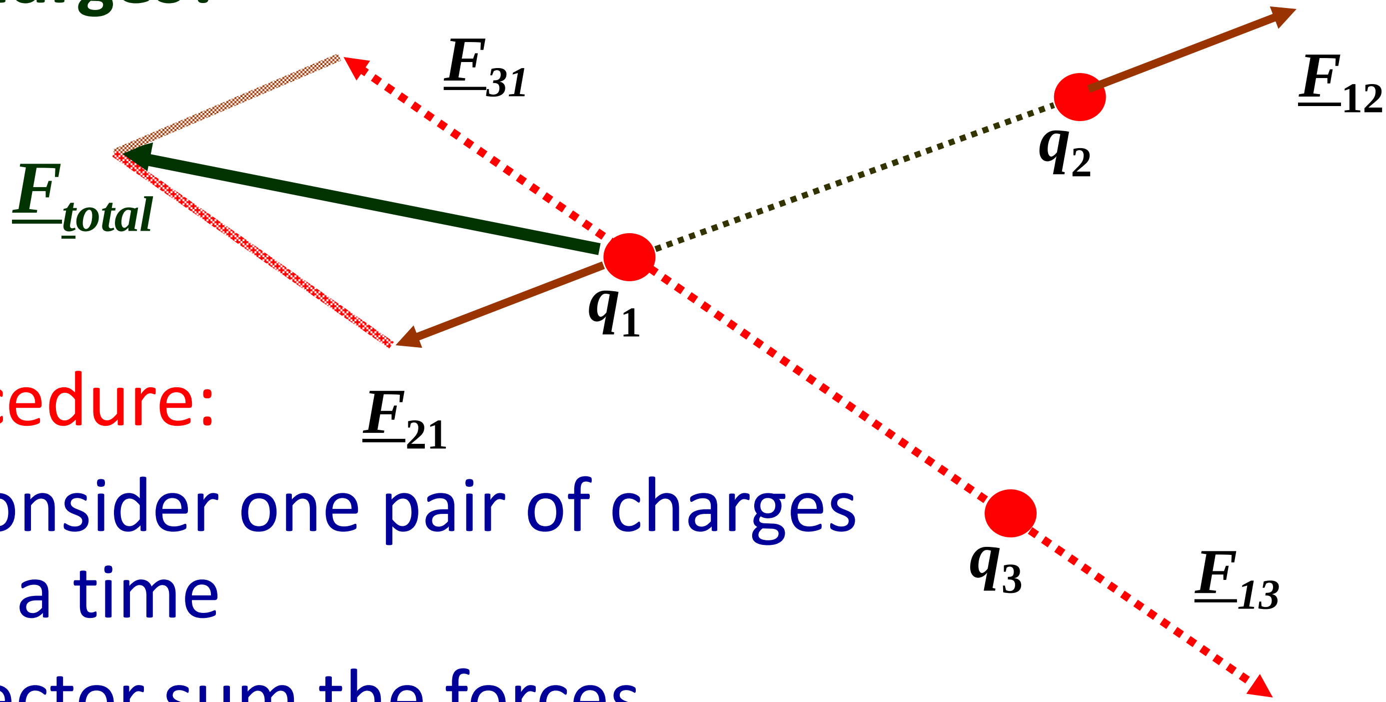
q_1 and q_2 are in Coulombs

r in metres

F in Newtons

Forces between many charges - the principle of superposition

- **What if you have more than two positive charges?**



Procedure:

- Consider one pair of charges at a time
- Vector sum the forces

Force due to Many Charges

- **In general the force on a charge *q* due to a collection of charges is the *vector sum* of all the individual forces due to all the others**

$$\underline{F} = \underline{F_1} + \underline{F_2} + \underline{F_3} + \ \ldots$$

$$= q \sum_i \frac{q_i}{4\pi\varepsilon_0 {r_i}^2} \underline{\hat{r}_i}$$

where r_i is the distance from q_i to q & $\underline{\hat{\boldsymbol{r}}}_i$ is the unit vector between them

- **This is known as the *principle of superposition of forces***

Electric Field due to many Charges

- But $\underline{F} = q\underline{E}$
- So electric field at the test charge, q must be:

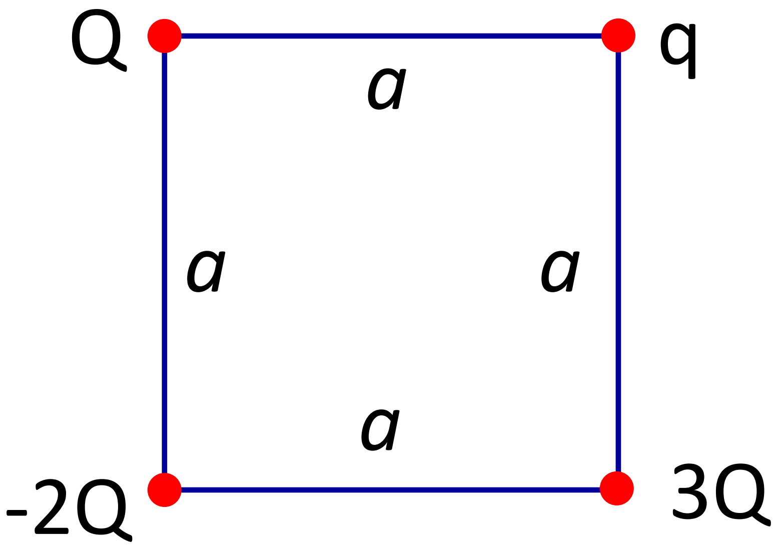
$$\underline{E} = \sum_i \frac{q_i}{4\pi\varepsilon_0 {r_i}^2}\underline{\hat{r}_i}$$

where r_i is the distance from q_i to q & $\underline{\hat{\boldsymbol{r}}}_i$ is the unit vector between them.

- Thus, the $\underline{E}$ at a certain point is equal to the *electric force per unit charge* at that point.
- I.e. the $\underline{E}$ at a certain point is equal to the vector sum of the E-fields from all the charges.

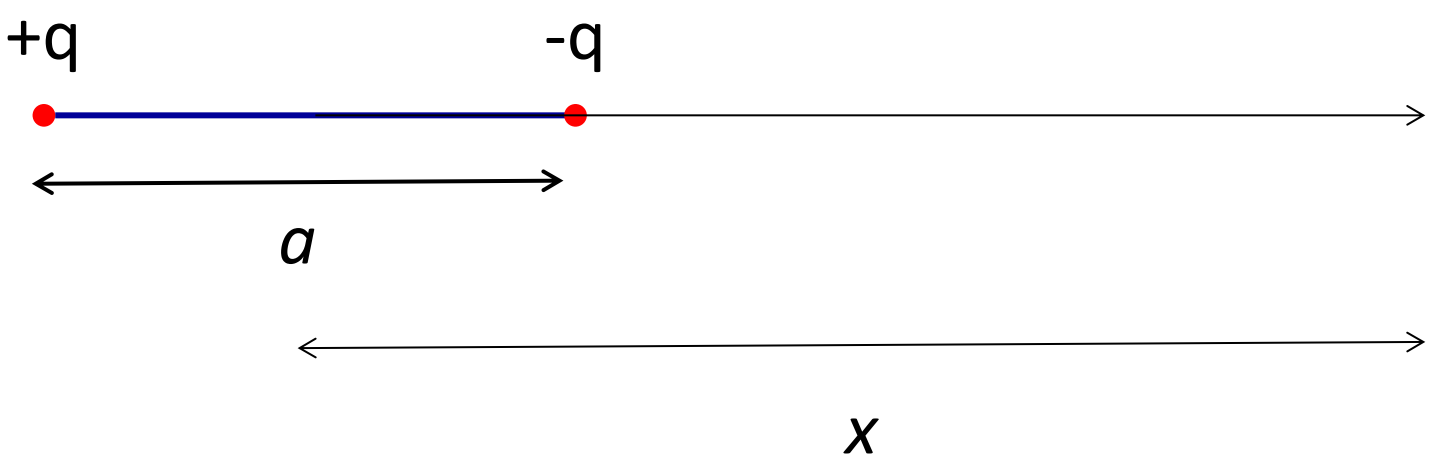
Example of Many Charges

1. What is the E-field at charge q?
2. Hence what is the force on q?



Example – electric dipole

- What is the E-field a distance x from the centre of an electric dipole (as shown)?



$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(x+a/2)^2} - \frac{q}{(x-a/2)^2}\right]$$

Example – electric dipole

$$E = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{(x + a/2)^2} - \frac{q}{(x - a/2)^2}\right]$$

$$E = \frac{q}{4\pi\varepsilon_0}\left[\frac{-2xa}{(x^2 - a^2/4)^2}\right]$$

What is the approx. E-field when *x* >> *a* ?

$$E = \frac{-2qa}{4\pi\varepsilon_0}\left[\frac{x}{x^4(1 - a^2/4x^2)^2}\right] \approx \frac{-2qa}{4\pi\varepsilon_0}\frac{1}{x^3}$$

The quantity *qa* is known as the electric dipole moment, *p*. *p* = *aq*