



Electromagnetism

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Lecture 19

Magnetic Materials

Week 10

(which is really Week 11)



Last Lecture

- Current in RL circuit when switched on

$$I(t) = I_0 \left(1 - e^{-\frac{t}{\tau}}\right)$$

- Mutual inductance from two coils:

$$M = \frac{N_2 \Phi_2}{i_1} = \frac{N_1 \Phi_1}{i_2}$$

$$\varepsilon_2 = -M \frac{di_1}{dt} \quad \& \quad \varepsilon_1 = -M \frac{di_2}{dt}$$

- AC Transformer $\frac{V_p(t)}{N_p} = \frac{V_s(t)}{N_s}$



This Lecture

- Magnetism and spin
- Paramagnetic materials
- Magnetic Susceptibility
- Relative permeability
- Ferromagnetic materials
- Diamagnetic materials

Magnetic Materials

Some materials are magnetic:

- Bar magnet, Compass. (permanent magnets)
- Some appear non-magnetic, but are attracted by a permanent magnet.
- Some are non-magnetic whatever.

WHY?

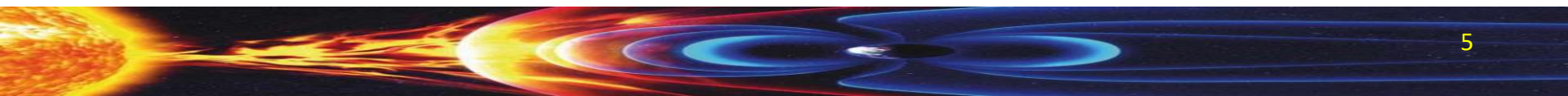
Magnetism & the Electron



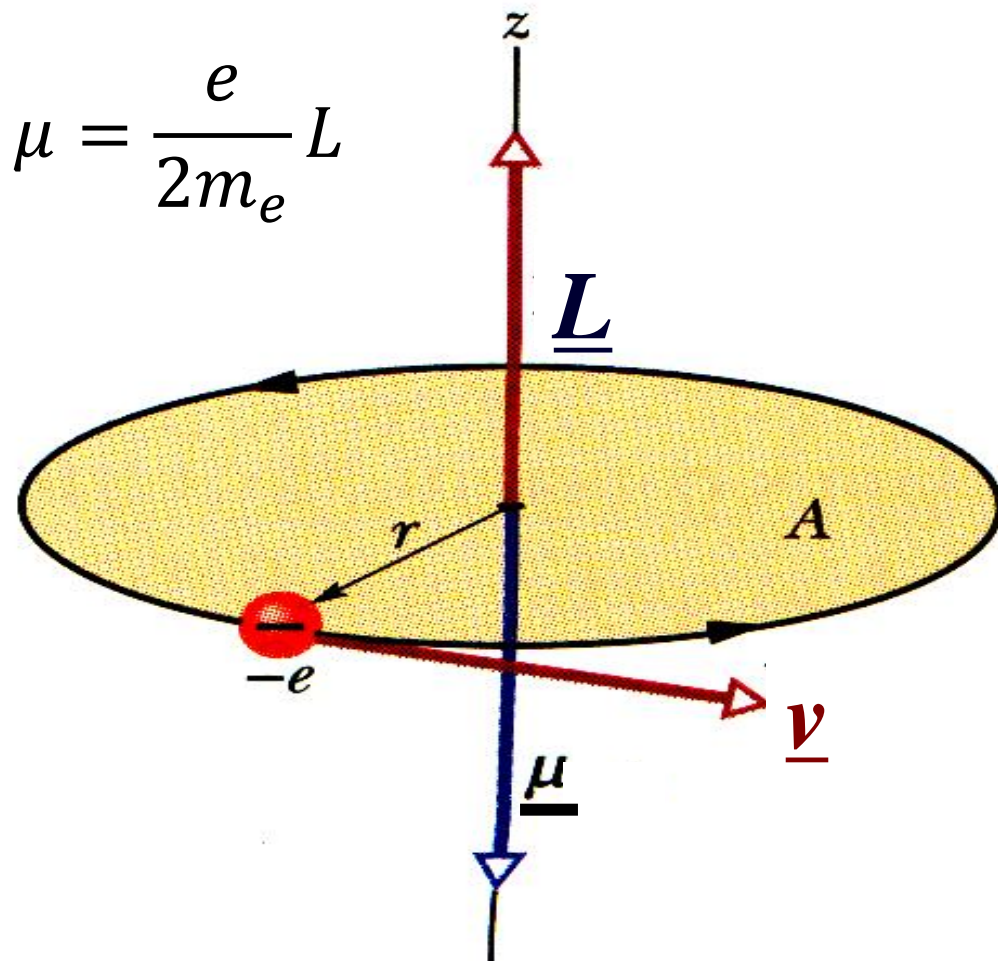
Magnetism is intimately linked to the behaviour of electrons in materials

Electrons generate magnetism in 3 ways:

1. Magnetism of Moving Charges (for example, electric current)
2. Magnetism of Orbital Motion(each electron is orbiting around the nucleus (classical view))
3. Magnetism and Intrinsic Spin(electrons have intrinsic magnetic moments)



Magnetic Dipole Moment of Electron in Atom



$$\mu = \frac{e}{2m_e} L$$

$$\mu = I \times \pi r^2$$

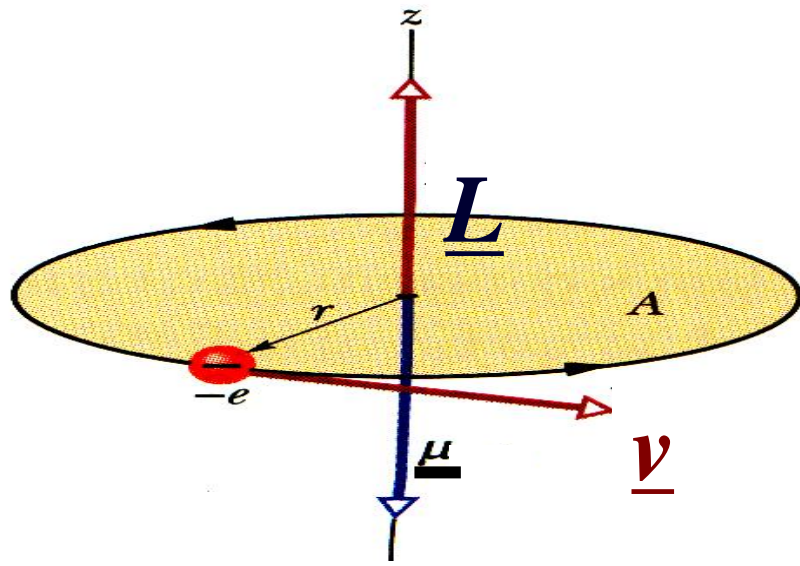
$$I = \frac{e}{T} = \frac{ev}{2\pi r}$$

$$\therefore \mu = \frac{evr}{2}$$

But angular momentum

$$\underline{L} = \underline{r} \wedge m_e \underline{v}$$

Magnetic Dipole Moment of Electron in Atom



$$\underline{\mu} = \frac{e}{2m_e} \underline{L}$$

Note: μ & L in opposite direction (as current in opposite direction to

electron) i.e. $\underline{\mu} = -\frac{e}{2m_e} \underline{L}$

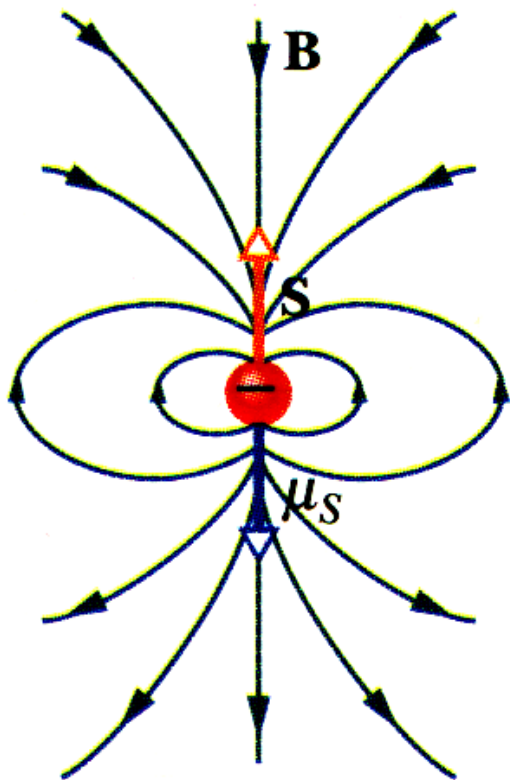
In QM L is quantised and fundamental unit is $L = \hbar$

Fundamental unit of magnetic moment:

$$\mu = \frac{e\hbar}{2m_e} = 9.27 \times 10^{-24} \text{ Am}^2 (\text{J/T}) \text{ (Bohr Magnetron)}$$

Magnetism and Spin

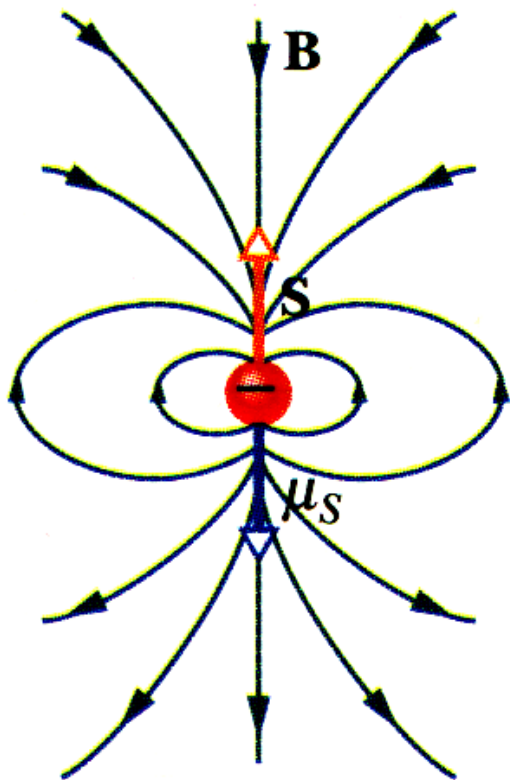
- The electron has an intrinsic magnetic moment!



$$\mu_S = \mu_B = \frac{e\hbar}{2m_e}$$

Magnetism and Spin

- Protons and Neutrons also have an intrinsic magnetic moment!



$$\mu_p = \frac{e\hbar}{2m_p}$$

$$\mu_n = \frac{e\hbar}{2m_n}$$

Approximately 2000 times smaller than the electron spin.

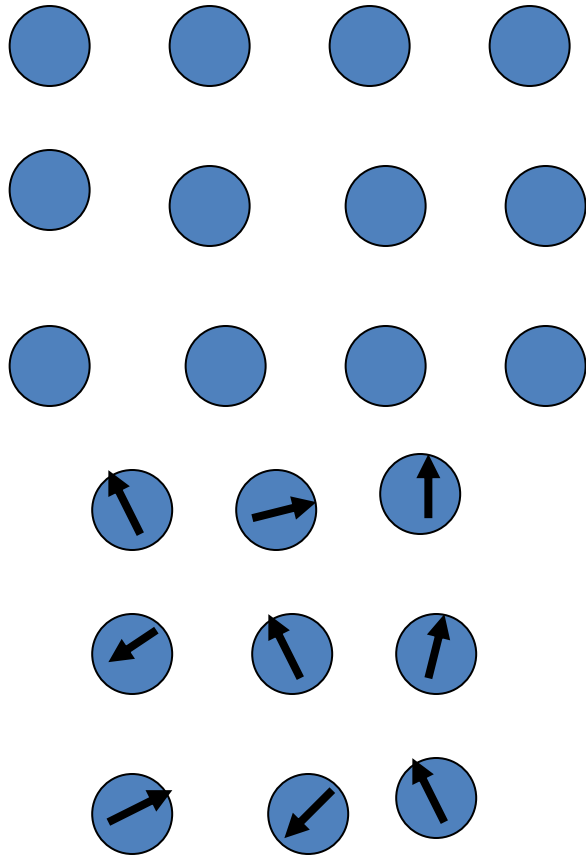
Magnetic Materials

- Net magnetic moment of an atom is obtained by combining both orbital and spin moments of all the electrons, taking into account the directions of these moments.
- (spin of nucleons negligible when considering bulk magnetic properties of matter).
- So why are most materials non-magnetic?

Magnetic Materials

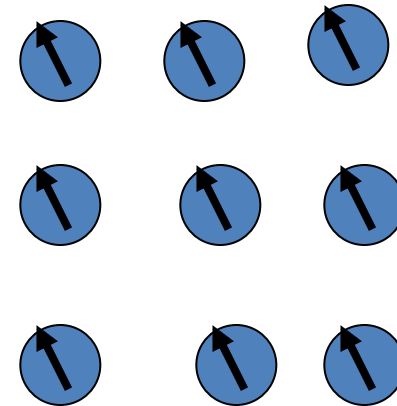
- Each atom contains a number of electrons, each electron contributes a magnetic moment, but the sum of all the electron contributions can be zero because magnetic moment is a vector.
- If not zero, then each atom can be treated as a magnetic dipole.
- The alignment of the “atomic dipoles” in a solid is important.

Difference Cases



Net moment zero

Magnetic moment
of each atom is zero
Total moment is zero.



Net moment non-zero

Types of Magnetism

Materials only exhibit “familiar” magnetic effects when:

- Atoms contain unpaired electrons

AND

- Large scale alignment of the dipole moments occurs
- Magnetic materials fall into three categories: Paramagnetic, Ferro-magnetic, or Diamagnetic

Paramagnetic & Ferromagnetic Materials

- Materials with net atomic magnetic moments are **paramagnetic** and **ferromagnetic** materials. When a magnetic field is applied to such a material, the magnetic dipoles try to line up parallel to the field.
- So the field inside these material increases, (which is the reversal of the electric case!)

Diamagnetic Materials

- Materials which have no net magnetic moment in the absence of applied B-field. When a magnetic field is applied Lorentz force on the orbiting electrons changes the orbit slightly so there is a tiny net magnetic moment opposite to the B-field. So the B-field inside is slightly reduced. These materials are called **Diamagnetic**.

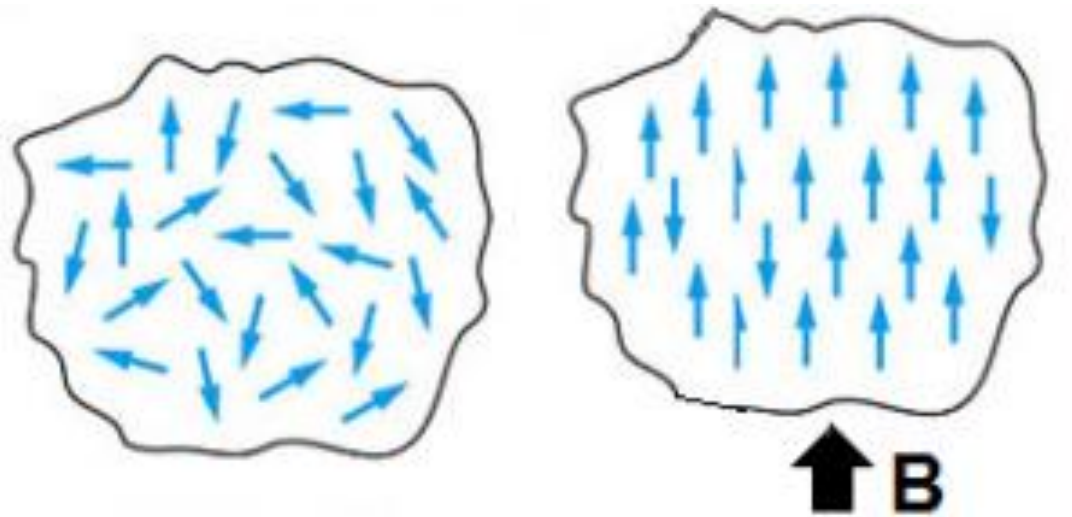
Paramagnetic Materials

- Atoms of a paramagnetic material have permanent magnetic dipole moments.
- These dipoles are randomly oriented - Magnetic fields average to zero.
- In an external field \underline{B}_0 , the dipoles tend to align with \underline{B}_0 - Results in an *additional* magnetic field B_m .
- Material with N atoms, $\mu_{\max} = \mu N$. Randomisation of the dipoles orientations by thermal collisions significantly reduces the total dipole moment (μ_{total})

Thermal Effects on Paramagnetism

- Compare: $\frac{3}{2}kT$ with $2\mu B$
- Example: $T = 300K$, $B = 1.5 T$ and $\mu = \mu_B$
- $U_T = \frac{3}{2}kT = 0.039 eV$
- $U_B = 2\mu_B B = 0.00017 eV$
- $1eV = 1.6 \times 10^{-19} J$
- Hence, thermal much greater than magnetic moment energy in most cases.

Paramagnet in External B-field



In Absence of Magnetic Field

In Presence of Magnetic Field

i.e. this is the total magnetic moment per unit volume.

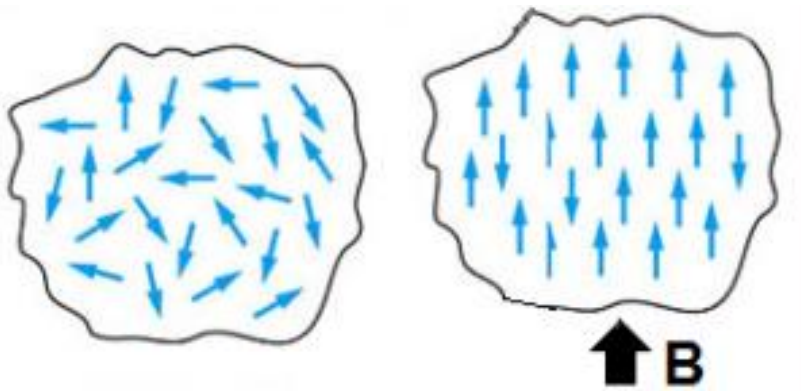
- Magnetisation is define as:
- $\underline{M} = n < \underline{\mu} >$
- Where n is the number of magnetic dipoles per unit volume.

Paramagnet in External B-field

The alignment of the magnetic dipoles produces an additional B-field B_m such that

$$B_m = \mu_0 M$$

- So total magnetic field inside paramagnet $B_T = B_0 + B_m$ where B_0 is the external B-field. i.e. $B_T = B_0 + \mu_0 M$.
- For “well behaved materials” $M \propto B_0$



In Absence of Magnetic Field

In Presence of Magnetic Field

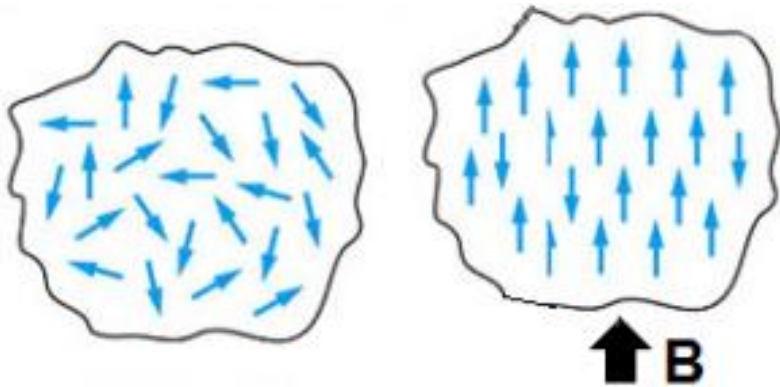
Paramagnet in External B-field

$$B_T = B_0 + \mu_0 M.$$

$M \propto B_0$ and we define the **magnetic susceptibility**,

χ_m (*dimensionless*) such that: $M = \frac{\chi_m B_0}{\mu_0}$

$$\text{Hence: } B_T = B_0 + \mu_0 \frac{\chi_m B_0}{\mu_0} = (1 + \chi_m) B_0$$



The **relative permeability** μ_r is defined as $\mu_r = (1 + \chi_m)$
So the B-field is increased by a factor μ_r

In Absence of Magnetic Field

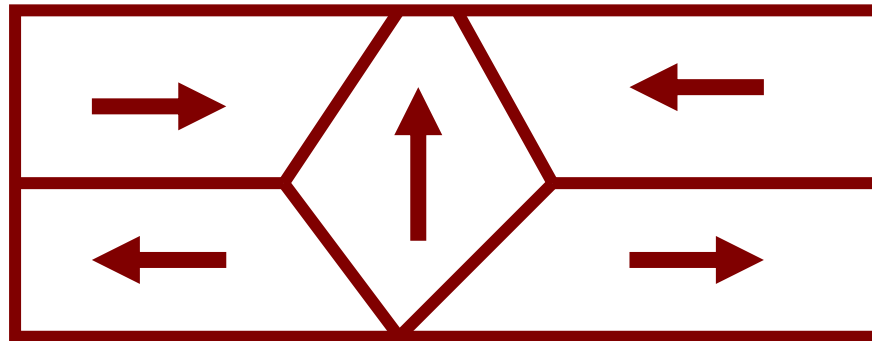
In Presence of Magnetic Field

Paramagnetic Material

- **Paramagnetic** materials have a small, positive susceptibility to magnetic fields. These materials are slightly attracted by a magnetic field and the material does not retain the magnetic properties when the external field is removed. Paramagnetic properties are due to the presence of some unpaired electrons, and from the realignment of the electron paths caused by the external magnetic field. Paramagnetic materials include magnesium, molybdenum, lithium, and tantalum.
- Paramagnetic materials have μ_r greater than but close to 1. μ_r doesn't change much with B-field strength but does with temperature.

Ferromagnetism

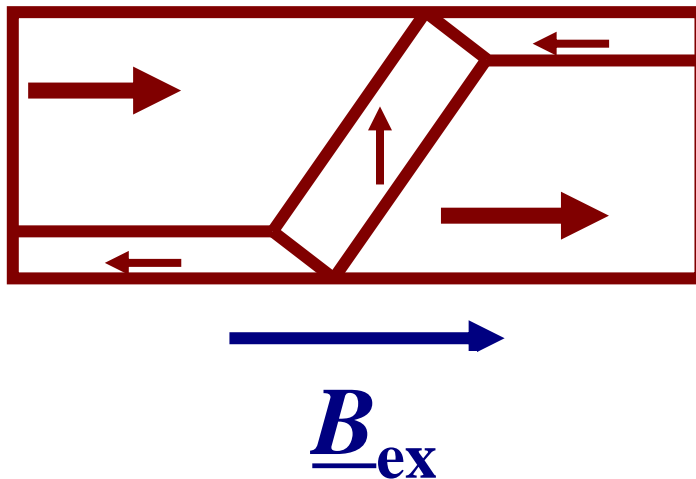
- Materials with atoms having unpaired electron spins (iron, cobalt, nickel, gadolinium, and dysprosium). Electron spins aligned $\sim 10^{10}$ atoms combine to form a *domain* ($l \sim 10^{-7}$ m) \Rightarrow large magnetic moment.



- When the domains are randomly arranged, the specimen as a whole is not magnetised.

Ferromagnetism

- Domains which are magnetized in the direction of an external magnetic field grow at the expense of those which are not aligned to the magnetic field.

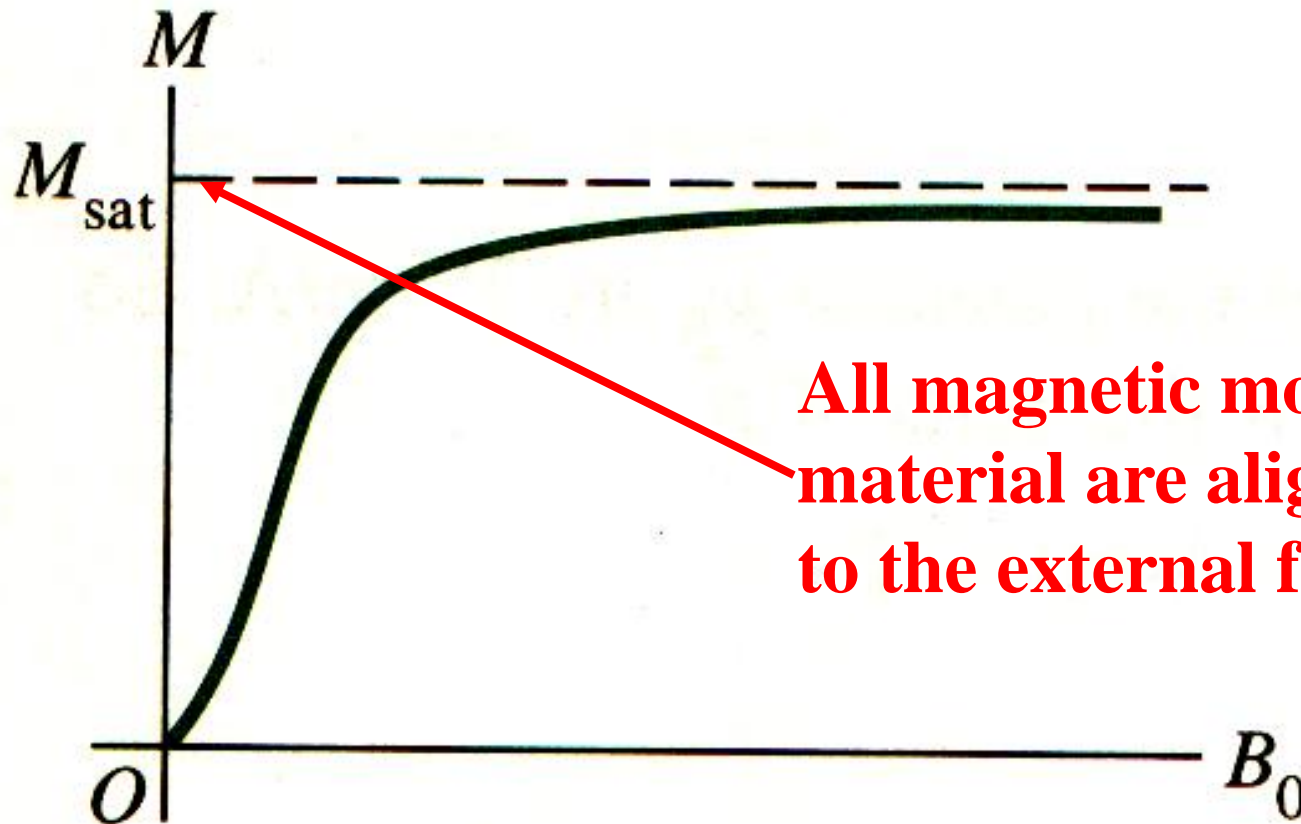


$$B_{internal} = \mu_r B_{ex}$$

- μ_r becomes very large $\sim 10^3 \rightarrow 10^5$

Ferromagnetism

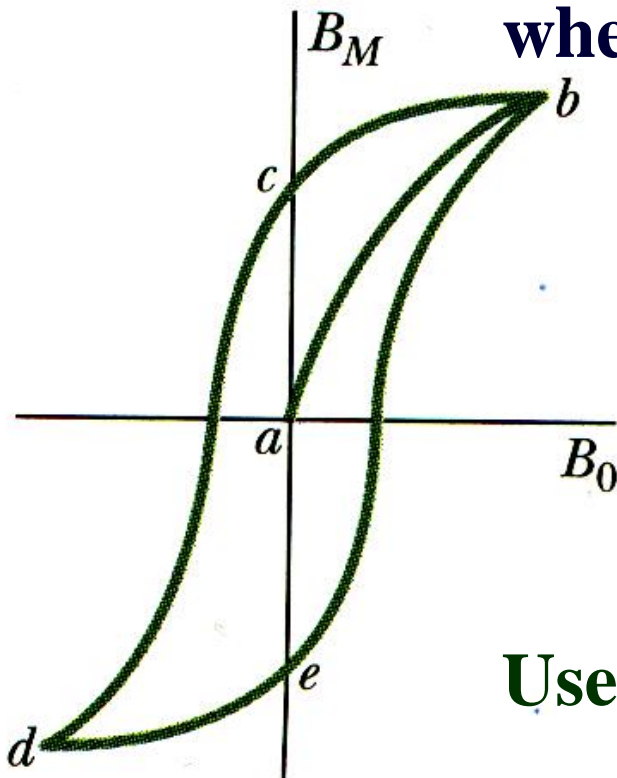
Magnetization Curve for a Ferromagnetic Material



All magnetic moments in the material are aligned parallel to the external field

Ferromagnetism: Hysteresis Loops

Magnetization is different when the external magnetic field is increasing from when it is decreasing - **hysteresis loop**



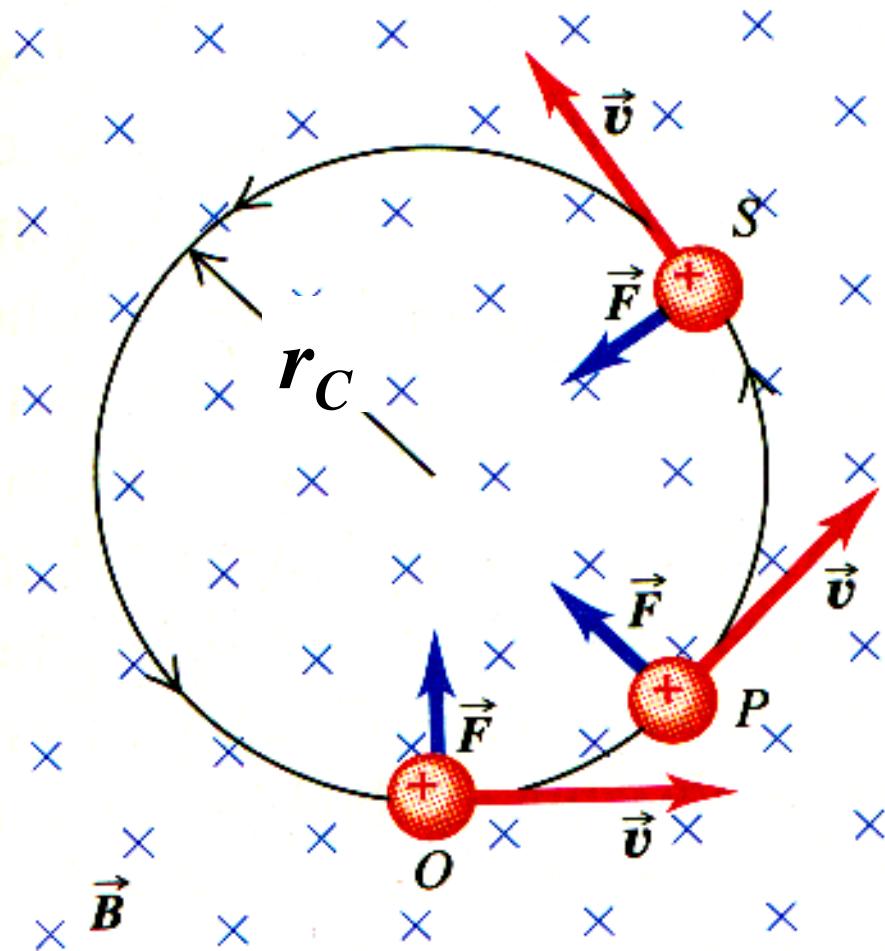
Explanation - reorientations of domain directions are not totally reversible

Uses - magnetic storage of information

Applications of Ferromagnetism

Material	$\mu/(\text{H m}^{-1})$	μ_r	Application
Ferrite U 60	1.00E-05	8	UHF chokes
Ferrite M33	9.42E-04	750	Resonant circuit RM cores
Nickel (99% pure)	7.54E-04	600	-
Ferrite N41	3.77E-03	3000	Power circuits
Iron (99.8% pure)	6.28E-03	5000	-
Ferrite T38	1.26E-02	10000	Broadband transformers
Silicon GO steel	5.03E-02	40000	Dynamos, mains transformers
supermalloy	1.26	1000000	Recording heads

Diamagnetism



- No intrinsic magnetic dipoles
- Dipole moments induced by an external magnetic field
- Induced B-field opposes the external field (Lenz's law) $B < B_0$
- i.e. $\mu_r < 1$ (slightly)

Relative Permeabilities

Paramagnetic		Diamagnetic		Ferromagnetic	
material	μ_r	material	μ_r	material	μ_r
Platinum	1.000265	Copper	0.999994	Iron (99.95% pure Fe annealed in H)	200,000
Aluminum	1.000022	Water	0.999992	Mu-metal	20,000
Air	1.00000037	Bismuth	$1 - 1.9 \times 10^{-5}$	Cobalt-iron (high permeability strip material)	18000
Oxygen	1.00133	Beryllium	$1 - 1.3 \times 10^{-5}$	Iron (99.8% pure)	5,000
Liquid Oxygen (-190°C)	1.00327	Methane	$1 - 3.1 \times 10^{-5}$	Ferrite (manganese zinc)	640+
Nickel Monoxide	1.000675	Glass	$1 - 1.5 \times 10^{-5}$	Carbon steel	100



Summary

- **Paramagnetic** materials are (weakly) attracted by a magnetic field, have intrinsic magnetic dipole moments that tend to line up with an external magnetic field, **thus enhancing (slightly) the field**. This tendency is interfered with by thermal agitation.
- **Diamagnetic** materials are (weakly) repelled by the pole of a strong magnet. The atoms of such materials do not have intrinsic magnetic dipole moments. A dipole moment may be induced, however, by an external magnetic field, its direction being **opposite that of the field**. Not temperature dependent.
- **$\mu_r \approx 1$ in both cases**



Summary

- **Ferromagnetic** materials result from spontaneous alignment of the dipole moment of atoms and the formation of large magnetic domains. Temperature dependant and Magnetic field dependant.
- $\mu_r \gg 1$