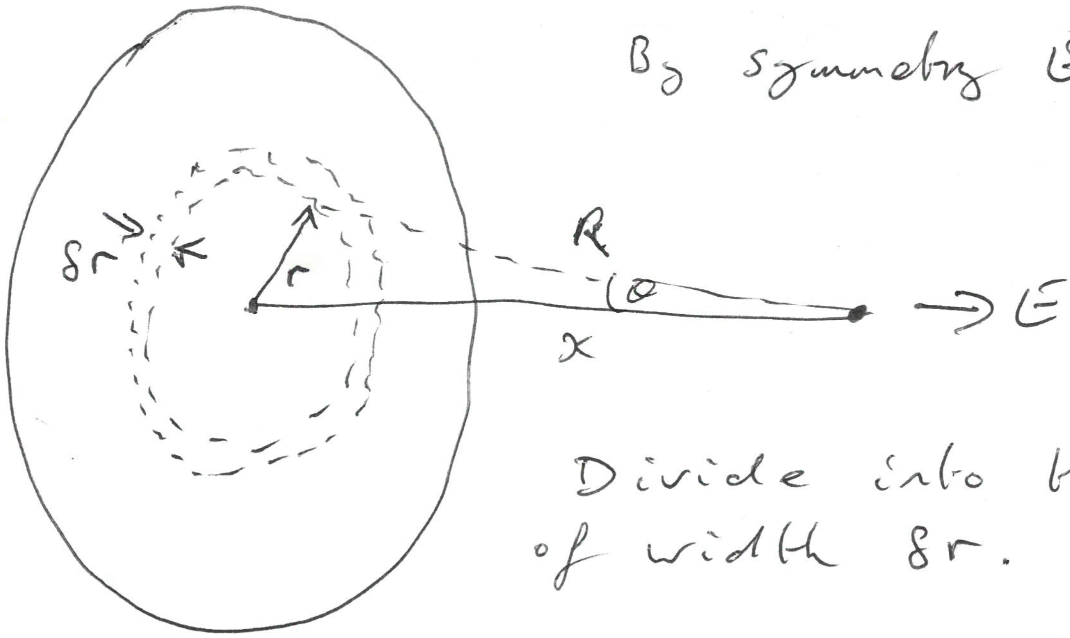
EM1 - lec 3

Example 3.1 uniform plane of surface charge density, σ.



By symmetry $E_y = 0$

Divide into thin rings of width δr.

From Example 2.2, E-field from thin ring

$$\delta E_{ring} = \frac{Q_{ring}}{4\pi\varepsilon_0} \frac{x}{[r^2 + x^2]^{3/2}}$$

$$Q_{ring} = \sigma \, 2\pi r \, \delta r$$

$$\delta E_{ring} = \frac{\sigma \, r \, \delta r}{2\varepsilon_0} \frac{x}{[r^2 + x^2]^{3/2}}$$

Ex 3.1

$$r = x \tan\theta \quad \Rightarrow \quad dr = \frac{x}{\cos^2\theta}\, d\theta$$

$$\cos\theta = \frac{x}{R} = \frac{x}{[r^2 + x^2]^{1/2}}$$

$$\therefore \frac{1}{[r^2 + x^2]^{3/2}} = \frac{\cos^3\theta}{x^3}$$

Sub in.

$$\delta E_{ring} = \frac{\sigma}{2\varepsilon_0} \cdot \Big(\underbrace{x \frac{\sin\theta}{\cos\theta}}_{r}\Big)\Big(\underbrace{\frac{x\,\delta\theta}{\cos^2\theta}}_{\delta r}\Big) x \cdot \frac{\cos^3\theta}{x^3}$$

$$= \frac{\sigma}{2\varepsilon_0} \sin\theta \,\delta\theta$$

$$\therefore E_{disc} = \frac{\sigma}{2\varepsilon_0} \int_0^{\theta_{max}} \sin\theta \, d\theta$$

Ex 3.1

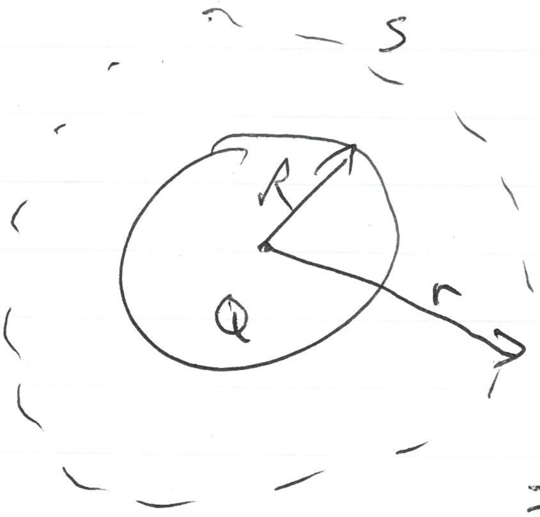
For infinite plane. $\theta_{max} = \pi/2$

$$E = \frac{\sigma}{2\varepsilon_0} \int_0^{\pi/2} \sin\theta \, d\theta$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

(Also true for finite plane if $x \ll$ radius of disc.)

Example 3.2



Gauss's law

$$\int_S \underline{E} \cdot d\underline{S} = \frac{Q_{en}}{\varepsilon_0}$$

By symmetry $\underline{E} \parallel d\underline{S}$

$$\Rightarrow \int_S \underline{E} \cdot d\underline{S} = \int_S E \, dS$$

Also, by symmetry E is constant for fixed r

$$\therefore \text{LHS}: \Rightarrow E \int_S dS = E \cdot 4\pi r^2$$

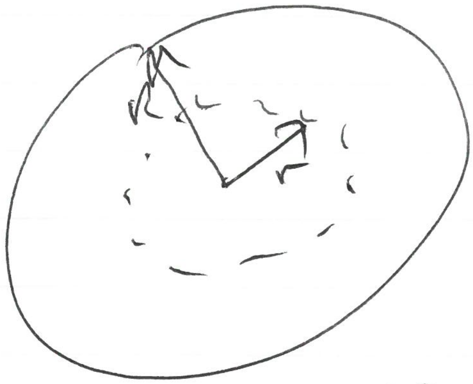
$$\therefore E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi \varepsilon_0 r^2}$$

ie. Coulomb's law!

$\boxed{\text{Ex 3.2}}$ (b)

Inside



$$\int_S \underline{E} \cdot d\underline{s} = \frac{Q_{enc}}{\varepsilon_0}$$

LHS $\Rightarrow$ $E \, 4\pi r^2$

as before $\Rightarrow$ symmetry.

$$Q_{enc} = Q \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} = \underline{\frac{Q r^3}{R^3}}$$

LHS = RHS

$$\therefore \quad E \, 4\pi r^2 = \frac{Q \, r^3}{\varepsilon_0 R^3}$$

$$\text{ie} \quad \underline{\underline{E = \frac{Q \, r}{4\pi \varepsilon_0 R^3}}}$$