L2 **The Ultraviolet Catastrophe**

Welcome to the UV catastrophe...

Blackbody radiation

A 'black body' is an idealised object that
- Does not reflect any light at any wavelength
- Absorbs internally all incident light (none shines out the other side)

→ ie is perfectly black ('vantablack' is close)

All bodies ~~m~~ emit electromagnetic energy.
Paul Hollywood (and other humans) emits at 300K → infrared
→ army night vision goggles

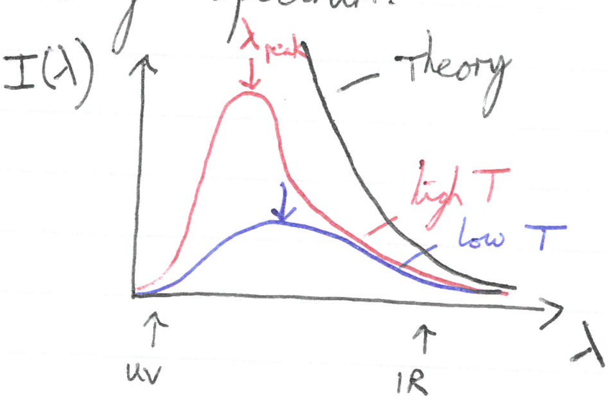
For the blackbody, the emission spectrum is only from this thermal emission (no reflection etc)

Hotter things are brighter and bluer (high energy, short wavelength)
↖ Think of hotter flames going blue

Blackbody Radiation Spectra

Classical thermodynamics gets the blackbody spectrum totally wrong - especially at small wavelengths
→ "UV catastrophe"

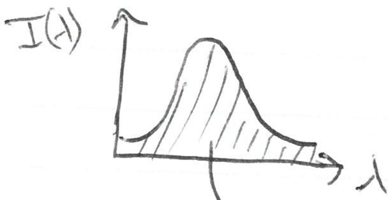
- Blackbody spectrum:



$I(\lambda)$ is the intensity per wavelength, at a wavelength λ

I, intensity, is the total energy per time, $I = \int_0^\infty I(\lambda)\, d\lambda$

in Wm^{-2} ie Power per area, adding up all colours



I is area under curve → total light

<u>Empirical results:</u>

- Stefan - Boltzmann Law: $I = \sigma T^4$

  ↑ Stefan - Boltzmann constant

  $\sigma = 5.67 \times 10^{-8}\ Wm^{-2} K^{-4}$

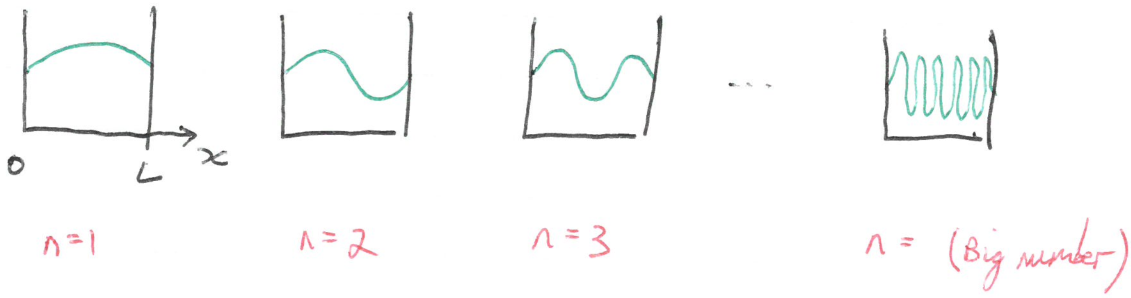
- Wien's Displacement Law: $\lambda_{peak} = \frac{b}{T}$ ← 2.898×10^{-3} Km

L2

- Why does classical thermodynamics break?

Model $I(\lambda)$ spectrum by slotting standing waves into a cavity

Consider a 1D cavity of length L



"Cavity modes" → the only allowed waves in there

Amplitude: $a(x) = \sin\left(\frac{n\pi x}{L}\right)$, $n = 1, 2, 3 \ldots \infty$

↖ 'Mode number'

- Relationship between n and λ:

$$\lambda = \frac{2L}{n}$$ (look at the pictures above!)

⇒ number of modes per wavelength,

$$n(\lambda) = \frac{2L}{\lambda}$$

Classically:

$$I(\lambda) \propto \frac{n(\lambda)}{\lambda} k_B T \propto \frac{1}{\lambda^2}$$

$n(\lambda) \propto \frac{1}{\lambda}$

density of modes at λ

average energy of modes

$I(\lambda) \propto \frac{1}{\lambda^2}$ $\Rightarrow$ for $\lambda \to 0$, $I(\lambda) \to \infty$

$\Rightarrow$ CATASTROPHE

- In 3D, same argument gives $n(\lambda) \propto \frac{1}{\lambda^3}$

$$\Rightarrow \quad I(\lambda) = \frac{2\pi c}{\lambda^4} k_B T$$

(Rayleigh-Jeans Law)

CATASTROPHE - works at large λ though

$\Rightarrow$ The problem is the 'k_BT' bit. The assumption is that all cavity modes have an average energy k_BT - "Equipartition Theorem" - you will meet this in later courses.

In brief: Probability distribution of energies is Boltzmann distribution:

$$p(E) = \frac{e^{-E/k_BT}}{k_BT}$$

k_B is Boltzmann constant

Average energy:

$$\bar{E} = \int_0^\infty E\, p(E)\, dE = k_BT$$

(see problem set)

- Planck's hypothesis (desperate, crazy!):

'Energy comes in discrete packets, 'quanta', that are proportional to frequency'

$$\Delta E = hf = \frac{hc}{\lambda}$$

[$h = 6.626 \times 10^{-34}$ Js
Planck's Constant

Sticking this into the Partition Function of statistical mechanics (will learn this in later courses, don't worry now!), we get an average energy:

$$\bar{E}(\lambda) = \frac{hc/\lambda}{e^{\frac{hc}{\lambda k_B T}} - 1}$$ instead of $k_B T$

Looking at limits:

→ $\bar{E}(\lambda \to \infty)$: $\frac{hc}{\lambda k_B T} \ll 1$

$$\therefore e^{\frac{hc}{\lambda k_B T}} \simeq 1 + \frac{hc}{\lambda k_B T} + \ldots$$

[Taylor series of e^x...

so $$\bar{E}(\lambda) \simeq \frac{\frac{hc}{\lambda}}{1 + \frac{hc}{\lambda k_B T} - 1} = k_B T$$

↓

classical behaviour recovered at low energy

$\rightarrow \bar{E}(\lambda \to 0)$: $e^{\frac{1}{0}} = e^{\infty} \to \infty$ (fast)

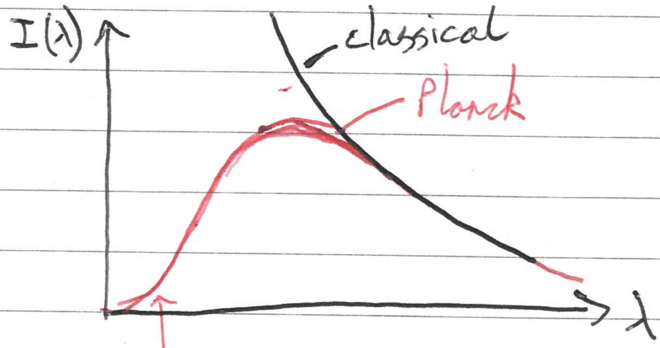
$$\therefore \bar{E}(\lambda \to 0) \simeq \frac{\frac{hc}{0}}{e^{1/0} - \cancel{1}}$$

small

Exp. term wins

$\bar{E}(\lambda \to 0) \to 0$ [Can do this rigorously via L'Hopital's Rule]

$\Rightarrow$ No catastrophe!



Energy per mode, $\bar{E}$, goes to 0 $\rightarrow$ brings curve down

|| Science Works!

Conclusion $\rightarrow$ This weird hypothesis fits the data

- Quantising energy means average energy of cavity modes is wavelength dependent
- Solves UV catastrophe
- Introduces idea of quantised 'packets' of energy