Year 1 Assessed Problems

Semester 1

Problem Sheet 8

SOLUTIONS TO BE SUBMITTED ON CANVAS

By 17:00hrs on Wednesday 27th November 2024

4 Assessed – Applications of differentiation

Problem 4.1 **Stationary Points of a Logarithmic Function**
Find and classify the stationary points of the function

$$f(x) = \ln\left(x^2 - x + 1\right) - x^2 + x \ .$$

What are the global maximum and minimum values of this function?

Assessed Problems 1

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03 33961 Introduction to Probability and Statistics

Problem 1. A busy shop has three queues of people. Unfortunately the three tills are somewhat unreliable.

- Till A fails with probability 0.1.
- Till B fails with probability 0.1.
- Till C fails with probability 0.05.

Once a till fails it must be reset, and then it continues to work. After an hour, 30 people used Till A, 35 used Till B and 35 used Till C.

1. What is the probability that someone (if selected at random) used till A or C? **[2]**
2. What is the probability of someone having a till failure if selected at random? **Hint**: Marginalise. **[2]**
3. Two poeple are selected at random. What is the probability that exactly one of them had a till failure? **[2]**
4. Thee people are selected at random. What is the probability that they all used different tills? State your sampling assumptions (with or without replacement). **[1]**
5. Given that someone experienced a failing till, what is the probability that the till they used was B? **[2]**
6. What is the probability that that person did not use Till B? **[1]**

Dynamical Systems Assessed Problem 1

Problem 1.3 **Range and Height of a Projectile**
A particle is projected from the origin in the xy-plane, where gravity acts in the negative y-direction. The particle initially has velocity v at angle of elevation θ with respect to the x-axis, and air resistance may be neglected. You may freely use the results for $x(t)$, $y(t)$ and $y(x)$ derived in lectures.

(a) Derive expressions for the horizontal range, X, and the maximum height, H, in terms of the parameters of the problem. **[3]**

(b) Derive an expression for the angle of elevation, θ, as a function of X and H. **[2]**

(c) Derive an expression for the velocity of projection, v, in terms of X, H and g. **[3]**

(d) Derive an expression for the time of flight, T, in terms of X, H and g. **[2]**