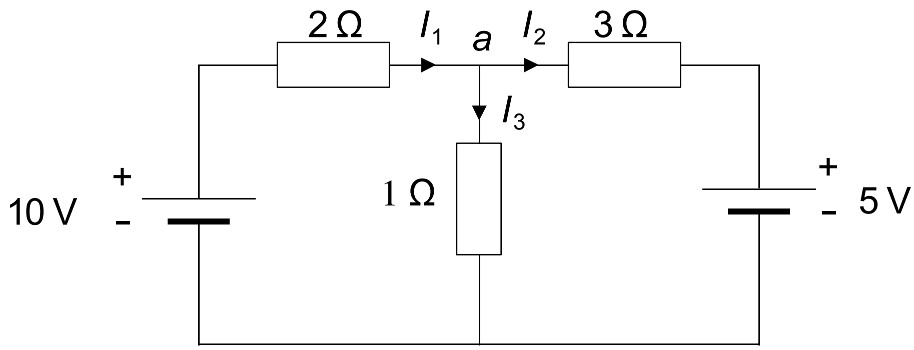
Electric Circuits
Solution non assessed Week 2

1. (a) Using Kirchhoff laws.



Applying Kirchhoff's current law at node *a*.

$$I_1 = I_2 + I_3 \qquad (1)$$

Applying Kirchhoff's voltage law in each internal loop.

$$10 - 2I_1 - I_3 = 0 \qquad (2)$$

$$I_3 - 3I_2 - 5 = 0 \qquad (3)$$

Substitute for I_1 in (2)

$$10 - 2I_2 - 3I_3 = 0 \qquad (4)$$

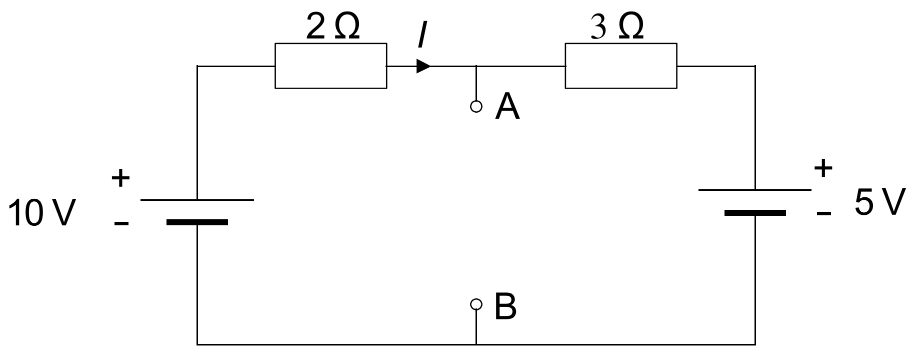
Solve to find I_3

The result is

$$I_3 = \frac{40}{11} = 3.64\,A$$

This is the current through the 1 Ω resistor, as required.

(b) Using Thévenin's theorem.



Remove the 1 Ω resistor and find the open circuit voltage V_{AB}. To do this, you first need to find the current. You can use Kirchhoff's voltage law to do this (or by inspection).

$$10 - 2I - 3I - 5 = 0$$

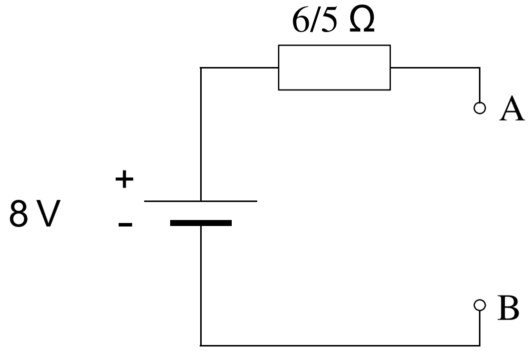
$$I = 1\mathrm{A}\,(\text{clockwise})$$

Now use Kirchhoff's voltage law to travel from A to B (either clockwise or anticlockwise) counting the potential rises and drops (and taking into account the direction of the current). Designating the potential at A as V_A and the potential at B as V_B, and going clockwise from A to B gives

$$V_A - 3I - 5 = V_B$$

$$V_A - V_B = 8\mathrm{V}$$

Point A is 8 V higher in potential than point B. (Crosscheck: in part (a) we assumed I_3 flowed from A to B and since we found I_3 was positive, this tells us that point A is at a higher potential than point B.) Therefore the Thévenin's equivalent voltage is 8 V. The resistance looking back into the terminals A and B is given by the *parallel* combination of the 2Ω and 3Ω resistors = 6/5Ω. (If this isn't completely obvious, consider what would happen if you injected some current at A when the other sources were switched off. The circuit provides two independent paths for charge to flow from A to B, one through the 2Ω resistor and one through the 3Ω resistor. Since a single charge carrier cannot flow through *both* resistors, the resistors must be in parallel,)



By adding the 1Ω resistor to the Thévenin equivalent circuit, the current is found by taking the ratio of the voltage to the total resistance:

$$I=\frac{V}{R}=\frac{8}{11/5}=\frac{40}{11}=3.64\,A$$