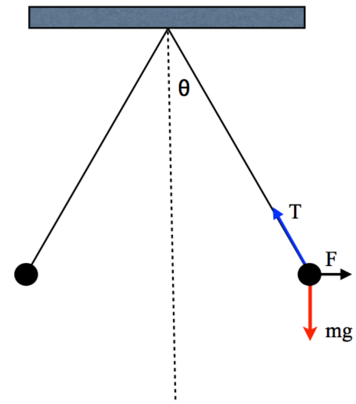
Electromagnetism I – Answers problem sheet 2

Problem 1.

1.1.a Considering the two balls with the same mass first. At equilibrium, the forces on the two masses (tension T, electrostatic force F and gravitational force mg) are the same.

Hence by symmetry the angles θ_1 and θ_2 are the same. Answer (a): $\theta_1 = \theta_2 = \theta$. **[1]**



Any sensible justification is fine i.e. both electrostatic and gravitation forces are the same on each ball. **1.1.b** The distance between the spheres is: $r = 2l\sin\theta$ where $\theta = \theta_1 = \theta_2$

So the elctrostatic force between the balls is:

$$F = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2} = \frac{1}{16\pi\epsilon_0}\frac{q^2}{l^2\sin^2\theta}$$

The horizontal force due to the tension is $F = T\sin\theta$
but the vertical force due to tension is $T\cos\theta = mg$ Hence, the gravitational force on each sphere in the horizontal direction is $F = mg\tan\theta$
Equating gravitational and electrostatic forces gives:

$$F = \frac{1}{16\pi\epsilon_0}\frac{q^2}{l^2\sin^2\theta} = F = mg\tan\theta$$

As long as they get this expression they get the 1 mark

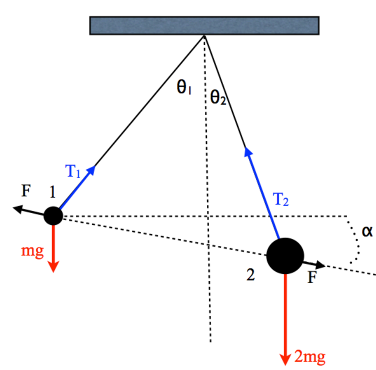
Hence:

$$\sin^2\theta\tan\theta = \frac{1}{16\pi\epsilon_0}\frac{q^2}{mgl^2}$$

[1]

1.2.a We now replace the sphere on the right hand side with a sphere twice as heavy. The diagramme on the right shown the forces on the two masses. **[1]**
As long as the diagram looks something like this, give them the 1 mark.

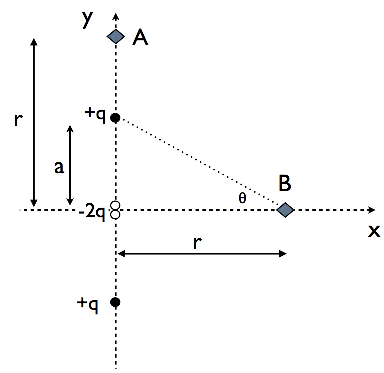
1.2.b The electrostatic force between the spheres remains the same but the gravitational force on the right sphere has increased. Hence the horzontal component would increase until the angle decreases to balance the foces. So, at equilibrium:



$$\theta_1 > \theta_2$$

The answer is (b) **[1]** Any sensible justification will do.

Problem 2.



Let's define

$$k = \frac{1}{4\pi\epsilon_0}$$

(just to avoid having to keep writing it) The students don't need to define k.

Considering point A. The electric field is radial from each charge, and therefore the x component is

$$E_x(A) = 0 \qquad [\mathbf{1}]$$

The y component is:

$$\begin{aligned} E_y(A) &= \frac{kq}{(r-a)^2} - 2\frac{kq}{r^2} + \frac{kq}{(r+a)^2} \qquad [\mathbf{1}] \\ &= \frac{kq}{r^2}\left[\frac{1}{(1-\frac{a}{r})^2} - 2 + \frac{1}{(1+\frac{a}{r})^2}\right] \end{aligned}$$

We now expand in terms of $\frac{a}{r}$ and keep only the leading order term:

$$\begin{aligned} E_y(A) &= \frac{kq}{r^2}\left[(1+2\frac{a}{r}+3\frac{a^2}{r^2}) - 2 + (1-2\frac{a}{r}+3\frac{a^2}{r^2})\right] + \mathcal{O}(\frac{a^3}{r^3}) \\ &\approx \frac{kq}{r^2}(6\frac{a^2}{r^2}) = \frac{3q}{2\pi\epsilon_0}\left(\frac{a^2}{r^4}\right) \qquad [\mathbf{1}] \end{aligned}$$

Provided students get the right answer, they get the marks

Considering now point B. E from the negative charges is along the x axis. Based on symmetry, the y components of the field from the positive charges are equal and opposite, and therefore cancel. Therefore the y component of the field is:

$$E_y(B) = 0 \qquad [\mathbf{1}]$$

The x component is now:

$$\begin{aligned}
E_x(B) &= \frac{2kq}{(r^2+a^2)}\cos\theta - \frac{2kq}{r^2} && [\mathbf{1}] \\
&= \frac{2kq}{(r^2+a^2)}\frac{r}{(r^2+a^2)^{1/2}} - \frac{2kq}{r^2} \\
&= 2kq\frac{r}{(r^2+a^2)^{3/2}} - \frac{2kq}{r^2} \\
&= 2kq\frac{r}{r^3(1+\frac{a^2}{r^2})^{3/2}} - \frac{2kq}{r^2} \\
&= \frac{2kq}{r^2}\left[(1+\frac{a^2}{r^2})^{-3/2} - 1\right] \\
&= \frac{2kq}{r^2}\left[(1-\frac{3}{2}\frac{a^2}{r^2} - 1\right] + \mathcal{O}(\frac{a^4}{r^4}) \\
&\approx \frac{2kq}{r^2}\left(-\frac{3}{2}\frac{a^2}{r^2}\right) = -\frac{3q}{4\pi\epsilon_0}\left(\frac{a^2}{r^4}\right) && [\mathbf{1}]
\end{aligned}$$

Provided students get the right answer, they get the marks