3 Probability revision – solutions

The problems are roughly in order of difficulty.

Problem 3.1 **More complicated outcomes**

1. There are 2 ways out of 6 possibilities, so $p = 1/3$.
2. There are 3 ways out of six, so $p = 1/2$.

Problem 3.2 **Many balls**

1. Total number of balls is 9, of which 3 are black, therefore $p(\text{black}) = 1/3$.
2. Number of ways to get not red is 5, therefore $p = 5/9$.

Problem 3.3 **Family probabilities**

1. There are 4 possible arrangements and 3 of these have at least one girl, therefore $p = 3/4$.
2. If at least one is a girl, we are now limited to 3 of the possible 4 arrangements: out of these 3, only one contains 2 girls, therefore $p = 1/3$.
3. $p = 1/2$. Chance has no memory!

Problem 3.4 **An irritating queue**
To be chosen on the third call requires you not to be chosen on any of the first 2. Therefore

$$\begin{aligned} p(\text{chosen on 3rd call, but not on first two}) &= p(\text{not being chosen on first 2 calls}) \times p(\text{chosen on 3rd}) \\ &= 9/10 \times 8/9 \times 1/8 = 1/10 \ . \end{aligned}$$

Problem 3.5 **Monkeys, typewriters and Shakespeare**
The probability of getting the first character right is 1/44.

The probability of getting 10^5 consecutive characters right is $(1/44)^{10^5}$.

Your calculator probably won't cope with this one, but you can take logs:

$$\log(\text{probability}) = 10^5 \times \log(1/44) = -164,345 \ .$$

so the probability of getting 10^5 consecutive characters right is $10^{-164,345}$.

Now we need to work out the number of attempts that the monkeys get. The monkeys can type 10×10^{18} characters in the age of the universe and so there are $10^{19} - 10^5 \simeq 10^{19}$ sequences of 10^5 successive characters in which Hamlet could occur. Also there are 10^{20} monkeys.

The probability of typing Hamlet is therefore $10^{-164,345} \times 10^{19} \times 10^{20} = 10^{-164,306}$.

A good enough definition of impossible?

Problem 3.6 **Penalty shoot-out**

1. $p(\text{exactly 2 goals}) = \frac{3}{4}\frac{3}{4}\frac{1}{4}\frac{1}{4}\frac{1}{4}\ {}^5C_2$, where the binomial coefficient ${}^5C_2$ accounts for the fact that it doesn't matter which 2 of the 5 penalties were scored.

$$ {}^5C_2 = \frac{5!}{3!2!} \quad \Rightarrow \quad p(\text{exactly 2 goals}) = \frac{90}{1024} \; . $$

2.
$$ p(\text{at least one goal}) = 1 - p(\text{no goals}) = 1 - \left(\frac{1}{4}\right)^5 = \frac{1023}{1024} \; . $$

Problem 3.7 **Bill and Ben, gambling men**

There are 6 different ways to throw a double out of a total of 36 possibilities. The probability of a double on a single throw is thus 1/6.

The probability that Bill is first to throw a double is the probability that he throws one straight away (1/6) plus the probability that neither he nor Ben throws a double in the first round ($5/6 \times 5/6$) multiplied by the probability that Bill throws a double on the second round (1/6), and so on .....

$$ \begin{aligned} p(\text{Bill wins}) &= \frac{1}{6} + \frac{1}{6}\left(\frac{5}{6}\right)^2 + \frac{1}{6}\left(\frac{5}{6}\right)^4 + \cdots \\ &= \frac{1}{6}\left[1 + \left(\frac{25}{36}\right) + \left(\frac{25}{36}\right)^2 + \cdots\right] \; . \end{aligned} $$

which is a geometric series, which we must sum to infinity:

$$ \begin{aligned} p(\text{Bill wins}) &= \frac{1}{6}\sum_{n=1}^{\infty}\left(\frac{25}{36}\right)^{n-1} \\ &= \frac{1}{6}\frac{1}{(1 - 25/36)} = \frac{2}{3} \; . \end{aligned} $$

Problem 3.8 **Combinatorics**
There are 7 ways (available microstates): taking the atoms in turn, we can have

$$\begin{array}{ccc}
1\epsilon & 1\epsilon & 1\epsilon \\
2\epsilon & 1\epsilon & 0 \\
2\epsilon & 0 & 1\epsilon \\
0 & 2\epsilon & 1\epsilon \\
1\epsilon & 2\epsilon & 0 \\
1\epsilon & 0 & 2\epsilon \\
0 & 1\epsilon & 2\epsilon
\end{array}$$

So 7 ways altogether.