Electromagnetism

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Lecture 4
Gauss's Law 2
More examples
Week 2

Last Lecture

- Some more examples of continuous charge distributions
  - Infinite plane
  - Inside charged hollow sphere
- Electric flux
- Gauss's Law
  - Examples using Gauss's Law

Gauss's Law

- This is Gauss's Law
  - You need to know this and know how to use it

$$\boxed{\int_S \underline{E} \cdot d\underline{S} = \frac{Q_{encl}}{\varepsilon_0}}$$

- Very useful for solving problems where there's symmetry (see examples).

Johann Carl Friedrich Gauss

Born: 30 April 1777 in Brunswick, Duchy of Brunswick

Died: 23 Feb 1855 in Göttingen, Hanover

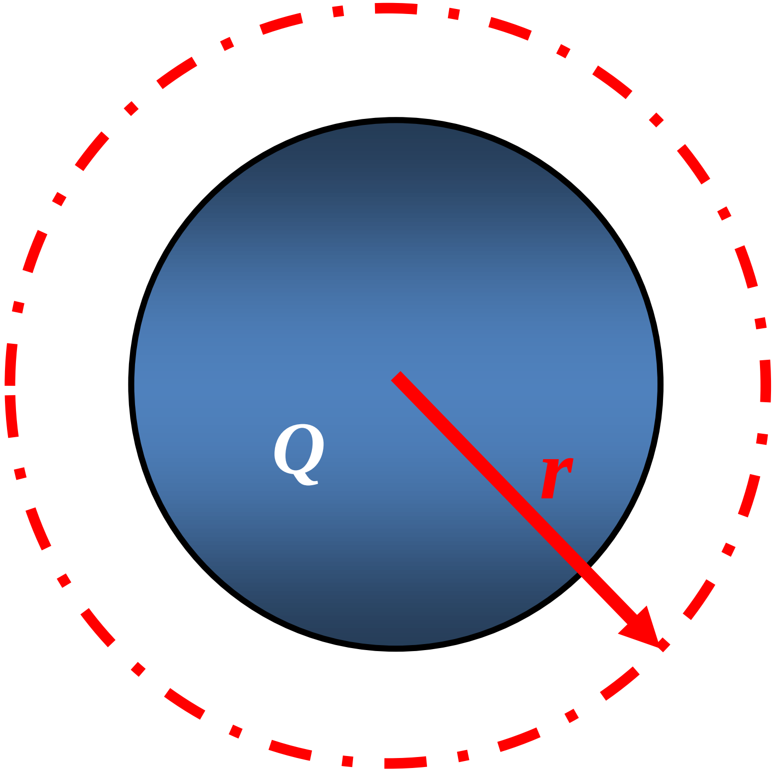
number theory, statistics, analysis, differential geometry, electrostatics, astronomy, and optics

Lecture 3 Content

- **More examples using Gauss's Law**

Solid Sphere with Uniform Charge

Example: Sphere of radius *R* uniformly charged throughout its volume. Total charge *Q*



(a) $\underline{E}$-field for $r > R$

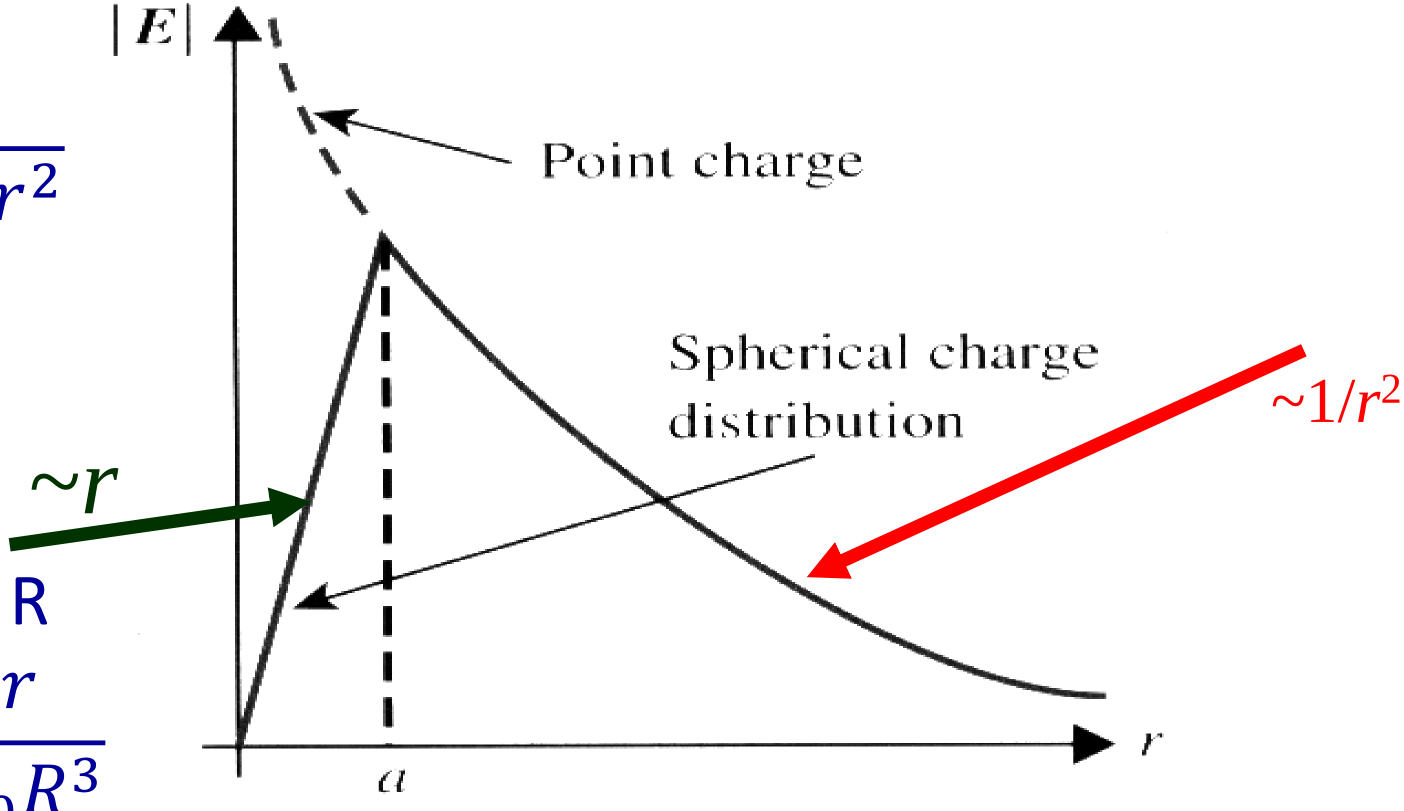
(b) $\underline{E}$-field for $r < R$

Method: set up an (imaginary) *Gaussian Surface* and use *symmetry*

E-field outside & inside uniform charged sphere

(a) Outside r > R

$$|E| = \frac{Q}{4\pi\varepsilon_0 r^2}$$



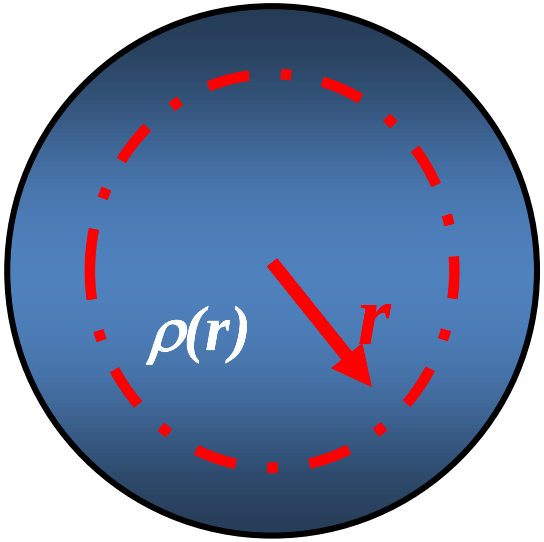
(b) Inside r < R

$$|E| = \frac{Q\,r}{4\pi\varepsilon_0 R^3}$$

What about Solid Sphere with Non-Uniform Charge

Example: Sphere of radius *R with charge density*

$$\rho(r) = \rho_0 r$$



$\underline{E}$-field for $r < R$

Now $Q_{encl} = \int_0^r \rho(r)dV$

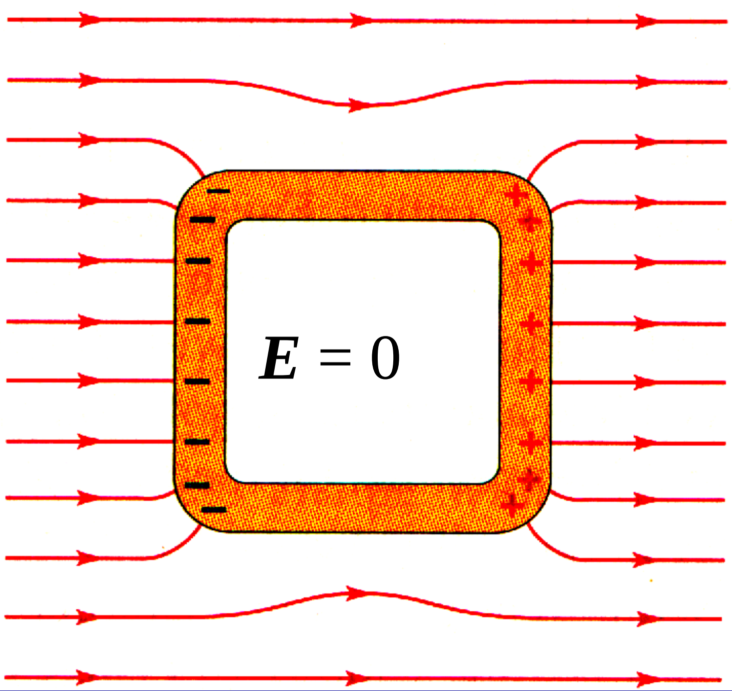
Use Gauss's Law and symmetry

$$\int_S \underline{E} \cdot d\underline{S} = \frac{1}{\varepsilon_0}\int_0^r \rho(r)dV$$

Let's solve this using the visualizer

E-fields in Conductors

- EXTERNAL $\underline{E}$-FIELD. Electrons are free to move, and exist in vast quantities.
- Free electrons drift to the surface until they create an equal but opposite E-field that cancels external E-field.



$\underline{E} = 0$ in a conductor

$$\int_S \underline{E} \cdot d\underline{S} = 0$$

Thus Q=0, free charge found on surface only

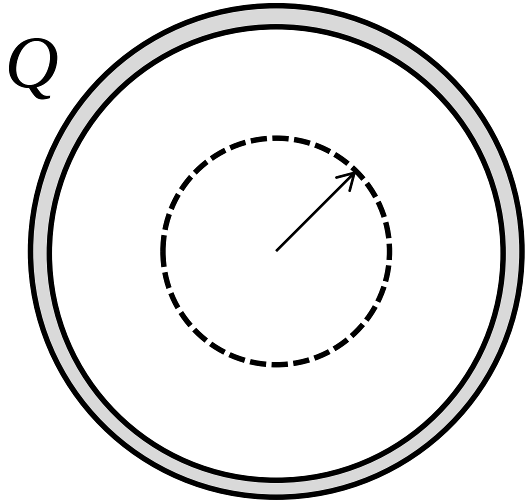
Application: shielding sensitive electronics from E-fields

Gauss's Law: More Examples

- Spherical Shell
- Infinite sheet of charge
  - Non-conducting
  - Conducting
- Infinite charged thin wire

E-field Inside Uniform Charged Shell

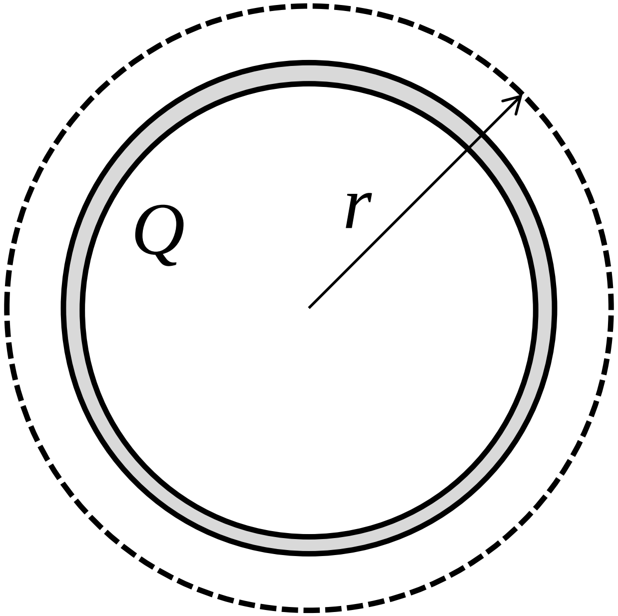
- We already know it's zero from before but now use Gauss's Law.



Inside shell (using Gauss's Law)

$$\int_S \underline{E} \cdot d\underline{S} = \frac{Q_{encl}}{\varepsilon_0} = 0 \;\rightarrow E = 0$$

E-field Inside Uniform Charged Shell



Inside shell (using Gauss's Law)

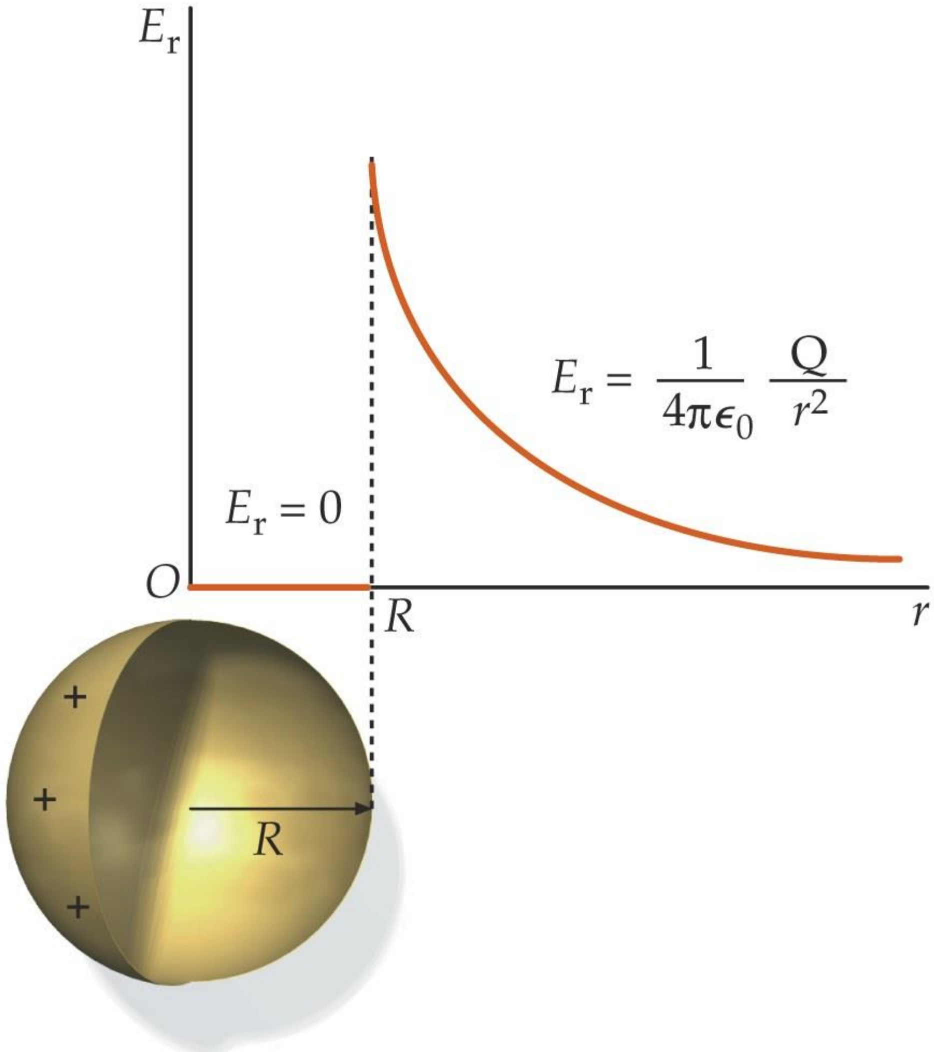
$$\int_S \underline{E} \cdot d\underline{S} = \frac{Q_{encl}}{\varepsilon_0} = 0 \;\rightarrow E = 0$$

Outside shell (using Gauss's Law)

$$\int_S \underline{E} \cdot d\underline{S} = \frac{Q}{\varepsilon_0} \rightarrow 4\pi r^2 E = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}$$

E-field Inside Uniform Charged Shell

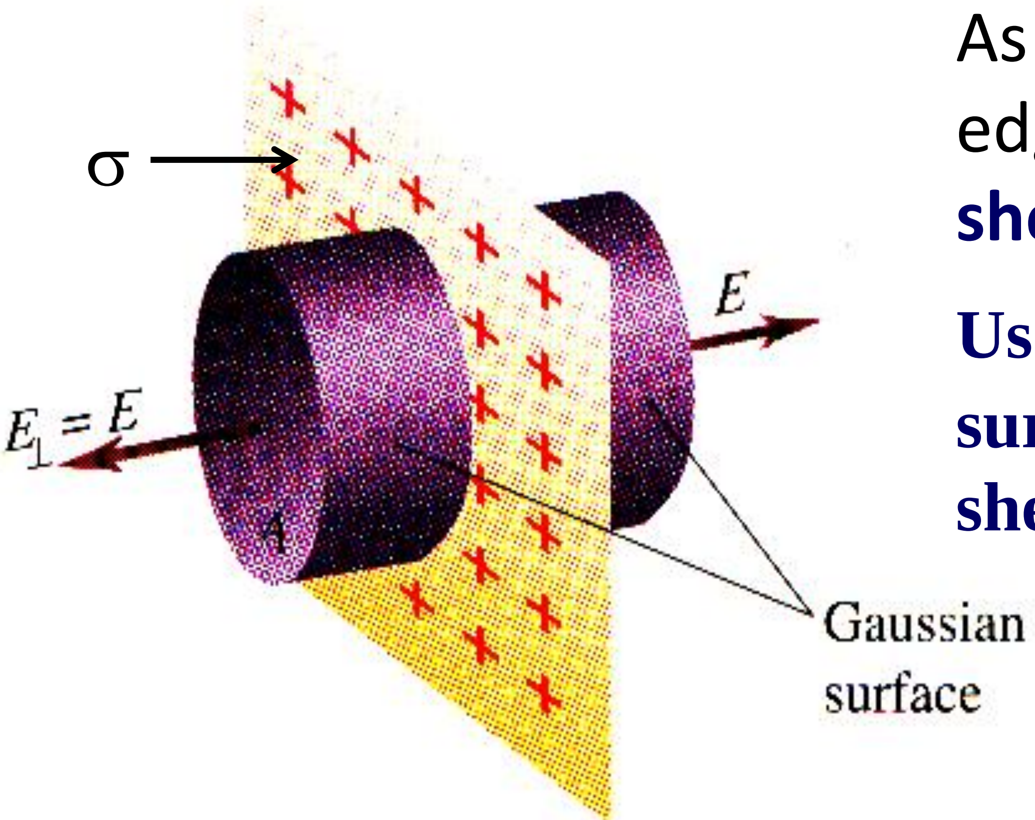


For ‘thin’ shell

Note: Same as for conducting material

Infinite Charged Sheet

- Non-Conducting (charge uniform throughout sheet) – surface charge density = σ



As sheet is infinite (i.e. no edge effects) ***E*** **is ⊥ to the sheet.**

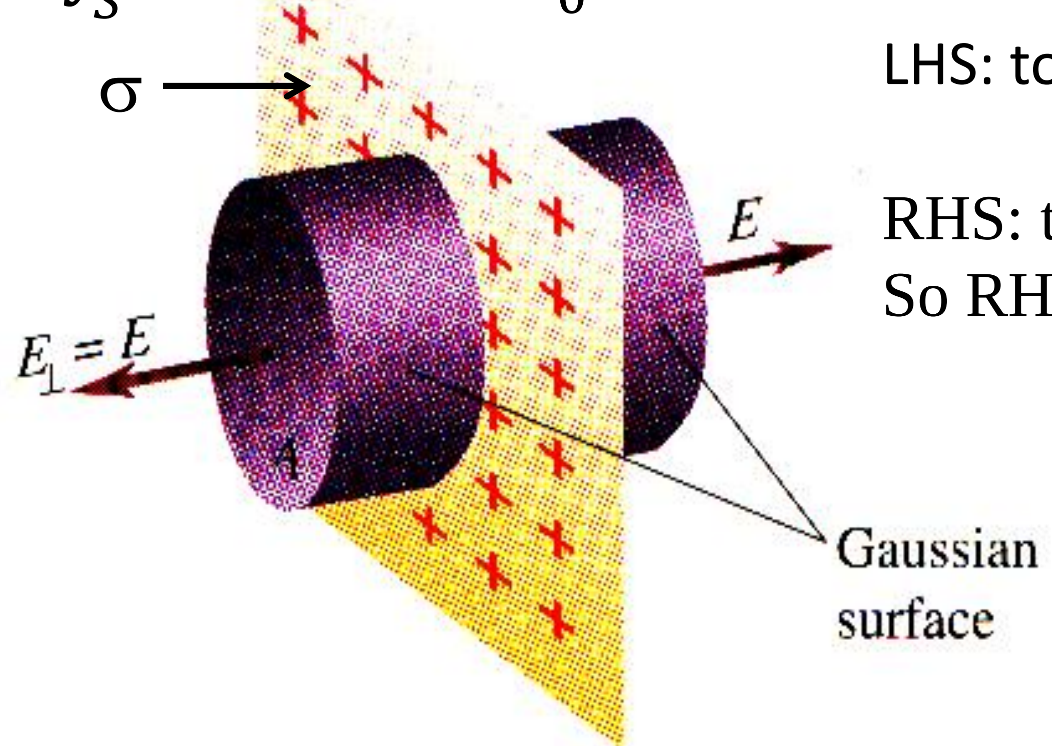
Use a cylindrical Gaussian surface - axis ⊥ to the sheet (as shown).

Infinite Charged Sheet

No E-field coming out of side walls, only ends

$$\int_S \underline{E} \cdot d\underline{S} = \frac{Q_{encl}}{\varepsilon_0}$$

Make area of end of cylindrical Gaussian surface = *A*



LHS: total flux, $\Phi = AE + AE = 2AE$

RHS: total charge on surface = $A\sigma$

So RHS = $A\sigma / \varepsilon_0$

$$E = \frac{\sigma}{2\varepsilon_0}$$

Infinite Charged Sheet

$$\boxed{E = \frac{s}{2e_0}}$$

The result agrees with the one we found by taking the infinite radius limit of a disk.

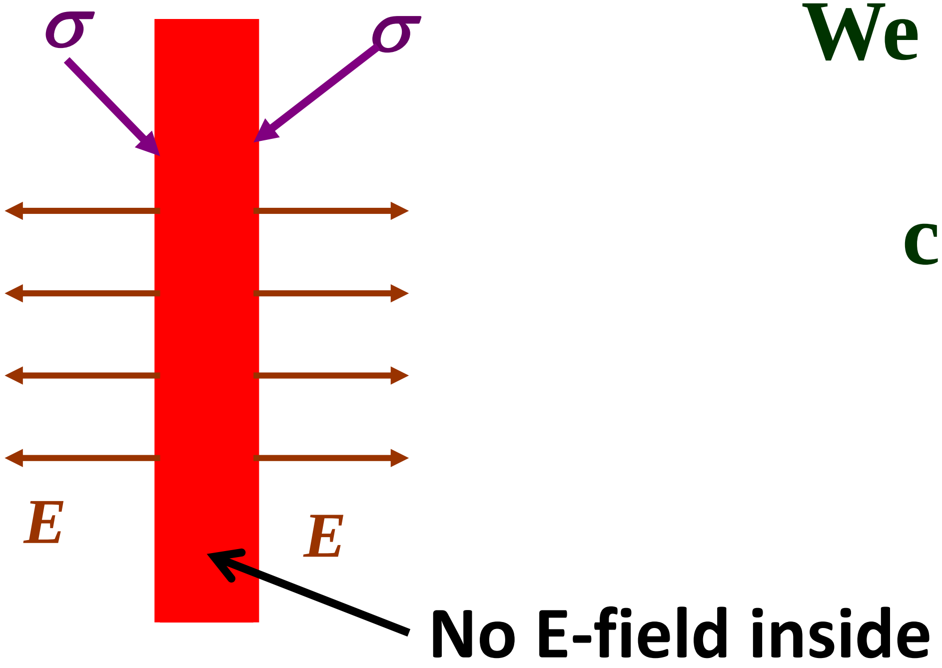
But much easier method (only works for infinite sheet, otherwise have to use integration as before).

Infinite Charged Sheet

- **Conducting** (charge only on surface, not inside conductor) – surface charge density = σ
- Charges move until $\underline{E} = 0$ inside a conductor
- No net charges exist inside a conductor
- **Free (extra) charges reside on surface**

E-field due to an infinite conducting sheet

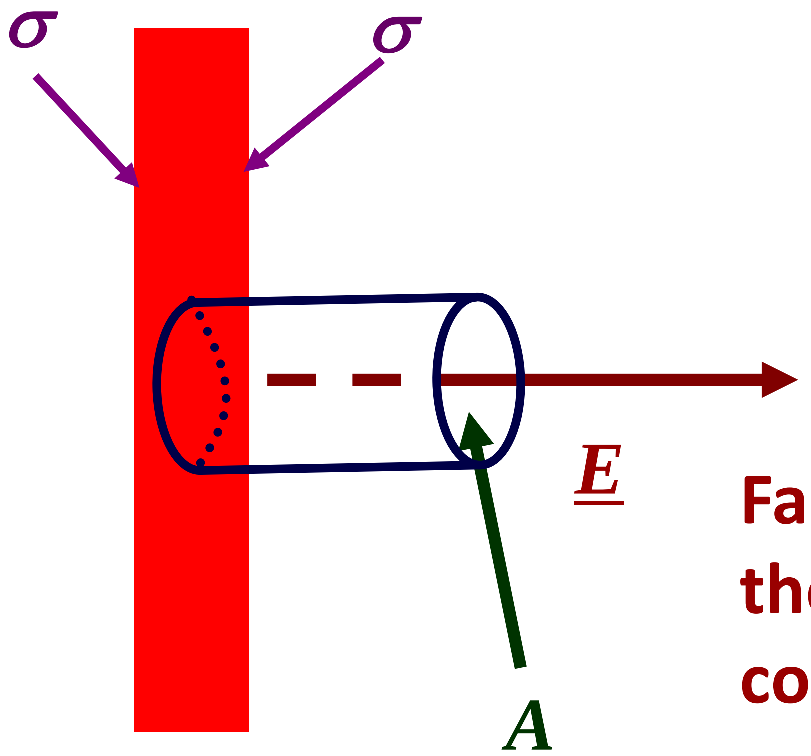
- Q distributes until there is a charge density σ on both surfaces



We now repeat the "pill box" construction.

E-field due to an infinite conducting sheet

- Apply Gauss's Law



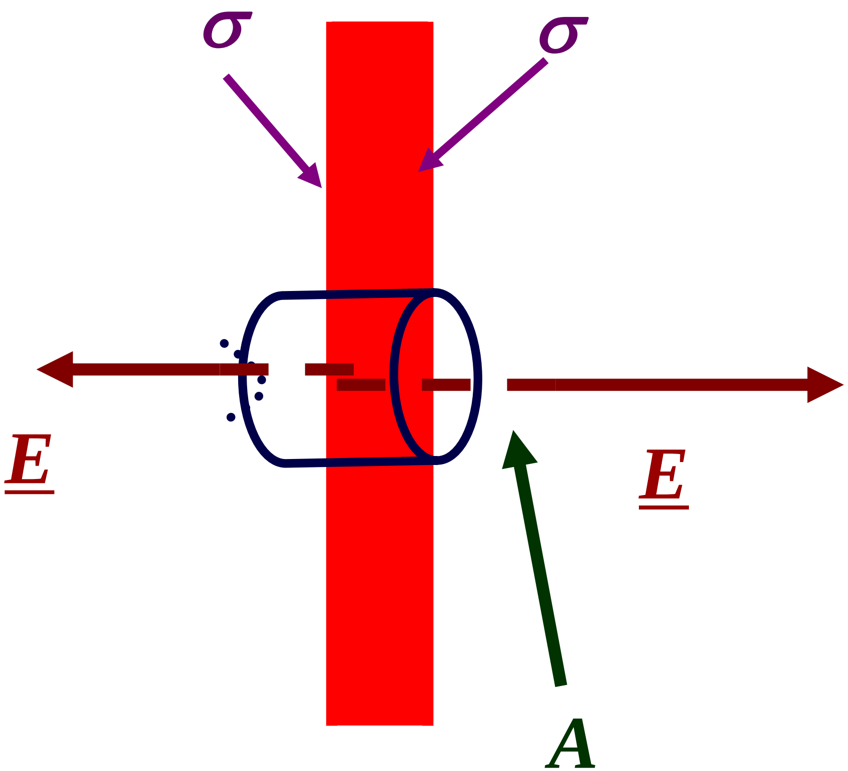
$$\Phi_E = EA = \frac{\sigma A}{\varepsilon_0}$$

$$\therefore \mathbf{E} = \frac{\boldsymbol{\sigma}}{\boldsymbol{\varepsilon_0}}$$

Factor of 2 difference between the fields of insulator and conducting infinite planes. Why?

$\underline{E}$-field due to an infinite conducting sheet

Let's put the Gaussian surface (pillbox) through the conducting sheet



$$f_E = 2EA = \frac{2sA}{e_0}$$

$$\backslash \quad E = \boxed{\frac{s}{e_0}}$$

E-field just out surface of charged conductor

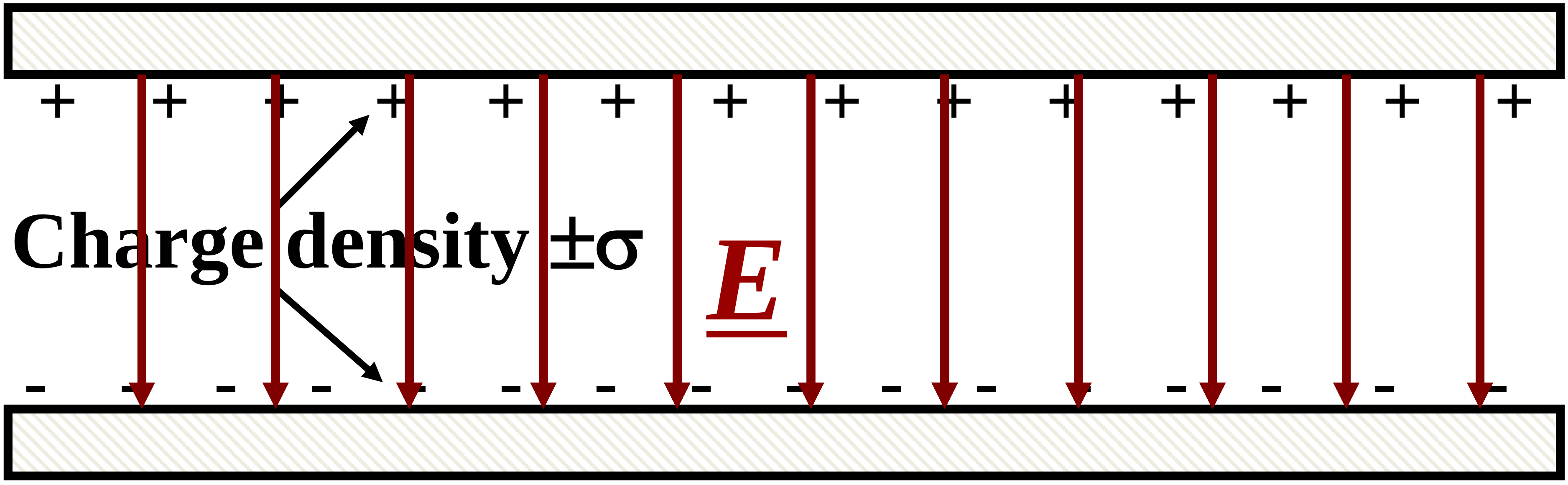
<u>*E*</u>-field just outside a charged conductor

$$\boxed{E = \frac{s}{e_0}}$$

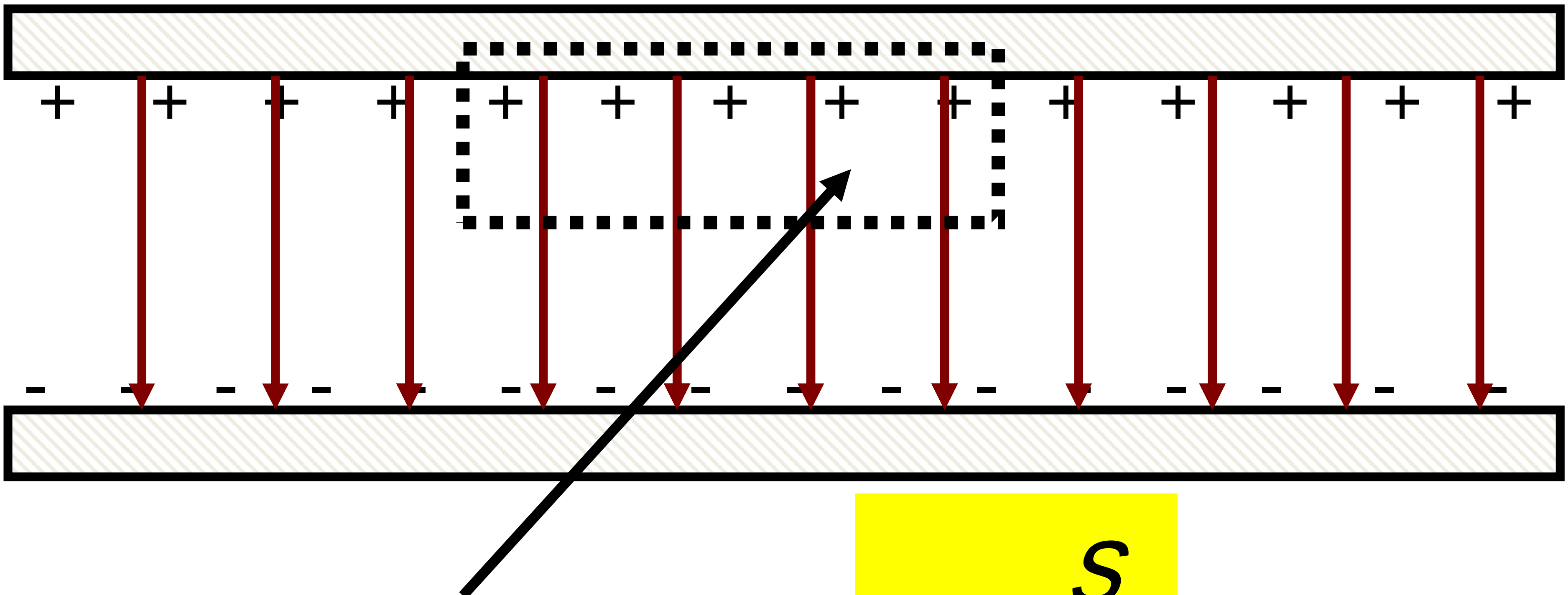
σ is the surface charge density.

E-field between Charged Metal Plates

E-field between parallel charged metal (i.e. conducting) plates



E-field between Charged Metal Plates

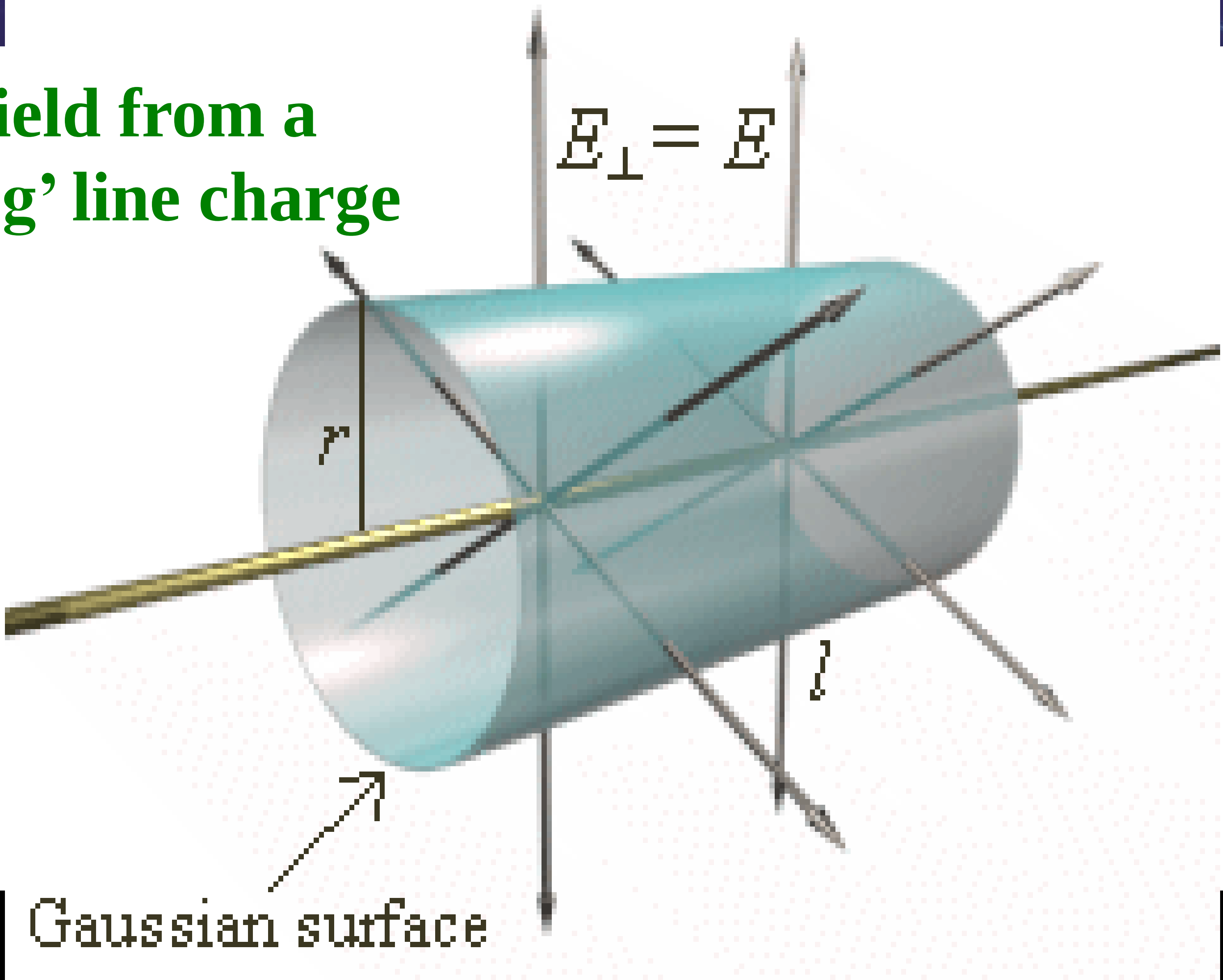


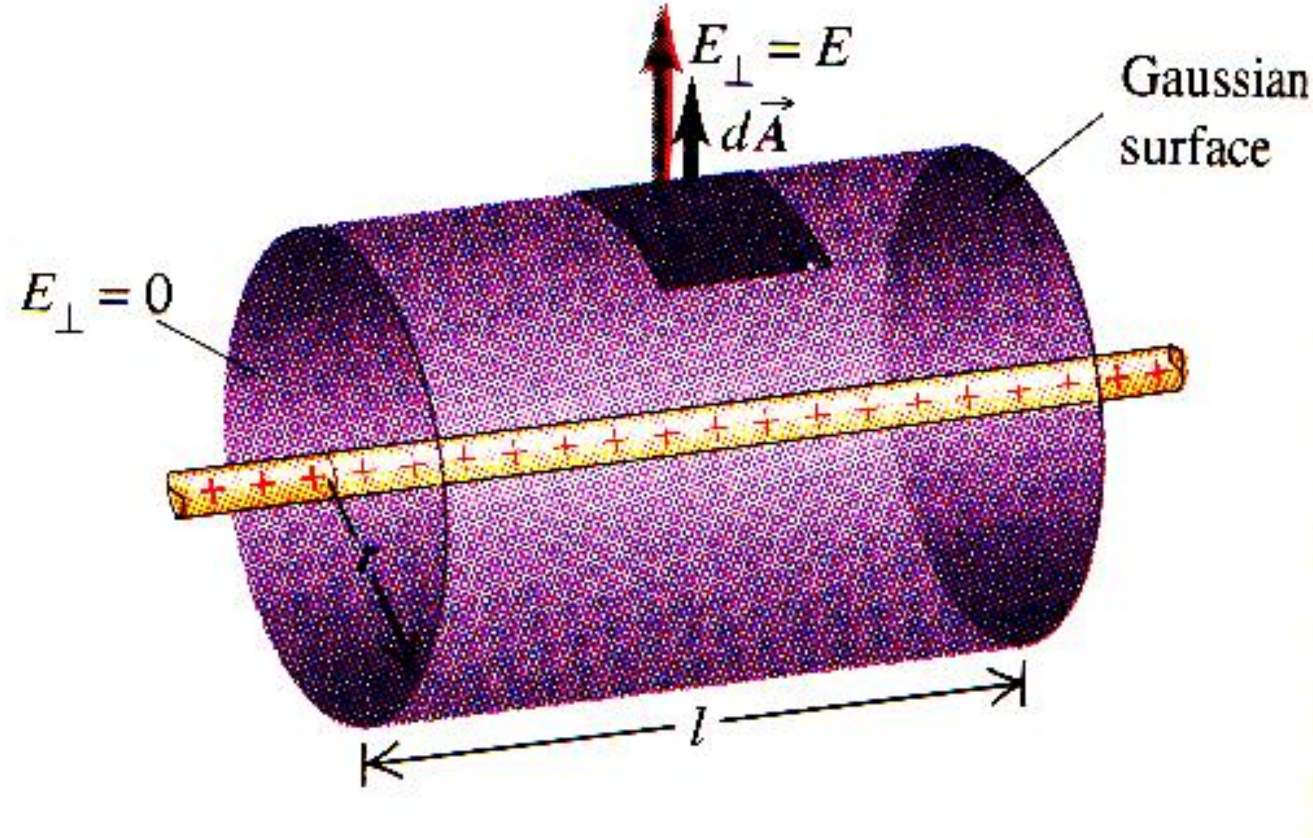
Cylindrical Gaussian Surface

$$E = \frac{\sigma}{\epsilon_0}$$

E-Field from an Infinite Line Charge

E **Field from a 'long' line charge**





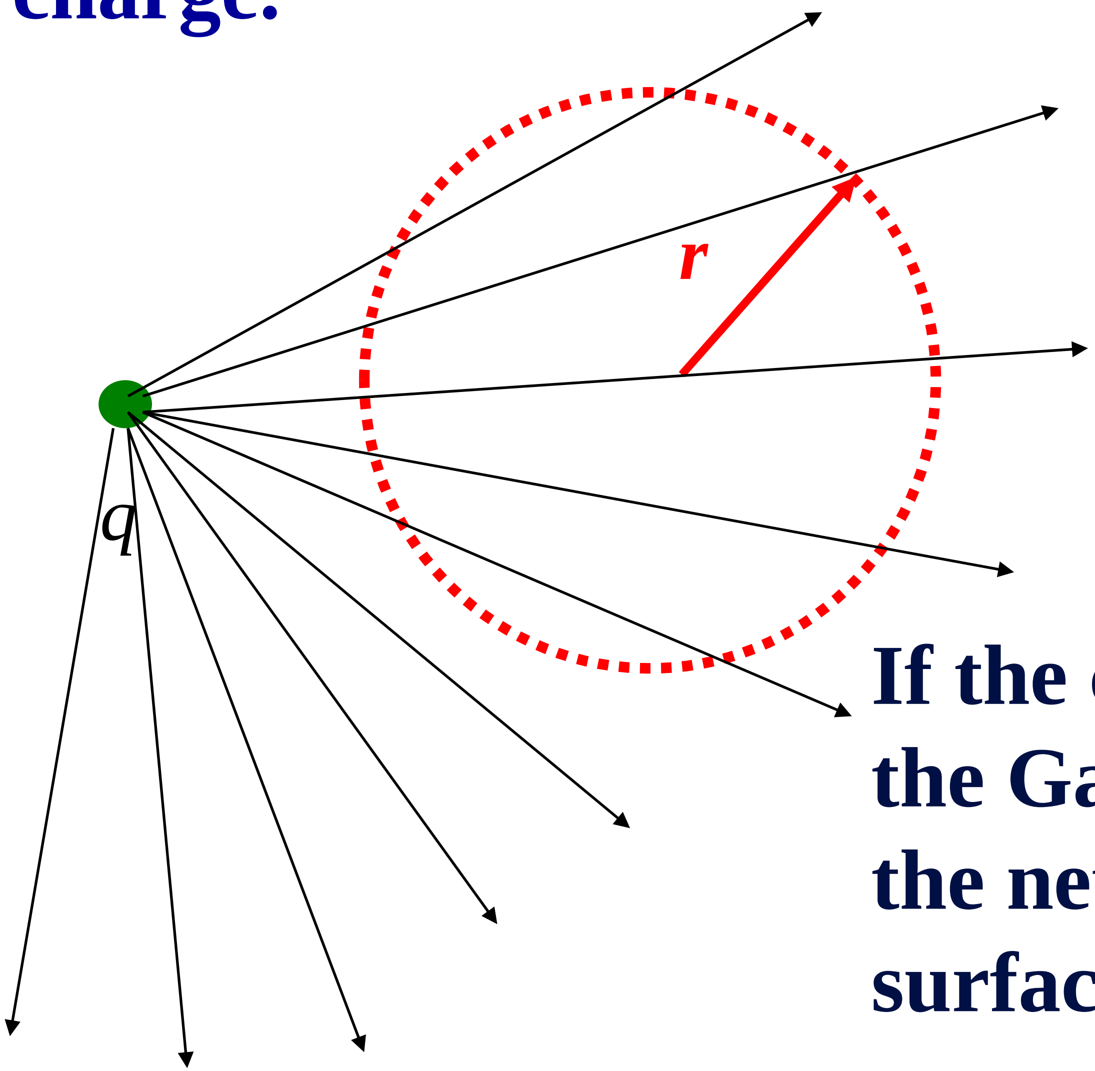
$$\oint \underline{E}.d\underline{A} = E \times 2\pi r l = \frac{\lambda l}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi \varepsilon_0 r}$$

Gaussian Surface in E-field but no enclosed charge.

$$\oint \vec{E} \cdot d\vec{A} = 0$$

$$\vec{E} = 0 \quad ??$$



No. In this case **E** is not parallel to d**A** and not constant around surface

If the charge falls outside the Gaussian Surface, the net flux through the surface is zero.

Summary on Gauss's Law

- **In situations of high symmetry (planar, spherical, cylindrical), Gauss' s law allows us to compute quantitatively the $\underline{E}$-field in a straightforward manner.**
- **Very useful.**

$$\boxed{\boxed{\int_S \underline{E} \cdot d\underline{S} = \frac{Q_{encl}}{\varepsilon_0}}}$$