UNIVERSITY OF BIRMINGHAM

School of Physics and Astronomy

DEGREE OF B.Sc. & M.Sci. WITH HONOURS

FIRST-YEAR EXAMINATION

03 19718

LC QUANTUM MECHANICS / OPTICS & WAVES

SUMMER EXAMINATION 2024

Time Allowed: 2 hours

Answer four questions from Section 1 and two questions from Section 2.

Section 1 consists of four questions and carries 44% of the marks for the examination. Answer ***all four*** questions from this Section.

Section 2 consists of two questions and carries 56% of the marks. Answer ***both*** questions in Section 2. Note that each question has two parts, of which only ***one part*** should be answered. If you answer both parts, credit will only be given for the best answer.

The approximate allocation of marks to each part of a question is shown in brackets **[ ]**.

PLEASE USE A SEPARATE ANSWER BOOK FOR SECTION 1 AND SECTION 2 QUESTIONS.

A formula sheet and a table of physical constants and units that may be required will be found at the end of this question paper.

SECTION 1

Answer **all four** *questions from this Section.*

1. A monoenergetic beam of electrons scatters from a crystal with planes of atoms separated by $0.335\ \mathrm{nm}$. Find the lowest electron accelerating voltage at which constructive interference will occur at an angle of 20° from the crystal planes. **[8]**

2. A photon of wavelength $13.5\ \mathrm{nm}$ is emitted when a single-electron atom makes a transition from its first excited state. How much energy is required to ionise the atom from its ground state? Express your answer in units of eV. What is the atomic number Z of the atom? **[8]**

3. The transverse displacement y at position x and time t of a harmonic wave on a string is given by
$$y = 0.006 \sin(0.1x - 2.2t + 0.1),$$
where x and y are measured in metres while t is measured in seconds. Find the amplitude, wavelength, frequency and period of this wave. Use the above equation to show that this wave travels in the $+x$ direction. What is the speed of this wave? **[8]**

4. Two taut strings of the same mass per unit length and held at the same tension are $2.10\,\mathrm{m}$ and $2.12\,\mathrm{m}$ long, respectively. The ends of each string are fixed. Given that the speed of waves along the strings is $1000\,\mathrm{m\,s^{-1}}$, what is the fundamental frequency of each string? Describe what is heard when both strings are oscillating at their fundamental frequencies. **[8]**

SECTION 2

*Answer **both** questions in this Section. Note that each question has two parts, of which only **one part** should be answered. If you answer both parts, credit will only be given for the best answer.*

5. **EITHER Part A**

(a) Using the time-independent Schrödinger equation (TISE) as an example, explain the terms *eigenvalue equation*, *eigenfunction* and *eigenvalue*. Identify the *operators* in the TISE. **[4]**

(b) When the potential function takes a constant positive value, $V(x) = V_0$, show that the wave function $\psi(x) = A\exp(ikx)$ is an eigenfunction of the TISE, and determine its eigenvalue. **[2]**

(c) A particle of mass, m, and energy, E, moving in the positive x-direction is incident upon a semi-infinite positive potential step at $x = 0$. The potential function is $V(x) = 0$ for $x < 0$ and $V(x) = V_0$ for $x \geq 0$. The solution of the TISE in the region $x < 0$ is

$$\psi(x) = A\exp(ikx) + B\exp(-ikx).$$

Consider the situation where $E < V_0$.

i. By solving the TISE, show that the wave function in the region $x \geq 0$ is a real exponential of the form

$$\psi(x) = C\exp(-\alpha x),$$

where C and α are constants. Find an expression for α in terms of the other constants in the problem. **[4]**

ii. Explain why the wave function, and the gradient of the wave function, must be continuous at the boundary at $x = 0$. Hence show that $|A|^2 = |B|^2$ and explain the significance of this result. **[5]**

iii. By normalising the wave function in the region $x \geq 0$, find an expression for the probability that a barrier-penetrating particle penetrates a depth $x \geq L$ into the potential step. **[5]**

OR Part B

(a) Describe two features of the photoelectric effect that are inconsistent with classical physics. For each feature, state what classical physics predicts and how the photon hypothesis explains the result. **[4]**

(b) When light of wavelength $200\ \mathrm{nm}$ is shone onto a metal surface the maximum electron energy is measured to be $2.41\ \mathrm{eV}$. What is the maximum wavelength for which electrons will be emitted? **[4]**

(c) The X-ray absorption spectrum of a particular element is found to have three absorption edges at $0.4\ \mathrm{keV}$, $2.6\ \mathrm{keV}$ and $20\ \mathrm{keV}$.

i. Explain what the appearance of edges in the absorption spectrum tells you about the electronic structure of atoms. **[4]**

ii. Use the data provided to find the atomic number of the element. **[4]**

(d) Find the longest incident wavelength for which the energy of a photon can be halved in the process of Compton scattering. **[4]**

6. **EITHER Part A**

(a) An object $1\,\mathrm{cm}$ high is placed $70\,\mathrm{cm}$ in front of a thin converging lens of focal length $40\,\mathrm{cm}$. Find the position and size of the image

i. qualitatively using a ray diagram.
ii. quantitatively using the lens equation. **[4]**

(b) The experimental setup described in a) is moved from air into a large bath of water of refractive index 1.33. If the lens is made of plastic of refractive index 1.20, determine the position and size of the image using both a ray diagram and the lens equation. The focal length f of a lens of refractive index n_l in the medium of refractive index n_m is given by

$$\frac{1}{f} = \left(\frac{n_l}{n_m} - 1\right)\left(\frac{1}{r_1} - \frac{1}{r_2}\right).$$

[4]

(c) Two thin lenses with focal lengths f_1 and f_2 are placed together to form a combination lens. Assuming that the distance between the two lenses is zero, show that the combined lens has an effective focal length f such that

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}.$$

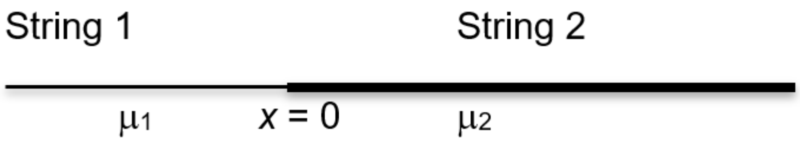
[4]

(d) A person has near point of $150\,\mathrm{cm}$. What is the power of the contact lens that is required to bring this near point to $25\,\mathrm{cm}$? **[3]**

(e) A diver finds when looking up at the surface of the water that it is possible to observe light from above the water only in a circular region on the water's surface. If the refractive index of the water is $n_w = 1.33$, what is the opening angle of the cone which is defined by the diver's eye and the circular region? You may assume that the medium above water is air with a refractive index $n_{air} = 1$. What does the diver see outside the cone? **[5]**

OR Part B

Two semi-infinite strings of different mass per unit length, μ_1 and μ_2, are joined at the position x = 0. The tension in the strings is T.



A wave, given by $y = A\cos(k_1 x - \omega t)$, travels along string 1 from left to right.

(a) Show that the amplitudes of the reflected and transmitted waves, C and B, respectively, are given by:

$$B = \frac{2k_1}{k_1 + k_2}A \text{ and } \mathrm{C} = \frac{\mathrm{k_1 - k_2}}{\mathrm{k_1 + k_2}}\mathrm{A}$$

where k_1 and k_2 are the wave-numbers for strings 1 and 2, respectively. State the boundary conditions that you used for the derivation. **[7]**

(b) What happens to the amplitudes of the reflected and transmitted waves if i) μ_2 tends to infinity and ii) μ_2 tends to zero? **[3]**

(c) For $\mu_2 = 0$, the resultant wave on string 1 can be obtained by superposition of the incident and the reflected waves. Express the resultant wave in terms of A, k_1 and ω. Where is the first node from $x = 0$? **[4]**

(d) For a particular combination of the two strings, $C = (-A)/2$ for the reflected wave. Assume that string 2 is infinitely long so that no reflected waves come from the right end of string 2, show that the resultant wave on string 1 can be expressed as the sum of a standing wave and a travelling wave. Are there any nodes on string 1 where the displacement is constantly zero? Comment on how energy transport from string 1 to string 2 is affected by the junction. **[6]**

Quantum Mechanics Formula Sheet

Stefan-Boltzmann Law

$$\frac{P}{A} = \sigma T^4$$

Wien's Displacement Law

$$\lambda_{\text{peak}} T = 2.898 \times 10^{-3}\,\text{Km}$$

Planck's Formula

$$\overline{E}\,(\lambda) = \frac{hc/\lambda}{\exp(hc/\lambda k_B T) - 1}$$

Einstein's Photoelectric Formula

$$\text{KE}_{\text{max}} = hf - \phi$$

Compton Shift Equation

$$\lambda_2 - \lambda_1 = \frac{h}{m_e c}\,(1 - \cos\theta)$$

Hydrogen Energy Levels

$$E_n = -\frac{13.6}{n^2}\,\text{eV}$$

Bragg's Law

$$2d\sin\theta = n\lambda$$

Moseley's Law

$$f_{\text{K}_\alpha} = \left(2.48 \times 10^{15}\,\text{Hz}\right) \times (Z-1)^2$$

Heisenberg's Uncertainty Principle

$$\Delta p_x \Delta x \geq \frac{h}{4\pi}$$

Free-Particle Wave Function

$$\Psi\,(x,t) = A\exp\left(\pm i\,(kx - \omega t)\right)$$

Momentum Operator

$$\hat{p}_x = -i\hbar\frac{\partial}{\partial x}$$

Energy Operator

$$\hat{E} = i\hbar\frac{\partial}{\partial t}$$

Expectation Value

$$\langle O \rangle = \int_{-\infty}^{+\infty} \Psi^* \hat{O} \Psi dx$$

Time-Independent Schrödinger Equation

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

Physical Constants and Units

Acceleration due to gravity	g	$9.81\,\mathrm{m\,s^{-2}}$
Gravitational constant	G	$6.674 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$
Ice point	T_{ice}	$273.15\,\mathrm{K}$
Avogadro constant	N_A	$6.022 \times 10^{23}\,\mathrm{mol^{-1}}$
		[*N.B.* 1 mole $\equiv 1$ *gram-molecule*]
Gas constant	R	$8.314\,\mathrm{J\,K^{-1}\,mol^{-1}}$
Boltzmann constant	k, k_B	$1.381 \times 10^{-23}\,\mathrm{J\,K^{-1}} \equiv 8.62 \times 10^{-5}\,\mathrm{eV\,K^{-1}}$
Stefan constant	σ	$5.670 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$
Rydberg constant	R_∞	$1.097 \times 10^{7}\,\mathrm{m^{-1}}$
	$R_\infty hc$	$13.606\,\mathrm{eV}$
Planck constant	h	$6.626 \times 10^{-34}\,\mathrm{J\,s} \equiv 4.136 \times 10^{-15}\,\mathrm{eV\,s}$
$h/2\pi$	$\hbar$	$1.055 \times 10^{-34}\,\mathrm{J\,s} \equiv 6.582 \times 10^{-16}\,\mathrm{eV\,s}$
Speed of light *in vacuo*	c	$2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$
	$\hbar c$	$197.3\,\mathrm{MeV\,fm}$
Charge of proton	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Mass of electron	m_e	$9.109 \times 10^{-31}\,\mathrm{kg}$
Rest energy of electron		$0.511\,\mathrm{MeV}$
Mass of proton	m_p	$1.673 \times 10^{-27}\,\mathrm{kg}$
Rest energy of proton		$938.3\,\mathrm{MeV}$
One atomic mass unit	u	$1.66 \times 10^{-27}\,\mathrm{kg}$
Atomic mass unit energy equivalent		$931.5\,\mathrm{MeV}$
Electric constant	ϵ_0	$8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Magnetic constant	μ_0	$4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$
Bohr magneton	μ_B	$9.274 \times 10^{-24}\,\mathrm{A\,m^2}$ ($\mathrm{J\,T^{-1}}$)
Nuclear magneton	μ_N	$5.051 \times 10^{-27}\,\mathrm{A\,m^2}$ ($\mathrm{J\,T^{-1}}$)
Fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	$7.297 \times 10^{-3} = 1/137.0$
Compton wavelength of electron	$\lambda_c = h/m_e c$	$2.426 \times 10^{-12}\,\mathrm{m}$
Bohr radius	a_0	$5.2918 \times 10^{-11}\,\mathrm{m}$
angstrom	Å	$10^{-10}\,\mathrm{m}$
barn	b	$10^{-28}\,\mathrm{m^2}$
torr (mm Hg at 0 °C)	torr	$133.32\,\mathrm{Pa}$ ($\mathrm{N\,m^{-2}}$)

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Do not complete the attendance slip, fill in the front of the answer book or turn over the question paper until you are told to do so.

Important Reminders

- Coats/outwear should be placed in the designated area.
- Unauthorised materials (e.g. notes or Tippex) must be placed in the designated area.
- Check that you do not have any unauthorised materials with you (e.g. in your pockets, pencil case).
- Mobile phones and smart watches **must** be switched off and placed in the designated area or under your desk. They must not be left on your person or in your pockets.
- You are not permitted to use a mobile phone as a clock. If you have difficulty seeing a clock, please alert an Invigilator.
- You are not permitted to have writing on your hand, arm or other body part.
- Check that you do not have writing on your hand, arm or other body part – if you do, you must inform an Invigilator immediately
- Alert an Invigilator immediately if you find any unauthorised item upon you during the examination.

Any students found with non-permitted items upon their person during the examination, or who fail to comply with Examination rules may be subject to Student Conduct procedures.