Example Sheet 3: Bases

1. Convert the following binary numbers into fractions

$$0.0101010\dot{0}\dot{1} \qquad 0.011001100\dot{1}1\dot{0} \qquad 0.101101\dot{1}0\dot{1}$$

2. Convert the following fractions into binary numbers

$$\frac{3}{7} \quad \frac{4}{9} \quad \frac{1}{15} \quad \frac{1}{17} \quad \frac{3}{4} \quad \frac{1}{11}$$

3. Convert the following ternary numbers into fractions

$$0.012012\dot{0}1\dot{2} \qquad 0.011001100\dot{1}1\dot{0} \qquad 0.101101\dot{1}0\dot{1} \qquad 0.001221\dot{0}0122\dot{1}$$

4. Convert the following fractions into ternary numbers

$$\frac{1}{2} \quad \frac{1}{8} \quad \frac{1}{9} \quad \frac{1}{10}$$

5. A function is defined by

$$\begin{aligned} f(x) &= 4x & x &\in \left[0, \frac{1}{4}\right] \\ &= 1 & x &\in \left[\frac{1}{4}, \frac{1}{2}\right] \\ &= 4\left[\frac{3}{4} - x\right] & x &\in \left[\frac{1}{2}, \frac{3}{4}\right] \\ &= 0 & x &\in \left[\frac{3}{4}, 1\right] \end{aligned}$$

Depict this function and show how to apply it to a number which is represented in base 4. Find a representation for the numbers which do not end up at zero eventually.

6. A function is defined by

$$\begin{aligned} f(x) &= 8x & x &\in \left[0, \frac{1}{8}\right] \\ &= 8\left[\frac{1}{4} - x\right] & x &\in \left[\frac{1}{8}, \frac{1}{4}\right] \\ &= 0 & x &\in \left[\frac{1}{4}, \frac{1}{2}\right] \\ &= 8\left[x - \frac{1}{2}\right] & x &\in \left[\frac{1}{2}, \frac{5}{8}\right] \\ &= 8\left[\frac{3}{4} - x\right] & x &\in \left[\frac{5}{8}, \frac{3}{4}\right] \\ &= 0 & x &\in \left[\frac{3}{4}, 1\right] \end{aligned}$$

Depict this function and show how to apply it to a number which is represented in base 8. Find a representation for the numbers which do not end up at zero eventually.

7. Consider the map

$$\begin{aligned} M[x] =& -2-3x & x \in& \left[-1, -\frac{1}{3}\right] \\ =& 3x & x \in& \left[-\frac{1}{3}, \frac{1}{3}\right] \\ =& 2-3x & x \in& \left[\frac{1}{3}, 1\right] \end{aligned}$$

Depict this map. Find a representation for this map in base 3. Find all the 1-cycles, 2-cycles and 3-cycles using base 3. Convert these cycles into fractions.

8. Consider the map

$$\begin{aligned} M[x] =& 4+5x & x \in& \left[-1, -\frac{3}{5}\right] \\ =& -2-5x & x \in& \left[-\frac{3}{5}, -\frac{1}{5}\right] \\ =& 5x & x \in& \left[-\frac{1}{5}, \frac{1}{5}\right] \\ =& 2-5x & x \in& \left[\frac{1}{5}, \frac{3}{5}\right] \\ =& 5x-4 & x \in& \left[\frac{3}{5}, 1\right] \end{aligned}$$

Depict this map. Find a base 5 representation for this map and use it to find all the 1-cycles.