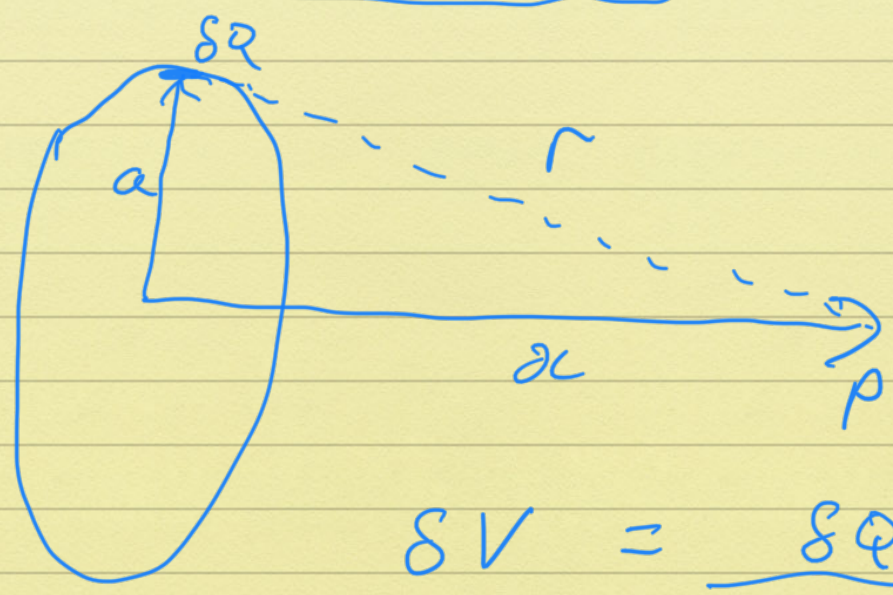
Lecture 6c

Example 6.4



$$\delta V = \frac{\delta Q}{4\pi\varepsilon_0 r}$$

$$= \frac{\delta Q}{4\pi\varepsilon_0} \frac{1}{(a^2+x^2)^{1/2}}$$

Hence $$V = \frac{1}{4\pi\varepsilon_0} \frac{1}{(a^2+x^2)^{1/2}} \oint dQ$$

$$V = \frac{Q}{4\pi\varepsilon_0} \frac{1}{(a^2+x^2)^{1/2}}$$

$$\underline{E} = -\nabla V = -\frac{\partial V}{\partial x}\underline{i} - \frac{\partial V}{\partial y}\underline{j} - \frac{\partial V}{\partial z}\underline{k}$$

$$\frac{\partial V}{\partial y} = \frac{\partial V}{\partial z} = 0$$

$$\Rightarrow \quad E_y = E_z = 0$$

$$E_{ox} = -\frac{dV}{dx} = -\frac{Q}{4\pi\varepsilon_0}\frac{d}{dx}\left(a^2+x^2\right)^{-1/2}$$

$$= -\frac{Q}{4\pi\varepsilon_0}\left(-\frac{1}{2}\cdot 2x\cdot\left(a^2+x^2\right)^{-3/2}\right)$$

$$E_{ox} = \frac{Q\,x}{4\pi\varepsilon_0}\frac{1}{\left(a^2+x^2\right)^{3/2}}$$

(Q.E.D)