Quantum Mechanics 1 – Solution 7

a) The de Broglie wavelength of the electrons is given by

$$\lambda = \frac{h}{p},$$

where,

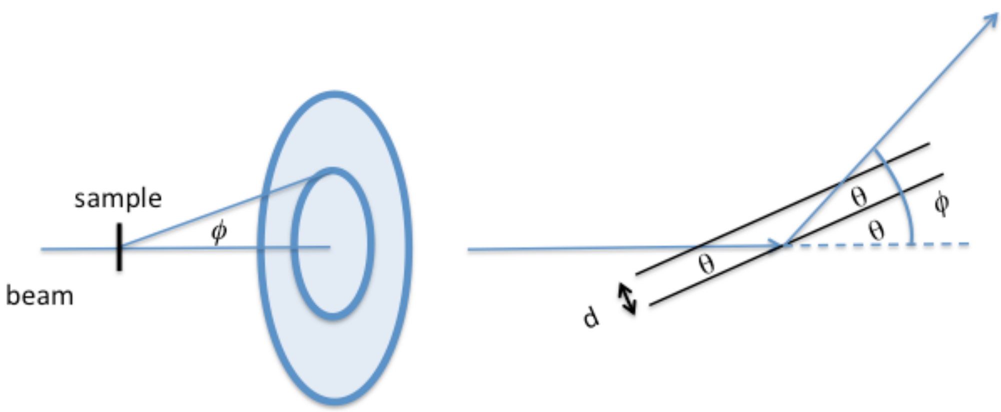
$$E = \frac{p^2}{2m} \Longrightarrow p = \sqrt{2mE}.$$

Working in SI units,

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 9.109 \times 10^{-31} \times 50 \times 1.6 \times 10^{-19}}} = 1.74 \times 10^{-10}\text{m}.$$

[2 marks]

b) The Bragg conditions for constructive interference from crystal planes within the sample are $2d \ \sin\theta \ = \ n\lambda$ and $\theta_{in} = \theta_{out}$.



The angle ϕ between the beam direction and the fringe position is not the angle θ in the Bragg formula. As shown in the figure, $\phi = \ 2\theta$. [2 marks]

c) From the Bragg formula, the diffraction maxima occur at angles satisfying

$$2d\ sin(\phi/2) = n\lambda.$$

Rearranging, the plane spacing is

$$d = \frac{n\lambda}{2\,sin(\phi/2)}.$$

[2 marks]

d) Some judgment is needed here. The smaller angle will correspond to the lowest order diffraction, n. Taking the smaller angle ($\phi = 20^{\circ}$) to be the first diffraction maximum ($n = 1$), the plane spacing is

$$d = \frac{n\lambda}{2\,sin(\phi/2)} = \frac{1.74 \times 10^{-10}}{2 \sin{(10^{o})}} = 5.0 \times 10^{-10}\mathrm{m}.$$

[2 marks]

This assumption should be confirmed by using $n = 2$ for the second peak.

$$d = \frac{n\lambda}{2\,sin(\phi/2)} = \frac{2 \times 1.74 \times 10^{-10}}{2 \sin{(20.35^{o})}} = 5.0 \times 10^{-10}\mathrm{m}.$$

[1 mark]

e) A photon of the same wavelength would have an energy $E = hc/\lambda = 7.1\ \mathrm{keV}\ (1.1 \times 10^{-15}\mathrm{J})$. Note that this is a very different energy.

[1 mark]