Any Calculator

UNIVERSITY OF BIRMINGHAM

School of Mathematics

Programmes in the School of Mathematics	First Examination
Programmes in the School of Mathematics	Second Examination
Programmes involving Mathematics	First Examination
Programmes involving Mathematics	Second Examination

1Mech 06 25661 Level C

LC Mechanics

1Mech2 06 27345 Level I

LI Mechanics

May/June Examinations 2021-22

One Hour and Thirty Minutes

Full marks will be obtained with complete answers to BOTH questions. Each question carries equal weight. You are advised to initially spend no more than 45 minutes on each question and then to return to any incomplete questions if you have time at the end. An indication of the number of marks allocated to parts of questions is shown in square brackets.

Calculators may be used in this examination but must not be used to store text. Calculators with the ability to store text should have their memories deleted prior to the start of the examination.

Turn over

Section A

1. (a) Is the following equation dimensionally homogeneous?

$$\frac{d\rho}{dx} = \frac{m}{A} + uV,$$

where ρ is density, x is distance, m is mass, A is area, u is velocity and V is volume. Justify your answer. [4]

(b) If a particle of mass m is subject to a force of the form $F = kt^2$, where k is a constant and t is time, find the acceleration, velocity and position of the particle, given the particle starts at $x = 0$ with constant velocity u. [12]

(c) Starting from Newton's Second Law, show that

$$\mathbf{r} \times \mathbf{F} = \frac{d}{dt}\left(\mathbf{r} \times m\dot{\mathbf{r}}\right),$$

where $\mathbf{r}$ is position, $\mathbf{F}$ is force, m is mass and t is time. Give the physical meaning of each term in the expression in words. [10]

(d) A particle of mass m is moving on a wire in the shape of a cosine wave in the x-z plane, where x, z are Cartesian coordinates in the horizontal and vertical directions respectively.

(i) If $x = a\theta(t)$, $z = c\cos\theta(t)$, where t is time and a, c are constants, find the position vector and hence the velocity of the particle.

(ii) Hence show that

$$\frac{1}{2}m\dot{\theta}^2\left(a^2 + c^2\sin^2\theta\right) + mgc\cos\theta = E,$$

where E is a constant. [You may assume that Conservation of Energy holds for this system.]

(iii) If the particle starts at the point $x = 0$, $z = c$ moving with speed v, find the value of E and hence show that the particle will never stop moving. [24]

Section B

2. A particle of mass m is attracted towards the origin O by a central force of magnitude

$$m\frac{av^2}{8}\left(\frac{1}{r^2}+\frac{3a}{r^3}\right),$$

where (r,θ) are polar coordinates and a, v are positive constants.

(a) Starting from Newton's Second Law, show that the particle's path satisfies

$$\frac{d^2u}{d\theta^2}+u \;=\; \frac{av^2}{8h^2}+\frac{3a^2v^2}{8h^2}u,$$

where $u=1/r$ and h is a constant which you should define. [You may use

$$\begin{aligned}\dot{\mathbf{r}} &= r\dot{\theta}\mathbf{e}_\theta+\dot{r}\mathbf{e}_r,\\ \ddot{\mathbf{r}} &= \left(\ddot{r}-r\dot{\theta}^2\right)\mathbf{e}_r+\frac{1}{r}\frac{d}{dt}\left(r^2\dot{\theta}\right)\mathbf{e}_\theta\end{aligned}$$

without proof.] [16]

(b) If the particle starts at a distance a from the origin, moving with velocity

$$\dot{\mathbf{r}}=-\frac{v}{\sqrt{2}}\mathbf{e}_r+\frac{v}{\sqrt{2}}\mathbf{e}_\theta,$$

find expressions for u and $\dfrac{du}{d\theta}$ at $\theta=0$, and the value of h. [10]

(c) Hence find the path of the particle. [14]

(d) Determine the minimum distance between the particle and the origin on the first revolution. How many revolutions are achieved before the particle flies off to infinity? What is the angle this happens at? [10]

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LC/LI Mechanics

Do not complete the attendance slip, fill in the front of the answer book or turn over the question paper until you are told to do so.

Important Reminders

- Coats and outer-wear should be placed in the designated area.
- Unauthorised materials (e.g. notes or Tippex) **MUST** be placed in the designated area.
- Check that you **DO NOT** have any unauthorised materials with you (e.g. in your pockets, pencil case).
- Mobile phones and smart watches **MUST** be switched off and placed in the designated area or under your desk. They must not be left on your person or in your pockets.
- You are **NOT** permitted to use a mobile phone as a clock. If you have difficulty in seeing a clock, please alert an Invigilator.
- You are **NOT** permitted to have writing on your hand, arm or other body part.
- Check that you do not have writing on your hand, arm or other body part – if you do, you must inform an Invigilator immediately.
- Alert an Invigilator immediately if you find any unauthorised item upon you during the examination.

Any students found with non-permitted items upon their person during the examination, or who fail to comply with Examination rules may be subject to the Student Conduct procedures.