# Year 1 Assessed Problems

Semester 1

Problem Sheet 10

**SOLUTIONS TO BE SUBMITTED ON CANVAS**

By 17:00hrs on Wednesday 11th December 2024

# 5 Assessed – Complex numbers

*Problem 5.1* **Standard and Exponential Forms**
Put the following examples in both standard (which should not use any trigonometric functions) and exponential form. [Hint: You may find one of the representations easier to work with when taking powers etc.]

(a) $1/\mathrm{i}$ ; (b) $(1+\mathrm{i})^2$ ; (c) $(1+\mathrm{i})^{10}$ ; (d) $\mathrm{e}^{\mathrm{i}\pi/8}$ ; (e) $1+\mathrm{i}+\mathrm{i}^2+\mathrm{i}^3$.

The associated marks are:
(a) 2, (b) 2, (c) 1, (d) 4 and (e) 1.

# Dynamical Systems Assessed Problem 2

*Problem 2.1* **A Rod on a String**
A uniform rod AB is supported at an angle $\alpha$ to the horizontal by a light string attached to A, with its lower end B on a rough horizontal plane with coefficient of friction, $\mu$ .

(a) Draw a diagram of the system, and mark on all the forces. [**3**]

(b) Derive the equations for force balance in the horizontal and vertical directions, and for torque balance around A. [**3**]

(c) Solve these equations to derive expressions for the various forces in the problem in terms of the weight, $mg$, of the rod. [**3**]

(d) Show that the greatest inclination, $\theta$, of the string to the vertical is given by

$$\cot\theta = \frac{1}{\mu} \pm 2\tan\alpha,$$

according to the direction in which the end B is going to move. [**1**]

# Assessed Problems 2

Richard Mason
03 33961 Introduction to Probability and Statistics

**Problem 1.** A quantum mechanical particle is confined to a region of the $x$-axis for which $0 \leq x \leq a$ (where $a > 0$). The particle has a Probability Density Function (PDF) describing the position given by

$$P(x) = \begin{cases} Ax^2(a^3 - x^3) & \text{if} \quad 0 \leq x \leq a \\ 0 & \text{otherwise} \end{cases}$$

1. For a PDF, $P(x)$, define $\langle x \rangle$, $\langle x^2 \rangle$, $\text{var}(x)$ and $\text{std}(x)$. The sample space for $x$ can be assumed to be the entire real line here. **[3]**
2. Calculate the value of $A$ such that the PDF is properly normalised. **[2]**
3. Calculate the expectation value of the position. **[2]**
4. Calculate the standard deviation of the position. **[2]**
5. Calculate the value $m$ such that

$$\frac{1}{2} = \int_0^m P(x)dx$$

otherwise known as the *median*. **[1]**