Electromagnetism

Professor D. Evans
d.evans@bham.ac.uk

Lecture 10

Dielectrics

Week 5

Last Lecture - Capacitance

- Earthing / Grounding
- The concept of capacitance, C
  - Definition of capacitance
- Energy stored in a capacitor
- To calculate C of ideal capacitors
  - Parallel plates
  - Co-axial cables
  - Spherical capacitors

Last Lecture Summary

- Capacitance defined as: $C = \frac{Q}{V}$
  - Unit of C - Farad (F)
- Energy stored in the electric field of capacitor:

$$U = \frac{1}{2} C V^2$$

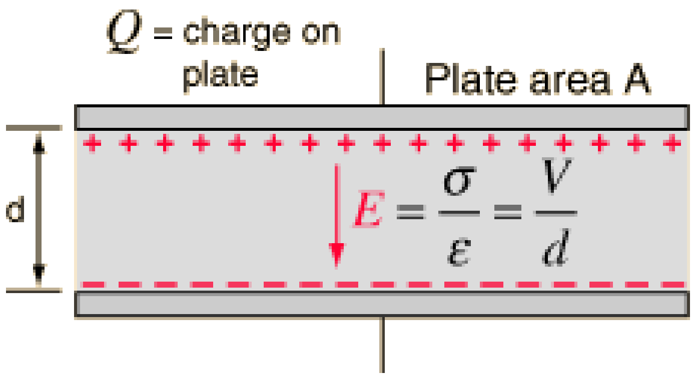
- How to find capacitance:
  1. Determine E (e.g. using Gauss's Law)
  2. Use $V = -\int \underline{E} \cdot d\underline{l}$
  3. Apply $C = Q / V$

This Lecture

- Energy density of Electric field
- Force between capacitor plates
- **Dielectrics**
  - Definition of
  - Polarisation, $\underline{\boldsymbol{P}}$
  - How external E-fields are modified inside dielectrics
  - Electric susceptibility χ_E
  - Relativity permittivity ε_r

Energy Density of Electric Field

- Consider a parallel plate capacitor
- Energy stored, $U = \frac{1}{2}CV^2$



- But $C = \frac{Q}{V} = \frac{\varepsilon_0 A}{d}$
- And $V = Ed$
- Hence $U = \frac{1}{2}\frac{\varepsilon_0 A}{d}E^2 d^2 = \frac{1}{2}\, Ad\, \varepsilon_0 E^2$
- i.e. stored energy, U = volume x $\frac{1}{2}\varepsilon_0 E^2$

Energy Density of Electric Field

- So **energy density** (energy stored per unit volume) of an electric field is:

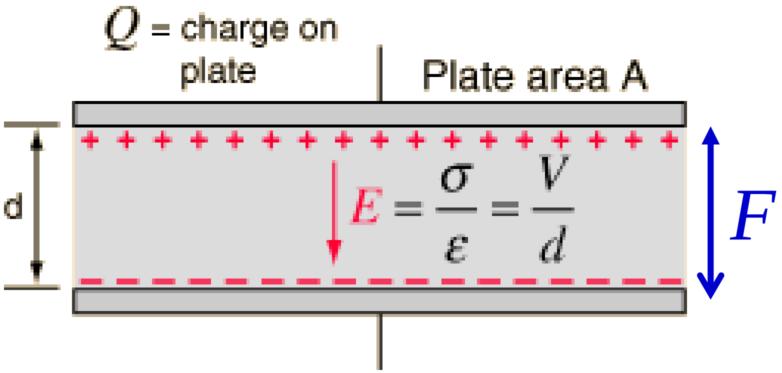
$$\boxed{u_E = \frac{1}{2}\varepsilon_0 E^2}$$

- This is universally valid for any E-field in a vacuum (not just for parallel plates).

Force between Capacitor Plates

- Can't use Coulomb's Law directly as integration gets very messy
- As $U = -\int \underline{\boldsymbol{F}} \cdot d\underline{\boldsymbol{x}}$
- Use $F = -\frac{dU}{dx}$
- Remember $U = \frac{1}{2} Ad\, \varepsilon_0 E^2$ so

$U(x) = \frac{1}{2} Ax\, \varepsilon_0 E^2$



Force between Capacitor Plates

- $U(x) = \frac{1}{2} Ax\, \varepsilon_0 E^2$
- $F = -\frac{dU}{dx} = -\frac{1}{2} A\varepsilon_0 E^2$
- But $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{A\varepsilon_0}$
- So $\boxed{F = -\frac{1}{2} A\varepsilon_0 E \frac{Q}{A\varepsilon_0} = -\frac{1}{2} QE}$
- Negative sign → attractive force

Summary

- Electrostatic potential energy of a charged capacitor given by:

$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{1}{2}\frac{Q^2}{C}$$

- U represents the work required to charge the capacitor.
- Energy density of electric field is

$$\mathrm{u(E)} = \frac{1}{2}\varepsilon_0 E^2$$

Dielectrics

Dielectrics

- **Definition of a dielectric:**

In a perfect conductor there is a lot of free charge and under an applied field, the charge moves rapidly along the field direction.

We will be concerned with poor conductors, called **dielectrics**, in which free charge is negligible. Under an applied electric field the charges in the atom are displaced while remaining bound. This effect is called **polarisation**.

Dielectrics

- A **dielectric** (or **dielectric** material) is an electrical insulator that can be polarized by an applied electric field.
- Where are they used?
  - They are placed between conducting surfaces in a capacitor
- What are the functions of a dielectric?
  - To keep charged surfaces physically separated (electrical insulation)
  - To raise the capacitance, C of a capacitor

Simple Media

- We will only consider “well behaved” dielectrics (true for EM2 as well) i.e. dielectrics that are (HILS):
- **Homogeneous** - same throughout
- **Isotropic** - same in all directions which means that **P** is parallel to **E**
- **Linear** - **P** is proportional to **E** (true for small enough fields)
- **Stationary** - volumes and areas do not move.

Polarisation

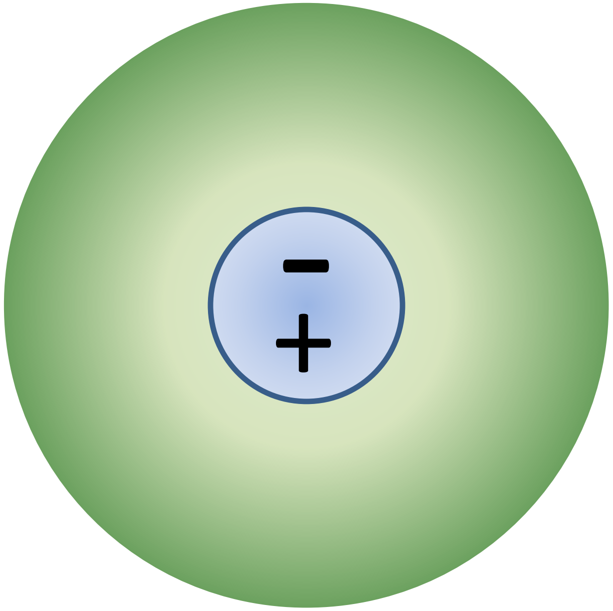
- **Polarisation** is the effect of an E-field on any dielectric to cause the positive charged nucleus to be pulled in one direction and the negative charged electron cloud to be pulled n the opposite direction.

- Consider a molecule in an E-field

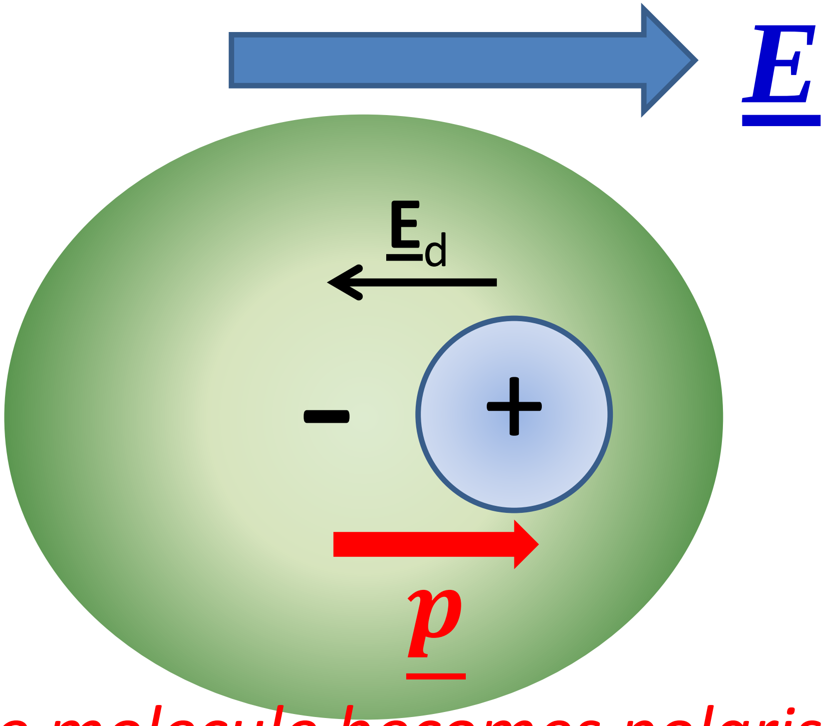
Molecule in E-field

Molecule, No E-field

Centre of -ve charge coincides with centre of +ve charge



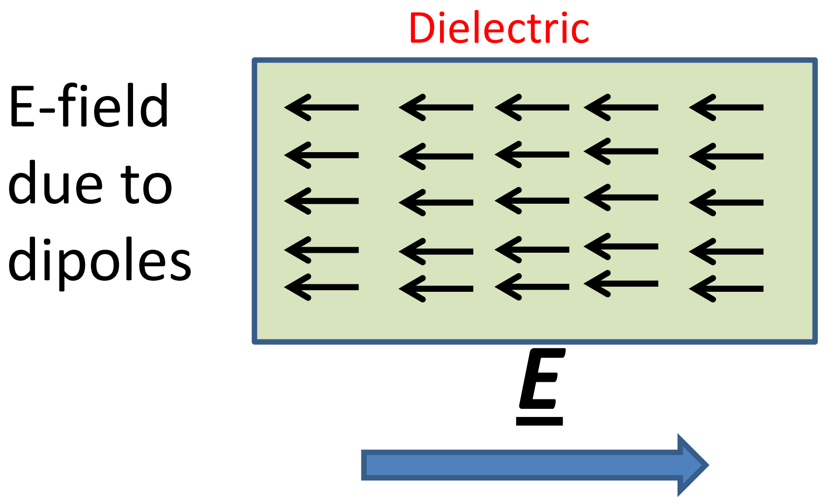
Apply E-field: charges become displaced.



The molecule becomes polarised

Polarisation

- The dipole's field acts oppositely to the applied field and this will reduce the field in the dielectric.



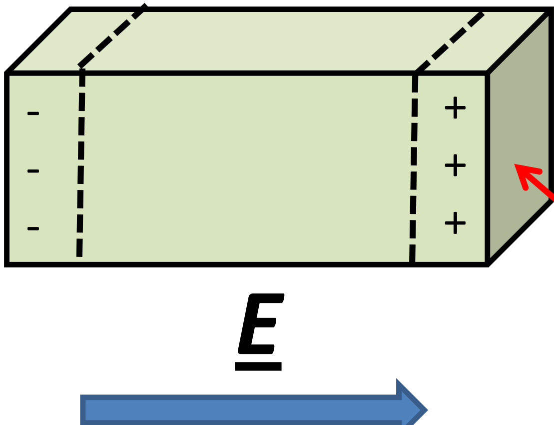
If the dielectric consists of N such dipoles (molecules) per unit volume, we define its total polarisation as:-

$$\underline{\boldsymbol{P}} = N\underline{\boldsymbol{p}} = Nq\underline{\boldsymbol{a}}$$

So the E-field inside the dielectric is **<u>less</u>** than the external applied E-field.

Polarisation

- Polarisation: $\underline{\boldsymbol{P}} = N\underline{\boldsymbol{p}} = Nq\underline{\boldsymbol{a}}$
- The displacement of bound charges causes a build up of charge on the external surfaces.
- Consider an applied E-field that is uniform and constant

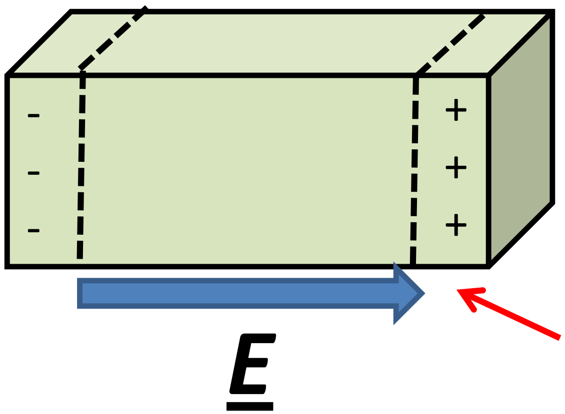


There is an accumulation of positive charge on the right hand side surface and of negative charges on the left hand side surface.

Depleted of –ve charges

Polarisation

- Polarisation: $\underline{\boldsymbol{P}} = N\underline{\boldsymbol{p}} = Nq\underline{\boldsymbol{a}}$
- Consider the surface area of a block of dielectric, S
- Surface charge = volume of depletion X molecular density X charge per molecule = $aS\ N\ q$
- But $P = Nqa$ so surface charge, $q_r = P\ S$

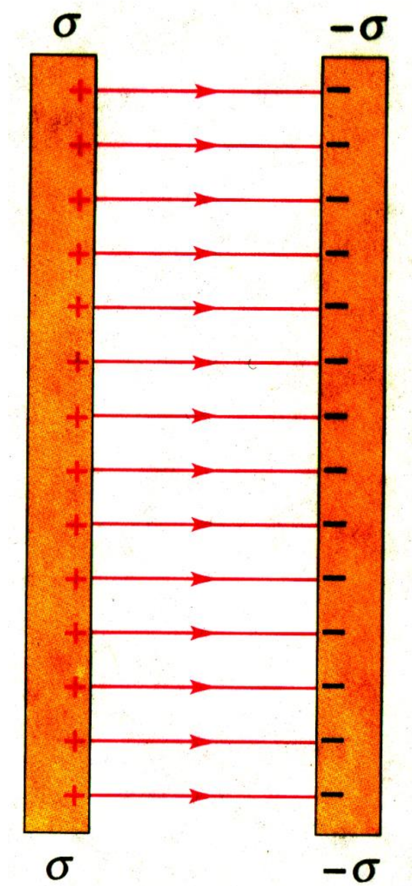


At left hand side: $q_l = -P\ S$

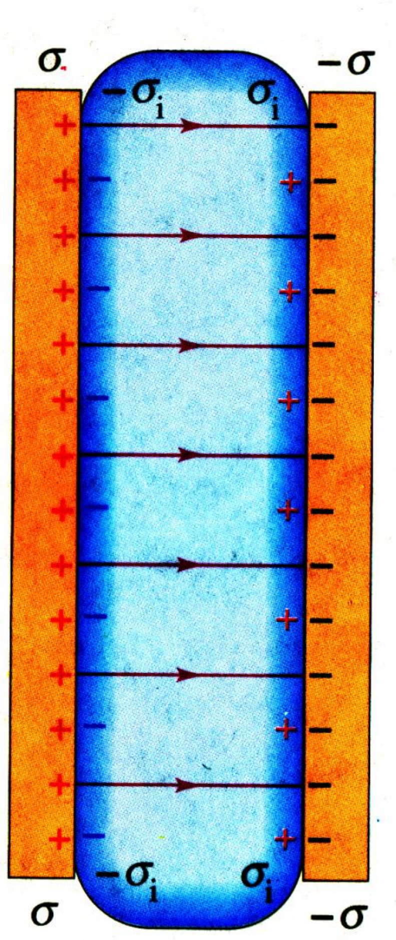
So surface charge density, $\boldsymbol{\sigma_i = \pm P}$

Depth of depletion = $\boldsymbol{a}$, size of dipoles

Dielectric Between Charged Plates



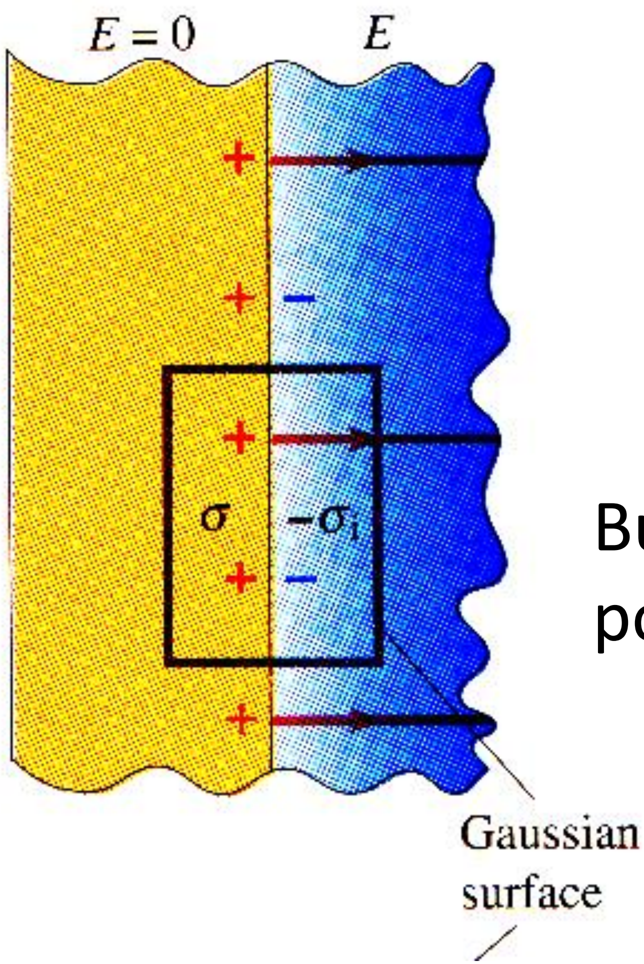
E_0



$E < E_0$

$\sigma_i = \pm P$

Dielectric Between Charged Plates



Net surface charge $= \sigma - \sigma_i$

For no Dielectric: $E_0 = \dfrac{\sigma}{\varepsilon_0}$

With dielectric: $E = \dfrac{\sigma - \sigma_i}{\varepsilon_0} = \dfrac{\sigma - P}{\varepsilon_0}$

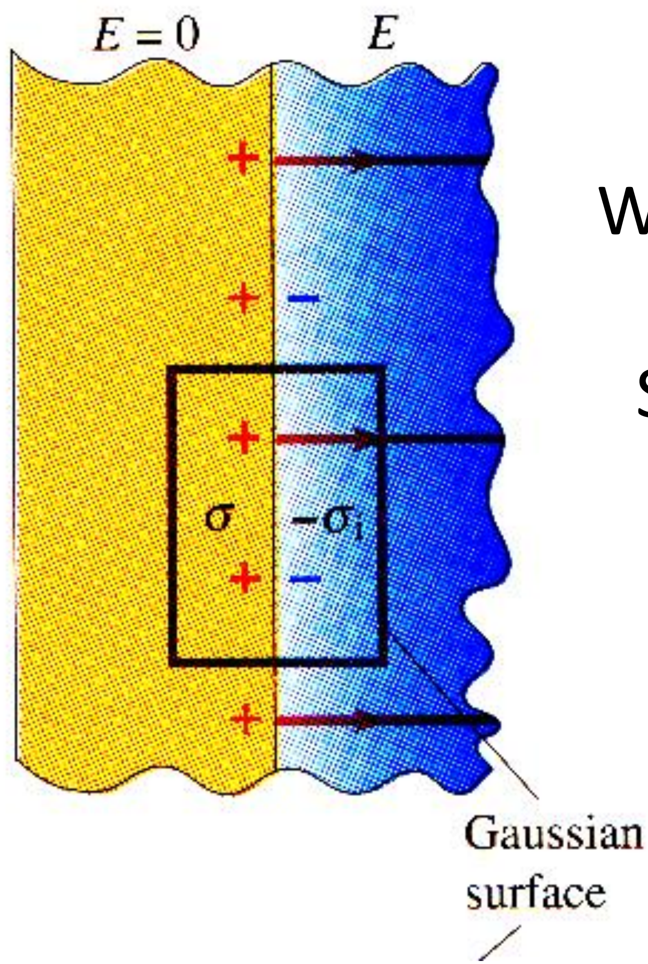
But for well behaved dielectrics polarisation $P \propto E$ i.e. $\boldsymbol{P} = \chi_{\boldsymbol{E}} \boldsymbol{\varepsilon_0} \boldsymbol{E}$

χ_E is called the electric susceptibility and is dimensionless

Electric Susceptibility

- Electric dipole moment, p has units C m
- Polarisation, P has units p/volume = C m^{-2}
- E-field has units: N C^{-1}
- Permittivity of free space ε_0 has units: C^2 m^{-2} N^{-1}
  So $\varepsilon_0 E$ has units: C m^{-2} same as polarisation, P
- Hence electric susceptibility χ_E is dimensionless
- Defined such that $\boldsymbol{P} = \chi_E \boldsymbol{\varepsilon_0} \boldsymbol{E}$ where $\boldsymbol{E}$ is electric field inside the dielectric.

Back to Dielectric Between Charged Plates



For no Dielectric: $E_0 = \frac{\sigma}{\varepsilon_0}$

With dielectric: $E = \frac{\sigma - \sigma_i}{\varepsilon_0} = \frac{\sigma - P}{\varepsilon_0}$

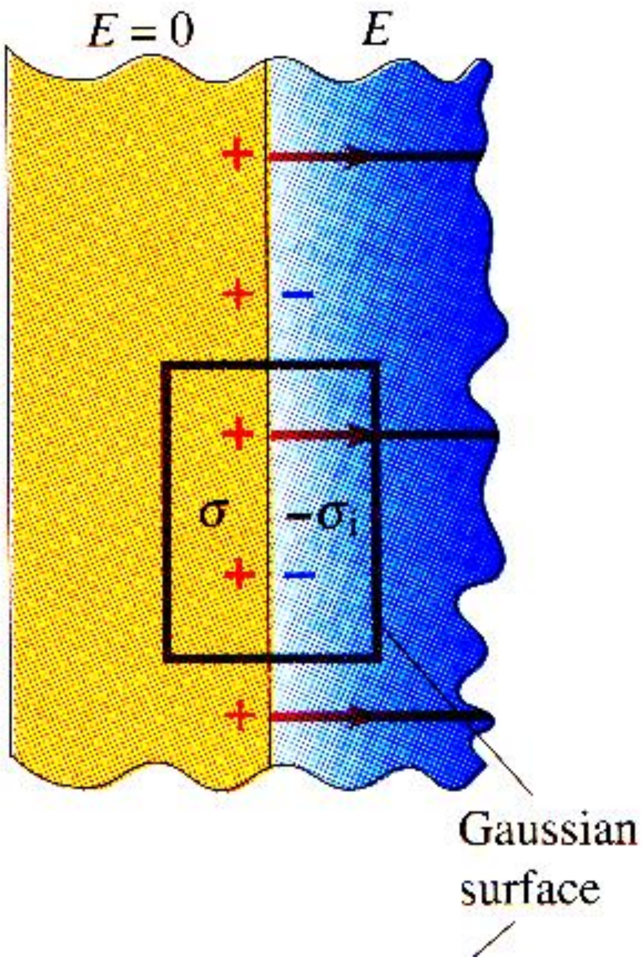
So $\varepsilon_0 E = \sigma - P = \varepsilon_0 E_0 - \chi_E \varepsilon_0 E$

i.e. $(1 + \chi_E)\varepsilon_0 E = \varepsilon_0 E_0$

We define the relative permittivity, $\varepsilon_r = (1 + \chi_E)$

So $\varepsilon_r \varepsilon_0 E = \varepsilon_0 E_0$

E-field in dielectrics



- $\varepsilon_r \varepsilon_0 E = \varepsilon_0 E_0$
- So E-field in dielectric

$$\boxed{E = \frac{E_0}{\varepsilon_r}}$$

- **i.e. the same as the E-field is free space divided by ε_r**

Capacitance when dielectric between parallel Plates

From Gauss's Law: $E = \frac{\sigma}{\varepsilon_r \varepsilon_0} = \frac{Q}{\varepsilon_r \varepsilon_0 A}$

$$V = V_+ - V_- = Ed = \frac{Qd}{\varepsilon_r \varepsilon_0 A}$$

$$\boxed{C = \frac{Q}{V} = \frac{\varepsilon_r \varepsilon_0 A}{d}}$$

$\varepsilon_r > 1$ so capacitance is increased

Relative Permittivity ε_r

- Relative permittivity, ε_r also called the *dielectric constant*. (It's so great it has 2 names)
- For E-fields in a dielectric, Gauss's Law becomes:

$$\int_S \underline{E} \cdot d\underline{S} = \frac{Q_{enc}}{\varepsilon_r \varepsilon_0}$$

- For oscillating E-fields (e.g. EM waves) ε_r generally varies with frequency of E-field (EM2)

Relative Permittivity ε_r

Material	ε_r
vacuum	1 (by definition)
air	1.00059
PTFR/Teflon	2.1
Polystyrene	2.4–2.7
Mylar	3.1
Concrete	4.5
Pyrex (glass)	4.7 (3.7–10)

Next Week

- Be begin ……..

MAGNETISM