Optics and Waves

Lecture 17

Lenses (Cont.)
Y&F 34.3-34.4

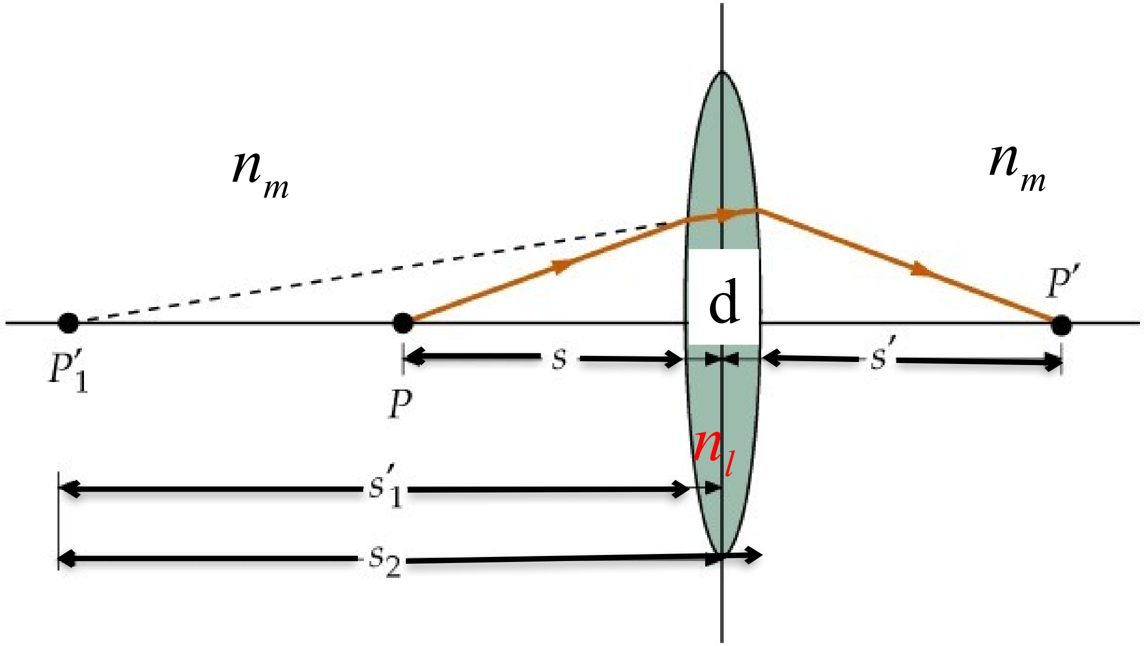
Refraction at a curved (spherical) interface

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{(n_2 - n_1)}{r}$$

Sign convention for refracting surfaces:

Radius of curvature: positive if centre of curvature on the same side of the outgoing ray. (convex towards object); otherwise, it is negative (concave towards object).

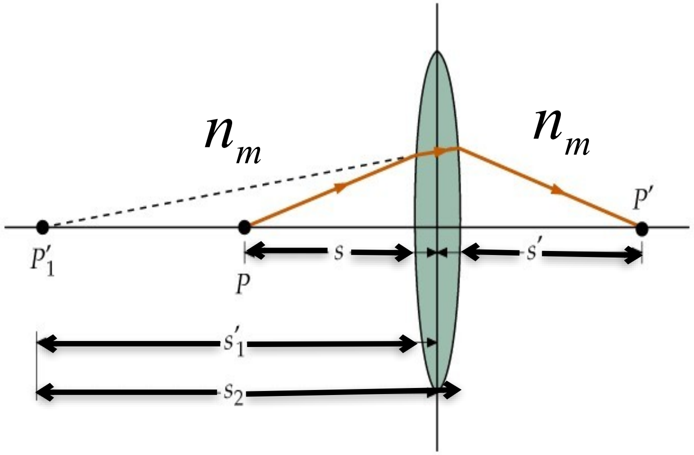
Thin lenses: Thickness of lens is much smaller than the radius.



Consider a lens of refractive index n_l with the refractive index of the medium n_m. Object is P.

Image P' is formed by refraction at each surface separately.

1st, Consider refraction at the first surface: It gives image P_1'. A virtual image.



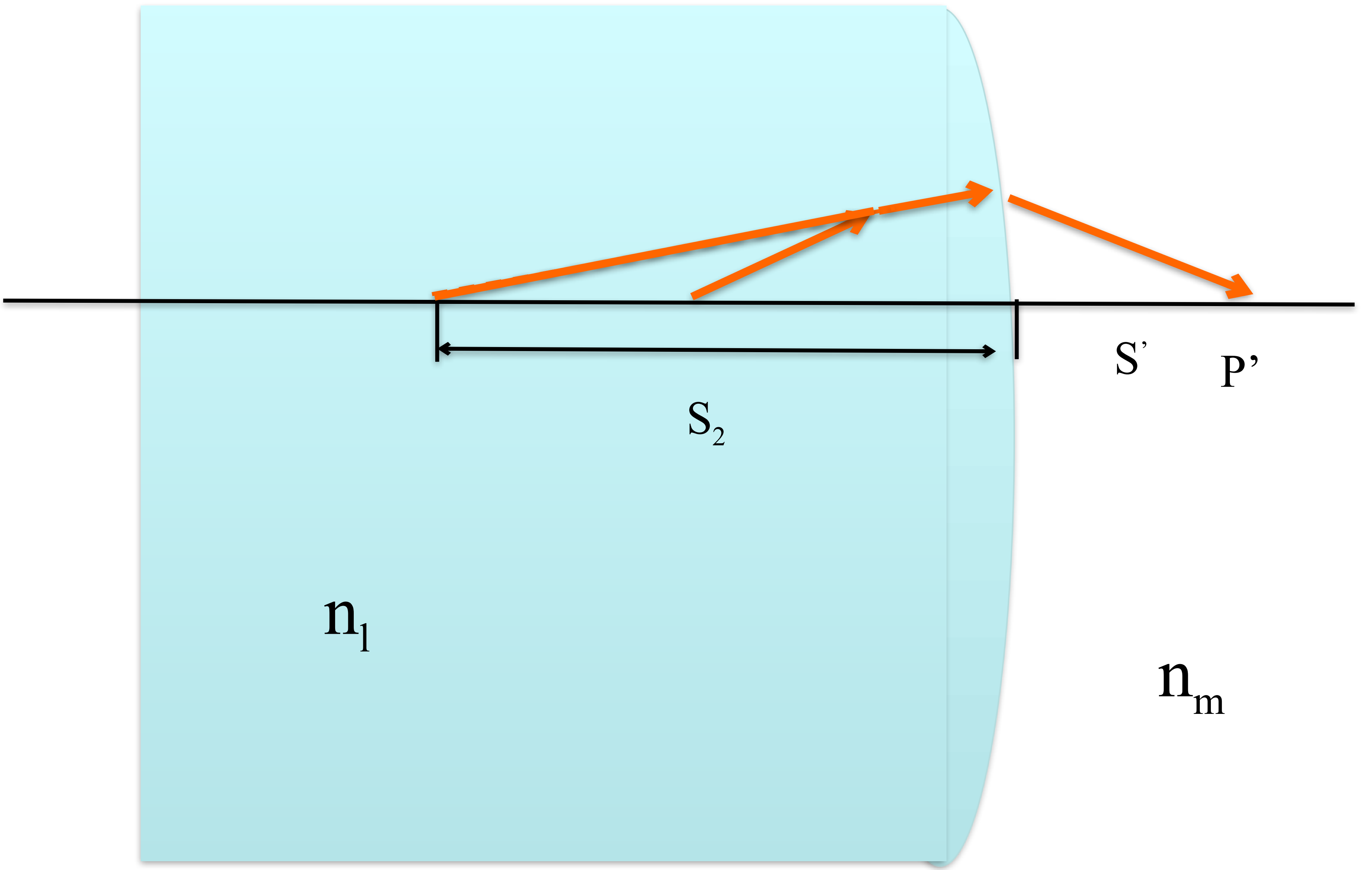
Applying usual equation for a surface….to the first surface

$$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{(n_2-n_1)}{r}$$

$$\frac{n_m}{s}+\frac{n_l}{s_1{}'}=\frac{n_l-n_m}{r_1} \qquad (A)$$

Equa. (A) gives you s_1'. In this case the image distance s_1' is negative (virtual image to the left).

So rays at second surface behave as if they came from P_1' in a straight line.



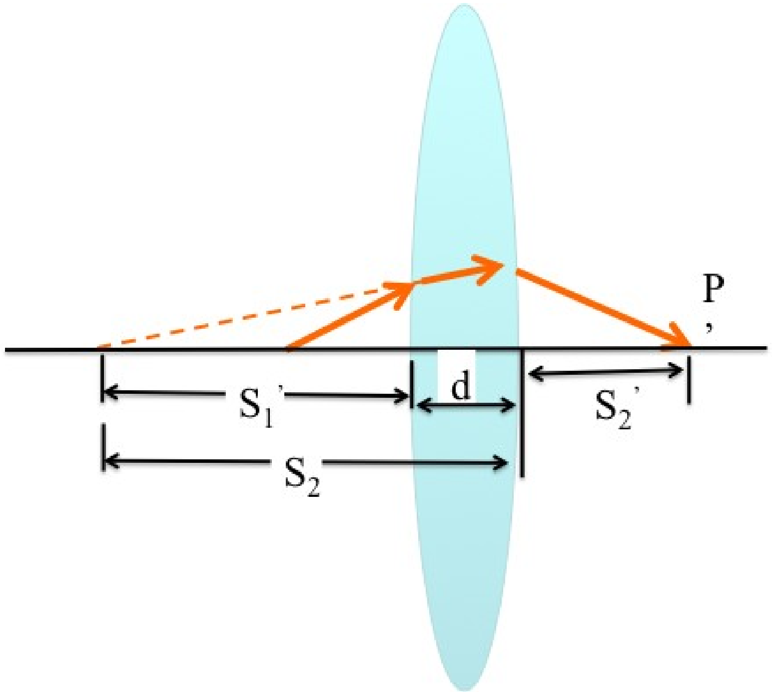
S'
P'
S_2
n_1
n_m

At second surface, medium on incident wave side has refractive index n_l and refracted side of n_m.

$$\frac{n_l}{s_2}+\frac{n_m}{s'}=\frac{n_m-n_l}{r_2}$$

$s_2 = |s_1'| + d = d - s_1'$

$$\frac{n_l}{d-s_1'}+\frac{n_m}{s'}=\frac{n_m-n_l}{r_2} \qquad (B)$$



Add $A+B$

$$\frac{n_m}{s}+\frac{n_l}{s_1'}+\frac{n_l}{d-s_1'}+\frac{n_m}{s'}=\frac{n_l-n_m}{r_1}+\frac{n_m-n_l}{r_2}$$

For a thin lens $d \rightarrow 0$

$$\frac{n_m}{s}+\frac{n_m}{s'}=\frac{(n_l-n_m)}{r_1}+\frac{(n_m-n_l)}{r_2}=(n_l-n_m)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

$$\frac{n_m}{s}+\frac{n_m}{s'}=(n_l-n_m)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

When s→∞

$$\frac{n_m}{s'}=(n_l-n_m)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

$$\frac{1}{s'}=\frac{(n_l-n_m)}{n_m}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

This s' is called the focal length of the lens f.

$$\frac{1}{f}=\frac{(n_l-n_m)}{n_m}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

This is called the lensmaker's equation!

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

The thin lens equation

If the medium is air then, $n_m = 1$

$$\frac{1}{f} = (n_l - 1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Summary

The lensmaker's equation

$$\frac{1}{f} = \frac{(n_l - n_m)}{n_m}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

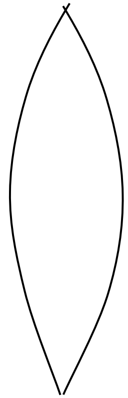
The thin lens equation

$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

Worked Examples of lenses

Y&F 34.3-34.4

Biconvex lens: $r_1>0$, $r_2<0$
Incoming ray assumed from the left.



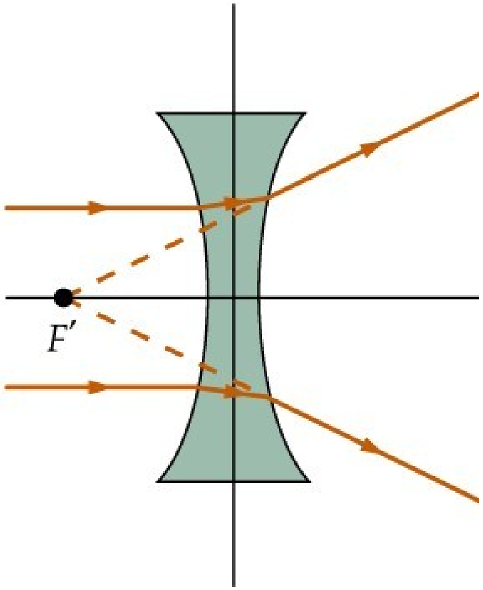
$$\frac{1}{f}=\frac{(n_l-n_m)}{n_m}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

$$=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)$$

$n=n_l/n_m$

If $n_l>n_m$, such as glass lens in air, f is +ve.

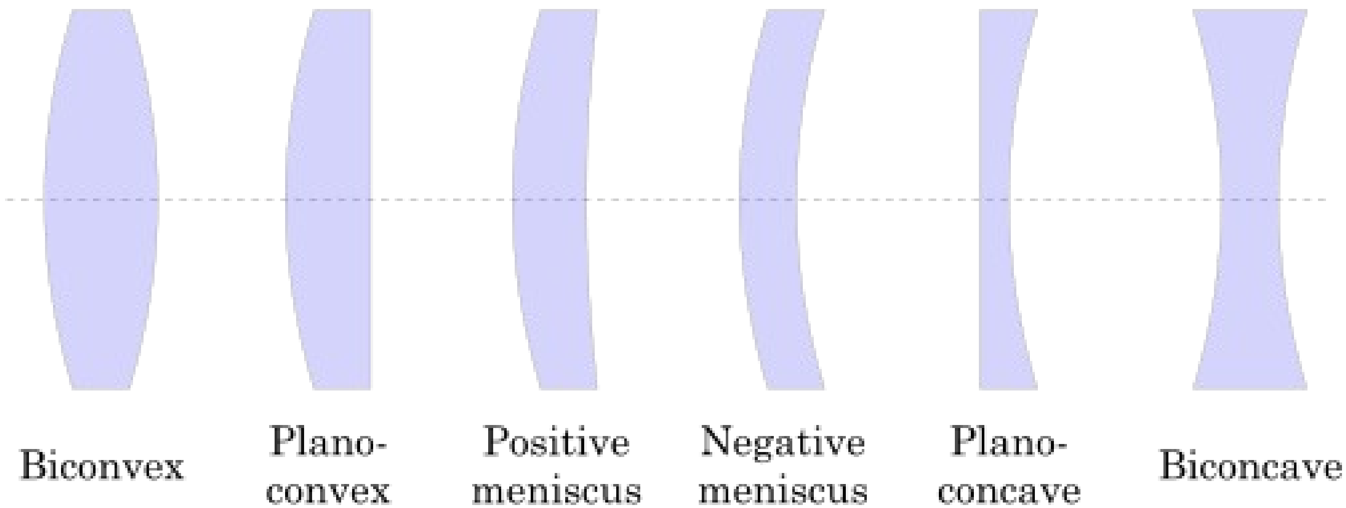
Biconcave lens



$r_1<0$, $r_2>0$

$$\frac{1}{f}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)<0 \; i.e. \; f \; -ve$$

Other types of lenses:



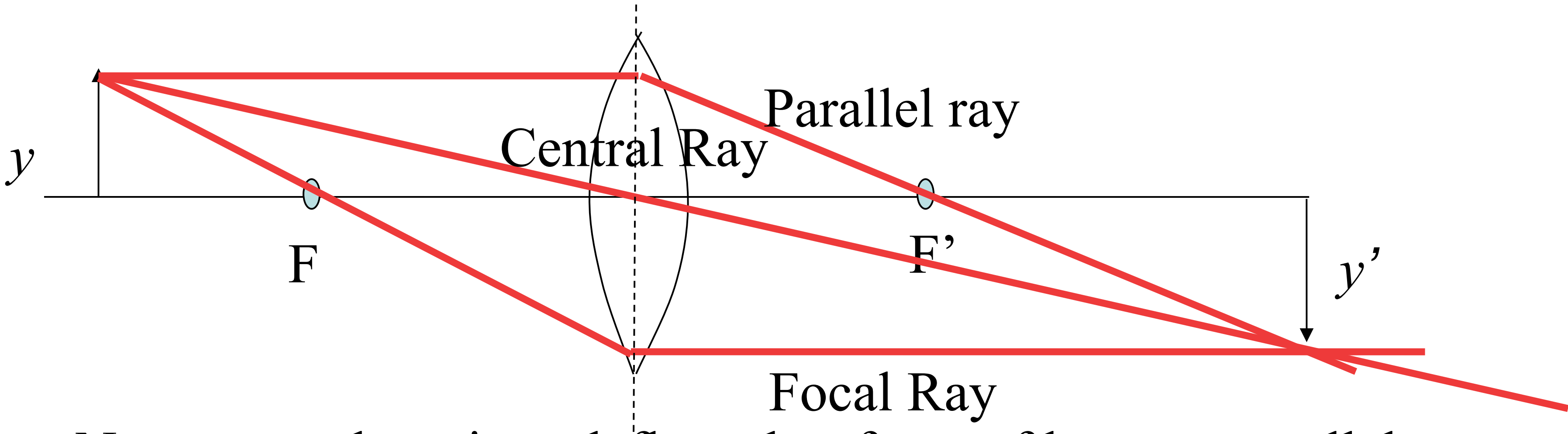
f can be calculated for each individual lens, and it depends on r_1 and r_2.

$P=1/f$ is called the power of the lens measured in units of dioptres (1/m)

A lens with a shorter focal length is a more powerful lens.

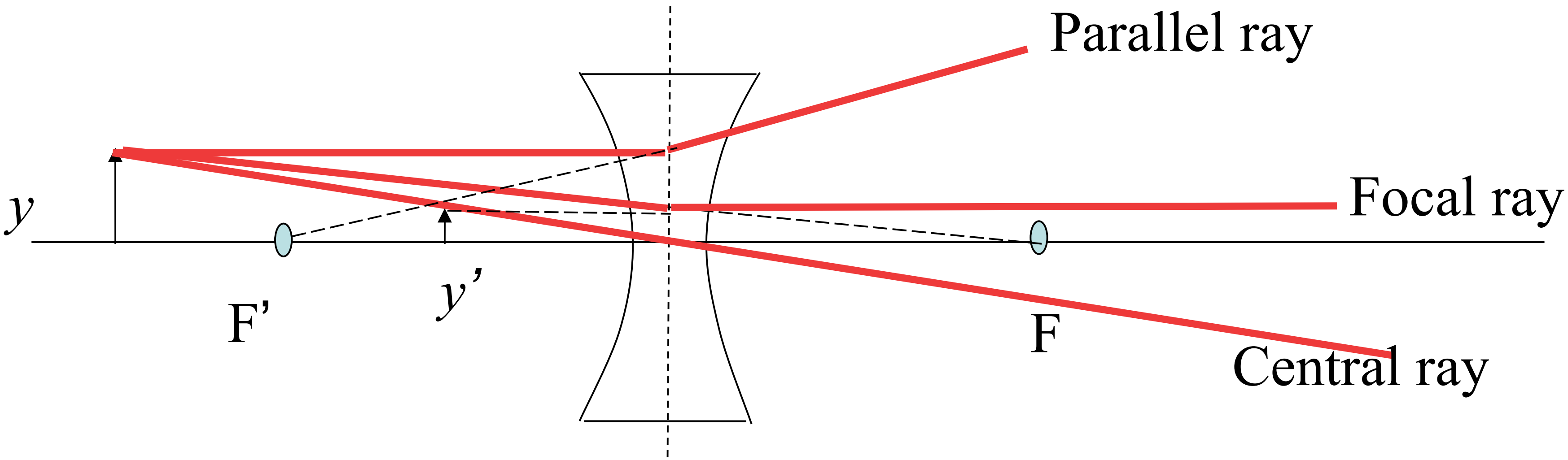
Ray tracing

Converging lens



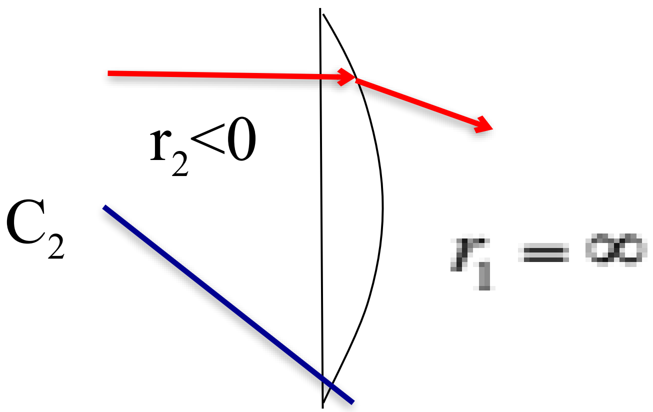
Note: central ray is undeflected as faces of lens are parallel – just like looking through a window (get slight displacement)

Diverging lens



Example 1

Plane – convex lens of refractive index of 1.5 and convex radius of curvature of 15 cm. What is the focal length?



$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\frac{1}{f} = (1.5-1)\left(\frac{1}{\infty} - \frac{1}{-15}\right) = \frac{0.5}{15}$$

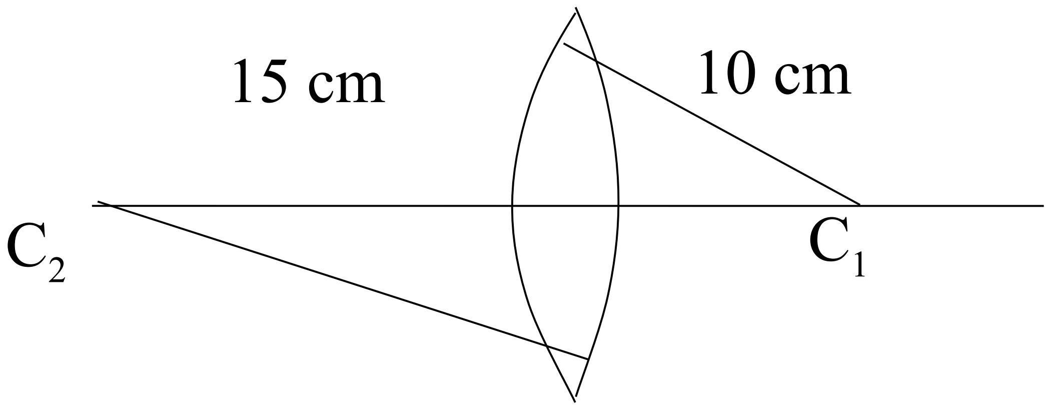
$$f = 30\ cm$$

$$P = 3.3 dioptres$$

This is a converging lens

Example 2

An object 1.2 cm high is placed 4 cm from a double convex lens (radii of curvature 10 and 15 cm, refractive index 1.5). Locate the image, and perform the ray tracing. Is the image real or virtual, and what is its height?



$r_2 = -15$ cm

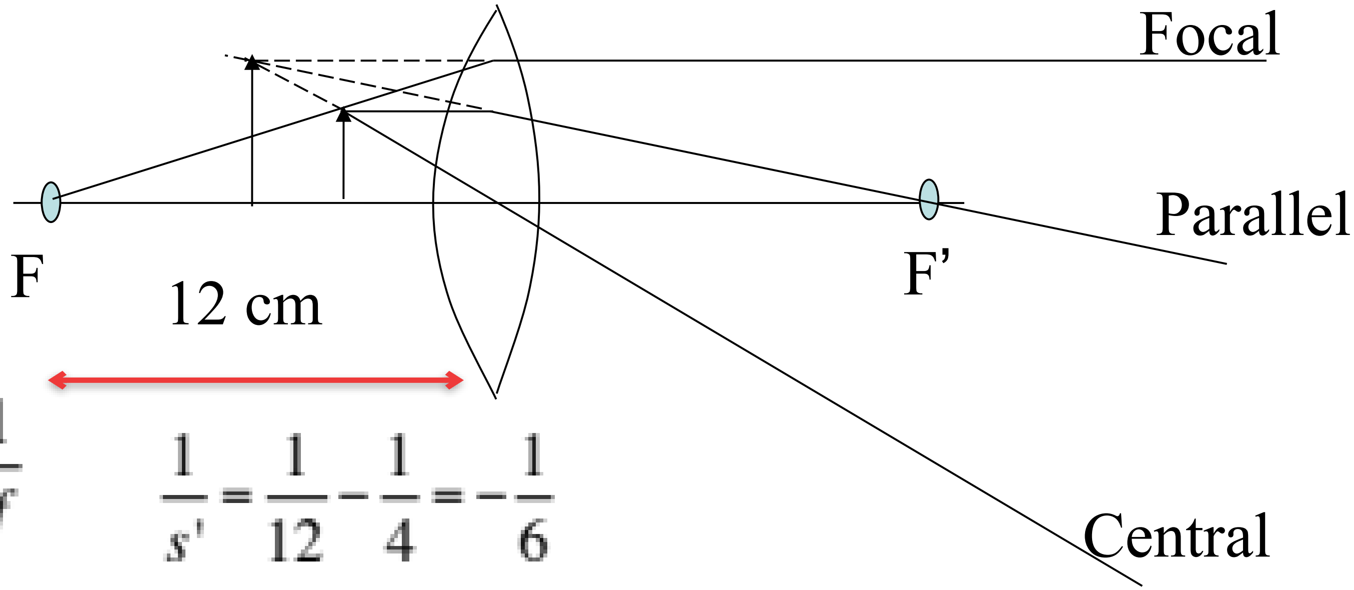
$r_1 = 10$ cm

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\frac{1}{f} = (1.5-1)\left(\frac{1}{10} - \frac{1}{-15}\right) = \frac{0.5}{6}$$

$$f = 12\ cm,\ P = 8.3\,dioptres$$

Diagram to scale



$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}$$

$$\frac{1}{s'}=\frac{1}{12}-\frac{1}{4}=-\frac{1}{6}$$

$$s'=-6\ cm$$

So image is to left of lens and thus virtual

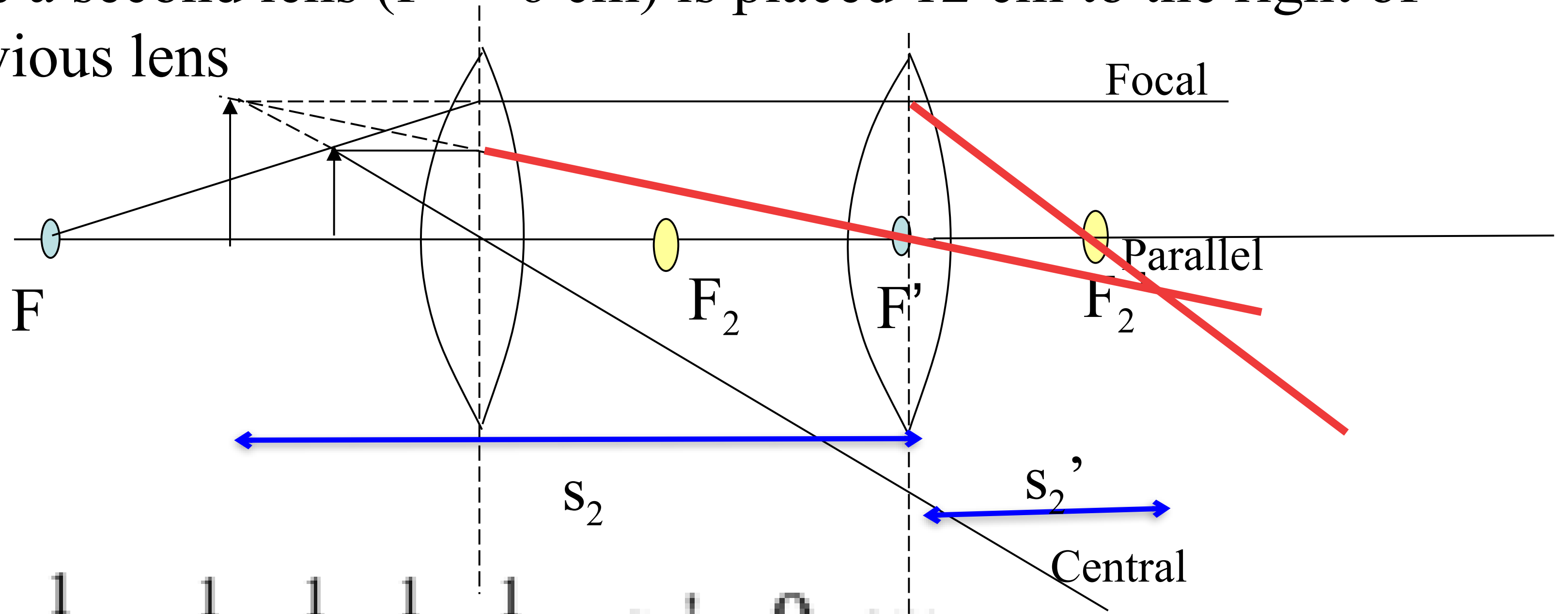
$$m=-\frac{s'}{s}=-\frac{-6}{4}=+1.5$$

Image is magnified

$$y'=1.5\times1.2=1.8\ cm$$

Systems with more than one lens

Imagine a second lens (f = +6 cm) is placed 12 cm to the right of the previous lens

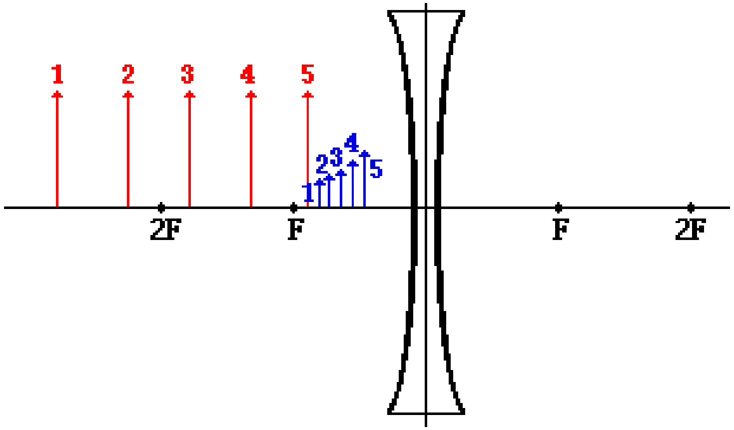


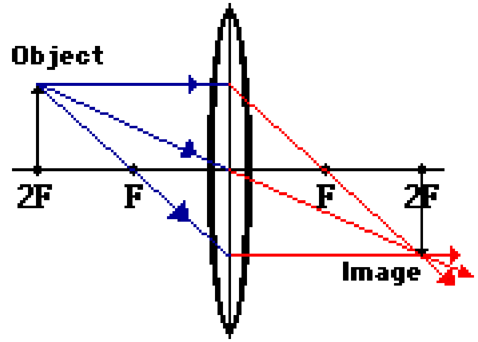
$$\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{f_2} \qquad \frac{1}{s_2'}=\frac{1}{6}-\frac{1}{18}=\frac{1}{9} \qquad s_2'=9\ cm$$

$$m_2=-\frac{s_2'}{s_2}=-\frac{9}{18} \qquad m_1=+1.5 \qquad m_{tot}=m_1\times m_2=1.5\times(-\frac{9}{18})=-0.75$$

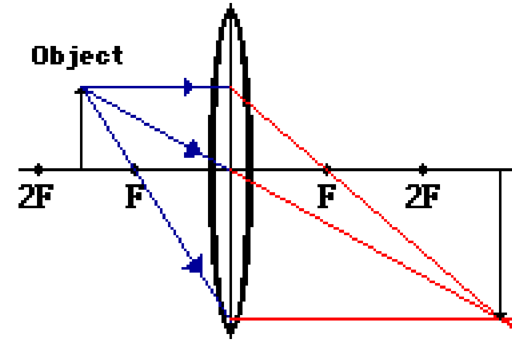
$$y'=-0.75\times 1.2=-0.9\,cm$$

Changed image from virtual to real



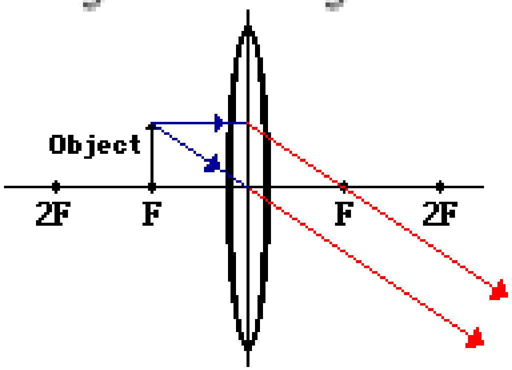


Ray Diagram for Object
Located at 2F

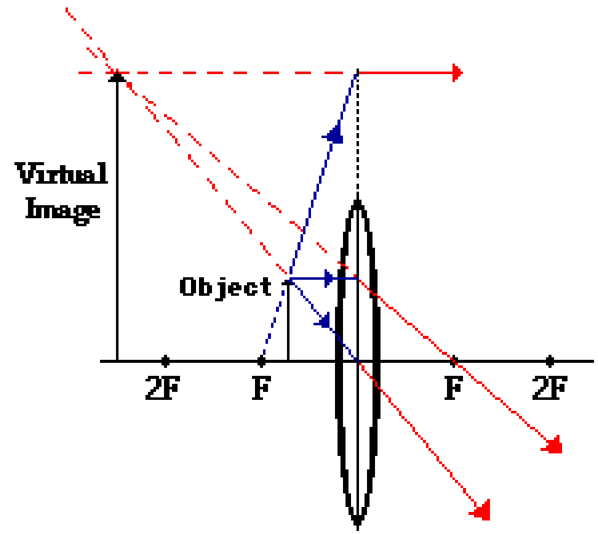


Ray Diagram for Object Located
Between F and 2F

$$\frac{1}{s}+\frac{1}{s'}=\frac{1}{f}, \quad \frac{1}{s'}=\frac{1}{f}-\frac{1}{s}=negative$$



Ray Diagram for Object Located at F
(an image is not formed)

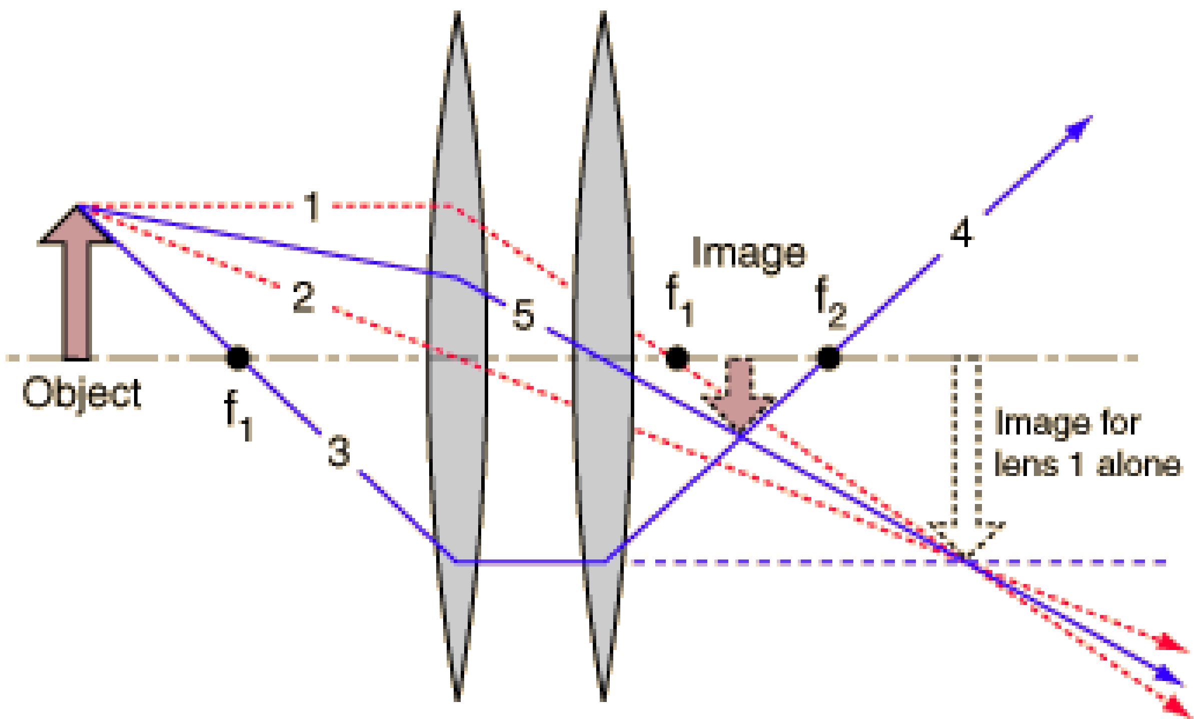


Ray Diagram for Object Located in Front of F

http://www.physicsclassroom.com/class/refrn/U14L5da.cfm

This page is for the very keen. The image formed by lens 1 falls on the right hand side of lens 2. The overall image due to both lenses is shown by the arrow in between F1 and F2.

1. The principal rays 1 and 2 are used to determine the location of the of the image for lens 1 alone.

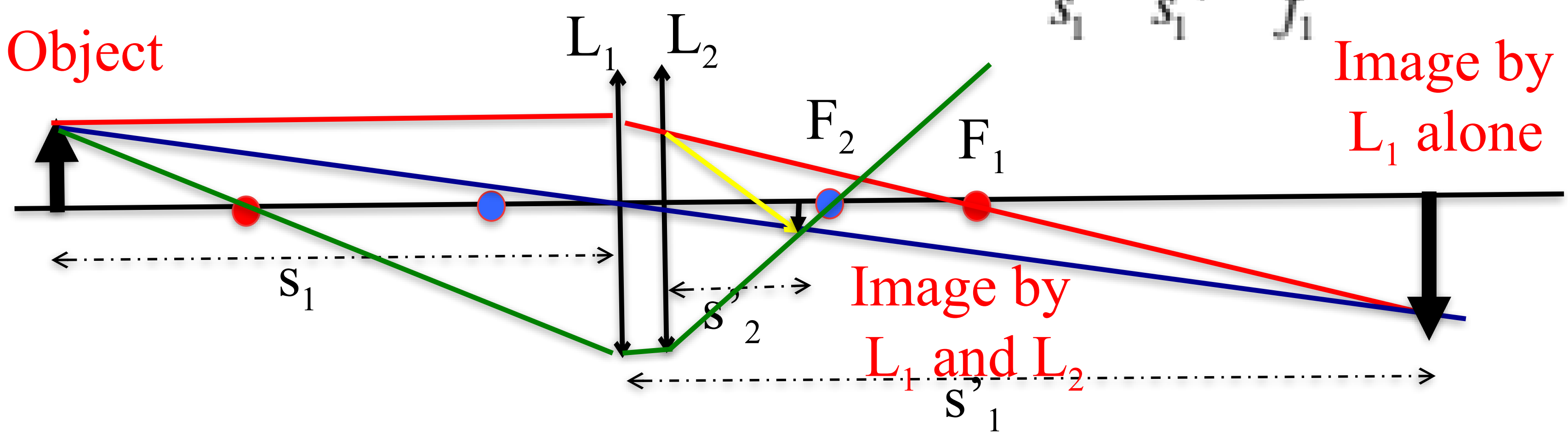


2. Ray 3 through f_1 will approach lens 2 parallel to the axis and will project through focal point f_2, forming one principal ray (4) for the final image.

3. Back projecting from the single lens image through the center of lens 2 will define the second needed ray (5) since that ray will be undeflected.

http://hyperphysics.phy-astr.gsu.edu/hbase/geoopt/raydiag.html

Two lenses are added together (no spacing) $\frac{1}{s_1}+\frac{1}{s_1'}=\frac{1}{f_1}$

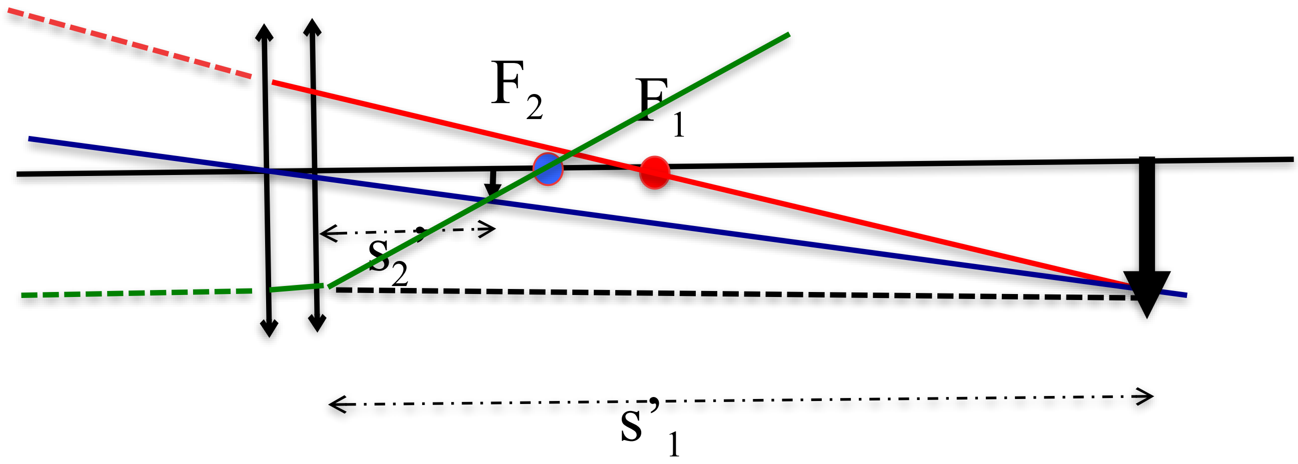


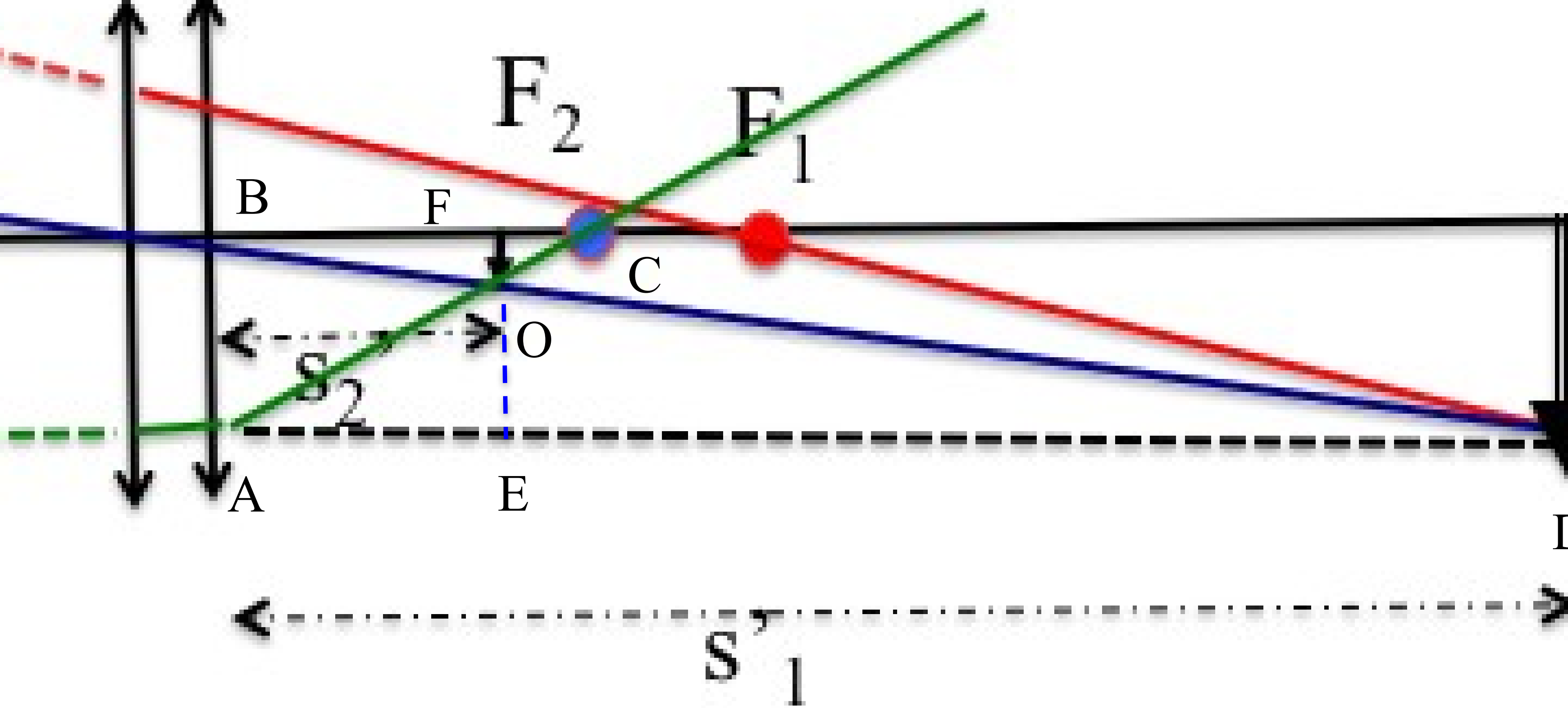
For the second lens, $s_2 = -s_1'$, s_1' is positive, but s_2 is negative.

$$\frac{1}{s_2}+\frac{1}{s_2'}=\frac{1}{f_2}\ ,\quad -\frac{1}{s_1'}+\frac{1}{s_2'}=\frac{1}{f_2}\ ,$$

$$\frac{1}{s_2'}=\frac{1}{s_1'}+\frac{1}{f_2}=\frac{1}{f_1}-\frac{1}{s_1}+\frac{1}{f_2},\quad \frac{1}{s_1}+\frac{1}{s_2'}=\frac{1}{f_1}+\frac{1}{f_2}$$

$$or\quad \frac{1}{s}+\frac{1}{s'}=\frac{1}{f_1}+\frac{1}{f_2}=\frac{1}{f}$$


F$_2$
F$_1$
s$_2$
s'$_1$



BCO/ADO: $f_2/s'_1=CO/AO$:   BC=f_2

riangles FCO and AEO: $CO/AO=CF/AE=(f_2-s'_2)/s'_2$

$f_2/s'_1=(f_2-s'_2)/s'_2$ $f_2/s'_1=(f_2/s'_2-1)$ $f_2/s'_1-f_2/s'_{2-}=(-1)$

$1/s'_1-1/s'_{2-}=-1/f_2$ $-1/s_2-1/s'_{2-}=-1/f_2$ $1/s_2+1/s'_{2-}=1/f_2$