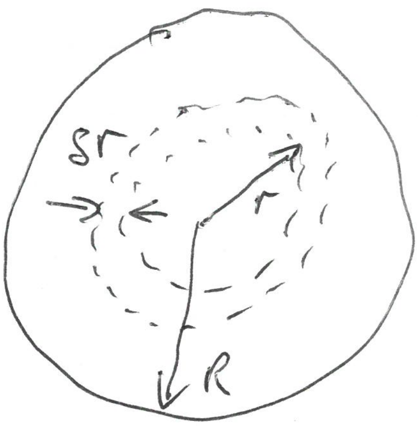
EM1 - Lec 4

Example 4-1



Sphere with

$$\rho(r) = \frac{\rho_0 r}{R}$$

Find E-field inside.

$$\int_S \underline{E} \cdot d\underline{S} = \frac{Q_{enc}}{\varepsilon_0} = \frac{1}{\varepsilon_0} \int_V \rho(r)\, dV$$

Divide sphere into thin shells of thickness δr.

Volume of shell, $\delta V = 4\pi r^2 \delta r$

$$\therefore \; Q_{enc} = \int \rho(r)\, dV = \int_0^r \frac{\rho_0}{R} r \cdot 4\pi r^2\, dr$$

$$= \frac{4\pi \rho_0}{R} \int_0^r r^3\, dr = \underline{\underline{\frac{\pi \rho_0}{R} r^4}}$$

Gauss's Law $\int_S \underline{E} \cdot d\underline{S} = \frac{\pi \rho_0}{\varepsilon_0 R} r^4$

LHS $= E\, 4\pi r^2$ by symmetry

$$\therefore \; E = \underline{\frac{\rho_0}{4\varepsilon_0 R} r^2}$$

N.B. $\int \underline{E} \cdot d\underline{S}$

By symmetry $\underline{E}$ must be radial

hence $\underline{E} \cdot d\underline{S} = E\,dS$

And, by symmetry E must be constant for a fixed r

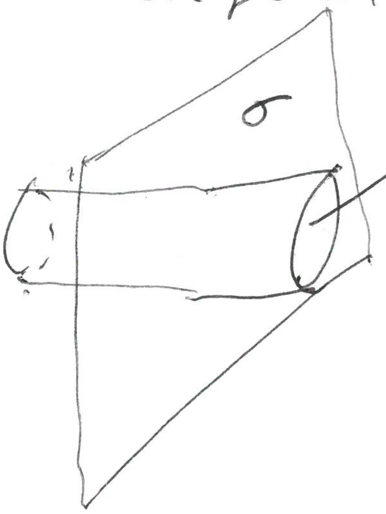
$$\therefore \int_S \underline{E} \cdot d\underline{S} = \int E\,dS = E \int dS$$

$\int_S dS$ over sphere = surface area of sphere

$= 4\pi r^2$

Example 4.2

infinite charged sheet.



make Gaussian surface.

$$\int_S \underline{E} \cdot d\underline{S} = \frac{Q_{en}}{\varepsilon_0}$$

LHS no field coming out the sides

only E-field from ends & they

are normal to surface.

Hence LHS : $EA + EA = 2EA$

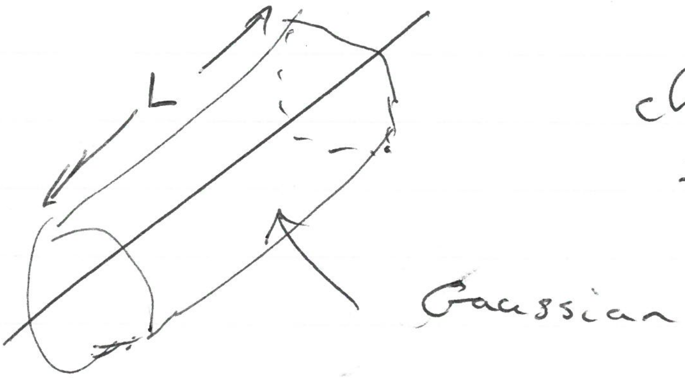
RHS : $\frac{Q_{enc}}{\varepsilon_0} = \frac{1}{\varepsilon_0} \sigma A$

$LHS = RHS \Rightarrow 2E\cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0}$

$$E = \frac{\sigma}{2\varepsilon_0}$$

much easier!

Example 4.3



infinite line of charge, charge density $= \lambda$

$$\int \underline{E} \cdot d\underline{S} = \frac{Q_{enc}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}.$$

$\underline{E} \parallel d\underline{S}$ & constant for fixed r

So $LHS = \int \underline{E} \cdot d\underline{S} = E \int dS = E \cdot 2\pi r L$

$LHS = RHS$. $E \cdot 2\pi r \cancel{L} = \frac{\lambda \cancel{L}}{\varepsilon_0}$

$$\Rightarrow E = \frac{\lambda}{2\pi\varepsilon_0 r}$$