Electromagnetism

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Lecture 6

Electrical Potential Examples

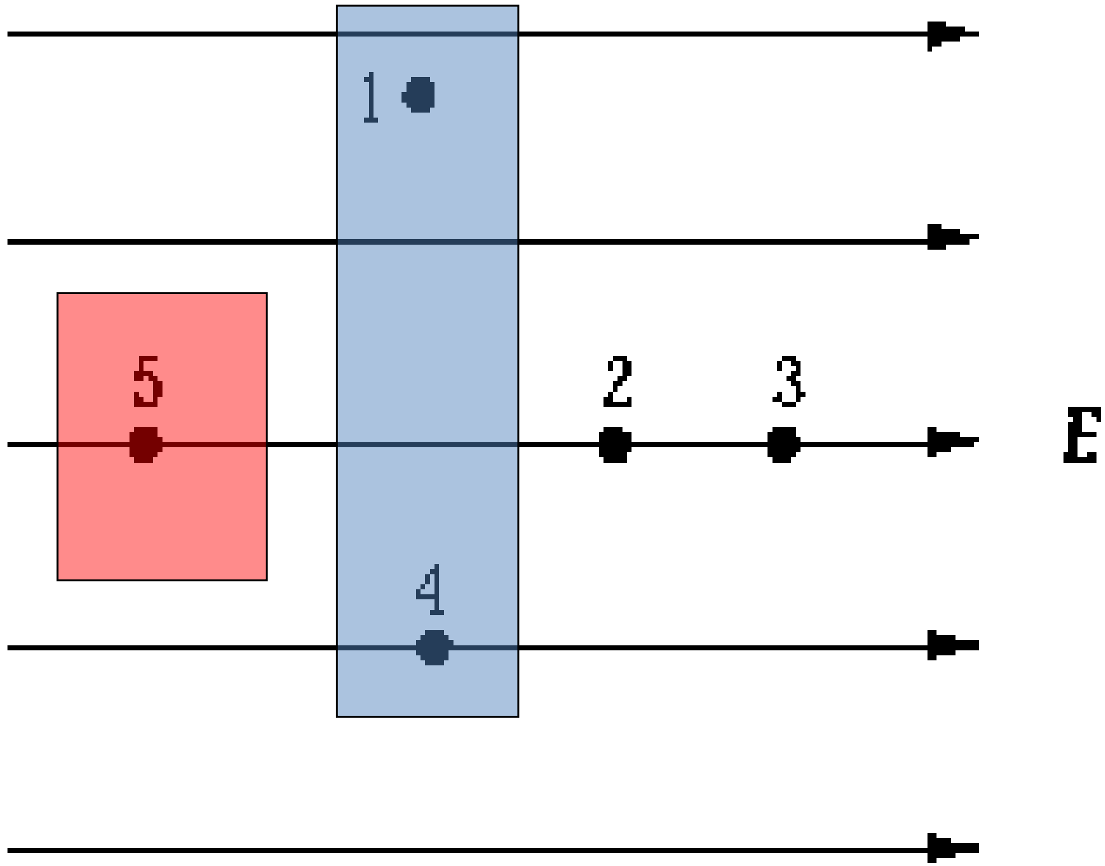
Week 3

Last Lecture

- **Electrical Potential**
  - **Definition**
  - **Relationship between electrical potential, V and Electric field $\underline{\mathrm{E}}$.**
  - **The Del operator, ∇**

This Lecture

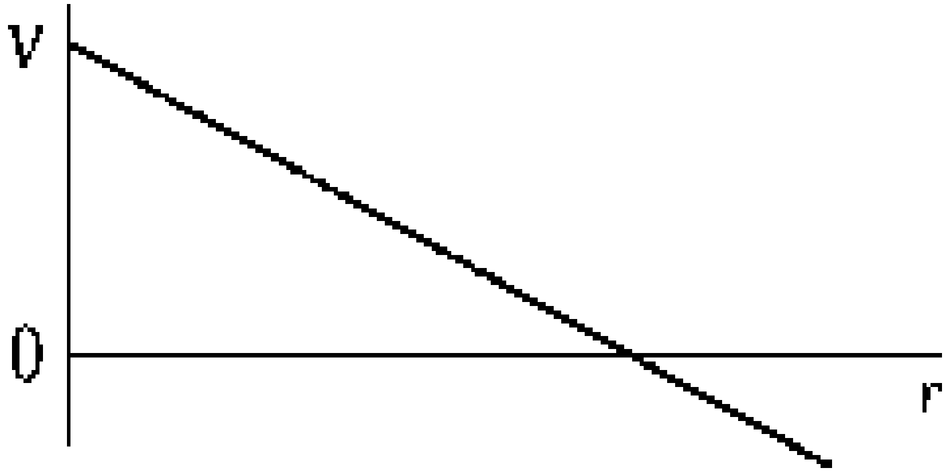
- Calculating electrical potential
- Calculating V if E-field known
- Calculating E-field from V
  - I.e. Using $\underline{E} = -\nabla V$
- Calculating change in potential energy



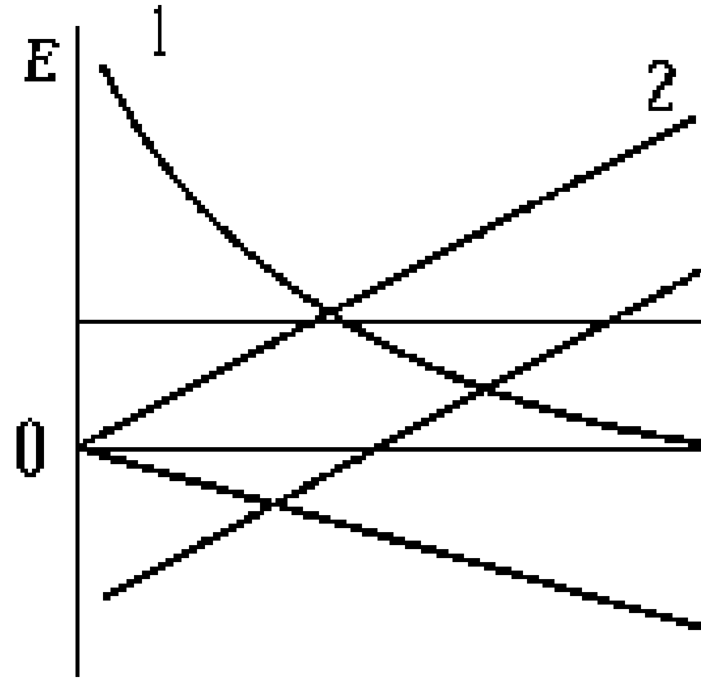
Which of the points shown in the diagram are at the same potential?

A) 2 and 5 B) 2, 3, and 5 C) 1 and 4 D) 1 and 5 E) 2 and 4

Which point in the electric field in the diagram is at the highest potential?

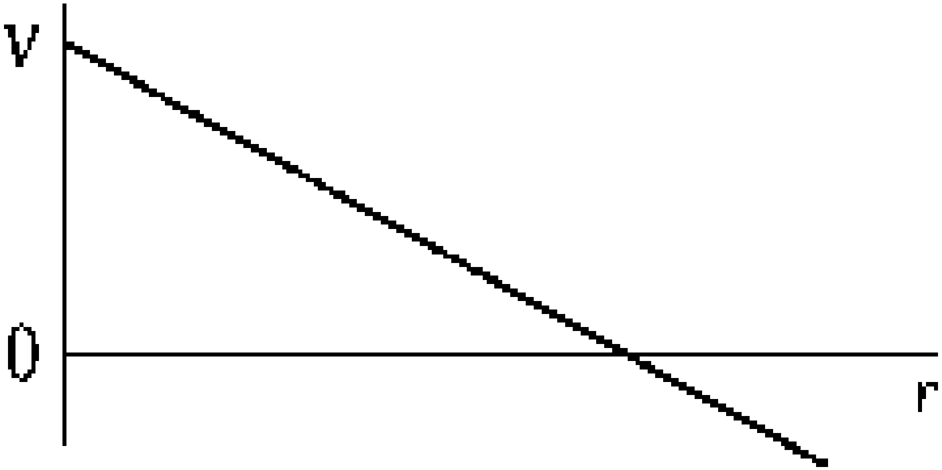


(1)

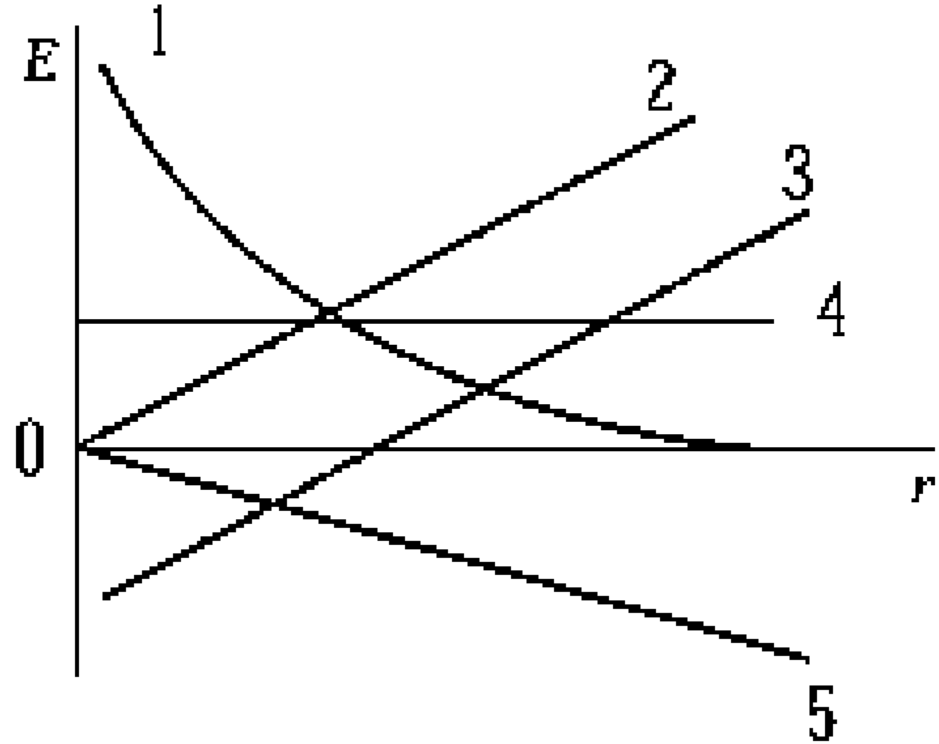


(2)

The electric potential as a function of distance along a certain line in space is shown in graph (1). Which of the curves in graph (2) is most likely to represent the electric field as a function of distance along the same line?



(1)



(2)

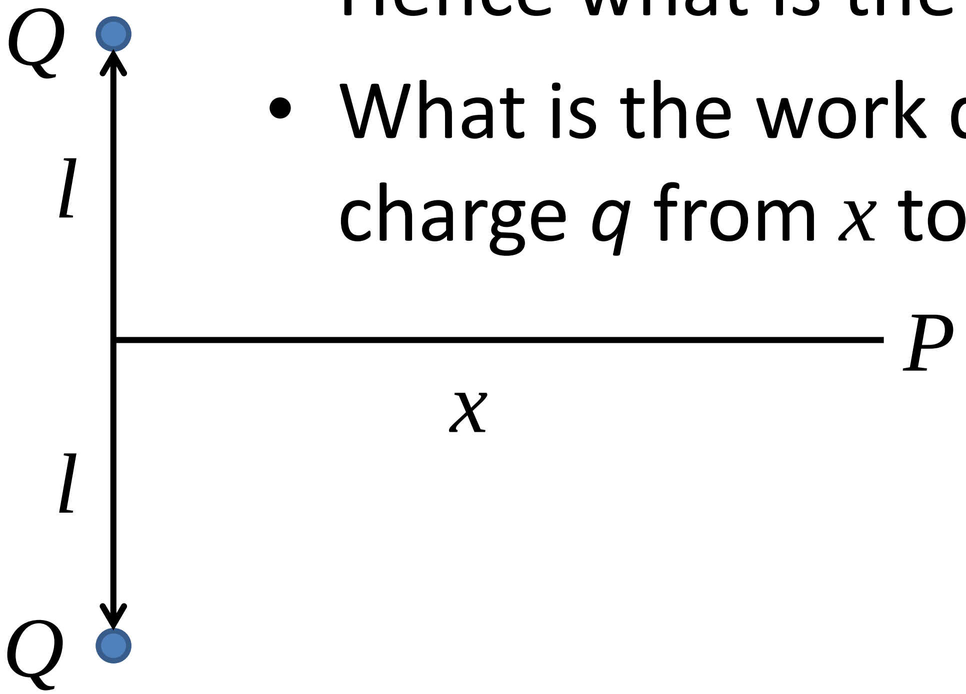
$$V = -mr + c$$

$$\underline{E} = -\nabla V = -\frac{dV}{dr}\underline{\boldsymbol{i}} = m\underline{\boldsymbol{i}}$$

$$E = \left|\underline{E}\right| = m$$ i.e. constant Line 4

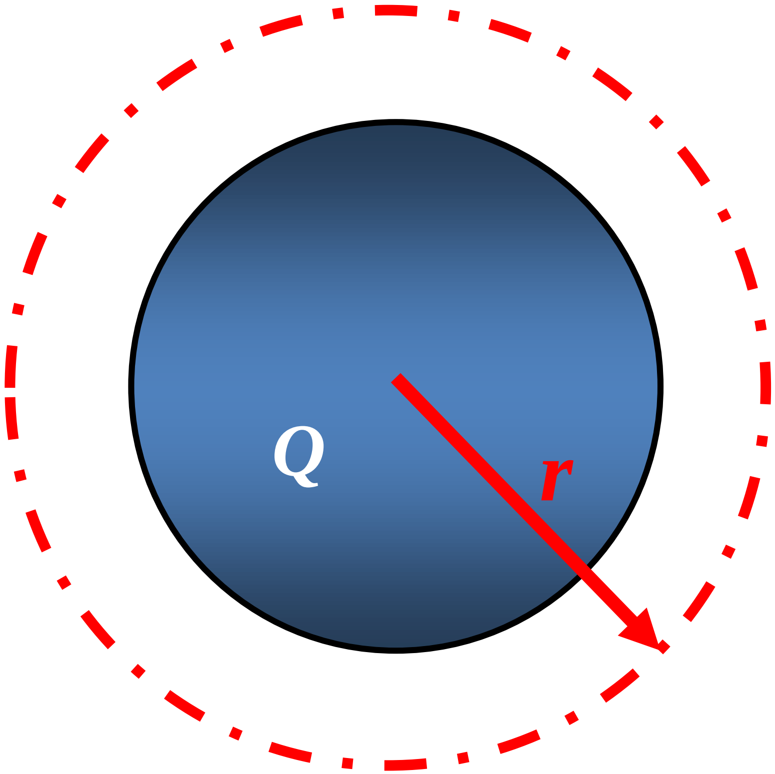
V from Two Charges

- What is the potential, V at point P?
- Hence what is the E-field at P?
- What is the work done in moving a charge q from x to $2x$?



V Inside and Outside Charged Sphere

Use: $V = -\int \underline{E} \cdot d\underline{r}$



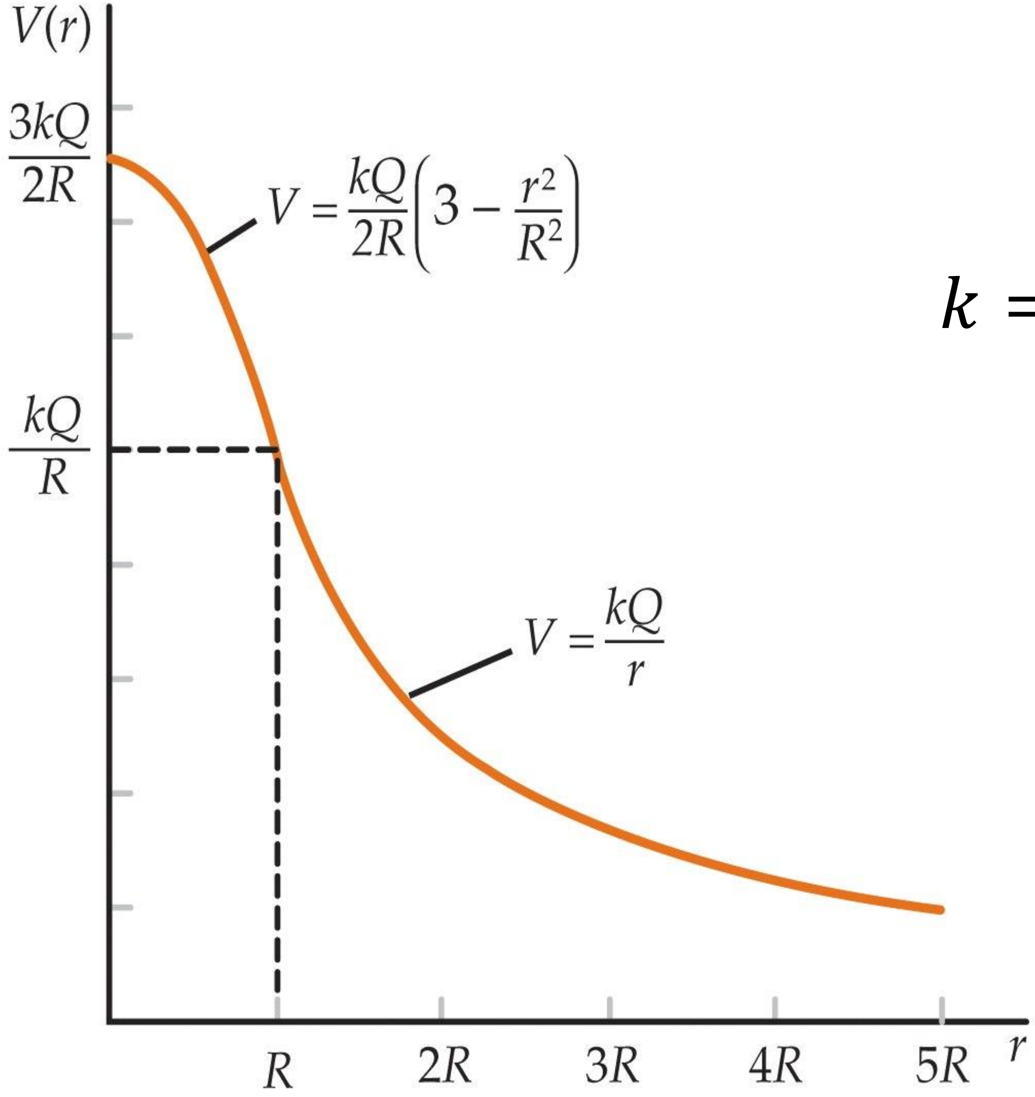
- Use Gauss's Law to find:
- **(a) *E*-field for *r* > *R***

$$|E| = \frac{Q}{4\pi\varepsilon_0 r^2}$$

- **(b) *E*-field for *r* < *R***

$$|E| = \frac{Q\,r}{4\pi\varepsilon_0 R^3}$$

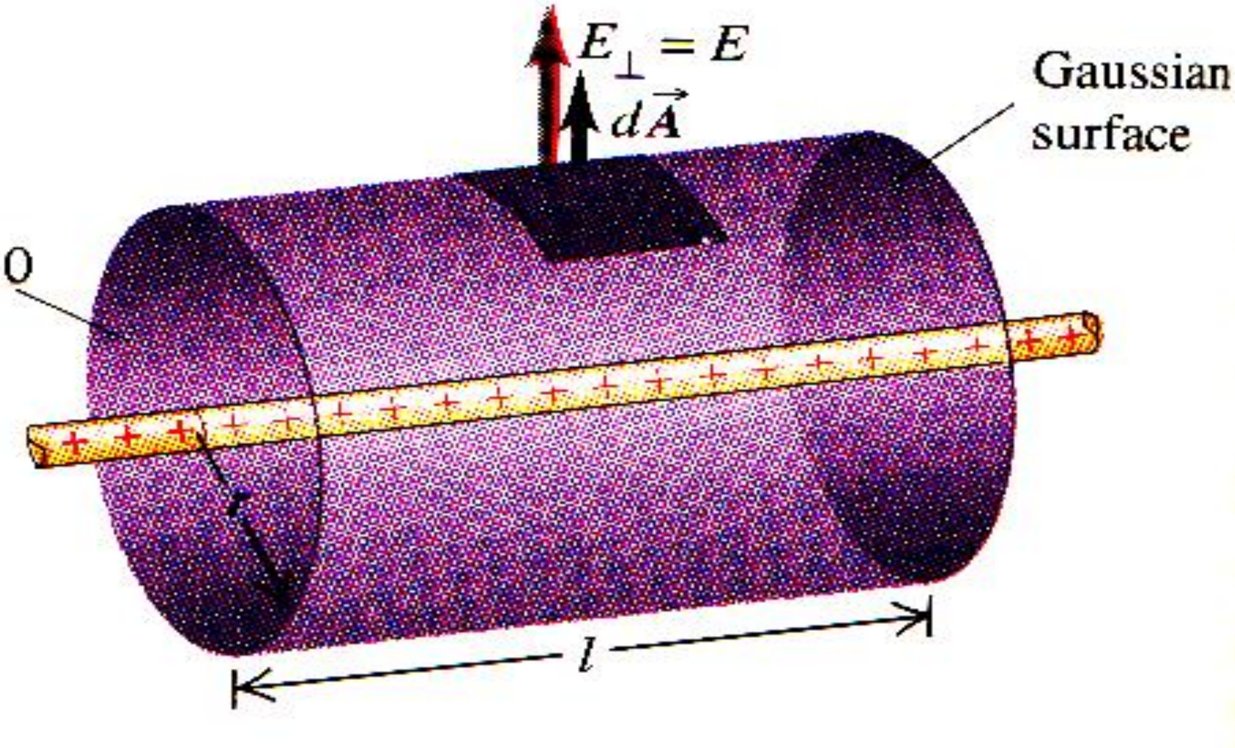
V Inside and Outside Charged Sphere



$$k = \frac{1}{4\pi\varepsilon_0}$$

V of infinite wire of uniform charge λ per unit length

Gauss's Law



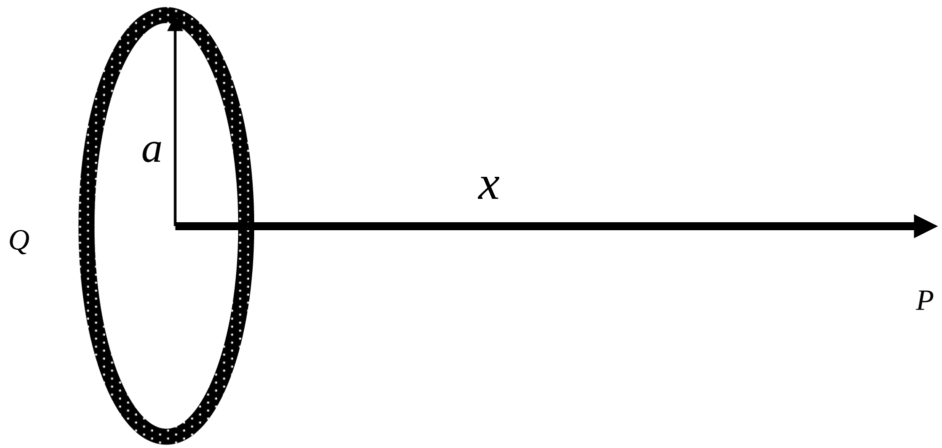
$$\int_S \underline{E} \cdot d\underline{S} = \frac{Q_{encl}}{\varepsilon_0}$$

$$E\ 2\pi r l = \frac{\lambda l}{\varepsilon_0}$$

$$\underline{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\underline{\hat{r}}$$

What is the potential outside wire?

A ring-shaped conductor, of radius *a*, carries a total charge *Q* uniformly distributed around it as illustrated below:



Find the potential at a point *P* on the ring axis at a distance *x* from the centre of the ring. (Hint consider the electric potential at *P* due to a small segment (effectively a point charge) carrying a charge d*Q*.)

Hence show that

$$E_x = \frac{Qx}{4\pi\varepsilon_0\left(a^2 + x^2\right)^{3/2}}$$