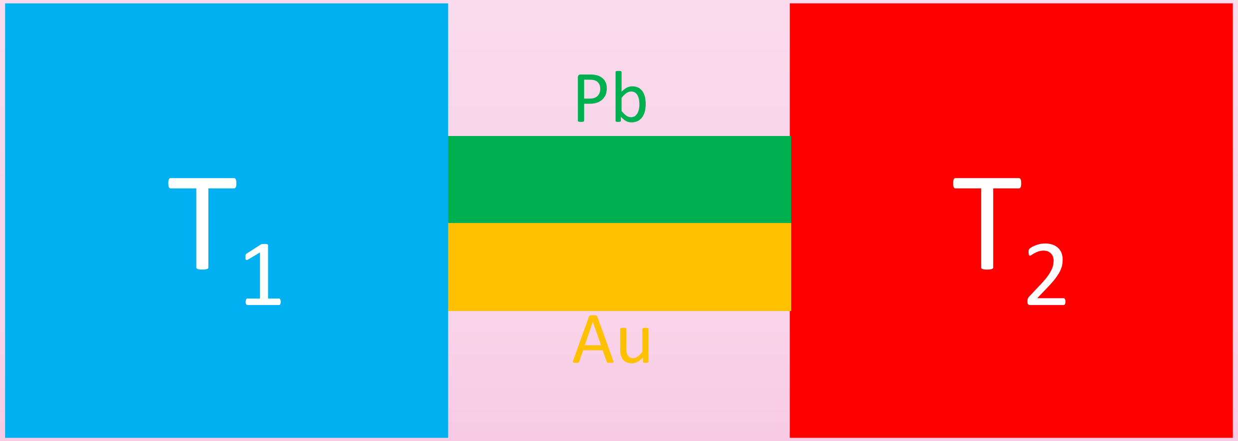
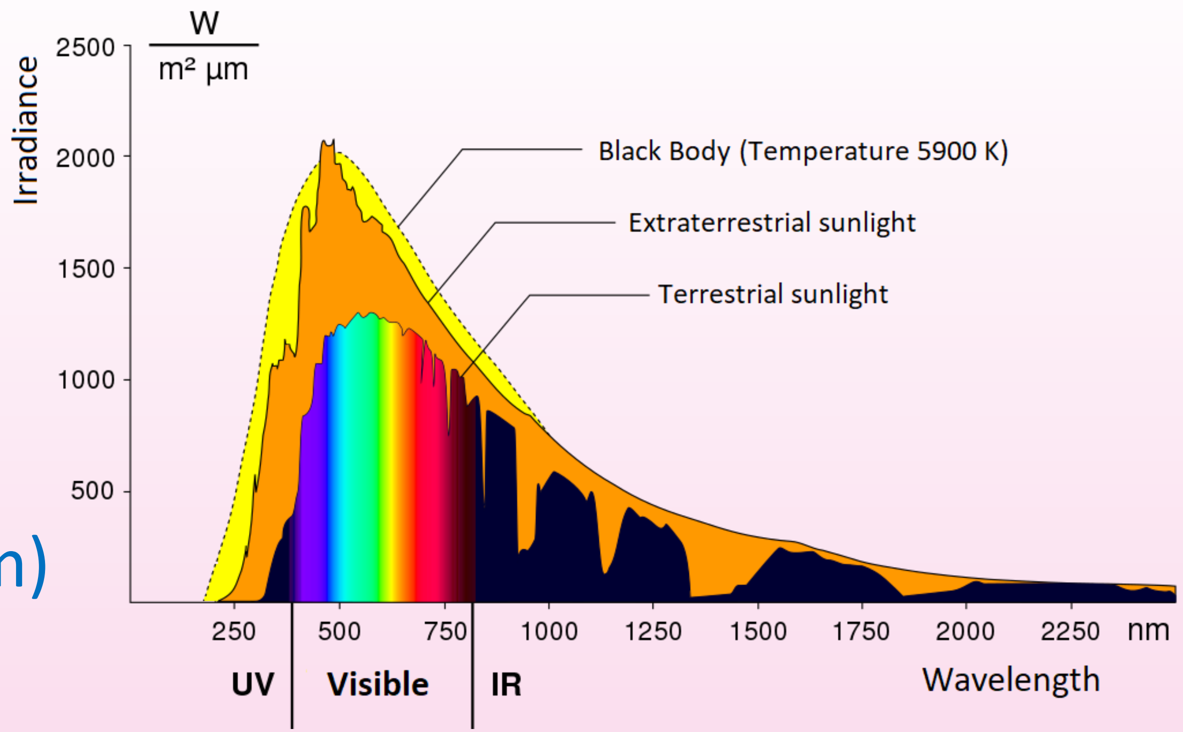
Recap from last time

Thermal resistances add like electrical resistances in series! $R = R_1 + R_2$

Thermal resistances add like electrical resistances in parallel! $1/R = 1/R_1 + 1/R_2$



Recap from last time



Black bodies emit and absorb EM radiation of all wavelengths, without reflecting any (unlike a white body, which reflect all light incident on them)

If a black body is in thermal equilibrium with its surroundings (both at some temperature T), they emit exactly the same power spectrum as they absorb

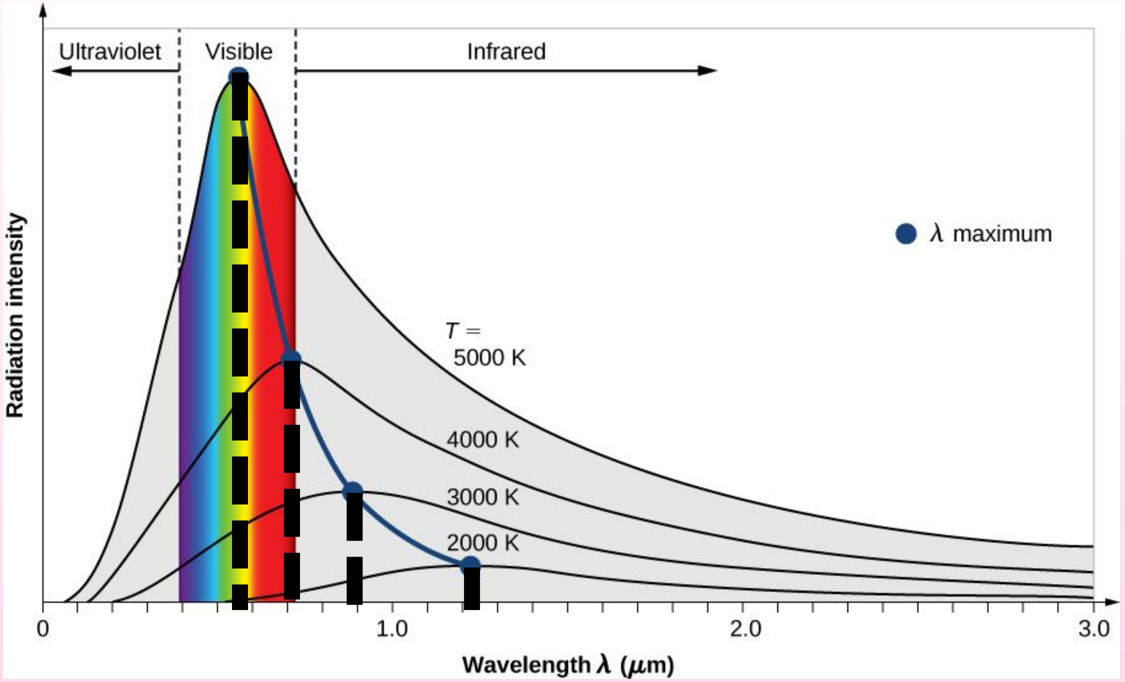
Planck's law

$$u(\lambda, T) = \frac{8\pi hc}{\lambda^5} \frac{1}{\left(e^{-\left(\frac{hc}{\lambda k_B T}\right)} - 1\right)}$$

Non-black bodies characterised by emissivity, ε (fraction of black body spectrum absorbed)

Recap from last time

By finding the maximum of the spectrum (through differentiation), you can find the wavelength corresponding to the maximal power output



https://phys.libretexts.org/@api/deki/files/15630/CNX_UPhysics_39_01_BBradcurve.jpg?revision=1

$$\lambda_{max} \approx \frac{hc}{5k_B T} \approx \frac{2.9\times10^{-3}\ \mathrm{mK}}{T}$$

Can also integrate the whole spectrum to give $\dot{Q}$, the total power output (energy per unit time)

$$\dot{Q} = \varepsilon\sigma A T^4$$

Temperature

Emissivity
(=1 for a black body)

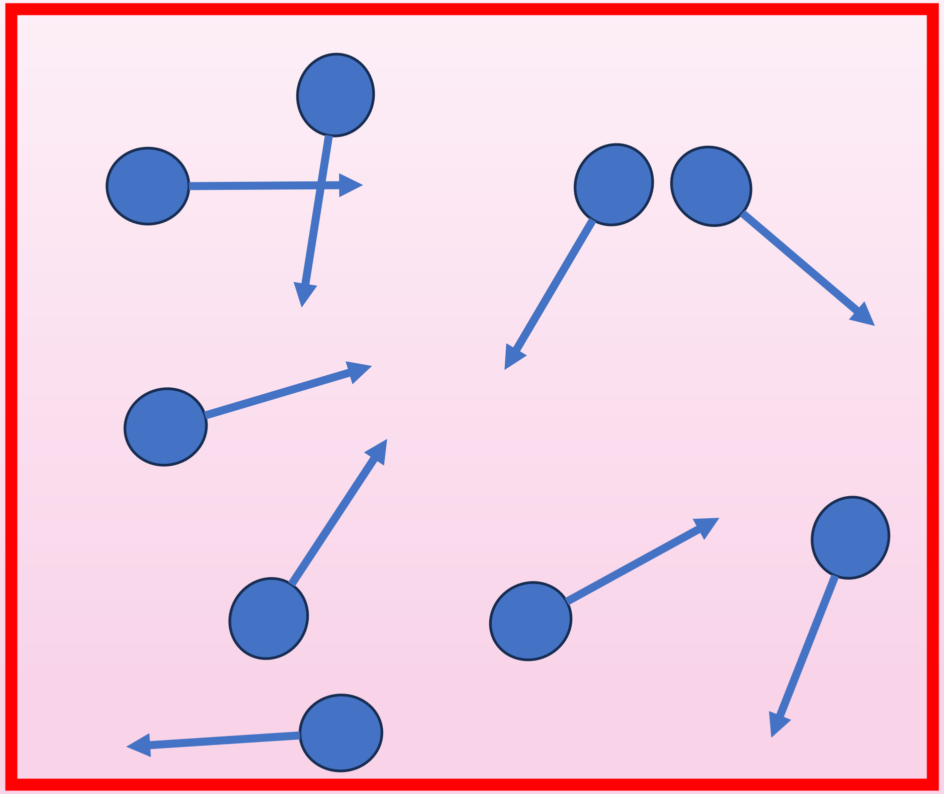
Stefan's constant
5.67 x 10^{-8} W m^{-2} K^{-1}

Surface area of body
(that is radiating power)

Kinetic theory of gases

Gas molecules move "randomly" – in that they can have vastly differing velocities (and positions)

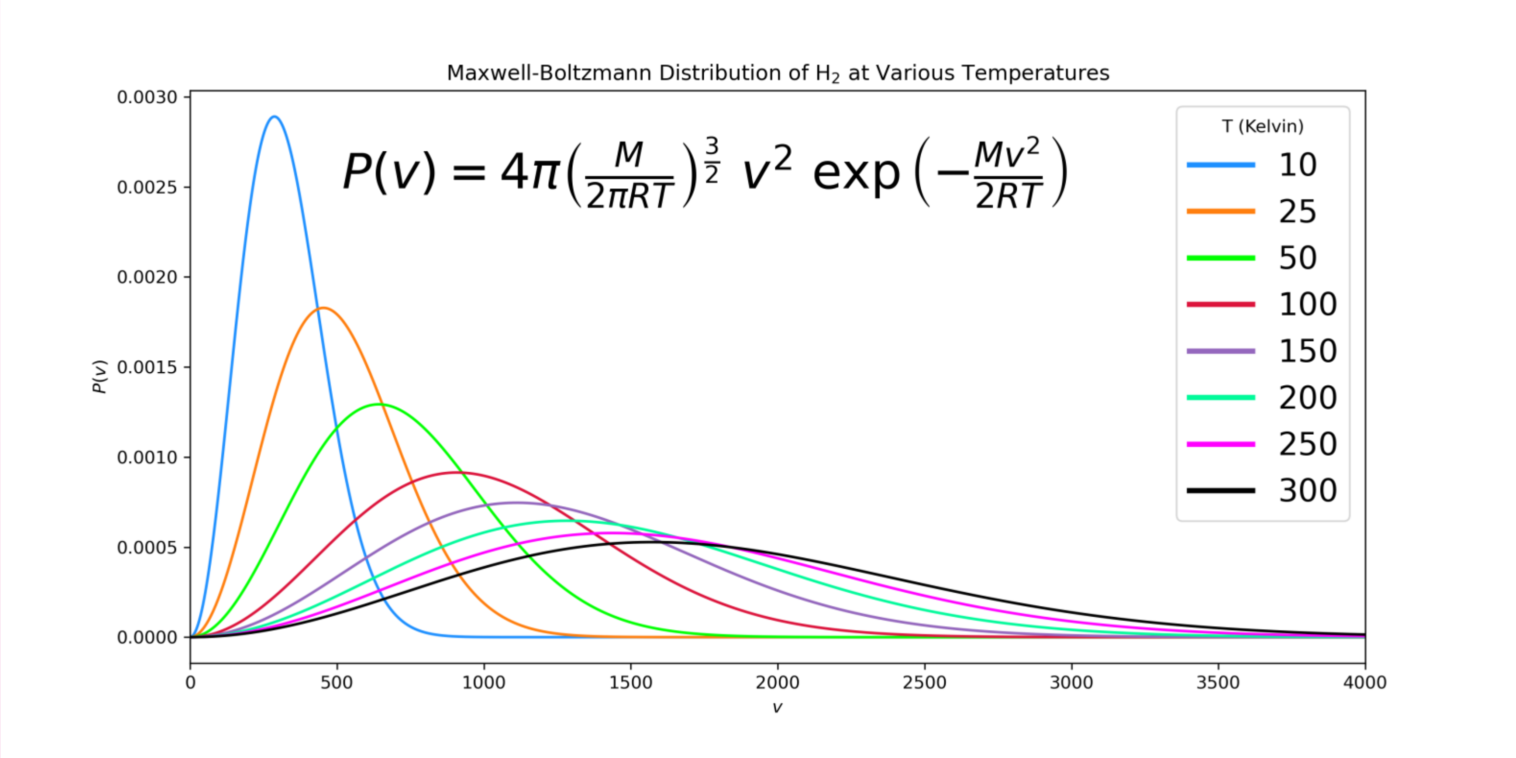
Ideally we would be able to describe a gas in terms of the individual molecules… but pretty hard to do that for 10^{23} molecules



Kinetic theory of gases

Easiest way to do this is statistically – like we have previously touched upon with temperature

With huge numbers of particles, statistical predictions improve (so can reliably work with probabilities of velocities)

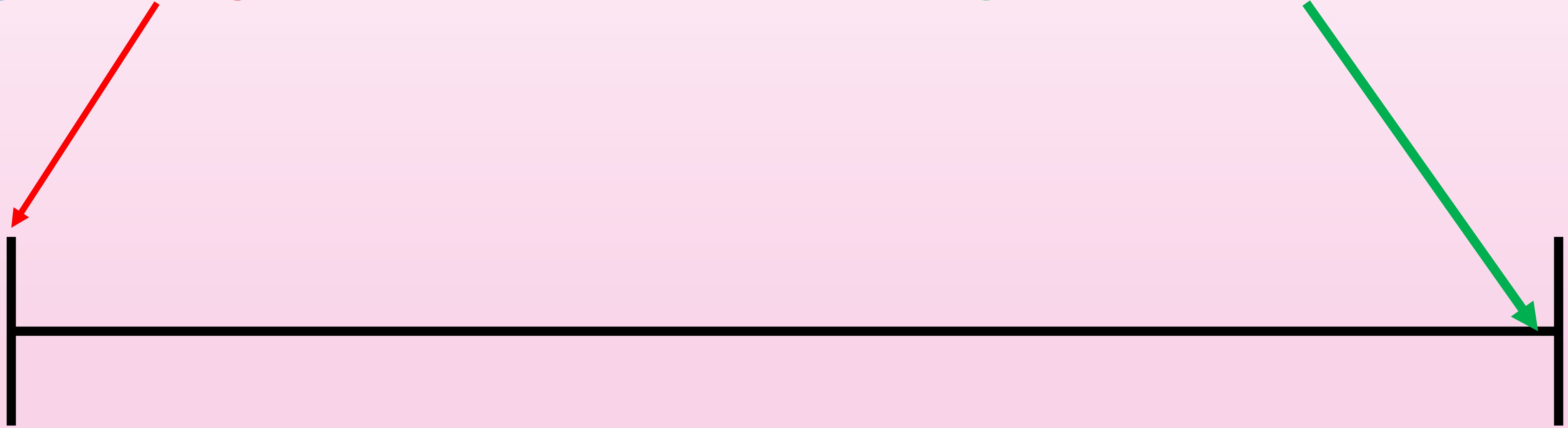


e.g. uncertainty of a Poisson distribution $\sigma = \frac{\sqrt{N}}{N}$

Probability

Probability exists on a number line between 0 and 1

e.g. throwing a 7 on a 6 sided die or throwing either a 1,2,3,4,5,6?



0
(will never happen)

1
(will happen)

Have to be careful in the real world with saying something is 100% certain ($P=1$)!

Probability

Probability of getting any specific number on a 6-sided die

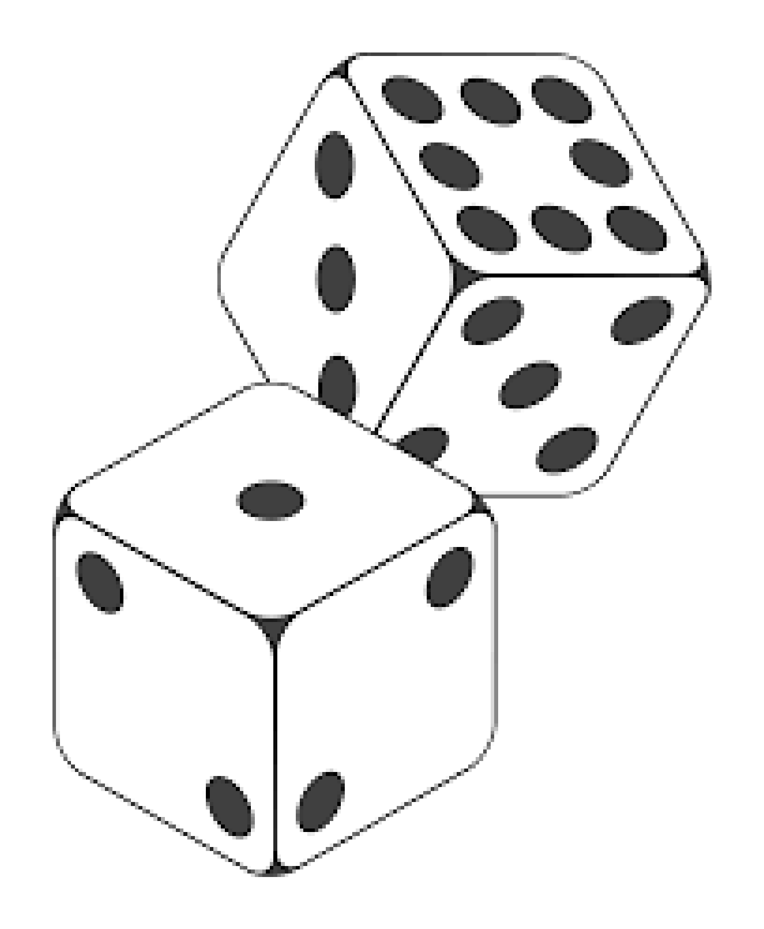
(basically) 1/6

Probability of getting two of the same number

$1/6 \times 1/6 = 1/36$ as these are INDEPENDENT

Probability of getting n of the same number?

$(1/6)^n$



Monkeys on a typewriter

How would we work out the probability that given 100 monkeys typing (essentially randomly) for the entire age of the universe, one would be able to write A Tale of Two Cities by Charles Dickens?

It was the best of times, it was the blurst of times?! You stupid monkey

How would we work out the probability that given 100 monkeys typing (essentially randomly) for the entire age of the universe, one would be able to write A Tale of Two Cities by Charles Dickens?

140000 words in ToTC
Average word length 5 characters
Age of the universe ~10^{18} seconds
44 keys on the typewriter
Monkeys can type 10 characters a second

Chance of a monkey getting the first character right: 1/44
Chance of a monkey getting all of the characters right: $(1/44)^{140000*5} = 10^{-1150417}$
Number of characters that a monkey can type: $10 \times 10^{18} = 10^{19}$
Number of sequences of 700000 characters: $10^{19} – 700000 = 10^{19}$
Probability of typing ToTC for 100 monkeys: $10^{-1150417} \times 10^{19} \times 100 = 10^{-1150396}$

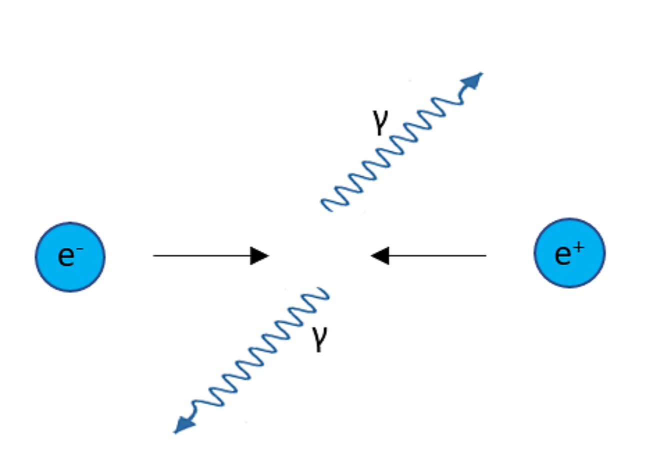
What about for infinity monkeys? $10^{-1150417} \times 10^{19} \times$ infinity = infinity

Probabilities greater than 1

Probabilities greater than 1 usually indicate that the event will occur multiple times during one measurement

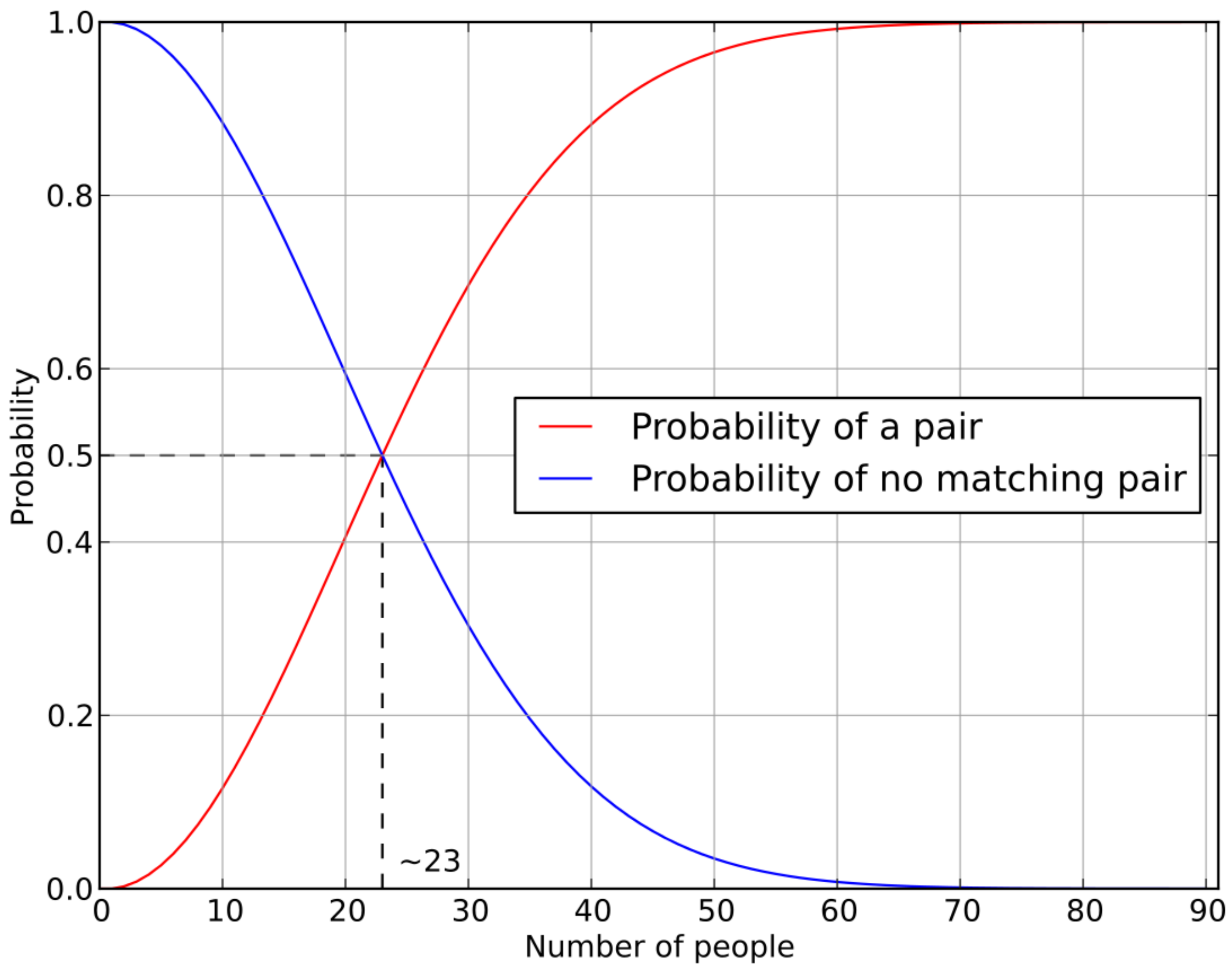
Gamma and X-ray radiation:

	Energy (keV)	Intensity (%)	Dose (MeV/Bq-s)
XR l	0.34	0.016 % *5*	5.4E-8 *18*
XR kα2	3.688	0.254 % *12*	9.4E-6 *4*
XR kα1	3.692	0.505 % *22*	1.87E-5 *8*
XR kβ1	4.013	0.053 % *3*	2.11E-6 *11*
XR kβ3	4.013	0.0268 % *13*	1.07E-6 *5*
Annihil.	511.0	188.556 % *22*	
	1157.022 *15*	99.8867 % *30*	1.15571 *4*
	1499.449 *15*	0.909 % *15*	0.01363 *22*
	2144.33 *10*	0.0037 % *7*	7.9E-5 *15*
	2150.840 *22* ?	0.00110 % *30*	2.4E-5 *6*
	2656.48 *4*	0.1119 % *30*	0.00297 *8*
	3301.35 *6*	0.0014 % *4*	4.6E-5 *13*



Probability of producing a positron: 0.94278

Probability of producing a 511 keV gamma is double



https://upload.wikimedia.org/wikipedia/commons/thumb/c/ca/Birthday_paradox_probability.svg/1280px-Birthday_paradox_probability.svg.png

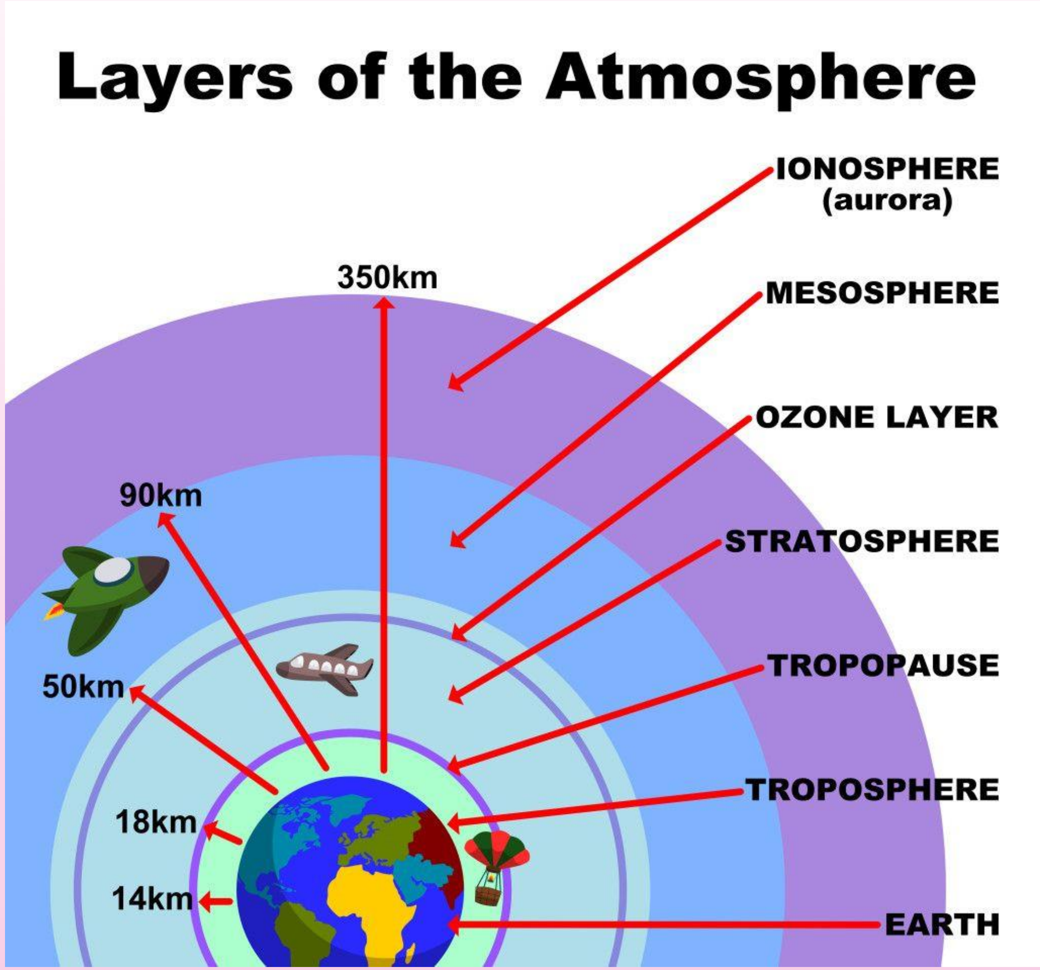
Probability of a pair of people having the same birthday reaches 50% for only 23 people – seems counterintuitive for 365 days: (23x22)/2 = 253 possible pairs!

The atmosphere

Ionosphere: essentially a mirror which reflects EM radiation of <40 MHz (> 7.5 metres)

Stratosphere: region of the atmosphere where the ozone layer exists

Troposphere: bottom few km of the atmosphere which we will focus on here



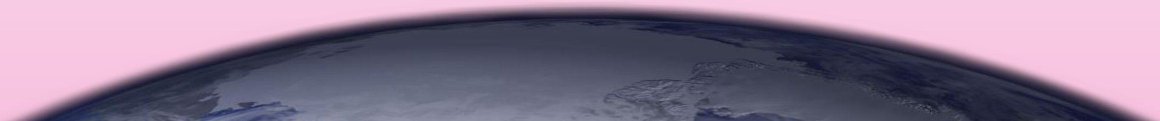
https://ilearncana.com/upload/original/8352cdd9e03ef06d44b860024116e885.jpg

Isothermal model of the atmosphere

Assumptions:

1) Atmosphere is isothermal
   (in reality, $dT/dh \sim 10\ \text{K km}^{-1}$) [5.8]
2) Treat the atmosphere as consisting of an ideal gas
   (good assumption for majority of component gases at low pressure – not water vapour)
3) The Earth is flat
   (~15 km << 6400 km)
4) The atmosphere is stationary and thus in mechanical equilibrium

Cross-sectional area A



$$P = \rho_N k_B T$$

Number density (N/V)

Forces: forces due to pressure and gravity (in equilibrium)

$$0 = P(0)A - P(\mathrm{d}h)A - m(\rho_N\,\mathrm{d}h)Ag$$

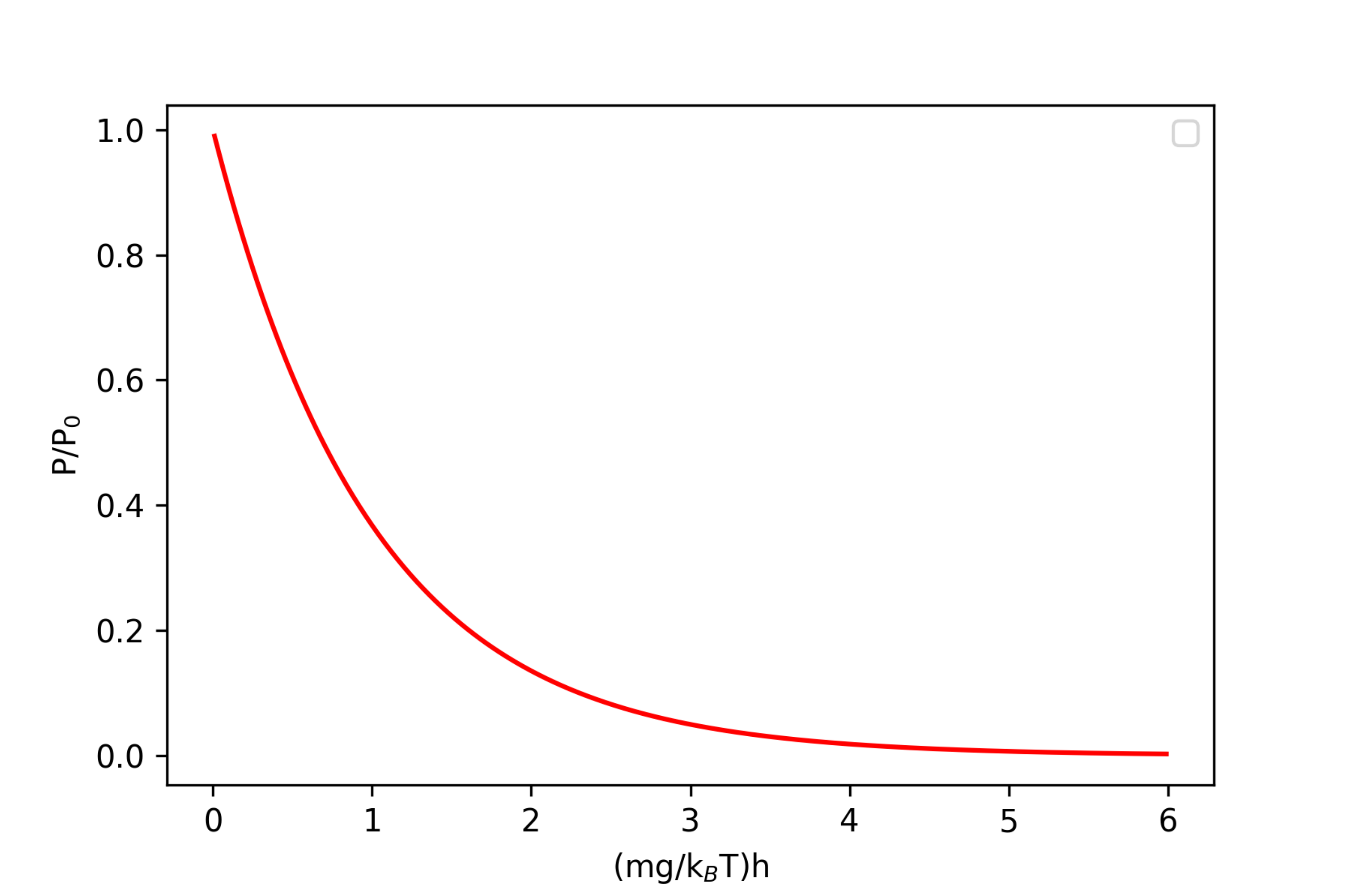
$$0 = dPA - m(\rho_N\,\mathrm{d}h)Ag$$

Isothermal model of the atmosphere

$$\rho_N(h) = \rho_N(0)\mathrm{e}^{-\left(\frac{mgh}{k_B T}\right)}$$

$$P(h) = P(0)\mathrm{e}^{-\left(\frac{mgh}{k_B T}\right)}$$

$\frac{mgh}{k_B T}$ must be dimensionless, and so $\frac{k_B T}{mg}$ must have dimensions of h -> scale height, h_0
(at which $P(h_0) = P(0)e^{-1}$)



Scale heights

Particle	Mass	Scale height
O_2 gas	32 amu = 5.3137e-26 kg	~10 km
N_2 gas	28 amu = 4.6495e-26 kg	~10 km
Covid virus	~10^{-18} kg	~0.5 mm
Smallest known virus	~10^{-21} kg	~50 cm
Bacteria	~10^{-15} kg	~5 µm
Dr Stuart Pirrie	~10^{2} kg	~50 ym (10^{-24} m is 1 ym)