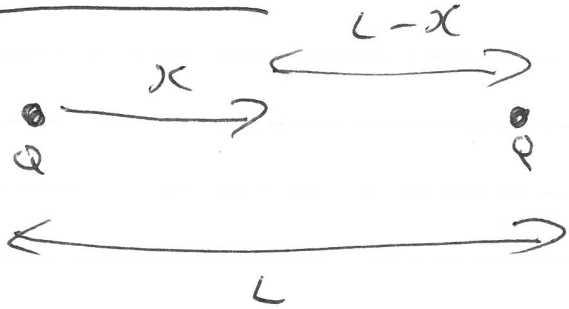
EM1 – Lec 7

Ex 7.1



potential at x:

$$V(x) = \frac{Q}{4\pi\varepsilon_0 x} + \frac{Q}{4\pi\varepsilon_0 (L-x)}$$

$$\underline{E} = -\nabla V = -\frac{dV}{dx}\,\underline{i}$$

$$= \frac{Q}{4\pi\varepsilon_0 x^2} - \frac{Q}{4\pi\varepsilon_0 (L-x)^2}\,\underline{i}$$

at $x = L/2$: $V\left(\frac{L}{2}\right) = \frac{2Q}{4\pi\varepsilon_0 L} + \frac{Q}{4\pi\varepsilon_0 \left(L - \frac{L}{2}\right)}$

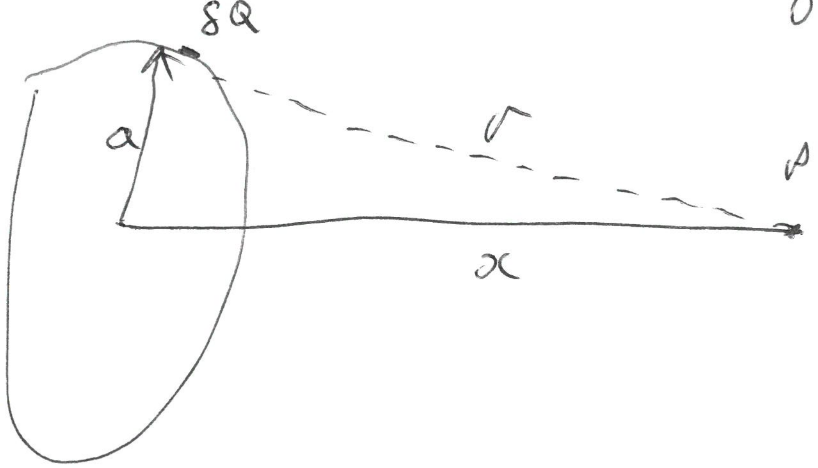
$$= \frac{Q}{\pi\varepsilon_0 L}$$

$$\underline{E}\left(x = \frac{L}{2}\right) = \frac{Q}{4\pi\varepsilon_0 \frac{L^2}{4}} - \frac{Q}{4\pi\varepsilon_0 \frac{L^2}{4}}\,\underline{i}$$

$$= 0$$

Ex 7.2

Thin ring, charge Q



potential at P from δQ

$$\delta V = \frac{\delta Q}{4\pi\varepsilon_0 r} = \frac{\delta Q}{4\pi\varepsilon_0 (a^2+x^2)^{1/2}}$$

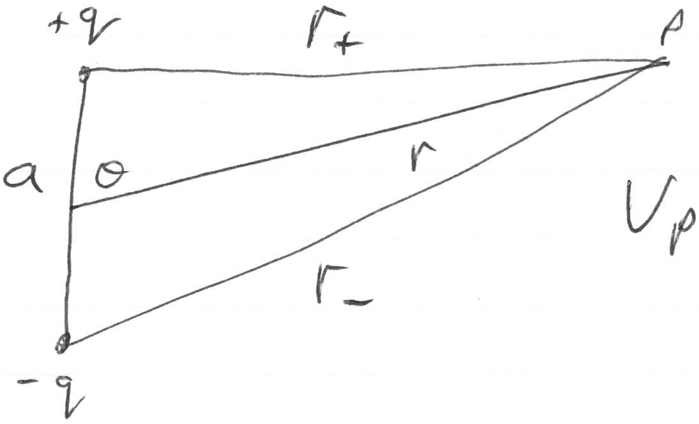
$$\therefore \text{ Total } V = \frac{1}{4\pi\varepsilon_0 (a^2+x^2)^{1/2}} \oint dQ$$

$$= \frac{Q}{4\pi\varepsilon_0 (a^2+x^2)^{1/2}}$$

$$\underline{E} = -\nabla V = -\frac{\partial V}{\partial x} = \frac{-Q}{4\pi\varepsilon_0} \cdot -\frac{1}{2} \cdot 2x (a^2+x^2)^{-3/2} \underline{i}$$

$$= \frac{Q}{4\pi\varepsilon_0} \frac{x}{(a^2+x^2)^{3/2}} \underline{i}$$

$\boxed{\text{Ex } 7 \cdot 3}$ V of Dipole



$$V_P = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_+} - \frac{1}{r_-}\right)$$

Consider case $r \gg a$

Using Triangle rule.

$$r_+^2 = r^2 + \left(\frac{a}{2}\right)^2 - ar\cos\theta$$

$$= r^2\left(1 + \frac{a^2}{4r^2} - \frac{a}{r}\cos\theta\right)$$

$$\therefore \frac{1}{r_+} = \frac{1}{r}\left(1 + \left[\frac{a^2}{4r^2} - \frac{a}{r}\cos\theta\right]\right)^{-1/2}$$

$$\approx \frac{1}{r}\left(1 - \frac{a^2}{8r^2} + \frac{a}{2r}\cos\theta\right)$$

& $r_-^2 = r^2 + \left(\frac{a}{2}\right)^2 + ar\cos\theta$

$$\frac{1}{r_-} \approx \frac{1}{r}\left(1 - \frac{a^2}{8r^2} - \frac{a}{2r}\cos\theta\right) \quad \boxed{(\cos(\pi - \theta) = -\cos\theta)}$$

$$\therefore\ V_p = \frac{q}{4\pi\varepsilon}\left(\frac{1}{r_+} - \frac{1}{r_-}\right) = \frac{q}{4\pi\varepsilon_0 r}\,\frac{a}{r}\cos\theta$$

$$\text{i.e. } V_p = \frac{qa\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

$$= \frac{\underline{p}\cdot\hat{\underline{r}}}{4\pi\varepsilon_0 r^2}$$

$\underline{E}$-field from dipole:

$$\underline{E} = -\left(\frac{\partial V}{\partial r}\,\hat{\underline{r}} + \frac{1}{r}\frac{dV}{d\theta}\,\hat{\theta}\right) \ ; \ V_p = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$$

$$\underline{E} = -\left(\frac{-p\cos\theta}{2\pi\varepsilon_0 r^3}\,\hat{\underline{r}} + \frac{1}{r}\,\frac{-p\sin\theta}{4\pi\varepsilon_0 r^2}\,\hat{\underline{\theta}}\right)$$

$$= \frac{p\cos\theta}{2\pi\varepsilon_0 r^3}\,\hat{\underline{r}} + \frac{p\sin\theta}{4\pi\varepsilon_0 r^3}\,\hat{\underline{\theta}}$$
