UNIVERSITY OF BIRMINGHAM

School of Physics and Astronomy

DEGREE OF B.Sc. & M.Sci. WITH HONOURS

FIRST-YEAR EXAMINATION

03 17483/ 03 20835

LC CHAOS IN NON-LINEAR SYSTEMS A

SEMESTER 2 EXAMINATIONS 2021/22

Time Allowed: 1 hour

Answer ***two*** questions. If you answer more than two questions, credit will only be given for the best two answers.

The approximate allocation of marks to each part of a question is shown in brackets **[ ]**.

All symbols have their usual meanings.

Calculators may be used in this examination but must not be used to store text. Calculators with the ability to store text should have their memories deleted prior to the start of the examination.

A formula sheet and a table of physical constants and units that may be required will be found at the end of this question paper.

Answer **two** *questions. If you answer more than two questions, credit will only be given for the best two answers.*

1. A forced damped pendulum is described by the equation

$$\frac{d^2\theta}{dt^2} + \frac{d\theta}{dt} + \sin\theta = R\cos t \tag{1}$$

(a) When $R \ll 1$ find the attractor to leading order in R. **[5]**

(b) Show that to order R^3 the attractor can be found approximately by solving the equation

$$\frac{d^2\theta}{dt^2} + \frac{d\theta}{dt} + \theta = R\cos t + A\sin t + B\sin 3t \tag{2}$$

where you should provide explicit representations for A and B. **[8]**

(c) Find a representation for the attractor which is correct to order R^3. **[8]**

(d) Describe how the attractor has been physically altered by the increasing of R. **[4]**

2. Consider the map
$$x_{n+1} = a x_n - x_n^2 \tag{3}$$
where a is a control parameter. Find all the possible 1-cycles and establish for which range of control parameter they are stable. **[8]**
Find all the possible 2-cycles and establish for which range of control parameter they are stable. **[13]**
Find the attractor when it is either a 1-cycle or a 2-cycle. **[4]**

3. Consider the map

$$x_{n+1} = 4x_n^3 - ax_n \tag{4}$$

where a is a control parameter.

(a) Show that

$$\cos 3\pi y = 4\cos^3 \pi y - 3\cos \pi y \tag{5}$$

and use it to find an exact solution for this map in terms of a linear map for a value of a that you should determine. **[5]**

(b) Depict the linear map in the restricted domain $y \in [0, 1]$. **[5]**

(c) Employ base 3 to find a useful representation for your linear map. **[10]**

(d) Find all the 1-cycles and 2-cycles for the linear map using base 3. **[5]**

Formula Sheet

1-cycles of a general map

$$x_{n+1} = M[x_n; r]$$

where r is a control parameter, satisfy

$$x^* = M[x^*; r]$$

and are stable if

$$-1 < |\frac{df}{\partial x}[x^*; r]| < 1$$

2-cycles of this map satisfy

$$y^* = M[x^*; r] \quad x^* = M[y^*; r]$$

and are stable if

$$-1 < |\frac{df}{\partial x}[x^*; r]\frac{df}{\partial x}[y^*; r]| < 1$$

Physical Constants and Units

Acceleration due to gravity	g	$9.81\,\mathrm{m\,s^{-2}}$
Gravitational constant	G	$6.674 \times 10^{-11}\,\mathrm{N\,m^2\,kg^{-2}}$
Ice point	T_{ice}	$273.15\,\mathrm{K}$
Avogadro constant	N_A	$6.022 \times 10^{23}\,\mathrm{mol^{-1}}$
		[*N.B.* 1 mole $\equiv 1$ *gram-molecule*]
Gas constant	R	$8.314\,\mathrm{J\,K^{-1}\,mol^{-1}}$
Boltzmann constant	k, k_B	$1.381 \times 10^{-23}\,\mathrm{J\,K^{-1}} \equiv 8.62 \times 10^{-5}\,\mathrm{eV\,K^{-1}}$
Stefan constant	σ	$5.670 \times 10^{-8}\,\mathrm{W\,m^{-2}\,K^{-4}}$
Rydberg constant	R_∞	$1.097 \times 10^{7}\,\mathrm{m^{-1}}$
	$R_\infty hc$	$13.606\,\mathrm{eV}$
Planck constant	h	$6.626 \times 10^{-34}\,\mathrm{J\,s} \equiv 4.136 \times 10^{-15}\,\mathrm{eV\,s}$
$h/2\pi$	$\hbar$	$1.055 \times 10^{-34}\,\mathrm{J\,s} \equiv 6.582 \times 10^{-16}\,\mathrm{eV\,s}$
Speed of light *in vacuo*	c	$2.998 \times 10^{8}\,\mathrm{m\,s^{-1}}$
	$\hbar c$	$197.3\,\mathrm{MeV\,fm}$
Charge of proton	e	$1.602 \times 10^{-19}\,\mathrm{C}$
Mass of electron	m_e	$9.109 \times 10^{-31}\,\mathrm{kg}$
Rest energy of electron		$0.511\,\mathrm{MeV}$
Mass of proton	m_p	$1.673 \times 10^{-27}\,\mathrm{kg}$
Rest energy of proton		$938.3\,\mathrm{MeV}$
One atomic mass unit	u	$1.66 \times 10^{-27}\,\mathrm{kg}$
Atomic mass unit energy equivalent		$931.5\,\mathrm{MeV}$
Electric constant	ϵ_0	$8.854 \times 10^{-12}\,\mathrm{F\,m^{-1}}$
Magnetic constant	μ_0	$4\pi \times 10^{-7}\,\mathrm{H\,m^{-1}}$
Bohr magneton	μ_B	$9.274 \times 10^{-24}\,\mathrm{A\,m^2}$ ($\mathrm{J\,T^{-1}}$)
Nuclear magneton	μ_N	$5.051 \times 10^{-27}\,\mathrm{A\,m^2}$ ($\mathrm{J\,T^{-1}}$)
Fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	$7.297 \times 10^{-3} = 1/137.0$
Compton wavelength of electron	$\lambda_c = h/m_e c$	$2.426 \times 10^{-12}\,\mathrm{m}$
Bohr radius	a_0	$5.2918 \times 10^{-11}\,\mathrm{m}$
angstrom	Å	$10^{-10}\,\mathrm{m}$
barn	b	$10^{-28}\,\mathrm{m^2}$
torr (mm Hg at 0 °C)	torr	$133.32\,\mathrm{Pa}$ ($\mathrm{N\,m^{-2}}$)

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Do not complete the attendance slip, fill in the front of the answer book or turn over the question paper until you are told to do so.

Important Reminders

- Coats/outwear should be placed in the designated area.
- Unauthorised materials (e.g. notes or Tippex) must be placed in the designated area.
- Check that you do not have any unauthorised materials with you (e.g. in your pockets, pencil case).
- Mobile phones and smart watches **must** be switched off and placed in the designated area or under your desk. They must not be left on your person or in your pockets.
- You are not permitted to use a mobile phone as a clock. If you have difficulty seeing a clock, please alert an Invigilator.
- You are not permitted to have writing on your hand, arm or other body part.
- Check that you do not have writing on your hand, arm or other body part – if you do, you must inform an Invigilator immediately
- Alert an Invigilator immediately if you find any unauthorised item upon you during the examination.

Any students found with non-permitted items upon their person during the examination, or who fail to comply with Examination rules may be subject to Student Conduct procedures.