1 Atoms, matter

The problems are roughly in order of difficulty. The ones with ♣ are the hardest ones, which might only occur as a "sting in the tail" at the end of a long examination question.

Problem 1.1 **Aluminium atomic spacing**
Aluminium has an Atomic Weight of 27 and a density of 2700 kg m^{-3}. Estimate the typical spacing of the Aluminium atoms.

Problem 1.2 **Lennard-Jones forces**
Plot out the *force* associated with the Lennard-Jones potential as a function of r. Indicate the relation of features in the curve with those in the LJ potential.

Problem 1.3 **Derivation of the size of an atom by dimensional analysis**
Consider the surface tension, γ, the Latent heat of vapourisation per kg, $\mathcal{L}$, and the density of a liquid, ρ. Write each of these in terms of powers of M, L and T, i.e. in the same manner as writing the dimensions of the speed of light as $[\mathrm{c}] = LT^{-1}$, where $[X]$ denotes the dimensions of a physical quantity X.

Derive a length scale ℓ, such that

$$\ell = \gamma^{\alpha}\mathcal{L}^{\beta}\rho^{\delta} \ .$$

and compare with the result for d in the lectures. This calculation provides no insight, unlike the previous derivation. But it is quick!

Problem 1.4 **Three Lennard-Jones atoms and fracture**
Consider three atoms, in one dimension, with positions $x_1 < x_2 < x_3$. Assume that they only interact with the adjacent atom(s), i.e atom 2 interacts with both atoms 1 and 3, but atoms 1 and 3 do *not* interact with each other.

Write down the total potential energy, $V^{\mathrm{tot}}(x_1, x_2, x_3)$, assuming the interaction is of Lennard-Jones form.

Assume that atoms 1 and 3 are fixed at $x_1 = -a$ and $x_3 = a$, and a is a general value (*not* r_0 in our standard terminology). Where would atom 2 be if atoms equally spaced? Is that point a point of equilibrium? Examine

$$\frac{\mathrm{d}^2 V^{\mathrm{tot}}}{\mathrm{d}x_2^2}$$

to deduce the stability of the point. Does this vary with a? Is any insight into fracture of an LJ chain of atoms provided and, if so, what is that insight?

♣*Problem 1.5* **Difference between sand and flour**
Both sand and flour can flow although they are not liquids. The description of this flow is an active area of research.

However sand (typically 1mm in size) and flour (typically 0.050 mm for fine powder) differ in that, even when dry, flour will clump but sand does not. Why?

Hint. The van der Waals interaction between two spheres of radius R, separated by $d \ll R$ is

$$V^{\text{Spheres}} \underset{d/R \to 0}{\sim} -n^2 \lambda \frac{R}{d} \simeq -U_0 \frac{R}{d} \ ,$$

where the interaction between two constituent atoms is $V^{\text{at}}(r) = -\lambda/r^6 = -U_0(r_0/r)^6$, and n is the number of atoms per unit volume. See Hamaker's calculation of the interaction between two spheres.

♣*Problem 1.6* **Bodies held together by gravity - the Roche limit**
We discounted gravity as the dominant interaction between atoms in a solid or a liquid, as the magnitude of the interaction was so small. However there are bodies where gravity is believed to be the only important interaction between their constituents (apart from a constraint that the constituents cannot overlap each other). An example is a "rubble-pile" asteroid such as 253 Mathilde. See figure below:

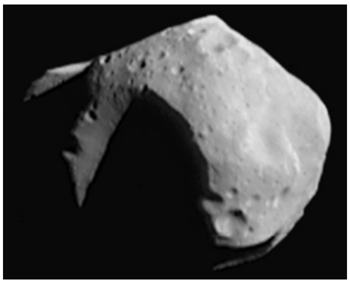

Figure 1.1: 253 Mathilde, note the irregularity of the shape.

Let us investigate the "strength" of the material in the presence of the gravitational field of, say, a planet - will it rupture?

The force due to the gravitational field of the planet, of mass M, on the body (an "asteroid") of mass $2m$ is, at a distance r:

$$\mathbf{F}_0(r) = -\frac{GM(2m)}{r^2} \hat{\mathbf{r}} \ .$$

Note the magnitude of the force varies with r. The asteroid is not a point particle but has an extent of order (because of irregular shape) $2a$, then the gravitational force will vary across the body.

Write down the gravitational force *per unit mass* at $r \pm a$. Taylor expand, assuming the body is small compared to the distance to the planet, i.e. $a \ll r$, to yield $\mathbf{F}(r \pm a) \simeq \mathbf{F}(r) \pm \delta \mathbf{f}$.

Consider a simple case where the asteroid may be represented by two point masses each of m which are $2a$ apart in a radial sense, i.e. m at $r - a$ and m at $r + a$.

Derive, from a force balance argument, a criterion that the mutual gravitational attraction of the two point masses is greater than the "tidal force" due to the difference of the planetary gravitational force on the two bodies. This is a simple version of the "Roche criterion" for the stability of gravitationally bound objects.

Almost all of Saturn's rings are inside Saturn's Roche radius - so may be due to satellites which could not resist Saturn's tidal forces.