University of Birmingham
School of Mathematics

1SAS | Sequences and Series | 2021-22

January Exam
Generic Feedback

Section A. Parts (a) and (b) were generally done well. Occasionally problems arose in Part (b) on bounding $|a_n - \frac{1}{2}|$. A small number of students bounded this from below. A more common problem arose when an upper bound was selected that didn't tend to zero, which made it impossible to find a suitable N.

Part (c) was mixed. This was about using series convergence tests. A common mistake was to mix up references to sequences and series: saying that "a_n converges" is quite different from saying that "$\sum a_n$ converges". Some argued, incorrectly, that a series $\sum a_n$ converges if (a_n) converges, or if $a_n \to 0$.

Familiarity and experience using the series convergence tests is important. Occasionally inequalities were generated in the wrong direction when applying the Comparison Test. The Root Test was generally applied without difficulty, although there were some errors of manipulation made in applying the Ratio Test. In the final series, which was an application of the Alternating Series Test, a significant minority were unable to argue that $\sqrt{2n+1} - \sqrt{2n-1}$ is decreasing to zero (I suggest using the identity $\sqrt{a} - \sqrt{b} = \frac{a-b}{\sqrt{a}+\sqrt{b}}$; see similar examples done in the module).

Section B. Part (a) was generally done well, with the inductive arguments set out well. In some cases inadequate justification was given in Part (a)(iii). For example, the use of the Monotone Convergence Theorem should be transparent, and the reasoning behind the selection of the solution to the quadratic equation should make some identifiable reference to the theorem on limits and order, as in similar examples done in the module.

In Part (b), a significant minority omitted to apply the definition of a convergent series to the particular series in hand ($\sum \frac{1}{n^{2/3}}$), and some attempted to show that it converges – see the lectures where a similar example is handled.

Part (c) was the most challenging. In Part (c)(i) many correctly observed that the series presented was a telescoping series, but didn't quite explain the "if and only if" claim in the question. Part (c)(ii) was only successfully done by a fairly small minority. This required a use of the Comparison Test to deduce that the series $\sum(a_{n+1} - a_n)$ is absolutely convergent, followed by an application of the Absolute Convergence Test. It was important to consider absolute convergence, as the series $\sum(a_{n+1} - a_n)$ will not in general have nonnegative terms, and so the Comparison Test will not apply.