Electromagnetism

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Lecture 2

Field Lines and Continuous Charges

Week 1

Last Lecture

- Coulomb's Law

$$\underline{\boldsymbol{E}} = \frac{Q}{4\pi\varepsilon_0 r^2}\underline{\hat{\boldsymbol{r}}}$$

- Force between two charges $\underline{\boldsymbol{F}} = \dfrac{q_1 q_2}{4\pi\varepsilon_0 r^2}\underline{\hat{\boldsymbol{r}}}_{\mathbf{12}}$

- E-field from multiple charges

$$\underline{E} = \sum_i \frac{q_i}{4\pi\varepsilon_0 {r_i}^2}\underline{\hat{r}_i}$$

Lecture 2 Content

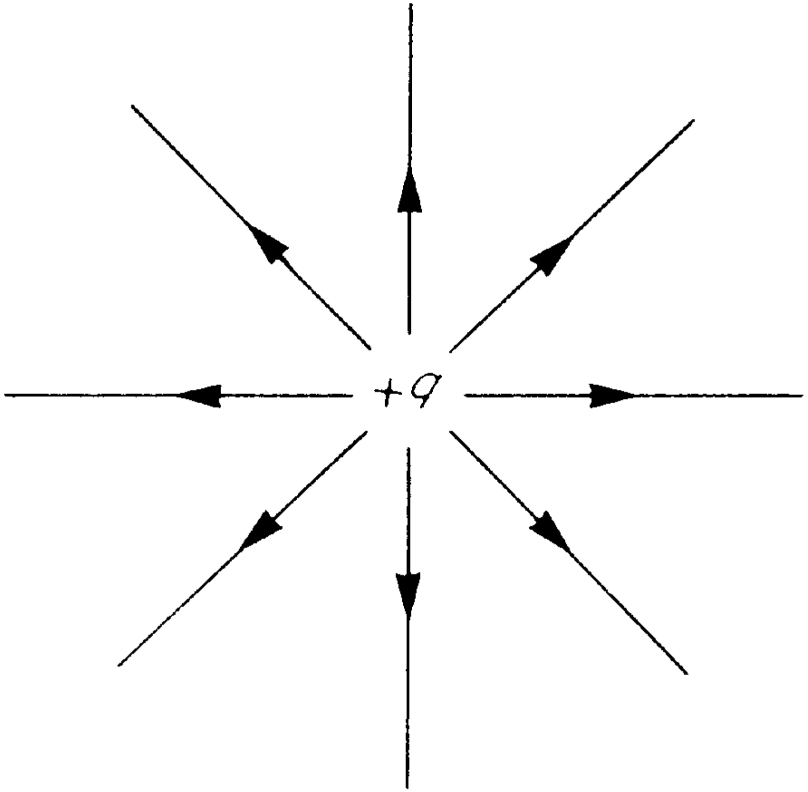
- Visualisation of E-fields
  - Electric field lines
- Continuous Charge Distributions
  - Using Coulomb's Law and integration
- **Some examples: E-field from …..**
  - **Line of continuous charge**
  - **Uniform charged thin ring**
  - **Uniform charged circular plane**
  - **Infinite plane**
  - **Inside charged hollow sphere**

Visualisation of E-fields

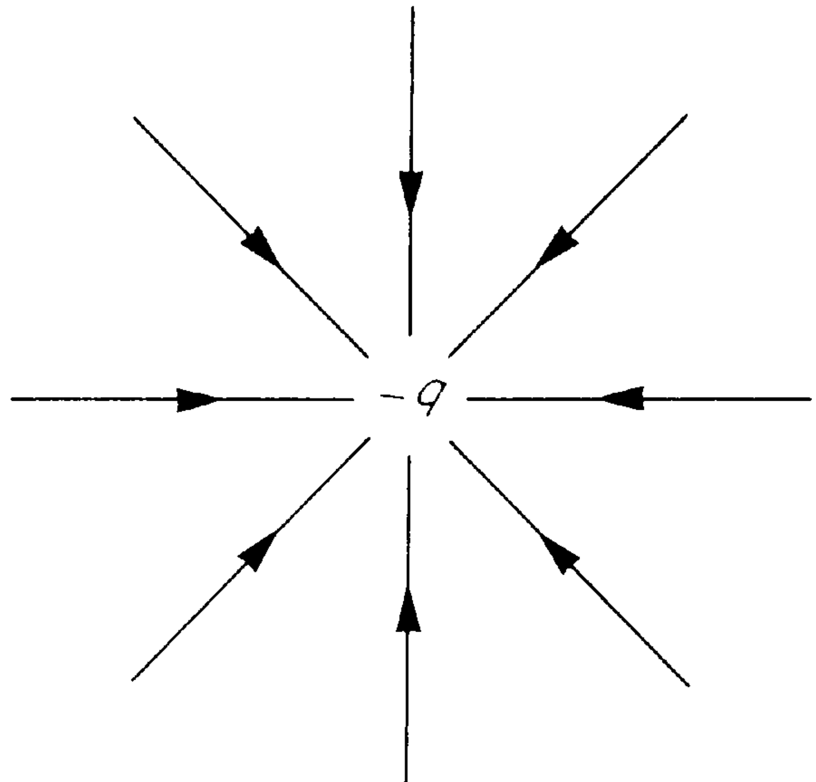
Use field lines of force

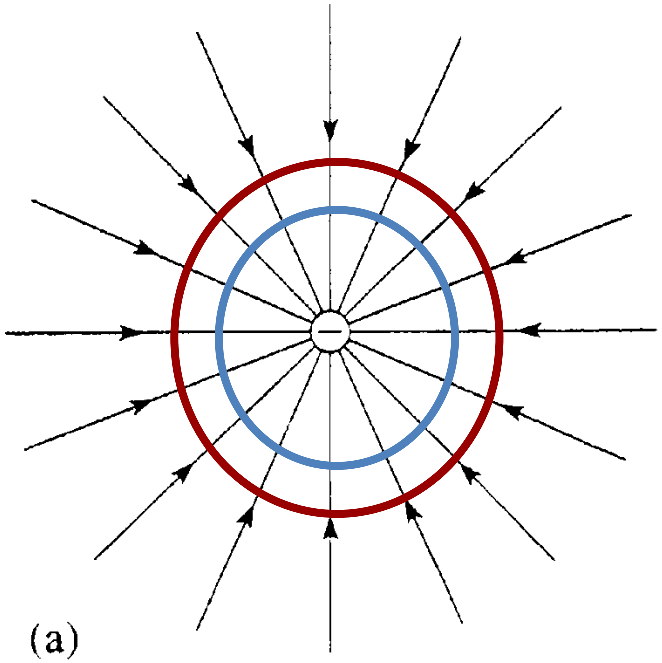
These are lines drawn in space so that the tangent to them at any point gives the direction of $\underline{E}$ at that point.

The *E* field near to a point charge

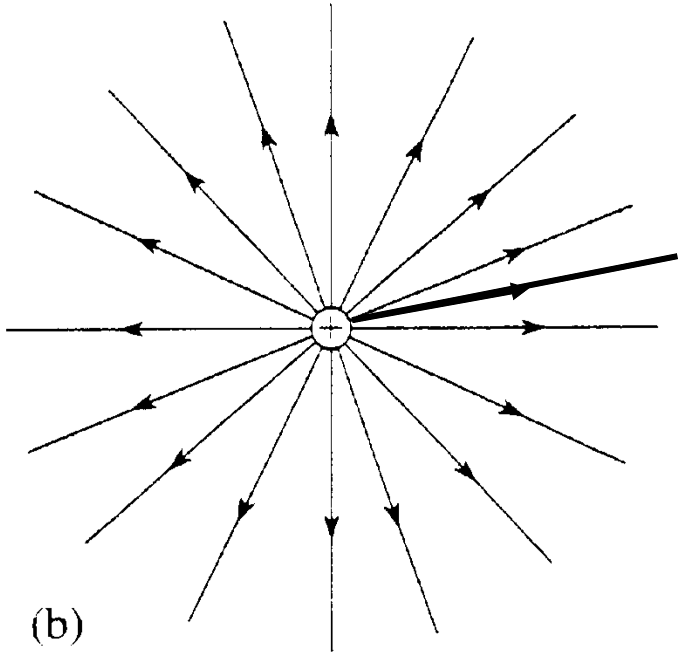


$$\underline{E} = \frac{q}{4\pi\epsilon_0 r^2}\,\underline{\hat{r}}$$





(a)



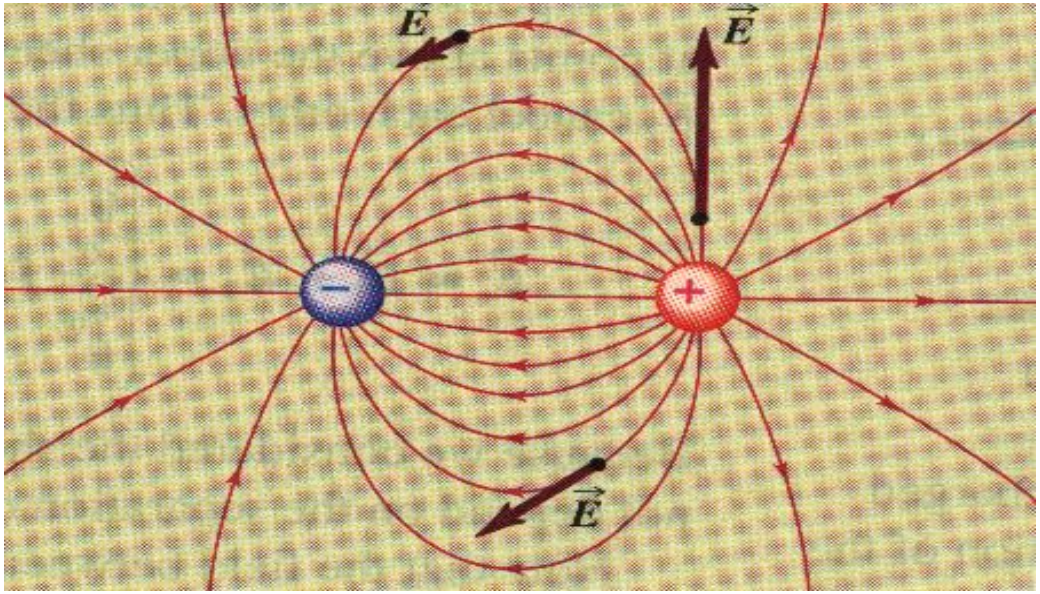
(b)

- **The density of the field lines is proportional to the strength of *E***

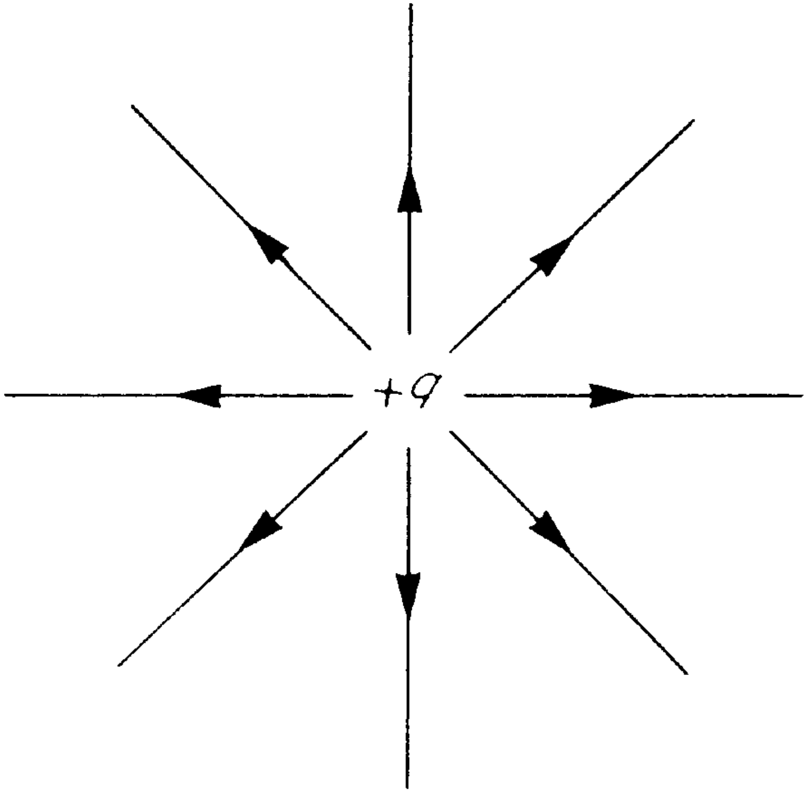
The absolute number of lines is not important. You cannot obtain the field strength by counting the number of lines. Adding lines must follow rules.

•Field lines only start on positive charge, they can only end on negative charge

Example: $\underline{E}$ field lines around an electric dipole

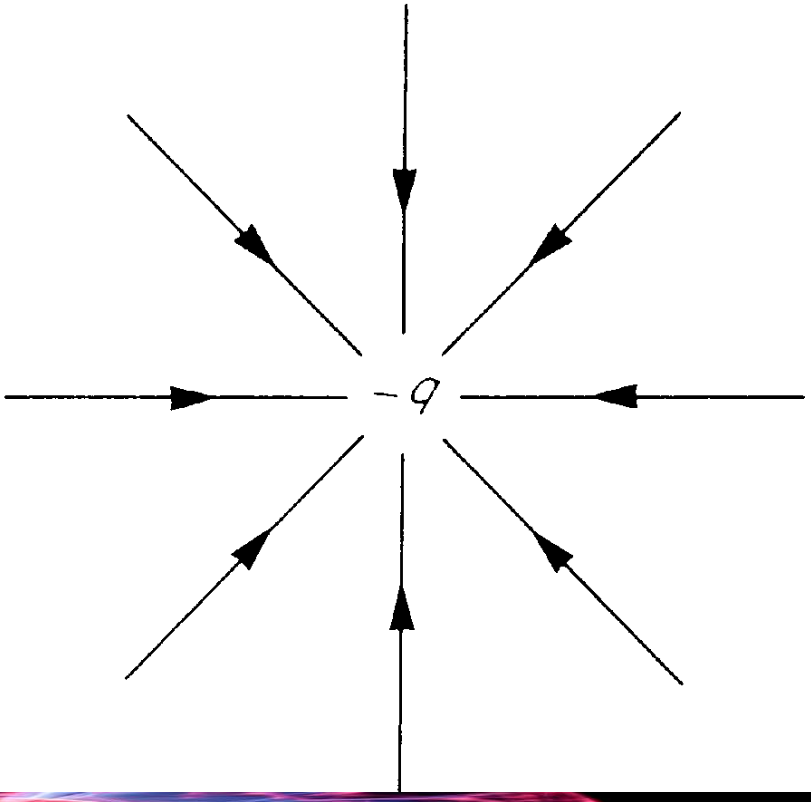


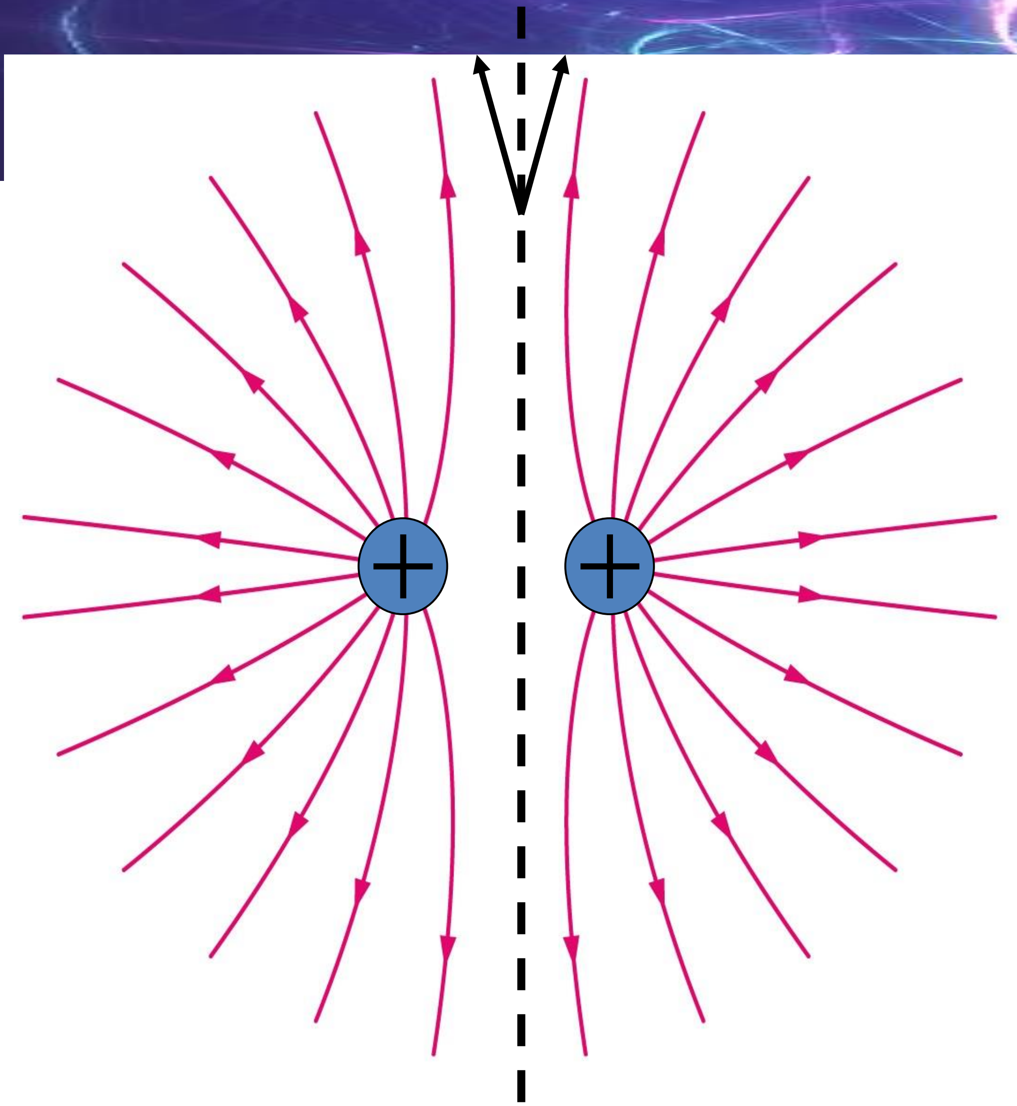
The tangent at any point gives the direction of $\underline{E}$ at that point.



Field lines terminate at distant negative charges

Field lines originate from distant positive charges





Field lines do not cross one another!

Field lines are continuous, only stop at charges!

Continuous Charge Distributions

- What do we do if the object is not a point particle?
- *Break the distribution into small (infinitesimal) pieces*
- *Treat each piece as a point charge and add up the separate* ***E****-fields (as a vector of course!)*
- *This can get messy - typically results in a 3D integral, evaluated numerically by computer*

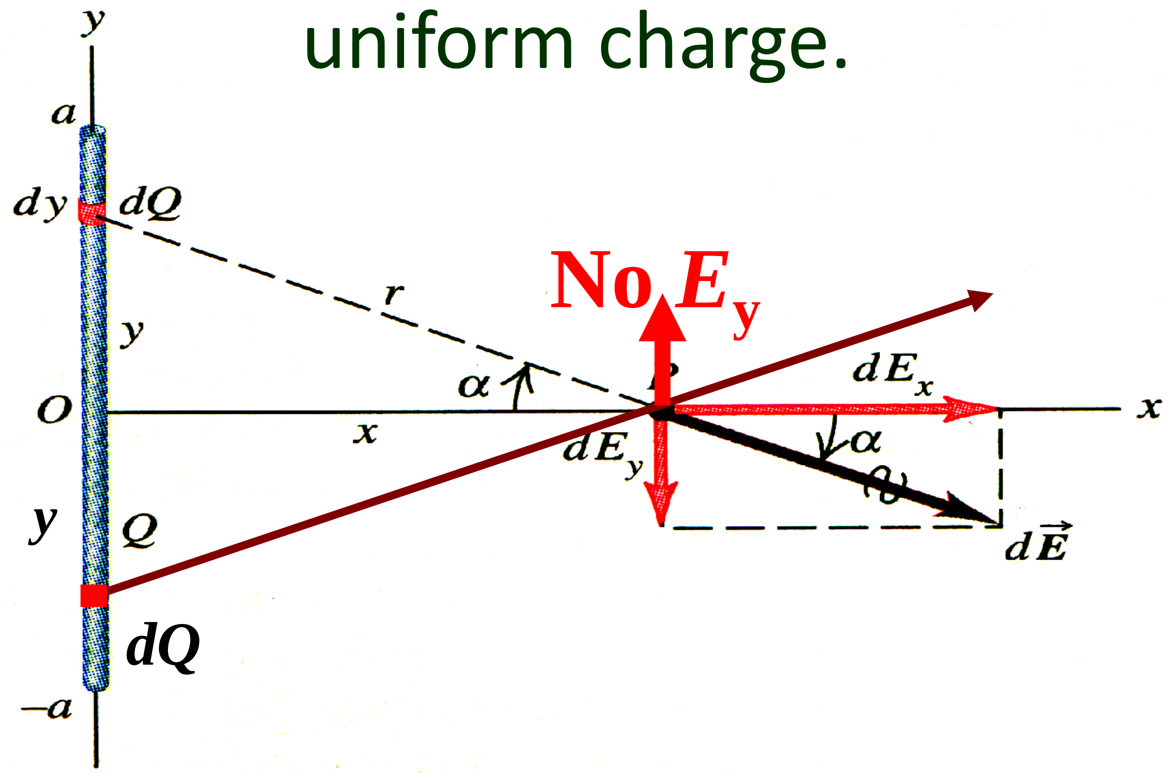
Charge Distributions

Charge can be distributed along a *line*, over a *surface*, or through a *volume*.

- *Use λ to represent charge/length over the element dl* $q = \int \lambda\, dl$
- *Use σ to represent the charge/unit area over a surface area element dA* $q = \int \sigma\, dA$
- *Use ρ to represent the charge/unit volume in a volume element dV* $q = \int \rho\, dV$

Example 2.1: E-field from line of charge

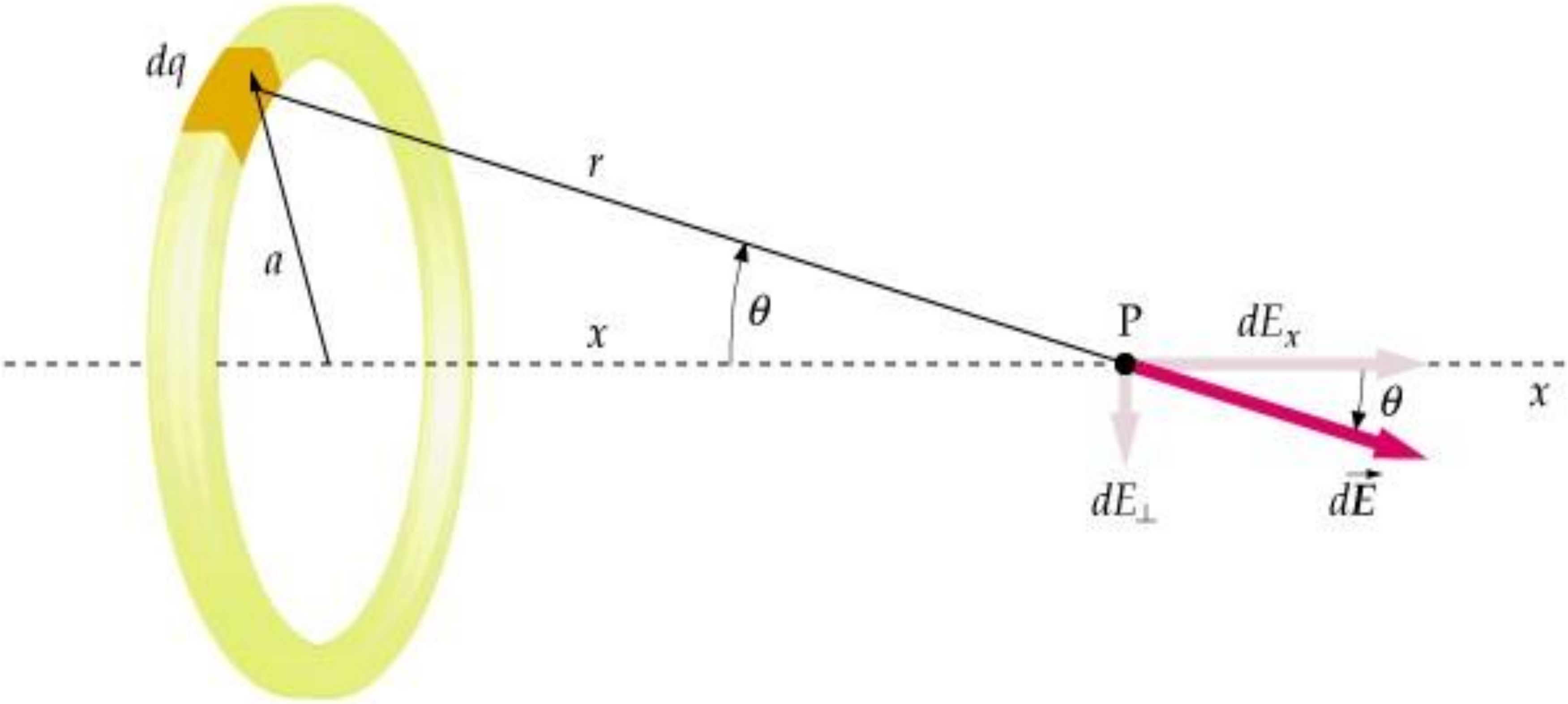
- Consider the *E*-field at a point along a line perpendicular to the mid point of line of uniform charge.



Take an element of charge $\delta Q = \lambda \delta y$ along line at *y*.
Use symmetry and consider element of charge at –*y*
Use Coulomb's Law and integrate.

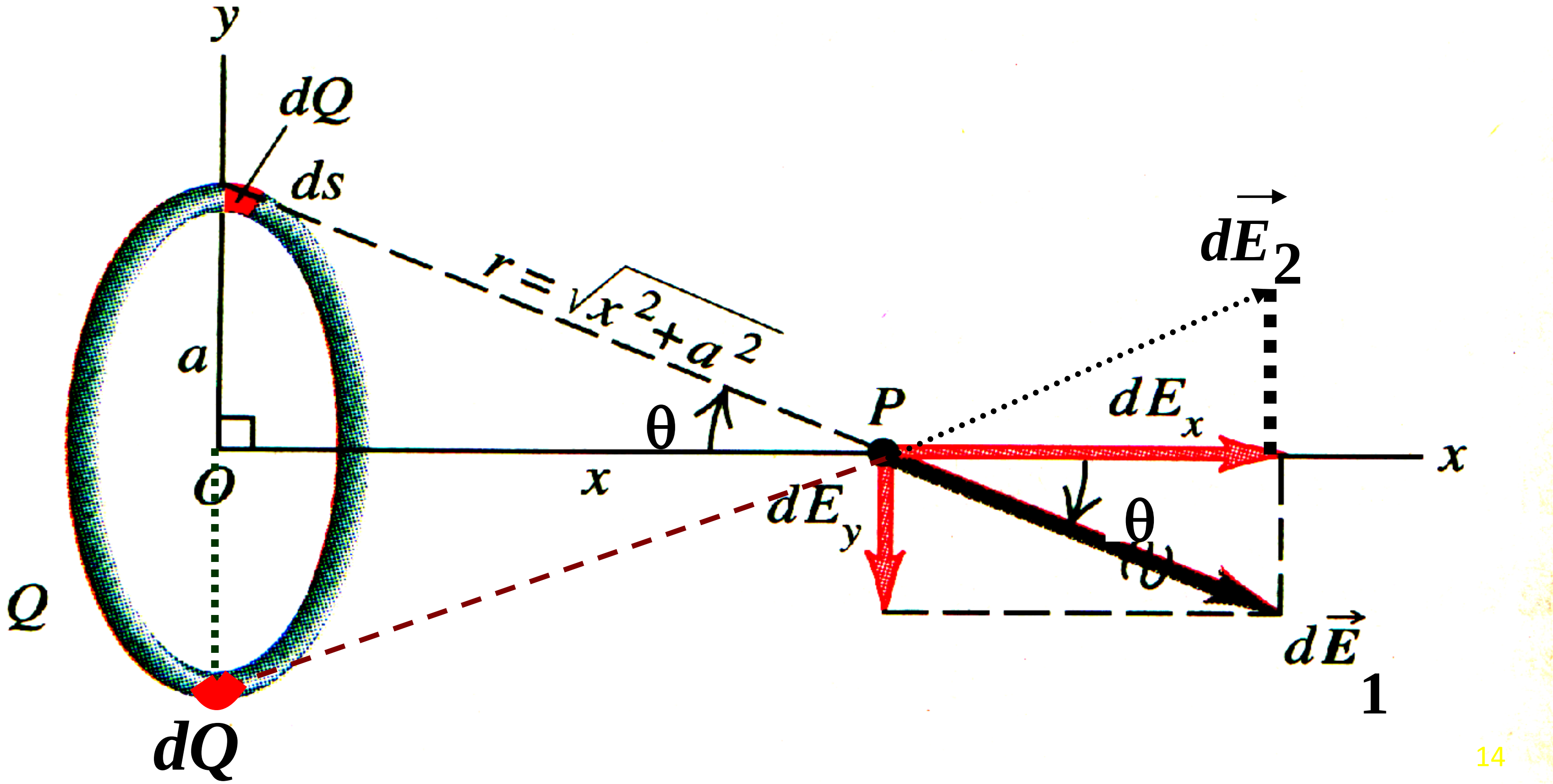
Example 2.2 - Calculate the *E*-field on the axis of a thin ring of charge Q.

- Represent the charge distribution as a series of small elements δs each with a charge δQ *(+ charge)*



Example 2.2 – use symmetry

- Use symmetry. Consider two ring elements which are diagonal to each other.



Example 2.2

Do example on the visualizer