Year 1 Assessed Problems

SEMESTER 1

Problem Sheet 2

SOLUTIONS TO BE SUBMITTED ON CANVAS
by 17:00 on Wednesday 16th October 2024

1 Assessed 1 – vectors

Problem 1.1 **Triangle**

Consider the triangle ABC where $A = (5, -3, 1)$, $B = (-2, 1, 5)$ and $C = (9, 5, 0)$.

(i) Find the lengths of the sides of the triangle.

(ii) Find the angles of the triangle.

In Euclidean geometry we define distance Δr between two points separated by Δx, Δx (we take two-dimensional space, a plane, for simplicity). The square of the distance is given by Pythagoras theorem:

$$\Delta r^2 = \Delta x^2 + \Delta y^2 \,.$$

The distance is *invariant* under rotations $\Delta x = \cos\alpha\Delta x' + \sin\alpha\Delta y'$, $\Delta y = -\sin\alpha\Delta x' + \cos\alpha\alpha\Delta y'$.

In relativistic Minkowski space we can define a similar quantity, called *interval* Δs between two events separated by distance Δx and time Δt. Its square (very often referred to as interval too) is

$$\Delta s^2 = c^2\Delta t^2 - \Delta x^2 \,.$$

(I am tempted to call this relation the relativistic Pythagoras theorem)

1. Show that interval Δs^2 is invariant under Lorentz transformations $(\Delta x, \Delta t) \to (\Delta x', \Delta t')$.

[2]

2. Unlike Euclidean distance, the interval Δs^2 can be positive (time-like) and negative (space-like). On Minkowski diagram with axes $\Delta t, \Delta x$ draw the " relativistic circle ", *i.e.* all points having the same interval. Consider cases of positive (time-like), negative (space-like) and zero (light-like) intervals Δs^2.

[2]

3. If events $E_1 = (0, 0)$, $E_2 = (\Delta x, \Delta t)$ are separated by a space-like interval, can the event E_1 be the cause for the event E_2?

[2]

In an inertial frame Σ two events are separated by a time interval of 3 s and occur at distance 8×10^8 m away from each other.

4. Can the earlier event be the cause for the later one?

[2]

5. Calculate the square of space-time invariant interval Δs^2 between the events.

[2]

probstat_ps4

October 9, 2024

1 Introduction to Probability and Statistics

2 Statistics - Problem Sheet 1

2.1 Question 1

Estimate the mean and variance of the population from which the following numbers were drawn: (1.4, 2.3, 4.7, 2.1, 3.2, 2.8)

Estimate the uncertainty on your mean value (sometimes called the error on the mean). [**1 Mark**]

2.2 Question 2

You measure the weight of a single M&M (a small sweet that you can assume all M&M's weigh the same) to be 2.1 ± 0.05 grams. What is your estimate of the weight of 370 M&M's from this single data point. [**1 mark**]

You know that you can reduce the uncertainty on your estimates by making multiple measurements of the weights of different M&M's. How many independent measurements do you need to achieve an uncertainty of better than 1 gram on the weight of 370 M&M's? [**1 mark**]

2.3 Question 3

You are stuck up a tree! There is a monster on the ground. The distribution of reach of a typical monster is normally distributed with mean $2.5\,\mathrm{m}$ and with a variance $0.25\,\mathrm{m}^2$. The monster has a step ladder that extends their reach by exactly 2m. What is the distribution of the reach of a typical monster standing on a step ladder? [**2 marks**]

2.4 Question 4

You are stuck up a tree! There are two monsters on the ground. The distribution of reach of a typical monster is normally distributed with mean $2.5\,\mathrm{m}$ and with a variance $0.25\,\mathrm{m}^2$. Monsters can lift each other up! A monster can lift another monster up as high as they can reach. What is the distribution of the reach of two monsters, one on top of the other? (The height of the second monster is independent of the height of the first monster) [**2 marks**]

2.5 Question 5

You are stuck up a tree! There are two monster siblings on the ground. The distribution of reach of a typical monster is normally distributed with mean $2.5\,\mathrm{m}$ and with a variance $0.25\,\mathrm{m}^2$. Monsters can lift each other up! A monster can lift another monster up as high as they can reach. What is the distribution of the reach of the two monster siblings, one on top of the other? Note, monster siblings are often very similar in height and the covariance between the two heights is $0.2\,\mathrm{m}^2$ (or the correlation is 0.8).

Explain in two sentences or less the difference between the answers to Question 4 and Question 5.

[3 marks]