UNIVERSITY OF BIRMINGHAM

School of Physics and Astronomy

DEGREE OF B.Sc. & M.Sci. WITH HONOURS

FIRST-YEAR EXAMINATION

03 19750

LC ELECTROMAGNETISM 1 / TEMPERATURE & MATTER / ELECTRIC CIRCUITS

SUMMER EXAMINATIONS 2018

Time Allowed: 3 hours

Answer Section 1 and three questions from Section 2.

Section 1 counts for 25% of the marks for the examination.
Full marks for this Section can be obtained by correctly answering ***five*** questions.
You may attempt more questions, but marks in excess of 25% will be disregarded.

Section 2 consists of three questions and carries 75% of the marks.
Answer ***all three*** questions from this Section. Note that each question has two parts, of which only ***one part*** should be answered. If you answer both parts, credit will only be given for the best answer.

The approximate allocation of marks to each part of a question is shown in brackets **[ ]**.

PLEASE USE A SEPARATE ANSWER BOOK FOR SECTION 1 AND SECTION 2 QUESTIONS.

Calculators may be used in this examination but must not be used to store text. Calculators with the ability to store text should have their memories deleted prior to the start of the examination.

A table of physical constants and units that may be required will be found at the end of this question paper.

SECTION 1

*Full marks for this section can be obtained by correctly answering **five** questions. You may attempt as many questions as you wish, but any marks in excess of 25% will be disregarded.*

1. A uniform metal rod of Young's modulus $Y = 2 \times 10^{11}$ Pa and coefficient of linear thermal expansion $\alpha = 10^{-5}$ K^{-1} is placed horizontally between two rigid vertical walls. At $10°$C, it just slips between the walls with no space to spare.
   Derive an expression for the thermal stress exerted by the rod on the walls as a function of Y, α and the increase in temperature ΔT.
   Calculate the thermal stress if the temperature is increased to $30°$C. **[5]**

2. Use the equipartition theorem to estimate the molar heat capacity of solid sodium chloride at high temperature. **[5]**

3. The interaction between a pair of xenon atoms is well described by the Lennard-Jones 6-12 potential with an equilibrium spacing of 3.98 Å and a binding energy $\epsilon = 0.020$ eV.
   Estimate, in SI units, the specific latent heat of vaporisation and the specific latent heat of fusion for xenon.

   [The molar mass of xenon is 131 g mol^{-1} and the density of solid xenon is approximately 3 g cm^{-3}] **[5]**

4. One mole of an ideal gas is compressed isothermally at a temperature T_0 until the original volume is halved.
   Sketch this process on a diagram of pressure versus volume.
   Derive an expression for the work done in terms of T_0 and state whether work is done on or by the system in this process. **[5]**

5. In a high-vacuum tunnel an electron is lifted just above the ground and shot at constant velocity $v = 1\,\mathrm{m\,s^{-1}}$ in a direction exactly parallel to the walls of the tunnel. By clever engineering a uniform magnetic field can be applied inside the tunnel in a direction perpendicular to the velocity of the electron. What is the strength and direction of the magnetic field that ensures that the electron never hits a tunnel wall? **[5]**

6. The electric field due to an infinite sheet of fixed charge is $E = \frac{\sigma}{2\epsilon_0}$, where σ is the surface charge density.

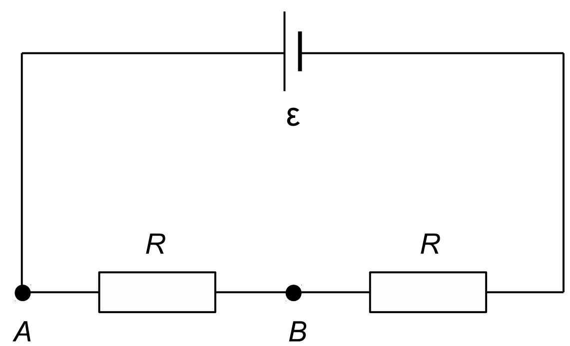
   Two sheets with the same charge density $\sigma/2$ are placed parallel to each other with a distance d between them.

   (a) What is the electric field between the sheets? **[3]**

   (b) If we put a positive charge q between the sheets, will it move if only the electrostatic force is considered? What happens if the charge is negative? **[2]**

7. In an alternate universe where space is two-dimensional, what form would Coulomb's force law take? Justify your answer. **[5]**

8. In the circuit shown below, a voltmeter, of internal resistance, r, is used to measure the potential difference across AB. Redraw the circuit showing the voltmeter connected and derive an expression for V_{AB} in terms of ε, R and r.



[5]

SECTION 2

*Answer **all three** questions from this Section. Note that each question has two parts, of which only **one part** should be answered. If you answer both parts, credit will only be given for the best answer.*

9. **EITHER Part A**

(a) The normalised Maxwell-Boltzmann distribution of the speeds v of molecules of mass m in a gas at temperature T is given by

$$P(v) = \left(\frac{m}{2\pi k_{\mathrm{B}}T}\right)^{\frac{3}{2}} 4\pi v^2 \exp\left(\frac{-mv^2}{2k_{\mathrm{B}}T}\right)$$

i. Explain the origins of the terms $4\pi v^2$ and $\exp\left(\frac{-mv^2}{2k_{\mathrm{B}}T}\right)$ in this expression. **[4]**

ii. Show that the most probable speed is $\sqrt{2k_{\mathrm{B}}T/m}$. **[3]**

iii. On the same axes, sketch the Maxwell-Boltzmann distributions for molecules at temperature T and at temperature $4T$, paying careful attention to the areas under each curve and annotating your sketch with the most probable speeds. **[4]**

(b) The normalised distribution of free paths x between collisions of molcules in a gas is given by $\frac{1}{l}\exp\left(\frac{-x}{l}\right)$

i. Show that l is the mean free path. **[2]**

ii. Find the median and root-mean-square free paths in terms of l. **[4]**

iii. Sketch the distribution of free paths and annotate your sketch with the median, mean, most probable and root-mean-square free paths. **[3]**

iv. 0.2 moles of helium are sealed in a box. The pressure of the helium is one tenth of an atmosphere. The helium atoms are at thermal equilibrium with the sides of the box, which is at room temperature. Estimate the mean free path of the helium atoms.
A single electron is added, at thermal equilibrium, to the centre of the box. Estimate its mean free path. **[5]**

OR Part B

You may assume throughout this question that all changes are reversible.

(a) A monatomic ideal gas at initial pressure P_0 is expanded adiabatically to 8 times its original volume V_0.
Sketch this change on a diagram of pressure versus volume.
Show that the final pressure is $P_0/32$. **[5]**

(b) Show that for an ideal gas $P^{1-\gamma}T^{\gamma}$ is constant in an adiabatic change, where $\gamma = C_{\mathrm{P}}/C_{\mathrm{V}}$ is the ratio of the heat capacities measured under constant pressure or constant volume conditions. **[2]**

(c) Show that in hydrostatic equilibrium, the pressure P of a fluid follows

$$\frac{\mathrm{d}P(h)}{\mathrm{d}h} = -\rho g$$

where h is the height, ρ is the density of the fluid and g is the acceleration due to gravity. **[5]**

(d) Using $P^{1-\gamma}T^{\gamma}$ is constant, or otherwise, show that for an adiabatic change of an ideal gas

$$(\gamma - 1)\frac{\mathrm{d}P}{P} = \gamma\frac{\mathrm{d}T}{T}$$ **[3]**

(e) In an adiabatic model of the atmosphere, a packet of air in the atmosphere does not exchange heat with surrounding packets of air.

i. Show that the rate of change of temperature with height is constant in an adiabatic model for the atmosphere. **[4]**

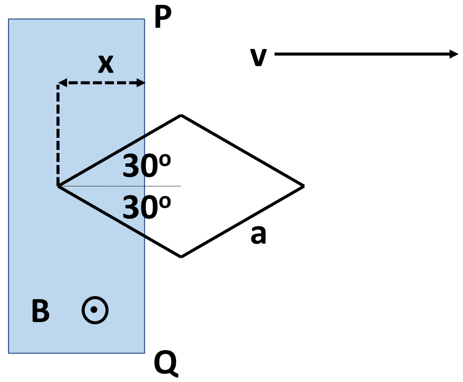
ii. Using an adiabatic model for the atmosphere, estimate the temperature and the pressure at the summit of Mount Everest (8848 m above sea level).

[You may assume that the average molar mass of air is $29\,\mathrm{g\,mol^{-1}}$] **[3]**

iii. Estimate the work done on a packet of air containing 6.022×10^{23} molecules that is swept by the wind from sea level over the Indian Ocean to the summit of Everest.
What becomes of this work done? **[3]**

10. **EITHER Part A**

A rhomboidal wire frame with side length a has total resistance R. It is being pulled with a speed v, perpendicular to PQ, out of a region where there is a uniform magnetic field B pointing out of the page. The extent of the B field corresponds to the shaded region in the figure below.



Consider the moment when the left corner of the wire frame is a distance $x \leq a\sqrt{3}/2$ inside the shaded area.

(a) What is the induced e.m.f. and current in the wire? Express your results in terms of B, x, v and R. **[6]**

(b) State whether the induced current flows clockwise or counter-clockwise. **[2]**

(c) Draw a diagram of the forces on each of the sides of the rhomboidal wire generated by the interaction between the magnetic field and the induced current. **[4]**

(d) Obtain an expression for the total net force. **[4]**

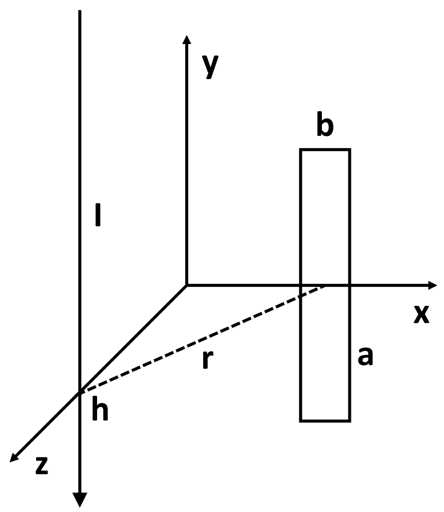
(e) Verify that the work you do from $x = a\sqrt{3}/2$ down to $x = 0$ equals the energy dissipated in the resistor. **[9]**

OR Part B

A long straight stationary wire in the (y, z) plane is parallel to the y axis and passes through the point $z = h$, see figure below.

A current I flows in the wire, returning by a remote conductor whose field we may neglect.

Lying in the (x, y) plane is a thin rectangular loop with two of its sides, of length a, parallel to the y axis. The length b of the other two sides is negligible.



(a) Use Ampere's law to obtain an expression for the magnetic field B generated by the current at the centre of the loop on the x axis, at a distance r from the wire as shown in the figure above. Verify your answer using the Biot-Savart law. Provide a sketch to illustrate the direction of the magnetic field. **[6]**

(b) Write down the algebraic expressions for the components of the magnetic field along the $\hat{x}$, $\hat{y}$, and $\hat{z}$ directions, in terms of I, x, h, r and any relevant constant. **[8]**

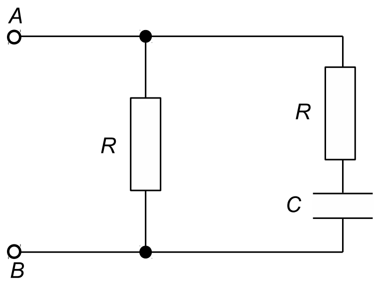
(c) Sketch a plot of the z component, $B_z(x)$, of the magnetic field as a function of x. **[8]**

(d) The loop slides with constant speed v in the positive x direction. Find the magnitude of the e.m.f. induced in the loop at the moment the centre of the loop reaches a position x on the x axis. **[8]**

11. **EITHER Part A**

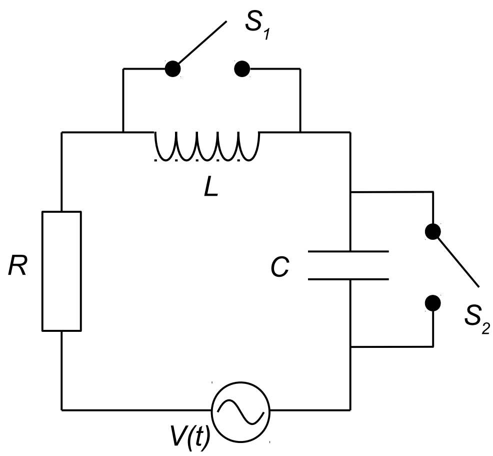
(a) State the AC analogue expression to Ohm's law and define any symbols used. **[3]**

(b) For the circuit shown below find the complex impedance between A and B and, hence, the impedance of the circuit.



Calculate the phase shift between an input alternating voltage applied across A and B and the resulting current. **[5]**

(c) In the circuit below, R = 2 kΩ, L = 20 mH and C = 136 μF. The sinusoidal voltage source is $V(t) = V_0 \sin(\omega t)$ with V_0 = 20 V and ω = 2000 rad/s.



In the following ignore the transient effects.

i. For both switches closed, find an expression for the current, $I(t)$, as a function of time and calculate the average power delivered to the circuit. **[3]**

ii. After only S_1 is opened, find an expression for the current as a function of time. **[4]**

iii. For both S_1 and S_2 open, calculate the resonant angular frequency, ω_0, with the current and voltage in phase. Calculate the impedance of the circuit.
Calculate the maximum energy stored in the capacitor during oscillations. **[10]**

OR Part B

(a) i. State Thévenin's theorem and explain how you find the Thévenin equivalent voltage. **[3]**

ii. Explain how you calculate Thévenin's resistance. **[2]**

iii. State Norton's theorem. **[2]**

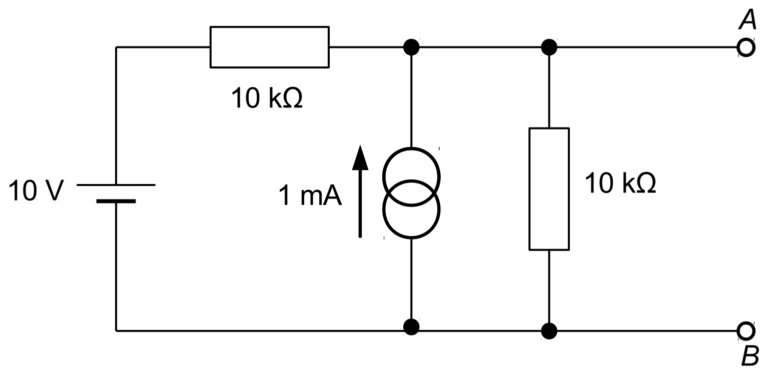
(b) State the superposition theorem and when it is used. **[3]**

(c) For the circuit below:

i. draw Thévenins equivalent circuit as seen at terminals A and B and find the Thévenin equivalent voltage and resistance; **[7]**

ii. find and draw Norton's equivalent circuit as seen at terminals A and B; **[4]**

iii. calculate the current through, and voltage across, an additional 4.7 kΩ resistor connected between terminals A and B. **[4]**



Physical Constants and Units

Acceleration due to gravity	g	9.81 m s^{-2}
Gravitational constant	G	6.674×10^{-11} N m^2 kg^{-2}
Ice point	T_{ice}	273.15 K
Avogadro constant	N_A	6.022×10^{23} mol^{-1}
		[*N.B.* 1 mole $\equiv 1$ *gram-molecule*]
Gas constant	R	8.314 J K^{-1} mol^{-1}
Boltzmann constant	k, k_B	1.381×10^{-23} J K^{-1} $\equiv 8.62 \times 10^{-5}$ eV K^{-1}
Stefan constant	σ	5.670×10^{-8} W m^{-2} K^{-4}
Rydberg constant	R_∞	1.097×10^{7} m^{-1}
	$R_\infty hc$	13.606 eV
Planck constant	h	6.626×10^{-34} J s $\equiv 4.136 \times 10^{-15}$ eV s
$h/2\pi$	$\hbar$	1.055×10^{-34} J s $\equiv 6.582 \times 10^{-16}$ eV s
Speed of light *in vacuo*	c	2.998×10^{8} m s^{-1}
	$\hbar c$	197.3 MeV fm
Charge of proton	e	1.602×10^{-19} C
Mass of electron	m_e	9.109×10^{-31} kg
Rest energy of electron		0.511 MeV
Mass of proton	m_p	1.673×10^{-27} kg
Rest energy of proton		938.3 MeV
One atomic mass unit	u	1.66×10^{-27} kg
Atomic mass unit energy equivalent		931.5 MeV
Electric constant	ϵ_0	8.854×10^{-12} F m^{-1}
Magnetic constant	μ_0	$4\pi \times 10^{-7}$ H m^{-1}
Bohr magneton	μ_B	9.274×10^{-24} A m^2 (J T^{-1})
Nuclear magneton	μ_N	5.051×10^{-27} A m^2 (J T^{-1})
Fine-structure constant	$\alpha = e^2/4\pi\epsilon_0\hbar c$	7.297×10^{-3}= 1/137.0
Compton wavelength of electron	$\lambda_c = h/m_e c$	2.426×10^{-12} m
Bohr radius	a_0	5.2918×10^{-11} m
angstrom	Å	10^{-10} m
barn	b	10^{-28} m^2
torr (mm Hg at 0 °C)	torr	133.32 Pa (N m^{-2})

Do not complete the attendance slip, fill in the front of the answer book or turn over the question paper until you are told to do so

Important Reminders

- Coats/outwear should be placed in the designated area.
- Unauthorised materials (e.g. notes or Tippex) must be placed in the designated area.
- Check that you do not have any unauthorised materials with you (e.g. in your pockets, pencil case).
- Mobile phones and smart watches **must** be switched off and placed in the designated area or under your desk. They must not be left on your person or in your pockets.
- You are not permitted to use a mobile phone as a clock. If you have difficulty seeing a clock, please alert an Invigilator.
- You are not permitted to have writing on your hand, arm or other body part.
- Check that you do not have writing on your hand, arm or other body part – if you do, you must inform an Invigilator immediately
- Alert an Invigilator immediately if you find any unauthorised item upon you during the examination.

Any students found with non-permitted items upon their person during the examination, or who fail to comply with Examination rules may be subject to Student Conduct procedures.