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THE UNIVERSITY OF BIRMINGHAM

Degree of B.Sc./M.Sci. with Honours

Programmes in the School of Mathematics and Statistics First examination

Programmes including Mathematics First examination

Degree of M.Eng. with Honours

Mathematical Engineering First examination

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MSM1C: COMPUTATIONAL AND APPLIED MATHEMATICS

May/June, 2003 2 hours

Full marks may be obtained with complete answers to ALL questions in Section A (worth a total of 50 marks) and TWO (out of THREE) questions from Section B (worth 25 marks each). Only the best TWO answers from Section B will be credited. Calculators may be used in this examination but must not be used to store text. Calculators with the ability to store text should have their memories deleted prior to the start of the examination.

Turn over

SECTION A

1. Consider the equation

$$\frac{\rho}{t^2} = 3\frac{mg}{A},$$

where ρ, t, m, A and g are density, time, mass, area and acceleration due to gravity respectively.

(a) What are the dimensions of ρ, t, m, A and g? [5]

(b) Is the equation dimensionally homogeneous? [5]

2. A particle of mass m has coordinates (x, y, z) at time t such that

$$x = Ut\cos\alpha,$$

$$y = 0$$

and

$$z = -\frac{1}{2}gt^2 + Ut\sin\alpha,$$

where g is the acceleration due to gravity, α is an angle and U is the initial speed of the particle.

(a) Determine expressions for the velocity $\mathbf{v}$ and the acceleration $\mathbf{a}$ of the particle. [4]

(b) Hence determine an expression for the kinetic energy of the particle in terms of the time t. [2]

(c) Determine an expression for the force $\mathbf{F}$ on the particle at time t. [2]

(d) Describe briefly what this set of equations represents. [2]

3. A satellite of mass $100kg$ orbits the earth on a circular orbit of radius $20,000km$. The satellite orbits the earth once every 2 hours. In which direction does the velocity vector of the satellite point? In which direction does the acceleration vector of the satellite point? What is the angular speed ω of the satellite? What is its speed v? What is its angular momentum $\mathbf{h}$? [12]

SECTION B

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4. A ball of mass m is dropped from rest at a height of h above the sea. The ball drops a distance h, hits the surface of the water and then carries on falling through the water (i.e. it does not float). Let $z = 0$ represent the point from which the ball is released, and $z = h$ be the point at which the ball hits the water, so that the z-axis points downwards. Let g be the acceleration due to gravity, and time $t = 0$ be the time at which the ball is released from rest.

(a) Ignoring air resistance, determine an expression for the speed of the ball when it hits the surface of the water. [6]

(b) The water produces a resistance (drag) force on the ball. A simple model for the magnitude of this force is

$$F = -km\rho v^2,$$

where ρ is the constant density of the water, v is the speed of the ball and k is a constant. This force acts in the opposite direction to the motion of the ball.

(i) Determine the SI units of k. [3]

(ii) Determine the speed of the ball as time $t \to \infty$? What occurs when $1 = 2k\rho h$? Sketch graphs of the speed of the ball against time for both $1 > 2k\rho h$ and $1 < 2k\rho h$. (You may either solve the equations directly or use the methods of equilibrium points and stability.) You may assume that the sea is sufficiently deep that the sea bed can be ignored. [11]

(c) Now suppose instead that the density of the water is not constant, but given by the expression

$$\rho = \alpha + \beta z,$$

where α and β are constants.

(i) Determine an expression for the pressure in the water p. [4]

(ii) If the density of the water increases with depth, will the ball's velocity increase or decrease relative to the solution found in part (b) (ii)? Explain your reasoning. [1]

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