Electromagnetism

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Lecture 16

Revision Lecture

Week 8

**Last Lecture of this term
Hoorah!**

Electrostatics

- Force between 2 charges $\underline{\boldsymbol{F}} = \frac{q_1 q_2}{4\pi\varepsilon_0 r^2}\underline{\hat{\boldsymbol{r}}}_{\mathbf{12}}$
- Gauss' Law $\int_S \underline{\boldsymbol{E}} \cdot d\underline{\boldsymbol{S}} = \frac{Q_{enc}}{\varepsilon_0} = \int_V \frac{\rho}{\varepsilon_0}\, dV$
- Coulomb' Law $\underline{\boldsymbol{E}} = \frac{Q}{4\pi\varepsilon_0 r^2}\underline{\hat{\boldsymbol{r}}}$
  – E-field from point charge (can derive from Gauss's Law)
- Coulomb Potential: $V = \frac{Q}{4\pi\varepsilon_0 r}$

Electrostatics

- Electric field is force per unit charge
- Electric potential is potential energy per unit charge
- Relationship: $\underline{\boldsymbol{E}} = -\nabla V$
- Electric dipole moment as $\underline{\boldsymbol{p}} = q\underline{\boldsymbol{a}}$
- For Dipole in external uniform E-field
- $\underline{\boldsymbol{\tau}} = q\underline{\boldsymbol{a}} \wedge \underline{\boldsymbol{E}} = \underline{\boldsymbol{p}} \wedge \underline{\boldsymbol{E}}$
- $\boldsymbol{U} = -\underline{\boldsymbol{p}} \cdot \underline{\boldsymbol{E}}$

Capacitance

- Capacitance defined as: $C = \frac{Q}{V}$
- Energy stored in the electric field of capacitor:

$$U = \frac{1}{2} C V^2$$

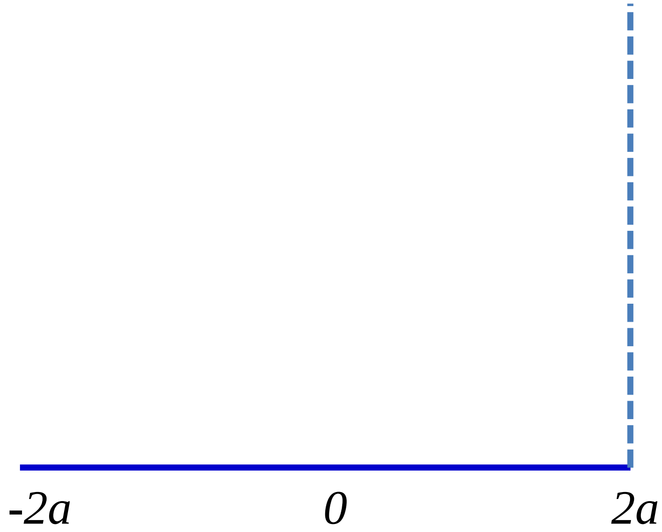
- Energy density of Electric field $u_E = \frac{1}{2} \varepsilon_0 E^2$
- Force on charge in electromagnetic field $\underline{\boldsymbol{F}} = q(\underline{\boldsymbol{E}} + \underline{\boldsymbol{v}} \wedge \underline{\boldsymbol{B}})$

Magneto-statics

- Force on current length l $\underline{\boldsymbol{F}} = I\,\underline{\boldsymbol{l}} \wedge \underline{\boldsymbol{B}}$
  - If B is perpendicular to I then $F = BIL$
- Current Loops and Magnetic Dipoles
  - $\underline{\mu} = I\underline{A}$ (magnetic dipole moment)
- Torque on magnetic dipole in B-field
  - $\underline{\boldsymbol{\tau}} = \underline{\boldsymbol{\mu}} \wedge \underline{\boldsymbol{B}}$
- Potential energy of magnetic dipole in B-field
  - $U = -\underline{\boldsymbol{\mu}} \cdot \underline{\boldsymbol{B}}$

Example 1

- Two point charges +Q and –Q are fixed at (x,y) coordinates ($-2a,0$) and ($2a,0$) respectively.
  - What is the potential at point P at ($2a,3a$) ?
  - What is the work required to move a point charge, q from ($2a,3a$) to ($-2a,3a$)?



Example 2 (2017 exam 10A)

(b) A ring-shaped conductor, of radius a, carries a total charge Q uniformly distributed around it. The ring is in the (y, z) plane, with its centre in the origin.

i. Find the potential at a point P on the ring axis at a distance x from the centre of the ring. [Hint: consider the electric potential at P due to a small segment (effectively a point charge) carrying a charge dQ.] **[5]**

ii. Hence, show that the only non-zero component of the electric field at P is along the axis of the ring and is given by:

$$E_x = \frac{Q\,x}{4\pi\epsilon_0(a^2 + x^2)^{3/2}} .$$

[8]

Example 2 (2017 exam 10A)

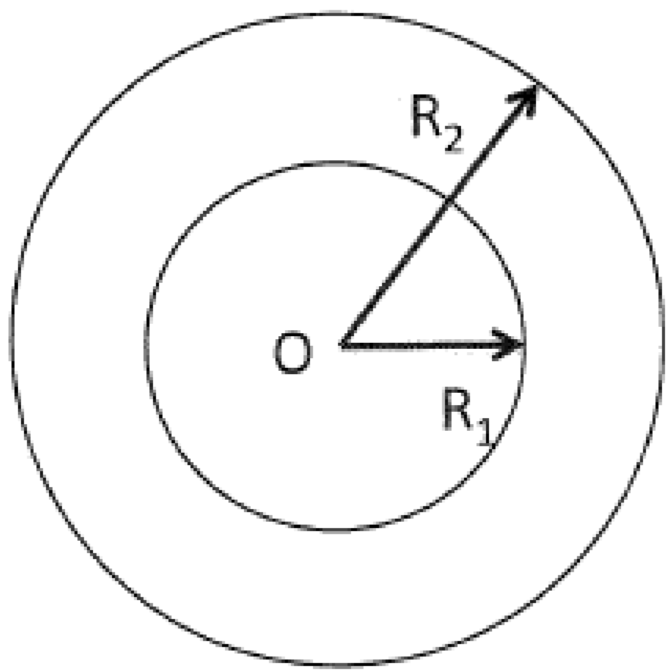
iii. If an electron is placed at the centre of the ring and then displaced by a small distance x along the x-axis (with $x \ll a$), show that the electron initially oscillates with a frequency f. Evaluate f for $a = 1\,\mathrm{m}$ and $Q = +1\,\mu\mathrm{C}$. **[7]**

Q3

(b) The figure below shows the cross section of an infinitely long hollow cylinder with inner radius R_1 and outer radius R_2. The charge distribution is $\rho(r) = \rho_0 r$ (C m^{-3}) for $R_1 \le r \le R_2$ and zero elsewhere.

Show that the total charge Q contained within a metre length of the cylinder can be expressed as:

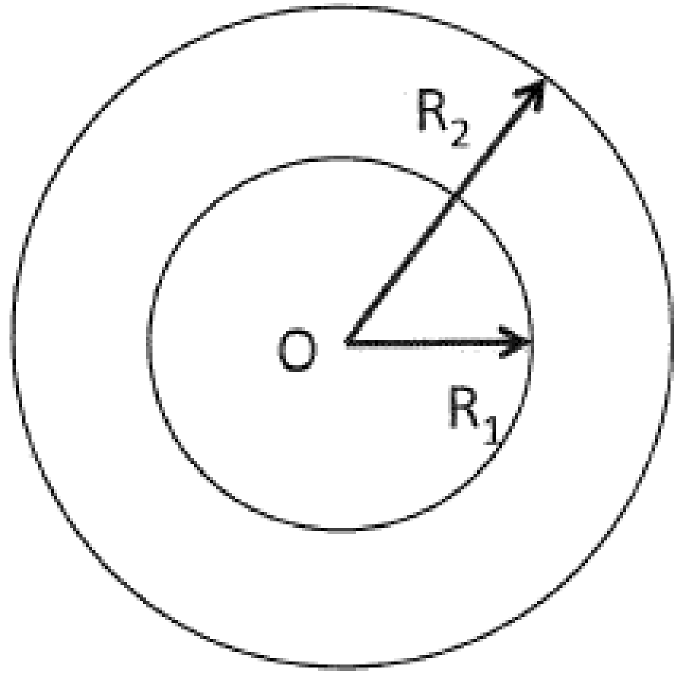
$$Q = \frac{2}{3}\pi\rho_0(R_2^3 - R_1^3).$$



[4]

$$Q = \frac{2}{3}\pi\rho_0(R_2^3 - R_1^3).$$

Q3 continued



(c) Calculate the electric field at a distance r from the centre, for i) $r > R_2$ and ii) $R_1 < r < R_2$. Write your expressions in terms of r and ρ_0. Sketch a graph to show how the electric field changes as a function of r.

[8]

(d) Use your answer in (c i)) to derive an expression for the electric potential V(r) for r > R_2.

[6]

8. **Q4** A wire bent into a semi-circle of radius R has a uniform positive charge density λ (C m^{-1}) along its length. Find the magnitude and direction of the electric field at the centre of this charged semi-circle. [5]