Electromagnetism

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Lecture 14

Ampere’s Law

Week 7

Last Lecture

- Magnetic field from moving charge
- Magnetic field from current element
- **Biot-Savart Law**
  - B-Field at centre of current loop (magnetic dipole)
  - B-field from line of current
  - B-field from infinite line of current
  - B-field along axis of current loop (magnetic dipole)

This Lecture

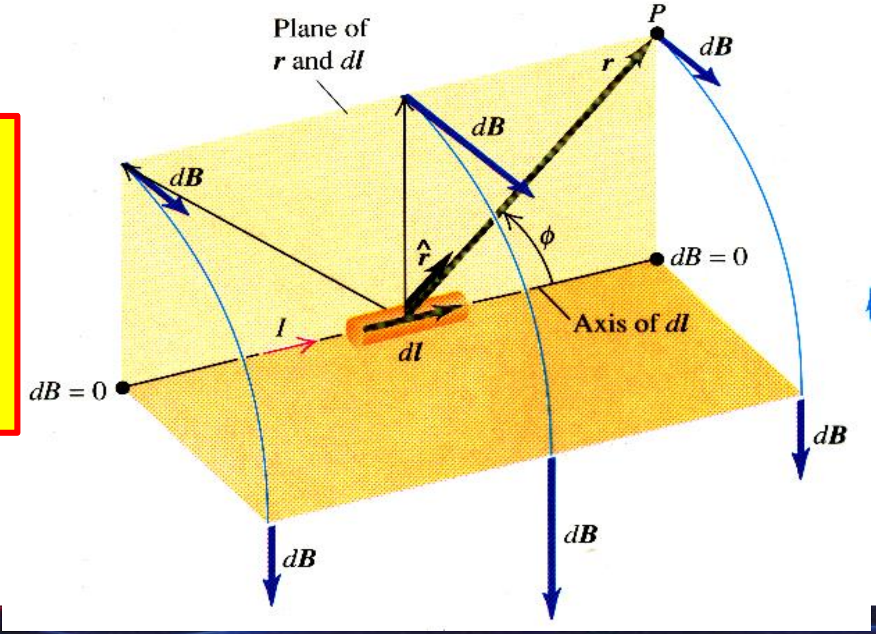
- Ampere's Law
  - B-fields inside and outside current carrying wires
  - B-fields inside solenoids
  - B-field from Toroidal Solenoid

- Force between two long parallel currents

Review – Biot-Savart Law

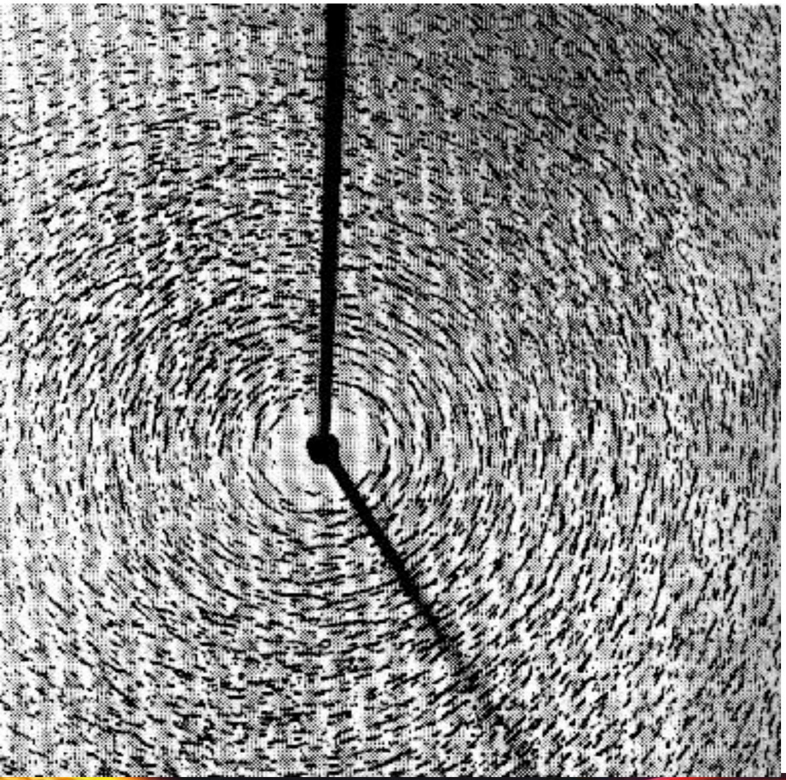
- The magnetic field set up by a current-carrying conductor can be found from the Biot-Savart law. This law asserts that the contribution $\delta\underline{\boldsymbol{B}}$ to the field set up by a current element $I\,\delta\underline{\boldsymbol{l}}$ at a point P, a distance $\underline{\boldsymbol{r}}$ from the current element, is:

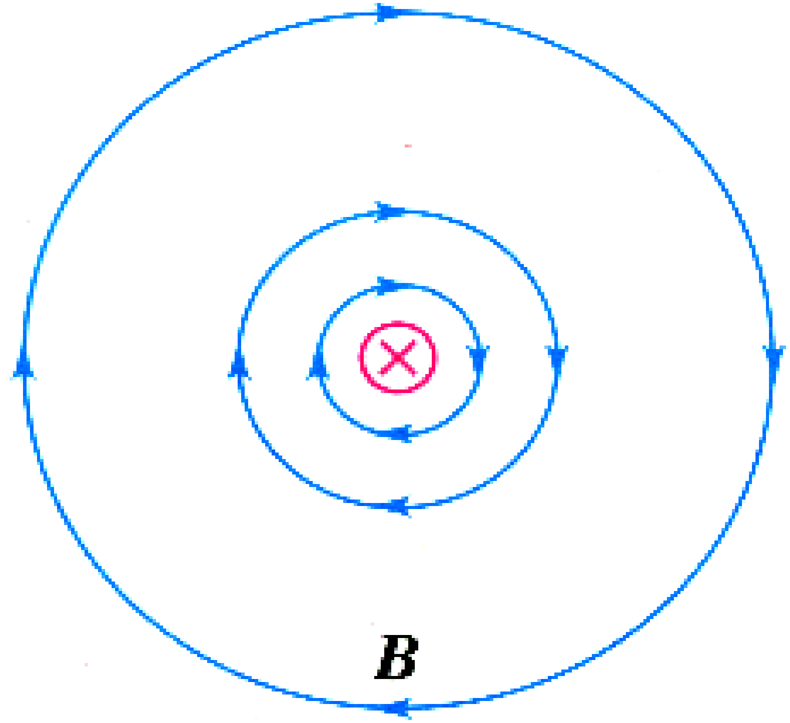
$$\delta\underline{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\,\frac{I\,\delta\underline{\mathbf{l}}\wedge\hat{\underline{\mathbf{r}}}}{r^2}$$



B-field from line of Current

B-field lines *encircle* the current that acts as their source. ***B***-field lines are continuous loops (lecture 11 - Law for Magnetism)





E- and B-Fields

- We already know the following:

Gauss's Law for E-fields

$$\int_S \underline{\boldsymbol{E}} \cdot d\underline{\boldsymbol{S}} = \frac{Q}{\varepsilon_0}$$

Gauss's Law for B-fields

$$\int_S \underline{\boldsymbol{B}} \cdot d\underline{\boldsymbol{S}} = 0$$

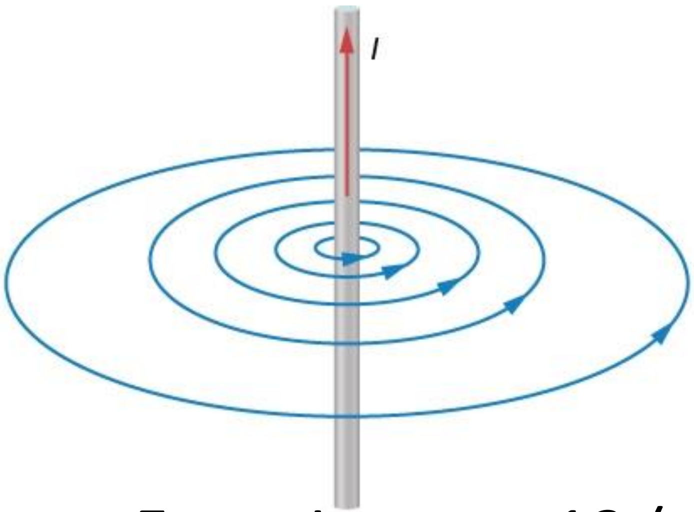
From Lecture 5: E-field is conservative *i.e.* If a charge in an E-field returns to its original position, by any route, NO WORK IS DONE.

$$\oint \underline{\boldsymbol{E}} \cdot d\underline{\boldsymbol{l}} = 0$$

So, what about: $\oint \underline{\boldsymbol{B}} \cdot d\underline{\boldsymbol{l}}$?

Ampere's Law

Consider circular path of the B-field around an infinite line of current at a radial distance r from the line.



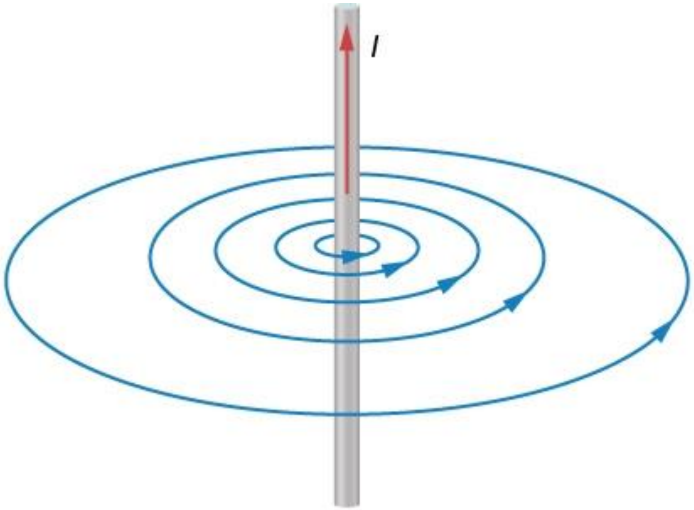
By symmetry, $\underline{\boldsymbol{B}}$ is parallel to $d\underline{\boldsymbol{l}}$ and constant for fixed r. Hence

$$\oint \underline{\boldsymbol{B}} \cdot d\underline{\boldsymbol{l}} = \oint B\, dl = B \oint dl = B\, 2\pi r$$

From Lecture 13 (ex 13.2, Eq 13.2) B-field from infinite line of current is:

$$B = \frac{\mu_0 I}{2\pi r}$$

Ampere's Law



$$\oint \underline{\boldsymbol{B}} \cdot d\underline{\boldsymbol{l}} = B\, 2\pi r$$

But from Lecture 13

$$B = \frac{\mu_0 I}{2\pi r}$$

Hence : $$\oint \underline{\boldsymbol{B}} \cdot d\underline{\boldsymbol{l}} = B\, 2\pi r = \frac{\mu_0 I}{2\pi r}\, 2\pi r = \mu_0 I$$

Ampere's Law

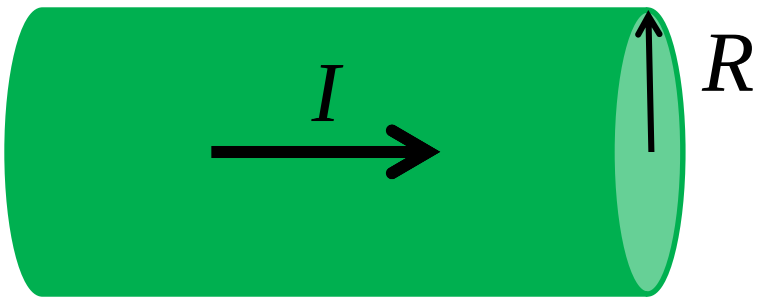
This is true for B-fields in general and is know as Ampere's Law:

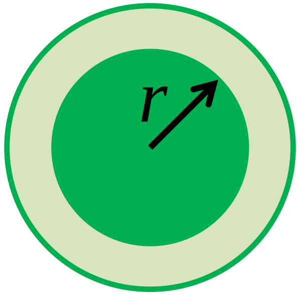
$$\oint \underline{\boldsymbol{B}} \cdot d\underline{\boldsymbol{l}} = \mu_0 I_{enc}$$

Where I is the current enclosed in the integration loop

First Example *(Ex 14.1)*

- B-Field Outside and Inside a Long Solid Cylindrical Conductor Carrying Uniformly Distributed Current

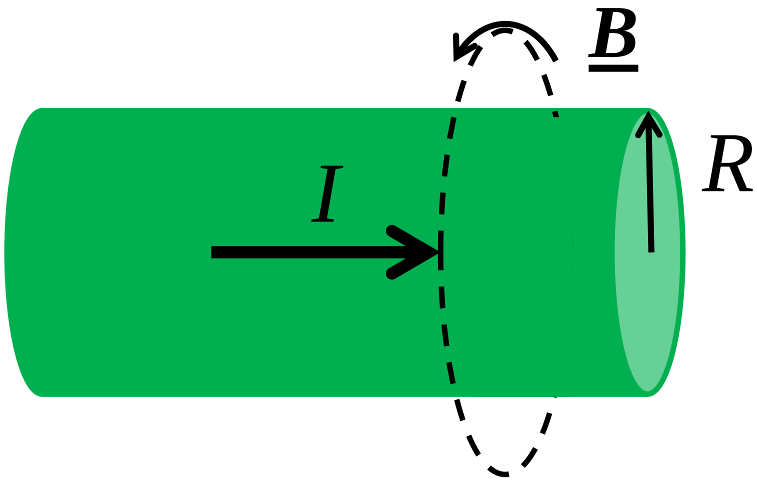




- Use Ampere's Law: $\oint \underline{\boldsymbol{B}} \cdot d\underline{\boldsymbol{l}} = \mu_0 I_{enc}$

First Example *(Ex 14.1)*

- Outside: $\oint \underline{\boldsymbol{B}} \cdot d\underline{\boldsymbol{l}} = \mu_0 I$
- By symmetry: $\underline{\boldsymbol{B}}$ is parallel to $d\underline{\boldsymbol{l}}$ and constant for fixed r.



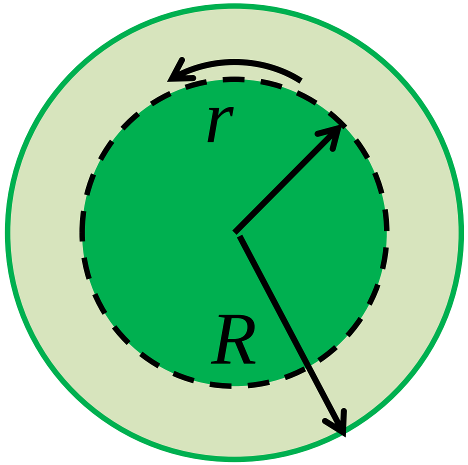
LHS: $B\,2\pi r$

RHS: $\mu_0 I$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r}$$

First Example *(Ex 14.1)*

- Inside: $\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 I_{enc}$
- By symmetry: $\boldsymbol{B}$ is parallel to $d\boldsymbol{l}$ and constant for fixed r.



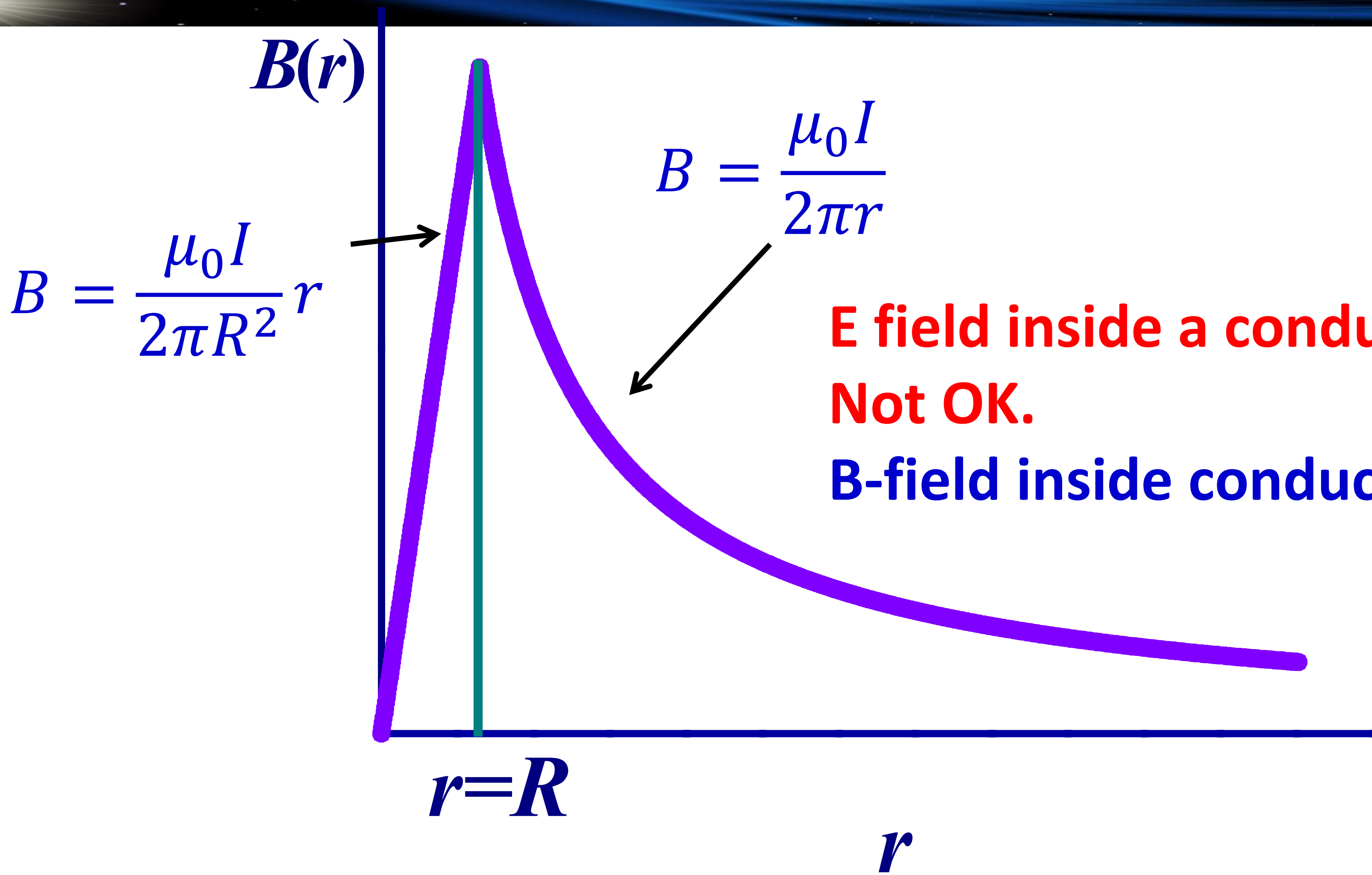
LHS: $B\,2\pi r$

RHS: $\mu_0 I_{enc}$

For uniform current $I_{enc} = \frac{\pi r^2}{\pi R^2} I$

$$\boxed{\Rightarrow B = \frac{\mu_0 I}{2\pi R^2} r}$$

First Example: Long Solid Cylindrical Conductor

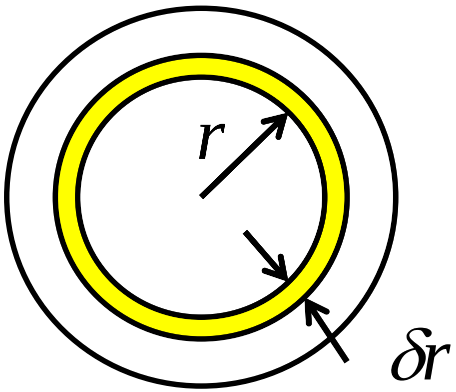


E field inside a conductor, Not OK.

B-field inside conductor , OK

Example 14.2

- B-Field Inside a Long Solid Cylindrical Conductor Carrying Non-Uniformly Current.
- Current density $J = J_0 \frac{r^2}{R^2}$
- Ampere's Law becomes: $\oint \underline{\boldsymbol{B}} \cdot d\underline{\boldsymbol{l}} = \mu_0 \int_0^r \underline{\boldsymbol{J}} \cdot d\underline{\boldsymbol{S}}$

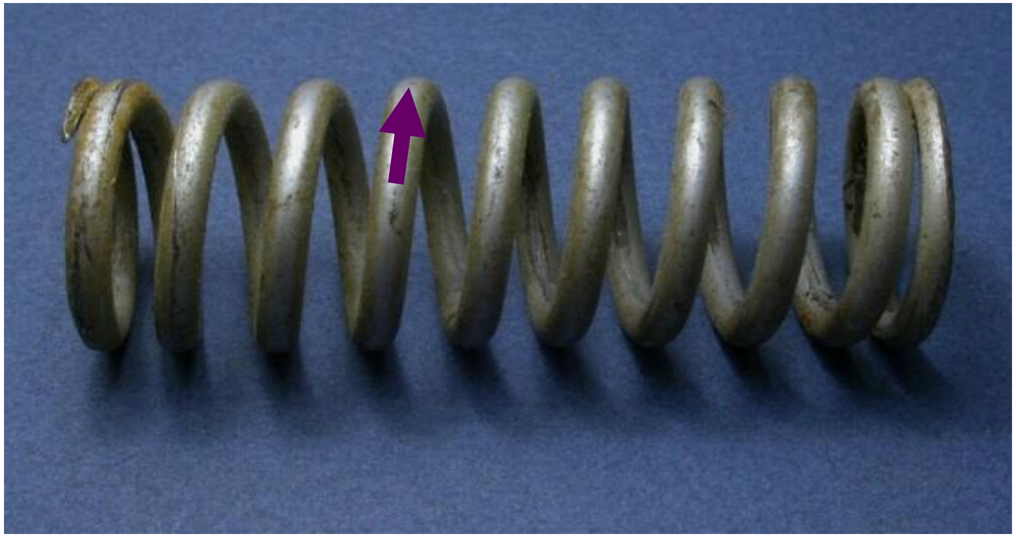


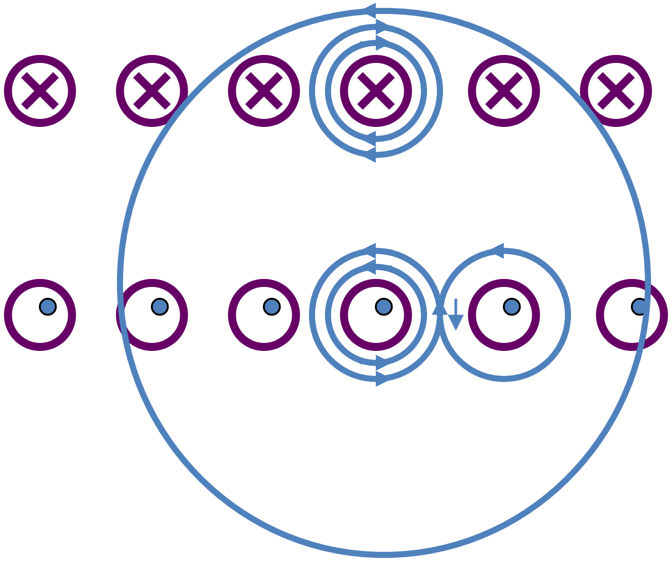
Element of area, $\delta S = 2\pi r\, \delta r$

Let's do it on the visualizer

Example 14.3

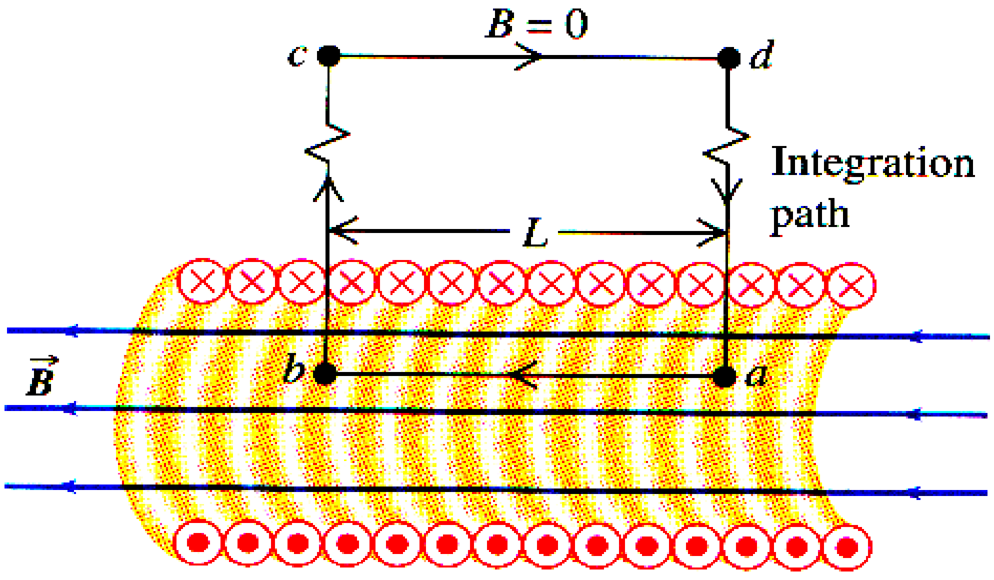
- B-Field Inside a Long Solenoid





- At first sight, this looks complicated – Don’t Panic!

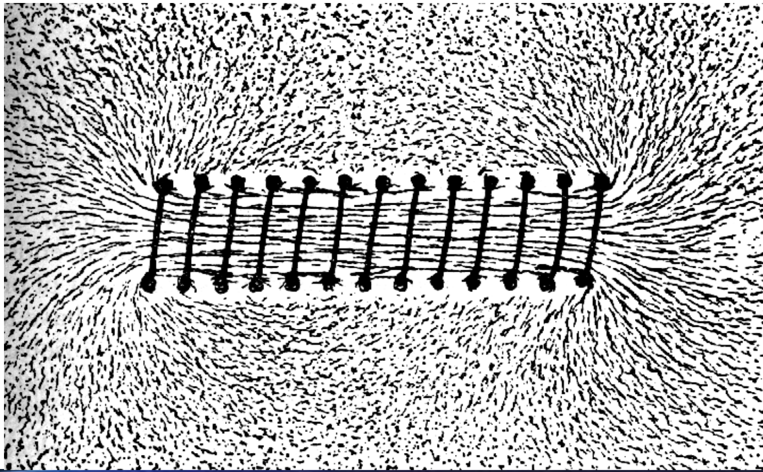
Example 14.3



Central part of solenoid

Choose integration path as shown

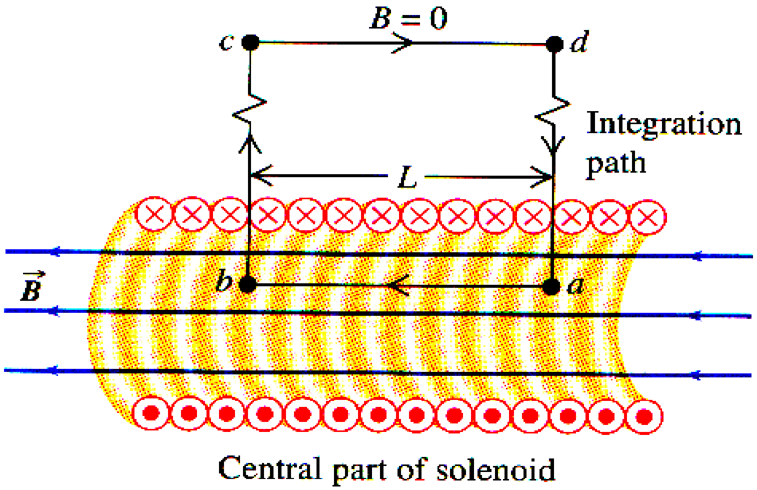
$$\oint \underline{\boldsymbol{B}} \cdot d\underline{\boldsymbol{l}} = BL$$



n = number of turns per unit length

So current enclosed in integration loop: $I_{enc} = nLI$

Example 14.3



$$\oint \underline{\boldsymbol{B}} \cdot d\underline{\boldsymbol{l}} = \mu_0 I_{enc}$$

LHS = $B\,L$

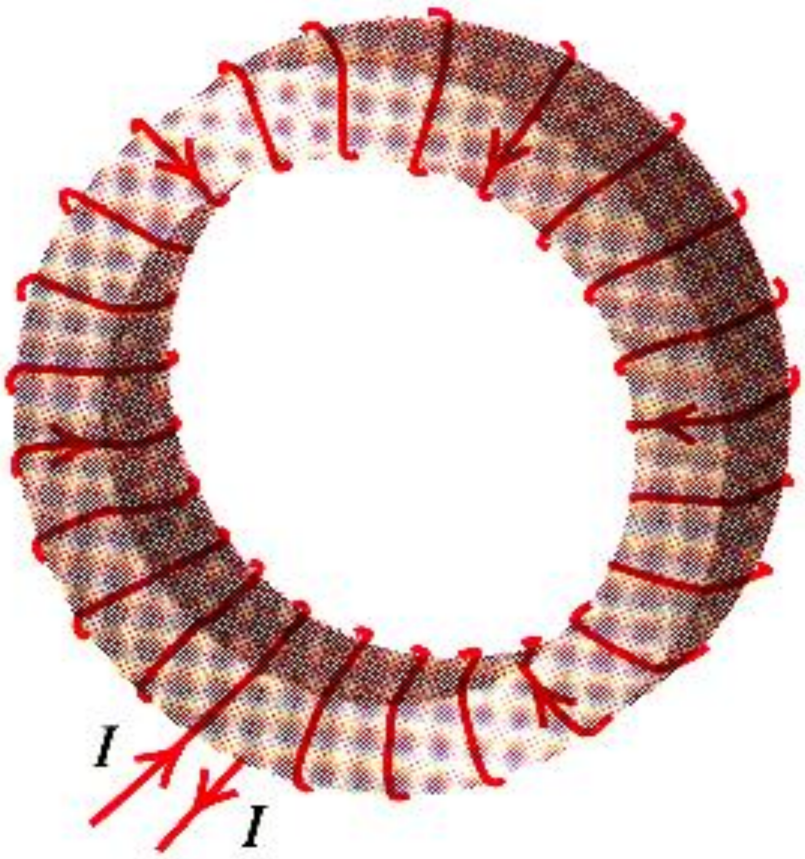
RHS = $\mu_0\, nL\, I$

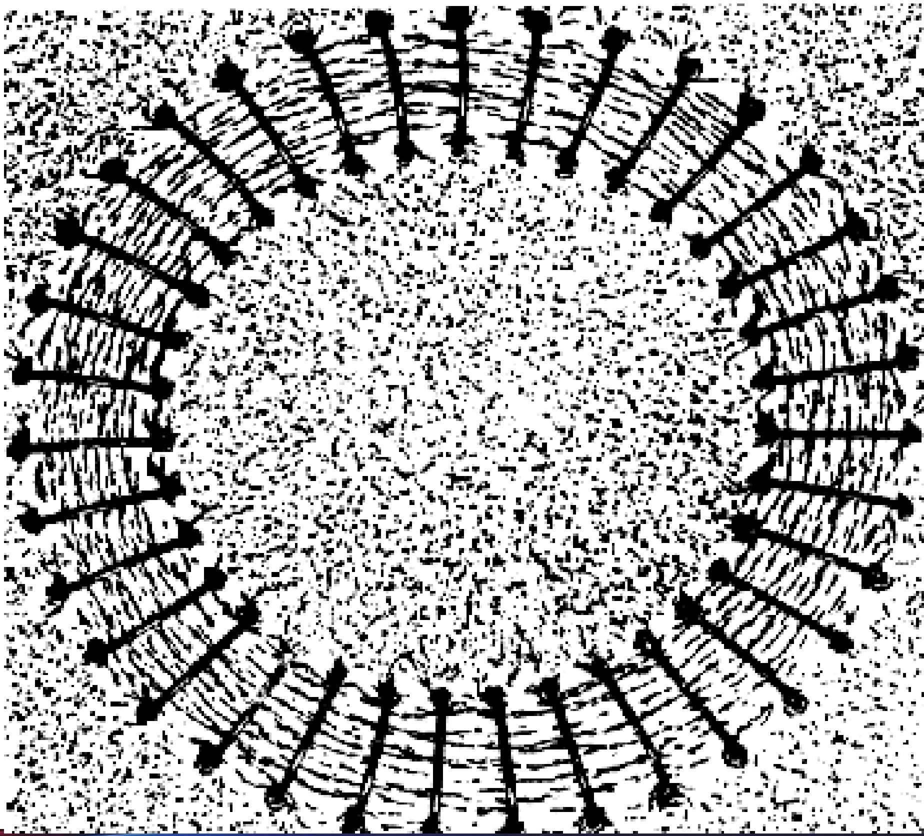
B-field inside long (i.e. neglecting end effects) Solenoid:

$$\boxed{B = \mu_0 n I}$$

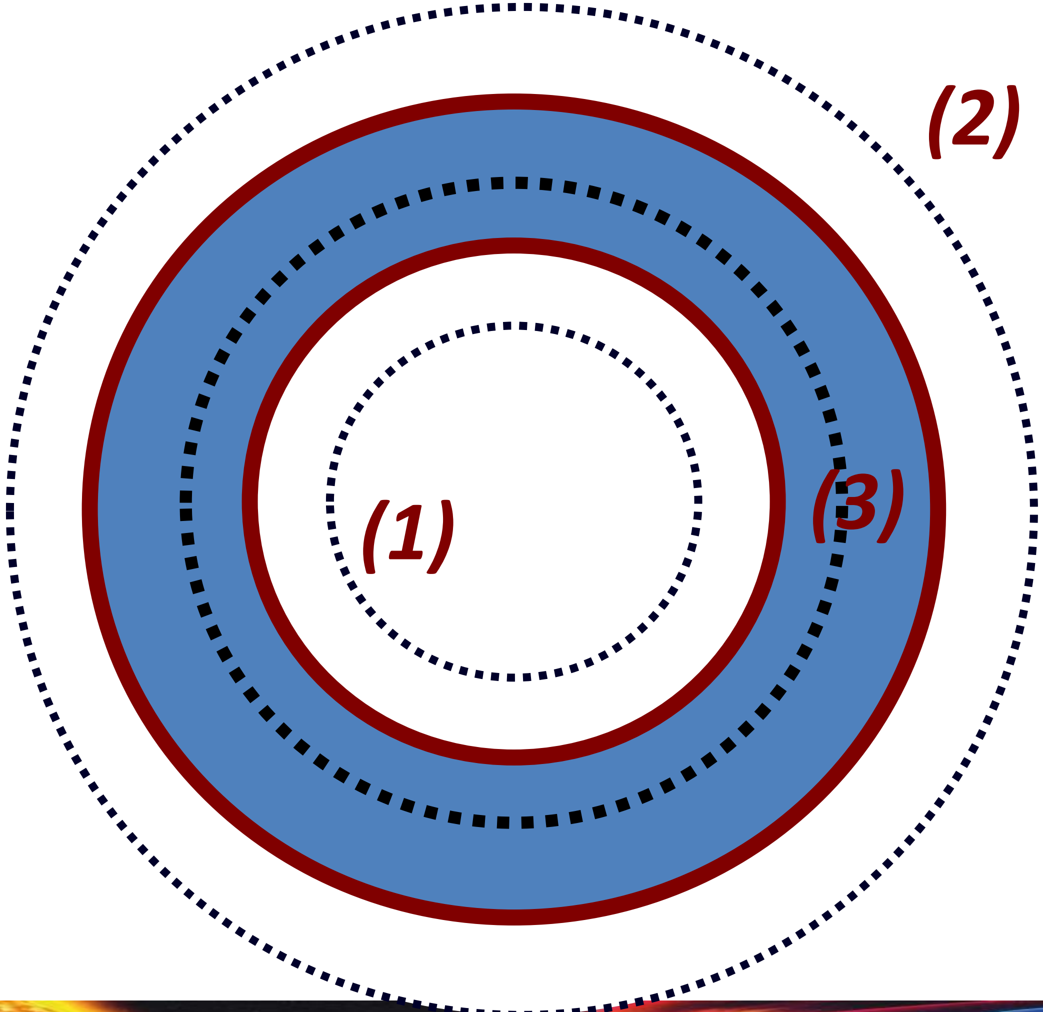
Example 14.4

- Field of a Toroidal Solenoid





Field of a Toroidal Solenoid with N Turns



Path 1 – no current enclosed: $\underline{\boldsymbol{B}} = 0$

Path 2 – no current enclose: $\underline{\boldsymbol{B}} = 0$

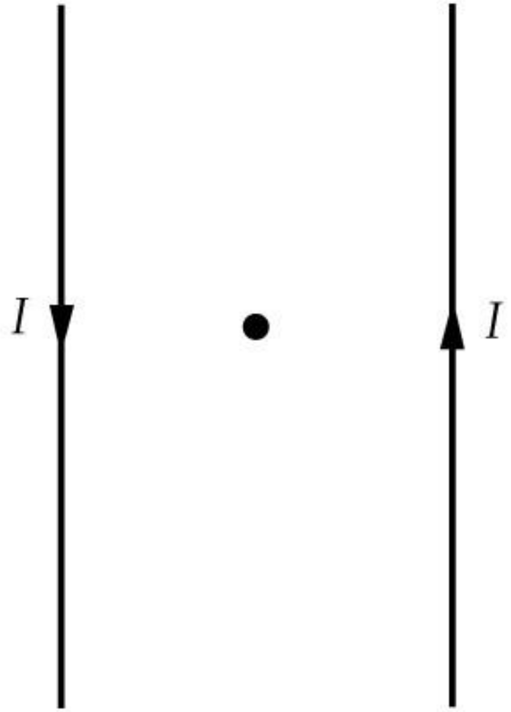
Path 3 – net current enclosed = NI

$$\oint \underline{\boldsymbol{B}} \cdot d\underline{\boldsymbol{l}} = B2\pi r = \mu_0 NI$$

$$\boxed{B = \frac{\mu_0 NI}{2\pi r}}$$

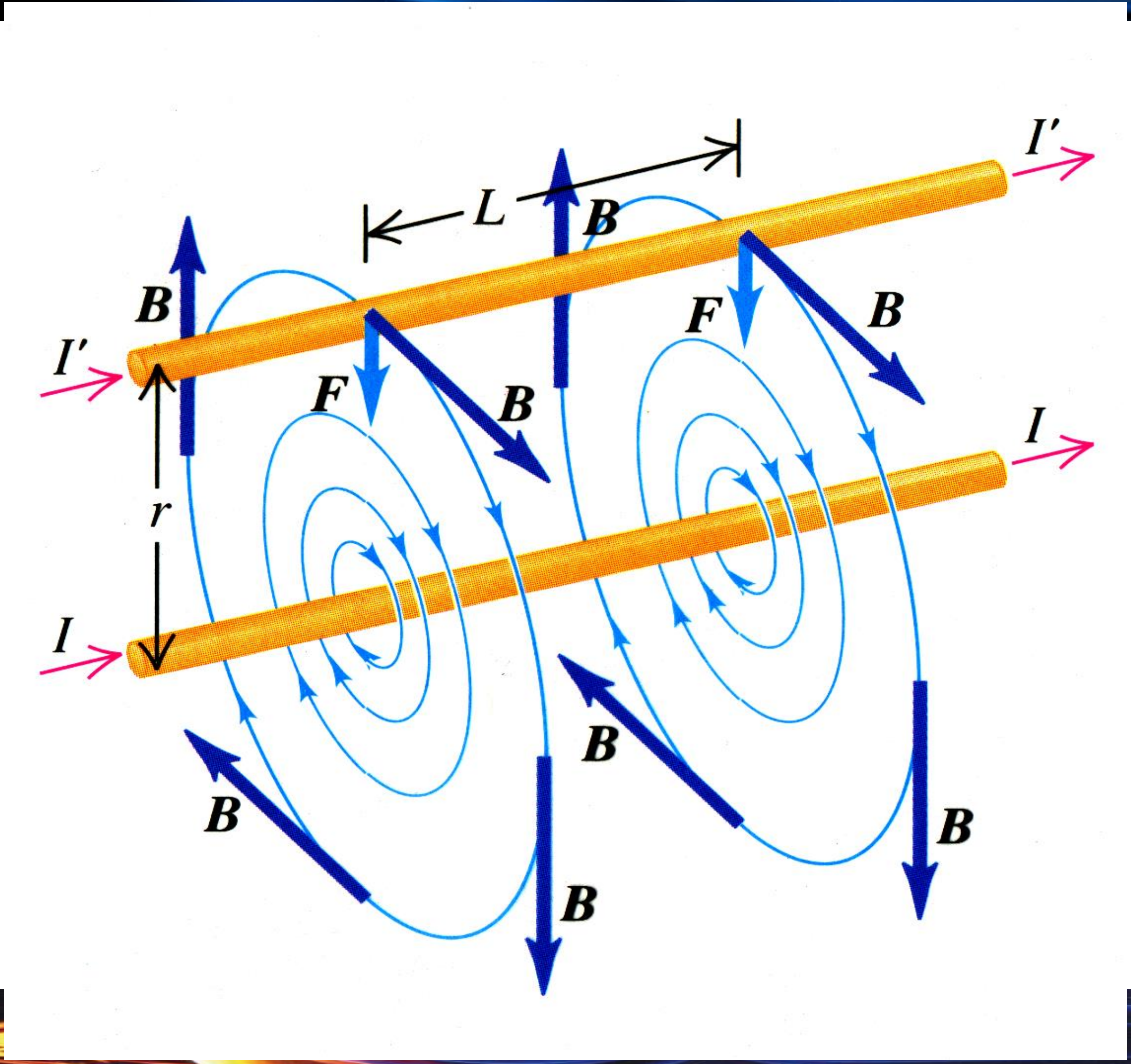
Quiz Time

- Two wires lie in the plane of the screen and carry equal currents in opposite directions. At a point midway between the wires, the magnetic field is

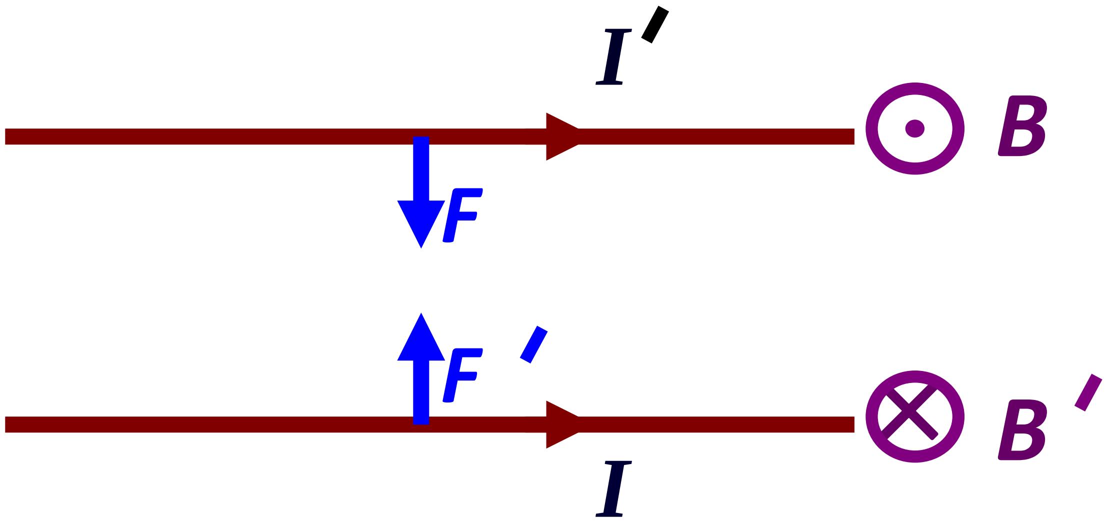


(a) zero
(b) into the screen
(c) out of the screen
(d) toward the top or bottom of the screen
(e) toward one of the wires

Force between Two Long Parallel Currents



Force between parallel Currents



$$B = \frac{\mu_0 I}{2\pi r}$$

$$B' = \frac{\mu_0 I'}{2\pi r}$$

Force F on a length L of the upper conductor is:

$$\boxed{F = I'LB = \frac{\mu_0 I I' L}{2\pi r}}$$

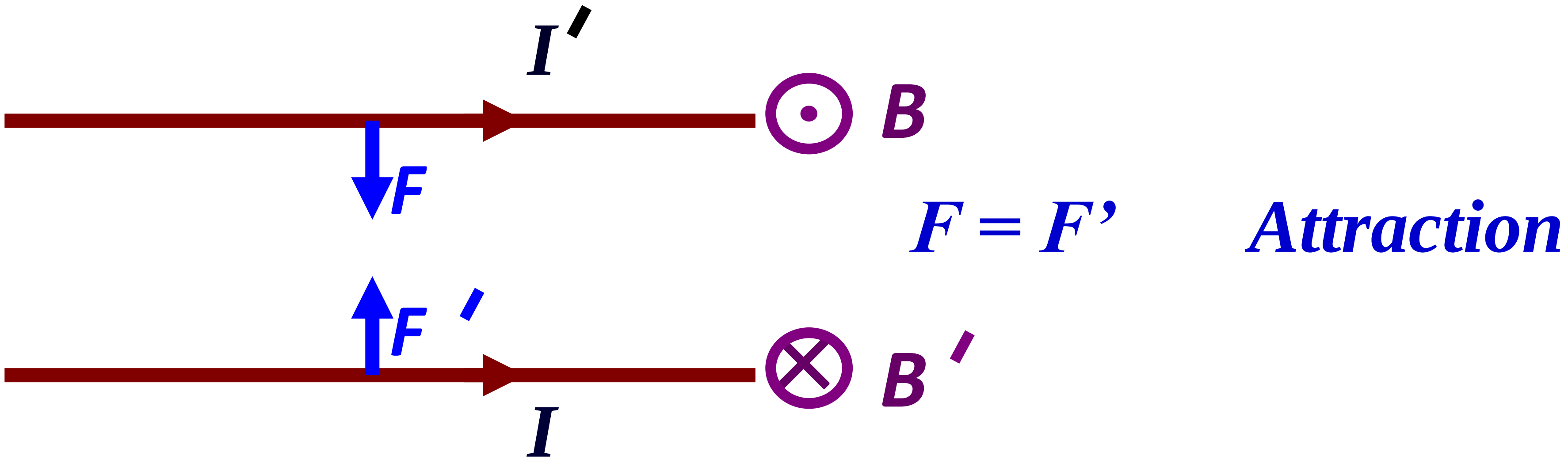
and

$$\boxed{F' = ILB' = \frac{\mu_0 I I' L}{2\pi r}}$$

$F = F'$

Attraction

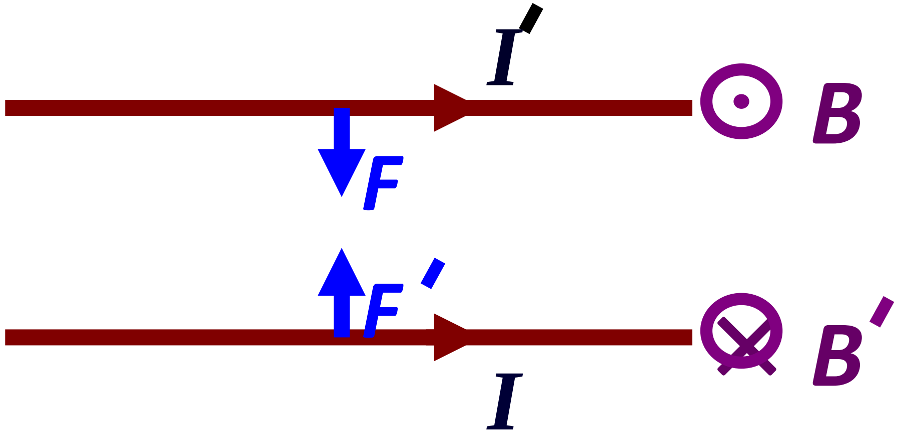
Force between parallel Currents



What happens when the currents are in opposite directions? (Ans. Repulsion)

$$\boxed{Force\ per\ unit\ length: \frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}}$$

Force between parallel Currents



$F = F'$ ***Attraction***

$$Force\ per\ unit\ length:\ \frac{F}{L} = \frac{\mu_0 I I'}{2\pi r}$$

This *fundamental magnetic effect* was first studied by Ampere (1822)

Definition of the Ampere

- The ampere is that steady current which, flowing in two infinitely long straight parallel conductors of negligible cross-sectional area placed 1 m apart in a vacuum, causes each wire to exert a force of 2 x10^{-7} N on each metre of the other wire.
- $\frac{F}{L} = \frac{\mu_0 I I'}{2\pi r} = \frac{4\pi\times10^{-7}\times1\times1}{2\pi\times1} = 2 \times 10^{-7}\ Nm^{-1}$
- Definition of the Coulomb: A current of one ampere carries a charge of one coulomb per second

Equations of Static Electric and Magnetic Fields

- For E- and B-fields that <u>don't</u> vary with time

Laws of Electrostatics

$$\int_S \underline{\boldsymbol{E}} \cdot d\underline{\boldsymbol{S}} = \frac{Q_{enc}}{\varepsilon_0}$$

$$\oint \underline{E} \cdot d\underline{l} = 0$$

Laws of Magnetostatics

$$\int_S \underline{\boldsymbol{B}} \cdot d\underline{\boldsymbol{S}} = 0$$

$$\oint \underline{B} \cdot d\underline{l} = \mu_0 I_{enc}$$

(Integrals over the closed surface)

Quiz Time

- Two parallel wires carry currents I_1 and I_2 (= $2I_1$) in the same direction. The forces F_1 and F_2 on the wires are related by:

(a) $\boxed{F_1 = F_2}$

(b) $F_1 = 2F_2$

(c) $2F_1 = F_2$

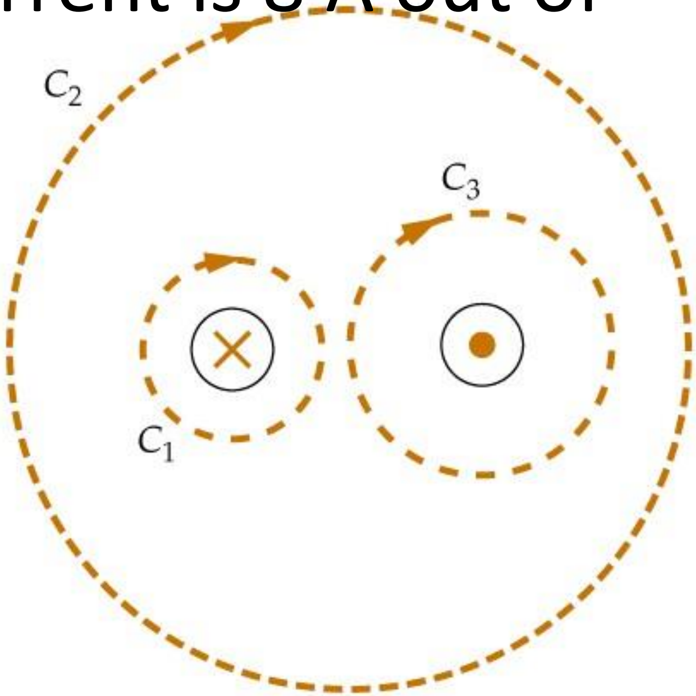
(d) $F_1 = 4F_2$

(e) $4F_1 = F_2$

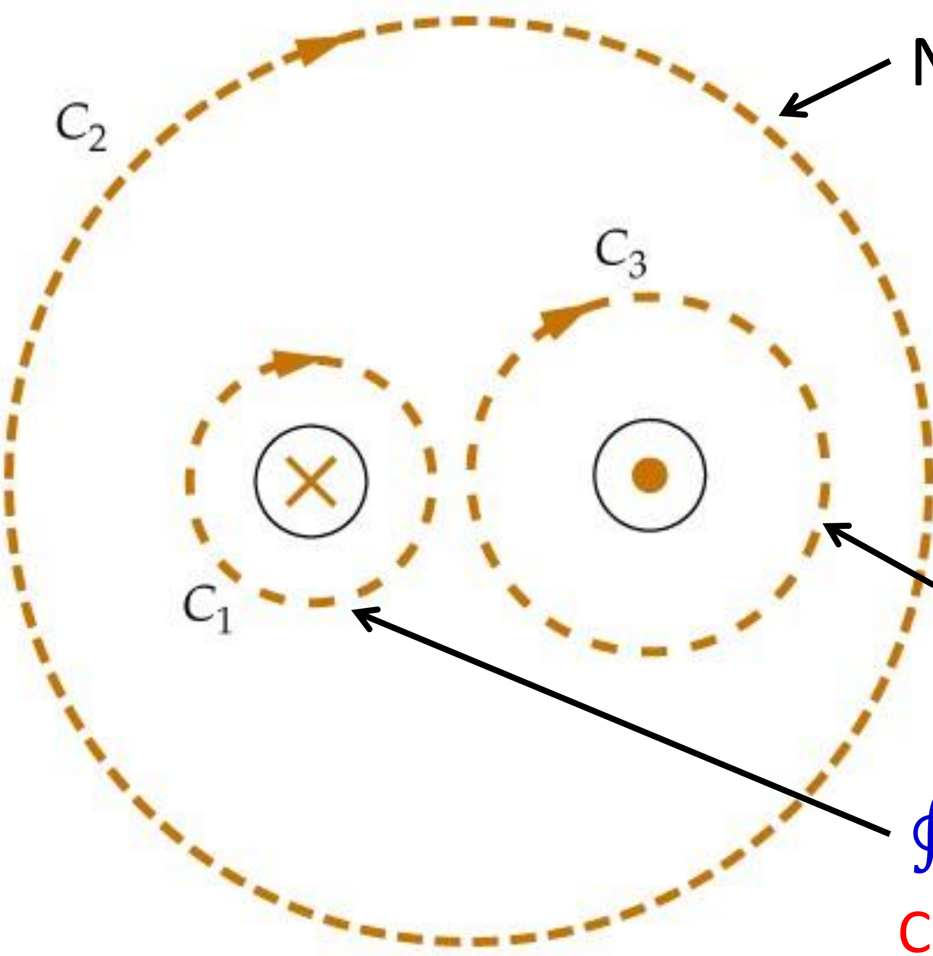
Short Exercise

- The diagram shows two currents associated with infinitely long wires, one current of 8 A into the screen, the other current is 8 A out of the screen. Find
- $\oint \underline{B} \cdot d\underline{l} = \mu_0 I_{enc}$ for each

path indicated.



Short Exercise



Net current = 0

Path C_2 $\oint \underline{B} \cdot d\underline{l} = 0$

But is B = 0 everywhere around path?

Path C_3

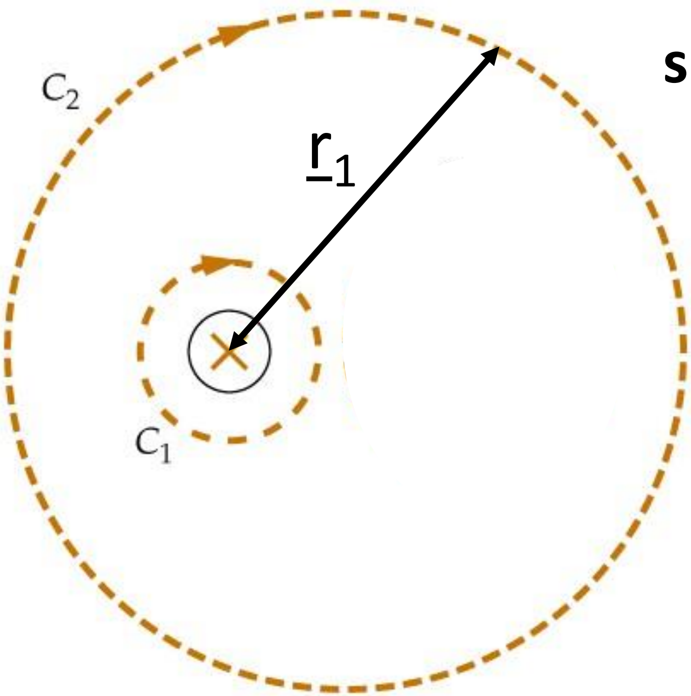
$$\oint \underline{B} \cdot d\underline{l} = \mu_0 I_{enc} = -8\mu_0$$

Path C_1

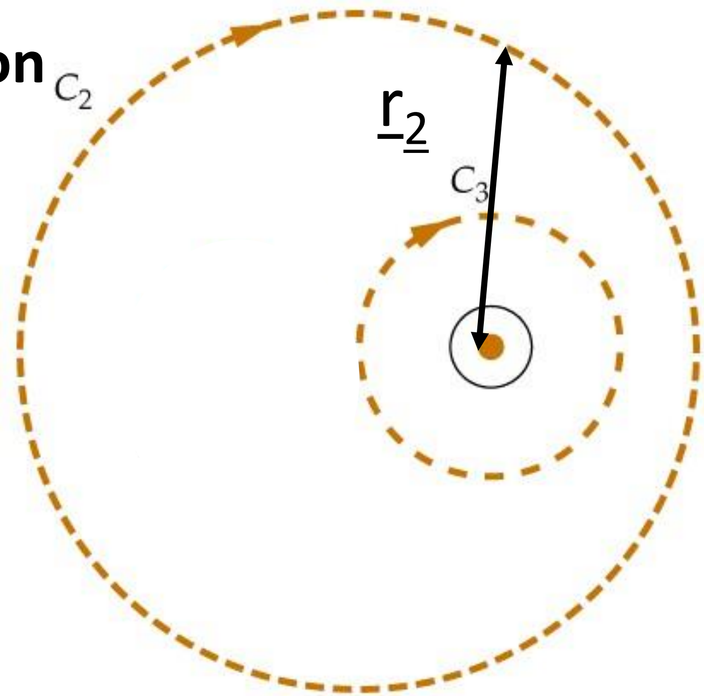
$$\oint \underline{B} \cdot d\underline{l} = \mu_0 I_{enc} = 8\mu_0$$

Can we obtain B from this?

Short Exercise



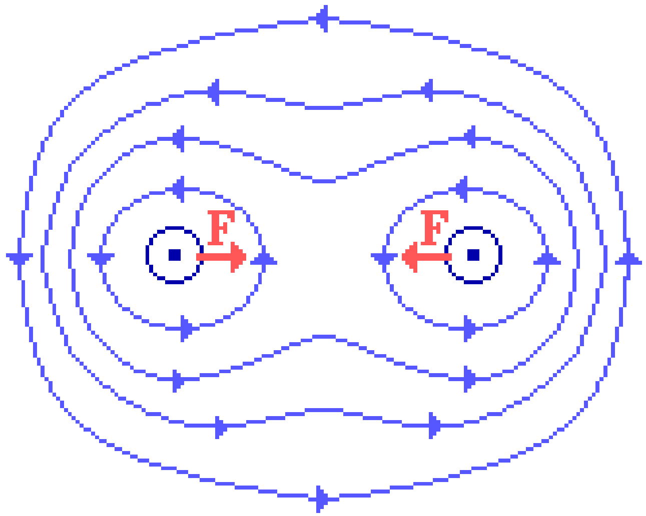
Use superposition principal



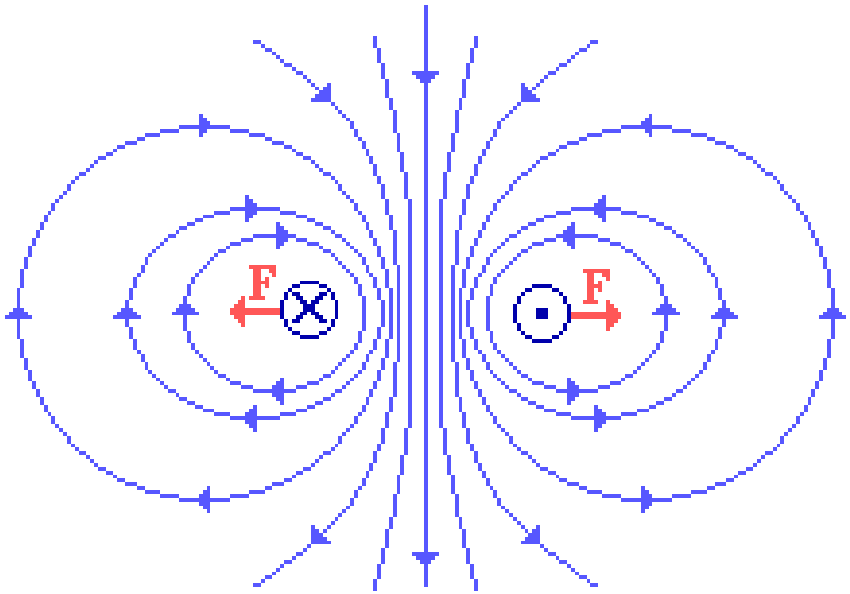
$$\boldsymbol{B} = \boldsymbol{B}_1 + \boldsymbol{B}_2 = \frac{\mu_0}{2\pi}\left\{\frac{\boldsymbol{I}\wedge\hat{\boldsymbol{r}}_1}{r_1} - \frac{\boldsymbol{I}\wedge\hat{\boldsymbol{r}}_2}{r_2}\right\}$$

Cross products to get direction of B-field

B-field from two Parallel Currents



Same direction



Opposite direction

Example

- Two straight rods 50 cm long and 1.5 mm apart carry a current of 15 A in opposite directions. One rod lies vertically above the other. What mass must be placed on the upper rod to balance the magnetic force of repulsion?
- $mg = I'LB = \frac{\mu_0 I I' L}{2\pi r} = \frac{4\pi \times 10^{-7} \times 15 \times 15 \times 0.5}{2\pi \times 1.5 \times 10^{-3}} = 0.015N$
- **Mass = 1.53 grams**
- **Note:** the magnetic force between two current-carrying wires is relatively small, even for currents as large as 15 A separated by only 1.5 mm.

Summary of Magetostatics

$$\boxed{\underline{\boldsymbol{F}}_m = q\underline{\boldsymbol{v}}\wedge\underline{\boldsymbol{B}}}$$

$$\boxed{\underline{\boldsymbol{F}} = I\ \underline{\boldsymbol{l}}\wedge\underline{\boldsymbol{B}}}$$

$$\underline{\mu} = I\underline{A}$$

$$U = -\underline{\boldsymbol{\mu}}\cdot\underline{\boldsymbol{B}}$$

$$\underline{\boldsymbol{\tau}} = \underline{\boldsymbol{\mu}}\wedge\underline{\boldsymbol{B}}$$

$$\underline{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{q}{r^2}\underline{\boldsymbol{v}}\wedge\underline{\hat{\boldsymbol{r}}}$$

$$\phi_m = \int_S \underline{\boldsymbol{B}}\cdot d\underline{\boldsymbol{S}} = 0$$

$$\boxed{\delta\underline{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{I\,\delta\underline{\boldsymbol{l}}\wedge\underline{\hat{\boldsymbol{r}}}}{r^2}}$$

$$\boxed{\oint \underline{B}\cdot d\underline{l} = \mu_0 I_{enc}}$$