University of Birmingham
School of Mathematics

Vectors, Geometry and Linear Algebra
VGLA

(Formative) Problem Sheet 3 (Semester 1)

This is a formative problem sheet and you do not submit your solutions. Do try the questions though and look at the model solutions when they are released. You will be able to try Questions 1–3 immediately. You will be able to attempt Question 4 after the last lecture in Week 7 (i.e., by the end of 12th November).

Remember that you should always carefully write out your solutions giving full explanations (not just stating the answer).

Q1. Suppose that $\mathbf{a} = (2, 1, 0)$ is a vector. Determine the scalar equation of the plane Π perpendicular to $\mathbf{a}$ containing the point $P = (1, 1, 1)$.

Q2. The aim of this question is to find the parametric form of the equation of the line of intersection of the planes

$$\Pi_1 = \{(x, y, z) \in \mathbb{R}^3 : x + 2y + z = 3\}$$

and

$$\Pi_2 = \{(x, y, z) \in \mathbb{R}^3 : x + y + 2z = 4\}.$$

To answer this question, we break the task into several sub-questions.

(i) Give a normal vector $\mathbf{n}_1$ to the plane Π_1 and a normal vector $\mathbf{n}_2$ to the plane Π_2.

(ii) Determine the vector product $\mathbf{n}_1 \times \mathbf{n}_2$.

(iii) Let L denote the line of intersection of Π_1 and Π_2. Explain why the vector $\mathbf{n}_1 \times \mathbf{n}_2$ is parallel to L. [Hint: to convince yourself this is true, you could draw a picture of two planes intersecting in a line.]

(iv) Use this fact to determine the parametric form of the equation of L.

Q3. Suppose that $z_1 = 4 - 4i$ and $z_2 = 5 + 2i$.

(i) Calculate (a) $z_1 z_2$, (b) $z_1 z_2^{-1}$ and (c) $z_1^{-1} + z_1^{-1} z_2$ giving your answer in the form $a + bi$ for appropriate real numbers a and b.

(ii) Determine the complex number α such that $z_1 \alpha = z_2$.

Q4. Use De Moivre's theorem to prove that $\sin 4\theta = 4 \cos\theta \sin\theta (\cos^2\theta - \sin^2\theta)$ for all $\theta \in \mathbb{R}$.