Optics and Waves

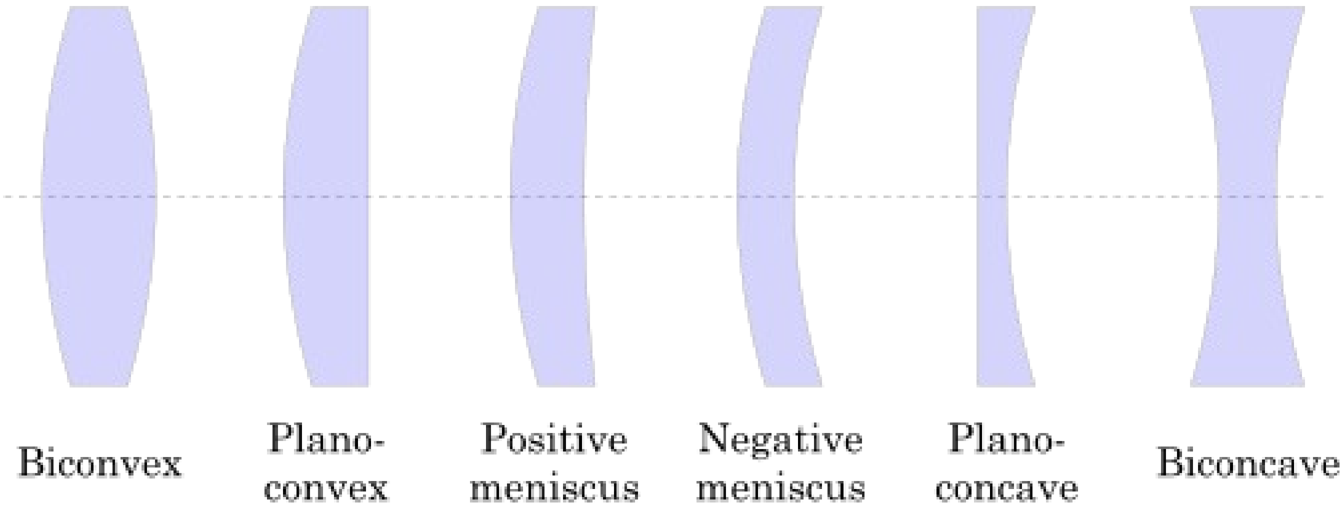
Lecture 16

Lenses

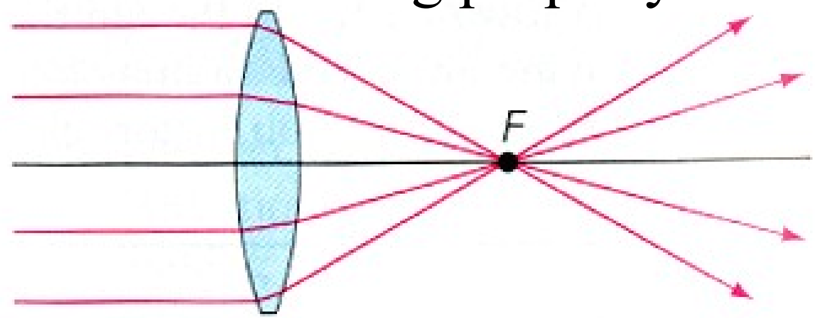
Y&F 34.3-34.4

A lens is a transparent object with two refracting surfaces whose central axes coincide.

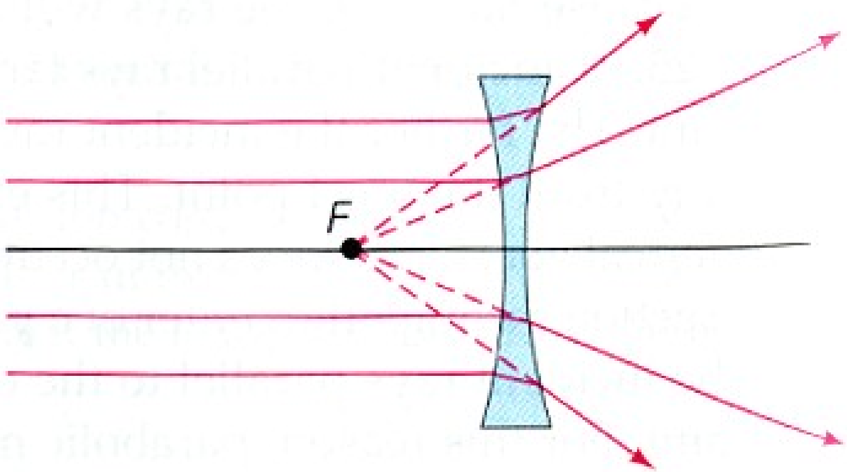
Types of lenses based on geometric shape:



Types of lenses based on focusing property:

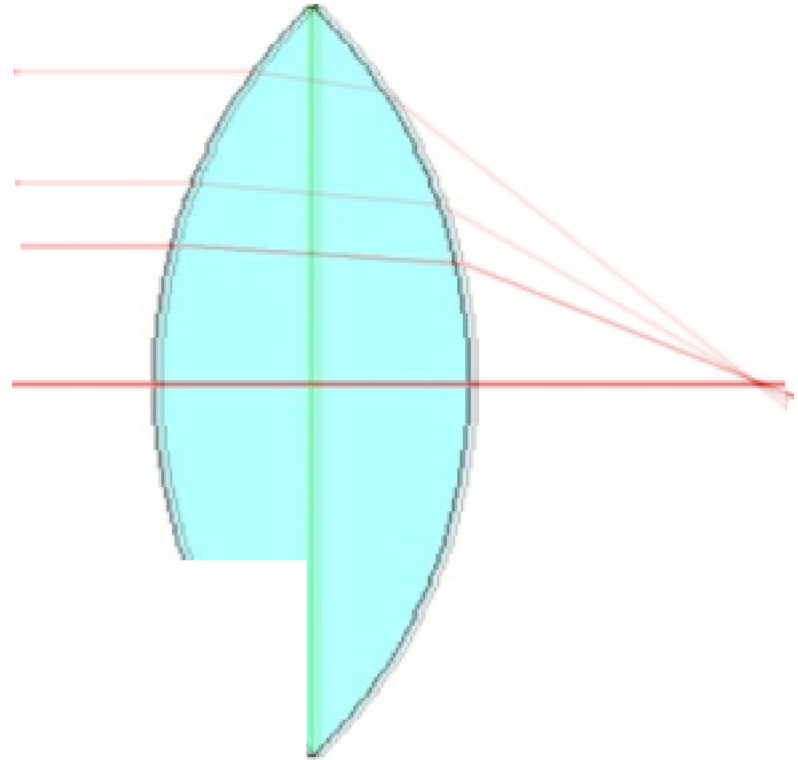


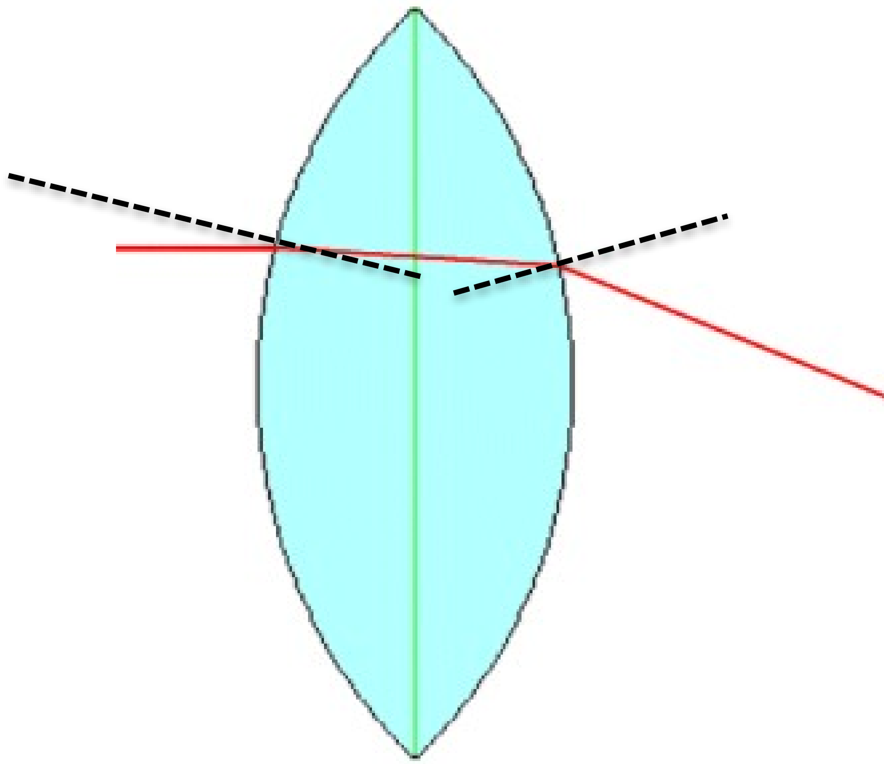
Converging lens



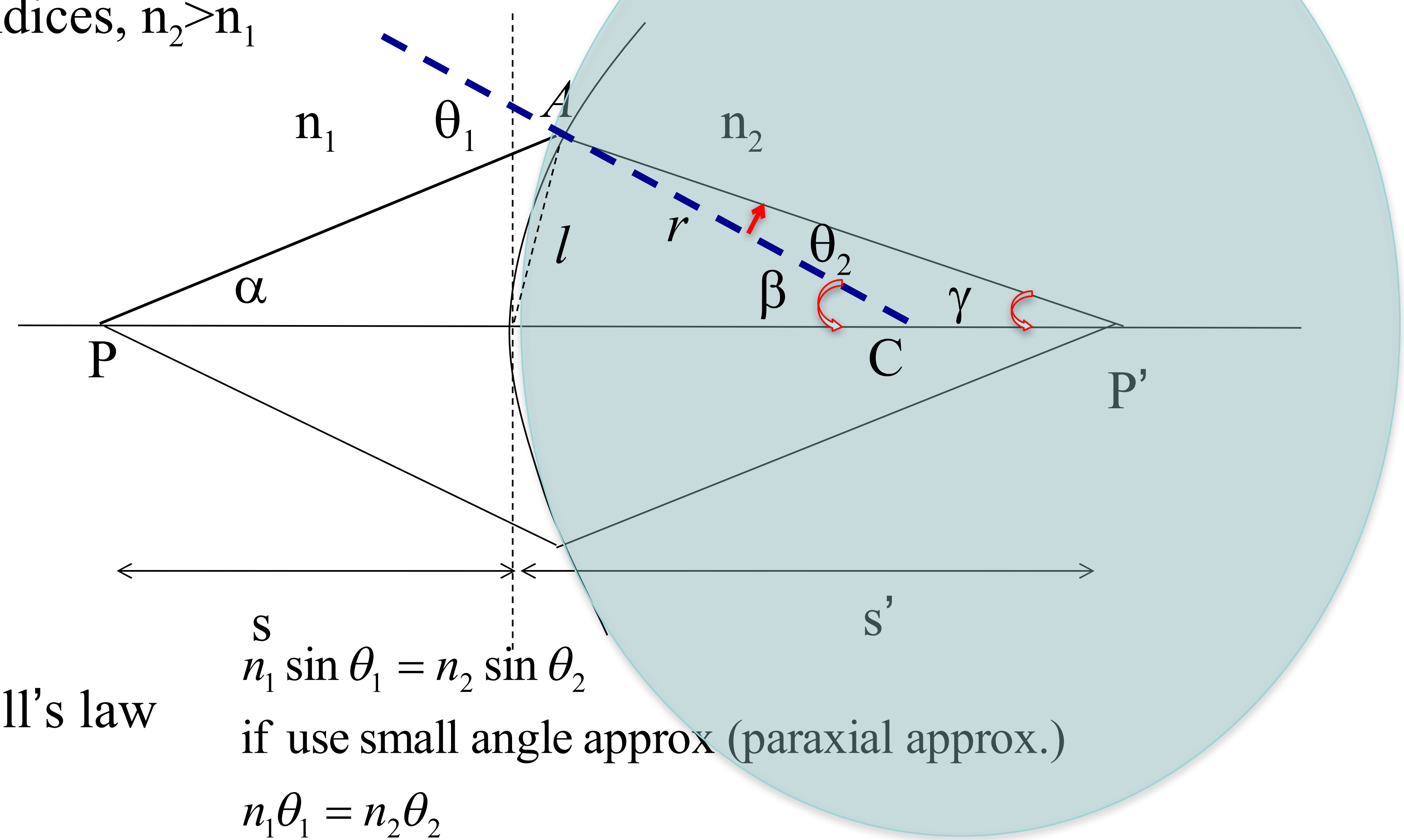
Diverging lens

Light ray is refracted twice when passing through a lens.





Images formed by refraction at a single surface
Y&F p 1126-1130
Consider a spherical surface separating two media of different refractive Indices, $n_2 > n_1$



Snell's law

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

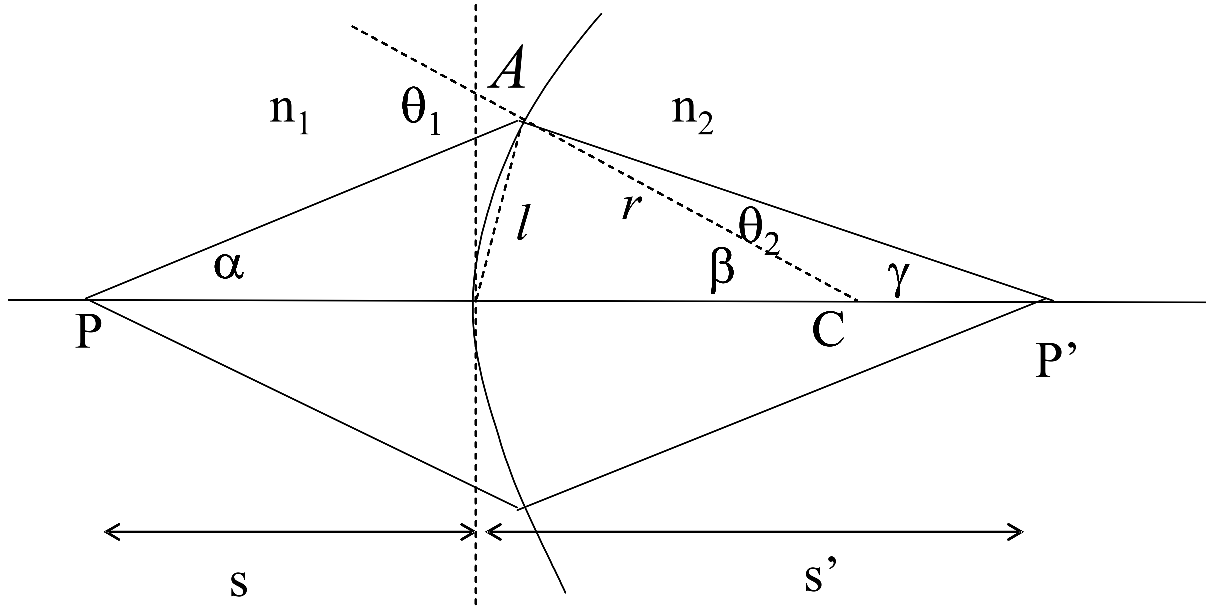
if use small angle approx (paraxial approx.)

$$n_1\theta_1 = n_2\theta_2$$

$From\ \Delta ACP' \quad \beta = \theta_2 + \gamma = \left(\frac{n_1}{n_2}\theta_1\right) + \gamma$ $\qquad From\ \Delta PAC \quad \theta_1 = \alpha + \beta$

$sub\ for\ \theta_1, \quad \beta = \frac{n_1}{n_2}(\alpha + \beta) + \gamma$ $\qquad n_2\beta = n_1\alpha + n_1\beta + n_2\gamma$

$$(n_2 - n_1)\beta = n_1\alpha + n_2\gamma$$



For small angles.

$$\alpha = \frac{l}{s}, \quad \beta = \frac{l}{r}, \quad \gamma = \frac{l}{s'}$$

$$(n_2 - n_1)\frac{l}{r} = n_1\frac{l}{s} + n_2\frac{l}{s'}$$

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{(n_2 - n_1)}{r}$$

If r=∞, $\quad \frac{n_1}{s} + \frac{n_2}{s'} = 0, \quad s' = -\frac{n_2}{n_1}s$

Sign convention for refracting surfaces:

1) Radius of curvature: positive if centre of curvature on the same side of the outgoing ray. (convex towards object); otherwise, it is negative (concave towards object).

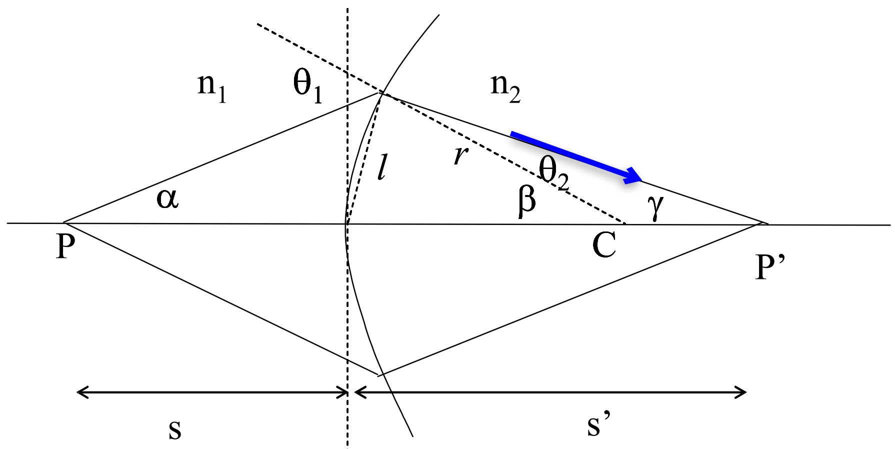
2) s’: Positive if image is formed on the same side of the outgoing ray. (Real image); otherwise, it is negative.

3) s: Positive if object on the same side of the incoming light.

Real images form on the side of a refracting surface that is opposite the object, and virtual images form on the same side of the object.

C on the same side of outgoing ray, r positive.

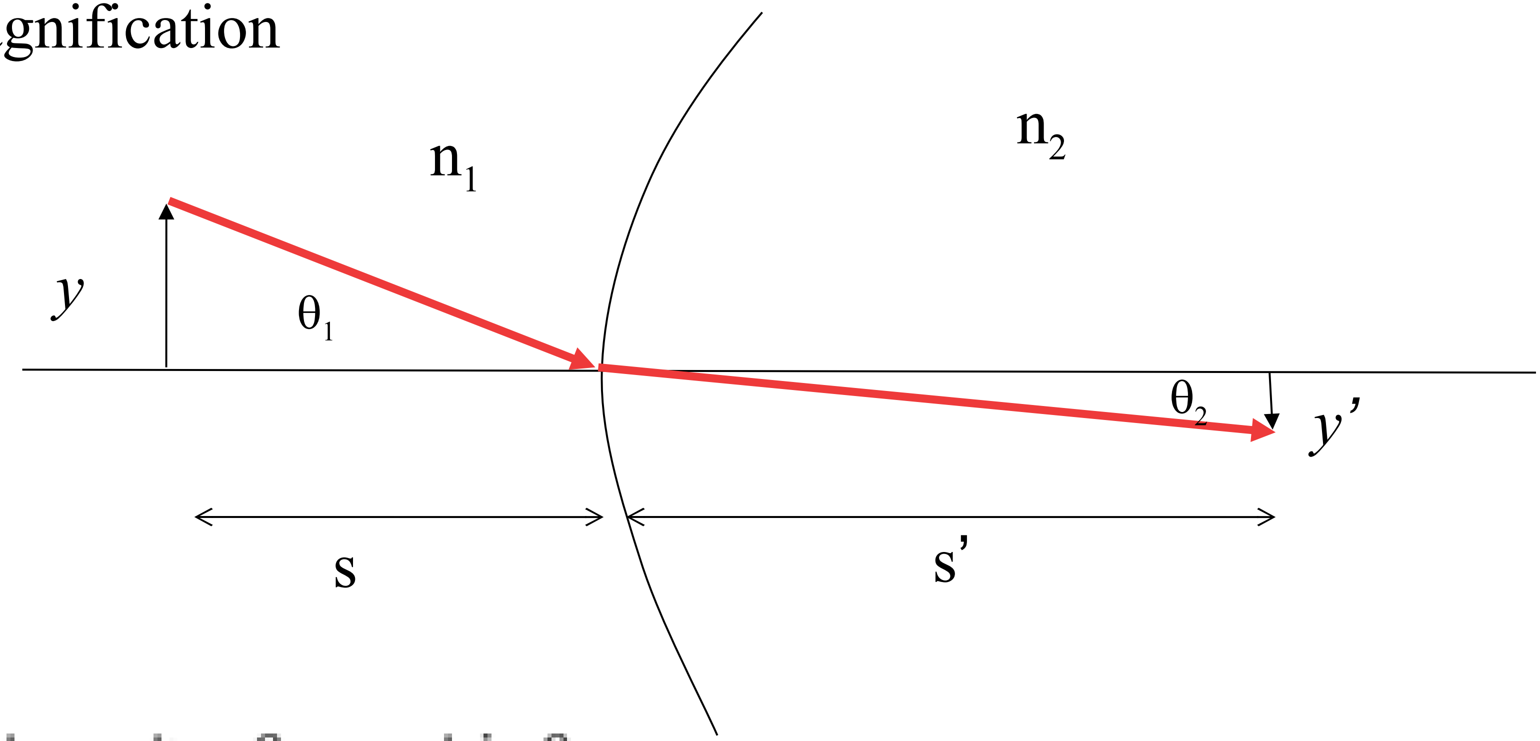
P' on the same side of outgoing ray, S' positive.



S on the same side of incoming ray, s positive.

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{(n_2 - n_1)}{r}$$

Magnification



$$m = \frac{y'}{y} = \frac{-s'\tan\theta_2}{s\tan\theta_1} \approx \frac{-s'\sin\theta_2}{s\sin\theta_1}$$

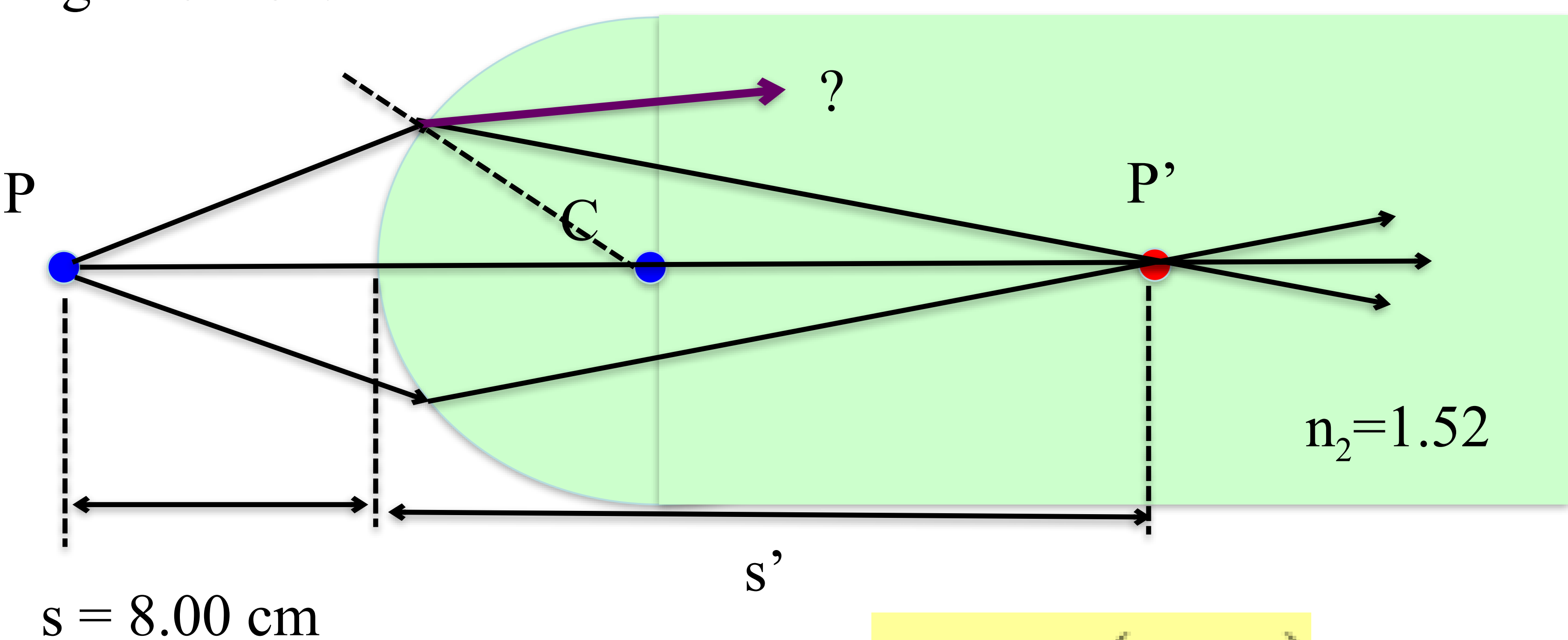
$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

i.e.

$$m = -\frac{n_1 s'}{n_2 s}$$

Example. Y&F p1129.

A cylindrical glass rod has index of refraction 1.52. It is surrounded by air. One end is grounded to a hemispherical surface with radius $r = 2.\,00$ cm. A small object is placed on the axis of the rod, 8.00 cm to the left of the vertex. Find the image distance and the lateral magnification.

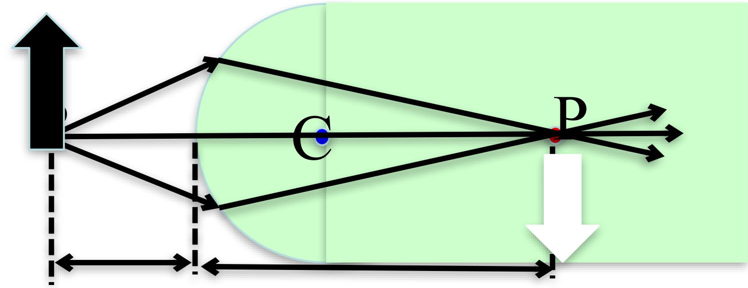


Solution, use

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{(n_2 - n_1)}{r}$$

$$\frac{n_1}{s}+\frac{n_2}{s'}=\frac{(n_2-n_1)}{r}$$

$n_1 = 1$, $n_2 = 1.52$,
$r = +2$cm, $s=+8$ cm



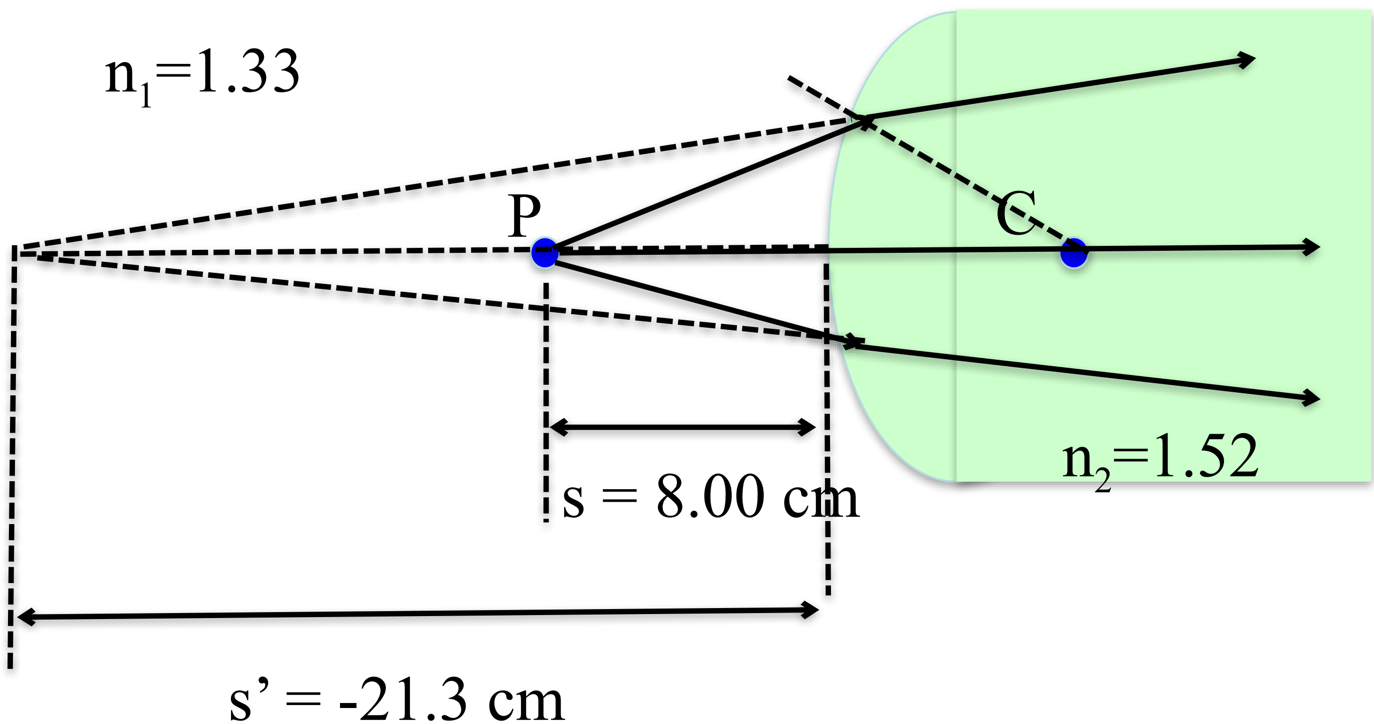
$$\frac{1}{8}+\frac{1.52}{s'}=\frac{(1.52-1)}{2}$$

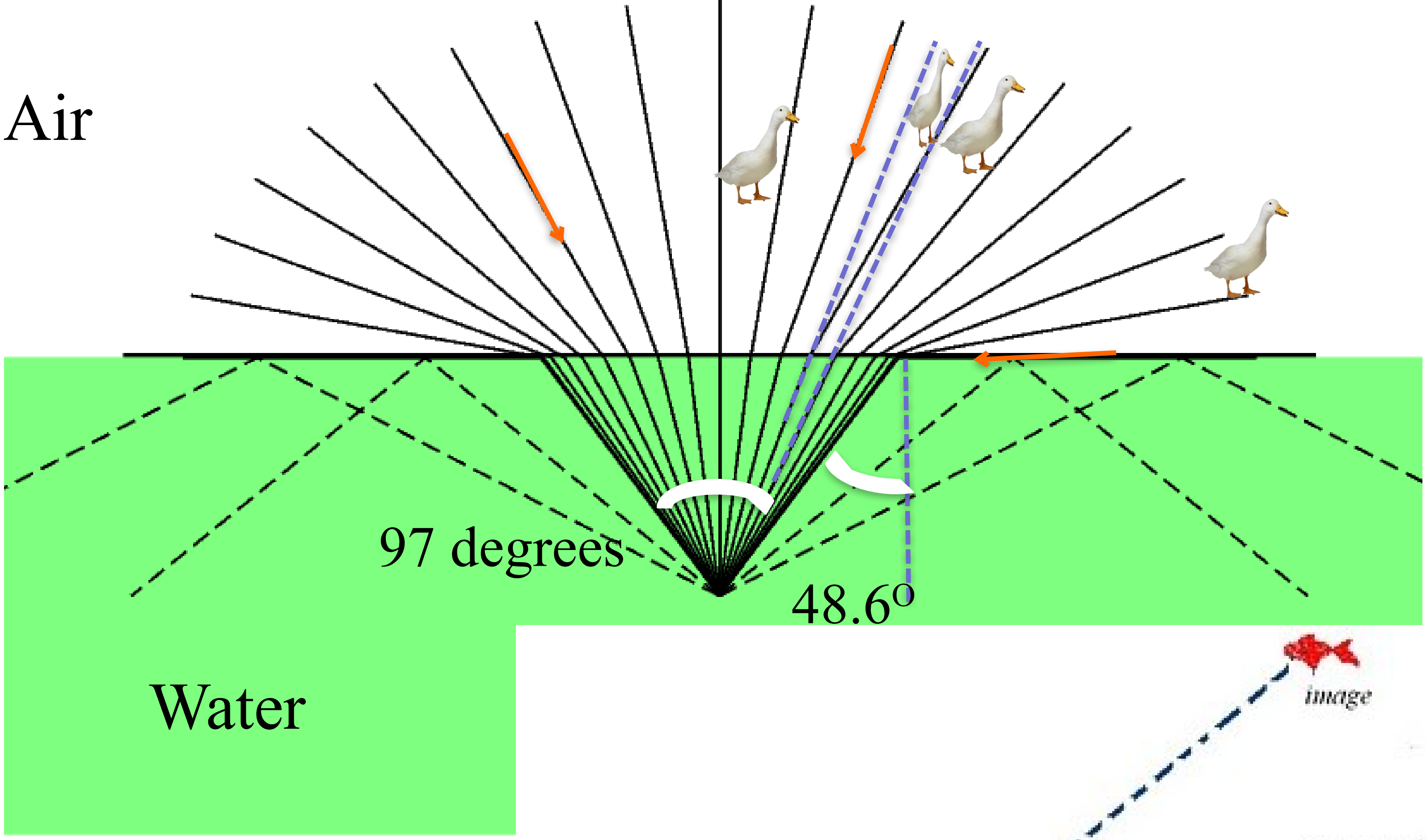
$s' = +11.3$ cm

$$m=-\frac{n_1 s'}{n_2 s}=-\frac{(1)(11.3)}{(1.52)(8)}=-0.929.$$

Because the image distance s’ is positive, the image is formed 11.3 cm to the right of the vertex (side of the outgoing ray). *m* is negative so image is inverted. Image is also smaller than object.

If the glass rod is placed in water with a refractive index n_1=1.33, then s'= –21.3 cm, a virtual image is formed 21.3 cm to the LEFT of the vertex.




Air
97 degrees
48.6°
Water

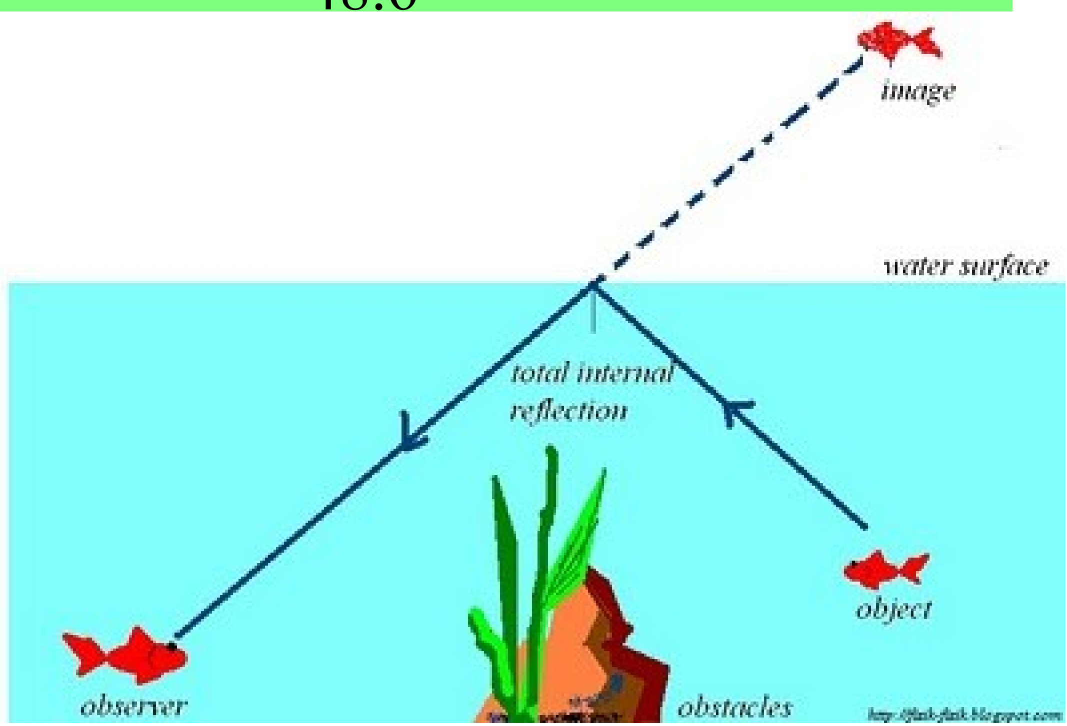
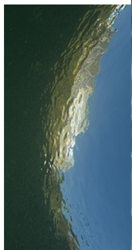
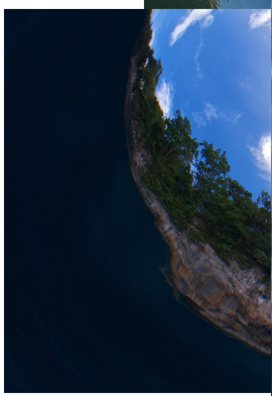
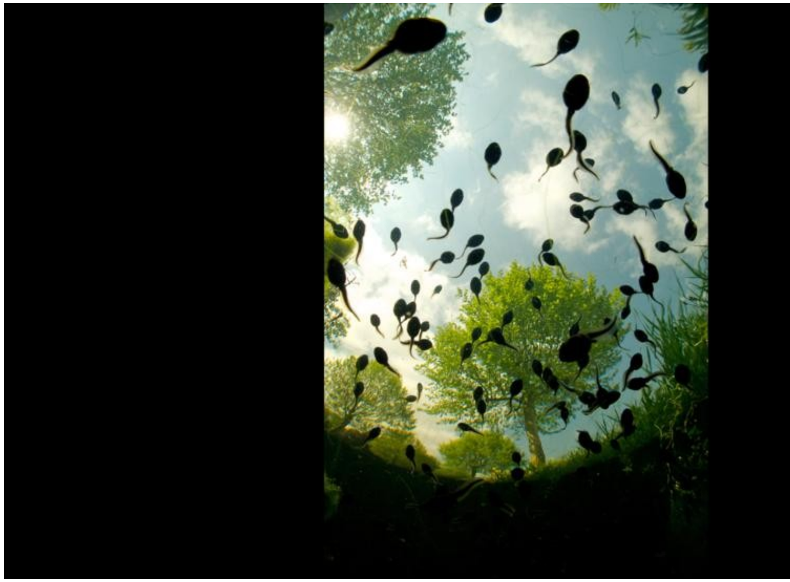


image
water surface
total internal
reflection
object
observer
obstacles

Snell’s window







[Royal Society Publishing photography competition.]

Tadpoles overhead, by Bert Willaert, Belgium ， Nov. 2015