Electromagnetism

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Lecture 15

Electromagnetic Induction

Week 8

Last Lecture

- B-field and E-field from dipoles

- Ampere's Law: $\oint \underline{\boldsymbol{B}} \cdot d\underline{\boldsymbol{l}} = \mu_0 I$
  - B-fields inside and outside current carrying wires
  - B-fields inside solenoids
  - B-field from Toroidal Solenoid

- Force between two long parallel currents

Summary of Magetostatics

$$\boxed{\underline{\boldsymbol{F}}_m = q\underline{\boldsymbol{v}}\wedge\underline{\boldsymbol{B}}}$$

$$\boxed{\underline{\boldsymbol{F}} = I\,\underline{\boldsymbol{l}}\wedge\underline{\boldsymbol{B}}}$$

$$\underline{\mu} = I\underline{A}$$

$$U = -\underline{\boldsymbol{\mu}}\cdot\underline{\boldsymbol{B}}$$

$$\underline{\boldsymbol{\tau}} = \underline{\boldsymbol{\mu}}\wedge\underline{\boldsymbol{B}}$$

$$\underline{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{q}{r^2}\underline{\boldsymbol{v}}\wedge\underline{\hat{\boldsymbol{r}}}$$

$$\phi_m = \int_S \underline{\boldsymbol{B}}\cdot d\underline{\boldsymbol{S}}$$

$$\boxed{\delta\underline{\boldsymbol{B}} = \frac{\mu_0}{4\pi}\frac{I\,\delta\underline{\boldsymbol{l}}\wedge\underline{\hat{\boldsymbol{r}}}}{r^2}}$$

$$\boxed{\oint \underline{B}\cdot d\underline{l} = \mu_0 I_{enc}}$$

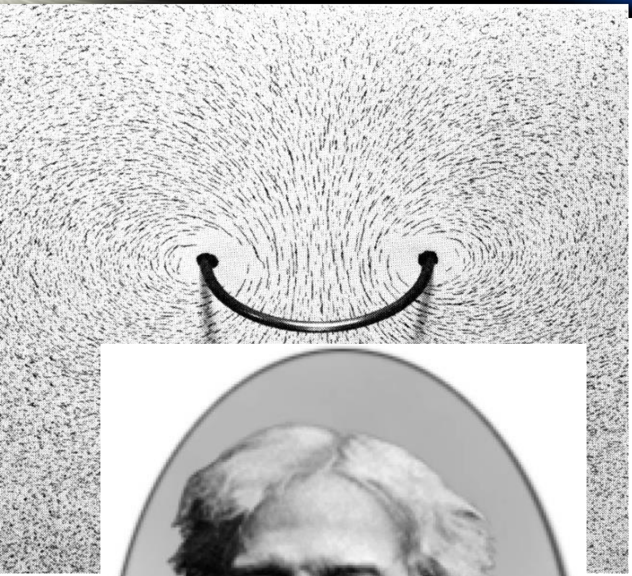
This Lecture

- Magnetic Inductance
  - Motion of conductor in B-field
  - Induced voltage (e.m.f)
  - Lenz’s Law (polarity of induced voltage)
- Induced E-fields
- Faraday’s Law

B-field from Current (Last lecture)

- Hans Christian Oersted
- (1777-1851)
- In 1820 Oersted demonstrated that a magnetic field exists near a current-carrying wire - first connection between electric and magnetic phenomena.

Michael Faraday



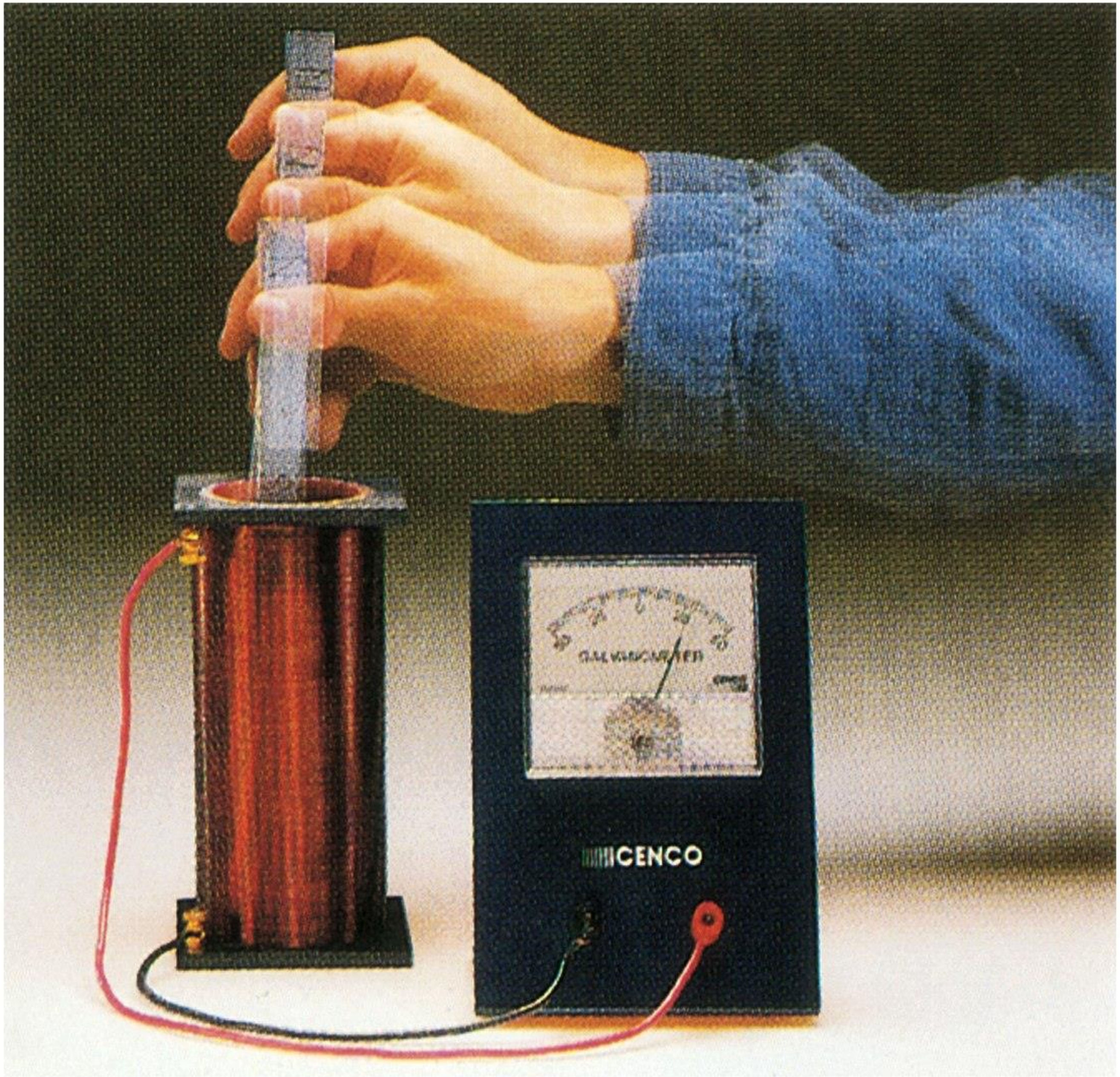
- So an electric current generates a magnetic field.

- Faraday thought that the reverse might be possible – to generate electricity from magnetism.

Michael Faraday 1791 - 1867

- Faraday discovered that whenever magnetic field lines move or change in anyway, they induce an electric field. This kind of electric field exerts the usual forces on charges .

- The unit for capacitance was named after Faraday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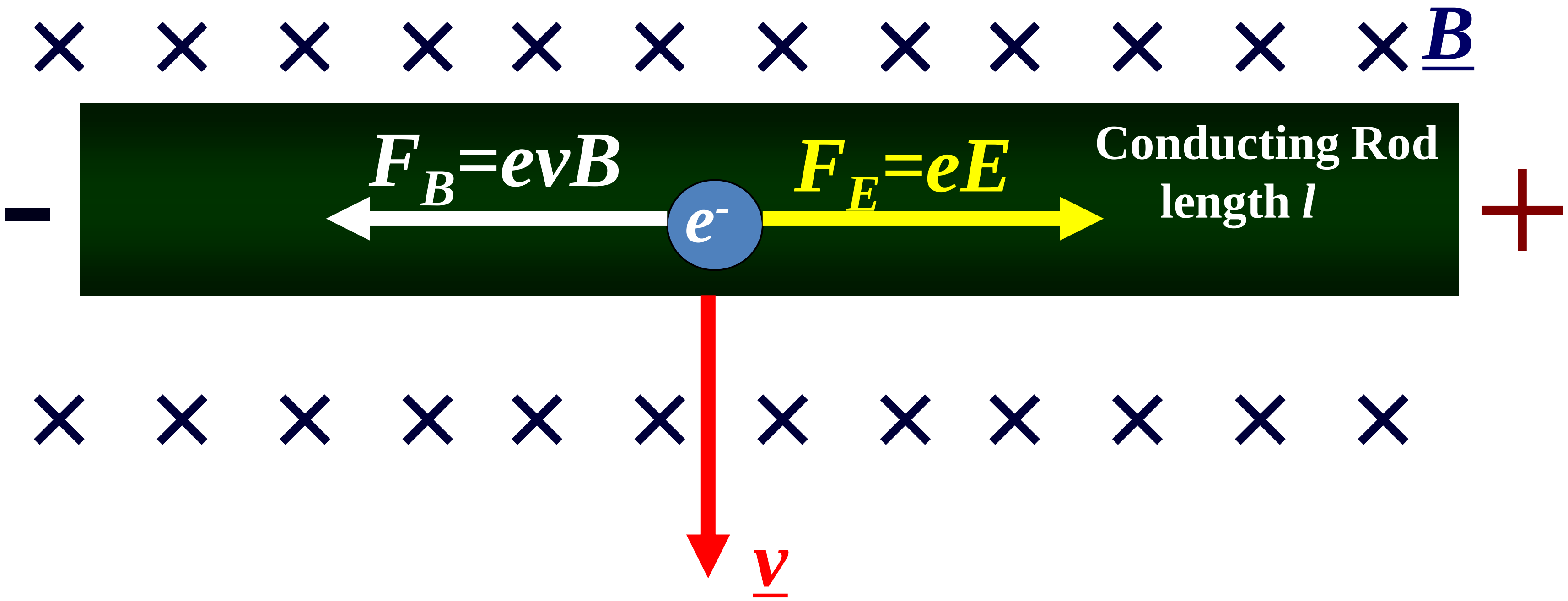
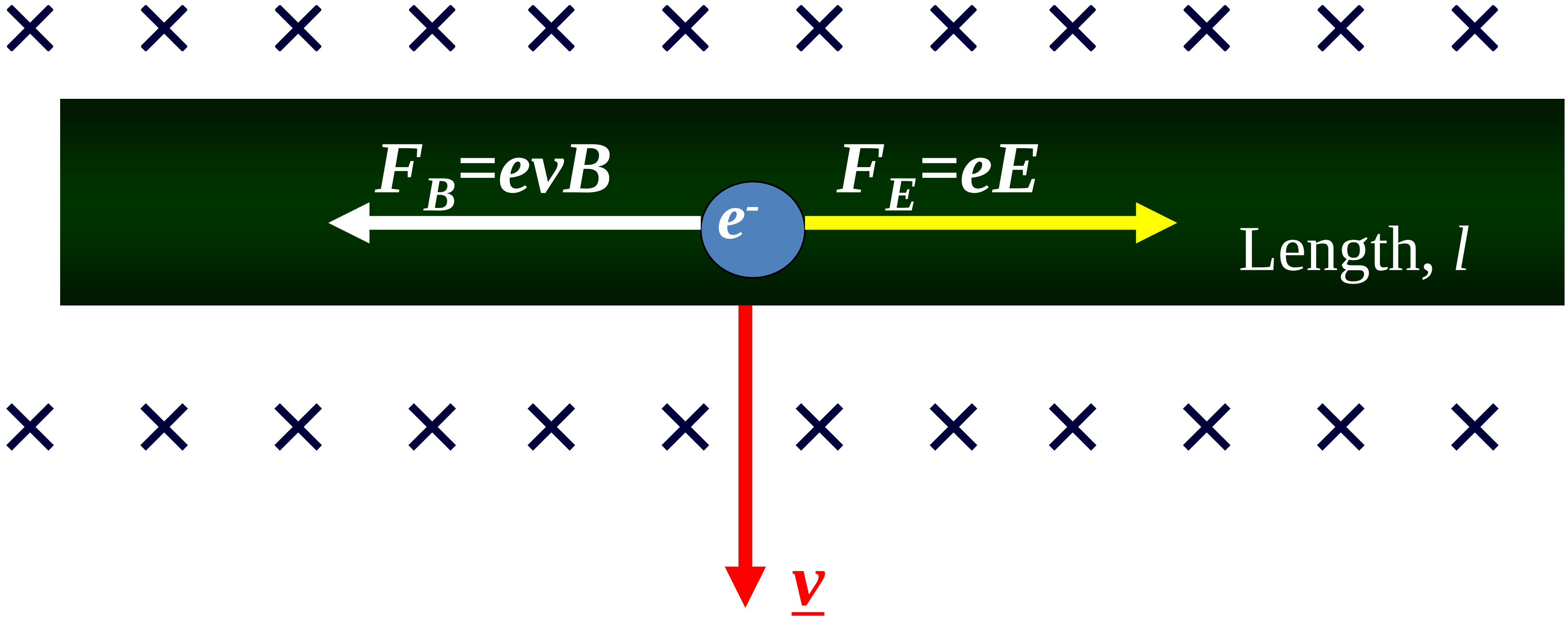
The Motion of a Conductor in a B Field



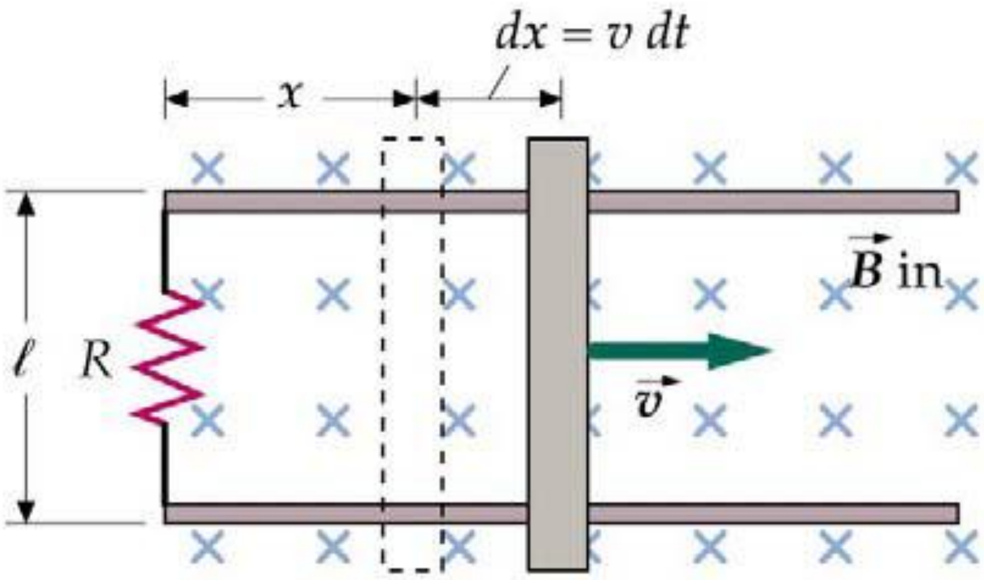
- Electrons accumulate until $F_E = F_B$
- Get an induced voltage ε across the ends of the conductor

The Motion of a Conductor in a B Field



- In equilibrium, $E = vB$
- Potential difference induced, $\boxed{\boldsymbol{\varepsilon = El = vBl}}$

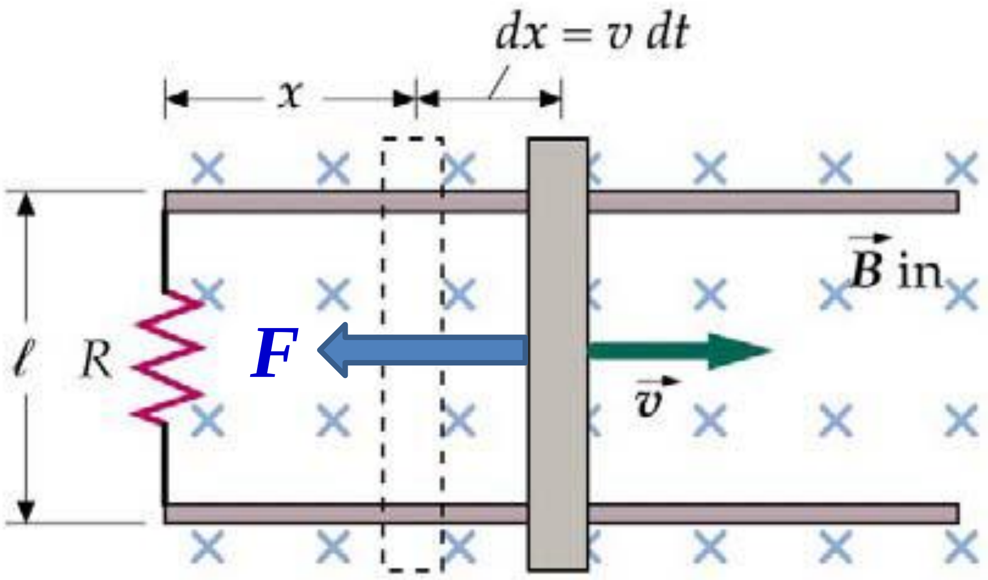
Sliding Rod in B-Field



The ends of the rod are in sliding contact with a pair of wires, a current will flow around the circuit

The moving rod has become a source of electrical energy

Sliding Rod in B-Field

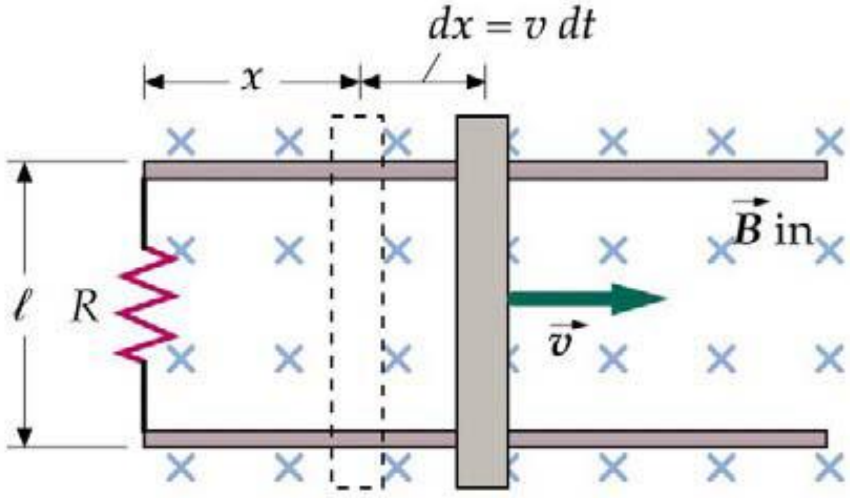


Force on rod is:

$F = BIl$

To maintain motion at constant speed a force of equal magnitude must be applied in the direction of $\underline{v}$ - Rate of work = $Fv = BIlv$

Sliding Rod in B-Field



Rate of work done by electrical energy = εI

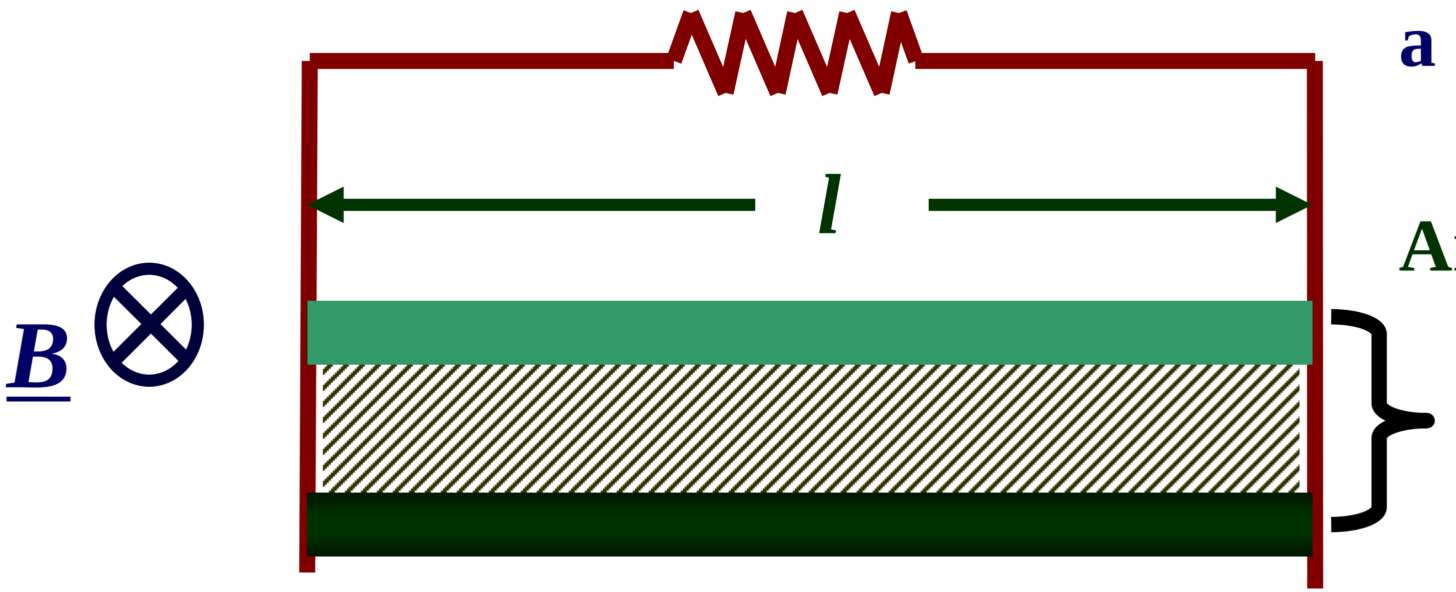
Conservation of Energy $BIlv = \varepsilon I$

Induced voltage $\boldsymbol{\varepsilon = Blv}$

Note Induce voltage called *Electro-motive force, e.m.f.*

Sliding Rod in B-Field

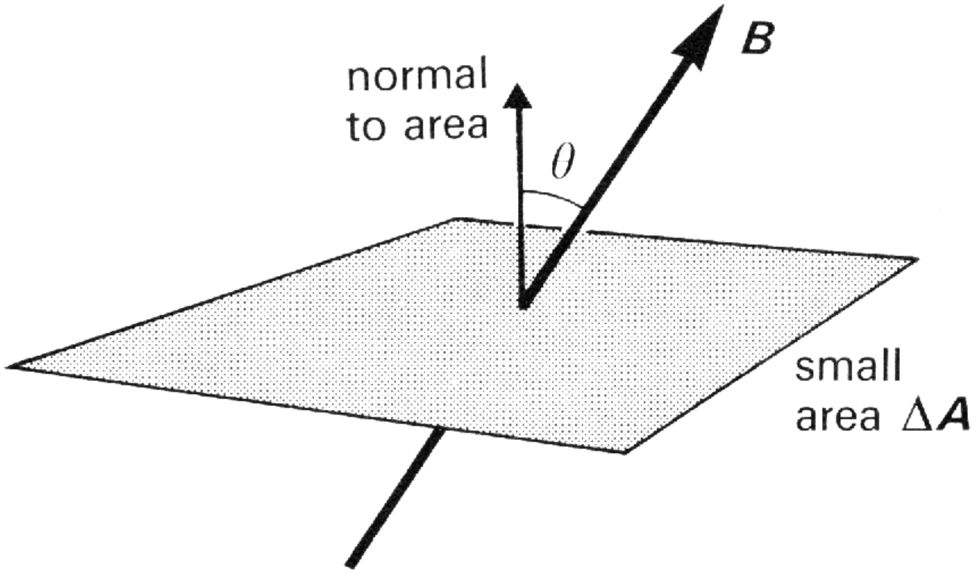
This can be given an interesting interpretation in terms of the MAGNETIC FLUX



In a time δt the rod travels a distance $v\delta t$

Area Change

Reminder: Magnetic Flux Φ_m



The magnetic flux $\Delta\Phi_m$ passing through the small area ΔA shown is defined by:

$$\Delta\Phi_m = B\cos\theta \times \Delta A$$

$$\Phi_m = \int_S \underline{B} \cdot d\underline{S}$$

Induced Voltage in terms of Magnetic Flux

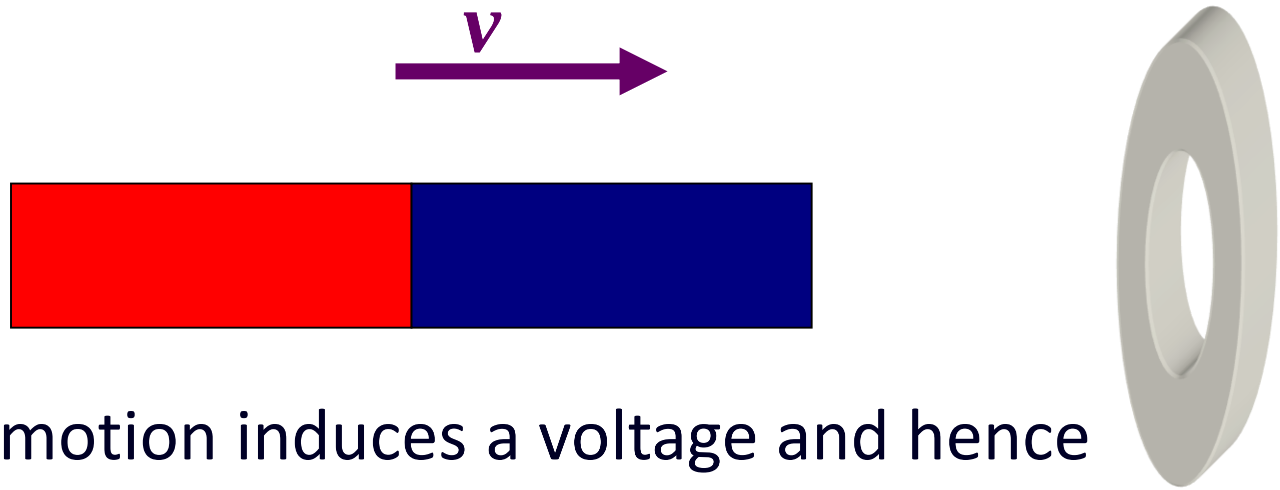
- In a time δt the conductor sweeps out an area $lv\delta t$. The flux change in time δt is:

$$\delta\phi_m = Blv\,\delta t \qquad \frac{d\phi_m}{dt} = Blv$$

But induced voltage (e.m.f), $\varepsilon = Blv$

Magnitude of induced voltage $$\varepsilon = \frac{d\phi_m}{dt}$$

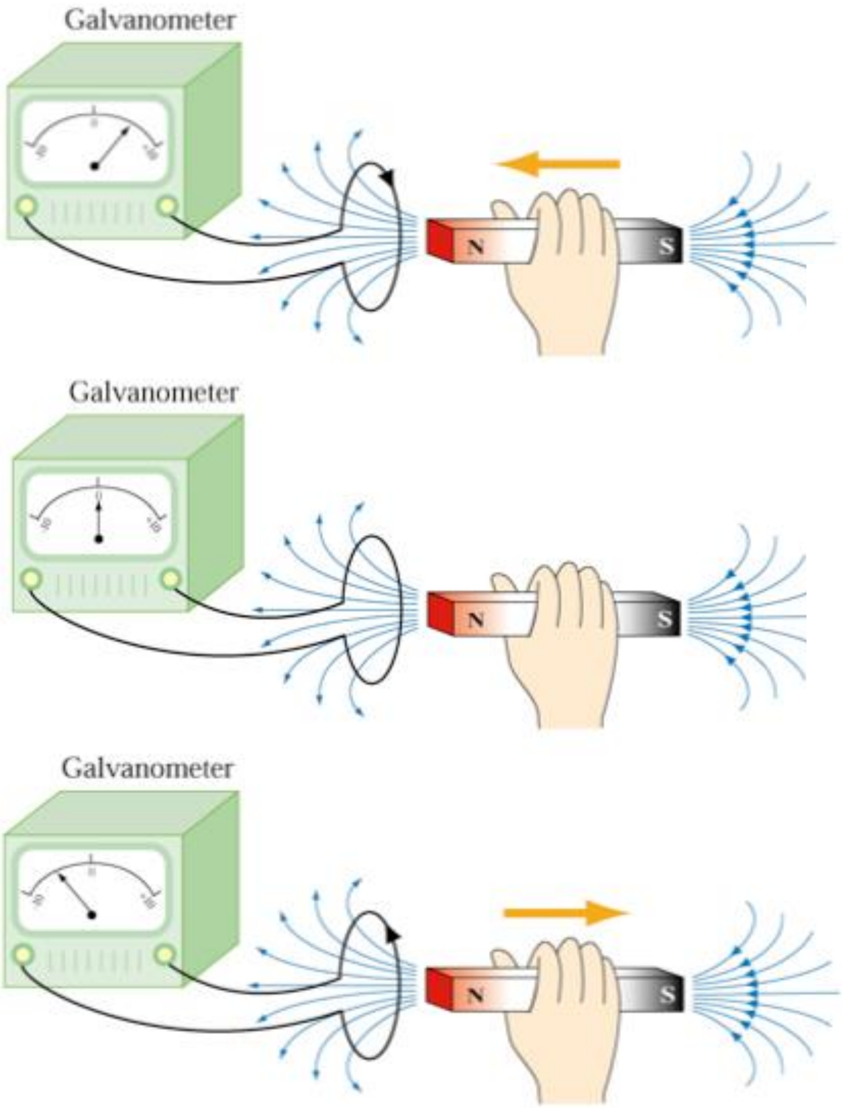
The Polarity of an Induced Voltage

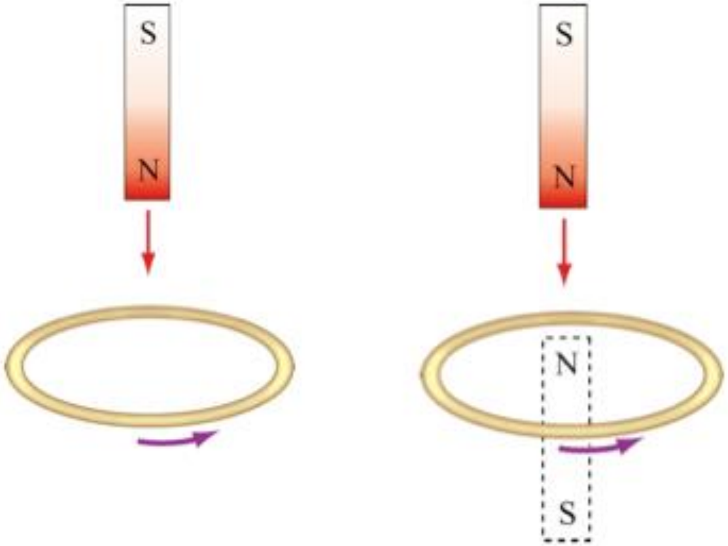


- The motion induces a voltage and hence a current in the metal ring. The current produces a magnetic field so that the ring behaves like a bar magnet.

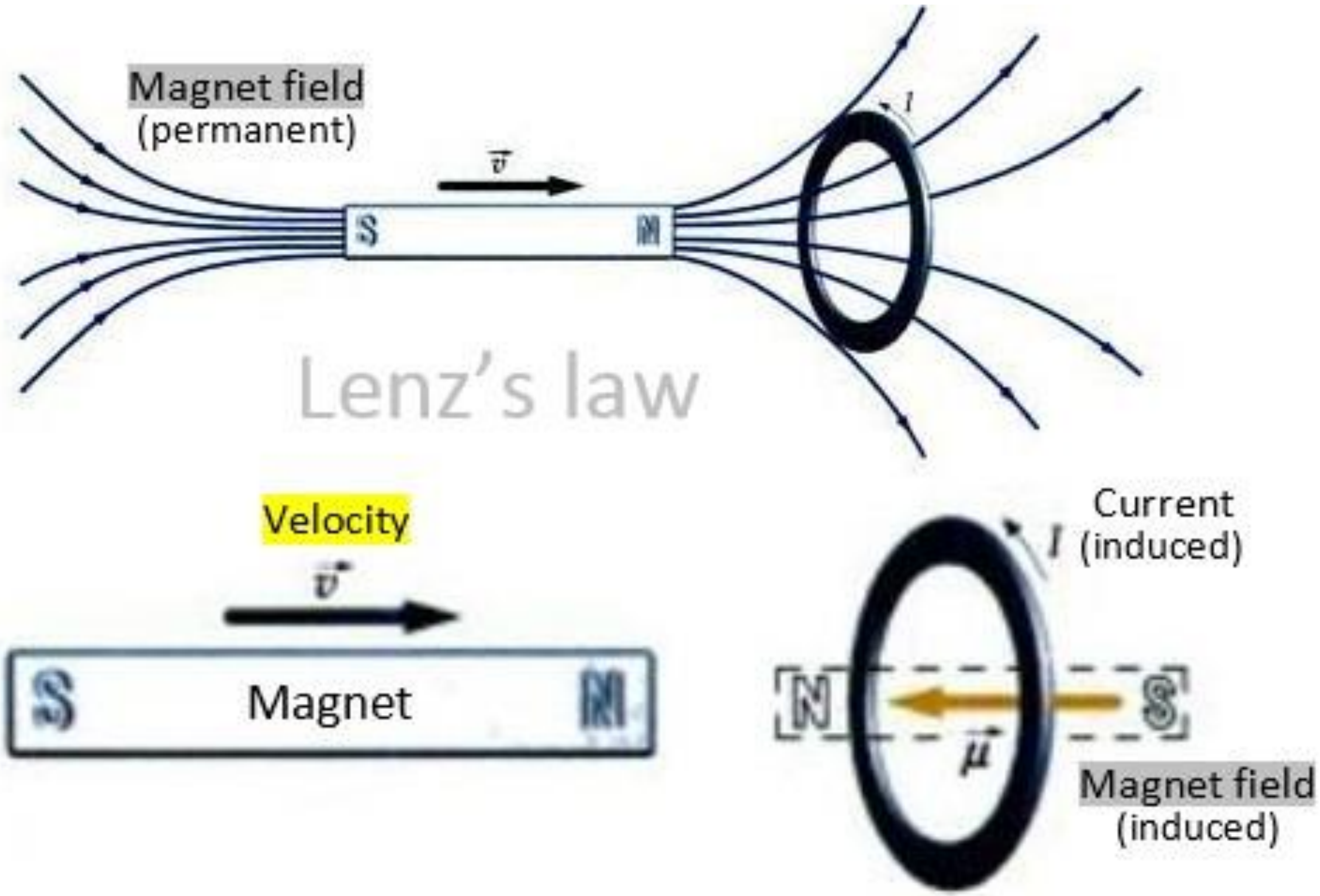
Question: The B-field from the induced current repels the approaching magnet, Yes or no?

The Polarity of an Induced Voltage





The Polarity of an Induced Voltage



Lenz's Law

- *"The direction of an induced current (if one were to flow) is such that its effect would oppose the change in magnetic flux which give rise to the current."*
- **It's all to do with the Conservation of Energy.**
- The magnetic field generated can only hinder the motion. Helping the motion would result in the creation of a perpetual motion machine, which violates the conservation of energy.

Mathematically

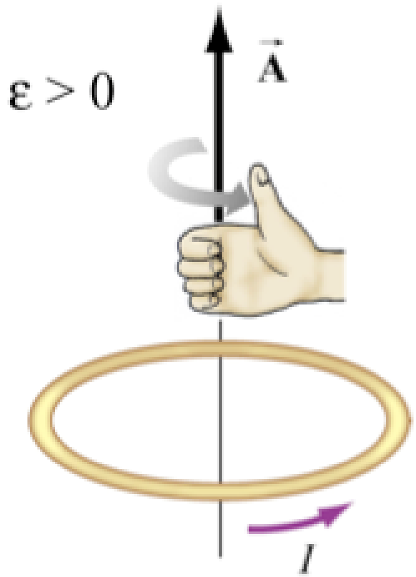
- Induced voltage (e.m.f):

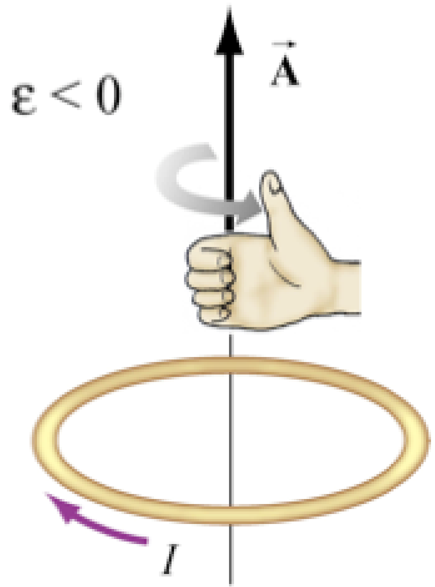
$$\boxed{\varepsilon = -\frac{d\Phi_m}{dt}}$$

Induced Voltage

1. Define positive direction of area vector A
2. (Assuming B is uniform) take the dot product of B and A
3. Obtain the rate of B-flux change
4. Determine the direction of the current using the right hand rule

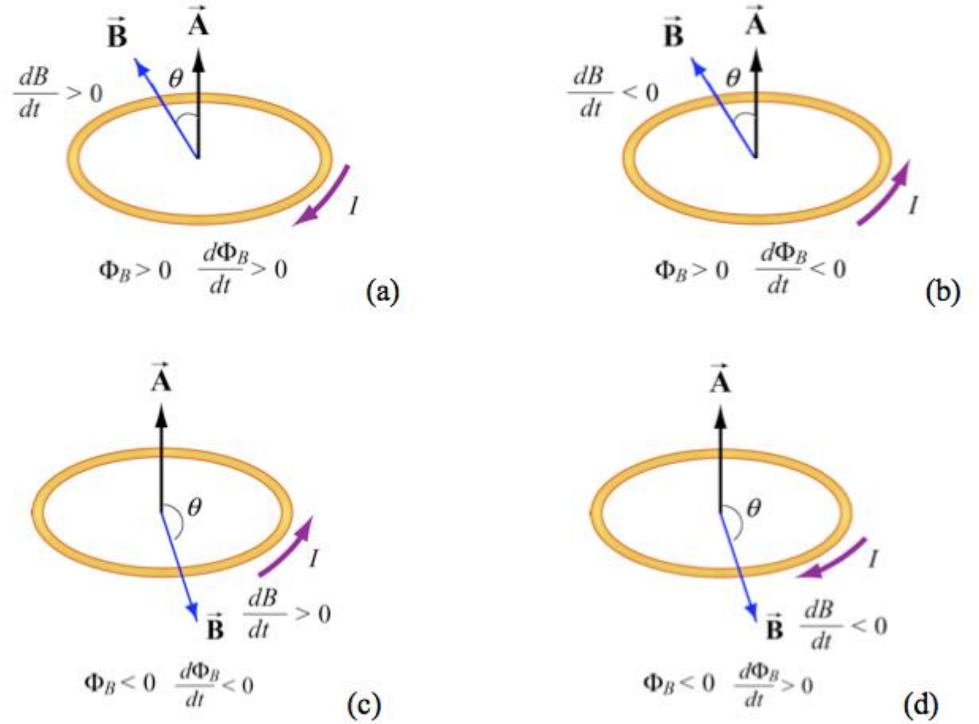
$$\frac{d\Phi_B}{dt}\begin{cases} >0 & \Rightarrow \text{ induced emf } \varepsilon < 0 \\ <0 & \Rightarrow \text{ induced emf } \varepsilon > 0 \\ =0 & \Rightarrow \text{ induced emf } \varepsilon = 0 \end{cases}.$$





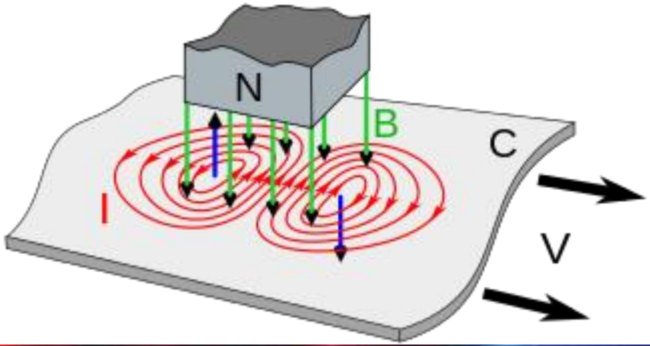
Sign of Induced Voltage

Φ_B	$d\Phi_B/dt$	ε	I
+	+	−	−
	−	+	+
−	+	−	−
	−	+	+



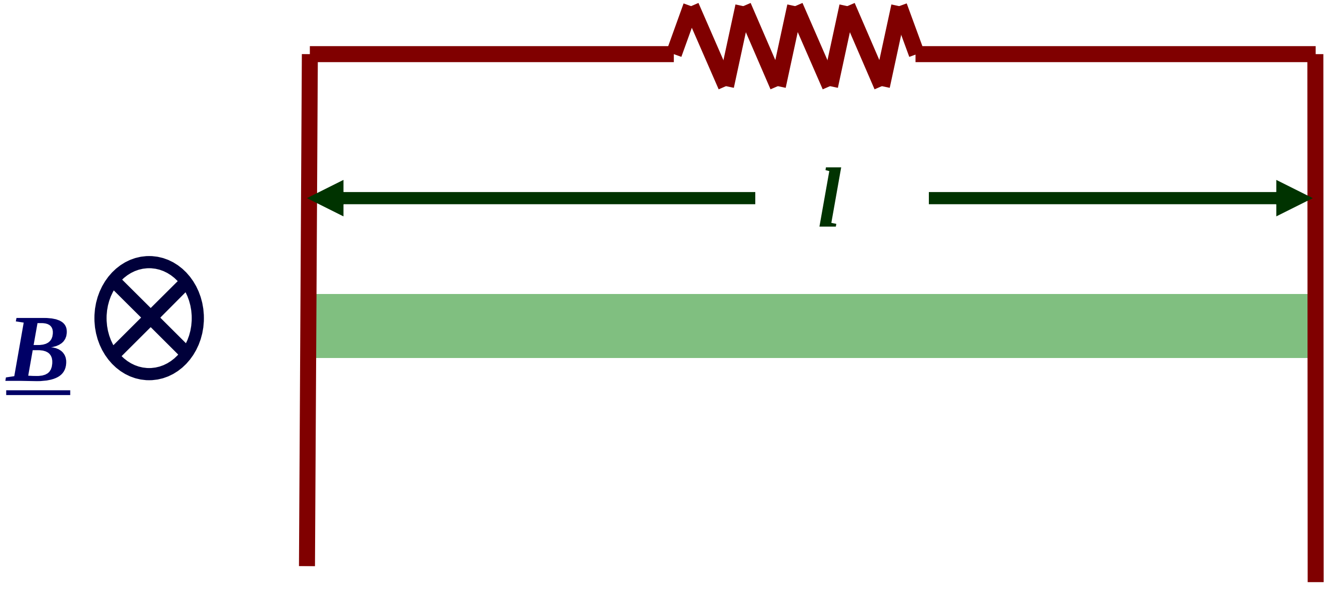
Aside: Eddy Currents

- When current is induced in a conductor such as the square piece of metal, the induced current often flows in small circles that are strongest at the surface and penetrate a short distance into the material.
- These current flow patterns are thought to resemble eddies in a stream. Because of this presumed resemblance, the electrical currents were named **eddy currents**.

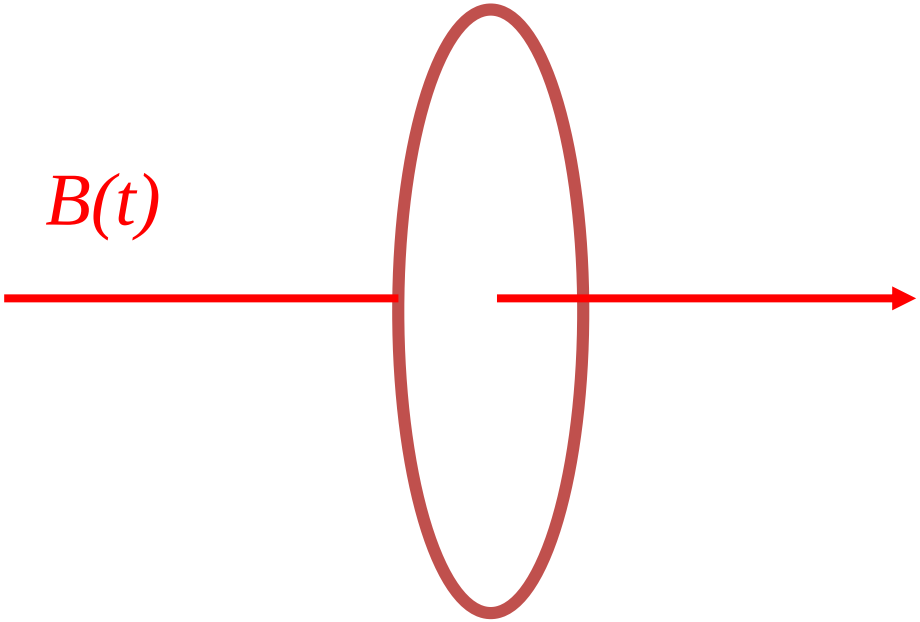


Change in Area or Field

- Note, that an induced voltage can be a result of a change in *area* or *magnetic field*, or *both*!



Example 15.1



$$B(t) = B_0 \cos \omega t$$

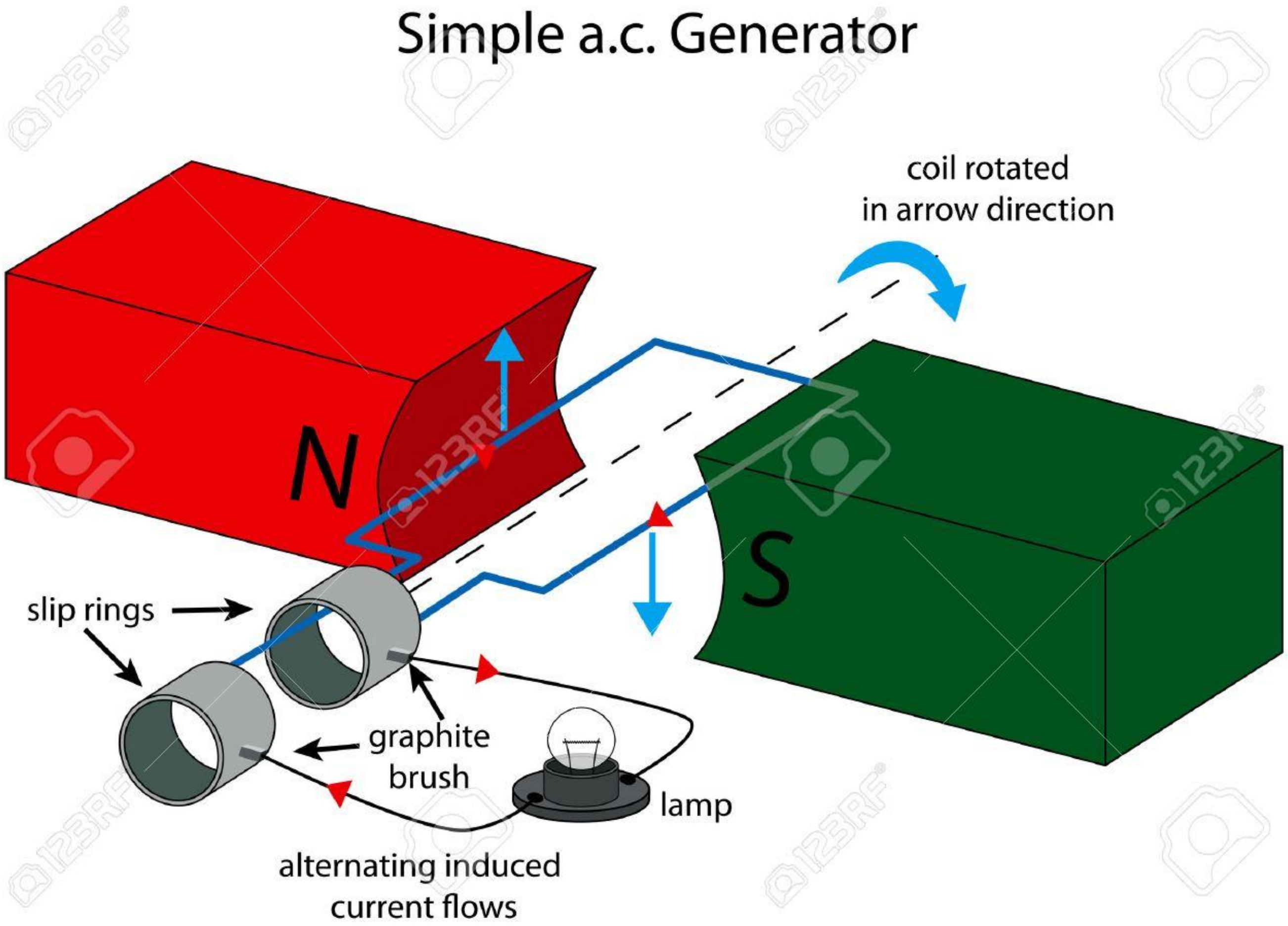
Area = πr^2

$$\Phi_m = \pi r^2 B_0 \cos \omega t$$

$$\varepsilon = -\frac{d\Phi_m}{dt} = -(-\pi r^2 \omega B_0 \sin \omega t) = \pi r^2 \omega B_0 \sin \omega t$$

If loop is connected to a circuit of resistance R, what is the current flowing through the loop?
Could also have fixed B-field and changing area e.g. dynamo

Simple A.C. Generator

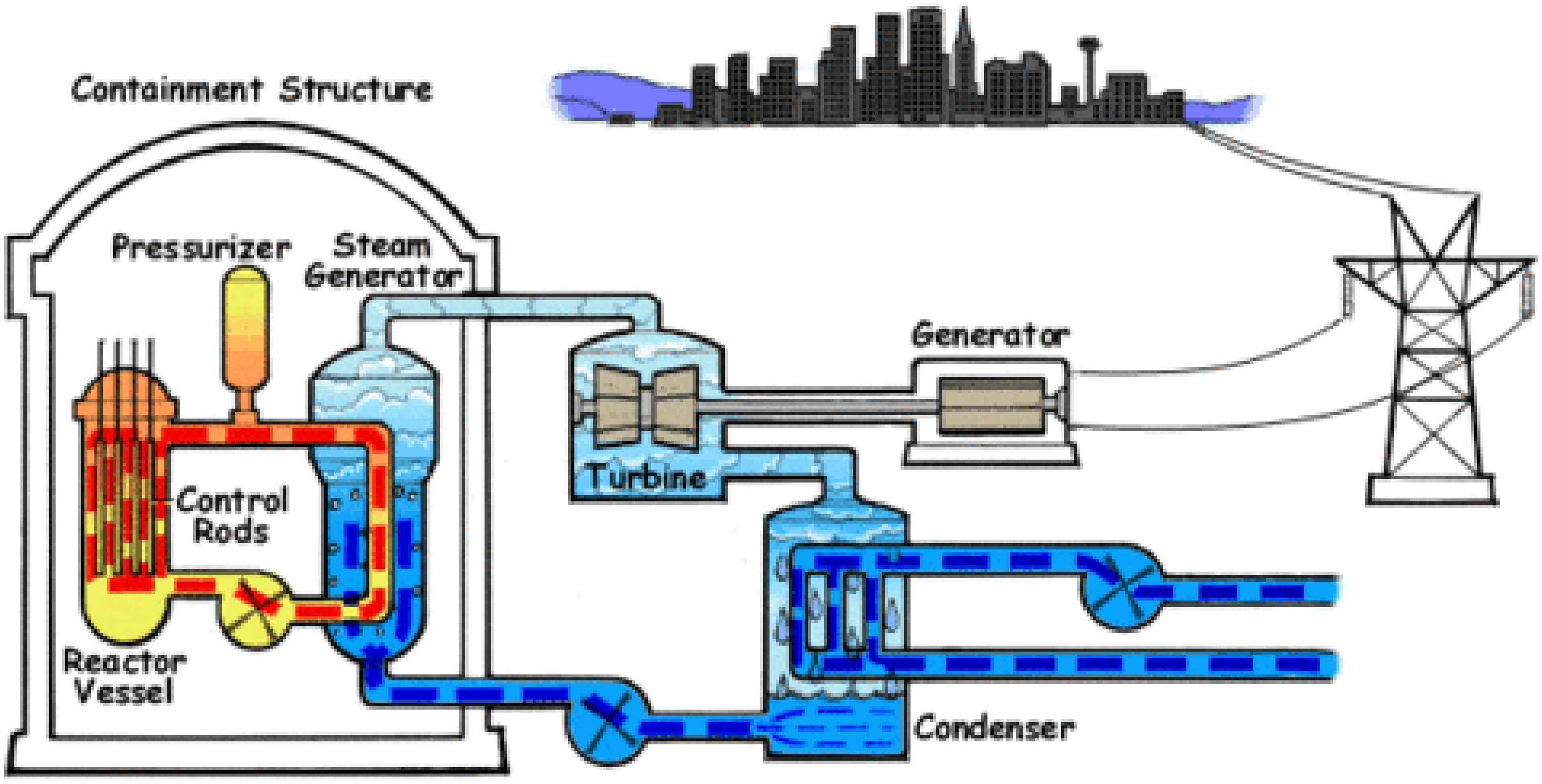


Major Application of Inductance

Production of Electricity

Conversion of one form of energy (e.g. wind, gravitational, chemical, nuclear) to electric energy.

E.G. Nuclear Power



Induced E-field

- If a changing magnetic flux produces an induced voltage, it must also induce an electric field.
- Indeed it is this electric field that makes a current flow (current flows in direction of E-field).
- Induced E-field is related to the induced voltage by: $\varepsilon = \oint \underline{\boldsymbol{E}} \cdot d\underline{\boldsymbol{l}}$
  - Positive as e.m.f. equals work done <u>by</u> the E-field in moving unit charge around circuit.

E-fields from Changing Magnetic fields

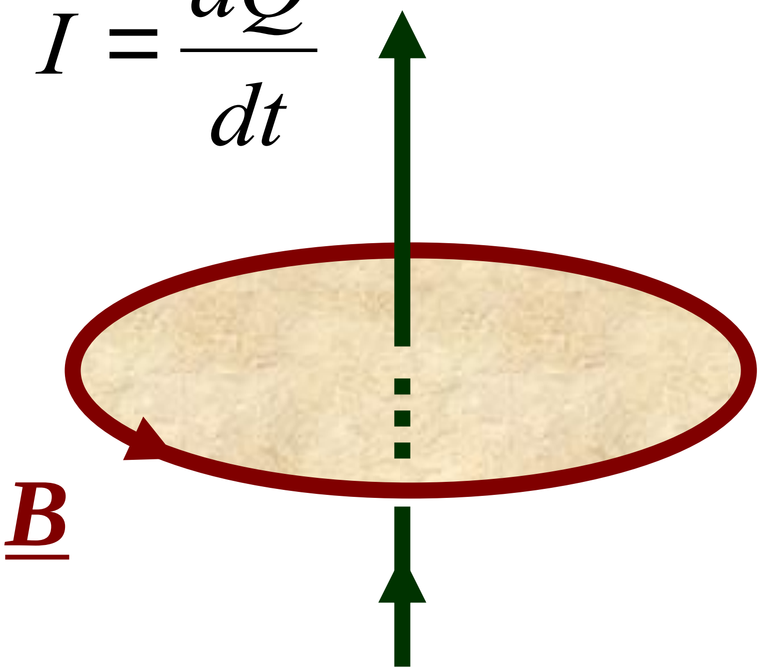
- Suppose we remove the conducting ring, and change the magnetic field.
- Will there be an electric field induced?

YES

- E-field is generated whether there is a conducting loop or not.
- Putting a conducting loop in the E-field is what causes a current to flow.

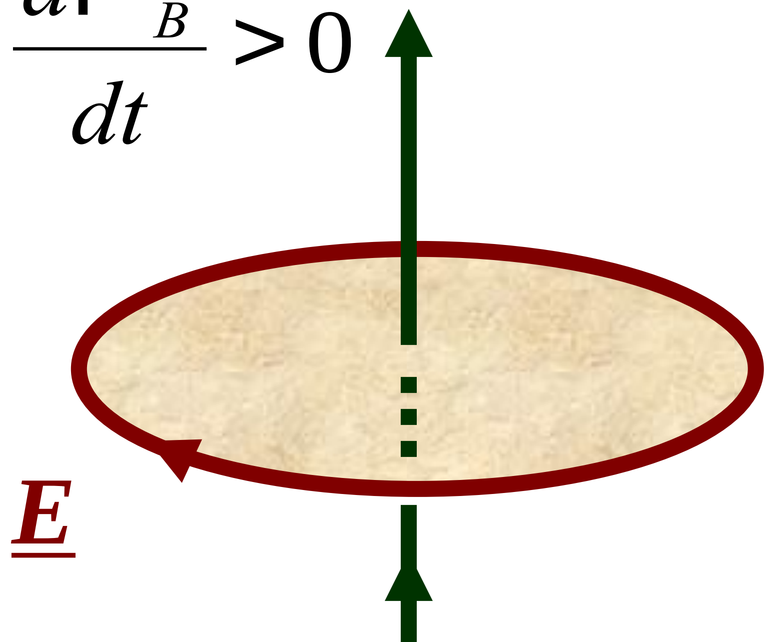
Induced E-field is Non-Conservative

$$I = \frac{dQ}{dt}$$



$$\varepsilon = \oint \underline{\boldsymbol{E}} \cdot d\underline{\boldsymbol{l}} \neq 0$$

$$\frac{dF_B}{dt} > 0$$



Electric field associated with an induced voltage. A non-conservative *E*-field.

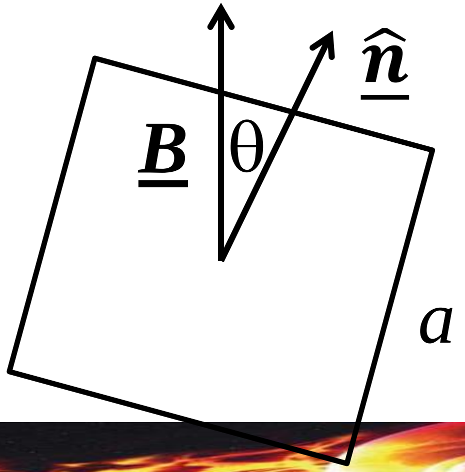
Faraday's Law of Electromagnetic Induction

$$\oint \underline{\boldsymbol{E}} \cdot d\underline{\boldsymbol{l}} = -\frac{d\Phi_m}{dt}$$

A time varying magnetic field induces a non-conservative electric field loop.

Exercise Time

- A magnetic field $B(t) = B_0 \cos \omega t$ is passing through a square coil, of side a and with N turns, at an angle θ to the normal of the plane of the coil.
- What is the e.m.f induced in the coil?
- If the coil were connected to a circuit of resistance R, what is the current flowing through the circuit?



N turns of coil

First Question

- A magnetic field $B(t) = B_0 \cos \omega t$ is passing through a square coil, of side a and with N turns, at an angle θ to the normal of the plane of the coil.
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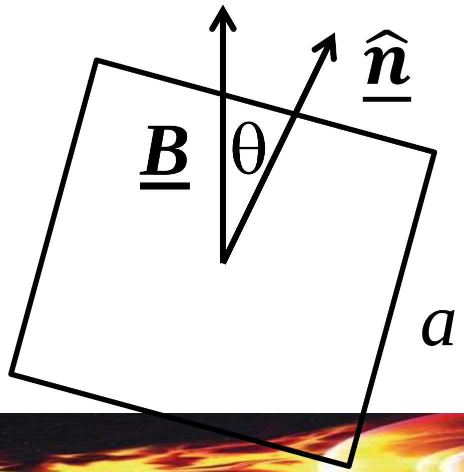

N turns of coil