Div, Grad, Curl and all that

Del operator

$$\nabla = \underline{i}\frac{\partial}{\partial x} + \underline{j}\frac{\partial}{\partial y} + \underline{k}\frac{\partial}{\partial z}$$

scaler ϕ

$$\underline{\text{grad}(\phi) = \nabla\phi = \underline{i}\frac{\partial\phi}{\partial x} + \underline{j}\frac{\partial\phi}{\partial y} + \underline{k}\frac{\partial\phi}{\partial z}}}$$

is a vector.

Vector $\underline{A}$

$$\underline{\text{div}(\underline{A}) = \nabla\cdot\underline{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}}$$

$$\text{curl}(\underline{A}) = \nabla\wedge\underline{A}$$

$$= \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$= \underline{i}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)$$

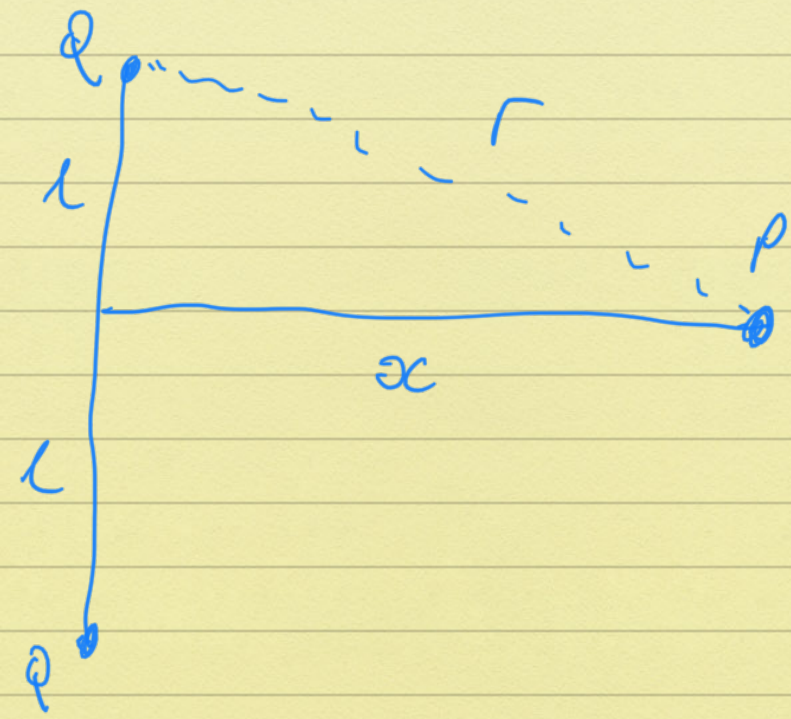
$$- \underline{j}\left(\frac{\partial A_z}{\partial x} - \frac{\partial A_x}{\partial z}\right)$$

$$+ \underline{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

For this course:

$$\vec{E} = -\nabla V$$

Example 6.1



① V at P ?

$$V = \frac{Q}{4\pi\varepsilon_0 r} + \frac{Q}{4\pi\varepsilon_0 r} = \frac{Q}{2\pi\varepsilon_0 r}$$

but $r = (l^2 + x^2)^{1/2}$

$$V = \frac{Q}{2\pi\varepsilon_0} \frac{1}{(l^2+x^2)^{1/2}}$$

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② Find $\underline{E}$ at P

$$\underline{E} = -\nabla V = -\frac{\partial V}{\partial x}\underline{i} - \frac{\partial V}{\partial y}\underline{j} - \frac{\partial V}{\partial z}\underline{k}$$

$$\frac{\partial V}{\partial y} = \frac{\partial V}{\partial z} = 0 \quad \therefore \underline{E} = -\frac{Q}{2\pi\varepsilon_0}\frac{d}{dx}(l^2+x^2)^{-\frac{1}{2}}\underline{i}$$

$$= -\frac{Q}{2\pi\varepsilon_0}\left(-\frac{1}{2}\cdot 2x(l^2+x^2)^{-3/2}\right)\underline{i}$$