Year 1 Assessed Problems

SEMESTER 1

Problem Sheet 1

SOLUTIONS TO BE SUBMITTED ON CANVAS
by 17:00 on Wednesday 9th October 2024

Classical mechanics and Relativity – Problem sheet 1

Problem 1.

Using only the constants $G = 6.67 \times 10^{-11}$ m^3/kg/s^2 and $c = 3.00 \times 10^8$ m/s and dimensional analysis, estimate the size of an event horizon of a black hole of one solar mass, $M_\odot = 1.99 \times 10^{30}$ kg. (Hint: the event horizon radius is a distance, so it should have units of meters.) **[2 marks]**

Problem 2.

Again using only the constants $G = 6.67 \times 10^{-11}$ m^3/kg/s^2 and $c = 3.00 \times 10^8$ m/s and dimensional analysis, estimate the peak power that was emitted in gravitational waves recently detected by LIGO from two merging black holes. (Hint: the SI unit of power is a watt, 1 W = 1 kg m^2 s^{-3}.) **[3 marks]**

Problem 3.

How does this compare to the total power emitted by all stars in the visible Universe? (Hint: Look up the luminosity of the Sun, which is a typical star; there are about 10^{21} stars in the visible Universe.) **[2 marks]**

Problem 4.

An object is moved vertically upwards by 2.6 km, then horizontally to the right by 4.0 km, and finally up and right by a total of 3.1 km at a 45-degree angle to the horizontal. Sketch the trajectory. How far was the object displaced in total relative to the original location? At what angle from the horizontal does the total displacement vector point? **[3 marks]**

Optics and Waves

Q1. A transverse wave travelling along a string is described by $y(x,t) = 0.10\cos(50x - 600t)$. Displacements are measured in metres and time is measured in seconds.

(a) Find the wavelength, frequency and period of this wave. [3]

(b) Find an expression for the transverse velocity of a particle at $x = 0.10$ m. [2]

(c) Find an expression for the acceleration of a particle at $x = 0.20$ m. [2]

(d) At one particular time t, a particle at $x = 0.10$ m is moving with the maximum speed in the +y direction, what is the velocity of the particle at $x = 0.20$ m at the same time? [3]

Quantum Mechanics 1 – Problem 1

a) An alpha particle (helium nucleus) with kinetic energy 7 MeV experiences a "head–on" collision with a gold nucleus. What is the distance of closest approach of the two nuclei? You may assume that the gold nucleus remains stationary as the alpha particle makes its approach. [6 marks]

b) If instead of an alpha particle, a 7 MeV proton experiences a head-on collision with a gold nucleus, what is the distance of closest approach of the proton to the gold nucleus? [2 marks]

c) How did Rutherford know that the radius of the gold nucleus is *smaller* than the distance of closest approach calculated in part a)? [2 marks]