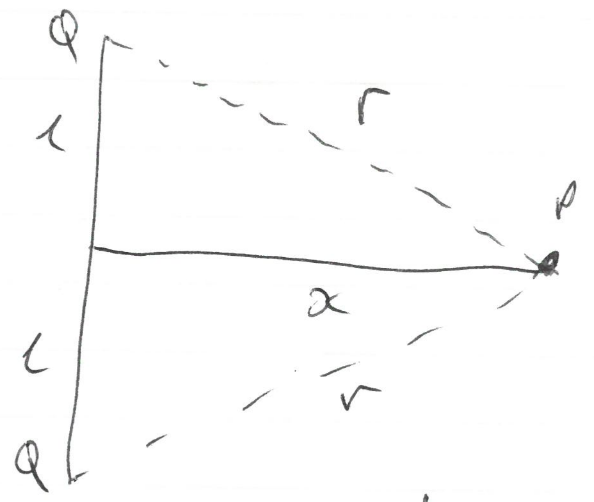
EM2 - Lec 6

Ex 6.1

① V at P?



$$V = \frac{Q}{4\pi\varepsilon_0 r} + \frac{Q}{4\pi\varepsilon_0 r}$$

$$= \frac{Q}{2\pi\varepsilon_0 r}$$

but $r = (l^2 + x^2)^{1/2}$

$$\therefore \quad V = \frac{Q}{2\pi\varepsilon_0} \frac{1}{(l^2+x^2)^{1/2}}$$

② Find $\underline{E}$ at P

$$\underline{E} = -\nabla V = -\frac{dV}{dx}\underline{i} - \frac{\partial V}{\partial y}\underline{j} - \frac{\partial V}{\partial z}\underline{k}$$

$$\frac{\partial V}{\partial y} = \frac{\partial V}{\partial z} = 0$$

$$\therefore \underline{E} = -\frac{Q}{2\pi\varepsilon_0}\left(-\frac{1}{2}\cdot 2x \cdot (l^2+x^2)^{-3/2}\right)\underline{i}$$

$$\underline{E} = \frac{Q}{2\pi\varepsilon_0} \frac{x}{(l^2+x^2)^{3/2}} \underline{i}$$

③ Work done $\underline{\text{by } E\text{-field}}$ moving charge q from $x \to 2x$.

Change in potential energy
= change in potential $\times q$

$$\therefore \Delta U = q\Delta V = q\left(V(2x) - V(x)\right)$$

$$\Delta V = \frac{Q}{2\pi\varepsilon_0}\left\{\frac{1}{(l^2+4x^2)^{1/2}} - \frac{1}{(l^2+x^2)^{1/2}}\right\}$$

$$\therefore \Delta W = -q\Delta V = \frac{Qq}{2\pi\varepsilon_0}\left\{\frac{1}{(l^2+x^2)^{1/2}} - \frac{1}{(l^2+4x^2)^{1/2}}\right\}$$

This is work done $\underline{\text{by}}$ E-field

$$\Delta W = -\Delta U.$$

$\boxed{Ex\ 6.2}$ V inside sphere uniform charge Q.

Inside $\underline{E}_{in} = \frac{Qr}{4\pi\varepsilon_0 R^3} \underline{\hat{r}}$

$$V_{in} = -\int \underline{E} \cdot d\underline{r} = -\frac{Q}{4\pi\varepsilon_0 R^3} \int r\, dr$$

$$= -\frac{Q}{4\pi\varepsilon_0 R^3} \left(\frac{r^2}{2} \right) + C \qquad (1)$$

Outside $\underline{E}_{out} = \frac{Q}{4\pi\varepsilon_0 r^2} \underline{\hat{r}}$

$$V_{out} = -\frac{Q}{4\pi\varepsilon_0} \int \frac{dr}{r^2} = \frac{Q}{4\pi\varepsilon_0 r} + D$$

But, we define $V=0$ @ $r \to \infty$

$$V_{out} = \frac{Q}{4\pi\varepsilon_0 r}$$

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at $r = R$ $\quad V_{in} = V_{out}$

i.e. $$-\frac{Q}{4\pi\varepsilon_0 R^3} \left(\frac{R^2}{2} \right) + C = \frac{Q}{4\pi\varepsilon_0 R}$$

(3)

$$\therefore C = \frac{Q}{4\pi\varepsilon_0}\left\{\frac{1}{R} + \frac{1}{2R}\right\} =$$

$$= \frac{Q}{4\pi\varepsilon_0}\,\frac{3}{2R}$$

Going back to 1

$$V_{in} = -\frac{Q}{4\pi\varepsilon_0 R^3}\,\frac{r^2}{2} + \frac{Q}{4\pi\varepsilon_0}\,\frac{3}{2R}.$$

$$V_{in} = \frac{Q}{8\pi\varepsilon_0 R}\left\{3 - \frac{r^2}{R^2}\right\}$$

$\boxed{\text{Ex } 6.3}$ infinite wire.

$$\underline{E} = \frac{\lambda}{2\pi\varepsilon_0 r} \hat{\underline{r}}$$

$$\Delta V = V_b - V_a = -\int_a^b \left(\frac{\lambda}{2\pi\varepsilon_0}\right) \frac{dr}{r}$$

$$= -\frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{b}{a}\right) = \underline{\underline{\frac{\lambda}{2\pi\varepsilon_0} \ln\left(\frac{a}{b}\right)}}$$

Note V does <u>not</u> tend to 0

as $r \to \infty$.