University of Birmingham School of Mathematics

Vectors, Geometry and Linear Algebra VGLA

(Summative) Problem Sheet 2 (Semester 1)

All the questions on this sheet contribute to your grade in this module. You should upload your solutions to this problem sheet on the VGLA Canvas page. The deadline is **5pm on 6th November 2024**.

Remember that you should always carefully write out your solutions giving full explanations (not just stating the answer). Also, submissions should consist of a single pdf file.

- **Q1**. Consider the following sets of points: U = (3,4,3), V = (5,6,7) and W = (8,9,13).
 - (a) Calculate \vec{UV} and \vec{UW} .
 - (b) Are U, V and W co-linear? Justify your answer.
 - (c) Determine the two unit-vectors that are parallel to \vec{UV} .
- **Q2**. Find the non-reflex angle between the vectors $\mathbf{u} = (2, 6, -9)$ and $\mathbf{v} = (0, -3, -2)$.
- **Q3.** Suppose that $\mathbf{a} = (4, 2, 10)$, $\mathbf{b} = (1, 2, 3)$ and $\mathbf{c} = (2, 2, 2)$ are vectors. Throughout your answers be careful to distinguish points, vectors and scalars.
 - (a) Calculate $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$.
 - (b) Calculate $(\mathbf{a} \cdot \mathbf{c})\mathbf{a} (\mathbf{b} \cdot \mathbf{c})\mathbf{b}$.
 - (c) Determine $\operatorname{proj}_{\mathbf{a}}(\mathbf{c})$.
 - (d) Find $\lambda \in \mathbb{R}$ such that $\mathbf{b} + \lambda \mathbf{c}$ is perpendicular to \mathbf{a} .
 - (e) Write down the set of points on the line which has direction vector parallel to \mathbf{a} and passes through the point D = (2, 3, 4).
- **Q4.** Suppose that $\mathbf{a} = (3, 2, -1)$ is a vector. Determine the scalar equation of the plane Π perpendicular to \mathbf{a} containing the point P = (1, -1, 1).