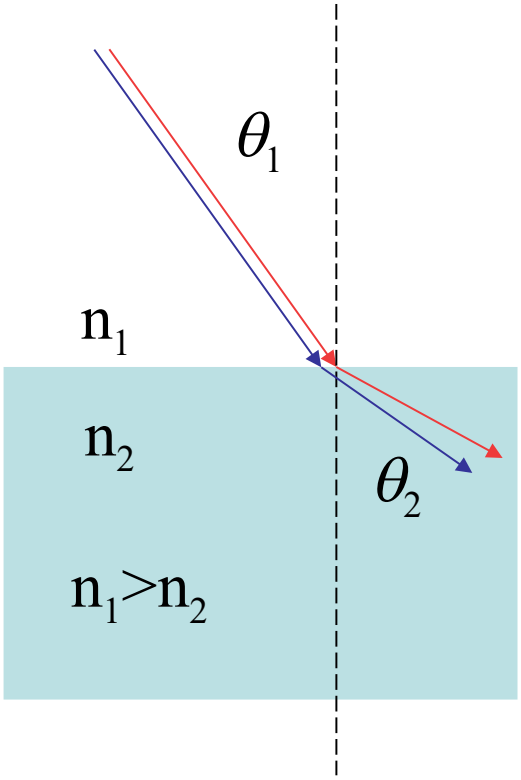
Optics and Waves

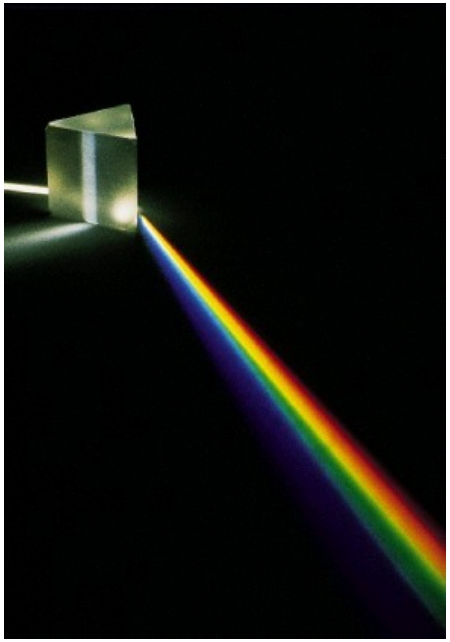
Lecture 12

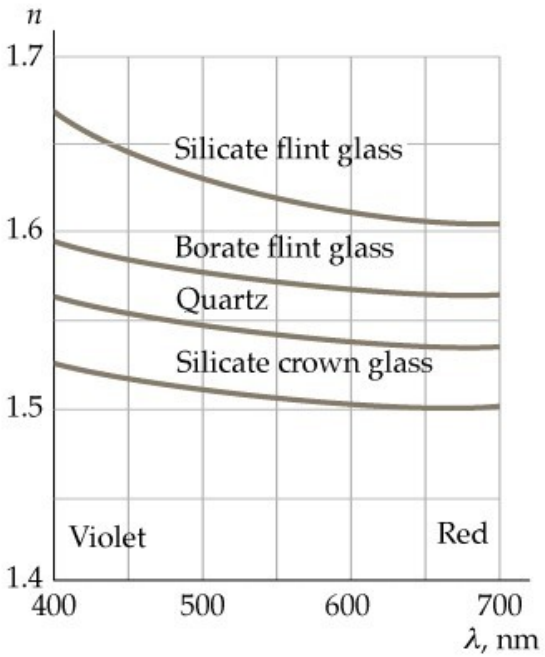
Physics inside a raindrop-
rainbows

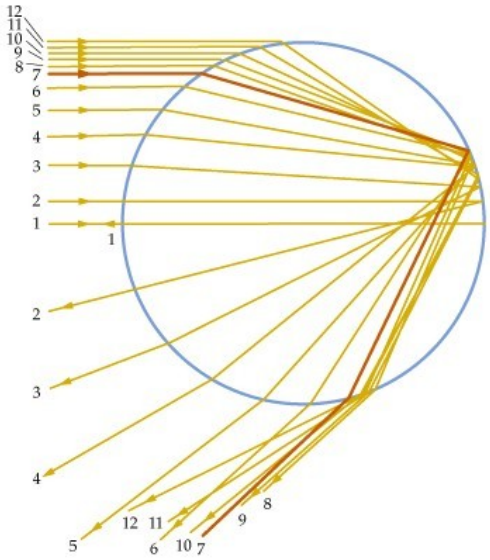
Dispersion

the refractive index is slightly different for different wavelengths









http://www.islandnet.com/~see/weather/elements/rainbow.htm

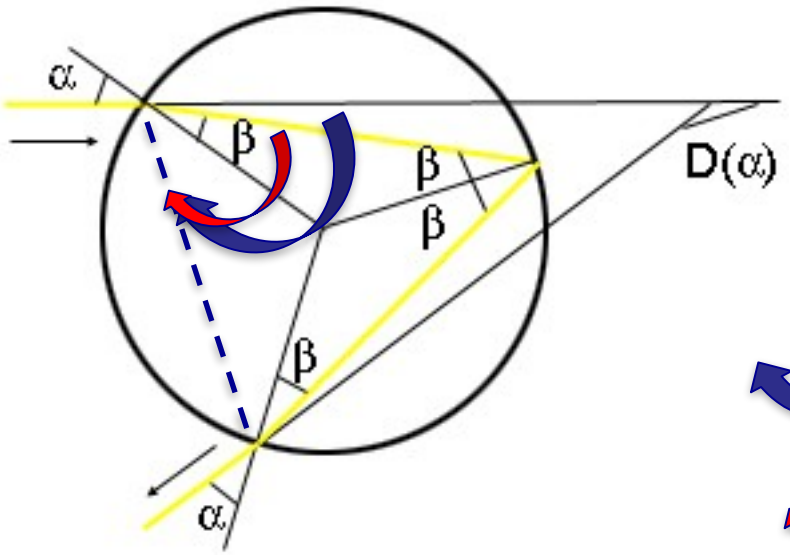
http://en.wikipedia.org/wiki/Rainbow

http://www.komonews.com/

http://plus.maths.org/content/rainbows

"By staring hard at figure 4 you can convince yourself that the deviation is given by the formula":

$$D(\alpha) = (\alpha - \beta) + (180^o - 2\beta) + (\alpha - \beta) = 180^o + 2\alpha - 4\beta$$



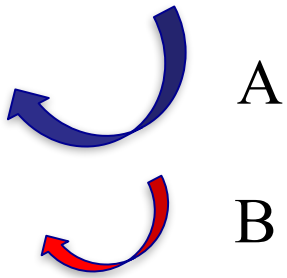
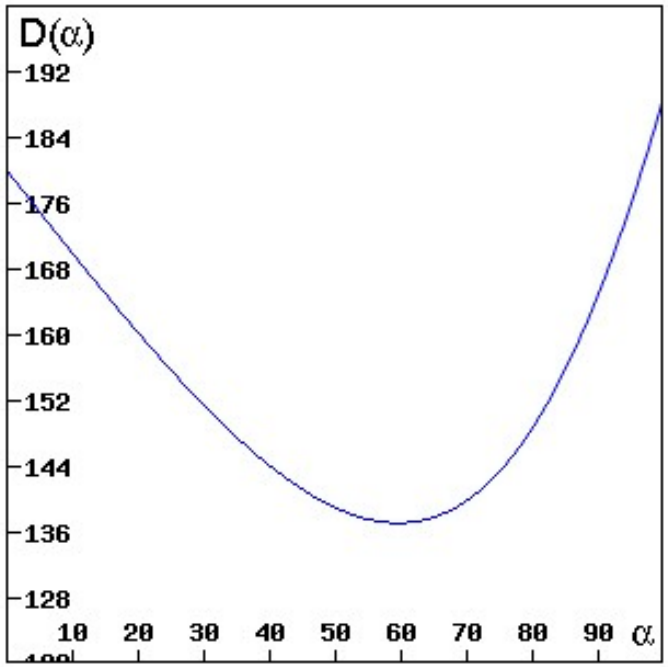
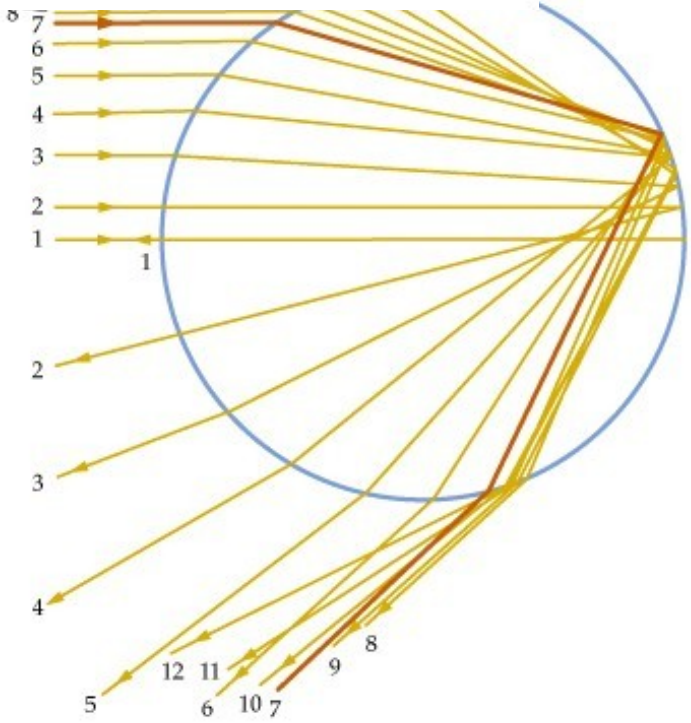


Figure 4

D=2A=2[B+(α-β)]

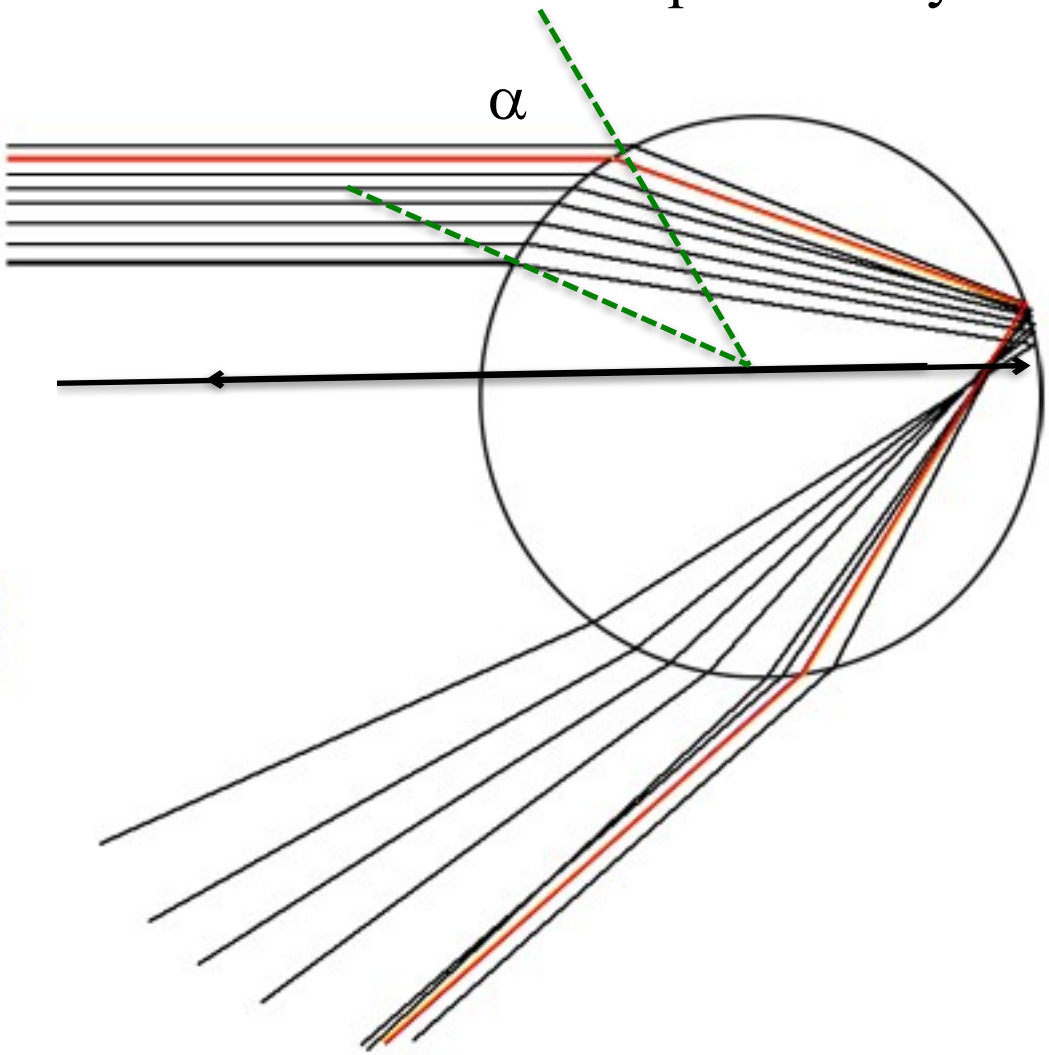
2B=[180-2β]





The red ray is the “rainbow ray”, corresponding to minimum D(α)

Different α for the parallel rays



$$D(\alpha) = 180^o + 2\alpha - 4\beta \qquad \beta = \sin^{-1}(\frac{\sin\alpha}{n})$$

$$\frac{dD(\alpha)}{d\alpha} = 2 - 4\frac{d\beta}{d\alpha}$$

$$\beta = \sin^{-1}(u), \quad \text{u}=\frac{\sin\alpha}{n}, \quad \frac{d\beta}{d\alpha} = \frac{1}{\sqrt{1-u^2}}\frac{du}{d\alpha}$$

$$\frac{d\beta}{d\alpha} = \frac{\cos\alpha}{n\sqrt{1-\frac{\sin^2\alpha}{n^2}}} = \frac{\cos\alpha}{\sqrt{n^2-\sin^2\alpha}}$$

$$\frac{dD(\alpha)}{d\alpha} = 2 - \frac{4\cos\alpha}{\sqrt{n^2-\sin^2\alpha}}$$

$$\text{let } \frac{dD(\alpha)}{d\alpha} = 2 - \frac{4\cos\alpha}{\sqrt{n^2 - \sin^2\alpha}} = 0$$

$$2\sqrt{n^2 - \sin^2\alpha} = 4\cos\alpha$$

$$4(n^2 - \sin^2\alpha) = 16\cos^2\alpha$$

$$4n^2 = 16\cos^2\alpha + 4\sin^2\alpha = 12\cos^2\alpha + 4$$

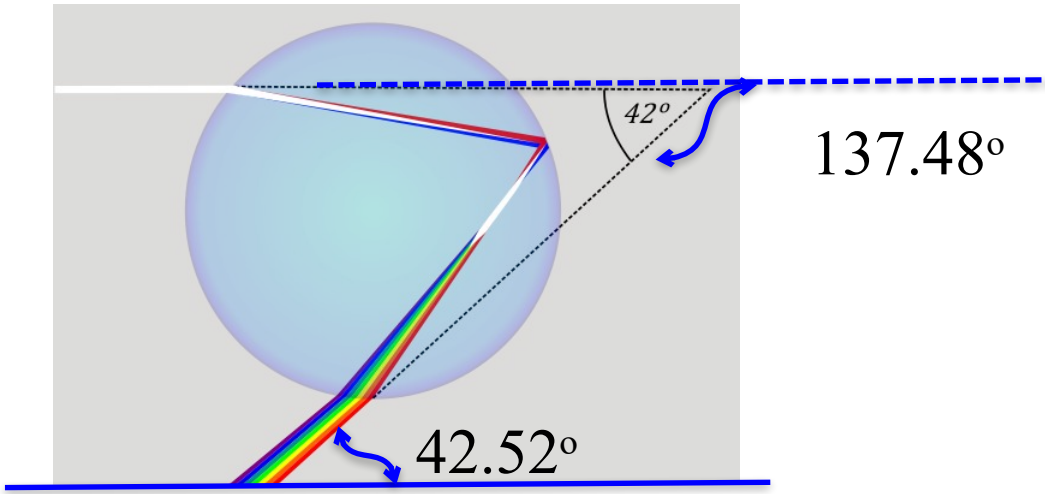
$$\cos^2\alpha = \frac{n^2 - 1}{3} \qquad \alpha_m = \cos^{-1}\sqrt{\frac{n^2 - 1}{3}}$$

$$\alpha_m = \cos^{-1}\sqrt{\frac{n^2-1}{3}},$$

$n = 1.33$ for water and red light,

$\alpha_m = 59.58^o$

$D(\alpha_m) = 137.48^o$

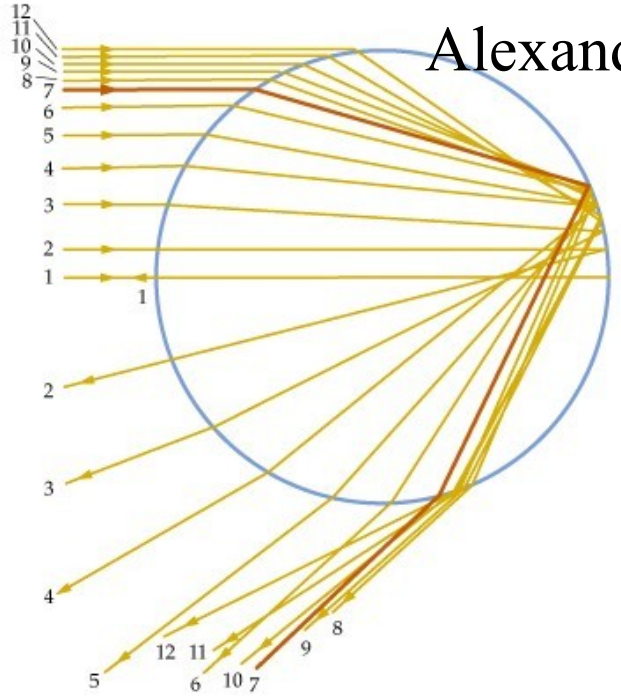


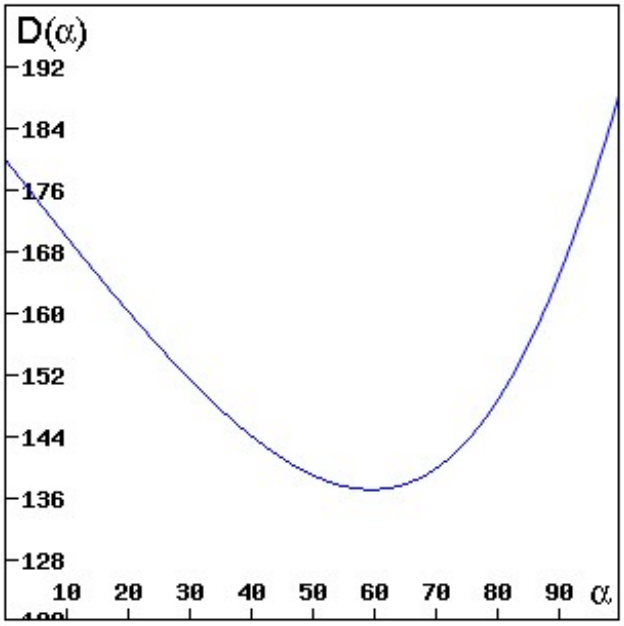
$n = 1.35$ for water and violet light,

$\alpha_m = 58.46^o$

$D(\alpha_m) = 144.44^o$

Alexander's dark band

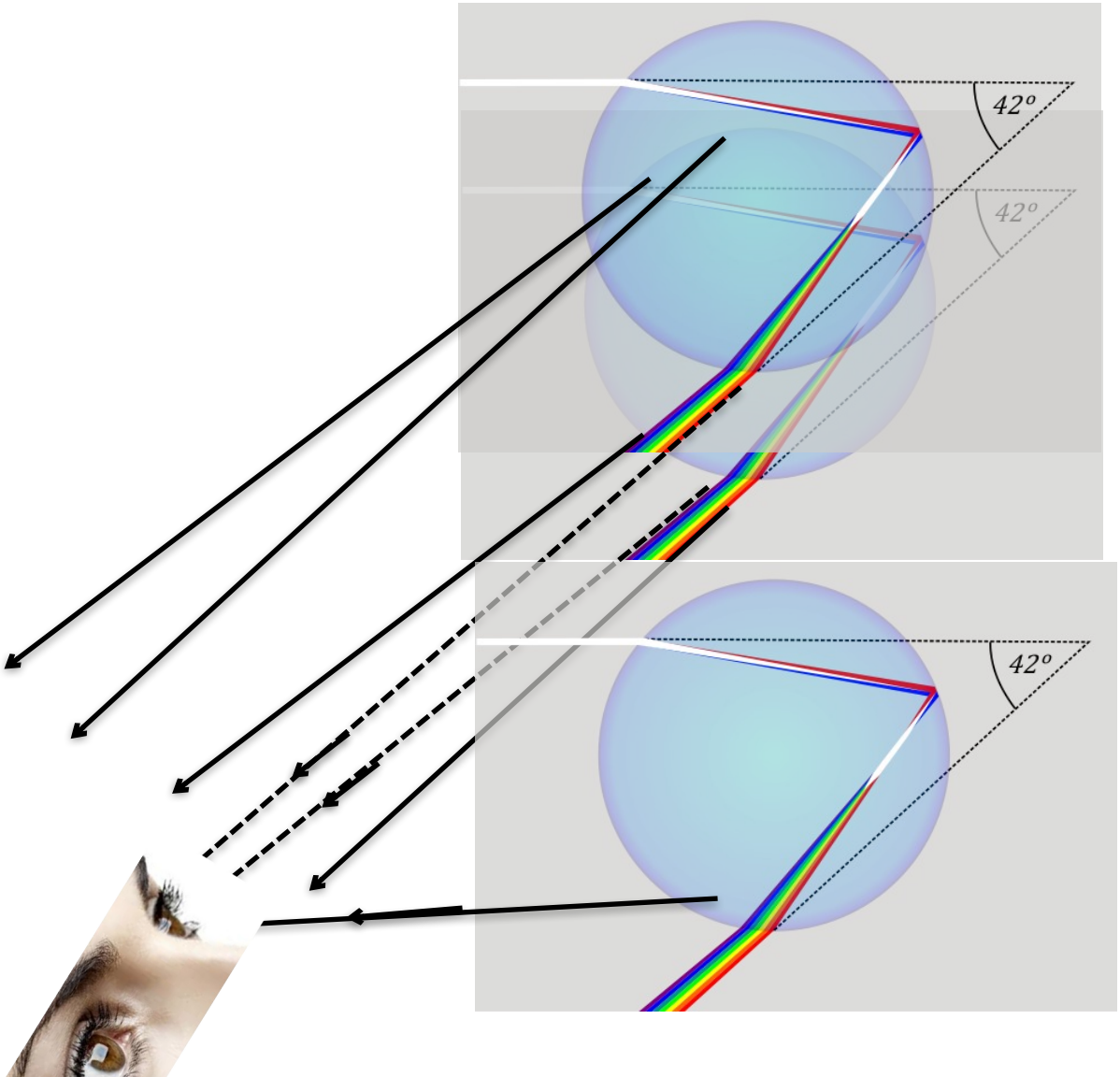




Why is there an Alexander's dark band?

1. No rain drops within the band
2. Rain drops are too small
3. Only Alexander knows

http://en.wikipedia.org/wiki/Rainbow




Photographer Owen Humphreys captured this dramatic view of a rainbow appearing over the lighthouse at Tynemouth, England.
Week in pictures: 21-27 March 2015