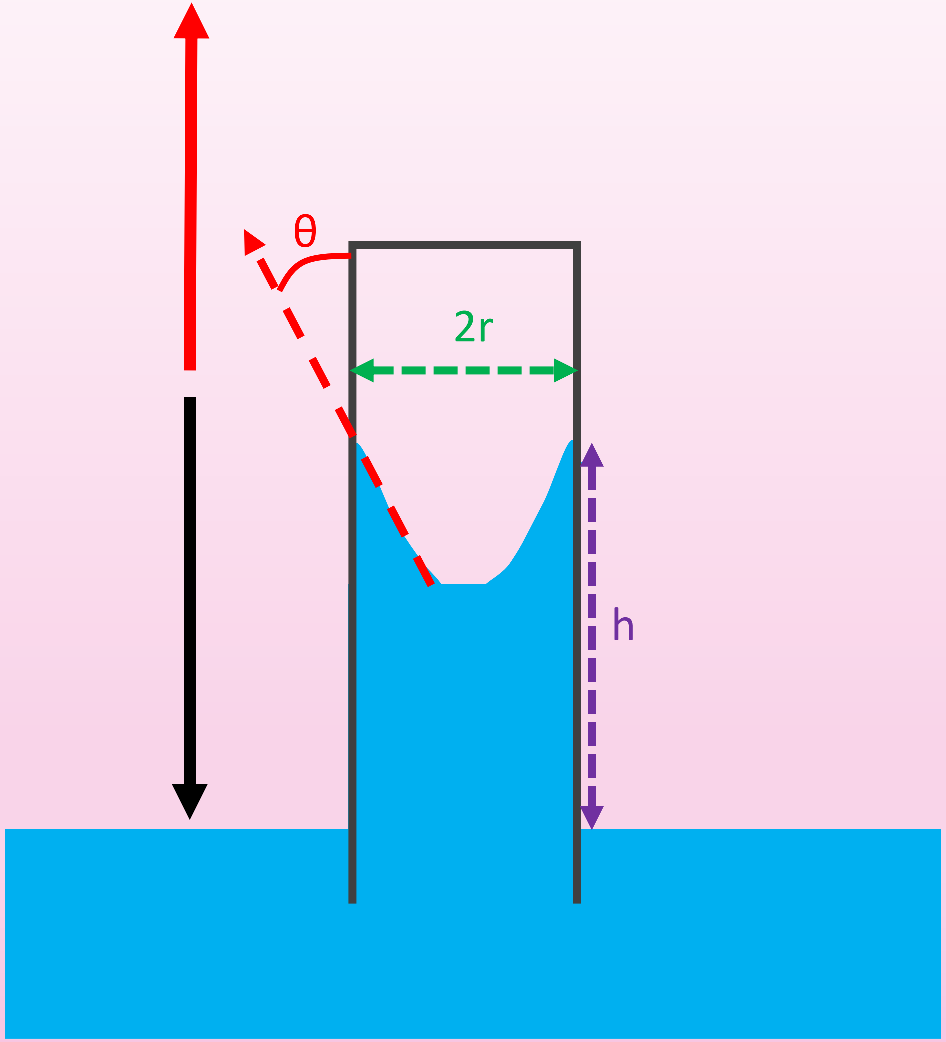
Recap from last lecture

Open tube placed in contact with the surface of a liquid causes the liquid to rise up the tube

$$h = \frac{2\gamma \cos\theta}{\rho g r}$$



Recap from last lecture

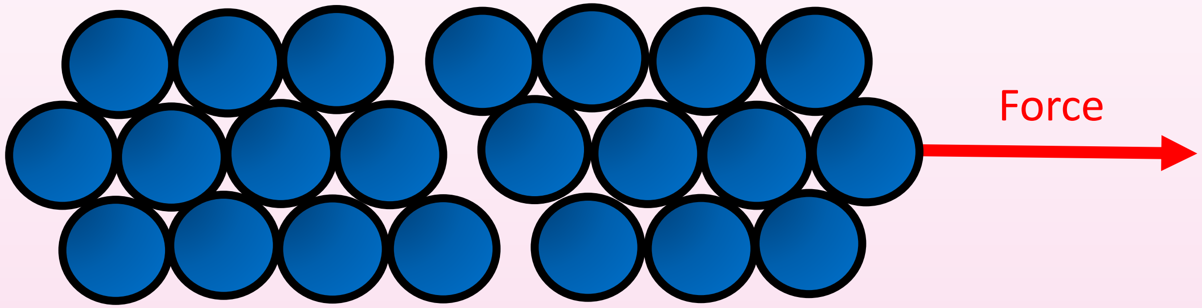
Elastic moduli given by stress/strain

Elastic moduli:

- Young’s modulus (linear force)
- Shear modulus (tangential force)
- Bulk modulus (material at a constant pressure)

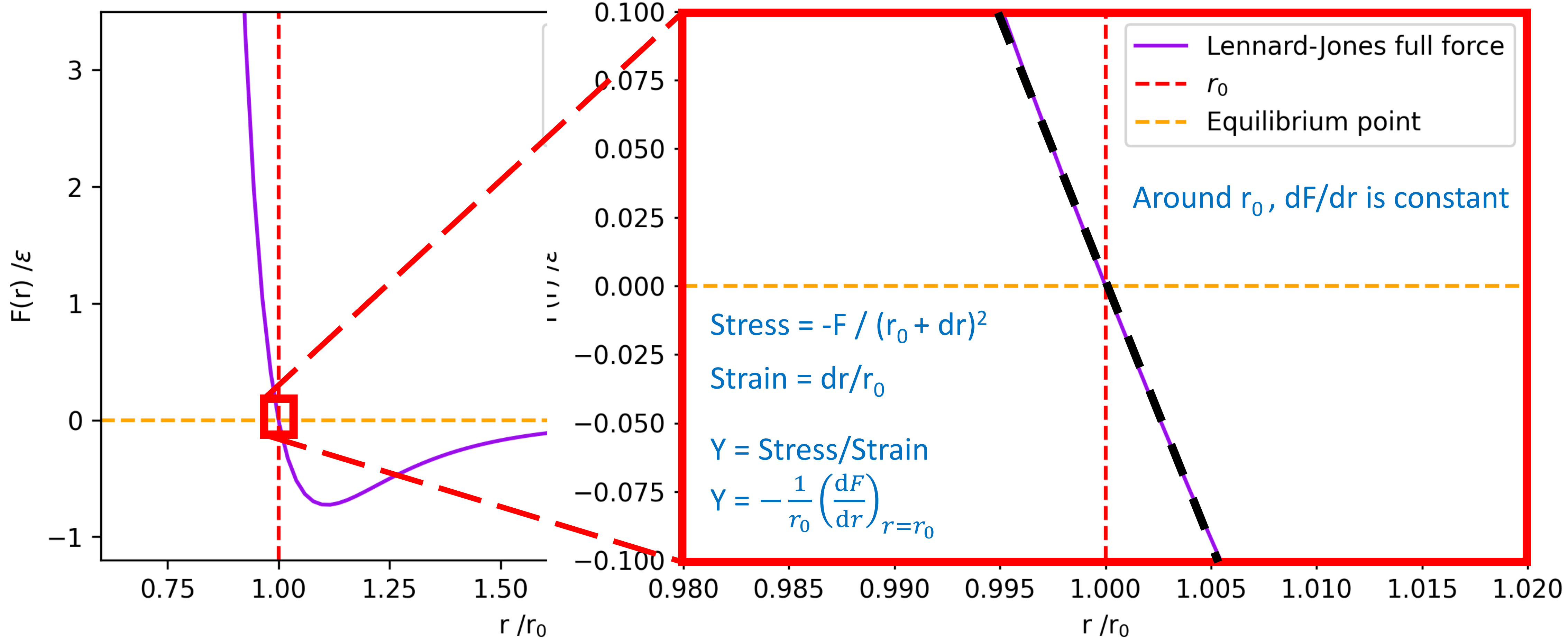
Young's modulus meets Lennard-Jones

Microscopically, a linear force extends the distance between neighbouring atoms



Changes distance from $r_0 \rightarrow r_0 + dr$

Near equilibrium point, expect the extension to be proportional to the force -> elastic region



F(r) /ε
r /r0
Lennard-Jones full force
r0
Equilibrium point
Around r0 , dF/dr is constant
Stress = -F / (r0 + dr)2
Strain = dr/r0
Y = Stress/Strain
$Y = -\frac{1}{r_0}\left(\frac{dF}{dr}\right)_{r=r_0}$

Young's modulus meets Lennard-Jones

$$Y = -\frac{1}{r_0}\left(\frac{\mathrm{d}F}{\mathrm{d}r}\right)_{r=r_0}$$

As F = -dV/dr, we can show that

$$Y = \frac{1}{r_0}\left(\frac{d^2V}{dr^2}\right)_{r=r_0}$$

Elastic modulus data

Material	Young's modulus (GPa)	Shear modulus (GPa)	Bulk modulus (GPa)
Aluminium	70	30	70

Thus:

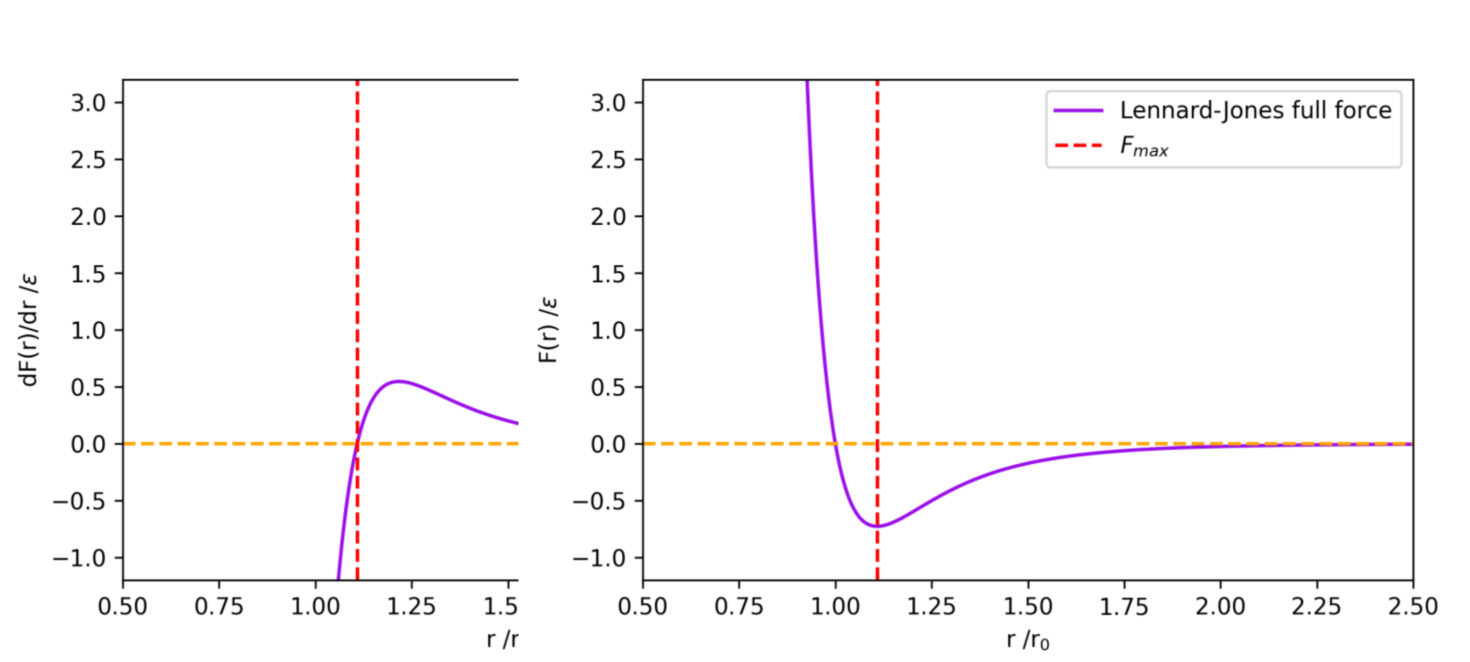
$$Y = \frac{72\varepsilon}{r_0^3} = 7.7\ \times 10^{10}\ Pa$$

Breaking materials

When do materials break? Think in terms of forces

Attractive force between atoms < Force applied to material

$$\frac{\mathrm{d}F}{\mathrm{d}r} = 0 \Rightarrow F = F_{max}$$



If we pull with a force greater than F_{max} continuously, we will pull the atoms apart –> material breaks

Breaking stress

Can define a maximum sustainable stress based on the maximum attractive force from LJ over the area of a single atom or molecule:

$$\text{Breaking stress} = \frac{F_{max}}{r_0^2}$$

Can calculate both F_{max} and hence the breaking stress from LJ:

$$\Gamma = \sqrt[6]{\frac{13}{7}}\, r_0 \qquad F_{max} = \frac{2.64\varepsilon}{r_0}$$

Macroscopic

Microscopic

$$\text{Breaking stress} = 0.037\, Y = \frac{F_{break}}{A} = \frac{F_{max}}{r_0^2}$$

Example

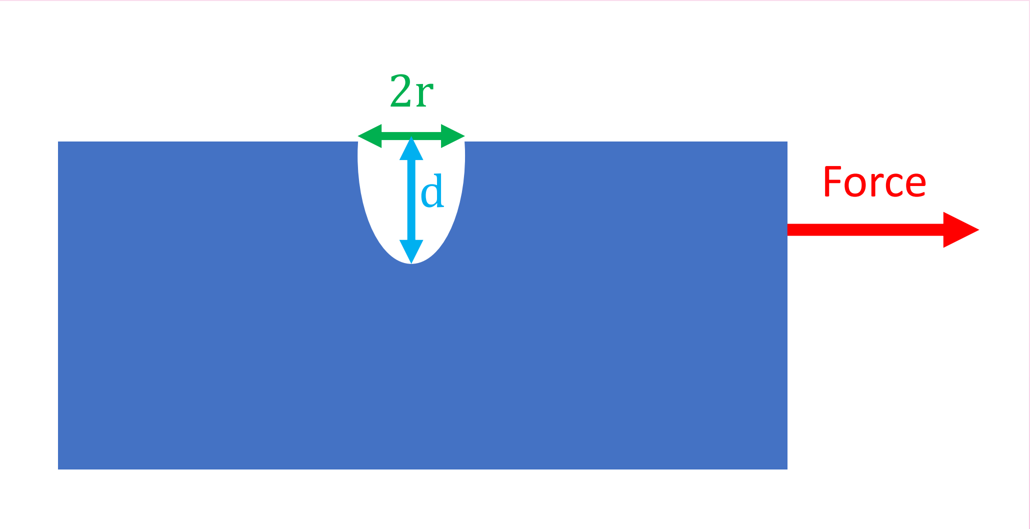
Q: A wire of aluminium has a Young's modulus of 70 GPa and a cross-sectional area of 0.5 mm^2 – how much force could you theoretically apply before you break it?

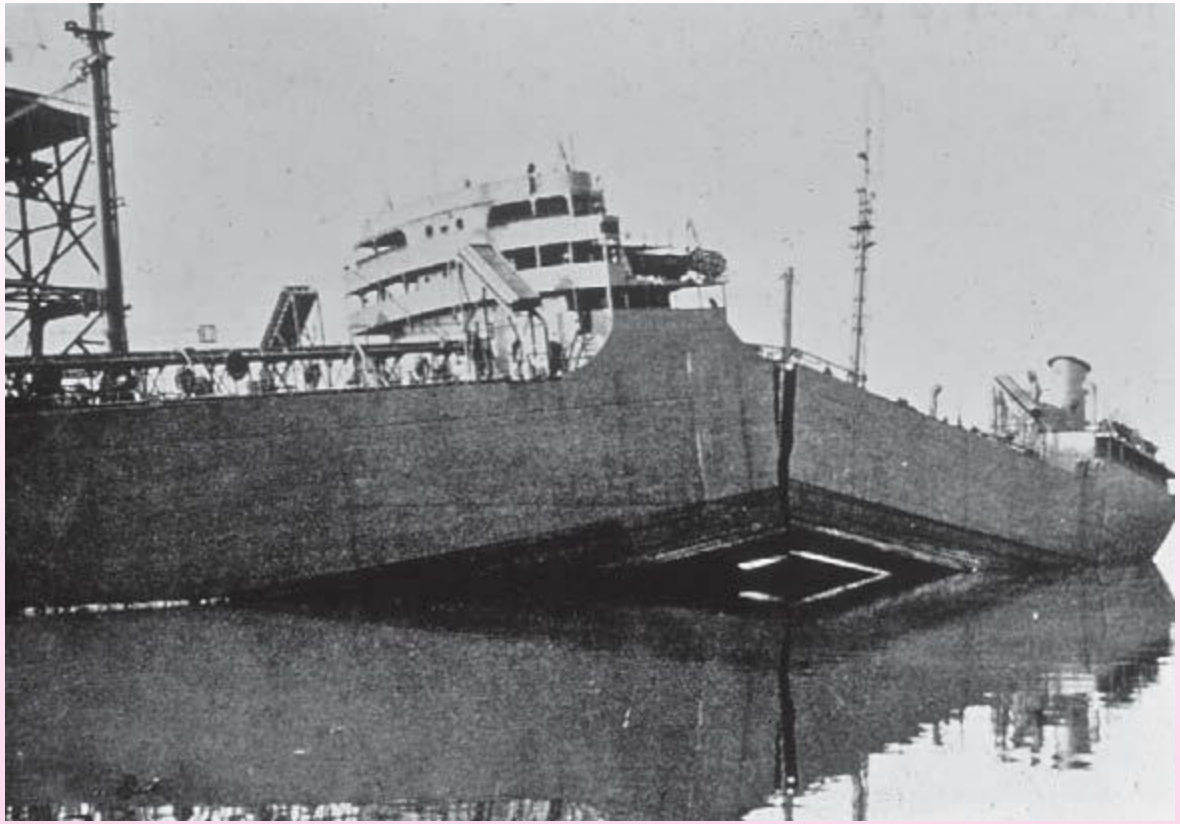
A: 1295 Newtons (high!)

In reality we probably expect a little lower than this due to the force not being applied equally and consistently across the whole material (due to imperfections, for example)

Fractures

Stress increases if there is a crack in the material by a factor of $\left(1 + 2\sqrt{\frac{d}{r}}\right)$





https://metallurgyandmaterials.files.wordpress.com/2015/12/liberty-ship-failure.jpg

Liberty ship fractures – not well understood that ductile steel could also split in half even during WWII

Force decreases with increasing d -> deeper grooves worse (makes sense)

Force increases with decreasing r -> atomic length scales cause huge increase in stress

https://www.trirentall.net/wp-content/uploads/2022/06/shutterstock_2059472675.jpg

https://img.freepik.com/premium-photo/worker-cuts-paving-slabs-with-gas-cutter-hand-saw_140282-63.jpg

Summary

Discussed Young’s modulus (macroscopic) using microscopic ideas

Investigated the maximum force and breaking force that materials can undergo

Discussed the physics of fractures and how these deviate from our understanding from a basic LJ interpretation