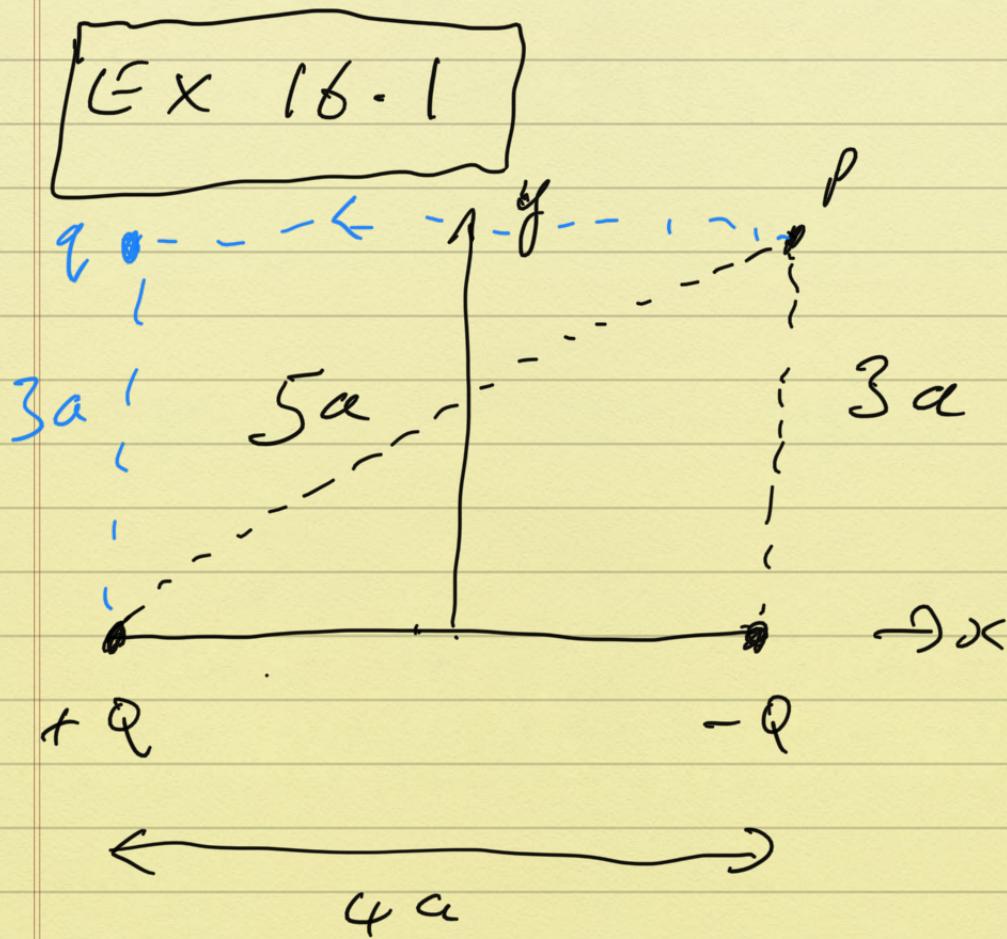


Lecture 16 - revision for weeks 1-6



$$V_p (+Q) = \frac{\Phi}{4\pi\epsilon_0 (5a)}$$

$$V_p (-Q) = \frac{-Q}{4\pi\epsilon_0 (3a)}$$

$$\therefore V_p = V_p (+Q) + V_p (-Q) = \frac{Q}{4\pi\epsilon_0} \left\{ \frac{1}{5a} - \frac{1}{3a} \right\}$$

$$= \frac{-Q}{4\pi\epsilon_0 a} \left(\frac{2}{15} \right)$$

$$W.D. = q \Delta V = q(V_f - V_i)$$

$$V_f(-2a, 3a) =$$

$$\frac{Q}{4\pi\epsilon_0(3a)} - \frac{Q}{4\pi\epsilon_0(5a)}$$

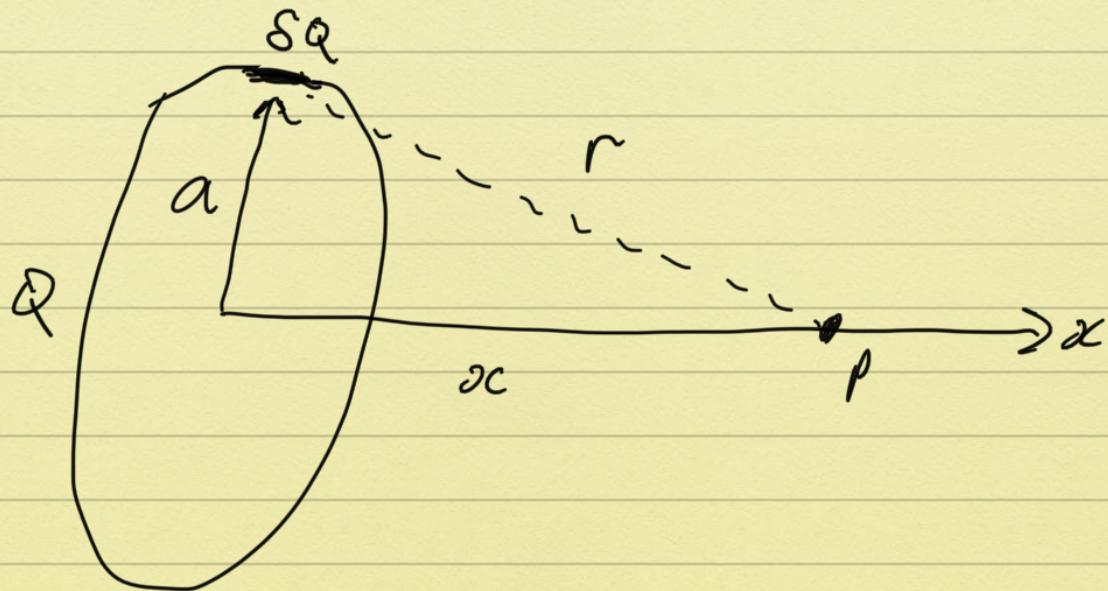
$$= \frac{Q}{4\pi\epsilon_0 a} \left(\frac{2}{15} \right)$$

$$W.D. = q(V_f - V_\infty)$$

$$= \frac{qQ}{4\pi\epsilon_0 a} \left(\frac{4}{15} \right)$$

$$= \frac{qQ}{15\pi\epsilon_0 a}$$

Ex 16.2



(i)

$$\delta V = \frac{\delta Q}{4\pi\epsilon_0 r} = \frac{\delta Q}{4\pi\epsilon_0 (a^2 + x^2)^{1/2}}$$

$$\therefore V = \frac{1}{4\pi\epsilon_0 (a^2 + x^2)^{1/2}} \int dQ$$

$$= \frac{Q}{4\pi\epsilon_0 (a^2 + x^2)^{1/2}}$$

(ii) $E = -\nabla V = -\frac{\partial V}{\partial x} i - \frac{\partial V}{\partial y} j - \frac{\partial V}{\partial z} k$

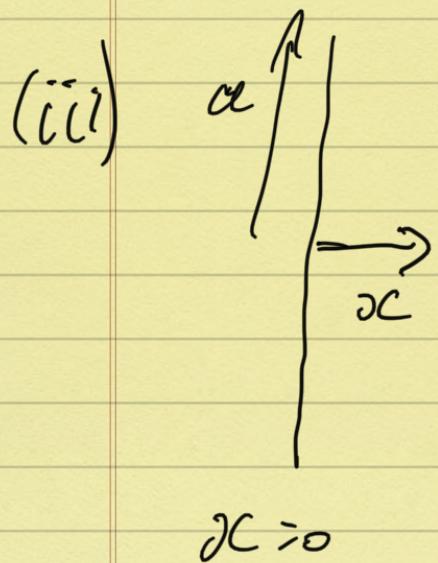
but no y or z in eq^n

$$= \frac{\partial V}{\partial y} = \frac{\partial V}{\partial z} = 0 \Rightarrow E_y = E_z = 0$$

$$\underline{E} = - \frac{\partial}{\partial x} \left(\frac{Q}{4\pi\epsilon_0} \frac{1}{(a^2 + x^2)^{1/2}} \right) \hat{x}$$

$$= - \frac{Q}{4\pi\epsilon_0} \frac{-x}{(a^2 + x^2)^{3/2}} \hat{x}$$

$$\therefore \underline{E} = \frac{Qx}{4\pi\epsilon_0 (a^2 + x^2)^{3/2}} \hat{x}$$



$x \ll a$

$$\text{Force} = -e\underline{E}$$

$$F = \frac{-eQx}{4\pi\epsilon_0 (a^2 + x^2)^{3/2}} \hat{x}$$

$$= \frac{-eQx}{4\pi\epsilon_0 a^3 \left(1 + \frac{x^2}{a^2}\right)^{3/2}} \hat{x}$$

$x \ll a$

$$F \approx \frac{-eQx}{4\pi\epsilon_0 a^3}$$

$$F = m \frac{d^2x}{dt^2} = -\left(\frac{eQ}{4\pi\epsilon_0 a^3}\right) x$$

$$\frac{d^2x}{dt^2} = -\left(\frac{eQ}{4\pi\epsilon_0 ma^3}\right) x$$

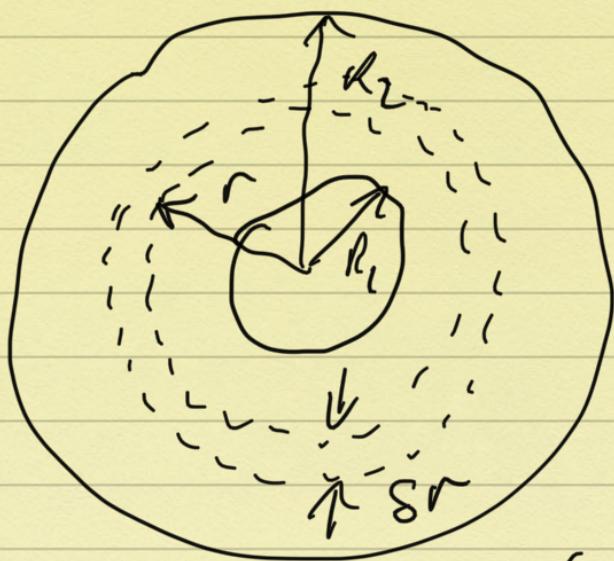
$$\text{Sol}^n \sim x(t) = x_0 \cos(\omega t)$$

$$\Rightarrow \omega = 2\pi f = \left(\frac{eQ}{4\pi\epsilon_0 ma^3}\right)^{1/2}$$

$$f = \frac{1}{2\pi} \left(\frac{eQ}{4\pi\epsilon_0 ma^3}\right)^{1/2}$$

$\boxed{L \times 16.3}$

$$\rho(r) = \rho_0 r$$



shell ar r &

thickness δr

$$\delta Q = \rho(r) \delta V$$

$$= \rho_0 r \cdot 2\pi r \delta r L$$

($L = \text{some length}$)

(a)

$$\delta Q = 2\pi \rho_0 L r^2 \delta r$$

and per unit length ($\delta Q/c$)

$$\delta Q = 2\pi \rho_0 r^2 \delta r$$

$$\therefore Q = 2\pi \rho_0 \int_{R_1}^{R_2} r^2 dr$$

$$= 2\pi \rho_0 \left[\frac{r^3}{3} \right]_{R_1}^{R_2}$$

$$= \frac{2\pi \rho_0}{3} (R_2^3 - R_1^3)$$

(b)
(i)

$$r > R_2$$

$$\int_S \underline{E} \cdot d\underline{S} = \frac{Q_{enc}}{\epsilon_s}$$

$$\text{By symmetry } \int_S \underline{E} \cdot d\underline{S} = \int_S E ds$$

$$= E \int_S ds = E \cdot 2\pi r L$$

$$E. 2\pi r L \geq \frac{Q_{enc}}{\epsilon_r} = \underbrace{Q}_{\epsilon_0} \left(\text{from } \epsilon_r = \epsilon_0 \right) L$$

$$E. 2\pi r = \frac{2\pi \ell_0}{3\epsilon_0} (R_2^3 - R_1^3)$$

$$E = \frac{\ell_0}{3\epsilon_0} (R_2^3 - R_1^3) \underbrace{\frac{1}{r}}_{\frac{1}{r}}$$

(b)
(ii) $R_1 < r < R_2$: LHS = E. $2\pi r$

$$\begin{aligned} Q_{enc} (R_1 < r < R_2) \\ = \int_{R_1}^r p \, dv = 2\pi \ell_0 \int_{R_1}^r r^2 dr \\ = \frac{2\pi \ell_0}{3} (r^3 - R_1^3) \end{aligned}$$

$$E. 2\pi r = \frac{2\pi \ell_0}{3\epsilon_0} (r^3 - R_1^3)$$

$$E = \frac{\ell_0}{3\epsilon_0} \left(r^2 - \frac{R_1^3}{r} \right)$$