Most probable speed (Maxwell-Boltzmann)

$$P_r(v) = 4\pi \left(\frac{m}{2\pi k_B T}\right)^{\frac{3}{2}} v^2 e^{-\left(\frac{mv^2}{2k_B T}\right)}$$

$$\boxed{f(x) = uv \qquad \frac{d\,f(x)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}}$$

most probable speed when

$$\frac{dP_r(v)}{dv} = 0$$

$$0 = 4\pi \left(\frac{m}{2\pi k_B T}\right)^{\frac{3}{2}} 2v\, e^{-\left(\frac{mv^2}{2k_B T}\right)} + 4\pi \left(\frac{m}{2\pi k_B T}\right)^{\frac{3}{2}} v^2 \times -\frac{2mv}{2k_B T} e^{-\left(\frac{mv^2}{2k_B T}\right)}$$

$$0 = \underbrace{4\pi \left(\frac{m}{2\pi k_B T}\right)^{\frac{3}{2}} 2v\, e^{-\left(\frac{mv^2}{2k_B T}\right)}}_{\text{clearly non-zero}} \left(1 + v^2 \times \frac{-m}{2k_B T}\right)$$

$$0 = \left(1 + v^2 \times \frac{-m}{2k_B T}\right)$$

$$\therefore 1 = \frac{mv^2}{2k_B T} \Rightarrow v^2 = \frac{2k_B T}{m}$$

$$\boxed{v_{\text{most probable}} = \sqrt{\frac{2k_B T}{m}}}$$