Week 2 Material

Kirchhoff's Laws

1. Definitions

Before we introduce Kirchhoff's laws, it will be useful to define some terms commonly used to describe a circuit (see Figure 2.1).

A **branch** is a series connection of one or more components. There are three branches in Figure 2.1: ***bafe***, ***be*** and ***bcde***.

A **node** is a point where *three* or more branches meet. (Note that a connection between two branches is just a series connection and so forms part of the same branch.) There are two nodes in Figure 2.1: ***b*** and ***e***.

A **mesh** or **loop** is a closed path that is made up of two or more branches. There are 3 loops in Figure 2.1: the two inner loops ***bafe***, ***bcde*** and the outer loop ***acdf***.

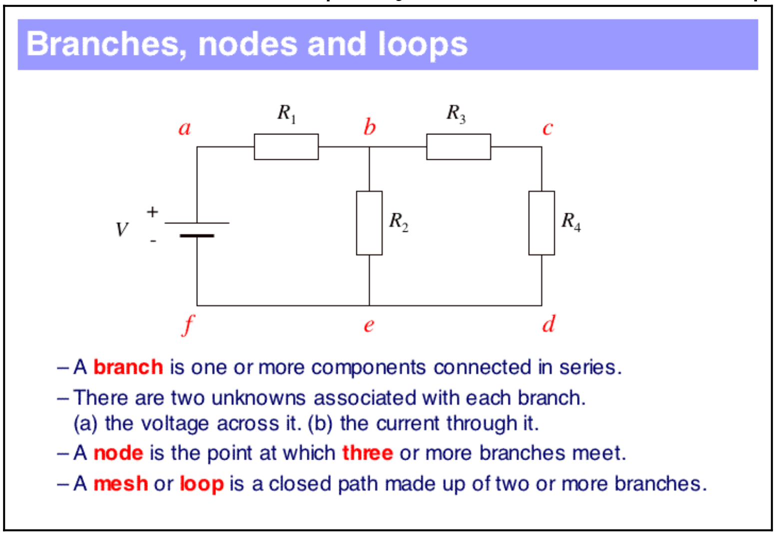

Figure 2.1: Definition of branches, nodes and loops.

2. Kirchhoff's laws

Kirchhoff's two circuit laws are stated in Figure 2.2 below.

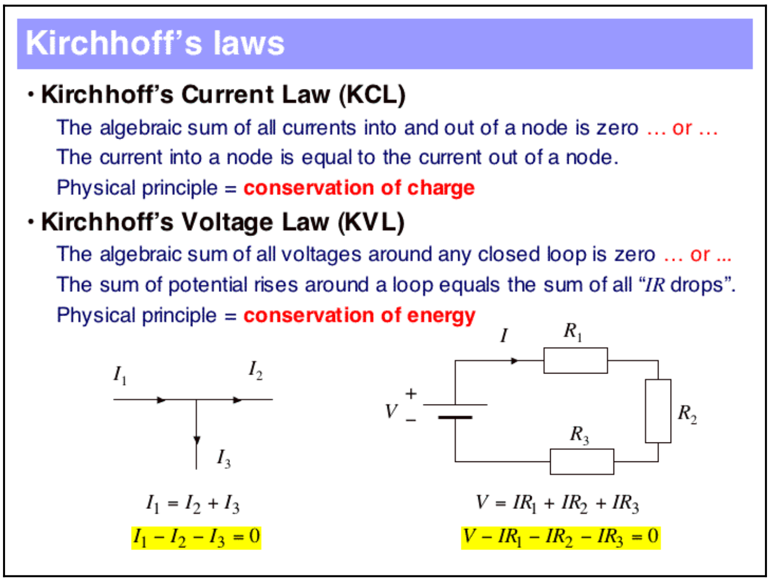

Figure 2.2: Statement of Kirchhoff's laws.

Does this look familiar?

We actually anticipated Kirchhoff's laws when we looked at combining resistors in series and parallel. (Go back and look at the arguments we used to justify this.) In simple circuits, energy and charge conservation are pretty straightforward, but as we will see, these basic principles can be applied to *any* loop or node.

Check that you follow the algebra in Figure 2.2. If current into a node equals the current out, then the algebraic sum of all the currents in and out must be zero. If the sum of all the potential rises (one in this case) equals the sum of all the voltage drops across the resistors (given by Ohm's law, $V = IR$), the algebraic sum of the potential rises and drops going around the loop must be zero.

Let's look at an example.

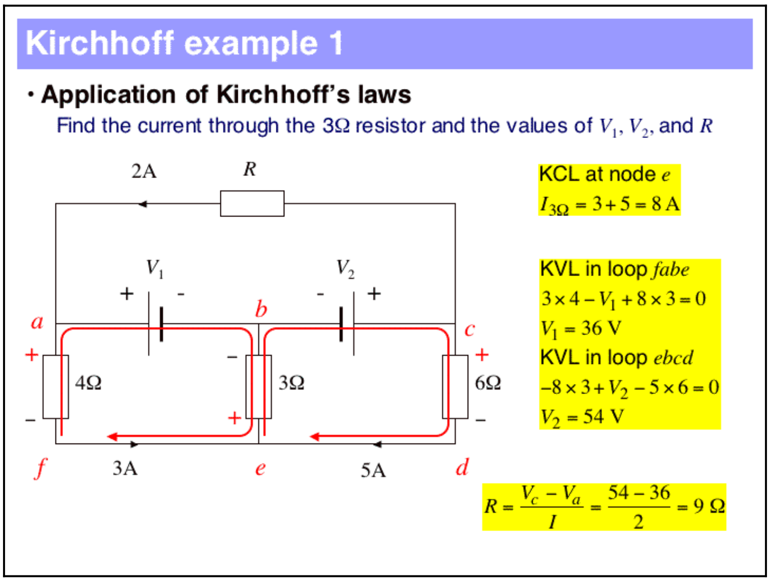

Figure 2.3: Kirchhoff example 1.

The problem asks us to find the current through the 3 Ohm resistor, the value of the voltage sources and the value of the unknown resistor, R.

To help us in this first example I have added labels to identify two of the inner loops and have drawn a clockwise path inside each of them. These are to here to show you how to solve the problem and generally wont be given to you. In the following, try to understand the physical principle that is being applied, rather than just treating Kirchhoff's laws as a recipe to follow blindly. This will help you to develop the physical intuition that will help you tackle the unseen problems at the end of this exercise.

Notice that some of the branch currents are given. At node e, we find 3 A and 5 A flowing in, so to satisfy Kirchhoff's Current Law (KCL from now on), the current in the 3 Ohm resistor must be 8 A directed upwards.

Since we now know the currents through all the resistors, we can apply Kirchhoff's Voltage Law (KVL from now on) to loop $fabe$ (and $ebcd$) to find the voltages, V_1 (and V_2). (**Attention!** – this is the tricky bit.) We know that resistors dissipate energy. This means that the end where the current leaves the resistor must be at a lower electrical potential than the end where the current enters. With this insight we can add a polarity to each of the resistors, indicating which end is at the higher electrical potential.

To apply KVL, pick a starting point in the loop $fabe$ and go around the loop in any direction adding up the potential rises and drops. (I started from point f and followed a clockwise path as shown, but you can start at any point and go either clockwise or anticlockwise. It doesn't matter.) The important thing to do is to note whether you encounter a potential rise or drop as you cross each component. In my case, $f \rightarrow a$ represents a potential rise; $a \rightarrow b$, a potential drop and $b \rightarrow e$, another potential rise. This determines the sign that you give to each term appearing in the sum of the potential rises and drops. Remember that the voltage across the resistors is given by Ohm's law, $V = IR$. The only remaining step is to solve for V_1, as shown in Figure 2.3.

Find V_2 by applying KVL in loop $ebcd$. Check your answer against that shown in Figure 2.3.

Finally to find the unknown resistor R, we need to know the potential difference between points a and c. Because the current direction is given, we know that node c is at a higher potential than node a. Going from $a \rightarrow b \rightarrow c$, we encounter a potential drop of V_1 and a potential rise of V_2. This means that the potential at c, relative to the potential at a, is given by $V_c = V_a - 36 + 54$. Rearranging, the potential difference $V_c - V_a = 18$ V. The resistance R is then simply found by applying Ohm's law.

3. Voltage conventions

The only tricky part in all this is getting the sign of the potential differences correct when applying KVL. So, let's summarise them again.

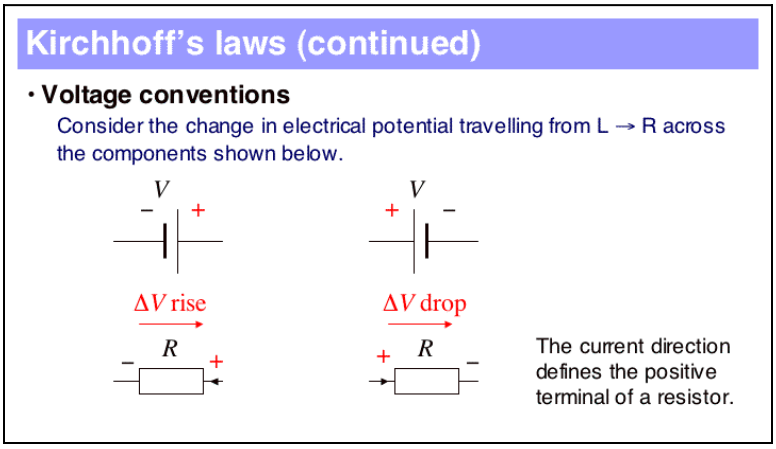

Figure 2.4: KVL voltage sign conventions.

In Figure 2.4, the change in potential across the battery is determined by the polarity of its terminals relative to the direction of travel. Negative to positive represents a potential rise (+), positive to negative a potential drop ˜(. For a resistor it depends on the direction of the current. We note a potential rise if we cross the resistor in the opposite direction to the current, a potential drop if it is the same direction. **This is the key point! Make sure you understand this before continuing.**

4. Equivalent circuits. Thévenin's theorem

We now turn to the main topic of this exercise. Let's start off with a statement of Thévenin's theorem, which is given in figure 2.5.

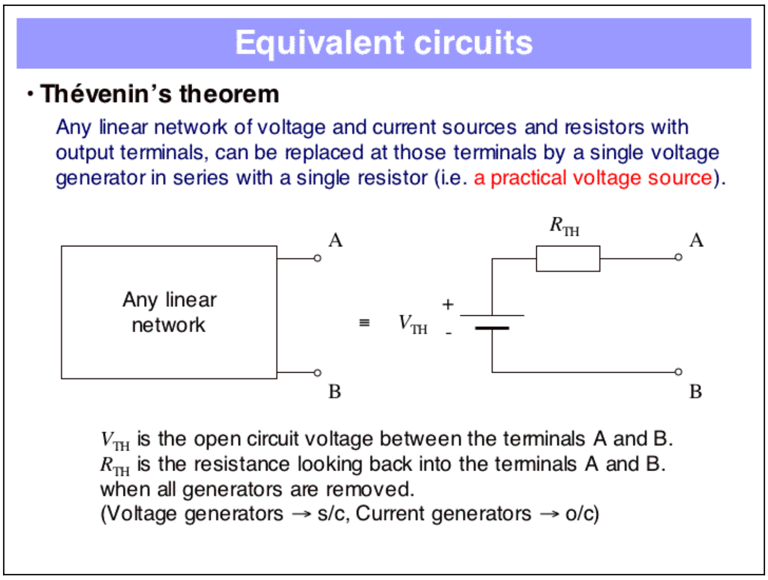

Figure 2.5: A statement of Thévenin's theorem
and the procedure for finding Thévenin's equivalent circuit.

Let that sink in for a moment. Thévenin's theorem states that for any circuit, no matter how complicated, as long as it is a linear circuit (elements within obey Ohm's law), there exists an equivalent circuit that comprises an ideal voltage source and a resistor connected in series. This is nothing more than a practical voltage source, as we have seen. That's astounding!

Thévenin's theorem is actually quite hard to prove for the general case, but we do have a specific case where we can see that this is true. Look back at figure 2.2 for a moment and ask yourself is there any way of knowing whether the source inside the box is a practical voltage source or a practical current source, from the point of view of the external circuit (i.e. the load)?

The short answer is no! All you could do is change the load and measure the voltage across it and the current flowing through it. You would find that there exists a linear relationship between the two quantities, but there is no way of deciding which type of source it might be. (You might think that the slope of the blue line might tell you something, but you could not be sure that you didn't have a voltage source with an unusually large internal resistance, or a current source with an unusually small internal resistance.) In this case, and Thévenin's theorem tells us every other case involving a linear circuit, both situations are equally well described by a practical voltage source.

Some of you may be thinking, wait a minute, what is special about the practical voltage source? The fact that the external circuit can't distinguish between a practical voltage source and a practical current source means that I could equally describe any linear circuit with a practical current source. And you'd be absolutely right! That is Norton's theorem. Thévenin came up with his theorem in 1883. Norton restated it for the current source in 1926.

OK, that's a neat trick, but is it useful? The answer is yes! Generally, we are only interested in what is happening in part of a circuit. On these occasions it simplifies things greatly if we can reduce the rest of the circuit to something more manageable. For example, you want to test the effects of different loads on a circuit, or you are trying to decide how to match the impedance of your load with the rest of your circuit.

I am sometimes asked why we don't study really complicated circuits in this course. The answer is that unless you are an electronic engineer, you are generally not interested in circuits at the individual component level. If you want to study what a circuit can do for you, take the simplest equivalent circuit that describes the one you've got. For linear circuits, where you can identify a pair of output terminals, this is nothing more than a practical voltage source. Now we will see what Thévenin's theorem can do for us.