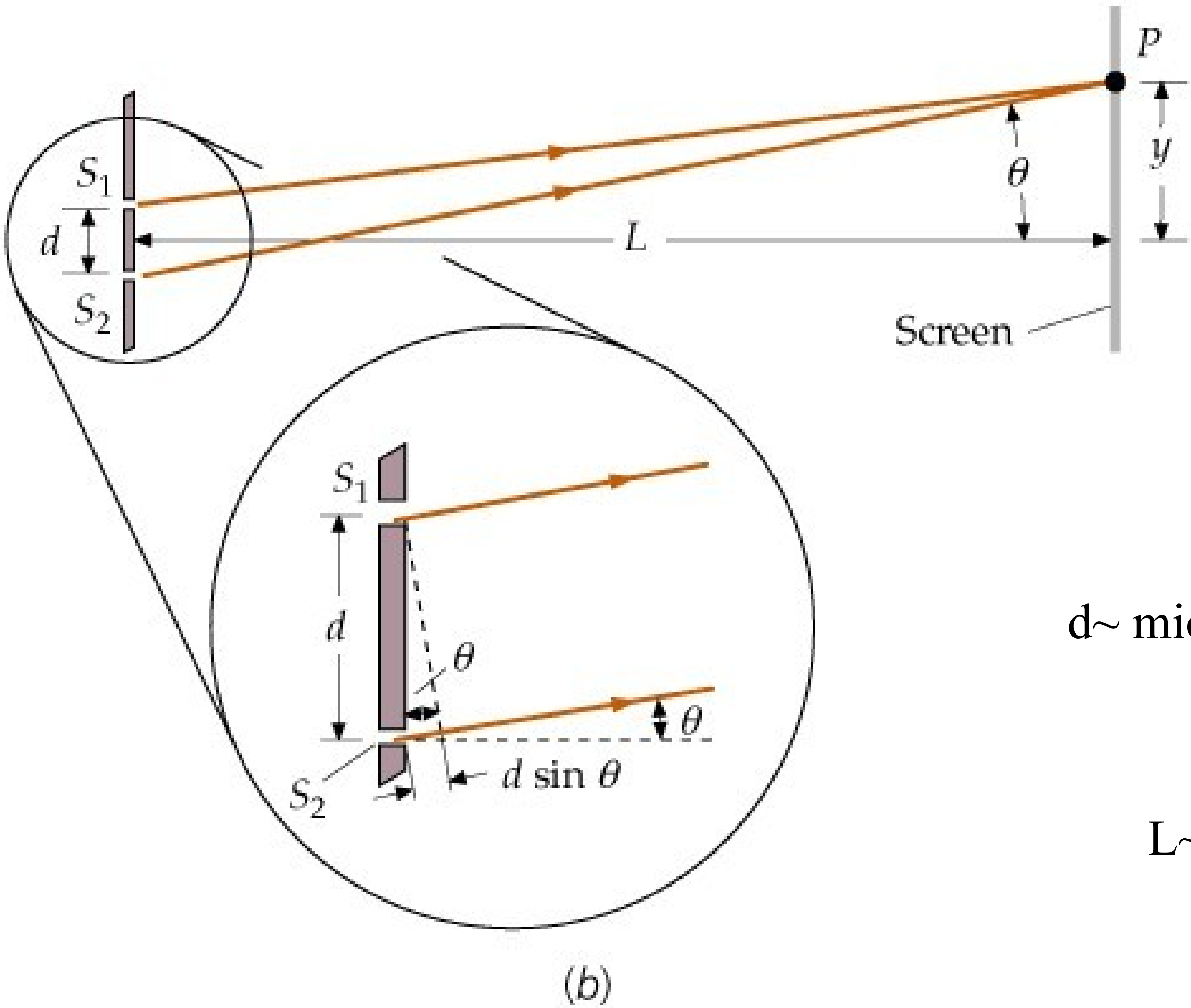
Lecture 20
Interference (Cont.)
Y&F Chapt 36.4

Young's two-slit interference experiment



d~ micrometer

L~ meter

the path difference

$$\Delta = d\sin\theta$$

$$L >> d \qquad i.e.\ \sin\theta \approx \tan\theta = \frac{y}{L}$$

Constructive interference occurs when path difference is an integral number of wavelengths

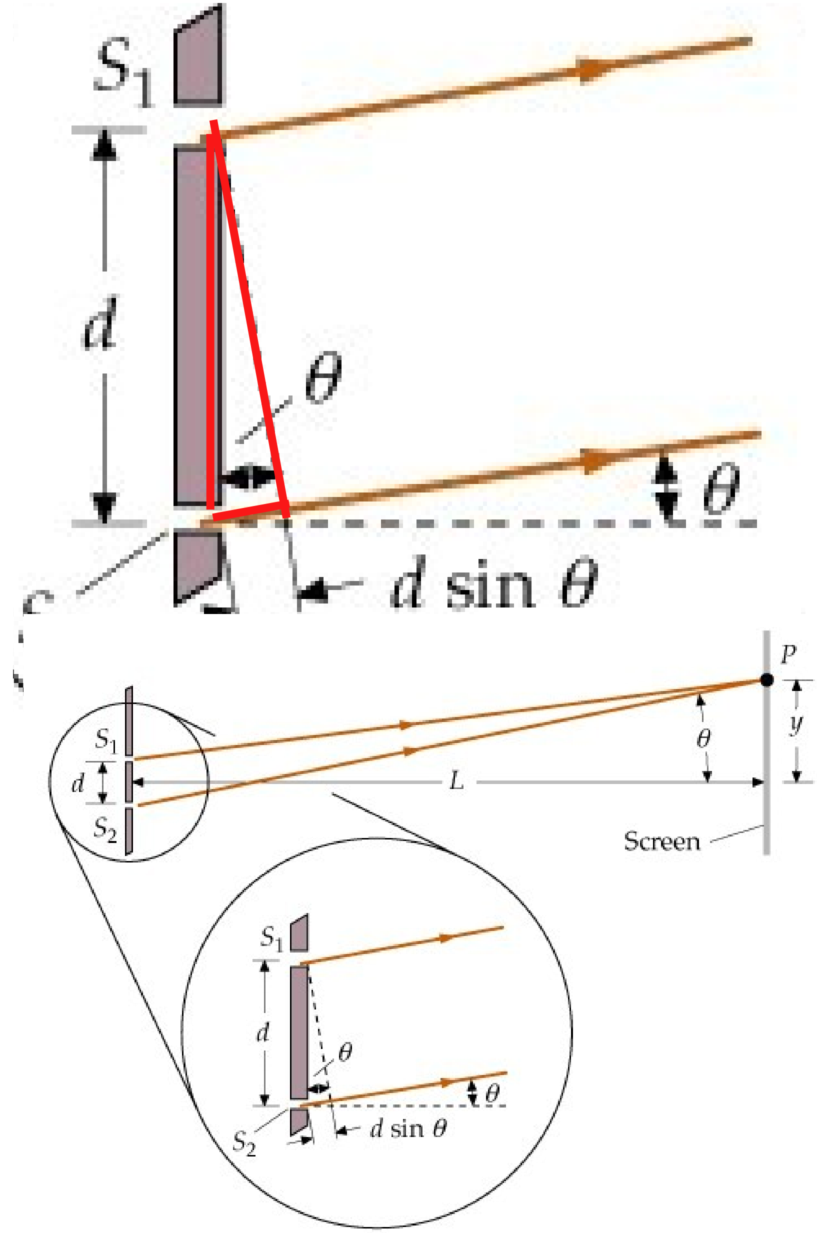
$$\Delta = d\sin\theta = m\lambda$$

$$d\frac{y_{max}}{L} = m\lambda$$

$$y_{max} = \frac{m\lambda L}{d}$$

This is the position of the bright fringes on the screen

m= 0, ±1, ±2, ±3,…



(b)

Separation between fringes $\Delta y = \dfrac{\lambda L}{d}$

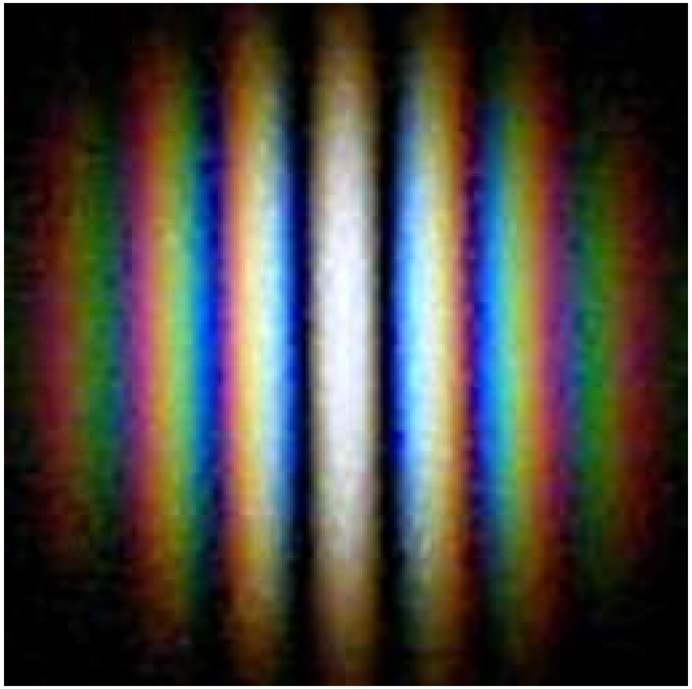
Destructive interference occurs when path difference is a half integral number of wavelengths

$$\Delta = d\sin\theta = (2m+1)\frac{\lambda}{2}, \quad m = 0, \pm 1, \pm 2, \ldots$$

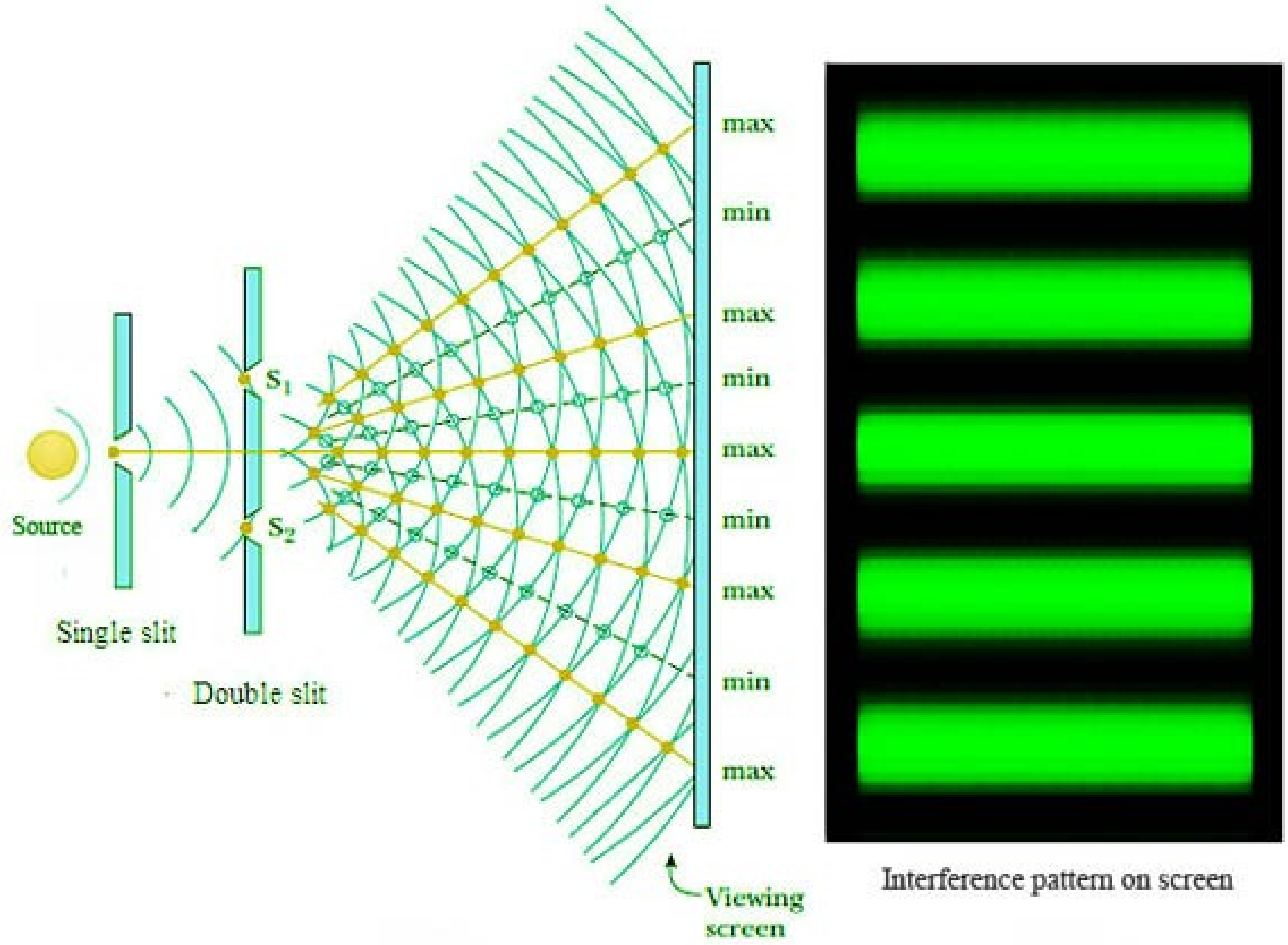
$$d\frac{y_{\min}}{L} = (2m+1)\frac{\lambda}{2}$$

$$y_{\min} = \frac{(2m+1)\frac{\lambda}{2}L}{d}$$

$$\Delta y = \frac{\lambda L}{d}$$



m=0, y=0, is where the central bright fringe located.



http://h2physics.org/?cat=48

What about the intensity pattern? (the intensity between the y_{max} and y_{min})

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

where $I_{1,2}$ is the intensity of source (slit) 1,2 alone

Let $I_1 = I_2 = I_0$

$$I = 2I_0(1+\cos\delta) = 4I_0\cos^2\left(\frac{\delta}{2}\right)$$

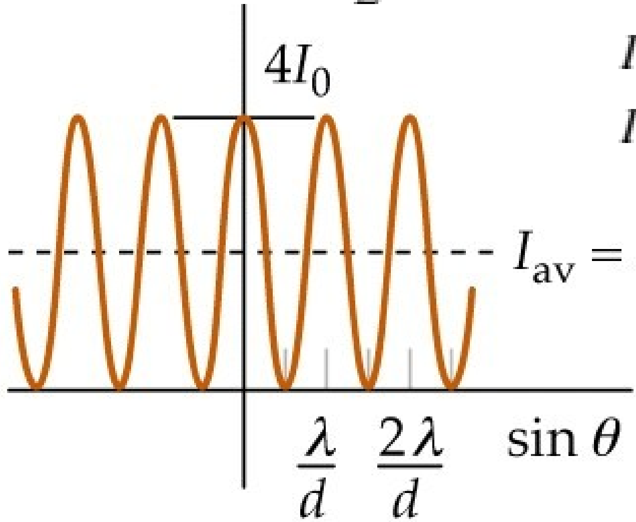
here we have used the identity : $(1+\cos\delta) = 2\cos^2(\frac{\delta}{2})$

here

$$\delta = \frac{2\pi}{\lambda} d\sin\theta$$

$$I = 4I_0\cos^2\left(\frac{\pi}{\lambda} d\sin\theta\right)$$

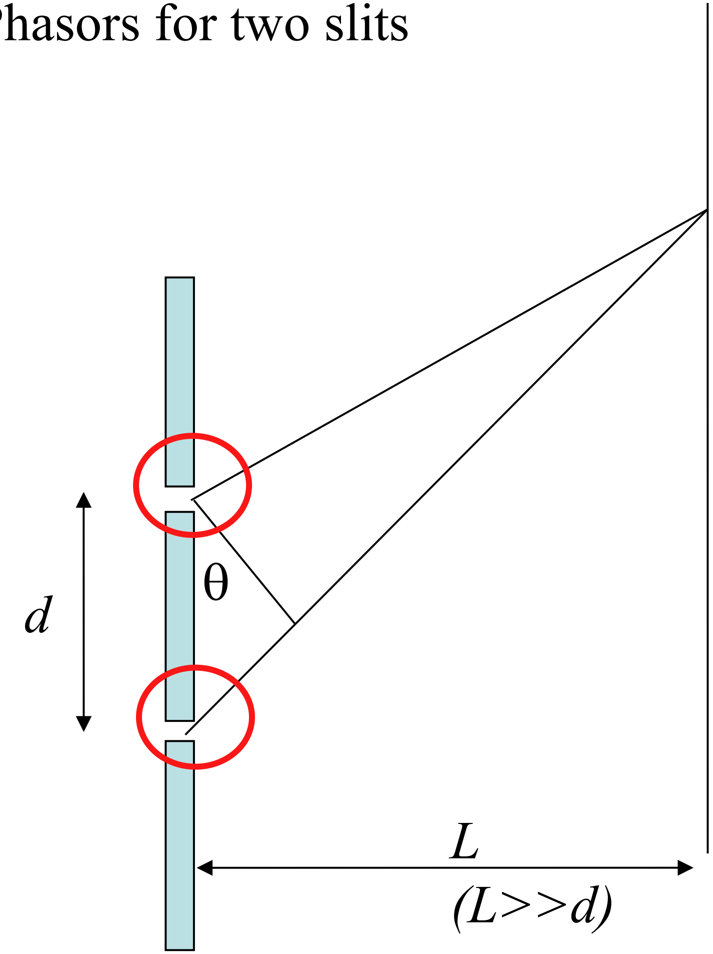
$$I = 4I_0\cos^2\left(\frac{\pi d}{\lambda}\frac{y}{L}\right)$$



$I_{max} = 4I_0$

$I_{min} = 0$

Phasors for two slits

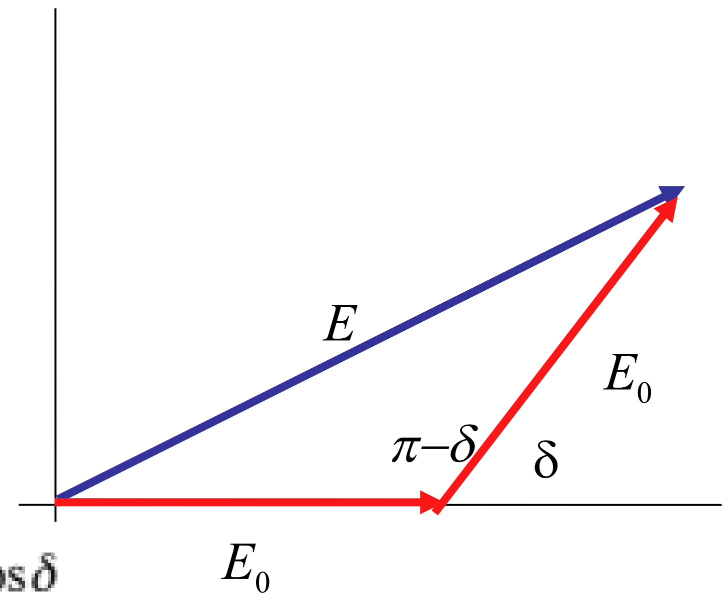


Path difference: $\Delta = d\sin\theta$

Phase diff:

$$\delta = \frac{2\pi}{\lambda}\Delta = \frac{2\pi}{\lambda}d\sin\theta$$

Waves received at screen is sum of all the waves from the slits



$$E^2 = E_0^2 + E_0^2 - 2E_0E_0\cos(\pi-\delta) = E_0^2 + E_0^2 + 2E_0E_0\cos\delta$$

$$= 2E_0^2(1+\cos\delta) = 4E_0^2\cos^2(\frac{\delta}{2})$$

here we have used : $(1+\cos\delta)=2\cos^2(\frac{\delta}{2})$

$$E^2 = 4E_0^2 \cos^2(\frac{\delta}{2})$$

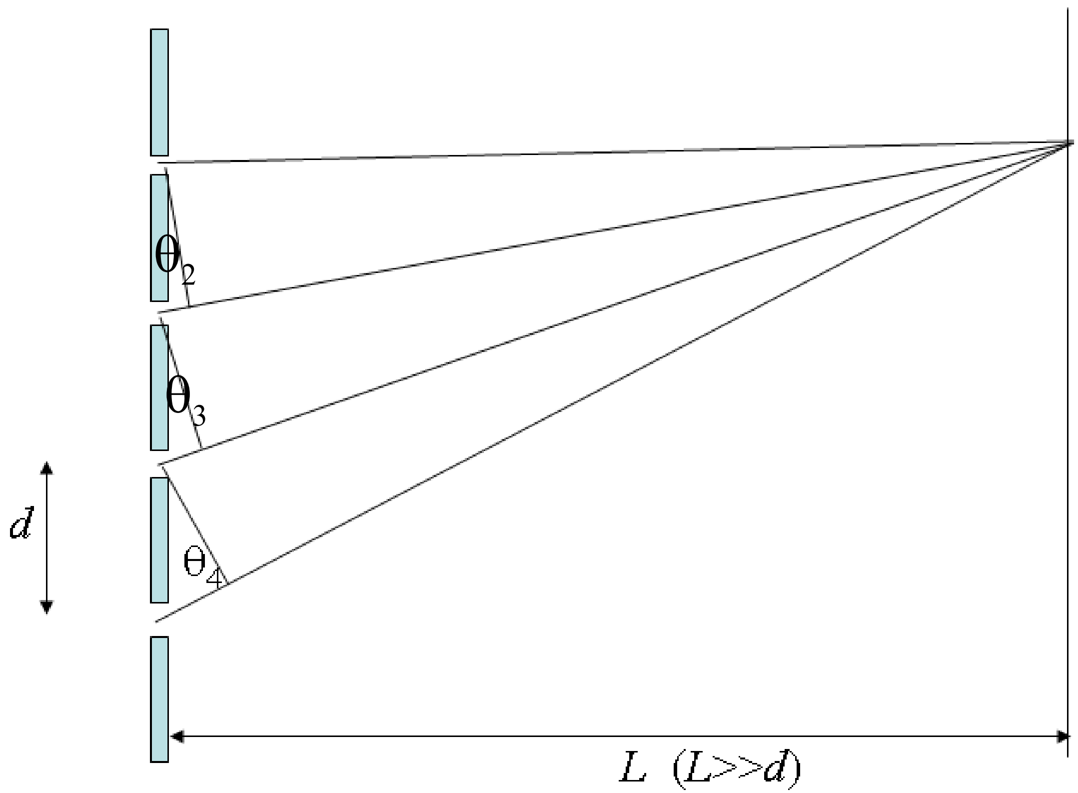
$$I = 4I_0 \cos^2(\frac{\delta}{2})$$

Earlier, we had a different looking equation:

$$I = E^2 = (2R)^2 \sin^2(\delta)$$

$$I = E_0{}^2 \frac{\sin^2(\delta)}{\sin^2\left(\frac{\delta}{2}\right)} = I_0 \frac{\sin^2(\delta)}{\sin^2\left(\frac{\delta}{2}\right)}$$

Applying phasors to multiple slits (of infinitesimal width)



if L>>d
then $\theta_2 \sim \theta_3 \sim \theta_4$

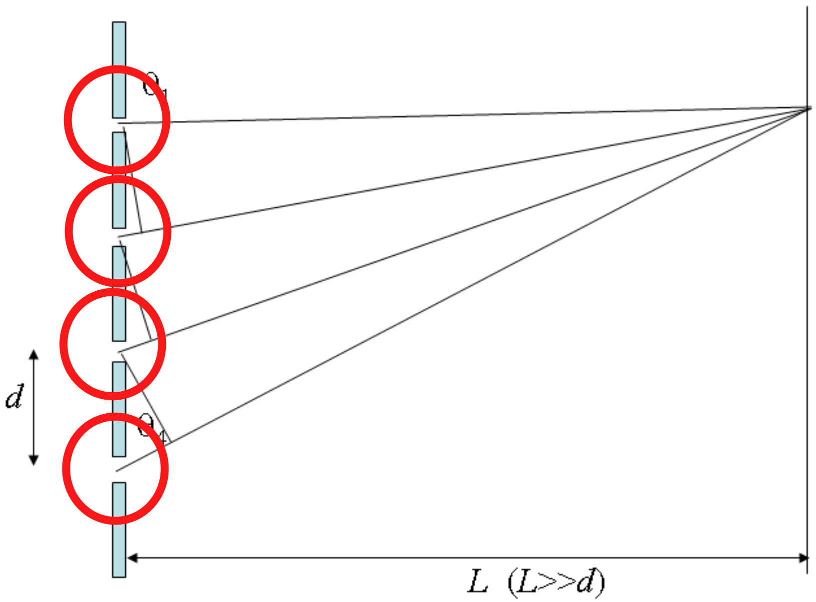
Path diff between adjacent slits

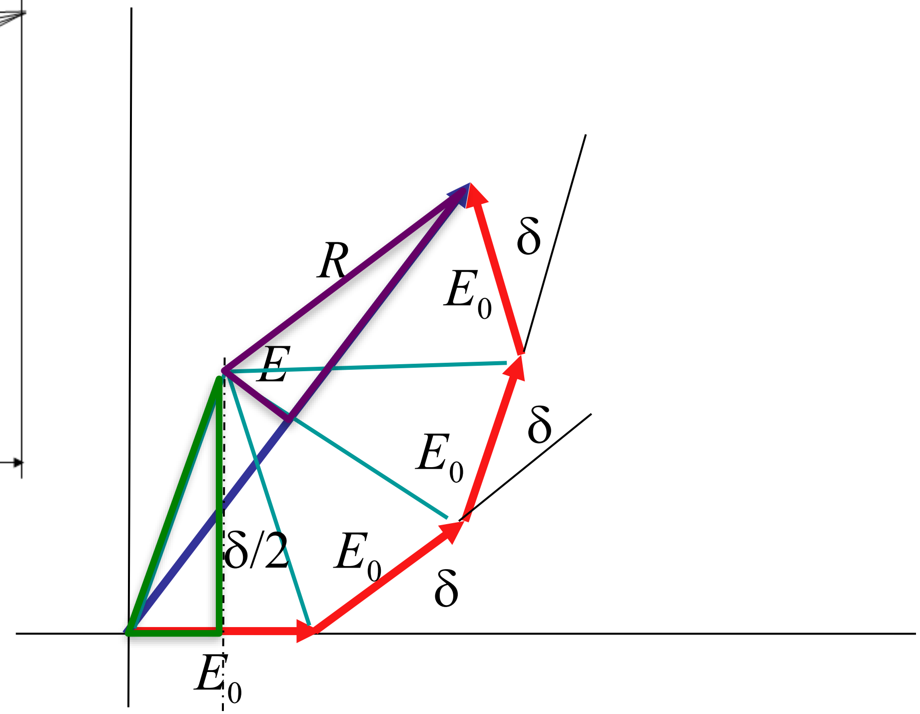
$$\Delta = d \sin \theta$$

Phase diff

$$\delta = \frac{2\pi}{\lambda}\Delta = \frac{2\pi}{\lambda} d \sin \theta$$

For four slits:





$$\frac{E}{2} = R\sin(2\delta) = R\sin\left(4\frac{\delta}{2}\right)$$

$$\frac{E_0}{2} = R\sin\left(\frac{\delta}{2}\right)$$

$$\frac{E}{E_0} = \frac{\sin\left(4\frac{\delta}{2}\right)}{\sin\left(\frac{\delta}{2}\right)}$$

$$\frac{I}{I_0} = \frac{E^2}{E_0^2} = \frac{\sin^2\left(4\frac{\delta}{2}\right)}{\sin^2\left(\frac{\delta}{2}\right)}$$

For N-slits

$$I = I_0 \frac{\sin^2\left(N\frac{\delta}{2}\right)}{\sin^2\left(\frac{\delta}{2}\right)}$$

numerator is zero when: $\frac{N\delta}{2} = m\pi \qquad m = 0, 1, 2, 3, \ldots$

i.e. minima when: $\delta = \frac{2m\pi}{N} \qquad m = 0, 1, 2, 3, \ldots$

$$= \underbrace{0}_{\text{principal maximum}}, \underbrace{\frac{2\pi}{N}, \frac{4\pi}{N}, \ldots\ldots\ldots,}_{\text{minima}} \underbrace{\frac{2N\pi}{N}}_{\text{principal maximum}}, \underbrace{\frac{2(N+1)\pi}{N}, \ldots\ldots}_{\text{minima}}$$

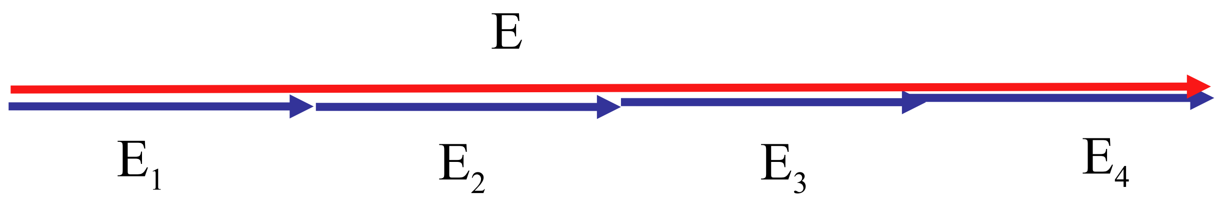
For $\delta = 0, 2\pi, 4\pi, \ldots$

we have $I = I_0 \frac{0}{0}$

We need to evaluate I differently

Using four slits as an example

$$\delta = 0$$



$$E = NE_0$$

$$I = N^2 E_0^2 = N^2 I_0$$

This is the same for $\delta = 2\pi, 4\pi, 6\pi, \ldots$

So for $\delta = 2m\pi, m = 0,1,2,3\ldots$

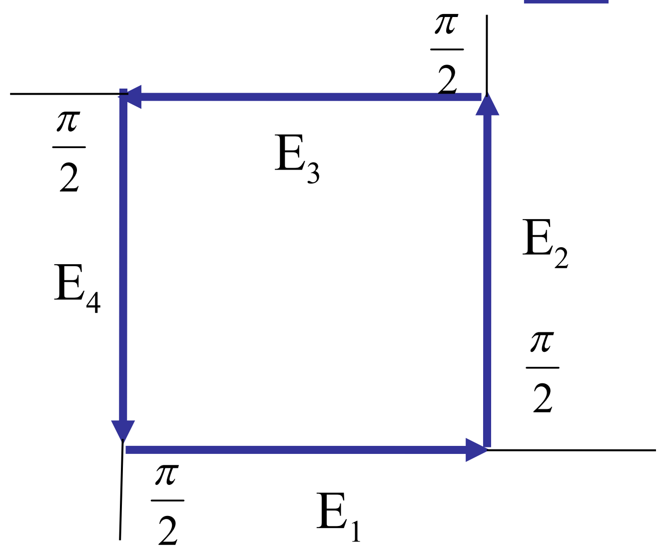
Intensity is maximum

Phasors: (for the case of 4 slits) $\delta = \frac{2m\pi}{N} = \frac{2m\pi}{4}$ $m = 0, 1, 2, 3, 4$

$= \boxed{0}, \boxed{\frac{2\pi}{4}, \frac{4\pi}{4}, \frac{6\pi}{4}}, \boxed{\frac{8\pi}{4}}, \boxed{\frac{10\pi}{4}, \frac{12\pi}{4} \ldots\ldots}$

First minimum:

$$\delta = \frac{2\pi}{4} = \frac{\pi}{2}$$

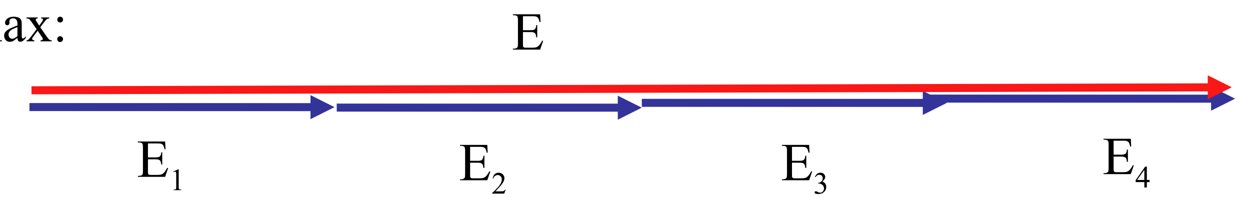


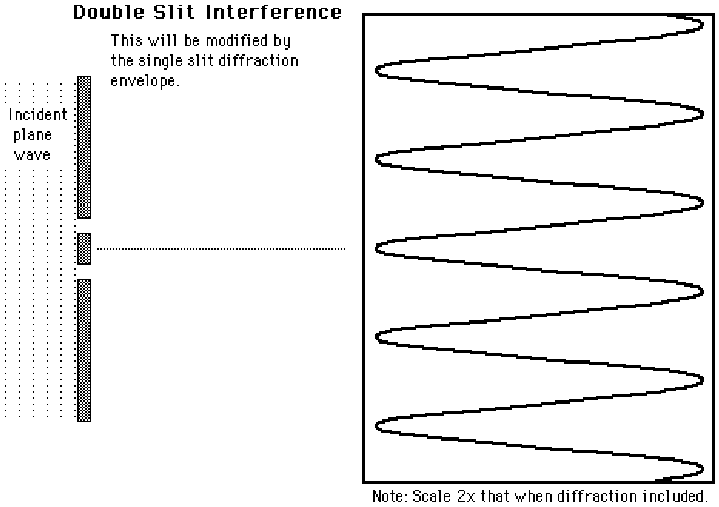
Second minimum:

$$\delta = \frac{4\pi}{4} = \pi$$

Principal max:

$$\delta = 0, 2\pi, 4\pi, \ldots$$





http://hyperphysics.phy-astr.gsu.edu/hbase/phyopt/mulslidi.html

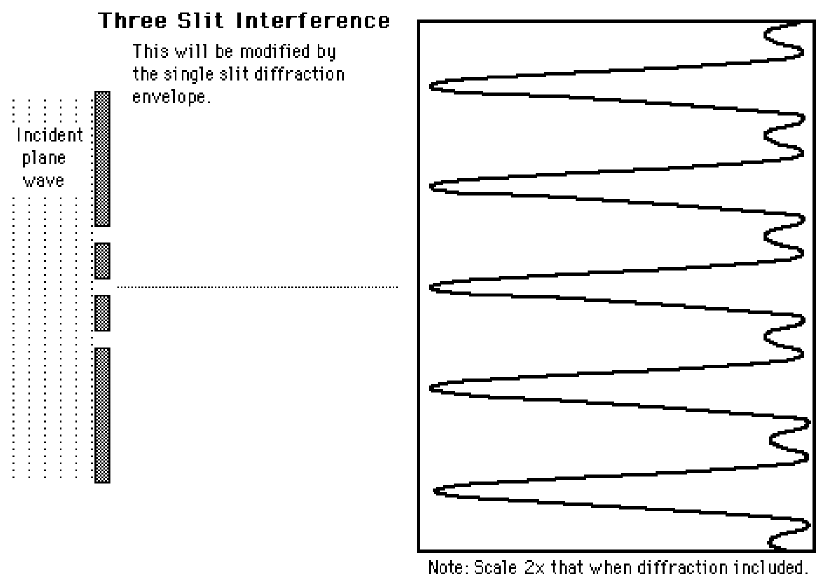
for the case of 2 slits

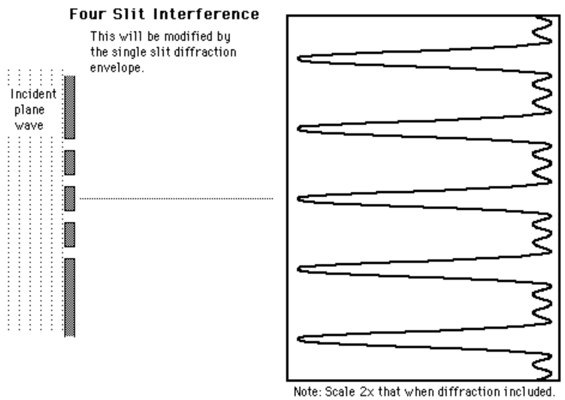
$$\delta = \frac{2m\pi}{N} = \frac{2m\pi}{2} \qquad m = 0, 1, 2, 3, 4$$

$$= \boxed{0,}\ \boxed{\pi,}\ \boxed{2\pi,}\ \boxed{3\pi,}\ \boxed{4\pi} \ldots.$$

For N slits:
There are (N-1) minima between each pair of principal maxima.

There are (N-2) secondary maxima





For 4 slits, we already know:

First minimum:

$$\delta = \frac{2\pi}{4} = \frac{\pi}{2}$$

Second minimum:

$$\delta = \frac{4\pi}{4} = \pi$$

First secondary max (between first and second minima):

$$\delta = (\frac{\pi}{2} + \pi)/2 = 0.75\pi = 135^o$$