Electromagnetism I – Solutions problem sheet 10

1. The emf is given by:

$$\epsilon = -\frac{d\Phi}{dt} \, .$$

The magnetic field is perpendicular to the area, A, of the loop. When the bar has reached a generic velocity v the magnitude of the emf is:

$$|\epsilon| = \frac{d\Phi}{dt} = B\frac{dA}{dt} \qquad \textbf{[1 mark]}$$

$$= B\frac{L\,v\,dt}{dt} = BLv \qquad \textbf{[1 mark]}$$

2. The terminal velocity v_t is reached when

$$F_g = F_B \, , \qquad \textbf{[1 mark]}$$

where:

$$F_g = mg\sin\alpha \qquad \textbf{[1 mark]}$$

and

$$F_B = BLI = BL\frac{\epsilon}{R} = BL\frac{BLv}{R} = \frac{(BL)^2 v}{R} \, . \qquad \textbf{[1 mark]}$$

Hence:

$$mg\sin\alpha = \frac{(BL)^2 v_t}{R}$$

and

$$v_t = \frac{R}{(BL)^2} mg\sin\alpha \qquad \textbf{[1 mark]}$$

3. The energy is dissipated at a rate:

$$P_e = \epsilon I = \frac{BLv_t}{R}\,BLv_t = \frac{(BLv_t)^2}{R} \qquad \textbf{[1 mark]}$$

and the rate at which the gravitational force is doing work is:

$$P_g = \vec{F}_g \cdot \vec{v}_t = mg(\sin\alpha)v_t \qquad \textbf{[1 mark]}$$

now substitution the value for v_t

$$P_e = \left[BL\frac{R}{(BL)^2} mg\sin\alpha\right]^2 \frac{1}{R}$$

$$= \frac{R}{(BL)^2}(mg\sin\alpha)^2 \qquad \textbf{[1 mark]}$$

and

$$\begin{aligned} P_g &= (mg\sin\alpha)\frac{R}{(BL)^2}(mg\sin\alpha) \\ &= \frac{R}{(BL)^2}(mg\sin\alpha)^2 \qquad [\mathbf{1\ mark}] \end{aligned}$$

which shows that:

$$P_e = P_g\,,$$

which are the same