Average Speed (Maxwell-Boltzmann)

$$P_r(v) = 4\pi\left(\frac{m}{2\pi k_B T}\right)^{\frac{3}{2}} v^2 e^{-\left(\frac{mv^2}{2k_B T}\right)}$$

$$\langle v \rangle = \int_0^\infty v\, P_r(v)\, dv$$

$$a = 4\pi\left(\frac{m}{2\pi k_B T}\right)^{\frac{3}{2}}$$

$$b = \frac{m}{2k_B T}$$

$$\langle v \rangle = a\int_0^\infty v^3 e^{-bv^2}\, dv$$

Substitute again $\quad u = v^2 \qquad du = 2v\,dv$

$$\langle v \rangle = \frac{a}{2}\int_0^\infty u\, e^{-bu}\, du$$

integrate by parts

$$\langle v \rangle = \frac{a}{2}\left(\overset{0}{\left[-u\,\frac{1}{b}\,e^{-bu}\right]_0^\infty} - \int_0^\infty -1 \times \frac{1}{b}\, e^{-bu}\, du\right)$$

$$\langle v \rangle = \frac{a}{2}\left(0 + \frac{1}{b}\int_0^\infty e^{-bu}\, du\right)$$

$$= \frac{a}{2b}\left[-\frac{1}{b}e^{-bu}\right]_0^\infty = \frac{a}{2b^2}\left[0 - -1\right]$$

$$\therefore \langle v \rangle = \frac{a}{2b^2}$$

$$\langle v \rangle = \frac{4\pi\left(\frac{m}{2\pi k_B T}\right)^{\frac{3}{2}}}{2\,\frac{m^2}{k_B^2 T^2}} = \sqrt{\frac{8k_B T}{\pi m}} = \cancel{\sqrt{\ }}\,\frac{2}{\sqrt{\pi}}\, v_{prob}$$

The average speed is $\frac{2}{\sqrt{\pi}}$ times larger than the most probable speed.