Electromagnetism

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Lecture 7

Electric Dipoles

Week 4

Last Lecture

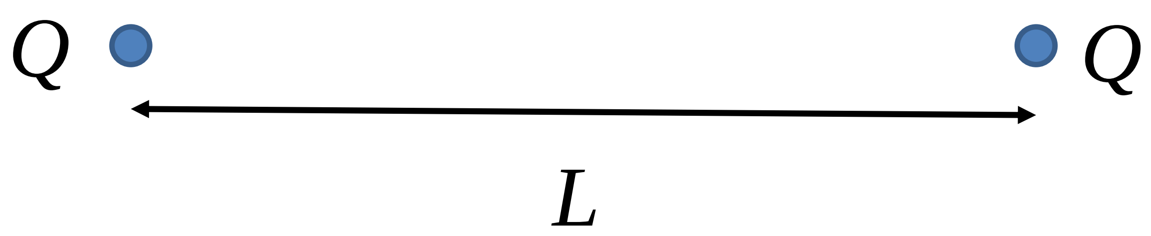
- Calculating electrical potential
- Calculating V if E-field known
- Calculating E-field from V
  - I.e. Using $\underline{E} = -\nabla V$
- Calculating change in potential energy

This Lecture

- Another example of calculating electric potential and then E-field.
- The electric dipole

Extra Example by Request

- Consider two identical charges, Q a distance L apart.
- What is the potential exactly in between the two charges?
- Use $\underline{E} = -\nabla V$ to calculate the E-field at this point

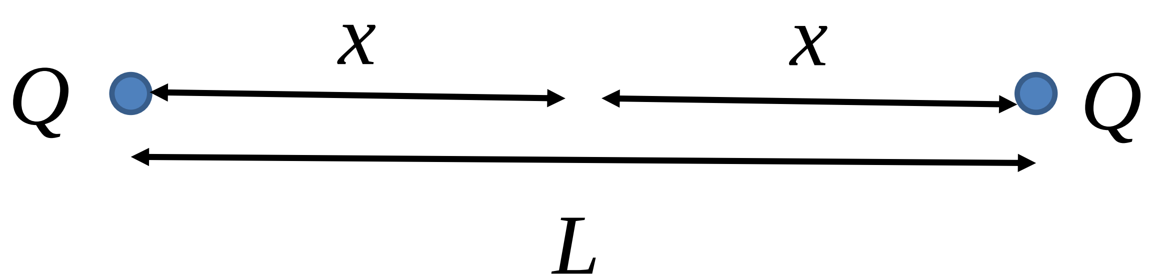


Extra Example by Request

- First let's see how NOT to solve this problem.
- Potential in the middle, a distance x from each charge (sum potential from each charge):

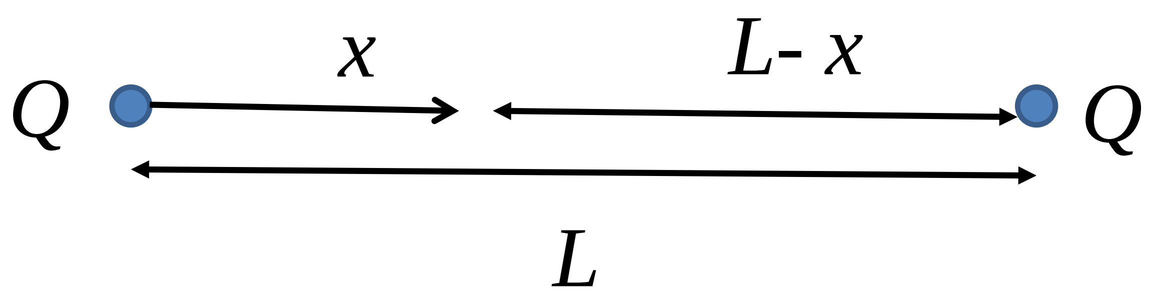
- $V = \dfrac{Q}{4\pi\varepsilon_0 x} + \dfrac{Q}{4\pi\varepsilon_0 x} = \dfrac{Q}{2\pi\varepsilon_0 x}$

$\underline{E} = -\dfrac{dV}{dx} = \dfrac{Q}{2\pi\varepsilon_0 x^2}$! But E should = 0 !!



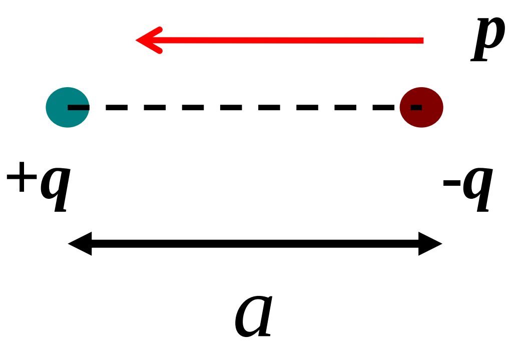
Extra Example by Request

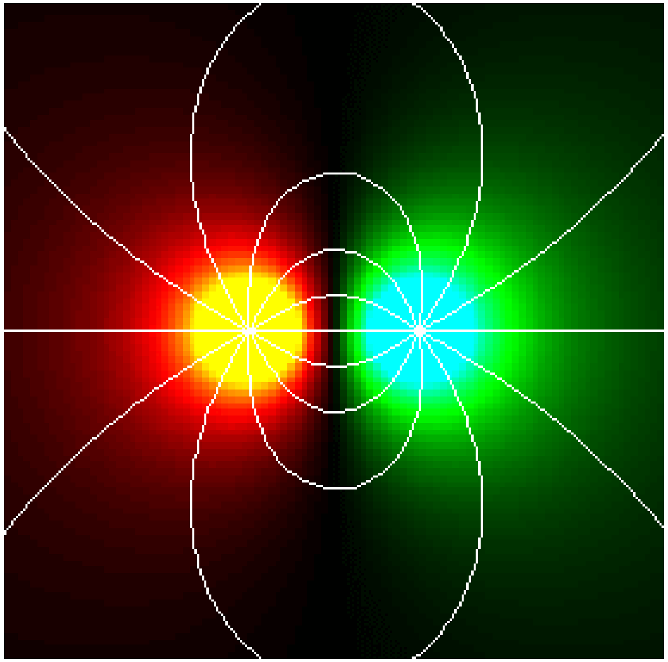
- By considering the point in the middle we have fixed x to $L/2$ i.e. made x a constant hence $\frac{dV}{dx} = 0$
- Instead, consider more general problem of potential a distance x from LHS charge.
- Let’s do this on the visualizer.



Electric Dipoles

An Electric Dipole



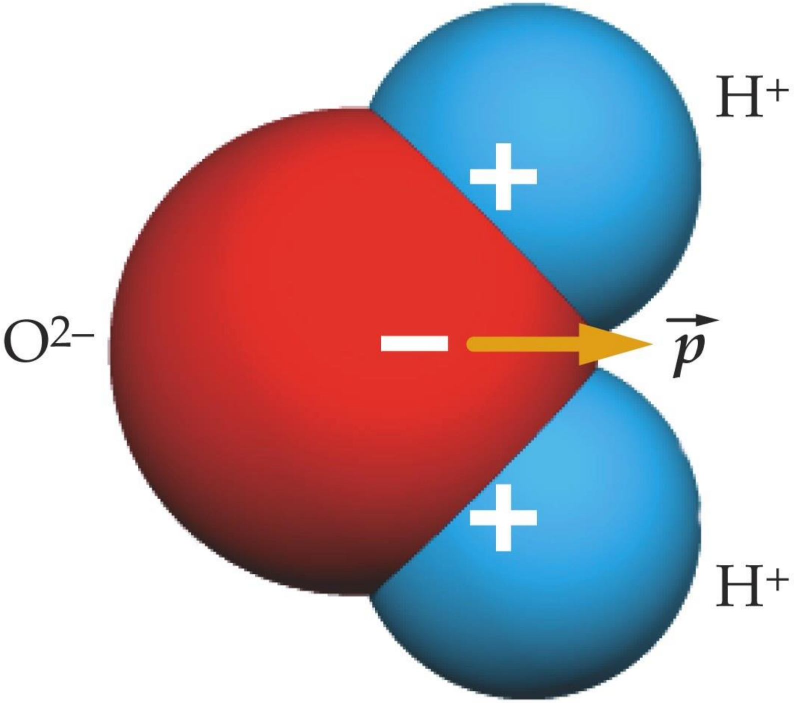


Define dipole moment as $\underline{\boldsymbol{p}} = q\underline{\boldsymbol{a}}$

The vector of **p** is drawn from the negative to the positive point charge **p** is a vector.

Natural Dipoles

- Many molecules form natural dipoles
- Molecular example: H_2O

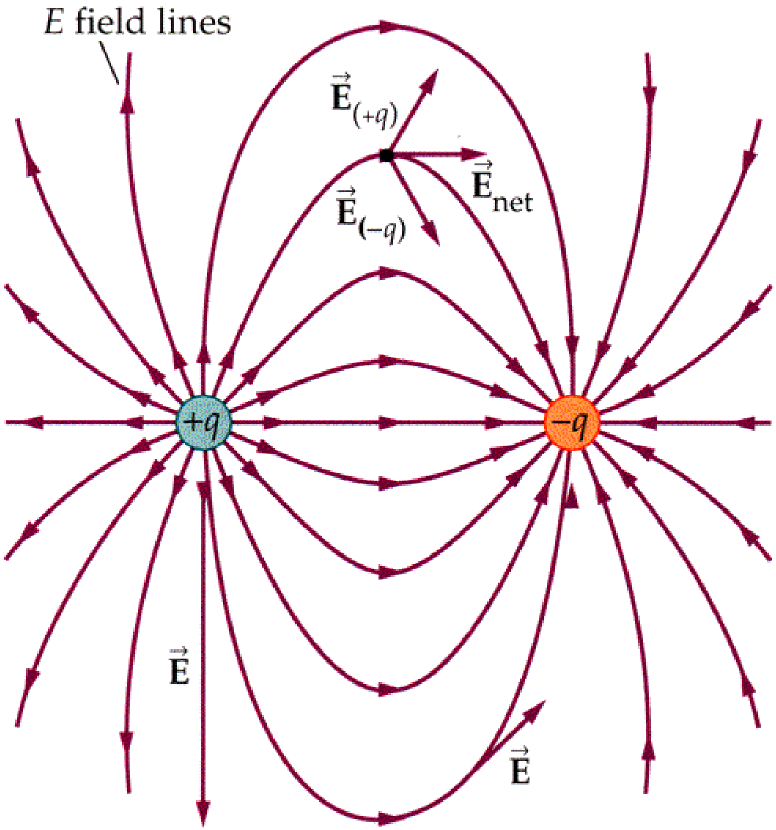


The electric dipole of water makes it an excellent solvent (able to dissolve other substances)

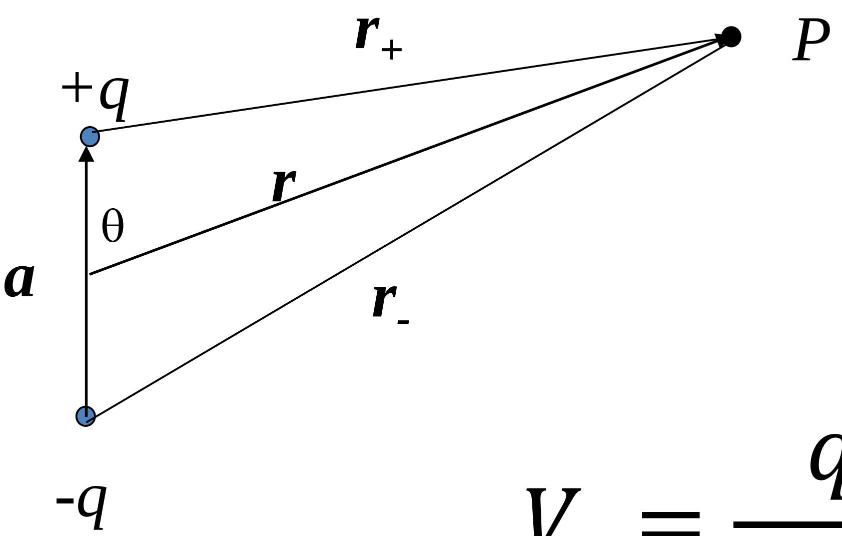
Used in heating food in a microwave oven
(see later)

p = 6.1 x 10^{-30} C m

Calculation of the E-field at an arbitrary (r, θ)



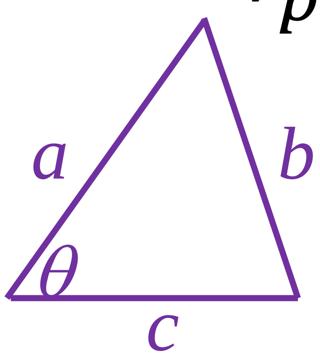
Potential due to Electric Dipole



- V at P due to two charges:
- $V_p = \dfrac{q}{4\pi\varepsilon_0 r_+} - \dfrac{q}{4\pi\varepsilon_0 r_-}$

$$V_p = \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r_+} - \frac{1}{r_-}\right)$$

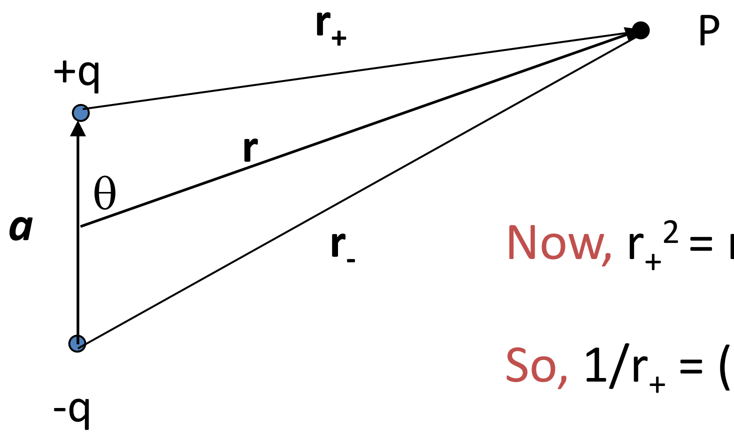
Use



$b^2 = a^2 + c^2 - 2ac\cos\theta$

Any triangle (GCSE maths)

Potential due to Electric Dipole



Potential at P is $(q/4\pi\varepsilon_0 r_+) + (-q/\ 4\pi\varepsilon_0 r_-)$

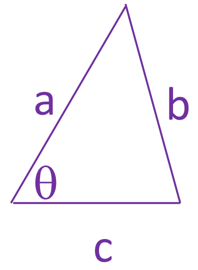
$= q/4\pi\varepsilon_0\ [\ (1/r_+) - (1/r_-)\ ]$

Now, $r_+^2 = r^2 + (a/2)^2 - ar\cos\theta$ and $r_-^2 = r^2 + (a/2)^2 + ar\cos\theta$

So, $1/r_+ = (r_+^2)^{-1/2} = (r^2[1 + a^2/4r^2 - (a/r)\cos\theta])^{-1/2}$

$= (1/r)\ (1 + a^2/4r^2 - (a/r)\cos\theta)^{-1/2}$

$= (1/r)\ (1 - (1/2)[a^2/4r^2 - (a/r)\cos\theta] + \text{higher order terms})$



$b^2 = a^2 + c^2 - 2ac\cos\theta$

Any triangle

Remember: $(1 + x)^n = 1 + nx + n(n\text{-}1)x^2/2! + \ldots\ldots$

For $x << 1$: $(1 + x)^n \approx 1 + nx$

Electric Dipole continued

- Consider the case when $r >> a$ (usually the case):

For r >> a we have $(1/r_+) \approx (1/r)\,(1 + (a/2r)\cos\theta) = 1/r + (a/2r^2)\cos\theta$

Similarly, $(1/r_-) \approx 1/r - (a/2r^2)\cos\theta$

So $V(r) = (q/4\pi\varepsilon_0)(\,[1/r + (a/2r^2)\cos\theta] - [1/r - (a/2r^2)\cos\theta]\,)$
$= (q/4\pi\varepsilon_0)(\,(a/r^2)\cos\theta\,)$

i.e. $V(r) = qa\cos\theta / 4\pi\varepsilon_0 r^2 = p\cos\theta / 4\pi\varepsilon_0 r^2$ (as p = qa)
$= pr\cos\theta / 4\pi\varepsilon_0 r^3$

$$\boxed{V(r) = \mathbf{p}.\mathbf{r} / 4\pi\varepsilon_0 r^3} \qquad (r>>a)$$

Electric Dipole continued

- $V_p \approx \frac{\underline{p} \cdot \underline{r}}{4\pi\varepsilon_0 r^3} = \frac{pr\cos\theta}{4\pi\varepsilon_0 r^3} = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$

- Note it drops off as ${}^{1}/_{r^2}$

- The sign of V_p depends on the angle θ

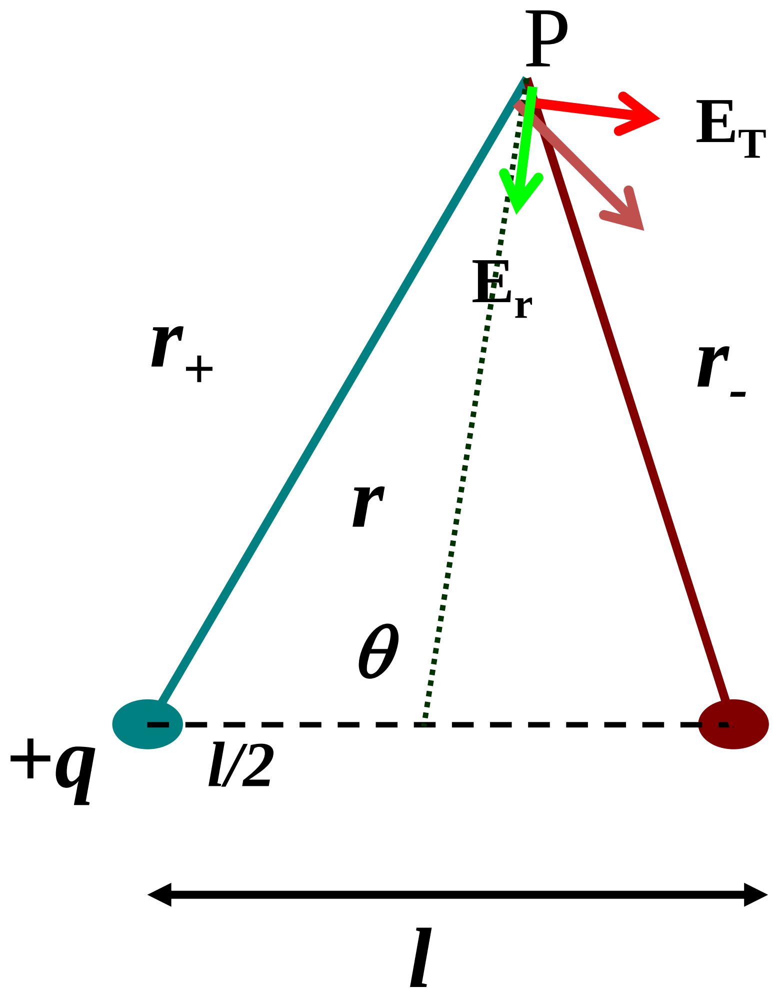
$\underline{E}$-field from Electric Dipole

- Now $\underline{\boldsymbol{E}} = -\nabla V$
- In plane polar coordinates:
- $\underline{E} = -\left(\frac{\partial V}{\partial r}\underline{\hat{\boldsymbol{r}}} + \frac{1}{r}\frac{dV}{d\theta}\underline{\widehat{\boldsymbol{\theta}}}\right)$
- $V_p \approx \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}$
- Let's do it on the visualizer
- $\underline{E} = -\left(\frac{-2\,p\cos\theta}{4\pi\varepsilon_0 r^3}\underline{\hat{\boldsymbol{r}}} + \frac{1}{r}\frac{-p\sin\theta}{4\pi\varepsilon_0 r^2}\underline{\widehat{\boldsymbol{\theta}}}\right)$

E-field from Electric Dipole

- $\underline{E} = -\left(\frac{-2\,p\cos\theta}{4\pi\varepsilon_0 r^3}\underline{\hat{\boldsymbol{r}}} + \frac{1}{r}\frac{-p\sin\theta}{4\pi\varepsilon_0 r^2}\underline{\widehat{\boldsymbol{\theta}}}\right)$ i.e.
- $\underline{E} = \left(\frac{2\,p\cos\theta}{4\pi\varepsilon_0 r^3}\underline{\hat{\boldsymbol{r}}} + \frac{p\sin\theta}{4\pi\varepsilon_0 r^3}\underline{\widehat{\boldsymbol{\theta}}}\right)$
- Note E-field drops as ${}^{1}/_{r^3}$ with r and has radial and transverse components.

E-field from Electric Dipole



$$\underline{\boldsymbol{E}}_r = \frac{2\,p\cos\theta}{4\pi\varepsilon_0 r^3}\,\underline{\hat{\boldsymbol{r}}}$$

$$\underline{\boldsymbol{E}}_T = \frac{p\sin\theta}{4\pi\varepsilon_0 r^3}\,\underline{\hat{\boldsymbol{\theta}}}$$

If $\theta=0$, $E_T=0$

$\theta=90$, $E_r=0$

Electrical Dipole Summary so far

- A dipole is two identical but opposite charges separated by a distance, say, $\underline{\boldsymbol{a}}$
- Define dipole moment as $\underline{\boldsymbol{p}} = q\underline{\boldsymbol{a}}$
- $V_p \approx \dfrac{p\cos\theta}{4\pi\varepsilon_0 r^2}$ (for $r >> a$)
- $\underline{\boldsymbol{E}} \approx \dfrac{2\,p\cos\theta}{4\pi\varepsilon_0 r^3}\underline{\hat{\boldsymbol{r}}} + \dfrac{p\sin\theta}{4\pi\varepsilon_0 r^3}\underline{\hat{\boldsymbol{\theta}}}$ (for $r >> a$)
- (Note: if I defined the θ as the angle between $\underline{\boldsymbol{r}}$ and the *negative* charge $cos\theta \rightarrow -\cos\theta$ and $sin\theta \rightarrow -sin\theta$)