Entangled Basis Finite Element Method PDE solver for Quantum Computer

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1. FINITE ELEMENT METHOD (FEM)

2. TENSOR NETWORK (TN)

3. TARGET EQUATION

In this work, we will focus on a class of analytic partial differential equation (A-PDE) of a function of time and space, $u(t, q_i | i \in \{1, 2, \cdots, D\})$, where we use with $\mathbf{q}$ be spatial vector, that is linear in time (LTA-PDE). This PDE can be written in the form

$$D_t u + h\left(t, \mathbf{q}, u, \partial_{\mathbf{q}} u, {\mathbf{D}_{\mathbf{q}}}^2 u, \cdots\right) = 0, \tag{3.1}$$

where D_t is time derivative, D_q is space derivative, and h is an analytic function which can be written as

$$h\left(t, q, u, D_q u, D_q^2 u, \cdots\right) = \sum_{p_t, p_q, p_0, \cdots} h_{p_t, p_q, p_0, \cdots} t^{p_t} q^{p_q} u^{p_0} \left(D_q u\right)^{p_1} \left(D_q^2 u\right)^{p_2} \cdots . \tag{3.2}$$

Notice that $h_{p_t, p_q, p_0, \cdots}$ is a rank $\prod_{r=0}(r \cdot p_r)$ tensors.

4. FEM REPRESENTATION OF LTA-PDE

5. TENSOR OPTIMIZATION

6. MATRIX PRODUCT STATE (MPS) REPRESENTATION

7. IMPLEMENTATION