MATH 325 QUIZ 10

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1. Solve the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2$, $a_0 = 1, a_1 = 0$.
   As part of your answer, include the following and related work for parts (b) and (e).

   (a) State the characteristic equation.

   The characteristic equation is: $r^2 = 5r^{n-1} - 6r^{n-2}$
   $\implies r^2 - 5r + 6 = 0$

   (b) What are the solutions to the characteristic equation?

   $(r-3)(r-2) = 0 \implies$ solutions: $3, 2$

   (c) Does Theorem 1 or Theorem 2 apply?

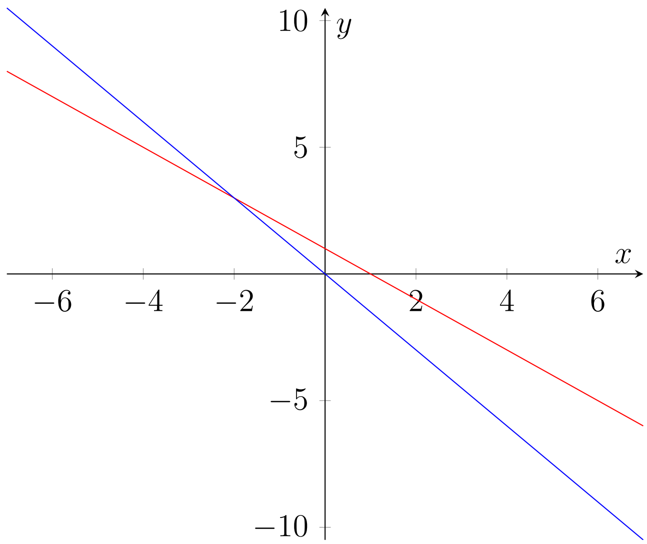
   Because there are two, unique roots, Theorem 1 applies.

   (d) Write out the general form of the solution given by the appropriate theorem.

   The general form of the equation using Theorem 1 would be:
   $a_n = \alpha_1 \cdot 3^n + \alpha_2 \cdot 2^n$

   (e) Give the final answer after solving for any constants.

   $a_0 = 1 = \alpha_1 \cdot 3^0 + \alpha_2 \cdot 2^0 = \alpha_1 + \alpha_2$
   $a_1 = 0 = \alpha_1 \cdot 3^1 + \alpha_2 \cdot 2^1 = \alpha_1 \cdot 3 + \alpha_2 \cdot 2$
   Therefore, we can solve this system using graphing.



Because we can see that the two lines intersect at $(-2, 3)$,
$\alpha_1 = -2$ and $\alpha_2 = 3$.
Our final answer is:
$a_n = (-2) \cdot 3^n + (3) \cdot 2^n$

2. Solve by defining an appropriate generating function:

   In how many ways can 25 identical donuts be distributed to four police officers so that each officer gets at least three but no more than seven donuts?

   The generating function is as follows: $(x^3 + x^4 + x^5 + x^6 + x^7)^4$
   There are 20 ways to distribute the 25 donuts.