

CISS430 Lecture 3: Data models

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Data models I

- **Data model**: Model/notation for managing data. Made up of 3 parts:
 - Structure
 - Operations
 - Constraints
- **Structure**:
 - Very high level description
 - Higher level than data structures
 - Also called **conceptual model**
- **Operations**:
 - Operations available on the structure
 - More limited than operations in standard programming langs

Data models II

- **Constraints**:
 - Limits on what data can be stored
- Two important data models:
 - Relational (includes object-relational model)
 - Semistructured (includes XML)

Relational data model (brief) I

- Structure:

Movies

title	year	length	genre
Gone with the wind	1939	231	drama
Star Wars	1977	124	scifi
Wayne's World	1992	95	comedy

- Operations:

- Relational algebra**
- Table-oriented operations: input is tables, output is table

Relational data model (brief) II

- Constraints:
 - Example: Uniqueness of a value in a column
 - Example: Type of values that can go into a column

Semistructured data model (brief) I

- XML – hierarchical, nested tagged elements
- Structure:

```
<Movies>
  <Movie title="Gone with the wind">
    <Year>1939</Year>
    <Length>231</Length>
    <Genre>drama</Genre>
  </Movie>
  <Movie title="Star Wars">
    <Year>1977</Year>
    <Length>124</Length>
    <Genre>scifi</Genre>
  </Movie>
  <Movie title="Wayne's World">
    <Year>1992</Year>
    <Length>95</Length>
```

Semistructured data model (brief) II

```
    <Genre>comedy</Genre>  
  </Movie>  
</Movies>
```

- Operations: Tree/forest operations. Follow path in tree.
- Constraints: Can limit value for a tag element, etc.

Other data models I

- Object-relational data model: Add object-oriented idea to relational data model. Values can be structure instead of just basic types like integers, strings, etc. Relations can have methods.
- Object-oriented data model: Data model is made up of objects.
- Hierarchical data model
- Network data model

Comparison I

- Semistructure appears to be more flexible than relations - graphs can be described using semistructured data models.
- But relational still preferred in DBMS
- Why?
- DB are large:
 - Efficiency of access and modification is important
 - Ease of use very important - productivity of programmers.
- Relational data model:
 - Very simple, very limited structure. Reasonably versatile.
 - Limited operations (you'll see that there are very few operations) but sufficiently useful.

Comparison II

- Limitations become features.
- Allows efficient implementation of SQL.
- Programs in SQL very short and compact.
- So SQL programs can be optimized and can be faster than code written by hand in other non-SQL languages.

Relational data model: structure I

- Recall example:

Movies

title	year	length	genre
Gone with the wind	1939	231	drama
Star Wars	1977	124	scifi
Wayne's World	1992	95	comedy

- Structure of relational data model is made up of two parts:
 - Attributes – set of names with data types
 - Data – set of tuples

Relational data model: structure II

- For each tuple, there is one data for each attribute
- Note: The attributes is treated as a *set*, not as a tuple. In implementation, attributes are usually ordered.
- Tuple: one value for each attribute of the relation. The value of a tuple is also called a component of the tuple. NOTE: The collection of tuples in a relation is considered a *set*.
- **Schema**: name of relation and attributes. Example:

`Movies(title, year, length, genre)`

- Relational model = one or more relations

Relational data model: structure III

- Database schema = set of schemas for all the relations in a database.
- **Domain**: The set of possible values for the values of an attribute.
 - Must be atomic such as integer or string type.
 - NOT record/struct/array/set/list.
 - Also called data type.
 - Example: Schema with data types
`Movies(title:string, year:integer,
length:integer, genre:string)`
- (Relation can change with time. An **instance** of a relation is the relation at one point in time. Conventional DB usually keeps only one instance of a relation.)

Relational data model: structure IV

- Given two relations R and S , you have two sets of attributes, one from R and one from S . The two sets of attributes are the same if their names are the same and their data types are the same.
- Given two relations R and S , you can only compare tuples $r \in R$ and $s \in S$ (by equality) only if R and S have the same attributes. $r = s$ if for each attribute, the corresponding values from r and s are the same.
- Given two relations R and S , we say that $R \subseteq S$ if R and S have the same attributes and each tuple of R is a tuple of S , i.e., for each tuple $r \in R$, there is some tuple $s \in S$ such that $r = s$.

Relational data model: structure V

- Given two relations R and S , $R = S$ if $R \subseteq S$ and $S \subseteq R$.
 - $R = S$ simply means R and S have the same attributes and they have the same set of tuples.

Relational data model: constraints I

- Many possible constraints.
- Key constraint.
 - **Key**: Set of attributes of a relation such that tuples in the relation are uniquely identified by the values of the attributes of a key.
- Example: If in the `Movies` relation, a movie is uniquely identified by `{title, year}`, then this is a key. This means that there are no two movies with the same title and same year. If `{title, year}` is a key of the `Movies` schema, then write

`Movies(title, year, length, genre)`

Relational data model: constraints II

- Example: If you have a schema that looks like this

Employee(fname, lname, ...)

you might want this:

Employee(fname, lname, ...)

But this means that you cannot have two employees with the same first and last name. Better to issue an employee id to each employee:

Employee(eid, fname, lname)

Relational data model: constraints III

- Example. C# is a corporate issued id.

Movies(title:string, year:integer, length:integer,
genre:string, studioName:string, producerC#)

MovieStar(name:string, address:string, gender:char,
birthdate:date)

StarsIn(movieTitle:string, movieYear:integer,
starName:string)

MovieExec(name:string, address:string, C#:integer,
netWorth:integer)

Studio(name:string, address:stirng, presC#:integer)

Relational data model: constraints IV

- In SQL there are two sublanguages:
 - **DDL** – data definition language. For managing schemas.
 - **DML** – data manipulation language. For managing data (tuples).
- SQL supports 3 relations
 - tables – Stored
 - views – Relation defined by computation. Not stored.
 - temporary tables – Create by SQL language processor. Not created by programs. Not stored.

Relational data model: relational algebra I

- **Relational algebra**: operations of relation data model.
 - Union, intersection, difference, cross/cartesian product
 - Selection, projection, rename
- \cup (union), \cap (intersection), $-$ (difference)
 - If R and S are relations, then $R \cup S$, $R \cap S$, $R - S$ is defined only when R and S have the same set of attributes.
 - If so, then the attributes of $R \cup S$, $R \cap S$, $R - S$ is the same as the attributes of R (or S).
- $R \cup S$ – Tuples are from R or S .
- $R \cap S$ – Tuples in both R and S .
- $R - S$ – Tuples in R but not in S .

Relational data model: relational algebra II

- $R \times S$ – Tuples of the form $(r_1, r_2, \dots, r_m, s_1, s_2, \dots, s_n)$ where (r_1, r_2, \dots, r_m) is a tuple in R and (s_1, s_2, \dots, s_n) is a tuple in S . If R has attribute a and S also has an attribute a , then distinguish them in $R \times S$ by calling them $R.a$ and $S.a$.

Relational data model: relational algebra III

- Example.

Relational data model: relational algebra IV

JohnFriends

id	name	bday
20	Tom	10/10/98
21	Mary	10/8/98
22	Harry	10/7/98

JaneFriends

id	name	bday
20	Tom	10/10/98
21	Sue	5/8/97
24	Harry	10/7/98

$\text{JohnFriends} \cup \text{JaneFriends}$

id	name	bday
20	Tom	10/10/98
21	Mary	10/8/98
22	Harry	10/7/98
21	Sue	5/8/97
24	Harry	10/7/98

$\text{JohnFriends} \cap \text{JaneFriends}$

id	name	bday
20	Tom	10/10/98

$\text{JohnFriends} - \text{JaneFriends}$

id	name	bday
21	Mary	10/8/98
22	Harry	10/7/98

Relational data model: relational algebra V

JohnFriends \times JaneFriends

JohnFriends.id	JohnFriends.name	JohnFriends.bday	JaneFriends.id	JaneFriends.name	JaneFriends.bday
20	Tom	10/10/98	20	Tom	10/10/98
20	Tom	10/10/98	21	Sue	5/8/97
20	Tom	10/10/98	24	Harry	10/7/98
21	Mary	10/8/98	20	Tom	10/10/98
21	Mary	10/8/98	21	Sue	5/8/97
21	Mary	10/8/98	24	Harry	10/7/98
22	Harry	10/7/98	20	Tom	10/10/98
22	Harry	10/7/98	21	Sue	5/8/97
22	Harry	10/7/98	24	Harry	10/7/98

Relational data model: relational algebra VI

- **Selection:**

$\sigma_c(R)$ = same as R but only keep tuples satisfying c

c is a propositional formula on the attributes of R .

- Assume boolean operators $=$ and \neq available for any data type. For numeric data types, assume $\leq, <, \geq, >$ available.
- Example. $\sigma_{\text{numfingers} > 5}(\text{Persons})$ gives relation of persons with more than 5 fingers.

Relational data model: relational algebra VII

- Example. If

Reserves

sid	bid	day
22	101	10/10/98
22	103	10/8/98
22	104	10/7/98

then $\sigma_{\text{bid} > 102}(\text{Reserves})$ is

sid	bid	day
22	103	10/8/98
22	104	10/7/98

Relational data model: relational algebra VIII

- **Projection:**

$\pi_{A_1, \dots, A_n}(R)$ = same as R but only keep attributes A_1, \dots, A_n

A_1, \dots, A_n must be attributes of R .

- Example. If

Reserves

sid	bid	day
22	101	10/10/98
22	103	10/8/98
22	104	10/7/98

then $\pi_{\text{sid}, \text{bid}}(\text{Reserves})$ is

Relational data model: relational algebra IX

sid	bid
22	101
22	103
22	104

Relational data model: relational algebra X

- **Rename:**

- $\rho(T, R)$: Make a copy of R and call it T .
- $\rho(T(A_1 \rightarrow A'_1, A_2 \rightarrow A'_2, \dots), R)$: Same as above and also rename attribute A_1 to A'_1 , etc.
- $\rho(T(1 \rightarrow A'_1, 2 \rightarrow A'_2, \dots), R)$: Same as above except first attribute is renamed A'_1 , second attribute is renamed as A'_2 , etc.

Relational data model: relational algebra XI

- Example. If

Reserves

sid	bid	day
22	103	10/8/98
22	104	10/7/98

then $\rho(X(\text{sid} \rightarrow s, \text{day} \rightarrow x), \text{Reserves})$ creates

X

s	bid	x
22	103	10/8/98
22	104	10/7/98

- Note that $R \cap S = R - (R - S)$. So \cap is not really necessary. But it's important and common enough to be a separate operator for clarity and optimization.

Relational data model: relational algebra XII

- **Join**:
 - **Inner join**: Shorthand for “selection of a cross product”:

$$R \bowtie_c S = \sigma_c(R \times S)$$

Also called **conditional join**.

- **Equi-join**: Join where c is the 'logical and' (\wedge) of one or more equalities. Example:

$$\text{Humans} \bowtie_{(\text{Humans.telescopes}=\text{Planet.moons})} \text{Planets}$$

i.e., humans match with planets where the number of telescopes owned by a human is exactly the number of moons (satellites) as the planet.

Relational data model: relational algebra XIII

- **Natural join**: Inner join where c involves the keys of R and S with the same name. Example: Assume that in the Cars and Bikes table, (owner, color) is a key.

$\text{Cars} \bowtie \text{Bikes}$

$= \text{Cars} \times_{\text{Cars.owner=Bikes.owner} \wedge \text{Cars.color=Bikes.color}} \text{Bikes}$

- **Left outer join**: Like inner join except that unmatched tuples of left-hand side relation is included and the right-hand side tuples set to NULL.
- **Right outer join**: Like inner join except that unmatched tuples of right-hand side relation is included and the left-hand side tuples set to NULL.
- **Outer join**: Union of left and right outer joins.

Examples I

- Will use the following relations.
- Schemas:

```
Sailors(sid: integer, sname: string, rating: integer,  
       age: real)
```

```
Boats(bid: integer, bname: string, color: string)
```

```
Reserves(sid: integer, bid: integer, date: date)
```

Examples II

- Data:

Sailors

sid	sname	rating	age
22	Dustin	7	45.0
29	Brutus	1	33.0
31	Lubber	8	55.5
32	Andy	8	25.5
58	Rusty	10	35.0
64	Horatio	7	35.0
71	Zorba	10	16.0
74	Horatio	9	35.0
85	Art	3	25.5
95	Bob	3	63.5

Boats

bid	bname	color
101	Interlake	blue
102	Interlake	red
103	Clipper	green
104	Marine	red

Reserves

sid	bid	date
22	101	10/10/98
22	102	10/10/98
22	103	10/8/98
22	104	10/7/98
31	102	11/10/98
31	103	11/6/98
31	104	11/12/98
64	101	9/5/98
64	102	9/8/98
74	103	9/8/98

Examples III

- Query 1. Names of all sailors who reserved boat with id 103.

Solution.

$$\pi_{\text{name}}(\sigma_{\text{bid}=103}(\text{Reserves}) \bowtie \text{Sailors})$$

Also:

$$\rho(\text{Temp}_1, \sigma_{\text{bid}=103}(\text{Reserves}))$$

$$\rho(\text{Temp}_2, \text{Temp}_1 \bowtie \text{Sailors})$$

$$\pi_{\text{name}}(\text{Temp}_2)$$

Examples IV

Note that

$$\pi_{\text{sname}}(\sigma_{\text{bid}=103}(\text{Reserves}) \bowtie \text{Sailors}) \quad (1)$$

and

$$\pi_{\text{sname}}(\sigma_{\text{bid}=103}(\text{Reserves} \bowtie \text{Sailors})) \quad (2)$$

gives the same result.

(2) is probably more efficient: Selection occurs earlier so that the join operation has a smaller left hand side parameter.

Query optimizer will choose the expression that optimizes the computation of the resulting relation.

Examples V

- Query 2. Names of sailors who reserved a red boat.

Solutions:

$$\pi_{\text{sname}} (\sigma_{\text{color}=\text{'red'}}(\text{Boats}) \bowtie \text{Reserves} \bowtie \text{Sailors})$$

or

$$\pi_{\text{sname}} (\pi_{\text{sid}} (\sigma_{\text{color}=\text{'red'}}(\text{Boats}) \bowtie \text{Reserves}) \bowtie \text{Sailors})$$

Which is better?

Examples VI

- Query 2. Names of sailors who reserved a red boat.

Solutions:

$$\pi_{\text{sname}} (\sigma_{\text{color}=\text{'red'}} (\text{Boats}) \bowtie \text{Reserves} \bowtie \text{Sailors})$$

or

$$\pi_{\text{sname}} (\pi_{\text{sid}} (\sigma_{\text{color}=\text{'red'}} (\text{Boats}) \bowtie \text{Reserves}) \bowtie \text{Sailors})$$

Which is better?

Examples VII

- Query 3. Colors of boats reserved by Lubber.

Solutions:

$$\pi_{\text{color}} (\sigma_{\text{sname}='Lubber'} (\text{Sailors}) \bowtie \text{Reserves} \bowtie \text{Boats})$$

Examples VIII

- Query 4. Names of sailors who reserved at least one boat.

Solutions:

$$\pi_{\text{sname}}(\text{Sailors} \bowtie \text{Reserves})$$

Note: There are two Sailors tuples with `sname = 'Horatio'` that have reserved a boat, but result is only one 'Horatio' since result is a relation, and relation is a set of tuples – no duplicates.

- Notice that joins are very common.

Examples IX

- Query 5. Names of sailors who reserved a red or green boat.

Solution:

$$\rho(T, \sigma_{\text{color}='red'}(\text{Boats}) \cup \sigma_{\text{color}='green'}(\text{Boats})) \\ \pi_{\text{sname}}(T \bowtie \text{Reserves} \bowtie \text{Sailors})$$

or

$$\rho(T, \sigma_{\text{color}='red' \vee \text{color}='green'}(\text{Boats})) \\ \pi_{\text{sname}}(T \bowtie \text{Reserves} \bowtie \text{Sailors})$$

Examples X

- Query 6. Names of sailors who reserved a red and a green boat.

Solution:

Tempting ...

$$\rho(T, \sigma_{\text{color}=\text{'red'}}(\text{Boats}) \cap \sigma_{\text{color}=\text{'green'}}(\text{Boats})) \\ \pi_{\text{sname}}(T \bowtie \text{Reserves} \bowtie \text{Sailors})$$

Wrong! T is relation of boats which are both red and green. This gives sailors s such that s reserved a red boat and s reserved a green boat – and there are no such boats. It does

Examples XI

not ask for s such that s reserved a boat that is both red and green (there are no such boats).

Therefore need to

- Find sailors who reserved a red boat
- Find sailors who reserved a green boat
- Intersect two relations above

Examples XII

But you have to be careful ...

$$\begin{aligned} & \rho(T_1, \pi_{\text{sname}}(\sigma_{\text{color}=\text{'red'}}(\text{Boats}) \bowtie \text{Reserves} \bowtie \text{Sailors})) \\ & \rho(T_2, \pi_{\text{sname}}(\sigma_{\text{color}=\text{'red'}}(\text{Boats}) \bowtie \text{Reserves} \bowtie \text{Sailors})) \\ & T_1 \cap T_2 \end{aligned}$$

Wrong! What if there are two sailors, both names John, one who reserved a red boat and the other reserved a green boat.

Examples XIII

So need sid.

$$\begin{aligned} & \rho(T_1, \pi_{\text{sid}, \text{sname}}(\sigma_{\text{color}=\text{'red'}}(\text{Boats}) \bowtie \text{Reserves} \bowtie \text{Sailors})) \\ & \rho(T_2, \pi_{\text{sid}, \text{sname}}(\sigma_{\text{color}=\text{'green'}}(\text{Boats}) \bowtie \text{Reserves} \bowtie \text{Sailors})) \\ & \pi_{\text{sname}}(T_1 \cap T_2) \end{aligned}$$

Or

$$\begin{aligned} & \rho(T_1, \pi_{\text{sid}}(\sigma_{\text{color}=\text{'red'}}(\text{Boats}) \bowtie \text{Reserves})) \\ & \rho(T_2, \pi_{\text{sid}}(\sigma_{\text{color}=\text{'green'}}(\text{Boats}) \bowtie \text{Reserves})) \\ & \pi_{\text{sname}}((T_1 \cap T_2) \bowtie \text{Sailors}) \end{aligned}$$

Examples XIV

- Query 7. Names of sailors who reserved at least two boats

Solution: We more or less want a relation containing sid, bid_1, bid_2 such that sailor with sid reserved boat with bid_1 and boat with bid_2 . The following

$$\begin{aligned} & \rho(R_1(sid \rightarrow sid_1, bid \rightarrow bid_1), \pi_{sid, bid}(Sailors \bowtie Reserves)) \\ & \rho(R_2(sid \rightarrow sid_2, bid \rightarrow bid_2), \pi_{sid, bid}(Sailors \bowtie Reserves)) \\ & \sigma_{sid_1=sid_2 \wedge bid_1 \neq bid_2}(R_1 \times R_2) \end{aligned}$$

gives us a table with four columns where row has values $(sid_1, sid_2, bid_1, bid_2)$ denoting the fact for sailor with sid_1 ($= sid_2$) reserved boats with bid_1 and bid_2 .

Examples XV

We need the sailor name, so we include that in R_1 :

$$\rho(R_1(\text{sid} \rightarrow \text{sid}_1, \text{bid} \rightarrow \text{bid}_1), \pi_{\text{sid}, \text{bid}, \text{sname}}(\text{Sailors} \bowtie \text{Reserves}))$$
$$\rho(R_2(\text{sid} \rightarrow \text{sid}_2, \text{bid} \rightarrow \text{bid}_2), \pi_{\text{sid}, \text{bid}}(\text{Sailors} \bowtie \text{Reserves}))$$
$$\pi_{\text{sname}}(\sigma_{\text{sid}_1 = \text{sid}_2 \wedge \text{bid}_1 \neq \text{bid}_2}(R_1 \times R_2))$$

Examples XVI

- Query 8. Find all sids of sailors with age over 20 who have not reserved a red boat.

Solution: The sids of sailors of age > 20 is

$$\pi_{\text{sid}}(\sigma_{\text{age} > 20}(\text{Sailors}))$$

The sids of sailors who reserved a red boat is

$$\pi_{\text{sid}}(\sigma_{\text{color} = \text{'red'}}(\text{Boats}) \bowtie \text{Reserves})$$

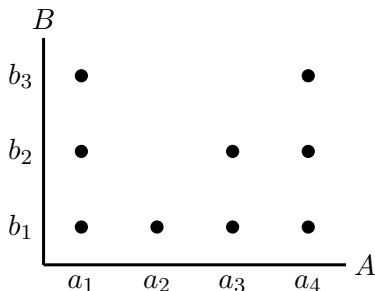
Therefore the answer is

$$\pi_{\text{sid}}(\sigma_{\text{age} > 20}(\text{Sailors})) - \pi_{\text{sid}}(\sigma_{\text{color} = \text{'red'}}(\text{Boats}) \bowtie \text{Reserves})$$

Examples XVII

- Query 9. Names of sailors who reserved all boats.

Solution: Look at this diagram ...



Examples XVIII

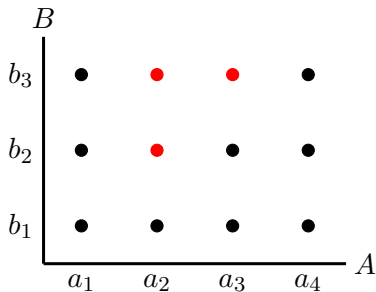
Think of A as the sailors (or their sids) and B as the boats (or their bids).

The set of black dots R represent the reservation relation. For instance the pair $(a_1, b_1) \in R$ represents the fact that sailor a_1 reserved b_1 .

So a_1 and a_4 have each reserved all boats. We want to compute $\{a_1, a_4\}$.

Note the following: The complement of R , i.e., $A \times B - R$ is denoted by the red dots:

Examples XIX



Examples XX

Furthermore, when you project down to A you get $\{a_2, a_3\}$, which is the A -complement of what we are after. Therefore what we are after is

$$\{a_2, a_3\} = \pi_A(A \times B - R)$$

Back to our schema, the sids of those who reserved all boats is given by

$$\pi_{\text{sid}}(\pi_{\text{sid,bid}}(\text{Sailors} \times \text{Boats}) - \pi_{\text{sid,bid}}(\text{Reserves}))$$

Examples XXI

To make it easy to read:

$$\rho(A, \pi_{\text{sid}}(\text{Sailors}))$$

$$\rho(B, \pi_{\text{bid}}(\text{Boats}))$$

$$\rho(R, \pi_{\text{sid}, \text{bid}}(\text{Reserves}))$$

$$\pi_{\text{sid}}(A \times B - R)$$

Examples XXII

But we need the snames. So we do this:

$$\begin{aligned} &\rho(A, \pi_{\text{sid}}(\text{Sailors})) \\ &\rho(B, \pi_{\text{bid}}(\text{Boats})) \\ &\rho(R, \pi_{\text{sid}, \text{bid}}(\text{Reserves})) \\ &\pi_{\text{sname}}(\pi_{\text{sid}}(A \times B - R) \bowtie \text{Sailors}) \end{aligned}$$

Note: If A and B are sets and R is a relation on $A \times B$ (i.e., $R \subseteq A \times B$), then the division A/B is defined as

$$A/B = \{a \mid (a, b) \in R \text{ for } \underline{\text{all}} \ b \in B\}$$

Examples XXIII

- Query 10. Names of all sailors who have reserved all boats named Interlake.

Solution: We use the same idea from Query 9:

$$\begin{aligned} & \rho(A, \pi_{\text{sid}}(\text{Sailors})) \\ & \rho(B, \pi_{\text{bid}}(\sigma_{\text{bname}='Interlake'}(\text{Boats}))) \\ & \rho(R, \pi_{\text{sid}, \text{bid}}(\text{Reserves})) \\ & \pi_{\text{sname}}(\pi_{\text{sid}}(A \times B - R) \bowtie \text{Sailors}) \end{aligned}$$

Aggregate functions I

- **Extended relational algebra** – the standard relational algebra with the aggregate operators below. The aggregate operator depends on 5 aggregate functions.
 - SUM – sum
 - MAX – maximum
 - MIN – minimum
 - AVG – average
 - COUNT – number of tuples in group

Aggregate functions II

- Suppose R is a relation containing attribute A which is numeric (integer or real). Then

$$G_{\text{SUM}(A)}(R)$$

is a relation with one attribute $\text{SUM}(A)$ and one tuple which is the sum of all values in column A of R .

Aggregate functions III

- Example.

R

A	B
2	b
5	a
1	c
9	a

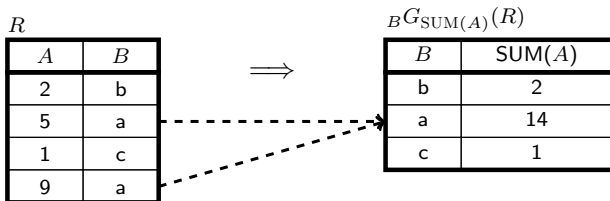


$G_{\text{SUM}(A)}(R)$

SUM(A)
17

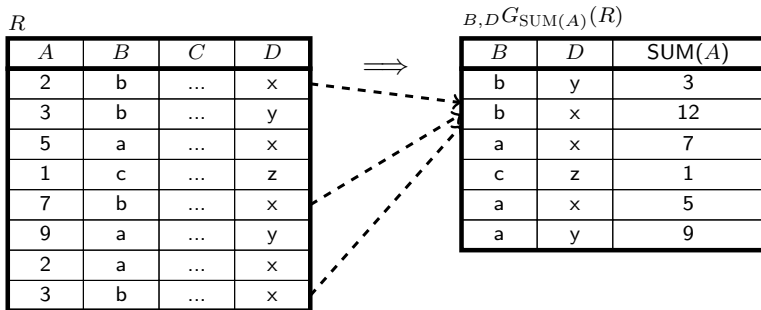
Aggregate functions IV

- You can group up tuples based on the values of attribute B .



Aggregate functions V

- You can group up tuples based on the values in attributes B and D and sum up the values of A .



Aggregate functions VI

- You can sum up the values in two columns:

 R

A	B	C
2	b	13
5	a	12
1	c	19
9	a	15

 $G_{\text{SUM}(A), \text{SUM}(C)}(R)$

$\text{SUM}(A)$	$\text{SUM}(C)$
17	59

Aggregate functions VII

- You can have multiple aggregate functions and group by multiple attributes

$$B, C G_{\text{SUM}(A), \text{MAX}(D), \text{SUM}(F)}(R)$$

where A, B, C, D, F are attributes of R and A, D are numeric. In this case, the resulting schema has 4 attributes:

- B
- C
- $\text{SUM}(A)$
- $\text{MAX}(D)$
- $\text{SUM}(F)$

Aggregate functions VIII

- The following are some other notations for $G_{\text{SUM}(A)}(R)$:

$$F_{\text{SUM}(A)}(R), \quad \mathcal{G}_{\text{SUM}(A)}(R), \quad \gamma_{\text{SUM}(A)}(R)$$

Identities I

- The following is an incomplete list of relational algebra identities.
- Set identities applies for $\cup, \cap, -, \times$. For instance

$$R \cup R' = R' \cup R$$

Identities II

- Selection: Let c, c' be predicates on attributes of R .

$$\sigma_c(\sigma_c(R)) = \sigma_c(R)$$

$$\sigma_c(\sigma_{c'}(R)) = \sigma_{c'}(\sigma_c(R)) = \sigma_{c' \wedge c}(R)$$

$$\sigma_c(R) \cap \sigma_{c'}(R) = \sigma_{c \wedge c'}(R)$$

$$\sigma_c(R) \cup \sigma_{c'}(R) = \sigma_{c \vee c'}(R)$$

$$\sigma_c(R) - \sigma_{c'}(R) = \sigma_{c \wedge \neg c'}(R)$$

$$\sigma_c(R \cup R') = \sigma_c(R) \cup \sigma_c(R')$$

$$\sigma_c(R \cap R') = \sigma_c(R) \cap \sigma_c(R')$$

$$\sigma_c(R - R') = \sigma_c(R) - \sigma_c(R')$$

Identities III

Suppose c is a predicate on attributes in R and c' is a predicate on attributes in R' and c'' is a predicate

$$R \bowtie_{c \wedge c' \wedge c''} R' = \sigma_c(R) \bowtie_{c''} \sigma_{c'}(R')$$

Identities IV

- Projection: Let A, B be sets of attributes of R .

$$\pi_A (R \cup R') = \pi_A(R) \cup \pi_A(R')$$

$$\pi_A (\pi_A(R)) = \pi_A(R)$$

$$\text{if } A \subseteq B, \pi_A (\pi_B(R)) = \pi_A(R)$$

- Selection and projection: If condition c only contains attributes in A , then

$$\pi_A (\sigma_c(R)) = \sigma_c(\pi_A(R))$$

- $R \cup S = S \cup R$ where R, S are relations with the same attributes.

Identities V

- $(R \cup S) \cup T = R \cup (S \cup T)$ where R, S, T are relations with the same attributes.
- $(R \times S) \times T = R \times (S \times T)$ where R, S, T are relations with the same attributes.
- $(R \bowtie S) \bowtie T = R \bowtie (S \bowtie T)$ if the joins are possible.