Synchronous Model Checking

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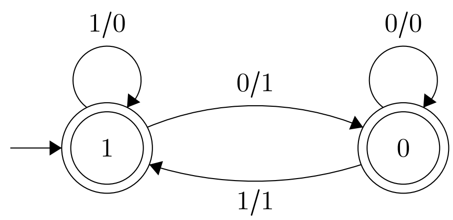
Usage

Defining an automaton

In this package, an automaton contains information about the names of the tapes it reads from. This means, for instance, that $x \rightarrowtriangle z$ and $y \rightarrowtriangle z$ are distinct automata, even though the relation $\rightarrowtriangle$ is the same in both. As such, we suggest defining an automaton as a function from tape names to an `Automaton` object, or, in our case, a suspension of an `Automaton` object. Here is an example for the successor automaton.




```
def auto_succ(v1, v2):
    d = {
        "alphabet": frozenset(["0", "1"]),
        "initial": frozenset([1]),
        "adjlist": {
            1: {
                "final": True,
                "edges": [
                    (0, {v1: "0", v2: "1"}),
                    (1, {v1: "1", v2: "0"})
                ]
            },
            0: {
                "final": True,
                "edges": [
                    (1, {v1: "1", v2: "1"}),
                    (0, {v1: "0", v2: "0"})
                ]
            }
        }
    }
    def susp(debug=0):
        if debug > 1:
            print("Auto_succ", d)
        return Automaton(d)
    return susp
```

The dictionary should be fairly understandable - at the top level, we store the alphabet, set of initial states, and an adjacency list to store the underlying graph. Edges are tuples of the form (v, l) where v is the destination vertex, and l is a dictionary that maps the tape name to the character expected on that tape. The rest of the syntax involved in building an automaton is boilerplate and can be copy-pasted. Please do make sure that all edge labels in your automaton are complete, in the sense that all the dictionaries that represent your labels should have the same set of keys.

Writing formulae

Once the atomic formulae are defined (via automata), building first-order formulae is easy. The syntax is self-explanatory. For example, if we use $\rightarrow$ to represent the successor relation, we can express injectivity, i.e. the formula

$$(\forall x)(\forall y)(\forall z)(x \rightarrow z \wedge y \rightarrow z \implies x = y)$$

as

```
ForAll("x", ForAll("y", ForAll("z", Implies(And(auto_succ("x", "z"),
auto_succ("y", "z")), auto_eq("x", "y")))))
```

The amount of computation performed when the formula is constructed is minimal. Actual evaluation of the formula is done by calling the `Eval` function. Note that this will throw an exception if the input formula is not a sentence. For a full list of the available constructors of first-order formulae, see `formulae.py`.

Debugging level

You can pass a keyword argument called `debug` to `Eval` to change verbosity. The default is 0 which prints nothing. If you pass 1, it will print the computations being performed and the size of the automaton at each step. If you pass 2, it will print everything included in 1 along with all intermediate automata.

Putting it all together

To try it yourself, we recommend you edit `tests.py` directly. There are a few useful automata already defined there for your use, and a few examples that you can use to learn the syntax. Hopefully everything works.

Implementation details

Organization

`automata.py` contains the routines that actually manipulate the machines. `tests.py` contains some actual automata and some tests. `formulae.py` contains wrapper functions for the routines in `automata.py` that are more readable and support lazy evaluation.

Algorithms

In order to process first-order formulae, in addition to dealing with the atomic formulae (in this context, defined via synchronous automata), we need to be able to handle the inductive constructors (quantifiers and boolean connectives). We discuss briefly how these are implemented. All functions mentioned are in `automata.py`.

1. (Disjunction) If our formula is of the form $A \vee B$ and we have automata $\mathcal{A}$ and $\mathcal{B}$ representing A and B respectively, we need to compute an automaton representing $A \vee B$. Since we allow for nondeterminism, this is easy - we just take the union of the automata $\mathcal{A}$ and $\mathcal{B}$ and make the set of initial states the union of the initial states in $\mathcal{A}$ and the initial states in $\mathcal{B}$. The actual code, implemented in `disj`, does this and a bit of pruning that is necessary to maintain invariants.

2. (Conjunction) If our formula is of the form $A \wedge B$ and we have automata $\mathcal{A}$ and $\mathcal{B}$ representing A and B respectively, we need to compute an automaton representing $A \wedge B$. For this we use the classic product construction, where we take the product of the state sets of $\mathcal{A}$ and $\mathcal{B}$ and simulate both in parallel. The naive way of doing this is quite wasteful, however, as many of the elements of this product are inaccessible from the initial states. As

such, we instead compute a closure under transitions of the initial state set. That is, if $I_{\mathcal{A}}$ and $I_{\mathcal{B}}$ are the initial state sets, we compute $\mathrm{Clos}(I_{\mathcal{A}} \times I_{\mathcal{B}}, \Delta)$ where $\Delta : Q_{\mathcal{A}} \times Q_{\mathcal{B}} \times \Sigma \to \mathcal{P}(Q_{\mathcal{A}} \times Q_{\mathcal{B}})$ (Σ is the set of all labels) via

$$\Delta(p, q, \sigma) = \delta_{\mathcal{A}}(p, \sigma) \times \delta_{\mathcal{B}}(q, \sigma)$$

This helps a lot to keep the automaton size down, which keeps our algorithms somewhat fast. The implementation of the closure algorithm is located in `product`, and the entire conjunction algorithm is in `conj`.

3. (Negation) Since we allow for nondeterminism, there is no straightforward way to negate an automaton without first determinizing. As usual, determinization is a potentially exponentially-expensive process, but we make it fairly fast in all but the most bizarre cases by using a closure algorithm similar to the one shown above. That is, instead of using the full powerset as the state set of the determinized automata, we start with the initial state set and only move to those elements of the powerset that are strictly needed. This is an extremely important optimization for many reasons. First, it makes the algorithm actually run in time (at least on the simple test cases we tested) - even just a couple full determinizations on an automaton with 5 states can and will blow it up. Second, the closure-based algorithm makes it so that if we perform two determinizations consecutively (which happens often when we have a bunch of universal quantifiers), the second is much cheaper than the first. The determinization code is found in `determinize`, and the small amount of extra code needed for negation can be found in `neg`.

4. (Existential Quantifier) Nondeterminism allows for an easy solution to reducing formulae of the form $(\exists x)(A(x))$ - just erase the x track from the automaton representing A. This is fast and so no optimization was considered. The code is in `exists`.

All other first-order formula constructors are implemented as macros in terms of these basic functions; see `formulae.py` for details on how this works.

We additionally considered optimizations involving running minimization algorithms after determinization made the automaton blow up too large, but we did not have time to implement them.

Weird annoyances

If you look at the key functions in `automata.py`, you might find some unnatural-looking stuff that deals with propositional logic directly. From experimentation, we found that in some cases, a fully erased automaton (unlabeled edges) did not play nice with the boolean connectives. We address this by immediately reducing to a boolean value and throwing out the automaton when the last track is erased, and then special casing in the functions that process boolean connectives.

Known Bugs

For some reason, the clearly true formula $(\forall x)(\exists y)(x \neq y)$ evaluates to false. We aren't sure why this is, but we believe that it has something to do with the empty string - for every other string, there is a string of the same length that is not equal to it, but this does not hold for the empty string.