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15-826: Multimedia Databases and Data Mining

Lecture #11: Fractals: M-trees and dim. curse (case studies – Part II)

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Must-read Material

- Alberto Belussi and Christos Faloutsos, Estimating the Selectivity of Spatial Queries Using the `Correlation' Fractal Dimension Proc. of VLDB, p. 299-310, 1995

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Optional Material

Optional, but **very** useful: Manfred Schroeder *Fractals, Chaos, Power Laws: Minutes from an Infinite Paradise* W.H. Freeman and Company, 1991

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Outline

Goal: 'Find similar / interesting things'

- Intro to DB
- Indexing - similarity search
- Data Mining

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Indexing - Detailed outline

- primary key indexing
- secondary key / multi-key indexing
- spatial access methods
  - z-ordering
  - R-trees
  - misc
- fractals
  - intro
  - applications
- text
- ...

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Indexing - Detailed outline

- fractals
  - intro
  - applications
    - disk accesses for R-trees (range queries)
    - dimensionality reduction
    - dim. curse revisited
    - quad-tree analysis [Gaede+]

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What else can they solve?

- ✓ separability [KDD' 02]
- forecasting [CIKM' 02]
- ✓ dimensionality reduction [SBBD' 00]
- non-linear axis scaling [KDD' 02]
- ✓ disk trace modeling [Wang+' 02]
- selectivity of spatial/multimedia queries [PODS' 94, VLDB' 95, ICDE' 00]
- ...

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Indexing - Detailed outline

- fractals
  - intro
  - applications
    - ✓ disk accesses for R-trees (range queries)
    - ✓ dimensionality reduction
    - dim. curse revisited
    - quad-tree analysis [Gaede+]

Dimensionality 'curse'

- Q: What is the problem in high-d?

Dimensionality 'curse'

- Q: What is the problem in high-d?
- A: indices do not seem to help, for many queries (eg., k-nn)
  - in high-d (& uniform distributions), most points are equidistant -> k-nn retrieves too many near-neighbors
  - [Yao & Yao, '85]: search effort ~ O($N^{(1-1/d)}$)

Dimensionality 'curse'

- (counter-intuitive, for db mentality)
- Q: What to do, then?

Dimensionality 'curse'

- A1: switch to seq. scanning
- A2: dim. reduction
- A3: consider the 'intrinsic'/fractal dimensionality
- A4: find approximate nn

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Dimensionality 'curse'

- A1: switch to seq. scanning
  - X-trees [Kriegel+, VLDB 96]
  - VA-files [Schek+, VLDB 98], 'test of time' award

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Dimensionality 'curse'

- A1: switch to seq. scanning
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- A3: consider the 'intrinsic' /fractal dimensionality
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Dim. reduction

a.k.a. feature selection/extraction:

- SVD (optimal, to preserve Euclidean distances)
- random projections
- using the fractal dimension [Traina+ SBBD2000]

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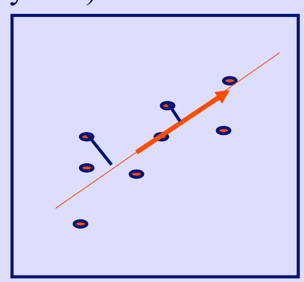
Singular Value Decomposition (SVD)

- SVD (~LSI ~ KL ~ PCA ~ spectral analysis...)



LSI: S. Dumais; M. Berry

KL: eg, Duda+Hart

PCA: eg., Jolliffe

MANY more details: soon

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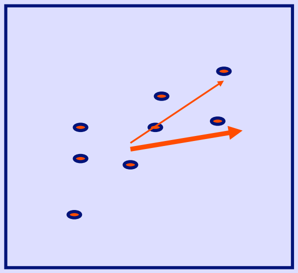
Random projections

- random projections(Johnson-Lindenstrauss thm [Papadimitriou+ pods98])



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Random projections

- pick 'enough' random directions (will be ~orthogonal, in high-d!!)
- distances are preserved probabilistically, within epsilon
- (also, use as a pre-processing step for SVD [Papadimitriou+ PODS98]

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Dim. reduction - w/ fractals

- Main idea: drop those attributes that don't affect the intrinsic ('fractal') dimensionality [Traina+, SBBD 2000]

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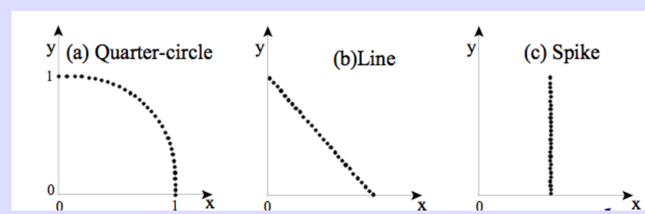
Dim. reduction - w/ fractals

global FD=1



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Dimensionality 'curse'

- A1: switch to seq. scanning
- A2: dim. reduction
- ➡ A3: consider the 'intrinsic' /fractal dimensionality
- A4: find **approximate** nn

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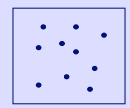
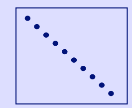
Intrinsic dimensionality

- before we give up, compute the intrinsic dim.:
- the lower, the better... [Pagel+, ICDE 2000]
- more details: in a few foils

intr. d = 2





intr. d = 1

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Dimensionality 'curse'

- A1: switch to seq. scanning
- A2: dim. reduction
- A3: consider the 'intrinsic' /fractal dimensionality
- ➡ A4: find approximate nn

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Approximate nn

- [Arya + Mount, SODA93], [Patella+ ICDE 2000]
- Idea: find k neighbors, such that the distance of the k-th one is guaranteed to be within epsilon of the actual.

Dimensionality 'curse'

- A1: switch to seq. scanning
- A2: dim. reduction
- A3: consider the 'intrinsic' /fractal dimensionality
- A4: find approximate nn

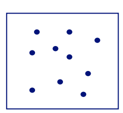
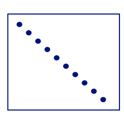
Dim. curse revisited

- (Q: how serious is the dim. curse, e.g.:)
- Q: what is the search effort for k-nn?
  - given N points, in E dimensions, in an R-tree, with k-nn queries ('biased' model)

[Pagel, Korn + ICDE 2000]


(Overview of proofs)

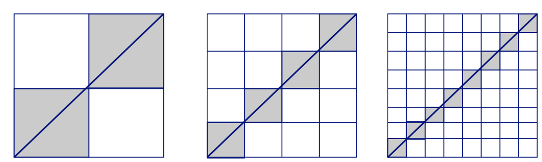
- assume that your points are uniformly distributed in a d-dimensional manifold (= hyper-plane)
- derive the formulas
- substitute d for the fractal dimension

DETAILS

Reminder: Hausdorff Dimension (D_0)

- r = side length (each dimension)
- $B(r)$ = # boxes containing points $\propto r^{D0}$



$r = 1/2$ $B = 2$ | $r = 1/4$ $B = 4$ | $r = 1/8$ $B = 8$

$\log r = -1$, $\log B = 1$ | $\log r = -2$, $\log B = 2$ | $\log r = -3$, $\log B = 3$

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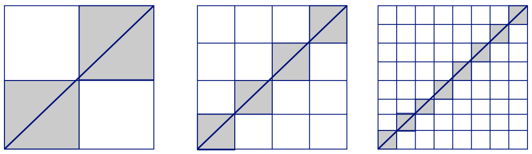
DETAILS

Reminder: Correlation Dimension (D_2)

- $S(r) = \sum p_i^2$ (squared % pts in box) $\propto r^{D2}$
  $\propto$ #pairs(within <= r)



$r = 1/2$ $S = 1/2$ | $r = 1/4$ $S = 1/4$ | $r = 1/8$ $S = 1/8$

$\log r = -1$, $\log S = -1$ | $\log r = -2$, $\log S = -2$ | $\log r = -3$, $\log S = -3$

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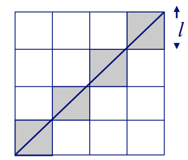
DETAILS

Observation #1

- How to determine avg MBR side l?
  - N = #pts, C = MBR capacity



Hausdorff dimension: $B(r) \propto r^{D0}$

$B(l) = N/C = l^{-D0} \Rightarrow \boxed{l = (N/C)^{-1/D0}}$

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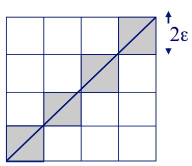
DETAILS

Observation #2

- k-NN query → ε-range query
  - For k pts, what radius ε do we expect?



Correlation dimension: $S(r) \propto r^{D2}$

$$S(\varepsilon) = \frac{k}{N-1} = (2\varepsilon)^{D2}$$

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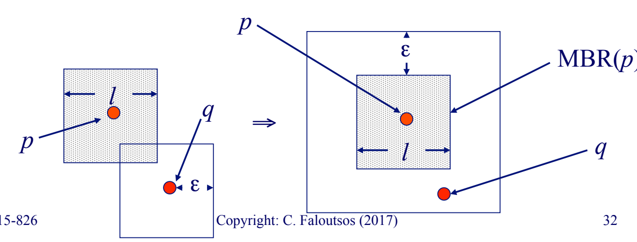
DETAILS

Observation #3

- Estimate avg # query-sensitive anchors:
  - How many **expected** q will touch **avg** page?
  - Page touch: q stabs ε-dilated MBR(p)



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Asymptotic Formula

- k-NN page accesses as N $\rightarrow \infty$
  - C = page capacity
  - D = fractal dimension (=$D0$ ~ $D2$)
  - h = height of tree

$$P_{all}^{L\infty}(k) \approx \sum_{j=0}^{h} \left\{ \frac{1}{C^{h-j}} + \left[1 + \left(\frac{k}{C^{h-j}} \right)^{1/D} \right]^{D} \right\}$$

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Asymptotic Formula

$$P_{all}^{L\infty}(k) \approx \sum_{j=0}^{h} \left\{ \frac{1}{C^{h-j}} + \left[1 + \left(\frac{k}{C^{h-j}} \right)^{1/D} \right]^{D} \right\}$$

- Observations?

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Asymptotic Formula

$$P_{all}^{L\infty}(k) \approx \sum_{j=0}^{h} \left\{ \frac{1}{C^{h-j}} + \left[1 + \left(\frac{k}{C^{h-j}} \right)^{1/D} \right]^{D} \right\}$$

- NO mention of the embedding dimensionality!!
- Still have dim. curse, but on f.d. D

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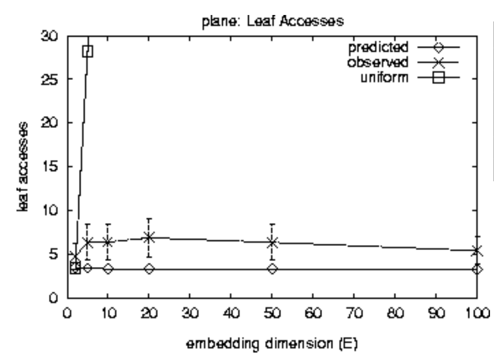
Embedding Dimension



E	*unif/ind*	*fractal*	*leaf*
2	3.49	3.49	4.75
5	28.26	3.45	6.40
10	847.26	3.34	6.42
20	All	3.36	6.9
50	All	3.32	6.37
100	All	3.32	5.43

plane
k = 50
L_{∞} dist

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Conclusions

- Dimensionality 'curse':
  - for high-d, indices slow down to ~O(N)
- If the **intrinsic** dim. is low, there is hope
- otherwise, do seq. scan, or sacrifice accuracy (approximate nn)

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Conclusions – cont'd

- Worst-case theory is **over-pessimistic**
- High dimensional data can exhibit good performance if **correlated, non-uniform**
- Many real data sets are **self-similar**
- Determinant is **intrinsic** dimensionality
  - multiple fractal dimensions (D_0 and D_2)
  - indication of how far one can go

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References

- Sunil Arya, David M. Mount: *Approximate Nearest Neighbor Queries in Fixed Dimensions*. SODA 1993: 271-280

  ANN library:

  http://www.cs.umd.edu/~mount/ANN/

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