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15-826: Multimedia Databases and Data Mining

Lecture #9: Fractals – examples & algo's
C. Faloutsos




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Must-read Material

- Christos Faloutsos and Ibrahim Kamel, [*Beyond Uniformity and Independence: Analysis of R-trees Using the Concept of Fractal Dimension*](#), Proc. ACM SIGACT-SIGMOD-SIGART PODS, May 1994, pp. 4-13, Minneapolis, MN.

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Recommended Material

optional, but **very** useful:

- Manfred Schroeder *Fractals, Chaos, Power Laws: Minutes from an Infinite Paradise* W.H. Freeman and Company, 1991
 - Chapter 10: boxcounting method
 - Chapter 1: Sierpinski triangle

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Outline

Goal: 'Find **similar** / **interesting** things'

- Intro to DB
- ➡ Indexing - similarity search
- Data Mining

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Indexing - Detailed outline

- primary key indexing
- secondary key / multi-key indexing
- spatial access methods
 - z-ordering
 - R-trees
 - misc
- fractals
 - intro
 - applications
- text

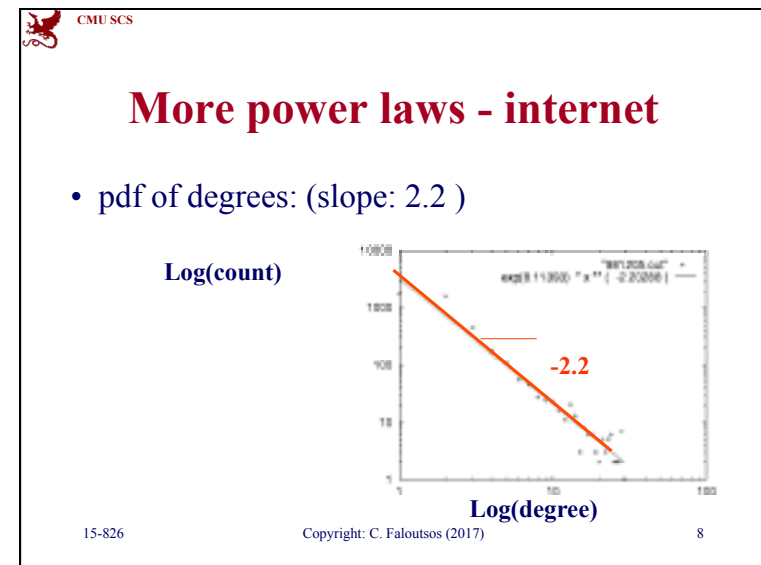
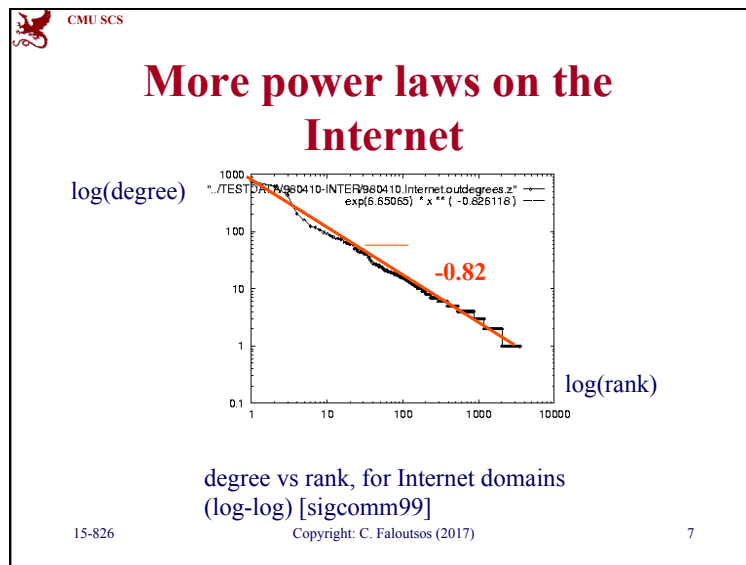
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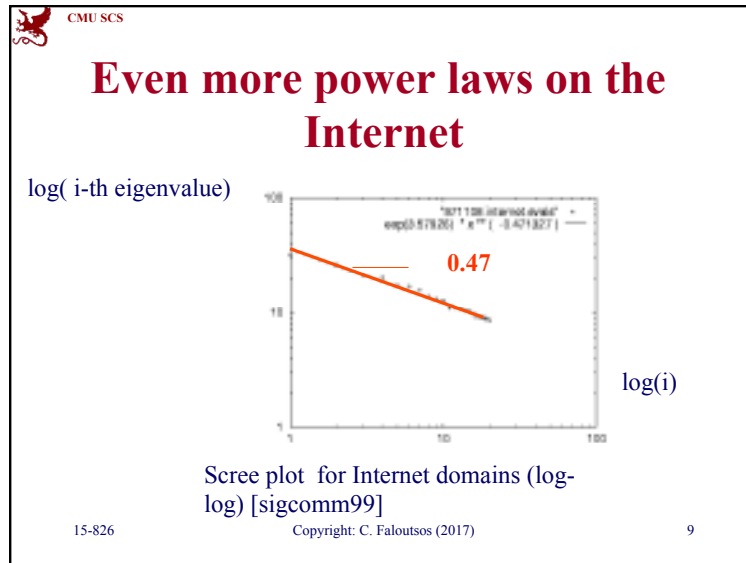
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Road map

- Motivation – 3 problems / case studies
- Definition of fractals and power laws
- Solutions to posed problems
- ➔ • More tools and **examples**
- Discussion - putting fractals to work!
- Conclusions – practitioner's guide
- Appendix: gory details - boxcounting plots

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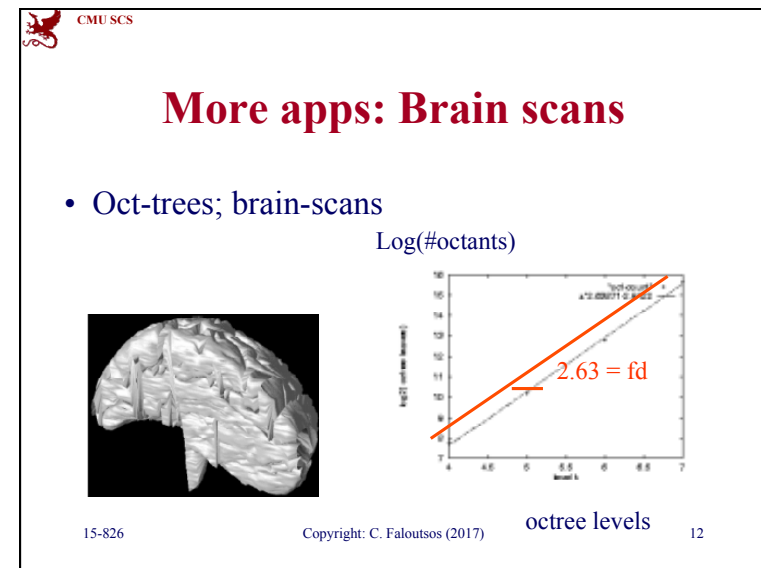
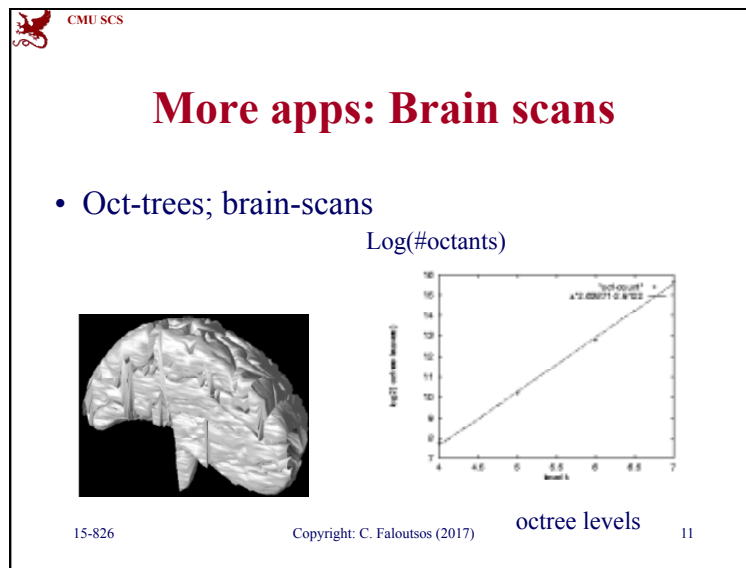
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Fractals & power laws:

appear in numerous settings:

- medical
- geographical / geological
- social
- computer-system related

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More apps: Medical images

[Burdett et al, SPIE '93]:


- benign tumors: $fd \sim 2.37$
- malignant: $fd \sim 2.56$

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More fractals:

- cardiovascular system: 3 (!)
- lungs: 2.9



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Fractals & power laws:

appear in numerous settings:


- medical
- **geographical / geological**
- social
- computer-system related

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More fractals:

- Coastlines: 1.2-1.58



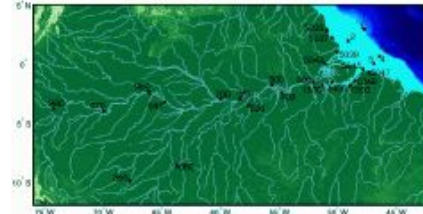
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More fractals:

- the fractal dimension for the Amazon river is 1.85 (Nile: 1.4)

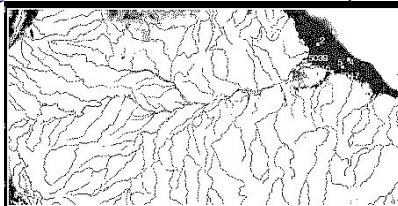
[ems.gphys.unc.edu/nonlinear/fractals/examples.html]



More fractals:

- the fractal dimension for the Amazon river is 1.85 (Nile: 1.4)

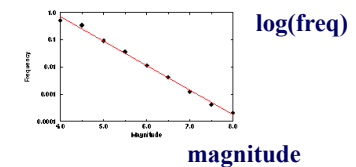
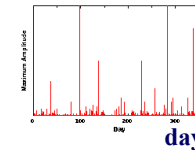
[ems.gphys.unc.edu/nonlinear/fractals/examples.html]



More power laws

- Energy of earthquakes (Gutenberg-Richter law) [simscience.org]

amplitude





Fractals & power laws:

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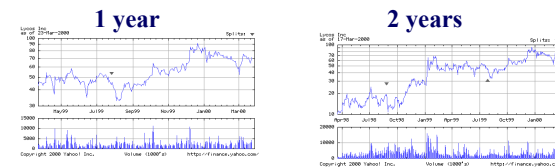
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More fractals:

stock prices (LYCOS) - random walks: 1.5



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Even more power laws:

- Income distribution (Pareto's law)
- size of firms
- publication counts (Lotka's law)

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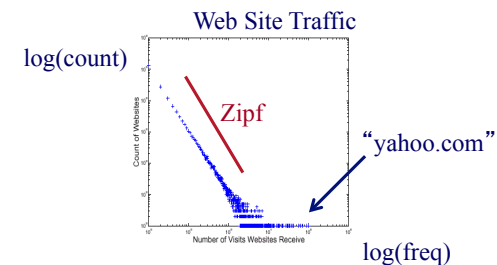
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Even more power laws:

- web hit counts [w/ A. Montgomery]



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Fractals & power laws:

appear in numerous settings:

- medical
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- social
- **computer-system related**

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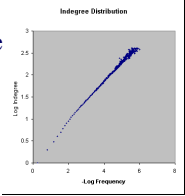
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Power laws, cont' d

- In- and out-degree distribution of web sites [Barabasi], [IBM-CLEVER]

log indegree

from [Ravi Kumar, Prabhakar Raghavan, Sridhar Rajagopalan, Andrew Tomkins]



- log(freq)

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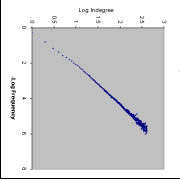
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Power laws, cont' d

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log indegree

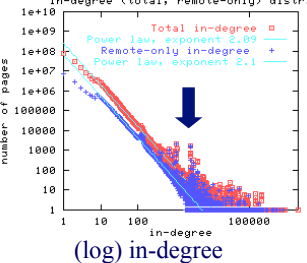
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“Foiled by power law”

- [Broder+, WWW' 00]

(log) count

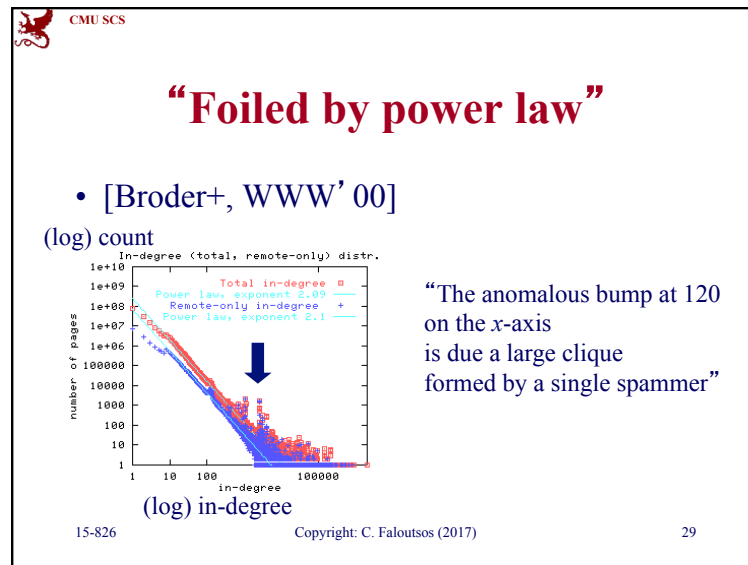


number of pages

in-degree

(log) in-degree

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Power laws, cont’ d

- In- and out-degree distribution of web sites [Barabasi], [IBM-CLEVER]
- length of file transfers [Crovella+Bestavros ‘96]
- duration of UNIX jobs [Harchol-Balter]

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Even more power laws:

- Distribution of UNIX file sizes
- web hit counts [Huberman]

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Road map

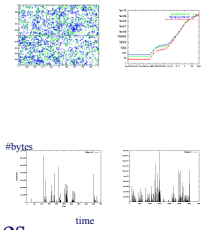
- Motivation – 3 problems / case studies
- Definition of fractals and power laws
- Solutions to posed problems
- More examples and tools
- ➔ Discussion - putting fractals to work!
- Conclusions – practitioner’s guide
- Appendix: gory details - boxcounting plots

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What else can they solve?

- ✓ separability [KDD' 02]
 - forecasting [CIKM' 02]
 - dimensionality reduction [SBBD' 00]
 - non-linear axis scaling [KDD' 02]
- ✓ disk trace modeling [Wang+' 02]
 - selectivity of spatial/multimedia queries [PODS' 94, VLDB' 95, ICDE' 00]
 - ...



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Settings for fractals:

Points; areas (-> fat fractals), eg:

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Settings for fractals:

Points; areas, eg:


- cities/stores/hospitals, over earth's surface
- time-stamps of events (customer arrivals, packet losses, criminal actions) over time
- regions (sales areas, islands, patches of habitats) over space

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Settings for fractals:

- customer feature vectors (age, income, frequency of visits, amount of sales per visit)



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Some uses of fractals:

- Detect non-existence of rules (if points are uniform)
- Detect non-homogeneous regions (eg., legal login time-stamps may have different fd than intruders')
- Estimate number of neighbors / customers / competitors within a radius

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Multi-Fractals

Setting: points or objects, w/ some value, eg:

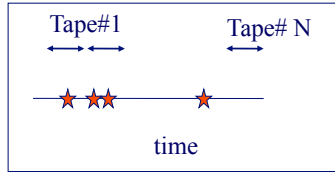
- cities w/ populations
- positions on earth and amount of gold/water/oil underneath
- product ids and sales per product
- people and their salaries
- months and count of accidents

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Use of multifractals:

- Estimate tape/disk accesses
 - *how many of the 100 tapes contain my 50 phonecall records?*
 - *how many days without an accident?*

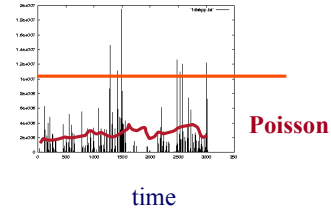


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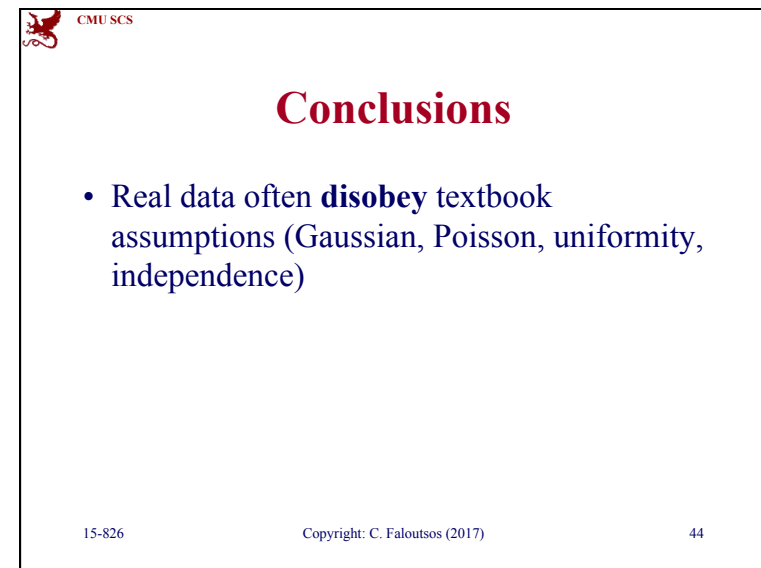
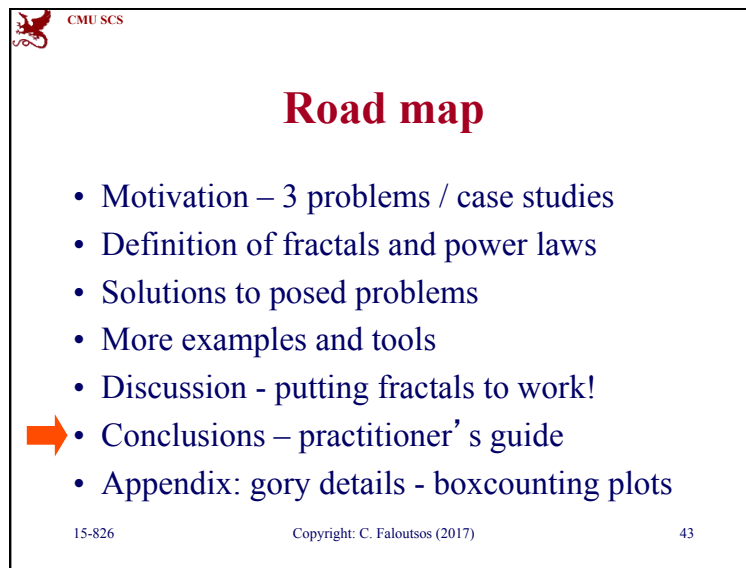
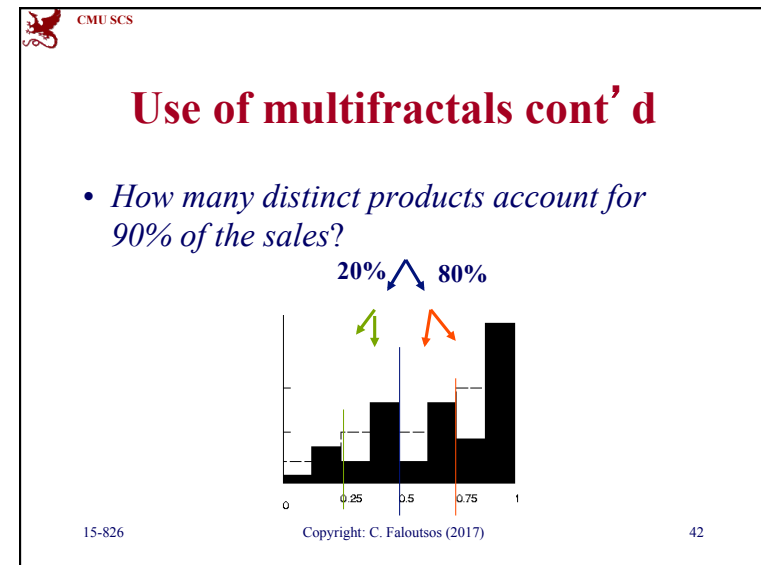
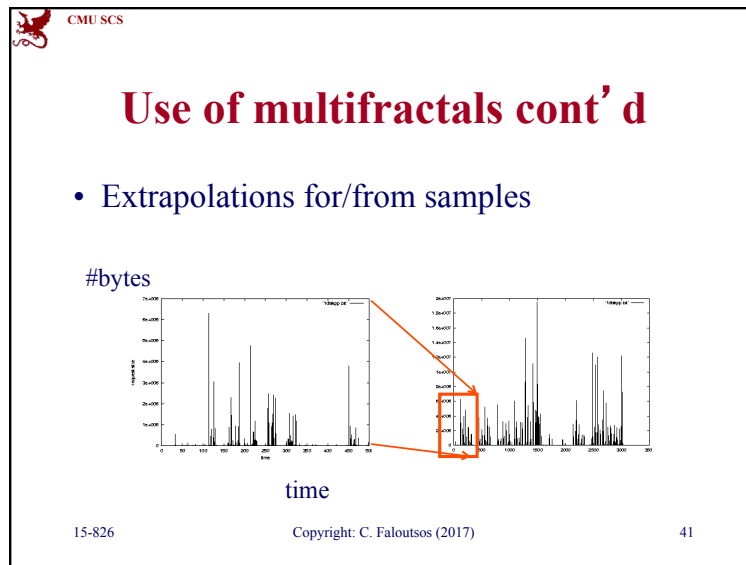
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Use of multifractals

- how often do we exceed the threshold?



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Conclusions

- Real data often **disobey** textbook assumptions (Gaussian, Poisson, uniformity, independence)

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Conclusions - cont' d

Self-similarity & power laws: appear in **many** cases

Bad news:
lead to skewed distributions
(no Gaussian, Poisson, uniformity, independence, mean, variance)


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Conclusions - cont' d

Self-similarity & power laws: appear in **many** cases

Bad news:
lead to skewed distributions
(no Gaussian, Poisson, uniformity, independence, mean, variance)

Good news: 

- 'correlation integral' for separability
- rank/frequency plots
- 80-20 (multifractals)
- (Hurst exponent, strange attractors, renormalization theory, ++)

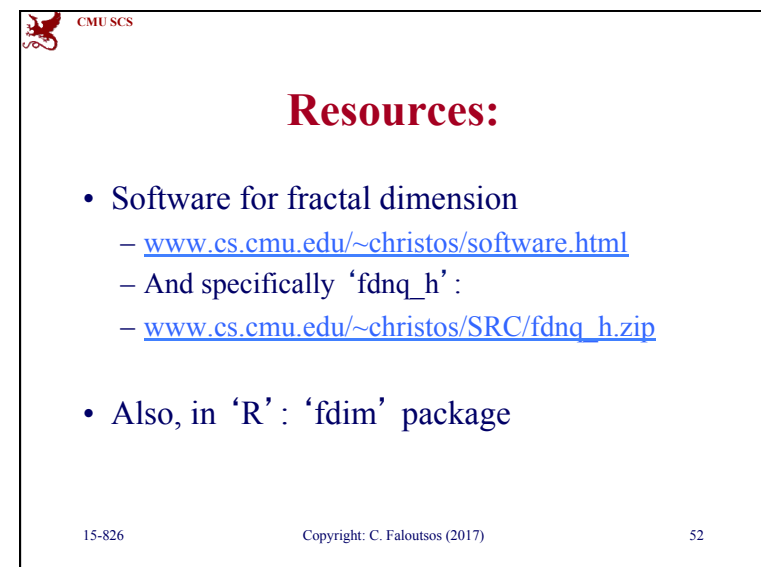
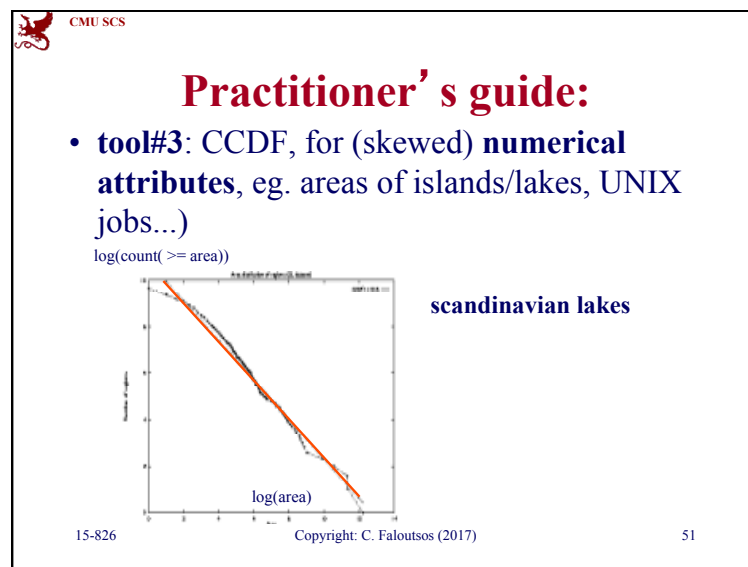
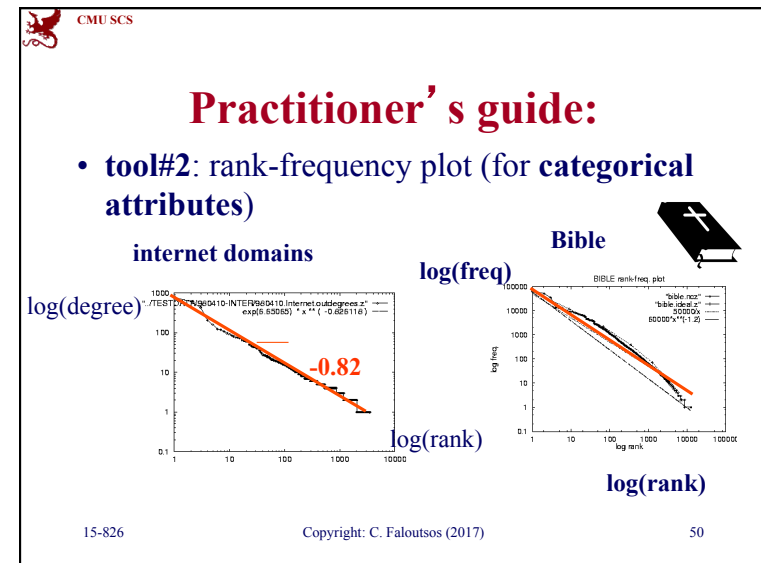
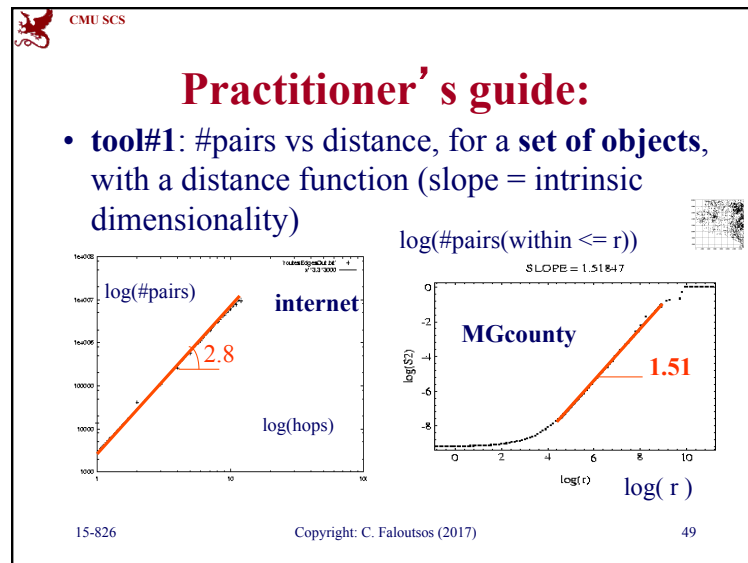
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Conclusions

- tool#1: (for points) 'correlation integral'**: (#pairs within $\leq r$) vs (distance r)
- tool#2: (for categorical values) rank-frequency plot** (a' la Zipf)
- tool#3: (for numerical values) CCDF**: Complementary cumulative distr. function (#of elements with value $\geq a$)

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Books

- Strongly recommended intro book:
 - Manfred Schroeder *Fractals, Chaos, Power Laws: Minutes from an Infinite Paradise* W.H. Freeman and Company, 1991
- Classic book on fractals:
 - B. Mandelbrot *Fractal Geometry of Nature*, W.H. Freeman, 1977


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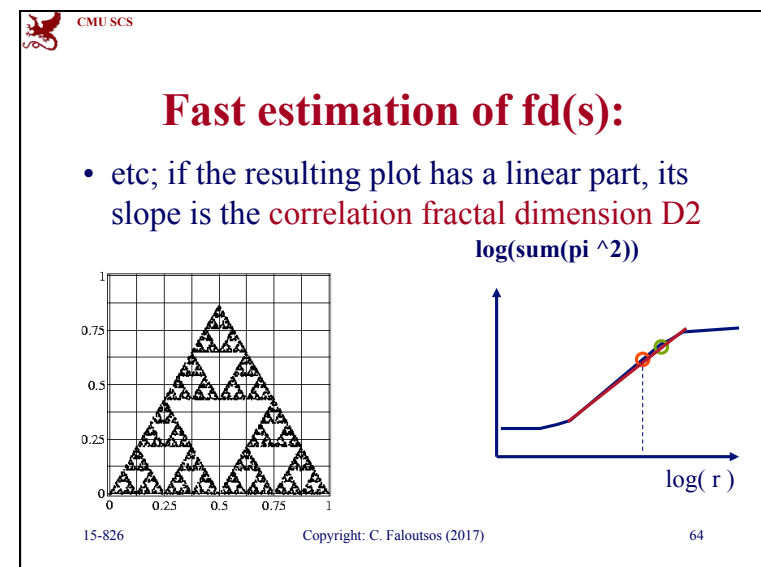
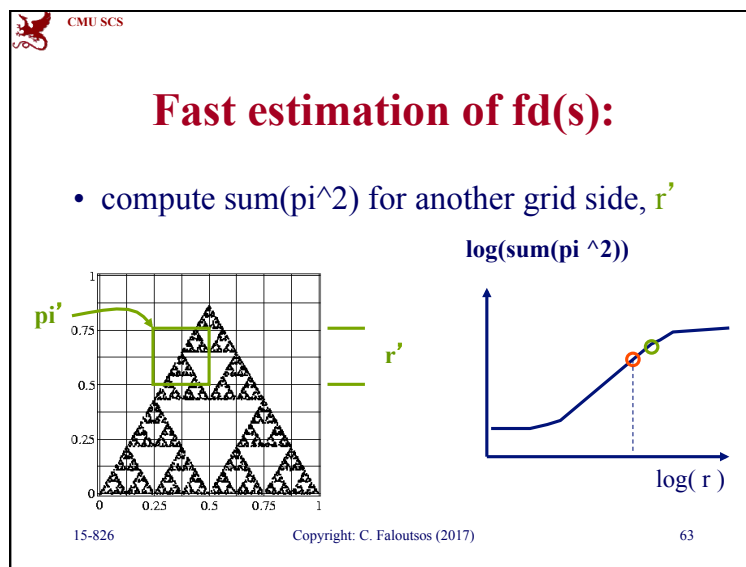
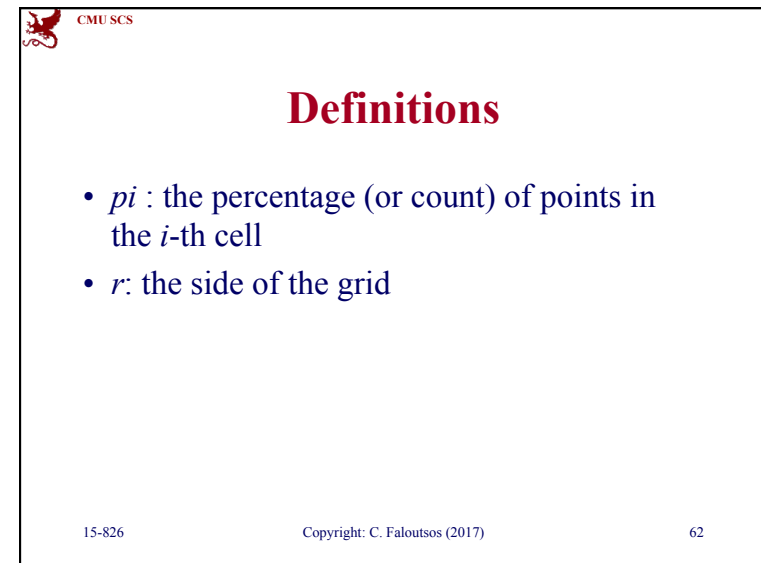
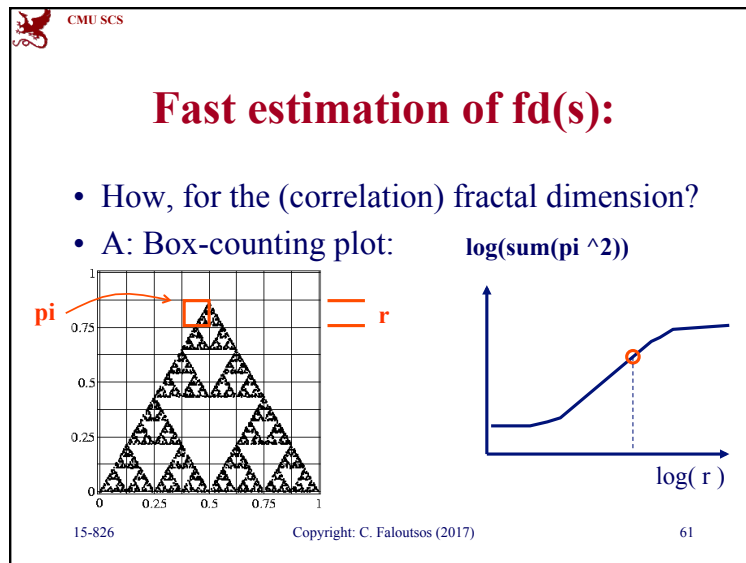
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Appendix - Gory details

- Bad news: There are more than one fractal dimensions
 - Minkowski fd; Hausdorff fd; Correlation fd; Information fd
- Great news:
 - they can all be computed fast!
 - they usually have nearby values

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Definitions (cont' d)

- Many more fractal dimensions D_q (related to Renyi entropies):

$$D_q = \frac{1}{q-1} \frac{\partial \log(\sum p_i^q)}{\partial \log(r)} \quad q \neq 1$$

$$D_1 = \frac{\partial \sum p_i \log(p_i)}{\partial \log(r)}$$

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Hausdorff or box-counting fd:

- Box counting plot: $\log(N(r))$ vs $\log(r)$
- r : grid side
- $N(r)$: count of non-empty cells
- (Hausdorff) fractal dimension D_0 :

$$D_0 = - \frac{\partial \log(N(r))}{\partial \log(r)}$$

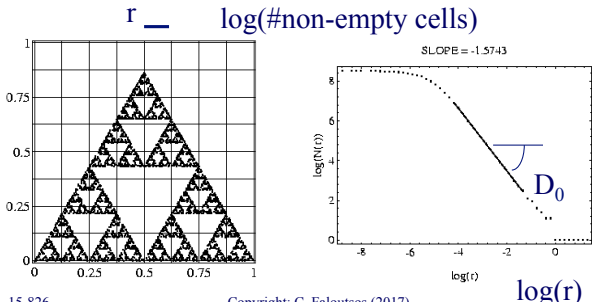
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Definitions (cont' d)

- Hausdorff fd:

$r \quad \log(\# \text{non-empty cells})$



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Observations

- $q=0$: Hausdorff fractal dimension
- $q=2$: Correlation fractal dimension (**identical** to the exponent of the number of neighbors vs radius)
- $q=1$: Information fractal dimension

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Observations, cont' d

- in general, the D_q 's take similar, but not identical, values.
- except for perfectly self-similar point-sets, where $D_q = D_{q'}$ for any q, q'

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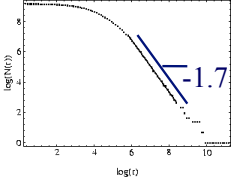
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Examples:MG county

- Montgomery County of MD (road end-points)

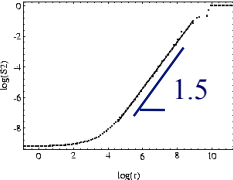
$q=0$

SLOPE = -1.71945



$q=2$

SLOPE = 1.51847



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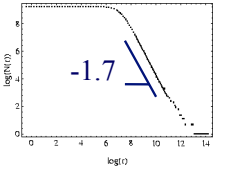
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Examples:LB county

- Long Beach county of CA (road end-points)

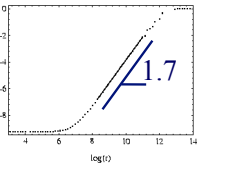
$q=0$

SLOPE = -1.72775



$q=2$

SLOPE = 1.73035



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Conclusions

- many fractal dimensions, with nearby values
- can be computed quickly ($O(N)$ or $O(N \log(N))$)
- (code: on the web:
 - www.cs.cmu.edu/~christos/SRC/fdnq_h.zip
 - Or 'R' ('fdim' package)

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