

- Natural Language Processing
- The Language Modeling Problem
 - We have some (finite) vocabulary, say $V = \{\text{the, a, man, telescope ...}\}$
 - We have an (infinite) set of strings, V^t
 - The STOP
 - The fan STOP
 - We have a training sample of example sentences in English
 - We need to “learn” a probability distribution p over the sentences in our language
 - Summation x element sentences of language
 - $P(x) = 1, p(x) \geq 0$ for all x element of sentences in language
 - Assign a probability to every sentence in the language
- Why on earth would we want to do this?
 - Speech recognition was the original motivation (related problems are optical character recognition, handwriting recognition.)
 - The estimation techniques developed for this problem will be very useful for other problems in NLP
- A Naïve Method
 - We have N training sentences
 - For any sentence $x_1 \dots x_n$ $c(x_1 \dots x_n)$ is the number of times the sentence is seen in our training data
 - A naïve estimate
 - $P(x_1 \dots x_n) = c(x_1 \dots x_n) / N$
- Trigram models
- Markov Processes
 - Consider a sequence of random variables X_1, X_2, \dots, X_n each random variable can take any value in a finite set V . For now we assume the length n is fixed (e.g., $n = 100$)
 - Markov process with states $V = \{0,1,2\}$ and length $n = 10$
 - Then 3^{10} sequences can be generated
- Our goal: model
 - $P(X_1 = x_1, X_2 = x_2, \dots, X_n = x_n)$
- First-Order Markov Processes

$$P(X_1 = x_1, X_2 = x_2, \dots, X_n = x_n) \\ = P(X_1 = x_1) \prod_{i=2}^n P(X_i = x_i | X_1 = x_1, \dots, X_{i-1} = x_{i-1})$$

$$P(A, B) = P(A) \times P(B|A) \\ P(A, B, C) = P(A) \times P(B|A) \times P(C|A, B)$$

$$P(X_1 = x_1, X_2 = x_2) = P(X_1 = x_1) P(X_2 = x_2 | X_1 = x_1) \\ P(X_1 = x_1, X_2 = x_2, X_3 = x_3) = \dots \times P(X_3 = x_3 | X_1 = x_1, X_2 = x_2)$$

- First order Markov assumption: For any $i \in \{2, \dots, n\}$ for any $x_1 \dots x_i$
 - $P(X_i = x_i | X_1 = x_1 \dots X_{i-1} = x_{i-1}) = P(X_i = x_i | X_{i-1} = x_{i-1})$

$$= P(X_1 = x_1) \prod_{i=2}^n P(X_i = x_i | X_{i-1} = x_{i-1})$$

The first-order Markov assumption: For any $i \in \{2 \dots n\}$, for any $x_1 \dots x_i$,

$$P(X_i = x_i | X_1 = x_1 \dots X_{i-1} = x_{i-1}) = P(X_i = x_i | X_{i-1} = x_{i-1})$$

- Second Order Markov Processes

Second-Order Markov Processes

$$P(X_1 = x_1, X_2 = x_2, \dots, X_n = x_n) \\ = P(X_1 = x_1) \times P(X_2 = x_2 | X_1 = x_1) \\ \times \prod_{i=3}^n P(X_i = x_i | X_{i-2} = x_{i-2}, X_{i-1} = x_{i-1}) \\ = \prod_{i=1}^n P(X_i = x_i | X_{i-2} = x_{i-2}, X_{i-1} = x_{i-1})$$

(For convenience we assume $x_0 = x_{-1} = *$ where $*$ is a special "start" symbol.)



- Modeling Variable Length Sequences
 - We would like the length of the sequence, n , to also be a random variable
 - A simple solution: always define $X_n = \text{STOP}$ where STOP is a special symbol
 - Then use a Markov process as before

- Generating the value of i 'th random variable on the two previous conditions

$$\begin{aligned}
 & \text{X}_n \in \text{STOP} \\
 & \downarrow \\
 & P(X_1 = x_1, X_2 = x_2, \dots, X_n = x_n) \\
 & = \prod_{i=1}^n P(X_i = x_i | X_{i-2} = x_{i-2}, X_{i-1} = x_{i-1})
 \end{aligned}$$

(For convenience we assume $x_0 = x_{-1} = *$, where $*$ is a special "start" symbol.)

- Trigram language Models
 - A trigram language model consists of
 - A finite set V vocabulary in the language model
 - A parameter $q(w|u,v)$ for each trigram u,v,w such that w element $V \cup \{\text{STOP}\}$, and u,v element $V \cup \{*\}$