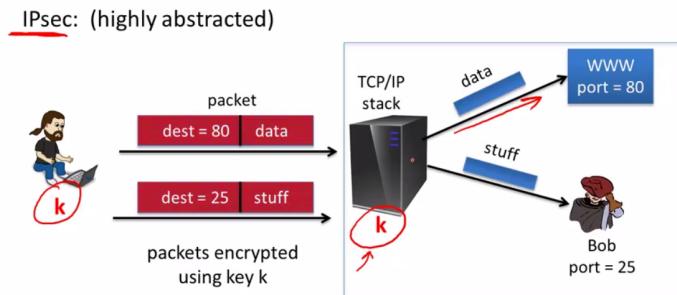


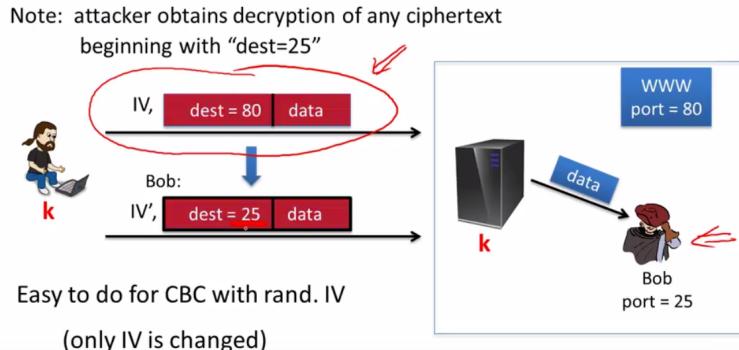
- Authenticated Encryption 1: why is it so important?
- Active Attacks on CPA-Secure Encryption
- Recap: the story so far

- Confidentiality: semantic security against a CPA attack
- Encryption secure against eavesdropping only
- Integrity:
- Existential unforgeability under a chosen message attack
  - CBC-MAC, HMAC, PMAC, CW-MAC
- This module: encryption secure against tampering.
- Ensuring both confidentiality and integrity

- Sample tampering attacks
  - TCP/IP: (highly abstracted)
    - send the data to the destination port. There are two people listening to the TCP/IP stack.
  - IPsec: (highly abstracted)
    - encrypts the packets between the sender and the recipient
    - packets encrypted using key k
    - TCP/IP stack decrypts the packet and sends it



- Reading someone else's data



- Example



Encryption is done with CBC with a random IV.

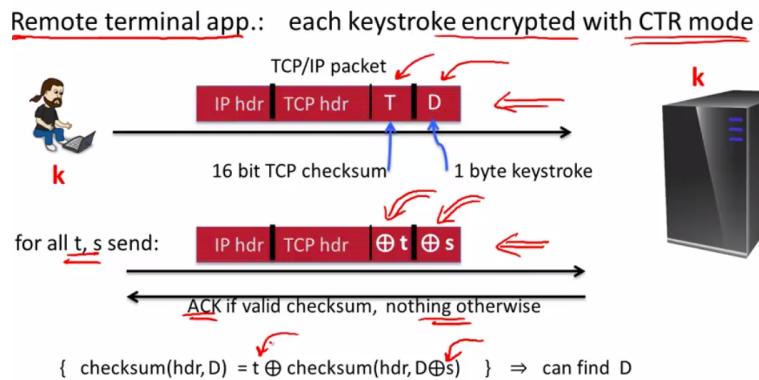
What should  $\text{IV}'$  be?

$$m[0] = D(k, c[0]) \oplus \text{IV} = \text{"dest } 80 \dots"$$

*dest = 25*

- $\text{IV}' = \text{IV} \oplus (\dots 25 \dots)$
- $\text{IV}' = \text{IV} \oplus (\dots 80 \dots)$
- $\text{IV}' = \text{IV} \oplus (\dots 80 \dots) \oplus (\dots 25 \dots)$
- It can't be done

- An attack using only network access



- The lesson

- active attacks attack modifying information in route
- CPA security cannot guarantee secrecy under active attacks
- Only use one of two modes:
  - If message needs integrity but no confidentiality: use a MAC
  - If message needs both integrity and confidentiality: use authenticated encryption modes (this module)

- Definitions

- Authenticated Encryption

- Goals

An **authenticated encryption** system  $(E, D)$  is a cipher where

As usual:  $E: K \times M \times N \rightarrow C$

but  $D: K \times C \times N \rightarrow M \cup \{\perp\}$

$\perp \notin M$

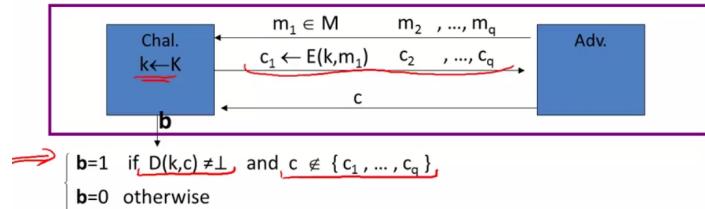
ciphertext  
is rejected

Security: the system must provide

- sem. security under a CPA attack, and
- ciphertext integrity:  
attacker cannot create new ciphertexts that decrypt properly

- Ciphertext integrity

Let  $(E, D)$  be a cipher with message space  $M$ .



Def:  $(E, D)$  has ciphertext integrity if for all “efficient” A:

$$\text{Adv}_{\text{CI}}[A, E] = \Pr[\text{Chal. outputs 1}] \text{ is “negligible.”}$$

Dan

- Authenticated encryption

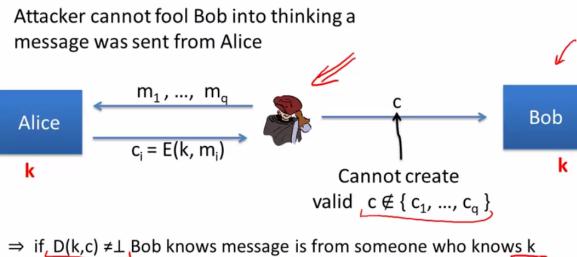
Def: cipher  $(E, D)$  provides authenticated encryption (AE) if it is

- (1) semantically secure under CPA, and
- (2) has ciphertext integrity

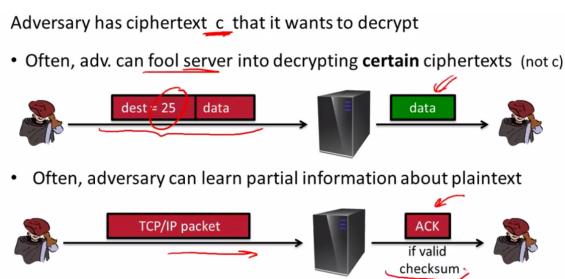
Bad example: CBC with rand. IV. does not provide AE

- $D(k, \cdot)$  never outputs  $\perp$ , hence adv. easily wins CI game

- Implication 1: authenticity

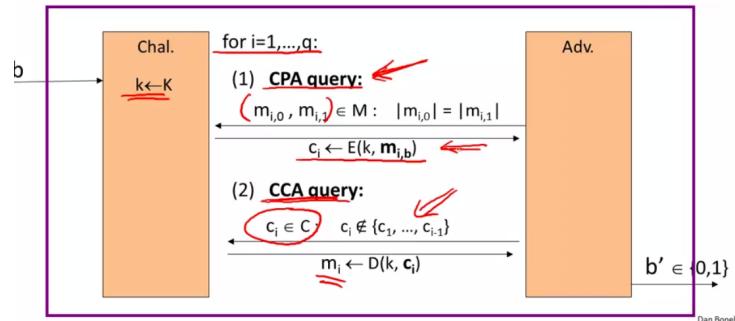


- (but message could be a replay)
- Implication 2
  - Authenticated encryption => Security against chosen cipher text attacks
- **Chosen Ciphertext attacks**
- Example chosen cipher text attacks



- Chosen cipher text security
  - Adversary's power: both CPA and CCA
    - can obtain the encryption of arbitrary messages of his choice
    - can decrypt any cipher text of his choice, other than challenge (conservative modeling of real life)
  - Adversary's goal: Break semantic security
- Chosen cipher text security: definition

$E = (E, D)$  cipher defined over  $(K, M, C)$ . For  $b=0, 1$  define  $\text{EXP}(b)$ :

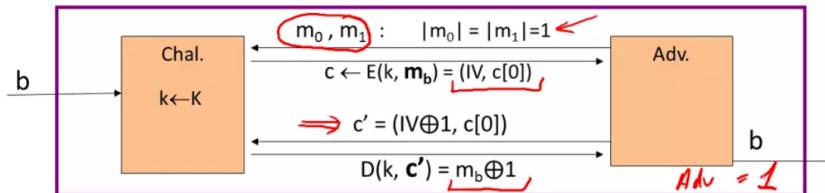


- Chosen cipher text security: definition

$E$  is CCA secure if for all "efficient"  $A$ :

$$\text{Adv}_{\text{CCA}}[A, E] = |\Pr[\text{EXP}(0)=1] - \Pr[\text{EXP}(1)=1]| \text{ is "negligible."}$$

Example: CBC with rand. IV is not CCA-secure



- Authenticated enc.  $\Rightarrow$  CCA security

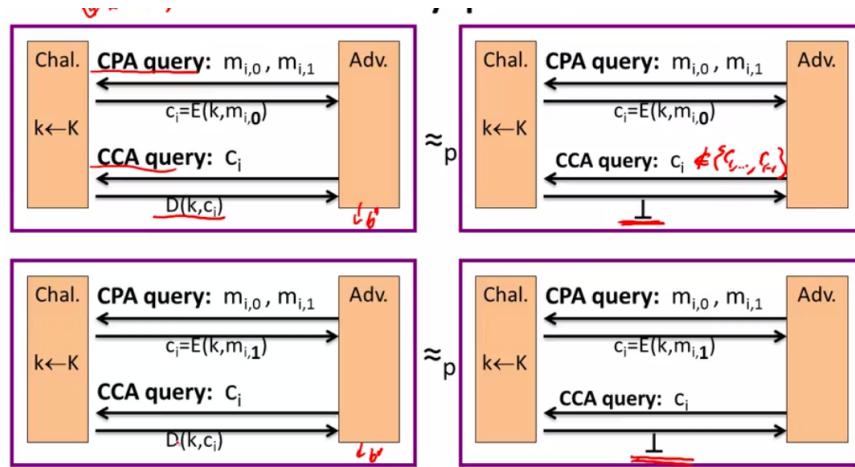
Thm: Let  $(E, D)$  be a cipher that provides AE.

Then  $(E, D)$  is CCA secure !

In particular, for any q-query eff.  $A$  there exist eff.  $B_1, B_2$  s.t.

$$\underbrace{\text{Adv}_{\text{CCA}}[A, E]}_{\text{neg.}} \leq 2q \cdot \underbrace{\text{Adv}_{\text{CI}}[B_1, E]}_{\text{neg.}} + \underbrace{\text{Adv}_{\text{CPA}}[B_2, E]}_{\text{neg.}}$$

- Proof by pictures
  - CCA secure



- remove queries since they don't provide any information since they always respond bottom.
- So what?
  - Authenticated encryption:
    - ensures confidentiality against an active adversary that can decrypt some cipher texts
  - Limitations:
    - does not prevent replay attacks
    - does not account for side channels (timing)
- **Authenticated Encryption 2: standard constructions**
- **Constructions from Ciphers and MACs**
- ... but first, some history

Authenticated Encryption (AE): introduced in 2000

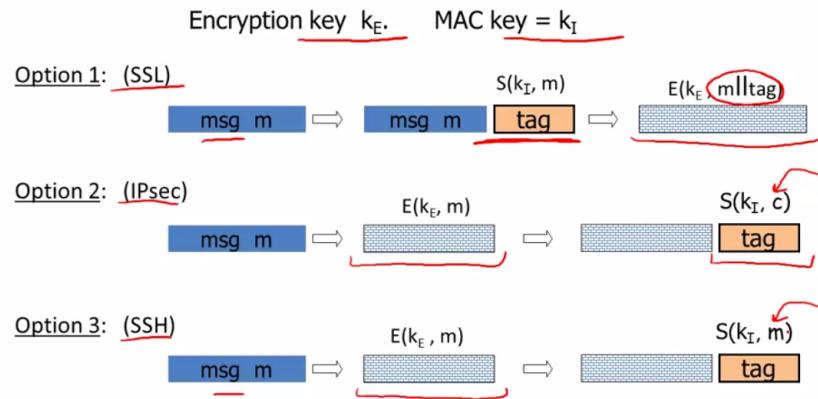
Crypto APIs before then: (e.g. MS-CAPI)

- Provide API for CPA-secure encryption (e.g. CBC with rand. IV)
- Provide API for MAC (e.g. HMAC)

Every project had to combine the two itself without a well defined goal

- Not all combinations provide AE ...

- Combining MAC and ENC (CCA)



- MACs are not designed for confidentiality only designed for integrity
- SSH (encrypt and mac)
  - tag concatenated after the message is encrypted
  - breaks CPA security
  - bits of the message are leaked in the cipher text
  - output of the mac signing algorithm might leak bits of the message
- SSL (mac then encrypt)
  - Bad interaction between the encryption scheme and mac algorithm
- **IPsec** (encrypt then mac)
  - always correct
  - message is encrypted and contents hidden in cipher text
  - compute tag of cipher text locks tag of cipher text and no one can produce a tag of the cipher text that will look valid
- A.E. Theorems

Let  $(E, D)$  be CPA secure cipher and  $(S, V)$  secure MAC. Then:

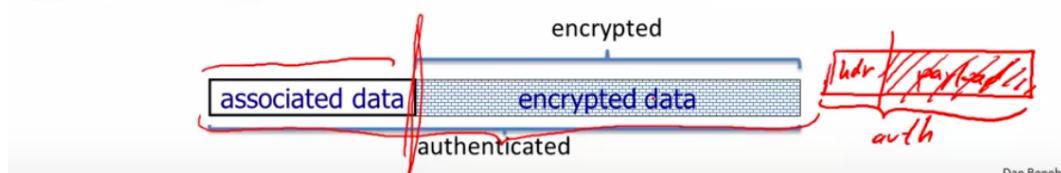
1. Encrypt-then-MAC: always provides A.E.
2. MAC-then-encrypt: may be insecure against CCA attacks  
however: when  $(E, D)$  is rand-CTR mode or rand-CBC  
M-then-E provides A.E.  
for rand-CTR mode, one-time MAC is sufficient

Dan Boneh

- Standards (at a high level)

- NIST*
- GCM: CTR mode encryption then CW-MAC  
(accelerated via Intel's PCLMULQDQ instruction)
  - CCM: CBC-MAC then CTR mode encryption (802.11i)  
*(AES)*
  - EAX: CTR mode encryption then CMAC

All support AEAD: (auth. enc. with associated data). All are nonce-based.



- An example API (OpenSSL)

```
Aead
```

```

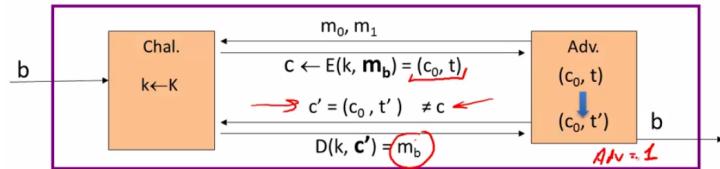
int AES_GCM_Init(AES_GCM_CTX *ain,
                    unsigned char *nonce, unsigned long noncelen,
                    unsigned char *key, unsigned int klen )

int AES_GCM_EncryptUpdate(AES_GCM_CTX *a,
                            → unsigned char *aad, unsigned long aadlen,
                            → unsigned char *data, unsigned long datalen,
                            unsigned char *out, unsigned long *outlen)

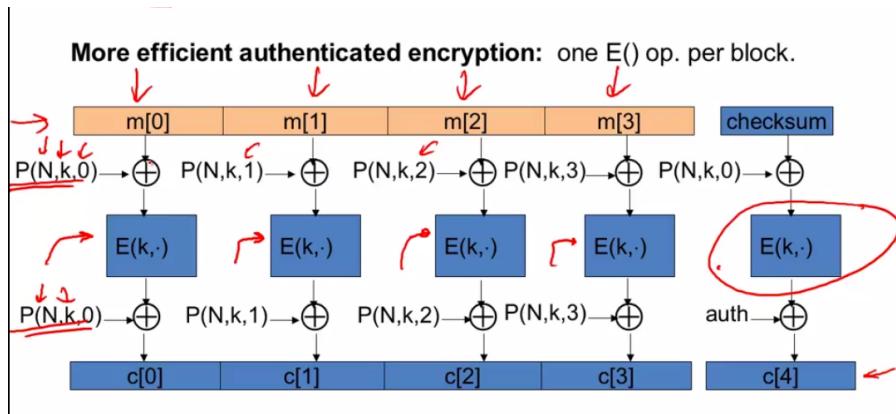
```

- MAC security — an explanation

Recall: MAC security implies  $(m, t) \not\Rightarrow (m, t')$  ↯  
 Why? Suppose not:  $(m, t) \rightarrow (m, t')$   
 Then Encrypt-then-MAC would not have Ciphertext Integrity !!



- OCB: a direct construction from a PRP



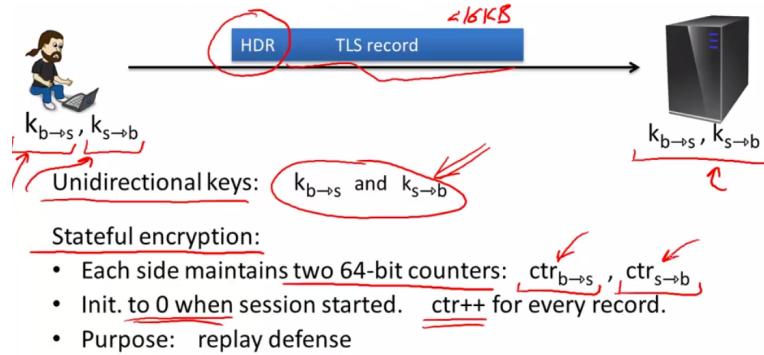
- Performance

AMD Opteron, 2.2 GHz (Linux)

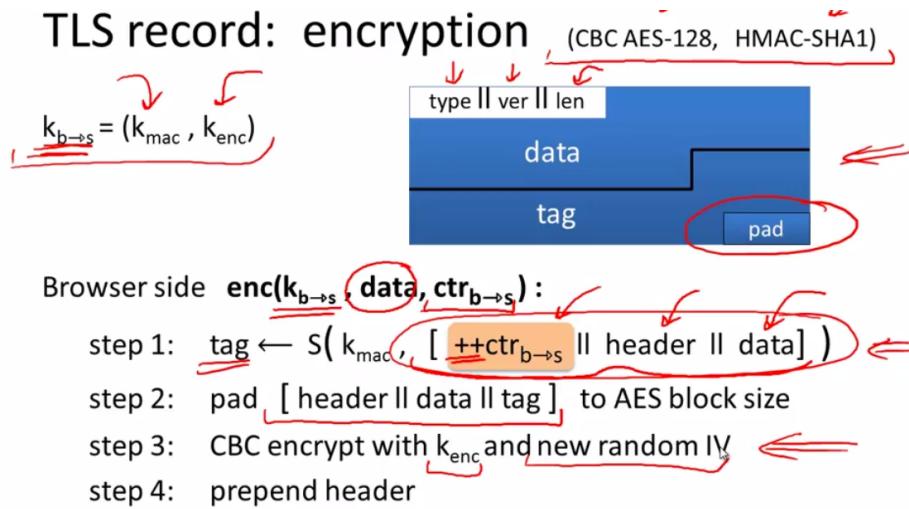
Cipher	code size	Speed (MB/sec)	
auth. enc.	AES/GCM	large **	108 <del>AES/CTR</del> 139
	AES/CCM	smaller	61 <del>AES/CBC</del> 109
	AES/EAX	smaller	61 <del>AES/CMAC</del> 109
AES/OCB	129*	HMAC/SHA1	147

\* extrapolated from Ted Kravitz's results    \*\* non-Intel machines

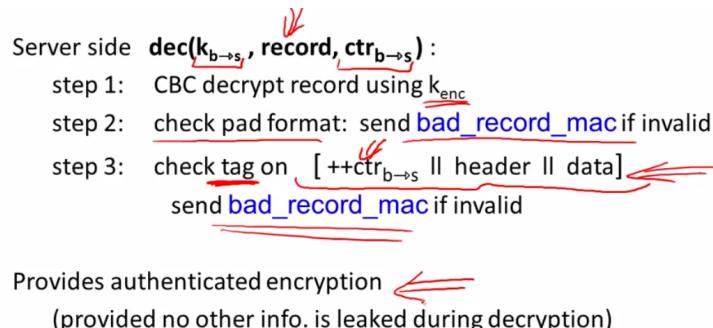
- Authenticated Encryption 3: pitfalls
- Case study: TLS 1.2
- The TLS Record Protocol



- TLS record: encryption



- TLS record: decryption



- Bugs in older versions

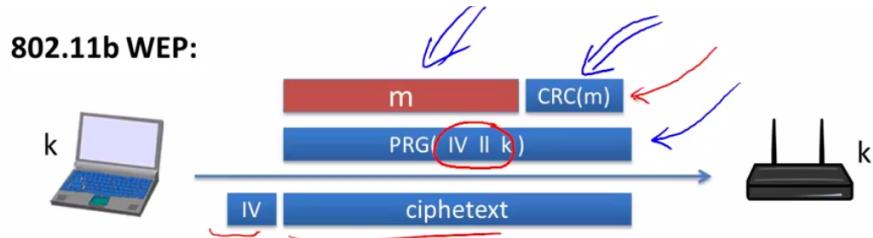
IV for CBC is predictable: (chained IV)  
IV for next record is last ciphertext block of current record.  
Not CPA secure. (a practical exploit: BEAST attack)

Padding oracle: during decryption  
 if pad is invalid send decryption failed alert  
 if mac is invalid send bad\_record\_mac alert  
 ⇒ attacker learns info. about plaintext (attack in next segment)

Lesson: when decryption fails, do not explain why

Dan Boneh

- 802.11b WEP:

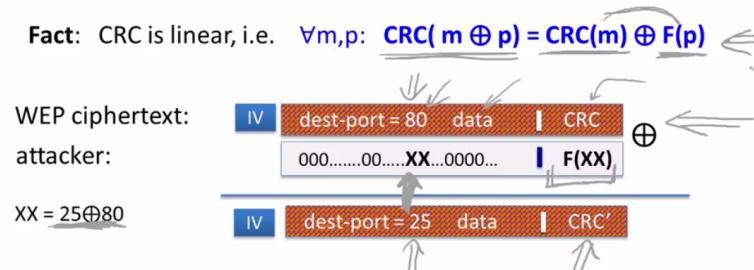


Previously discussed problems:

two time pad and related PRG seeds  
 $\begin{bmatrix} \text{Zv} \\ \text{||} \\ \text{K} \end{bmatrix}$

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- Active attacks



Upon decryption: CRC is valid, but ciphertext is changed !!

- CRC check sum provides no integrity at all against active attacks

- CBC Padding Attacks
- Recap

Authenticated encryption: CPA security + ciphertext integrity

- Confidentiality in presence of active adversary
- Prevents chosen-ciphertext attacks

Limitation: cannot help bad implementations ... (this segment)

Authenticated encryption modes:

- Standards: GCM, CCM, EAX
- General construction: encrypt-then-MAC.

- The TLS record protocol (CBC encryption)

Decryption:  $\text{dec}(k_{b \rightarrow s}, \text{record}, \text{ctr}_{b \rightarrow s})$ :

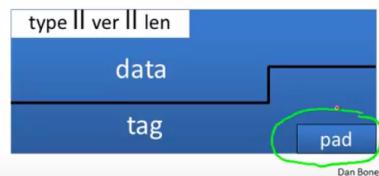
step 1: CBC decrypt record using  $k_{\text{enc}}$

step 2: check pad format: abort if invalid

step 3: check tag on  $[++\text{ctr}_{b \rightarrow s} \parallel \text{header} \parallel \text{data}]$   
abort if invalid

Two types of error:

- **padding error**
- **MAC error**



- important that the advisory is no told that a pad or mac error occurred
- Padding oracle

Suppose attacker can differentiate the two errors

(pad error, MAC error):

⇒ Padding oracle:

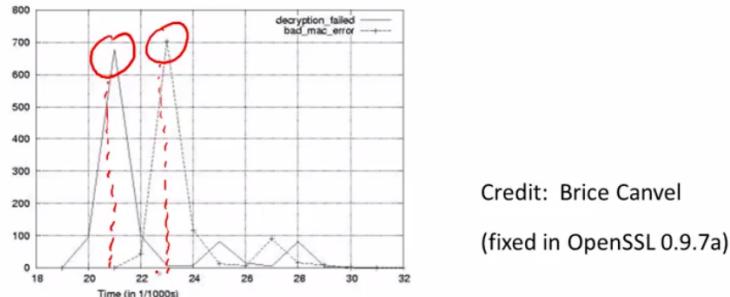
attacker submits ciphertext and learns if  
last bytes of plaintext are a valid pad

Nice example of a  
**chosen ciphertext attack**



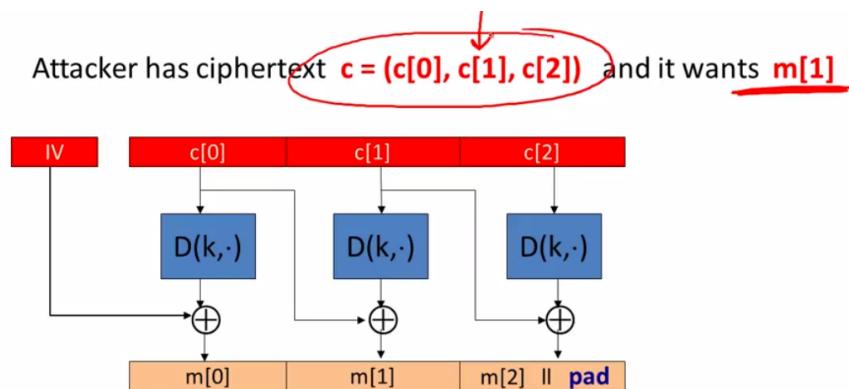
- advisory is allowed to learn something about the resulting plaintext

- Padding oracle via timing OpenSSL



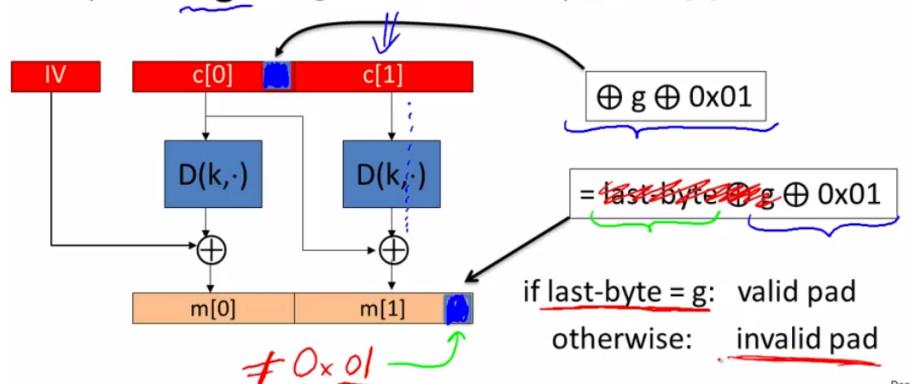
In older TLS 1.0: padding oracle due to different alert messages.

- if the time was short knows that the pad was invalid
- if the time was long knows that the pad was valid but the mac was invalid
- still has a padding oracle to tell if the pad was valid or invalid
- Using a padding oracle (CBC encryption)



- Using a padding oracle (CBC encryption)

step 1: let  $g$  be a guess for the last byte of  $m[1]$



Dan Boneh

- if our guess for the last pad is correct we learn that our guess is correct
- if our guess for the last byte is not correct we learn that our guess is not correct
- Using a padding oracle (CBC encryption)

Attack: submit  $(IV, c[0], c[1])$  to padding oracle  
 $\Rightarrow$  attacker learns if last-byte = g

Repeat with  $g = 0, 1, \dots, 255$  to learn last byte of  $m[1]$

Then use a  $(02, 02)$  pad to learn the next byte and so on ...

- IMAP over TLS

**Problem:** TLS renegotiates key when an invalid record is received

Enter IMAP over TLS: (protocol for reading email)

- Every five minutes client sends login message to server:  
LOGIN "username" "password"
- Exact same attack works, despite new keys  
 $\Rightarrow$  recovers password in a few hours.

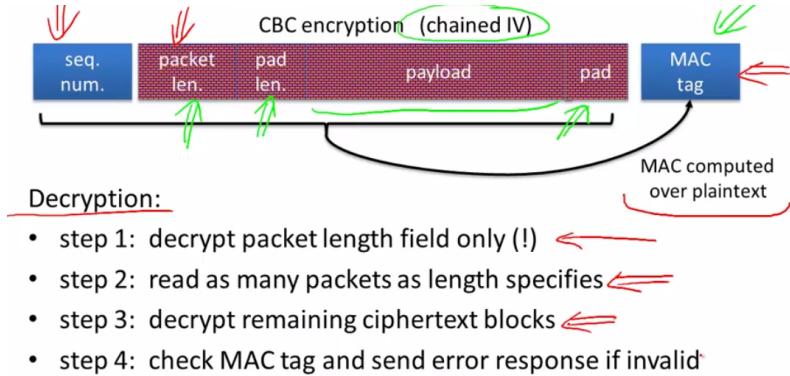
- Lesson
  - 1. Encrypt-then-MAC would completely avoid this problem:
    - MAC is checked first and cipher text discarded if invalid
  - 2. MAC-then-CBC provides A.E., but padding oracle destroys it
- Example

Will this attack work if TLS used counter mode instead of CBC?

(i.e. use MAC-then-CTR)

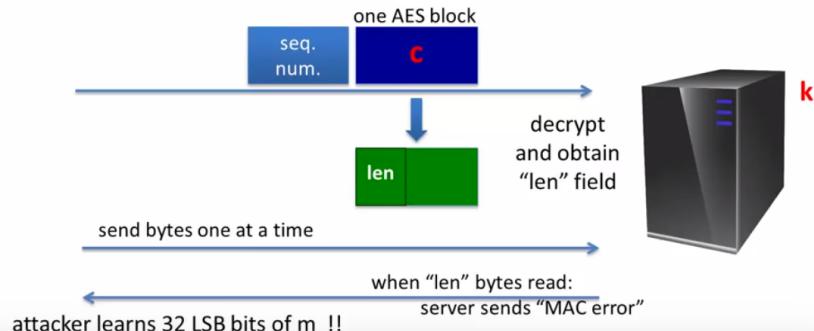
- Yes, padding oracles affect all encryption schemes
- It depends on what block cipher is used
- No, counter mode does not use padding

- Attacking Non-Atomic Decryption
- SSH Binary Packet Protocol



- An attack on the ecn. length field

Attacker has one ciphertext block  $c = \text{AES}(k, m)$  and it wants  $m$



- send packet to the server
- server decrypts the first few bytes as the length field of the packet
- server expects this many bytes before it checks that this is valid
- attack feeds server one byte at a time
- server reads as many bytes as the length field specifies
- the attack was counting the number of bytes
- learns the length
- Lesson

The problem: (1) non-atomic decrypt  
 (2) len field decrypted and used it before it is authenticated

How would you redesign SSH to resist this attack?

- ○ Send the length field unencrypted (but MAC-ed)
- Replace encrypt and MAC by encrypt then MAC
- ○ Add a MAC of (seq-num, length) right after the len field
- Remove the length field and identify packet boundary by verifying the MAC after every received byte

- Key Derivation
- Deriving many keys from one

**Typical scenario.** a single source key (SK) is sampled from:

- Hardware random number generator ←
- A key exchange protocol (discussed later) ←

Need many keys to secure session:

- unidirectional keys; multiple keys for nonce-based CBC.

**Goal:** generate many keys from this one source key



- When source key is uniform

F: a PRF with key space K and outputs in {0,1}<sup>n</sup>

Suppose source key SK is uniform in K

- Define Key Derivation Function (KDF) as:

$$\boxed{\text{KDF}(\text{SK}, \text{CTX}, L) := F(\text{SK}, (\text{CTX} \parallel 0)) \parallel F(\text{SK}, (\text{CTX} \parallel 1)) \parallel \dots \parallel F(\text{SK}, (\text{CTX} \parallel L))}$$

- CTX: a string that uniquely identifies the application
- Example

$$\boxed{\text{KDF}(\text{SK}, \text{CTX}, L) := F(\text{SK}, (\text{CTX} \parallel 0)) \parallel F(\text{SK}, (\text{CTX} \parallel 1)) \parallel \dots \parallel F(\text{SK}, (\text{CTX} \parallel L))}$$

What is the purpose of CTX?

- 
- Even if two apps sample same SK they get indep. keys
  - It's good practice to label strings with the app. name
  - It serves no purpose
  -

- What if source key is not uniform?

Recall: PRFs are pseudo random only when key is uniform in K

- SK not uniform ⇒ PRF output may not look random

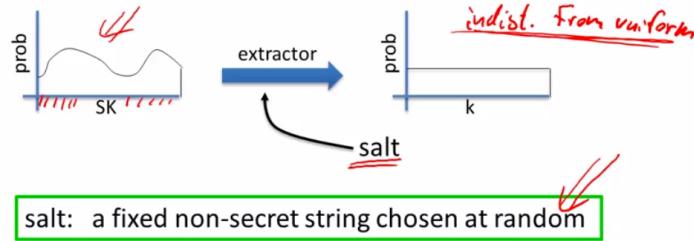
Source key often not uniformly random:



- Key exchange protocol: key uniform in some subset of K
- Hardware RNG: may produce biased output

- Extract-then-Expand paradigm

**Step 1:** extract pseudo-random key  $k$  from source key  $\text{SK}$



**step 2:** expand  $k$  by using it as a PRF key as before

- HKDF: a KDF from HMAC

Implements the extract-then-expand paradigm:

- extract: use  $k \leftarrow \text{HMAC}(\text{salt}, \text{SK})$ 
  - $\text{HMAC key}$  (red handwritten note)
  - $\text{HMAC data}$  (red handwritten note)
- Then expand using  $\text{HMAC}$  as a PRF with key  $k$

- Password-Based KDF (PBKDF)

Deriving keys from passwords:

- Do not use HKDF: passwords have insufficient entropy
- Derived keys will be vulnerable to dictionary attacks  
(more on this later)

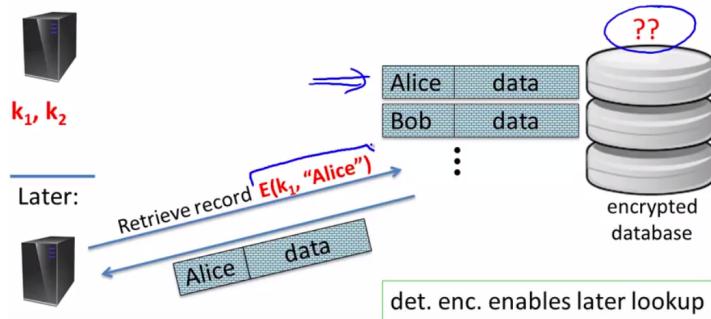
PBKDF defenses: salt and a slow hash function

Standard approach: PKCS#5 (PBKDF1)

$H^{(c)}(\text{pwd} \parallel \text{salt})$ : iterate hash function  $c$  times

- slow hash function is taking a hash function and iterate the function many many times
- **Odds and Ends 2: searching on encrypted data**
- **Deterministic Encryption**

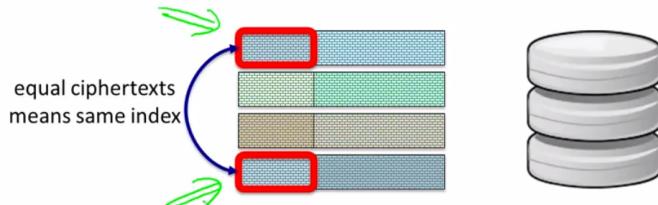
- The need for deterministic Encryption (no nonce)



- encrypts the index with  $k_1$  and encrypts the data with  $k_2$  the database has no knowledge of the data that is stored in it
- Problem: deterministic encryption cannot be CPA secure

The problem: attacker can tell when two ciphertexts encrypt the same message  $\Rightarrow$  leaks information

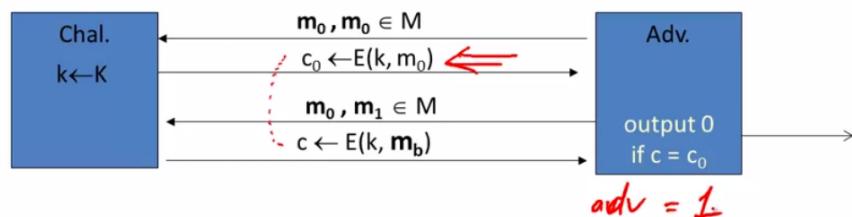
Leads to significant attacks when message space  $M$  is small.  $\leftarrow$



- Problem: deterministic encryption cannot be CPA secure

The problem: attacker can tell when two ciphertexts encrypt the same message  $\Rightarrow$  leaks information

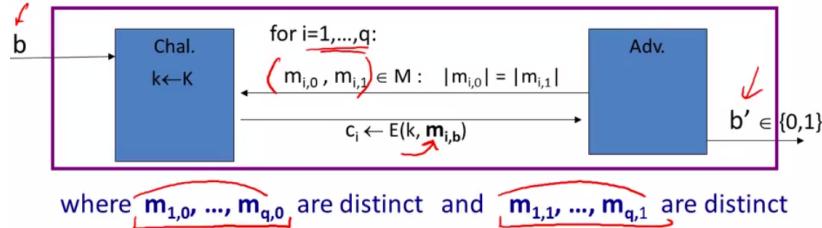
Attacker wins CPA game:



- A solution: the case of unique messages
  - Suppose encryptor never encrypts same message twice: the pair  $(k, m)$  never repeats

- This happens when encryptor:
  - chooses messages at random from large message space (e.g. keys)
  - message structure ensures uniqueness (e.g. unique user ID)
- Deterministic CPA security

$E = (E, D)$  a cipher defined over  $(K, M, C)$ . For  $b=0,1$  define  $\text{EXP}(b)$  as:



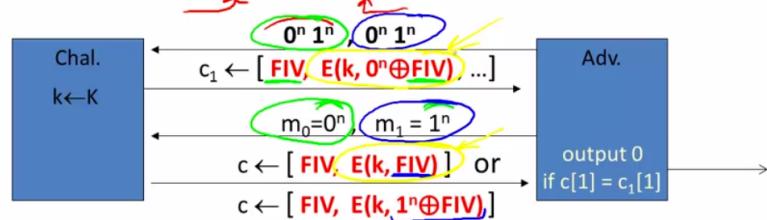
Def:  $E$  is sem. sec. under det. CPA if for all efficient  $A$ :

$$\text{Adv}_{\text{dCPA}}[A, E] = |\Pr[\text{EXP}(0)=1] - \Pr[\text{EXP}(1)=1]| \text{ is negligible.}$$

- If the two probabilities are close then the advantage is negligible
- A Common mistake

### CBC with fixed IV is not det. CPA secure.

Let  $E: K \times \{0,1\}^n \rightarrow \{0,1\}^n$  be a secure PRP used in CBC



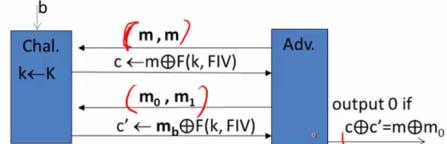
Leads to significant attacks in practice.

- Example
  - two time pad attack

Is counter mode with a fixed IV det. CPA secure?



- Yes  
 No  
 It depends



- Deterministic Encryption: SIV and Wide PRP
- Deterministic Encryption
  - Needed for maintaining an encrypted database index
    - lookup records by encrypted index
  - Deterministic CPA security
    - Security if never encrypt same message twice using same key:
      - the pair (key,msg) is unique
  - Formally: we defined deterministic CPA security game
- Construction 1: Synthetic IV (SIV)
  - CPA that doesn't use nonces has to be randomized

Let  $(E, D)$  be a CPA-secure encryption.  $E(k, m ; r) \rightarrow c$

Let  $F: K \times M \rightarrow R$  be a secure PRF

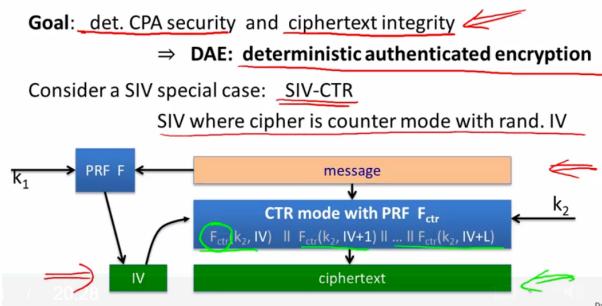
Define:  $E_{\text{det}}(k_1, k_2, m) = \begin{cases} r \leftarrow F(k_1, m) \\ c \leftarrow E(k_2, m; r) \\ \text{output } c \end{cases}$

Thm:  $E_{\text{det}}$  is sem. sec. under det. CPA.

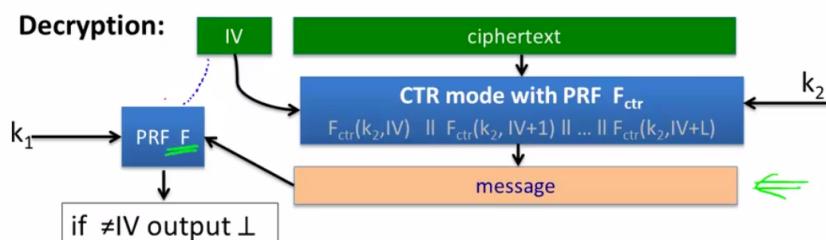
Proof sketch: distinct msgs.  $\Rightarrow$  all r's are indist. from random

Well suited for messages longer than one AES block (16 bytes)

- Ensuring ciphertext integrity
  - ciphertext integrity - attack gets to ask for encryptions of messages of his choice and shouldn't be able to produce another ciphertext that decrypts to a valid message



- Det. Auth. Enc. (DAE) for free



Thm: if F is a secure PRF and CTR from  $F_{ctr}$  is CPA-secure  
then SIV-CTR from  $F, F_{ctr}$  provides DAE

- Construction 2: just use a PRP

Let  $(E, D)$  be a secure PRP.  $E: K \times X \rightarrow X$

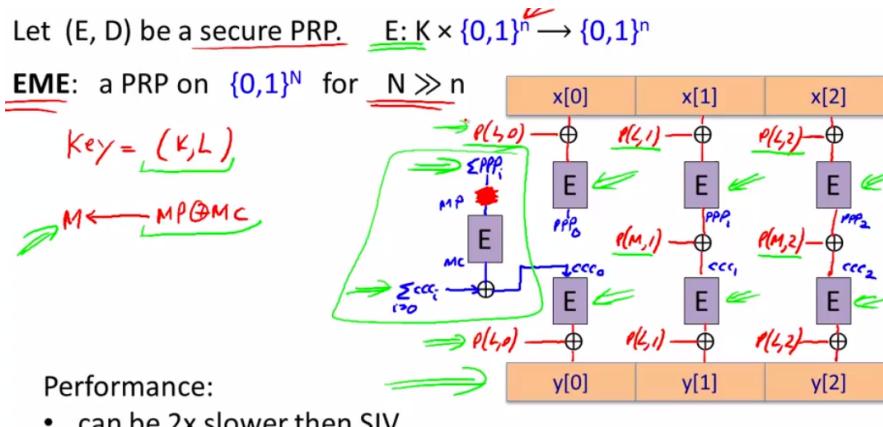
**Thm:**  $(E, D)$  is sem. sec. under det. CPA.

Proof sketch: let  $f: X \rightarrow X$  be a truly random invertible func.

in  $\text{EXP}(0)$  adv. sees:  $f(m_{1,0}), \dots, f(m_{q,0})$   $\xrightarrow{\text{q random values in } X}$   
 in  $\text{EXP}(1)$  adv. sees:  $f(m_{1,1}), \dots, f(m_{q,1})$

**Using AES:** Det. CPA secure encryption for 16 byte messages.

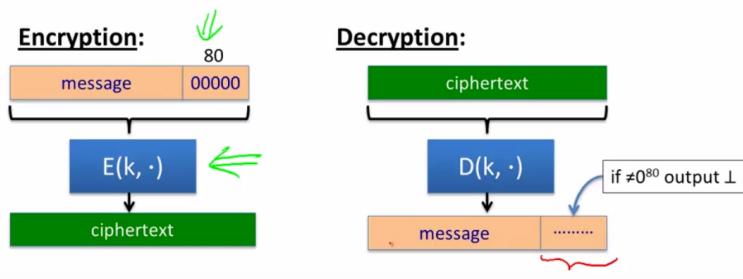
- if longer then 16 byte messages use SIV
- can we construct PRP that have message spaces that are larger then 16 bytes
- EME: constructing a wide block PRP
  - function P() padding function
  - EME construction very parallel



- PRP-based Deterministic Authenticated Encryption

**Goal:** det. CPA security and ciphertext integrity

$\Rightarrow$  DAE: deterministic authenticated encryption



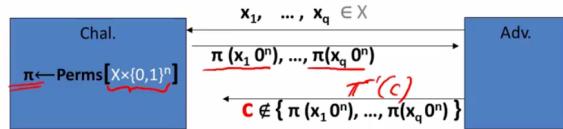
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- PRP-based Deterministic Authenticated Encryption

Let  $(E, D)$  be a secure PRP.  $E: K \times (X \times \{0,1\}^n) \rightarrow X \times \{0,1\}^n$

**Thm:**  $\frac{1}{2^n}$  is negligible  $\Rightarrow$  PRP-based enc. provides DAE

Proof sketch: suffices to prove ciphertext integrity



But then  $\Pr[\text{LSB}_n(\pi^{-1}(c)) = 0^n] \leq 1/2^n$  ↪ ~~no~~

- SIV

- encrypt index in database if large

- PRP

- encrypt index in database if small
- append zeros which will be used as an integrity check
- (wide block PRP from narrow PRP called EME)

- **Odds and Ends 3: disk encryption and credit card encryption**

- **Tweakable Encryption**

- Disk encryption: no expansion

Sectors on disk are fixed size (e.g. 4KB)

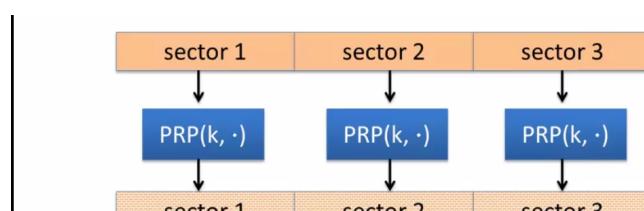
$\Rightarrow$  encryption cannot expand plaintext (i.e.  $M = C$ )

$\Rightarrow$  must use deterministic encryption, no integrity

Lemma: if  $(E, D)$  is a det. CPA secure cipher with  $M = C$   
then  $(E, D)$  is a PRP.

$\Rightarrow$  every sector will need to be encrypted with a PRP

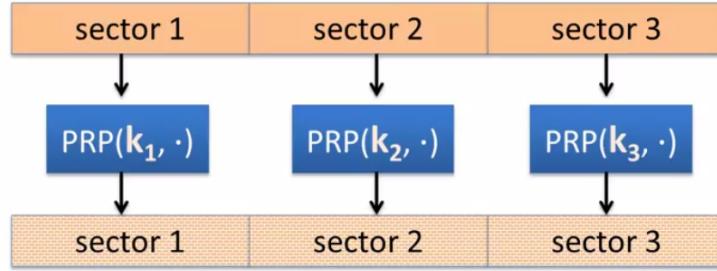
- encryption using a PRP



Problem: sector 1 and sector 3 may have same content  
• Leaks same information as ECB mode

Can we do better?

- can we do better



Avoids previous leakage problem

- ... but attacker can tell if a sector is changed and then reverted

Managing keys: the trivial construction  $\mathbf{k_t = PRF(k, t)}$ ,  $t=1,\dots,L$

#### - Teakable block ciphers

Goal: construct many PRPs from a key  $k \in K$ .

Syntax:  $E, D: K \times T \times X \rightarrow X$

for every  $t \in T$  and  $k \in K$ :

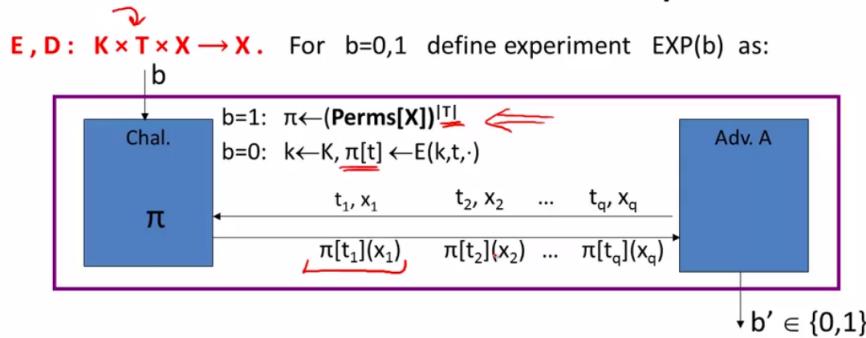
$E(k, t, \cdot)$  is an invertible func. on  $X$ , indist. from random

Application: use sector number as the tweak

⇒ every sector gets its own independent PRP

#### - Secure tweakable block ciphers

- did he interact with truly random functions or pseudo random functions



- Def:  $E$  is a secure tweakable PRP if for all efficient  $A$ :

$$\text{Adv}_{\text{tPRP}}[A, E] = |\Pr[\text{EXP}(0)=1] - \Pr[\text{EXP}(1)=1]| \text{ is negligible.}$$

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- Example 1: the trivial construction

$$\begin{array}{c} \text{X} \xrightarrow{\sim} \text{X} \\ \text{X} \xrightarrow{\sim} \text{X} \end{array}$$

Let  $(E, D)$  be a secure PRP,  $E: K \times X \rightarrow X$ .

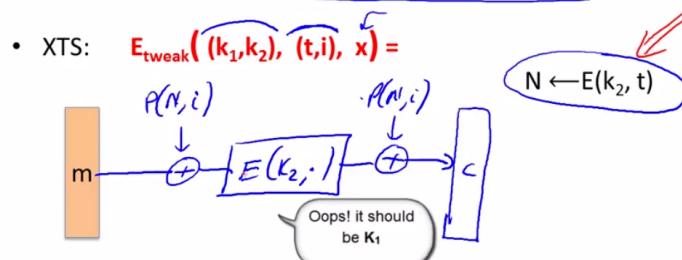
- The trivial tweakable construction: (suppose  $K = X$ )

$$E_{\text{tweak}}(k, t, x) = E(E(k, t), x)$$

⇒ to encrypt  $n$  blocks need  $2n$  evals of  $E(\dots)$

- 2. the XTS tweakable block cipher

Let  $(E, D)$  be a secure PRP,  $E: K \times \{0,1\}^n \rightarrow \{0,1\}^n$ .



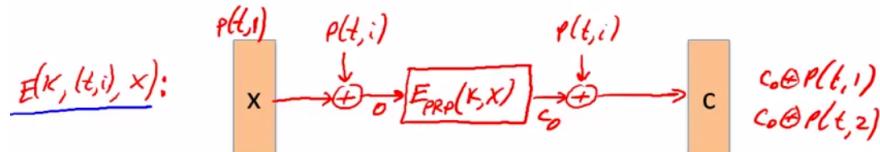
⇒ to encrypt  $n$  blocks need  $n+1$  evals of  $E(\dots)$

Dan F

- Example

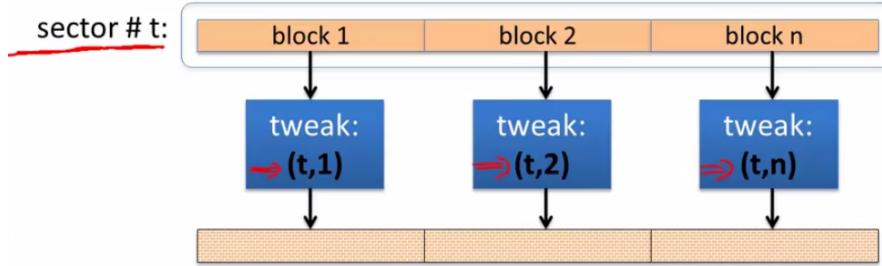
Is it necessary to encrypt the tweak before using it?

That is, is the following a secure tweakable PRP?



- Yes, it is secure
- No:  $E(k, (t, 1), P(t, 2)) \oplus E(k, (t, 2), P(t, 1)) = P(t, 1) \oplus P(t, 2)$
- ⇒ ○ No:  $E(k, (t, 1), P(t, 1)) \oplus E(k, (t, 2), P(t, 2)) = P(t, 1) \oplus P(t, 2)$
- No:  $E(k, (t, 1), P(t, 1)) \oplus E(k, (t, 2), P(t, 2)) = 0$

- Disk encryption using XTS



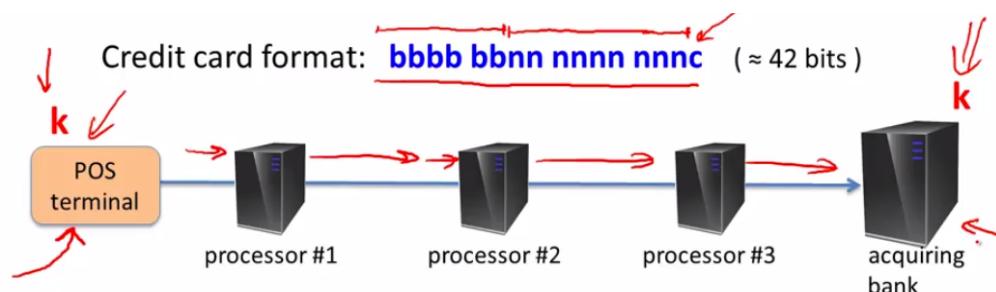
- note: block-level PRP, not sector-level PRP.
- Popular in disk encryption products:  
Mac OS X-Lion, TrueCrypt, BestCrypt, ...

- Summary

- Use tweakable encryption when you need many independent PRPs from one key
- XTS is more efficient than the trivial construction
  - Both are narrow block: 16 bytes for AES
- EME (previous segment) is a tweakable mode for wide block
  - 2x slower than XTS

- Format Preserving Encryption

- Encrypting credit card numbers



Goal: end-to-end encryption

- intermediate processors expect to see a credit card number => encrypted credit card should look like a credit card

- Format preserving encryption (FPE)

$$s \leq 2^{42}$$

This segment: given  $0 < s \leq 2^n$ , build a PRP on  $\{0, \dots, s-1\}$

from a secure PRF  $F: K \times \{0,1\}^n \rightarrow \{0,1\}^n$  (e.g. AES)  
 $n=128$

Then to encrypt a credit card number: ( $s = \text{total } \# \text{ credit cards}$ )

1. map given CC# to  $\{0, \dots, s-1\}$
2. apply PRP to get an output in  $\{0, \dots, s-1\}$
3. map output back a to CC#

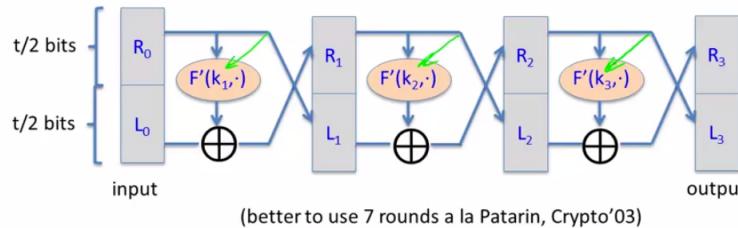
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- Step 1

**Step 1: from  $\{0,1\}^n$  to  $\{0,1\}^t$  ( $t < n$ )**

Want PRP on  $\{0, \dots, s-1\}$ . Let  $t$  be such that  $2^{t-1} < s \leq 2^t$ .  $t = 42$

Method: Luby-Rackoff with  $F': K \times \{0,1\}^{t/2} \rightarrow \{0,1\}^{t/2}$  (truncate F)



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- Step 2

Given PRP  $(E, D): K \times \{0,1\}^t \rightarrow \{0,1\}^t$   $\xleftarrow{2^{t-1} < s \leq 2^t}$

we build  $(E', D'): K \times \{0, \dots, s-1\} \rightarrow \{0, \dots, s-1\}$   $\xleftarrow{s > \frac{2^t}{2}}$

$E'(k, x)$ : on input  $x \in \{0, \dots, s-1\}$ , do:

$y \leftarrow x$ ; do  $\{y \leftarrow E(k, y)\}$  until  $y \in \{0, \dots, s-1\}$ ; output y



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- Security

Step 2 is tight:  $\forall A \exists B: \text{PRP}_{\text{adv}}[A, E] = \text{PRP}_{\text{adv}}[B, E']$

Intuition:  $\forall$  sets  $Y \subseteq X$ , applying the transformation to a random perm.  $\pi: X \rightarrow X$  ↪  
gives a random perm.  $\pi': Y \rightarrow Y$  ↪

Step 1: same security as Luby-Rackoff construction

(actually using analysis of Patarin, Crypto'03)

note: no integrity