Base GARCH Model Comparision

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Introduction

The goal of this document is to describe the thought process behind the base GARCH model specification. Information about the data and the calculation of returns can be found in the data section of the draft. I will not review that, instead beginning with the OLS results. Just to remind of the ols specification:

$$R_t^e = \alpha + \beta R_t^a + \epsilon_t$$

where R^e is the return of the asset and R^a is the return of the asset. Other literature defined the difference between these two returns as *tracking difference* which is analogous to setting α to 0 and β to 1. By incorporating α and β we capture systematic differences in returns such as those caused by the management fee or so-called *cash drag*. For further discussions on definitions of tracking ability, see Davidson *et al.*

We define *tracking error* thus as ϵ_t, but we may give more thought to the specific language of that. It will be important to define exactly what we mean and what we are measuring. Your input on this matter is greatly appredicated

The general outline for what follows is such: I will go one-by-one through each ETF investigating the tracking error (ϵ_t). I will first test for ARCH effects by analyzing the autocorrelation of squared tracking error. I will then go on to test multiple specifications of a GARCH(1,1) model, including those with ARCH in Mean, those with an ARMA process in the mean, and those with external variables in the mean. For fitting the ARMA process, I will utilize the

Just some thoughts before I get going: based off all of my prior model runs, I suspect fitting an ARMA process will greatly improve model fit. I believe that wihtout an ARMA process, no external variance variables will be significant.

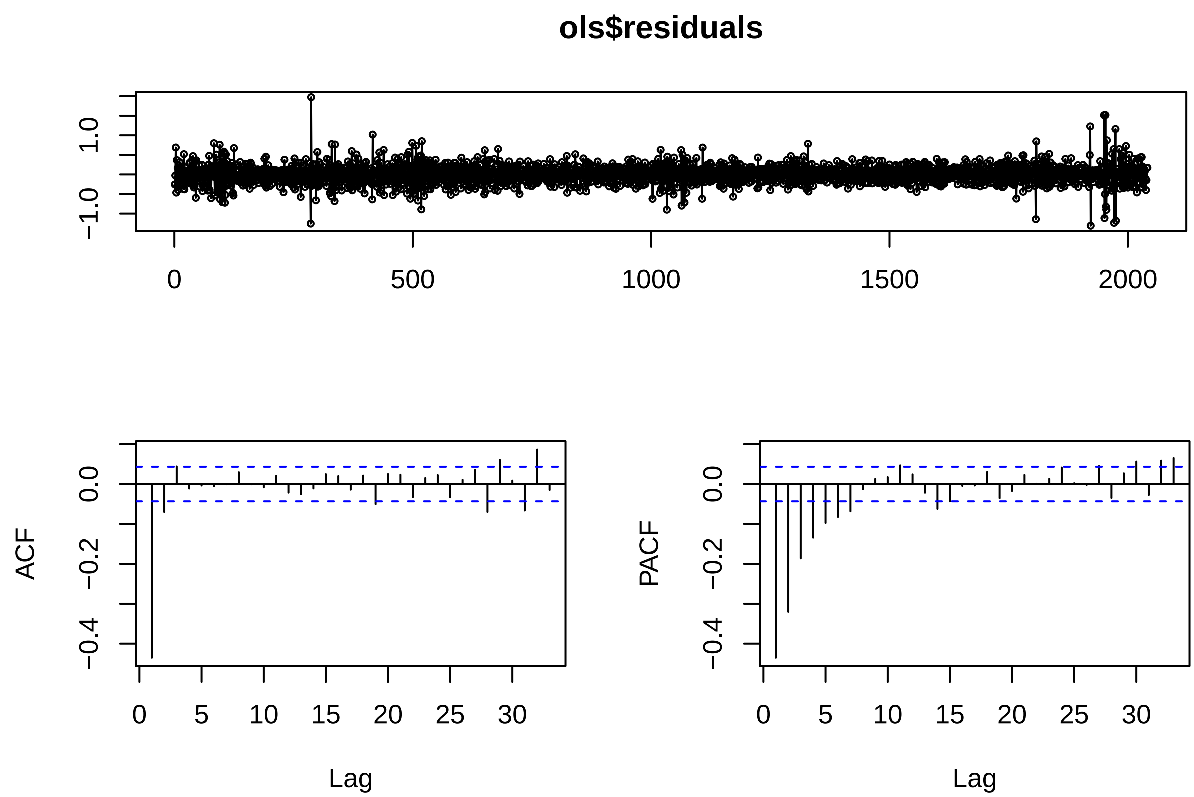
Corn

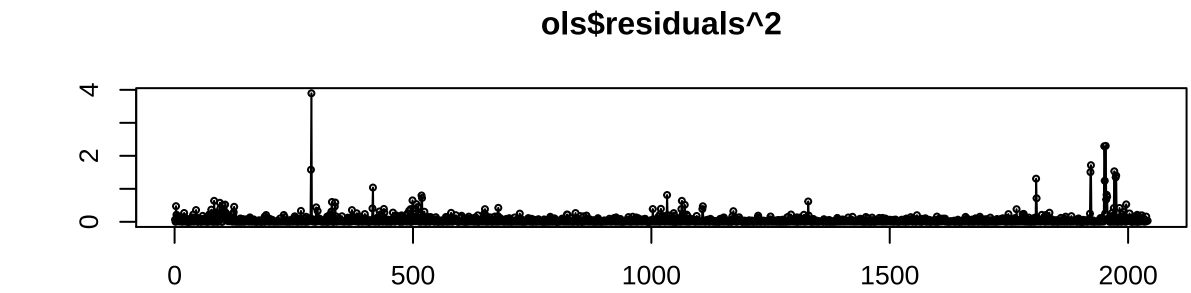
Lets begin by looking at the OLS results for CORN. We would reject both of our null hypothesis ($\alpha = 0$ and $\beta = 1$)

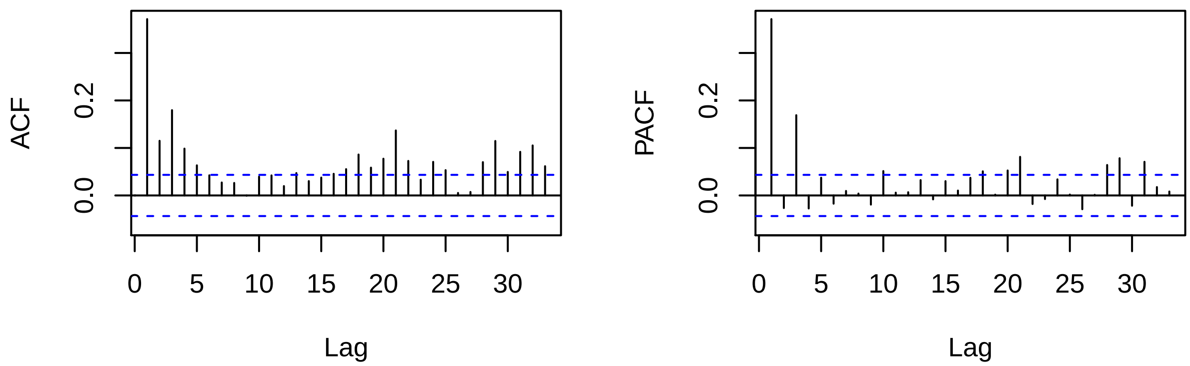
```
##
## Call:
## lm(formula = x$per_ETF_return ~ x$per_asset_return)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.31006 -0.14012  0.00163  0.13317  1.97356
##
## Coefficients:
```

```
##                        Estimate Std. Error t value Pr(>|t|)
## (Intercept)           -0.015100   0.005459  -2.766  0.00572 **
## x$per_asset_return  0.981419   0.004491 218.534  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2465 on 2039 degrees of freedom
## Multiple R-squared:  0.9591, Adjusted R-squared:  0.959
## F-statistic: 4.776e+04 on 1 and 2039 DF,  p-value: < 2.2e-16
```

Now we investigate the residuals. There is clearly evidence of ARCH effects in the residuals based off the Ljung-Box Test.








```
##
##  Box-Ljung test
##
## data:  ols$residuals^2
## X-squared = 281.81, df = 1, p-value < 2.2e-16
```

Now a base model without an ARMA process. The simplist GARCH(1,1) model where we do not include the unconditional mean would be produce these robust coefficients

```
## [1] "No unconditional mean Robust Coefficients"
```

```
##          Estimate  Std. Error  t value   Pr(>|t|)
## omega  0.01092029 0.009491288 1.150559 0.24991371
## alpha1 0.26358304 0.106548925 2.473822 0.01336764
## beta1  0.55980200 0.256602734 2.181590 0.02913979
```

```
##
## Akaike       -0.1425104
## Bayes        -0.1342480
## Shibata      -0.1425147
## Hannan-Quinn -0.1394797
```

Including the unconditional mean does not improve model fit.

```
## [1] "Including Unconditional Mean Robust Coefficients"
```

```
##             Estimate   Std. Error      t value    Pr(>|t|)
## mu     5.861664e-16 0.0025165006 2.329292e-13 1.000000000
## omega  7.929258e-05 0.0003110415 2.549261e-01 0.798780197
## alpha1 7.660923e-02 0.0263013204 2.912752e+00 0.003582588
## beta1  9.171640e-01 0.0377124625 2.431992e+01 0.000000000
```

```
##
## Akaike       -0.08983103
## Bayes        -0.07881448
## Shibata      -0.08983869
## Hannan-Quinn -0.08579013
```

We can now investigate an ARCH in means. The coefficient estimate for archm is statistically insignificant and inclusion does not improve model fit.

```
## [1] "Robust Coefficients"
```

```
##             Estimate  Std. Error      t value   Pr(>|t|)
## mu     5.861664e-16 0.014680937 3.992704e-14 1.00000000
## archm  1.516357e-02 0.072097926 2.103191e-01 0.83341865
## omega  1.095387e-02 0.009667316 1.133083e+00 0.25717936
## alpha1 2.648803e-01 0.108440151 2.442641e+00 0.01458025
## beta1  5.580403e-01 0.261552708 2.133567e+00 0.03287822
```

```
##
## Akaike       -0.1408368
## Bayes        -0.1270661
## Shibata      -0.1408488
## Hannan-Quinn -0.1357857
```

Currently the best model does not include the unconditional mean or arch in mean. While we have this very basic models, lets try some other flavors or GARCH to see if there are any large difference.

```
## [1] "Asymmetric Power ARCH (apARCH) Robust Coefficients"
```

```
##              Estimate   Std. Error       t value     Pr(>|t|)
## mu     -5.861662e-16 0.0027961961 -2.096299e-13 1.000000e+00
## omega   2.244999e-04 0.0002624647  8.553526e-01 3.923560e-01
## alpha1  5.267609e-02 0.0395601737  1.331543e+00 1.830102e-01
## beta1   8.797586e-01 0.0498613607  1.764410e+01 0.000000e+00
## gamma1  1.247528e-01 0.1020708197  1.222218e+00 2.216250e-01
## delta   3.248620e+00 0.5904199135  5.502219e+00 3.750409e-08
```

```
##
## Akaike       -0.1242651
## Bayes        -0.1077403
## Shibata      -0.1242823
## Hannan-Quinn -0.1182038
```

```
## [1] "GJR ARCH Robust Coefficients"
```

```
##            Estimate   Std. Error      t value     Pr(>|t|)
## mu     5.861664e-16 0.0467853865 1.252883e-14 1.000000e+00
## omega  6.234863e-05 0.0006596097 9.452352e-02 9.246933e-01
## alpha1 5.190862e-02 0.0542870449 9.561880e-01 3.389773e-01
## beta1  8.943478e-01 0.1209879557 7.392040e+00 1.445510e-13
## gamma1 6.876850e-02 0.1795447106 3.830160e-01 7.017079e-01
```

```
##
## Akaike       -0.04238521
## Bayes        -0.02861452
## Shibata      -0.04239718
## Hannan-Quinn -0.03733409
```

```
## [1] "eARCH Robust Coefficients"
```

```
##             Estimate  Std. Error       t value     Pr(>|t|)
## mu     -5.861663e-16 0.003922555 -1.494348e-13 1.000000e+00
## omega  -3.309038e-01 0.120758433 -2.740212e+00 6.139948e-03
## alpha1 -5.485943e-02 0.039101749 -1.402992e+00 1.606193e-01
## beta1   8.855129e-01 0.041648463  2.126160e+01 0.000000e+00
## gamma1  4.134670e-01 0.079038375  5.231219e+00 1.683961e-07
```

```
##
## Akaike       -0.1396372
## Bayes        -0.1258665
## Shibata      -0.1396491
## Hannan-Quinn -0.1345861
```

No asymmetric model we have tested has a better fit than the base. We may also try some different distributions. Ramos 2015 used a student-t distribution which introduces two new coefficients: *shape* and *skew*

```
## [1] "Base Robust Coefficients"
```

```
##          Estimate  Std. Error   t value     Pr(>|t|)
## omega  0.01561239 0.006862938  2.274885 2.291283e-02
## alpha1 0.30144171 0.068734038  4.385625 1.156533e-05
## beta1  0.43475888 0.173653971  2.503593 1.229393e-02
## skew   1.01749586 0.034982753 29.085643 0.000000e+00
## shape  8.30839188 1.622101605  5.121992 3.023243e-07
```

```
##
## Akaike       -0.1794744
## Bayes        -0.1657037
## Shibata      -0.1794864
## Hannan-Quinn -0.1744233
```

```
## [1] "Asymmetric Power ARCH (apARCH) Robust Coefficients"
```

```
##             Estimate   Std. Error       t value     Pr(>|t|)
## mu     -5.861664e-16 2.585796e-03 -2.266870e-13 1.000000e+00
```

```
## omega   3.978239e-04 5.612632e-05  7.088009e+00 1.360467e-12
## alpha1  8.385658e-02 2.397494e-02  3.497676e+00 4.693302e-04
## beta1   7.999772e-01 1.010321e-01  7.918049e+00 2.442491e-15
## gamma1  7.748081e-02 8.781845e-02  8.822839e-01 3.776233e-01
## delta   3.262139e+00 5.331607e-01  6.118490e+00 9.446619e-10
## skew    9.945410e-01 3.673930e-02  2.707022e+01 0.000000e+00
## shape   8.167403e+00 1.170388e+00  6.978375e+00 2.986056e-12
```

```
##
## Akaike       -0.1644853
## Bayes        -0.1424522
## Shibata      -0.1645158
## Hannan-Quinn -0.1564035
```

```
## [1] "GJR ARCH Robust Coefficients"
```

```
##            Estimate   Std. Error      t value  Pr(>|t|)
## mu     5.861664e-16 0.0121616236 4.819804e-14 1.0000000
## omega  6.227106e-05 0.0002323755 2.679760e-01 0.7887178
## alpha1 5.152725e-02 0.0364002739 1.415573e+00 0.1569005
## beta1  8.910399e-01 0.0216392306 4.117706e+01 0.0000000
## gamma1 7.807932e-02 0.0630398837 1.238570e+00 0.2155047
## skew   9.406428e-01 0.0239329789 3.930321e+01 0.0000000
## shape  4.684057e+00 0.1930865501 2.425885e+01 0.0000000
```

```
##
## Akaike       -0.06871350
## Bayes        -0.04943453
## Shibata      -0.06873691
## Hannan-Quinn -0.06164193
```

```
## [1] "eARCH Robust Coefficients"
```

```
##             Estimate  Std. Error       t value   Pr(>|t|)
## mu     -5.861664e-16 0.006193691 -9.463926e-14 1.00000000
## omega  -3.400788e-01 0.208742768 -1.629176e+00 0.10327568
## alpha1 -2.976734e-02 0.073112660 -4.071434e-01 0.68390265
## beta1   8.846453e-01 0.071073006  1.244699e+01 0.00000000
## gamma1  3.964546e-01 0.140866049  2.814408e+00 0.00488671
## skew    1.005594e+00 0.064053809  1.569921e+01 0.00000000
## shape   6.241083e+00 0.548748027  1.137331e+01 0.00000000
```

```
##
## Akaike       -0.1691387
## Bayes        -0.1498597
## Shibata      -0.1691621
## Hannan-Quinn -0.1620671
```

Using a student-t distribution significantly improves model fit. Within the models we test, the base, non asymmetric model still has the best fit.

Before we switch gears and try this out with an ARMA process, there are two external variables which I believe have very strong support for being included in the mean process. The first is the volatility of asset returns. *Is the tracking error time varying* found that tracking error increased significantly in times of high volatility. The second variable that I believe is appropriate in the mean is backwardation. The shape of the futures curve is a major concern for ETFs which need to roll before contract expiration and rebalance. In highly backwarded markets, ETP managers may more aggressively rebalance in the back of the curve.

There are multiple ways we might try to capture the volatility of the underlying. Here I try three ways. In each of the results mxreg1 is the coefficient in question. The best model fit of this group is using the absolute value of the asset return, which is the methodology of the original Neff paper. Still, the fit is not better than the base and the coefficient is not statistically significant at the 5% level.

```
## [1] "Asset Percent Return"

##           Estimate  Std. Error    t value     Pr(>|t|)
## mxreg1 -0.00144283 0.005090103 -0.2834579 7.768258e-01
## omega   0.01554436 0.007020065  2.2142762 2.680979e-02
## alpha1  0.30070179 0.070031169  4.2938279 1.756186e-05
## beta1   0.43679673 0.177806564  2.4565838 1.402651e-02
## skew    1.01717566 0.034932730 29.1181265 0.000000e+00
## shape   8.26465592 1.616726796  5.1119682 3.188196e-07

##
## Akaike       -0.1785564
## Bayes        -0.1620316
## Shibata      -0.1785736
## Hannan-Quinn -0.1724951

## [1] "Absolute Value of Percent Asset Return"

##           Estimate  Std. Error    t value     Pr(>|t|)
## mxreg1 0.001839579 0.002838350  0.6481155 5.169103e-01
## omega  0.015689391 0.006886032  2.2784372 2.270054e-02
## alpha1 0.301895690 0.068592485  4.4012940 1.076072e-05
## beta1  0.433114371 0.173809494  2.4918913 1.270649e-02
## skew   1.016669754 0.034785303 29.2269918 0.000000e+00
## shape  8.280511486 1.614351319  5.1293119 2.908031e-07

##
## Akaike       -0.1786173
## Bayes        -0.1620925
## Shibata      -0.1786345
## Hannan-Quinn -0.1725560

## [1] "Squared Asset Percent Returns"

##            Estimate  Std. Error     t value     Pr(>|t|)
## mxreg1 0.0001423182 0.001525659  0.09328312 9.256786e-01
## omega  0.0156334364 0.006882470  2.27148615 2.311756e-02
## alpha1 0.3015178461 0.068668068  4.39094699 1.128581e-05
## beta1  0.4343551949 0.173873899  2.49810464 1.248593e-02
## skew   1.0171624317 0.034729165 29.28842168 0.000000e+00
## shape  8.2949582036 1.619575215  5.12168754 3.028134e-07
```

```
##
## Akaike       -0.1784992
## Bayes        -0.1619744
## Shibata      -0.1785165
## Hannan-Quinn -0.1724379
```

I will now add the backwardation external variable to two models: one with no other mean external variables, the other which includes the absolute value of the mean. Here I define backwardation only with regard to the second and third contract to expire. For these models mxreg1 is the backwardation coefficient, mxreg2 is the absolute asset return where applicable

```
## [1] "Only Backwardation included"
```

```
##           Estimate  Std. Error   t value     Pr(>|t|)
## mxreg1 -0.01572100 0.004696885 -3.347111 8.165836e-04
## omega   0.01528495 0.006586818  2.320536 2.031191e-02
## alpha1  0.30403722 0.067739764  4.488312 7.178962e-06
## beta1   0.43821158 0.167732090  2.612569 8.986468e-03
## skew    1.01856014 0.035227457 28.913814 0.000000e+00
## shape   8.45851429 1.685467821  5.018496 5.207745e-07
```

```
##
## Akaike       -0.1796233
## Bayes        -0.1630985
## Shibata      -0.1796405
## Hannan-Quinn -0.1735620
```

```
## [1] "Backwardation and Absolute Value of Percent Asset Return"
```

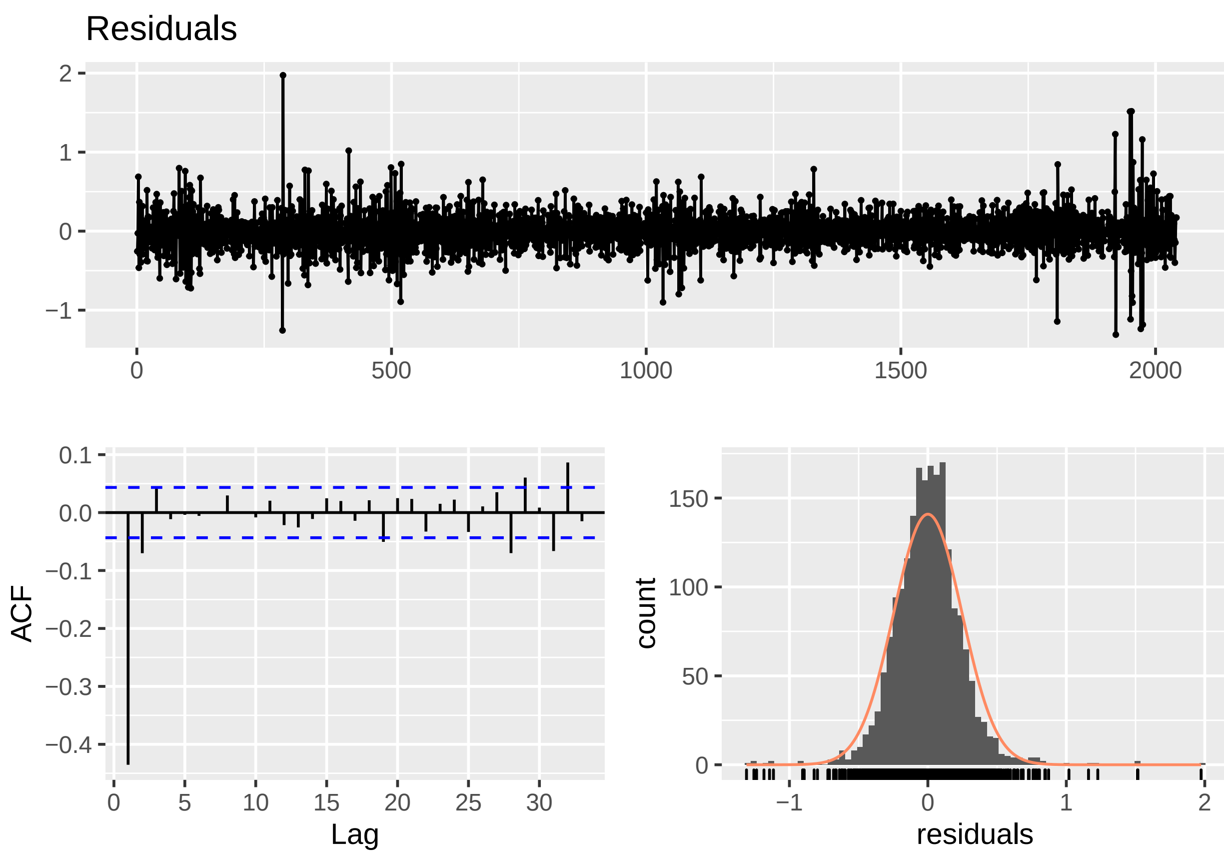
```
##            Estimate  Std. Error   t value     Pr(>|t|)
## mxreg1 -0.020010587 0.005624371 -3.557836 3.739233e-04
## mxreg2  0.004322194 0.003103969  1.392473 1.637791e-01
## omega   0.015370874 0.006535164  2.352026 1.867148e-02
## alpha1  0.305750627 0.067082760  4.557812 5.168921e-06
## beta1   0.435463075 0.165971897  2.623716 8.697635e-03
## skew    1.016712787 0.034972888 29.071456 0.000000e+00
## shape   8.432151386 1.675838772  5.031601 4.864017e-07
```

```
##
## Akaike       -0.1792412
## Bayes        -0.1599623
## Shibata      -0.1792646
## Hannan-Quinn -0.1721697
```

Now lets start over and fit an ARMA process to the residuals. I utilize the Box- Jenkins procedure to fit the model. A good specification would have no autocorrelation in the residuals, but autocorrelation in the squared residuals.

The autocorrelation of the residuals is significant, but dies off very slowly. With a max lag of 10, an ARMA(0,1) model has the characteristics we are searching for.

```
## Warning in modeldf.default(object): Could not find appropriate degrees of
## freedom for this model.
```




```
##
##  Box-Ljung test
##
## data:  a$residuals
## X-squared = 15.972, df = 10, p-value = 0.1004
```

```
##
##  Box-Ljung test
##
## data:  a$residuals^2
## X-squared = 214.69, df = 10, p-value < 2.2e-16
```

Now let's start over from the beginning with out base GARCH model and see if we should include the unconditional mean. Immediately huge improvements in model fit by including an MA process. The fit does not improve when including the unconditional mean, so I will leave it out.

```
## [1] "No Unconditional Mean Robust Coefficients"
```

```
##            Estimate   Std. Error    t value   Pr(>|t|)
## ma1    -0.691656498 0.0180056287 -38.413349 0.00000000
## omega   0.001018287 0.0009117043   1.116905 0.26403503
## alpha1  0.058749271 0.0275329803   2.133778 0.03286095
## beta1   0.917256948 0.0474984558  19.311300 0.00000000
```

```
##
## Akaike       -0.4598195
## Bayes        -0.4488029
## Shibata      -0.4598272
## Hannan-Quinn -0.4557786
```

```
## [1] "Including Unconditional Mean Robust Coefficients"
```

```
##             Estimate   Std. Error       t value   Pr(>|t|)
## mu     -5.861664e-16 0.0014019919 -4.180954e-13 1.00000000
## ma1    -6.916566e-01 0.0181034168 -3.820586e+01 0.00000000
## omega   1.018330e-03 0.0009179855  1.109309e+00 0.26729669
## alpha1  5.875096e-02 0.0275446903  2.132932e+00 0.03293027
## beta1   9.172542e-01 0.0476408637  1.925352e+01 0.00000000
```

```
##
## Akaike       -0.4588396
## Bayes        -0.4450689
## Shibata      -0.4588516
## Hannan-Quinn -0.4537885
```

As in the non ARMA specification, the archm coefficient is not statistically significant and its inclusion does not help model fit.

```
## [1] "No ARCH in Mean Robust Coefficients"
```

```
##            Estimate   Std. Error    t value   Pr(>|t|)
## ma1    -0.691656498 0.0180056287 -38.413349 0.00000000
## omega   0.001018287 0.0009117043   1.116905 0.26403503
## alpha1  0.058749271 0.0275329803   2.133778 0.03286095
## beta1   0.917256948 0.0474984558  19.311300 0.00000000
```

```
##
## Akaike       -0.4598195
## Bayes        -0.4488029
## Shibata      -0.4598272
## Hannan-Quinn -0.4557786
```

```
## [1] "ARCH in Mean Robust Coefficients"
```

```
##            Estimate   Std. Error     t value   Pr(>|t|)
## ma1    -0.691862828 0.0180432374 -38.3447169 0.00000000
## archm   0.003481459 0.0073015419   0.4768115 0.63349639
## omega   0.001008979 0.0009013884   1.1193607 0.26298631
## alpha1  0.058705044 0.0273814448   2.1439717 0.03203515
## beta1   0.917545922 0.0470512078  19.5010068 0.00000000
```

```
##
## Akaike       -0.4589713
## Bayes        -0.4452006
## Shibata      -0.4589832
## Hannan-Quinn -0.4539201
```

Now lets compare normal distribution with student-t. Again we find student-t distribution to have a much better fit.

```
## [1] "Normal Distribution Robust Coefficients"

##            Estimate   Std. Error    t value   Pr(>|t|)
## ma1    -0.691656498 0.0180056287 -38.413349 0.00000000
## omega   0.001018287 0.0009117043   1.116905 0.26403503
## alpha1  0.058749271 0.0275329803   2.133778 0.03286095
## beta1   0.917256948 0.0474984558  19.311300 0.00000000

##
## Akaike       -0.4598195
## Bayes        -0.4488029
## Shibata      -0.4598272
## Hannan-Quinn -0.4557786

## [1] "Student t Distribution Robust Coefficients"

##            Estimate   Std. Error    t value     Pr(>|t|)
## ma1    -0.685107436 0.0168386235 -40.686665 0.000000e+00
## omega   0.001122145 0.0005603066   2.002734 4.520588e-02
## alpha1  0.056113502 0.0188312480   2.979808 2.884291e-03
## beta1   0.913906675 0.0308041765  29.668272 0.000000e+00
## skew    1.000865135 0.0315679831  31.705071 0.000000e+00
## shape   7.086754867 1.2750114274   5.558189 2.725877e-08

##
## Akaike       -0.5300421
## Bayes        -0.5135173
## Shibata      -0.5300593
## Hannan-Quinn -0.5239808
```

As we saw with the GARCH process with an ARMA process, asymmetric model specifications do not improve model fit

```
## [1] "Standard GARCH Robust Coefficients"

##            Estimate   Std. Error    t value     Pr(>|t|)
## ma1    -0.685107436 0.0168386235 -40.686665 0.000000e+00
## omega   0.001122145 0.0005603066   2.002734 4.520588e-02
## alpha1  0.056113502 0.0188312480   2.979808 2.884291e-03
## beta1   0.913906675 0.0308041765  29.668272 0.000000e+00
## skew    1.000865135 0.0315679831  31.705071 0.000000e+00
## shape   7.086754867 1.2750114274   5.558189 2.725877e-08

##
## Akaike       -0.5300421
## Bayes        -0.5135173
## Shibata      -0.5300593
## Hannan-Quinn -0.5239808
```

```
## [1] "Asymmetric Power GARCH (apGARCH) Robust Coefficients"

##            Estimate  Std. Error     t value     Pr(>|t|)
## ma1    -0.685290233 0.016836499 -40.7026554 0.000000e+00
## omega   0.001857159 0.001706568   1.0882426 2.764880e-01
## alpha1  0.065361354 0.023424139   2.7903418 5.265242e-03
## beta1   0.911909579 0.029055431  31.3851683 0.000000e+00
## gamma1  0.047069601 0.092525182   0.5087221 6.109471e-01
## delta   1.697285303 0.426072415   3.9835606 6.789034e-05
## skew    1.000232403 0.031609098  31.6438139 0.000000e+00
## shape   7.085018371 1.274009146   5.5611990 2.679275e-08

##
## Akaike       -0.5285093
## Bayes        -0.5064762
## Shibata      -0.5285399
## Hannan-Quinn -0.5204275

## [1] "GJR GARCH Robust Coefficients"

##            Estimate   Std. Error     t value     Pr(>|t|)
## ma1    -0.684955374 0.0168846634 -40.5667177 0.000000e+00
## omega   0.001126926 0.0005552519   2.0295758 4.239967e-02
## alpha1  0.052655271 0.0195278134   2.6964243 7.008832e-03
## beta1   0.913560570 0.0307892904  29.6713746 0.000000e+00
## gamma1  0.007362779 0.0200091720   0.3679702 7.128955e-01
## skew    1.000747422 0.0315135486  31.7561007 0.000000e+00
## shape   7.102428206 1.2810614701   5.5441744 2.953444e-08

##
## Akaike       -0.5291412
## Bayes        -0.5098623
## Shibata      -0.5291647
## Hannan-Quinn -0.5220697

## [1] "Exponential Power GARCH Robust Coefficients"

##            Estimate  Std. Error     t value     Pr(>|t|)
## ma1    -0.685562562  0.01614707 -42.4573857 0.000000e+00
## omega  -0.097662661  0.04206509  -2.3217032 2.024892e-02
## alpha1 -0.008631583  0.01341937  -0.6432181 5.200826e-01
## beta1   0.970921598  0.01242107  78.1672973 0.000000e+00
## gamma1  0.147509287  0.02934731   5.0263299 4.999555e-07
## skew    0.999216376  0.03136540  31.8572808 0.000000e+00
## shape   7.012819657  1.25531141   5.5865179 2.316677e-08

##
## Akaike       -0.5287558
## Bayes        -0.5094768
## Shibata      -0.5287792
## Hannan-Quinn -0.5216842
```

I will not test three specifications of external variables: one with only Absolute Percent Asset Returns, one with only Backwardation, and one with both. We find exactly what we found for the non ARMA specification: asset return is not significant by itself, backwardation is, and together both are. The best model in terms of AIC includes both.

```
## [1] "Absolute Percentage Asset Returns Robust Coefficients"
```

```
##              Estimate   Std. Error     t value     Pr(>|t|)
## ma1    -6.850963e-01 0.0168741565 -40.6003307 0.000000e+00
## mxreg1  2.705352e-05 0.0014607101   0.0185208 9.852234e-01
## omega   1.121697e-03 0.0005635934   1.9902587 4.656244e-02
## alpha1  5.610699e-02 0.0188565037   2.9754715 2.925385e-03
## beta1   9.139253e-01 0.0309069074  29.5702597 0.000000e+00
## skew    1.000997e+00 0.0312131832  32.0696973 0.000000e+00
## shape   7.086929e+00 1.2757161687   5.5552555 2.772061e-08
```

```
##
## Akaike       -0.5290624
## Bayes        -0.5097834
## Shibata      -0.5290858
## Hannan-Quinn -0.5219908
```

```
## [1] "Backwardation Robust Coefficients"
```

```
##             Estimate   Std. Error    t value     Pr(>|t|)
## ma1    -0.695221895 0.0163322635 -42.567394 0.000000e+00
## mxreg1 -0.012173730 0.0028225814  -4.312977 1.610706e-05
## omega   0.001218413 0.0006107548   1.994929 4.605057e-02
## alpha1  0.059283661 0.0200584765   2.955542 3.121206e-03
## beta1   0.908235944 0.0333297264  27.250027 0.000000e+00
## skew    0.985025118 0.0314750782  31.295399 0.000000e+00
## shape   6.951626206 1.2203327217   5.696501 1.222914e-08
```

```
##
## Akaike       -0.5358571
## Bayes        -0.5165781
## Shibata      -0.5358805
## Hannan-Quinn -0.5287855
```

```
## [1] "Absolute Asset returns (mxreg1) and Backwardation (mxreg2) Robust Coefficients"
```

```
##             Estimate   Std. Error    t value     Pr(>|t|)
## ma1    -0.696385412 0.0161421745 -43.140744 0.000000e+00
## mxreg1  0.002329618 0.0015631843   1.490303 1.361446e-01
## mxreg2 -0.014465265 0.0031429501  -4.602448 4.175536e-06
## omega   0.001210053 0.0006092354   1.986183 4.701306e-02
## alpha1  0.059582269 0.0202676554   2.939771 3.284547e-03
## beta1   0.908024767 0.0335251414  27.084890 0.000000e+00
## skew    0.993780358 0.0311689841  31.883630 0.000000e+00
## shape   6.951411034 1.2181360988   5.706596 1.152577e-08
```

```
##
## Akaike       -0.5361893
## Bayes        -0.5141562
## Shibata      -0.5362199
## Hannan-Quinn -0.5281076
```

CORN Summary

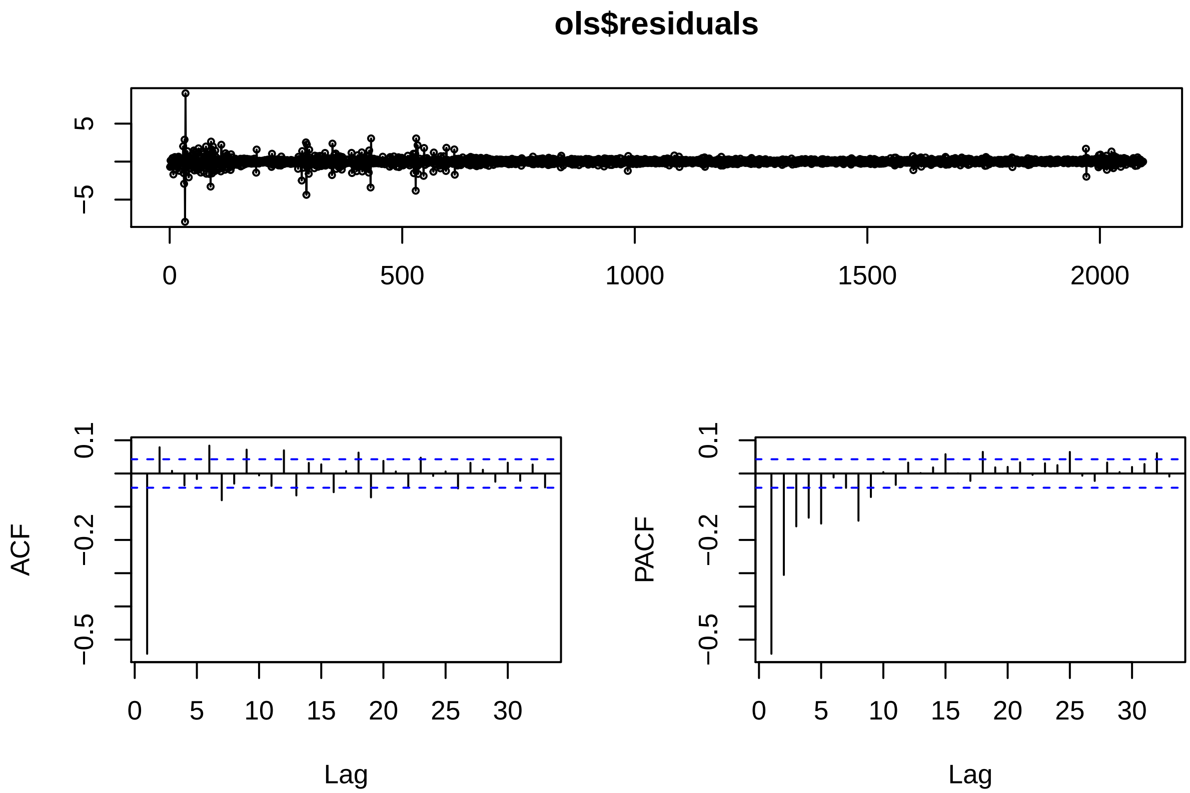
We find that include ARCH in mean or the unconditional mean does not improve model fit. The assumed distribution is important: the student t distribution is a much better fit than the normal. Regardless of distribution, asymmetric model specifications do not improve model fit. Fitting an ARMA process (in this case an MA1 process) greatly improves model fit. Regardless of ARMA process, the best fitting model includes both the absolute value of the asset return and a dummy variable for backwardation in the mean equation.

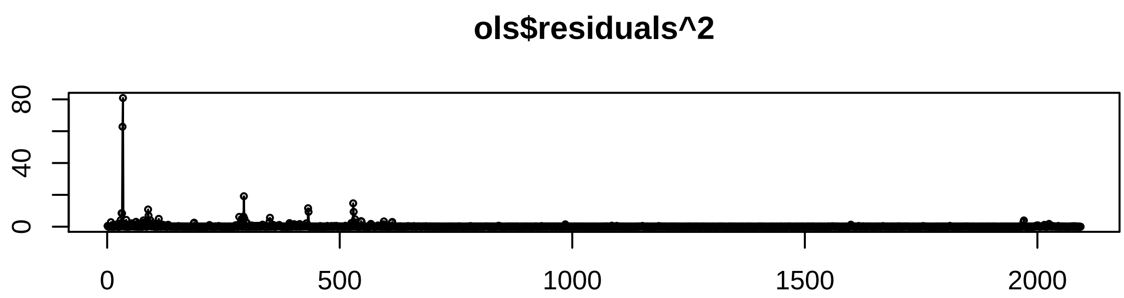
Soybeans

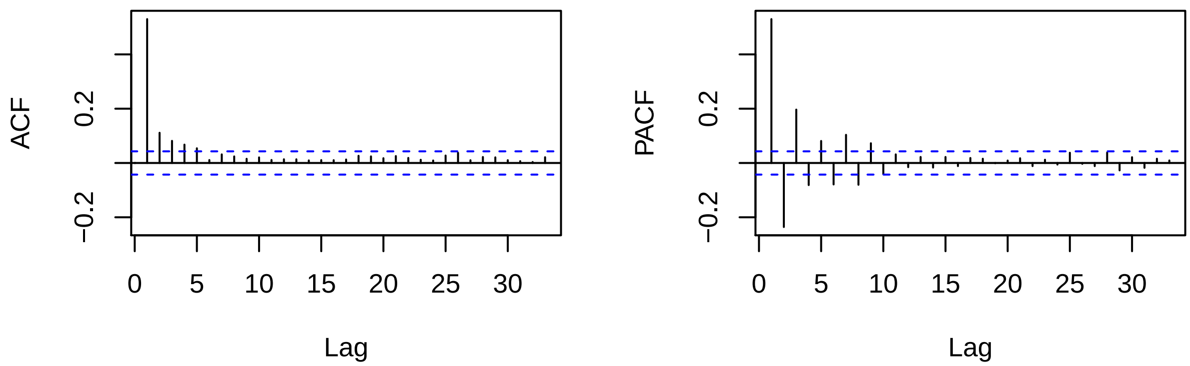
Lets begin by looking at the OLS results for soybeans. Unlike CORN, we fail to reject both of our null hypothesis ($\alpha = 0$ and $\beta = 1$). This is interesting considering the large size of tracking error in 2013.

```
##
## Call:
## lm(formula = x$per_ETF_return ~ x$per_asset_return)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -7.9248 -0.1541  0.0020  0.1674  8.9927
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)        -0.01730    0.01127  -1.535    0.125
## x$per_asset_return  0.99137    0.01095  90.559   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5155 on 2091 degrees of freedom
## Multiple R-squared:  0.7968, Adjusted R-squared:  0.7967
## F-statistic:  8201 on 1 and 2091 DF,  p-value: < 2.2e-16
```

Just like CORN, there evidence of ARCH effects in the residuals based off the Ljung-Box Test.


ols$residuals
5
−5
0
500
1000
1500
2000
ACF
0.1
−0.2
−0.5
0
5
10
15
20
25
30
Lag
PACF
0.1
−0.2
−0.5
0
5
10
15
20
25
30
Lag






```
##
##  Box-Ljung test
##
## data:  ols$residuals^2
## X-squared = 588.76, df = 1, p-value < 2.2e-16
```

Just as in CORN, including the unconditional mean is unimformative.

```
## [1] "No unconditional mean Robust Coefficients"
```

```
##           Estimate  Std. Error  t value     Pr(>|t|)
## omega  0.004532856  0.00444878 1.018899 3.082511e-01
## alpha1 0.220664891  0.12217967 1.806069 7.090760e-02
## beta1  0.775293032  0.12756877 6.077452 1.221075e-09
```

```
##
## Akaike       0.5397242
## Bayes        0.5478174
## Shibata      0.5397201
## Hannan-Quinn 0.5426890
```

```
## [1] "Including Unconditional Mean Robust Coefficients"
```

```
##            Estimate   Std. Error      t value     Pr(>|t|)
```

```
## mu     1.209873e-15 0.0034063703 3.551795e-13 1.000000e+00
## omega  5.627180e-04 0.0003966214 1.418779e+00 1.559636e-01
## alpha1 9.723188e-02 0.0245659321 3.957997e+00 7.558101e-05
## beta1  8.954777e-01 0.0264351504 3.387451e+01 0.000000e+00
```

```
##
## Akaike       0.5838772
## Bayes        0.5946681
## Shibata      0.5838699
## Hannan-Quinn 0.5878303
```

The ARCH in mean coefficient is not statistically significant at the 5% level, and including it help model fit.

```
## [1] "ARCH in Mean Robust Coefficients"
```

```
##             Estimate  Std. Error       t value     Pr(>|t|)
## mu     -1.209873e-15 0.012459508 -9.710439e-14 1.000000e+00
## archm   1.440355e-02 0.048191998  2.988784e-01 7.650328e-01
## omega   4.550660e-03 0.004487516  1.014071e+00 3.105489e-01
## alpha1  2.213534e-01 0.123518378  1.792069e+00 7.312196e-02
## beta1   7.746810e-01 0.128768611  6.016070e+00 1.787018e-09
```

```
##
## Akaike       0.5413869
## Bayes        0.5548755
## Shibata      0.5413755
## Hannan-Quinn 0.5463283
```

As with CORN, lets compare different asymmetric specifications, still assuming a normal distribution.

```
## [1] "Standard GARCH Robust Coefficients"
```

```
##           Estimate  Std. Error  t value     Pr(>|t|)
## omega  0.004532856  0.00444878 1.018899 3.082511e-01
## alpha1 0.220664891  0.12217967 1.806069 7.090760e-02
## beta1  0.775293032  0.12756877 6.077452 1.221075e-09
```

```
##
## Akaike       0.5397242
## Bayes        0.5478174
## Shibata      0.5397201
## Hannan-Quinn 0.5426890
```

```
## [1] "Asymmetric Power ARCH (apARCH) Robust Coefficients"
```

```
##             Estimate   Std. Error       t value     Pr(>|t|)
## mu      1.209873e-15 0.0042495881  2.847035e-13 1.000000e+00
## omega   7.088178e-04 0.0001868279  3.793961e+00 1.482629e-04
## alpha1  1.065969e-01 0.0206308423  5.166869e+00 2.380484e-07
## beta1   7.939969e-01 0.1045524117  7.594247e+00 3.086420e-14
## gamma1 -2.603372e-04 0.0873878702 -2.979100e-03 9.976230e-01
## delta   3.150204e+00 0.7343162093  4.289983e+00 1.786870e-05
```

```
##
## Akaike       0.5431401
## Bayes        0.5593265
## Shibata      0.5431237
## Hannan-Quinn 0.5490697

## [1] "GJR ARCH Robust Coefficients"

##            Estimate  Std. Error      t value   Pr(>|t|)
## mu     1.209873e-15 0.010099780 1.197920e-13 1.00000000
## omega  3.405613e-04 0.000336028 1.013491e+00 0.31082575
## alpha1 6.989875e-02 0.030133374 2.319646e+00 0.02036006
## beta1  9.092623e-01 0.032746143 2.776701e+01 0.00000000
## gamma1 1.631497e-02 0.040121912 4.066349e-01 0.68427619

##
## Akaike       0.6184553
## Bayes        0.6319440
## Shibata      0.6184440
## Hannan-Quinn 0.6233967

## [1] "eARCH Robust Coefficients"

##             Estimate  Std. Error       t value     Pr(>|t|)
## mu      1.209871e-15  0.00130356  9.281285e-13 1.000000e+00
## omega  -6.420969e-02  0.06486623 -9.898786e-01 3.222335e-01
## alpha1 -1.970296e-02  0.02846576 -6.921637e-01 4.888345e-01
## beta1   9.553909e-01  0.03105808  3.076143e+01 0.000000e+00
## gamma1  4.521855e-01  0.10196939  4.434522e+00 9.227694e-06

##
## Akaike       0.5470358
## Bayes        0.5605244
## Shibata      0.5470244
## Hannan-Quinn 0.5519771
```

Based on AIC, apARCH has the best fit, followed by the standard. I am going to keep this in mind, but continue with the base model. Again, I will replace the normal distribution with the student t distribution.

```
## [1] "Standard GARCH Robust Coefficients"

##          Estimate  Std. Error   t value     Pr(>|t|)
## omega  0.01678569  0.00488964  3.432909 5.971422e-04
## alpha1 0.47875454  0.07962713  6.012455 1.827344e-09
## beta1  0.47357690  0.07663659  6.179514 6.429912e-10
## skew   0.97771747  0.02708225 36.101787 0.000000e+00
## shape  4.34809811  0.53233279  8.168007 2.220446e-16

##
## Akaike       0.3649545
## Bayes        0.3784432
## Shibata      0.3649431
## Hannan-Quinn 0.3698959
```

```
## [1] "Asymmetric Power ARCH (apARCH) Robust Coefficients"
```

```
##            Estimate   Std. Error      t value     Pr(>|t|)
## mu     1.209873e-15 5.638894e-03 2.145585e-13 1.000000e+00
## omega  4.112796e-04 8.065721e-05 5.099105e+00 3.412624e-07
## alpha1 5.720701e-02 1.478371e-02 3.869599e+00 1.090146e-04
## beta1  8.649644e-01 1.600955e-02 5.402803e+01 0.000000e+00
## gamma1 2.496903e-03 4.440919e-01 5.622491e-03 9.955139e-01
## delta  2.924473e+00 1.917789e-01 1.524919e+01 0.000000e+00
## skew   1.017007e+00 6.092746e-02 1.669209e+01 0.000000e+00
## shape  4.879139e+00 3.320706e-01 1.469308e+01 0.000000e+00
```

```
##
## Akaike       0.4310281
## Bayes        0.4526099
## Shibata      0.4309990
## Hannan-Quinn 0.4389343
```

```
## [1] "GJR ARCH Robust Coefficients"
```

```
##            Estimate   Std. Error      t value    Pr(>|t|)
## mu     1.209873e-15 0.0219344947 5.515845e-14 1.000000000
## omega  3.238547e-04 0.0002251383 1.438470e+00 0.150300610
## alpha1 5.553123e-02 0.1455832603 3.814397e-01 0.702877028
## beta1  9.032853e-01 0.2756990344 3.276346e+00 0.001051598
## gamma1 5.056149e-02 0.1862635108 2.714514e-01 0.786043861
## skew   9.862278e-01 0.0594588178 1.658674e+01 0.000000000
## shape  5.036800e+00 2.5626550162 1.965462e+00 0.049360829
```

```
##
## Akaike       0.4448413
## Bayes        0.4637254
## Shibata      0.4448190
## Hannan-Quinn 0.4517592
```

```
## [1] "eARCH Robust Coefficients"
```

```
##             Estimate  Std. Error       t value     Pr(>|t|)
## mu      1.209873e-15  0.00377224  3.207306e-13 1.000000e+00
## omega  -3.434919e-01  0.13148246 -2.612454e+00 8.989469e-03
## alpha1 -4.271488e-02  0.04043438 -1.056400e+00 2.907855e-01
## beta1   8.558247e-01  0.05531800  1.547100e+01 0.000000e+00
## gamma1  6.345544e-01  0.09352454  6.784897e+00 1.161693e-11
## skew    9.681967e-01  0.03849345  2.515224e+01 0.000000e+00
## shape   4.317580e+00  0.55063086  7.841152e+00 4.440892e-15
```

```
##
## Akaike       0.3696923
## Bayes        0.3885765
## Shibata      0.3696701
## Hannan-Quinn 0.3766103
```

As with CORN, using a student-t distribution significantly improves model fit. However, unlike CORN, the asymmetric models have better fit based off AIC. The intution I have built up is making me cautious of going towards an asymmetric approach, but I will continue to compare these models as I move forward. I now add in the two external variables, just as I did with CORN. I will again start by testimg which crude metric for asset volatility does the best job. Again absolute percent return is the preferred metric based off AIC and the statistical significance of the value.

```
## [1] "Asset Percent Return"
```

```
##           Estimate  Std. Error    t value     Pr(>|t|)
## mxreg1 0.004965145 0.006012443  0.8258117 4.089109e-01
## omega  0.016915554 0.004910118  3.4450405 5.709738e-04
## alpha1 0.481157550 0.079824281  6.0277092 1.663000e-09
## beta1  0.471340072 0.076561550  6.1563549 7.443832e-10
## skew   0.978208143 0.027289367 35.8457612 0.000000e+00
## shape  4.332480154 0.529670090  8.1795824 2.220446e-16
```

```
##
## Akaike       0.3654555
## Bayes        0.3816418
## Shibata      0.3654391
## Hannan-Quinn 0.3713851
```

```
## [1] "Absolute Value of Percent Asset Return"
```

```
##           Estimate  Std. Error   t value     Pr(>|t|)
## mxreg1 0.009313404 0.003092492  3.011617 2.598598e-03
## omega  0.016445273 0.004810571  3.418570 6.295123e-04
## alpha1 0.476472112 0.079561893  5.988697 2.115282e-09
## beta1  0.477380270 0.076406866  6.247871 4.160843e-10
## skew   0.984683525 0.027958252 35.219781 0.000000e+00
## shape  4.351762271 0.533363984  8.159085 4.440892e-16
```

```
##
## Akaike       0.3640203
## Bayes        0.3802067
## Shibata      0.3640039
## Hannan-Quinn 0.3699499
```

```
## [1] "Squared Asset Percent Returns"
```

```
##           Estimate  Std. Error   t value     Pr(>|t|)
## mxreg1 0.004120266 0.001654836  2.489833 1.278031e-02
## omega  0.016496007 0.004796391  3.439254 5.833205e-04
## alpha1 0.476546233 0.079163736  6.019754 1.746822e-09
## beta1  0.477087716 0.075916206  6.284399 3.291247e-10
## skew   0.980256004 0.027392257 35.785880 0.000000e+00
## shape  4.349522653 0.530559407  8.197994 2.220446e-16
```

```
##
## Akaike       0.3641407
## Bayes        0.3803271
## Shibata      0.3641243
## Hannan-Quinn 0.3700704
```

I will now add the backwardation external variable. Just as in CORN, the best fitting model included both external variables.

```
## [1] "No Externals Included Robust Coefficients"

##          Estimate  Std. Error   t value     Pr(>|t|)
## omega  0.01678569  0.00488964  3.432909 5.971422e-04
## alpha1 0.47875454  0.07962713  6.012455 1.827344e-09
## beta1  0.47357690  0.07663659  6.179514 6.429912e-10
## skew   0.97771747  0.02708225 36.101787 0.000000e+00
## shape  4.34809811  0.53233279  8.168007 2.220446e-16

##
## Akaike       0.3649545
## Bayes        0.3784432
## Shibata      0.3649431
## Hannan-Quinn 0.3698959

## [1] "Only Absolute Value of Percent Asset Return"

##           Estimate  Std. Error   t value     Pr(>|t|)
## mxreg1 0.009313404 0.003092492  3.011617 2.598598e-03
## omega  0.016445273 0.004810571  3.418570 6.295123e-04
## alpha1 0.476472112 0.079561893  5.988697 2.115282e-09
## beta1  0.477380270 0.076406866  6.247871 4.160843e-10
## skew   0.984683525 0.027958252 35.219781 0.000000e+00
## shape  4.351762271 0.533363984  8.159085 4.440892e-16

##
## Akaike       0.3640203
## Bayes        0.3802067
## Shibata      0.3640039
## Hannan-Quinn 0.3699499

## [1] "Only Backwardation included"

##            Estimate  Std. Error   t value     Pr(>|t|)
## mxreg1 -0.007821599 0.004054035 -1.929337 5.368905e-02
## omega   0.016893439 0.004903211  3.445383 5.702518e-04
## alpha1  0.481692904 0.080015348  6.020006 1.744102e-09
## beta1   0.471117163 0.076566161  6.153073 7.599590e-10
## skew    0.974691182 0.027530723 35.403762 0.000000e+00
## shape   4.347258519 0.531999664  8.171544 2.220446e-16

##
## Akaike       0.3655208
## Bayes        0.3817072
## Shibata      0.3655044
## Hannan-Quinn 0.3714504

## [1] "Backwardation and Absolute Value of Percent Asset Return"
```

```
##           Estimate  Std. Error   t value     Pr(>|t|)
## mxreg1 -0.01769686 0.004952639 -3.573218 3.526213e-04
## mxreg2  0.01316913 0.003576837  3.681779 2.316118e-04
## omega   0.01654377 0.004767887  3.469833 5.207816e-04
## alpha1  0.48249962 0.079568093  6.063984 1.327906e-09
## beta1   0.47278388 0.075548112  6.258050 3.898204e-10
## skew    0.98062393 0.028242241 34.721888 0.000000e+00
## shape   4.35454030 0.531823672  8.187940 2.220446e-16
```

```
##
## Akaike       0.3633018
## Bayes        0.3821859
## Shibata      0.3632795
## Hannan-Quinn 0.3702197
```

Before we go on to an ARMA process, lets revisit our asymmetric flavor of GARCH but this time including the two external variables in the Mean.

```
## [1] "Standard GARCH Robust Coefficients"
```

```
##            Estimate  Std. Error    t value     Pr(>|t|)
## mxreg1 -0.022730331 0.012438223 -1.8274581 6.763091e-02
## mxreg2  0.010554326 0.005586454  1.8892710 5.885553e-02
## omega   0.004531634 0.004713080  0.9615016 3.363000e-01
## alpha1  0.221951154 0.130043433  1.7067463 8.786921e-02
## beta1   0.774292443 0.135828107  5.7005318 1.194343e-08
```

```
##
## Akaike       0.5390674
## Bayes        0.5525561
## Shibata      0.5390560
## Hannan-Quinn 0.5440088
```

```
## [1] "Asymmetric Power ARCH (apARCH) Robust Coefficients"
```

```
##             Estimate   Std. Error       t value     Pr(>|t|)
## mu      1.209873e-15 0.0042495881  2.847035e-13 1.000000e+00
## omega   7.088178e-04 0.0001868279  3.793961e+00 1.482629e-04
## alpha1  1.065969e-01 0.0206308423  5.166869e+00 2.380484e-07
## beta1   7.939969e-01 0.1045524117  7.594247e+00 3.086420e-14
## gamma1 -2.603372e-04 0.0873878702 -2.979100e-03 9.976230e-01
## delta   3.150204e+00 0.7343162093  4.289983e+00 1.786870e-05
```

```
##
## Akaike       0.5431401
## Bayes        0.5593265
## Shibata      0.5431237
## Hannan-Quinn 0.5490697
```

```
## [1] "GJR ARCH Robust Coefficients"
```

```
##            Estimate  Std. Error      t value   Pr(>|t|)
## mu     1.209873e-15 0.010099780 1.197920e-13 1.00000000
## omega  3.405613e-04 0.000336028 1.013491e+00 0.31082575
## alpha1 6.989875e-02 0.030133374 2.319646e+00 0.02036006
## beta1  9.092623e-01 0.032746143 2.776701e+01 0.00000000
## gamma1 1.631497e-02 0.040121912 4.066349e-01 0.68427619
```

```
##
## Akaike       0.6184553
## Bayes        0.6319440
## Shibata      0.6184440
## Hannan-Quinn 0.6233967
```

```
## [1] "eARCH Robust Coefficients"
```

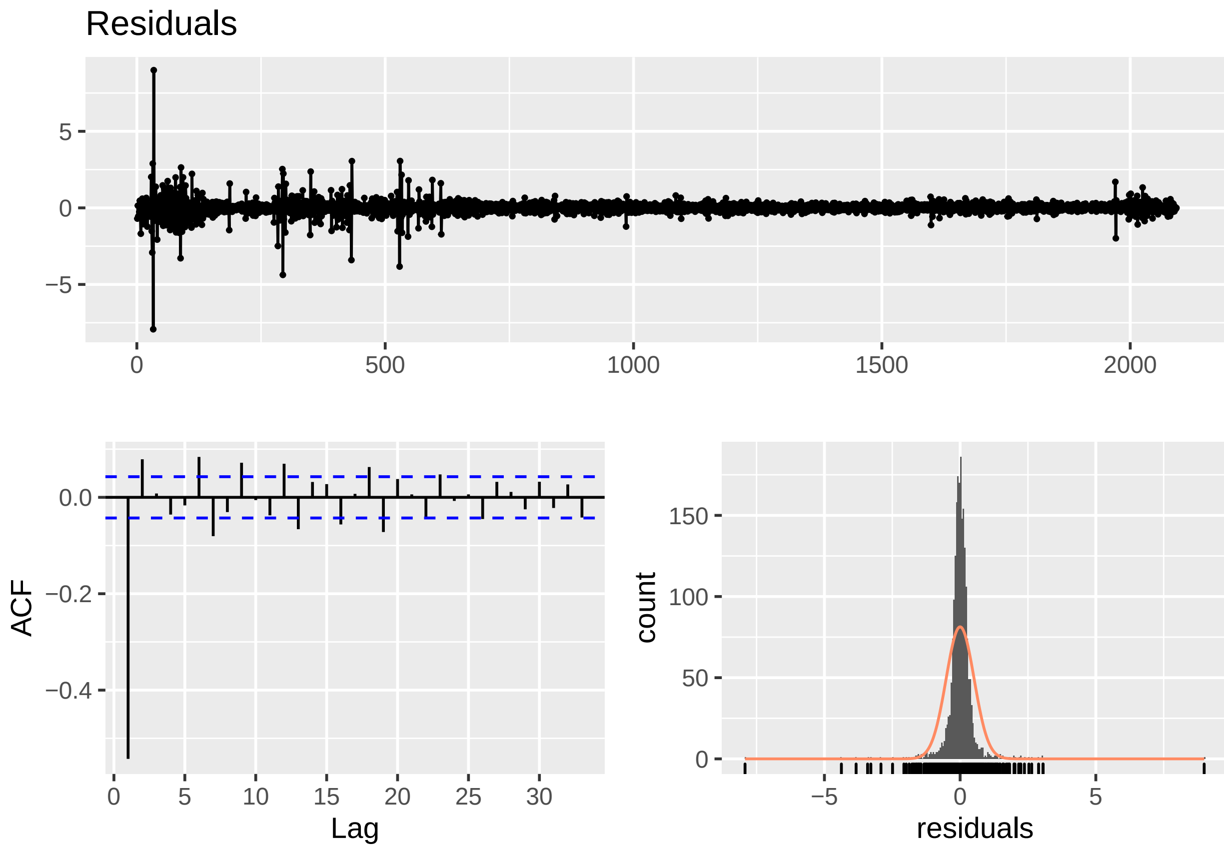
```
##             Estimate  Std. Error       t value     Pr(>|t|)
## mu      1.209871e-15  0.00130356  9.281285e-13 1.000000e+00
## omega  -6.420969e-02  0.06486623 -9.898786e-01 3.222335e-01
## alpha1 -1.970296e-02  0.02846576 -6.921637e-01 4.888345e-01
## beta1   9.553909e-01  0.03105808  3.076143e+01 0.000000e+00
## gamma1  4.521855e-01  0.10196939  4.434522e+00 9.227694e-06
```

```
##
## Akaike       0.5470358
## Bayes        0.5605244
## Shibata      0.5470244
## Hannan-Quinn 0.5519771
```

!!!! Now lets start over and fit an ARMA process to the residuals. I utilize the Box- Jenkins procedure to fit the model. A good specification would have no autocorrelation in the residuals, but autocorrelation in the squared residuals.

The autocorrelation of the residuals is significant, but dies off very slowly. With a max lag of 10, an ARMA(0,1) model has the characteristics we are searching for.

```
## Warning in modeldf.default(object): Could not find appropriate degrees of
## freedom for this model.
```




```
##
##  Box-Ljung test
##
## data:  a$residuals
## X-squared = 57.218, df = 10, p-value = 1.213e-08
```

```
##
##  Box-Ljung test
##
## data:  a$residuals^2
## X-squared = 196.65, df = 10, p-value < 2.2e-16
```

Now let's start over from the beginning with out base GARCH model and see if we should include the unconditional mean. Immediately huge improvements in model fit by including an MA process. The fit does not improve when including the unconditional mean, so I will leave it out.

```
## [1] "No Unconditional Mean Robust Coefficients"
```

```
##             Estimate   Std. Error    t value   Pr(>|t|)
## ma1    -0.730469527 0.0221367279 -32.998080 0.00000000
## omega   0.001318428 0.0008587402   1.535305 0.12470899
## alpha1  0.111956130 0.0446236436   2.508897 0.01211088
## beta1   0.887043868 0.0394194931  22.502671 0.00000000
```

```
##
## Akaike       0.2115807
## Bayes        0.2223716
## Shibata      0.2115734
## Hannan-Quinn 0.2155338
```

```
## [1] "Including Unconditional Mean Robust Coefficients"
```

```
##             Estimate   Std. Error       t value   Pr(>|t|)
## mu      1.209873e-15 0.0030550888  3.960189e-13 1.00000000
## ma1    -7.305587e-01 0.0396450136 -1.842751e+01 0.00000000
## omega   1.240377e-03 0.0008725079  1.421622e+00 0.15513593
## alpha1  1.147048e-01 0.0489745621  2.342129e+00 0.01917407
## beta1   8.872899e-01 0.0416258590  2.131583e+01 0.00000000
```

```
##
## Akaike       0.2124362
## Bayes        0.2259249
## Shibata      0.2124249
## Hannan-Quinn 0.2173776
```

As in the non ARMA specification, the archm coefficient is not statistically significant and its inclusion does not help model fit.

```
## [1] "No ARCH in Mean Robust Coefficients"
```

```
##            Estimate   Std. Error    t value   Pr(>|t|)
## ma1    -0.730469527 0.0221367279 -32.998080 0.00000000
## omega   0.001318428 0.0008587402   1.535305 0.12470899
## alpha1  0.111956130 0.0446236436   2.508897 0.01211088
## beta1   0.887043868 0.0394194931  22.502671 0.00000000
```

```
##
## Akaike       0.2115807
## Bayes        0.2223716
## Shibata      0.2115734
## Hannan-Quinn 0.2155338
```

```
## [1] "ARCH in Mean Robust Coefficients"
```

```
##            Estimate   Std. Error    t value    Pr(>|t|)
## ma1    -0.747242433 0.0254254094 -29.389593 0.000000000
## archm   0.018848675 0.0063448551   2.970702 0.002971198
## omega   0.001245088 0.0008534818   1.458833 0.144610964
## alpha1  0.107335379 0.0463050060   2.318008 0.020448880
## beta1   0.891664620 0.0411428143  21.672427 0.000000000
```

```
##
## Akaike       0.2070284
## Bayes        0.2205171
## Shibata      0.2070171
## Hannan-Quinn 0.2119698
```

Now lets compare normal distribution with student-t. Again we find student-t distribution to have a much better fit.

```
## [1] "Normal Distribution Robust Coefficients"

##            Estimate   Std. Error    t value   Pr(>|t|)
## ma1    -0.730469527 0.0221367279 -32.998080 0.00000000
## omega   0.001318428 0.0008587402   1.535305 0.12470899
## alpha1  0.111956130 0.0446236436   2.508897 0.01211088
## beta1   0.887043868 0.0394194931  22.502671 0.00000000

##
## Akaike       0.2115807
## Bayes        0.2223716
## Shibata      0.2115734
## Hannan-Quinn 0.2155338

## [1] "Student t Distribution Robust Coefficients"

##            Estimate  Std. Error    t value  Pr(>|t|)
## ma1    -0.747201414 0.020426620 -36.579787 0.0000000
## omega   0.002885552 0.002416543   1.194083 0.2324456
## alpha1  0.089204495 0.067536262   1.320839 0.1865552
## beta1   0.873626932 0.086708250  10.075476 0.0000000
## skew    0.894847824 0.033037893  27.085500 0.0000000
## shape   3.933650061 0.474092874   8.297214 0.0000000

##
## Akaike       -0.02801273
## Bayes        -0.01182634
## Shibata      -0.02802910
## Hannan-Quinn -0.02208308
```

As we saw with the GARCH process with an ARMA process, asymmetric model specifications do not improve model fit

```
## [1] "Standard GARCH Robust Coefficients"

##            Estimate  Std. Error    t value  Pr(>|t|)
## ma1    -0.747201414 0.020426620 -36.579787 0.0000000
## omega   0.002885552 0.002416543   1.194083 0.2324456
## alpha1  0.089204495 0.067536262   1.320839 0.1865552
## beta1   0.873626932 0.086708250  10.075476 0.0000000
## skew    0.894847824 0.033037893  27.085500 0.0000000
## shape   3.933650061 0.474092874   8.297214 0.0000000

##
## Akaike       -0.02801273
## Bayes        -0.01182634
## Shibata      -0.02802910
## Hannan-Quinn -0.02208308
```

```
## [1] "Asymmetric Power GARCH (apGARCH) Robust Coefficients"

##            Estimate  Std. Error    t value     Pr(>|t|)
## ma1    -0.752183694 0.021064180 -35.709138 0.000000e+00
## omega   0.008603123 0.004931569   1.744500 8.107191e-02
## alpha1  0.108923848 0.035750272   3.046798 2.312933e-03
## beta1   0.877672855 0.037741662  23.254749 0.000000e+00
## gamma1  0.306781490 0.146376100   2.095844 3.609602e-02
## delta   1.242560417 0.251297368   4.944582 7.630746e-07
## skew    0.890097802 0.030362351  29.315839 0.000000e+00
## shape   3.857348662 0.440908705   8.748633 0.000000e+00

##
## Akaike       -0.031386243
## Bayes        -0.009804385
## Shibata      -0.031415315
## Hannan-Quinn -0.023480050

## [1] "GJR GARCH Robust Coefficients"

##            Estimate  Std. Error     t value  Pr(>|t|)
## ma1    -0.747377458 0.020129743 -37.1280186 0.0000000
## omega   0.003356978 0.002594327   1.2939687 0.1956762
## alpha1  0.073683344 0.055687554   1.3231564 0.1857834
## beta1   0.864106653 0.087588922   9.8654788 0.0000000
## gamma1  0.037555758 0.038312956   0.9802365 0.3269694
## skew    0.893054662 0.032046460  27.8674983 0.0000000
## shape   3.916277078 0.471438642   8.3070770 0.0000000

##
## Akaike       -0.027901654
## Bayes        -0.009017528
## Shibata      -0.027923926
## Hannan-Quinn -0.020983735

## [1] "Exponential Power GARCH Robust Coefficients"

##           Estimate  Std. Error    t value     Pr(>|t|)
## ma1    -0.75422610 0.020508119 -36.776953 0.000000e+00
## omega  -0.07580163 0.026890358  -2.818915 4.818628e-03
## alpha1 -0.05539803 0.019952248  -2.776531 5.494240e-03
## beta1   0.97083973 0.009912061  97.945296 0.000000e+00
## gamma1  0.16540309 0.035949420   4.600995 4.204785e-06
## skew    0.89468625 0.031061903  28.803330 0.000000e+00
## shape   3.90248942 0.441531121   8.838538 0.000000e+00

##
## Akaike       -0.03243545
## Bayes        -0.01355132
## Shibata      -0.03245772
## Hannan-Quinn -0.02551753
```

I will not test three specifications of external variables: one with only Absolute Percent Asset Returns, one with only Backwardation, and one with both. We find exactly what we found for the non ARMA specification: asset return is not significant by itself, backwardation is, and together both are. The best model in terms of AIC includes both.

```
## [1] "Absolute Percentage Asset Returns Robust Coefficients"

##            Estimate  Std. Error    t value     Pr(>|t|)
## ma1    -0.765138043 0.022166796 -34.517304 0.000000e+00
## mxreg1  0.006803954 0.001318781   5.159276 2.479067e-07
## omega   0.002171906 0.001698009   1.279089 2.008656e-01
## alpha1  0.076385945 0.056845781   1.343740 1.790325e-01
## beta1   0.889821576 0.072873491  12.210497 0.000000e+00
## skew    0.949150667 0.034860130  27.227399 0.000000e+00
## shape   4.074141080 0.473330924   8.607384 0.000000e+00

##
## Akaike       -0.03815744
## Bayes        -0.01927331
## Shibata      -0.03817971
## Hannan-Quinn -0.03123952

## [1] "Backwardation Robust Coefficients"

##            Estimate  Std. Error     t value     Pr(>|t|)
## ma1    -0.747233500 0.020494459 -36.4602697 0.000000e+00
## mxreg1 -0.001155842 0.002708435  -0.4267563 6.695568e-01
## omega   0.002962982 0.002550729   1.1616216 2.453892e-01
## alpha1  0.090799877 0.069775722   1.3013105 1.931522e-01
## beta1   0.871856652 0.089119249   9.7830341 0.000000e+00
## skew    0.890787439 0.035230831  25.2843154 0.000000e+00
## shape   3.911557330 0.487617136   8.0217799 1.110223e-15

##
## Akaike       -0.027172812
## Bayes        -0.008288686
## Shibata      -0.027195084
## Hannan-Quinn -0.020254893

## [1] "Absolute Asset returns (mxreg1) and Backwardation (mxreg2) Robust Coefficients"

##            Estimate  Std. Error    t value     Pr(>|t|)
## ma1    -0.770090865 0.022493895 -34.235550 0.000000e+00
## mxreg1  0.008602850 0.001341524   6.412742 1.429254e-10
## mxreg2 -0.006709971 0.002627614  -2.553637 1.066044e-02
## omega   0.002304832 0.001815080   1.269824 2.041474e-01
## alpha1  0.080056376 0.058770723   1.362181 1.731407e-01
## beta1   0.884996906 0.075495756  11.722472 0.000000e+00
## skew    0.939623088 0.035497199  26.470344 0.000000e+00
## shape   4.023441255 0.464571237   8.660547 0.000000e+00
```

```
##
## Akaike       -0.04108565
## Bayes        -0.01950380
## Shibata      -0.04111473
## Hannan-Quinn -0.03317946
```

SOYB Summary

We find that include ARCH in mean or the unconditional mean does not improve model fit. The assumed distribution is important: the student t distribution is a much better fit than the normal. Regardless of distribution, asymmetric model specifications do not improve model fit. Fitting an ARMA process (in this case an MA1 process) greatly improves model fit. Regardless of ARMA process, the best fitting model includes both the absolute value of the asset return and a dummy variable for backwardation in the mean equation.