Maximizing Utility with Commodity Futures Diversification

The greater the risk aversion, the higher the utility.

Mark J.P. Anson

Commodity futures are becoming accepted as a legitimate asset class for portfolio diversification. While much of the research on commodity futures focuses on managed, leveraged futures vehicles such as commodity pools, more recent attention has begun to consider unleveraged futures investments benchmarked to a commodity futures index. An investment in a commodity futures index may offer the diversifying properties that investors seek to balance their exposure to stocks and bonds.[1]

The amount of portfolio assets allocated to commodity futures, however, will depend on the value of portfolio diversification to the investor. In other words, the utility of investing in commodity futures for diversification purposes will be different for different investors.

The purpose of this article is to determine the utility of investing in commodity futures benchmarked to a commodity futures index. We examine relative levels of risk aversion to determine how the addition of commodity futures to a diversified portfolio will affect an investor's utility. Despite the popular perception of commodity futures as a risky investment, we conclude that the utility of investing in commodity futures is higher, the more risk-averse the investor.

COMMODITY FUTURES INDEXES AS A SOURCE OF PORTFOLIO DIVERSIFICATION

Anson [1998b] summarizes the benefits of investing in a commodity futures index. Commodity

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futures indexes invest in only non-financial futures; are designed to be "long-only" investments; and are generally tied to the same long-run economic fundamentals as stocks and bonds. Commodity futures indexes have demonstrated significant positive correlation with the rate of inflation as well as changes in the rate of inflation. Consequently, commodity futures may be an excellent source of portfolio diversification.[2]

There is a variety of research on the diversification benefits of commodity futures. Ankrim and Hensel [1993], Lummer and Siegel [1993], and Kaplan and Lummer [1997] all conclude that an allocation to the Goldman Sachs Commodity Index (GSCI) provides a good diversifier for stocks and bonds in a mean-variance framework. Froot [1995] finds that the GSCI is an effective portfolio diversification tool both as an initial hedge and as a secondary hedge after other real assets have already been added to the investment portfolio.

Satyanarayan and Varangis [1996] find that an allocation to the GSCI in an international stock portfolio improves portfolio returns for a given level of risk. Similarly, Greer [1994] concludes that a fifty-fifty allocation to the Daiwa Physical Commodity Index and the S&P 500 results in a portfolio that outperforms the return to the S&P 500 with a lower standard deviation.[3] Finally, Anson [1998a] finds that an investment in non-financial futures contracts benchmarked to one of four unleveraged commodity futures indexes improves the Sharpe ratios for a diversified portfolio of domestic and foreign stocks and bonds.

DATA

To simplify the analysis, we maximize utility over four asset classes: large-capitalization stocks, small-capitalization stocks, long-term bonds, and commodity futures. For traditional asset classes, we use the S&P 500 and Nasdaq total return indexes for domestic large-cap and small-cap stock exposure, and Lehman Brothers Long-Term Government Bond index for domestic fixed-income exposure.[4]

Diversification with commodity futures may be accomplished by investing in an unleveraged portfolio of futures contracts benchmarked to a commodity futures index. For this study we choose four indexes: the Goldman Sachs Commodity Index (GSCI), the Chase Physical Commodity Index (CPCI), the Investable Commodity Index (ICI), and the J.P. Morgan Commodities Index (JPMCI). All these indexes offer unleveraged exposure to the commodities markets. None of them invests in financial or currency futures. The four indexes are described in Appendix A.[5]

Exhibit 1 presents the quarterly expected returns, standard deviations, and Sharpe ratios for the asset classes and commodity futures indexes over the time period 1974-1997.[6] As can be seen, the Sharpe ratios of the four commodity futures indexes are lower than those for stocks and, except in the case of the JPMCI, bonds. Consequently, commodity futures may not necessarily make a good stand-alone investment. Commodity futures should not be considered in isolation, however; their value is best achieved as part of a diversified portfolio.[7]

The diversification potential of commodity futures is demonstrated in Exhibit 2, which presents the quarterly correlations between the returns to the four commodity futures indexes and the returns to domestic stocks and bonds. All four commodity futures indexes demonstrate significant negative correlation with the returns to stocks and bonds over the full period of 1974-1997.[8] Therefore, commodity futures have the potential to reduce risk in a portfolio of stocks and bonds. This risk reduction potential should have utility for investors.

DESCRIBING UTILITY

Studies of commodity futures investing generally examine an investor's optimal portfolio policy within a mean-variance framework along an efficient investment frontier. I examine how an investor's investment policy changes when maximizing expected utility is the objective function.

When presented with various outcomes of portfolio return and volatility based on numerous asset allo-

EXHIBIT 1
QUARTERLY SHARPE RATIOS 1974-1997 (%)

Asset Class	Expected Return	Standard Deviation	Sharpe Ratio
LBLT	2.58	6.21	12.18
S&P 500	3.73	8.02	23.77
Nasdaq	4.16	10.85	21.47
GSCI	2.74	9.97	9.17
CPCI	2.49	8.45	7.82
ICI	2.20	6.31	5.86
JPMCI	3.89	10.56	19.55

EXHIBIT 2
QUARTERLY CORRELATIONS 1974–1997

	LBLT	S&P 500	Nasdaq	GSCI	CPCI	ICI	JPMCI	CPI
LBLT	1.000000							
S&P 500	0.397199	1.000000						
Nasdaq	0.353568	0.907692	1.000000					
GSCI	–0.173729	–0.316072	–0.289226	1.000000				
CPCI	–0.254889	–0.231901	–0.215407	0.900544	1.000000			
ICI	–0.264567	–0.202769	–0.147398	0.673162	0.783785	1.000000		
JPMCI	–0.254117	–0.190322	–0.223752	0.614631	0.678753	0.653455	1.000000	
CPI	–0.341878	–0.239569	–0.177841	0.151006	0.099886	0.236637	0.315426	1.000000

A correlation coefficient equal to 0.156 (0.22) in absolute value is significant at the 5% (1%) level.

cations, investors must choose the "best" portfolio for their own personal satisfaction. To make this choice, the expected utility of each portfolio can be calculated. The preferred portfolio is the one that provides the greatest expected utility. The question to be answered is whether commodity futures in a portfolio of stocks and bonds provide sufficient utility to add them to the preferred portfolio.

We define an investor's objective function in terms of quadratic utility, which is measured over the investor's end-of-period return. Quadratic utility functions have the advantage of being described in a linear format and can be defined in terms of mean-variance inputs. If it is assumed that one-period returns are normally distributed, so that only the first two moments of the distribution are necessary, then an investor's utility function may be described as:

$$E(U_i) = E(R_p) - \sigma^2(R_p)A_i$$

where

$E(U_i)$ is the expected utility for the i-th investor;
$E(R_p) = \Sigma_i E(R_i)$ is the expected return on the portfolio;
$\sigma^2(R_p) = \Sigma\Sigma x_i x_j \rho_{ij} \sigma_i \sigma_j$ is the variance of the portfolio returns;
A_i is a measure of relative risk aversion of the i-th investor; and
x_i and x_j are the portfolio weights of the i-th and j-th asset class.

The expected utility in the Equation may be viewed as the expected return on the portfolio minus a risk penalty. The risk penalty is equal to the risk of the portfolio multiplied by the investor's relative risk aversion. Consequently, the expected utility is a risk-adjusted expected rate of return for the portfolio, where the risk adjustment depends on the level of risk aversion. This form of utility function is consistent with the integrated asset allocation approach of Sharpe [1987], where the optimization depends on the expected return and standard deviation of return on current assets.

Whether we call the Equation the expected utility or the risk-adjusted return, solving this function requires quadratic programming. This is because solving for E(U) involves both squared terms (the individual asset variances) as well as multiplicative terms (the covariances of the various asset classes). The important point to realize is that quadratic solutions recognize that the risk of the portfolio depends on the interactions among the asset classes.[9]

There are two problems with determining the utility of commodity futures investing. First, utility functions are hard to define in terms of all of the inputs that affect investors' behavior. Second, even if a utility function can be specified for each investor, these functions would be as varied and as different as the investors they attempt to describe. Consequently, there is no single answer to the utility of commodity futures investing.

Instead of trying to determine the unique benefits of investing in commodity futures for every investor, we develop a simple scale to measure risk aversion. We set the value of A_i in the Equation to four different levels of relative risk aversion ranging from a risk-neutral investor to the most risk-averse investor. A risk-averse investor may be defined as one who prefers a higher level of return for a given level of risk, or a lower level of risk for a given level of return. A risk-neutral investor concentrates only on the expected return of the portfolio; the level of risk is unimportant.

To examine the utility of commodity futures diversification, we set A_i equal to 0 (risk-neutral), 0.33 (low risk aversion), 0.67 (moderate risk aversion), and 1.0 (high risk aversion) to define levels of relative risk aversion. At $A_i = 0$, the investor is neutral to risk, and expected return is all that matters (the risk adjustment term falls out of the Equation). Conversely, at $A_i = 1$, the full impact of the portfolio volatility is considered in the investor's utility function.

As the value of A_i increases from 0 to 1, and the associated level of risk aversion with it, we should see a larger investment in commodity futures. The greater the risk aversion of the investor, the greater should be the allocation to commodity futures to dampen the negative impact of portfolio volatility on expected utility.[10]

ANALYSIS

A constrained optimization program is run to solve the Equation for each commodity futures index at each level of risk aversion. The results for each commodity futures index over the time period 1974-1997 are presented in Exhibits 3-6.[11]

Exhibit 3 demonstrates that, given our description of utility, as an investor's risk aversion increases, so does the allocation to commodity futures. The most risk-averse investor ($A_i = 1$) allocates approximately 23.4% of the portfolio to commodity futures represented by the GSCI. Furthermore, as the level of commodity futures investment increases, so does the value of the Sharpe ratio for the investment portfolio.[12]

A risk-neutral investor who cares only about expected return makes no allocation to commodity futures. The reason is that the risk-neutral investor does not care whether commodity futures can reduce the risk of the portfolio; expected return is all that matters. This investor invests in the assets that yield the highest expected return: small-cap stocks. Higher absolute portfolio returns may be achieved by the risk-neutral investor, but the best risk-adjusted returns are achieved when commodity futures are added to the investment portfolio.

The results in Exhibits 3-6 indicate that the marginal utility of investing in commodity futures is highest for the most risk-averse investor. This conclusion may seem counterintuitive because commodity futures are usually perceived as a risky investment. Commodity futures should not be analyzed as a stand-alone investment; their true benefit is derived in a balanced portfolio that includes stocks and bonds.

This portfolio effect is demonstrated in Appendix B, where it is shown that the marginal utility of commodity futures investing is greater, the more risk-averse the investor.

The results for the CPCI, the ICI, and the JPMCI in Exhibits 4-6 are similar to those for the GSCI. The very risk-averse investor allocates 15.3% of the

EXHIBIT 3
MAXIMIZING UTILITY WITH THE GSCI 1974–1997

Risk-Neutral			
LBLT 0.00%	S&P 500 0.00%	Nasdaq 100.00%	GSCI 0.00%
Expected Utility 4.16%	Expected Return 4.16%	Standard Deviation 10.85%	Sharpe Ratio 21.51%
Low Risk Aversion			
LBLT 0.00%	S&P 500 0.00%	Nasdaq 100.00%	GSCI 0.00%
Expected Utility 3.77%	Expected Return 4.16%	Standard Deviation 10.85%	Sharpe Ratio 21.51%
Moderate Risk Aversion			
LBLT 0.00%	S&P 500 0.00%	Nasdaq 84.61%	GSCI 15.39%
Expected Utility 3.42%	Expected Return 3.94%	Standard Deviation 8.86%	Sharpe Ratio 23.87%
High Risk Aversion			
LBLT 0.00%	S&P 500 28.13%	Nasdaq 48.48%	GSCI 23.39%
Expected Utility 3.21%	Expected Return 3.71%	Standard Deviation 7.02%	Sharpe Ratio 26.77%

EXHIBIT 4
MAXIMIZING UTILITY WITH THE CPCI 1974–1997

Risk-Neutral			
LBLT	S&P 500	Nasdaq	CPCI
0.00%	0.00%	100.00%	0.00%
Expected Utility	Expected Return	Standard Deviation	Sharpe Ratio
4.16%	4.16%	10.85%	21.51%
Low Risk Aversion			
LBLT	S&P 500	Nasdaq	CPCI
0.00%	0.00%	100.00%	0.00%
Expected Utility	Expected Return	Standard Deviation	Sharpe Ratio
3.77%	4.16%	10.85%	21.51%
Moderate Risk Aversion			
LBLT	S&P 500	Nasdaq	CPCI
0.00%	26.42%	72.90%	0.68%
Expected Utility	Expected Return	Standard Deviation	Sharpe Ratio
3.38%	4.04%	9.86%	22.40%
High Risk Aversion			
LBLT	S&P 500	Nasdaq	CPCI
0.00%	44.43%	40.27%	15.29%
Expected Utility	Expected Return	Standard Deviation	Sharpe Ratio
3.14%	3.71%	7.56%	24.96%

EXHIBIT 5
MAXIMIZING UTILITY WITH THE ICI 1974–1997

Risk-Neutral			
LBLT	S&P 500	Nasdaq	ICI
0.00%	0.00%	100.00%	0.00%
Expected Utility	Expected Return	Standard Deviation	Sharpe Ratio
4.16%	4.16%	10.85%	21.51%
Low Risk Aversion			
LBLT	S&P 500	Nasdaq	ICI
0.00%	0.00%	100.00%	0.00%
Expected Utility	Expected Return	Standard Deviation	Sharpe Ratio
3.77%	4.16%	10.85%	21.51%
Moderate Risk Aversion			
LBLT	S&P 500	Nasdaq	ICI
0.00%	27.62%	72.38%	0.00%
Expected Utility	Expected Return	Standard Deviation	Sharpe Ratio
3.38%	4.04%	9.91%	22.35%
High Risk Aversion			
LBLT	S&P 500	Nasdaq	ICI
0.00%	68.56%	29.54%	1.90%
Expected Utility	Expected Return	Standard Deviation	Sharpe Ratio
3.11%	3.83%	8.49%	23.56%

portfolio to the CPCI, 1.9% to the ICI, and 46.3% to the JPMCI, while the risk-neutral investor commits all its assets to small-cap stocks. Also, the level of commodity futures investment as well as the portfolio Sharpe ratio increase as the investor's risk aversion increases.[13]

To provide a benchmark as to the incremental value of commodity futures in portfolio management, we also run a constrained utility function for an investor with low, moderate, and high risk aversion (A_i = 0.33, 0.67, and 1.0) but with the allocation to commodity futures constrained to equal zero (w_c = 0). The expected utilities and Sharpe ratios for the resulting portfolios are presented in Exhibit 7. These values are dominated by the values presented in Exhibits 3-6.

CONCLUSION

A common perception is that commodity futures may be too risky for the typical risk-averse investor. This research demonstrates that commodity futures, when considered in their proper portfolio context, are a valuable asset class for risk-averse investors. Because of their excellent diversification potential over long periods of time, commodity futures are found to have greater utility, the more risk-averse the investor.

The actual level of commodity futures investment for a risk-averse investor will depend on the investor's individual utility function, level of risk tolerance, and current portfolio composition.

EXHIBIT 6
MAXIMIZING UTILITY WITH THE JPMCI
1974–1997

Risk-Neutral

LBLT	S&P 500	Nasdaq	JPMCI
0.00%	0.00%	100.00%	0.00%
Expected Utility	Expected Return	Standard Deviation	Sharpe Ratio
4.16%	4.16%	10.85%	21.51%

Low Risk Aversion

LBLT	S&P 500	Nasdaq	JPMCI
0.00%	0.00%	63.48%	36.52%
Expected Utility	Expected Return	Standard Deviation	Sharpe Ratio
3.89%	4.06%	7.10%	31.47%

Moderate Risk Aversion

LBLT	S&P 500	Nasdaq	JPMCI
0.00%	0.00%	56.08%	43.92%
Expected Utility	Expected Return	Standard Deviation	Sharpe Ratio
3.73%	4.04%	6.78%	32.69%

High Risk Aversion

LBLT	S&P 500	Nasdaq	JPMCI
0.00%	0.00%	53.71%	46.29%
Expected Utility	Expected Return	Standard Deviation	Sharpe Ratio
3.58%	4.04%	6.72%	32.88%

EXHIBIT 7
MAXIMIZING UTILITY
WITHOUT COMMODITIES

Risk Aversion Level	A_j	Expected Utility	Sharpe Ratio
Low	0.33	3.77%	21.51%
Moderate	0.67	3.38%	22.35%
High	1.00	3.10%	23.45%

APPENDIX A
DESCRIPTION OF COMMODITY FUTURES INDEXES

Goldman Sachs Commodity Index (GSCI)

The GSCI is a world production-weighted index of twenty-six commodities in five basic categories: energy, agriculture, livestock, industrial metals, and precious metals. The weights of each commodity are determined each year according to its relative importance in the global production cycle. Not surprisingly, energy is the largest category, accounting for approximately 47% of the index, followed by agriculture (28%), livestock (12%), industrial metals (9%), and precious metals (4%). Once commodity weights are established at the outset of each year, the percentage allocation to each commodity within the index is allowed to vary depending on its current market price.

The index value is based on the nearby, actively traded futures contract for each commodity. Each month, the index must roll its positions in the expiring futures contracts to the next-nearby month. This results in the roll yield described above. Separately, a futures contract trades on the GSCI index on the Chicago Mercantile Exchange.

Chase Physical Commodity Index (CPCI)

Formerly the Daiwa Physical Commodity Index, the CPCI is based on the historical value (instead of quantity) of worldwide physical production. The CPCI uses an algorithm to dampen the very high weights and enhance the very low weights. This dampening/enhancing effect allows the index weights to come closer together while taking advantage of the diversification effect across different commodities due to less-than-perfect correlation.

Once the value-weighting of each commodity is established, it remains constant for the full year. In other words, commodities that have increased in value are sold to maintain their equivalent weights, while commodities that have declined in value are purchased. This systematic rebalancing allows the CPCI to produce additional returns by selling "rich" commodities and purchasing "cheap" commodities.

The CPCI comprises nineteen physical commodity components in five markets: energy (approximately 43%), livestock (20%), grains (17%), food/fiber (12%), and metals (8%). The CPCI does not publicly disclose which nearby futures contracts it uses to construct its index, to prevent a "trading footprint" to be discerned in the marketplace.

Investable Commodity Index (ICI)

The ICI includes five major commodity groups (energy, grains, metals, livestock, and food and fiber) and sixteen commodities. Equal weighting is given to each commodity to provide an unbiased measure of performance. Equal weighting is maintained so that successive price changes do not change a commodity's relative weight in the ICI. Only the most active and liquid U.S. exchange-traded commodity markets are included in the ICI. Therefore, the ICI represents an unbiased basket of commodity futures.

The investability objective of the ICI requires that only actively traded contract delivery months be used in its calculation. Therefore, non-cycle months are excluded as they are generally illiquid. Furthermore, prices for each of the ICI's constituents are represented by continuous average price series calculated from the compounded change in the average price of the three active commodity delivery month contracts. The use of three delivery month contracts seeks to avoid possible price distortions and potential volatility implicit in using only the nearest futures contract.

J.P. Morgan Commodity Index (JPMCI)

The JPMCI index comprises only three commodity sectors: energy, industrial metals, and precious metals. The index includes eleven commodity futures and is calculated on the basis of the two nearby futures contracts. Each component of the index is rebalanced monthly according to an internal scoring system that seeks to maximize the risk and return performance of the index; its usefulness as a hedge against unexpected inflation; its hedging ability against stock and bond investments; and its correlation with economic statistics.

The JPMCI is an optimized index (the other indexes are not optimized). The weightings assigned to its components were chosen after studying their correlation with stocks and bonds as well as their return potential. The JPMCI is designed to maximize the negative correlation with stocks and bonds while maximizing the return of the index.

APPENDIX B
THE MARGINAL UTILITY OF COMMODITY FUTURES DIVERSIFICATION

To demonstrate that the marginal utility of commodity futures investing is greater, the more risk-averse the investor, we continue to assume that an investor maximizes expected utility by investing the portfolio across the four asset classes discussed in the text.

Expanding the formula for expected utility we can express the Equation in the text as:

$$E(U) = \Sigma w_i E(R_i) - A_i \Sigma\Sigma w_i w_j \sigma_i \sigma_j \rho_{ij} \qquad \text{(B-1)}$$

where the terms are defined as before. To determine the marginal utility of commodity futures investing, we differentiate Equation (B-1) by the amount allocated to commodity futures, or w_c:

$$\partial E(U)/\partial w_c = E(R_c) - A_i \sigma_c \Sigma w_j \sigma_j \rho_{cj} \qquad \text{(B-2)}$$
for $0 < A_i \leq 1$ (risk-averse investor)

$$\partial E(U)/\partial w_c = E(R_c) \qquad \text{(B-3)}$$
for $A_i = 0$ (risk-neutral investor)

where

$E(R_c)$ is the expected return to commodity futures;
σ_c is the volatility of the returns to commodity futures; and
ρ_{cj} is the correlation coefficient of commodity futures returns with the returns to the j-th asset class.

The correlation coefficients presented in Exhibit 2 demonstrate that the returns to commodity futures are consistently negatively correlated with the returns to stocks and bonds. Therefore, the terms ρ_{cj} are negative, and the whole expression $A_i \sigma_c \Sigma w_j \sigma_j \rho_{cj}$ is negative. Since this negative amount is *subtracted* in Equation (B-2), it in fact *adds* value to the expected return to commodities. Furthermore, the value of Equation (B-2) increases as an investor's risk aversion, as measured by the value of A_i, increases. In other words, the diversification potential of commodity futures adds value to an investor's utility function, and this value increases the greater an investor's risk aversion, A_i.

This follows from the valuable diversification benefits offered by commodity futures. When added to a portfolio of stocks and bonds, commodity futures can reduce portfolio volatility because of their negative correlation with stock and bond returns. Therefore, the marginal utility of commodity futures increases as the risk aversion of the investor increases.

When we examine Equation (B-3), we see that the expression $A_i \sigma_c \Sigma w_j \sigma_j \rho_{cj}$ falls out of the marginal utility equation. This is because for a risk-neutral investor $A_i = 0$. This means that a risk-neutral investor is indifferent to risk — the investor does not derive any utility from either reducing risk or increasing risk; only expected return matters.

In summary, because the correlation coefficients, ρ_{cj}, are negative, the value of Equation (B-2) is greater, the higher the value of A_i. Furthermore, for any value of $A_i > 0$, the value of Equation (B-2) will always be greater than the value of Equation (B-3). This demonstrates that the marginal utility of commodity futures diversification is greater, the more risk-averse the investor.

ENDNOTES

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[1]For background, see Schneeweis, Spurgin, and Potter [1997], Edwards and Park [1996], and McCarthy, Schneeweis, and Spurgin [1996]. See also Anson [1998a, 1998b], Kaplan and Lummer [1997], Ankrim and Hensel [1993], and Lummer and Siegel [1993].

[2]See Anson [1998a], Halpern and Warsager [1998], Bodie [1983], and Greer [1978].

[3]The Daiwa Physical Commodity Index was purchased by Chase Manhattan Bank and renamed the Chase Physical Commodity Index in March 1998.

[4]We also maximized utility over a subset of data that

include foreign bonds and equities with domestic stocks and bonds over the period 1985–1997 (data on the Salomon Smith Barney World Government Bond Index begin in 1985), and found substantially the same results as presented for domestic stocks and bonds. In fact, neither foreign stocks (represented by Morgan Stanley's EAFE index) nor foreign bonds were selected in our utility maximization program.

[5]The GSCI was created in 1990 and measures from January 1970. The CPCI was created in 1993 and measures from January 1970. The ICI was created in 1991 and measures from December 31, 1971. The JPMCI was created in 1993 and measures from January 1974.

As Schneeweis and Spurgin [1997] point out, historical returns and correlations prior to the official launch date of an index are less reliable indicators of future performance than returns since the launch date. The reason is that the historical period of returns must inevitably overlap with the time period used to construct and, in some cases, optimize the index. Unfortunately, given the relatively short history of commodity futures indexes, it is necessary to include some of this overlap period to provide analysis of sufficient duration.

[6]Anson [1998a] observes that the significance of the negative correlation between commodity futures returns and the returns to stocks and bonds is greatest with quarterly returns compared to monthly or annual returns. This observation may be tied to the quarterly reporting of corporate results, and the impact that commodity prices have on the profit margins of major corporations.

[7]We also examine the Sharpe ratios of the four asset classes for three subperiods: 1974–1980, a period of high inflation; 1981–1986, a period of declining inflation rates; and 1987–1997, a period of moderate growth and low inflation. Over the period 1974–1980, commodity futures are the dominant performing asset class, while during periods of moderate inflation or disinflation, they underperform stocks and bonds.

[8]The negative correlation between the returns to commodity futures and the returns to stocks and bonds is consistent across the subperiods of high inflation, disinflation, and low inflation.

[9]Another important advantage of defining expected utility as a risk-adjusted return is that the absolute risk aversion of the investor decreases with expected return. That is, absolute risk aversion declines as an investor earns a higher return on the portfolio. Absolute risk aversion is measured as:

$$-U''(R)/U'(R)$$

If we define the variance of the portfolio as

$$\sigma^2(R_p) = E(R_p^2) - [E(R_p)]^2$$

then

$$U'(R) = 1 + 2A_iE(R_p) \text{ and } U''(R) = 2A_i$$

Therefore

$$-U''(R)/U'(R) = -2A_i/[1 + 2A_iE(R_p)]$$

For a typical risk-averse investor, A_i is positive as is $E(R_p)$. Consequently, the minus sign before the last equation makes the absolute aversion negative.

[10]We could also set $A_i < 0$ to represent a risk-seeking investor. In fact, we ran our optimization formula for each commodity futures index where $A_i = -1$. Not surprisingly, risk-seeking investors do not select commodity futures for their portfolios because of their risk-reducing properties.

[11]To solve the Equation in the text, we program an optimization as follows: Maximize $E(U) = E(R_p) - \sigma^2(R_p)A_i$, subject to constraints $\Sigma w_i = 1$, and $0 \leq w_i \leq 1$, where $A_i = 0$, 0.33, 0.67, and 1.0 for different levels of relative risk aversion.

The first constraint ensures that the portfolio is fully invested, and the second constraint ensures that the allocation to each asset class is positive and that no asset class is leveraged. In other words, investors cannot short an asset class and cannot leverage their returns through borrowing. These two conditions in fact result in nine constraints that must be programmed into the optimization.

[12]We also examine three subperiods of high inflation, low inflation, and disinflation. During the subperiod of high inflation, the allocation to commodity futures is increased to reflect their role as the dominant asset class during periods of rising prices. In the disinflationary period, we find commodities to be a poor choice for utility-maximizing investors, even though the correlations of commodity futures returns to the returns of stocks and bonds remain negative. Over the period of moderate inflation, we find results consistent with those in the text.

[13]Note that bonds are not selected in any of the optimized portfolios, although fixed-income investments are often held as a portfolio diversifier to balance stock market exposure. There are three possible reasons for this result. First, the simple behavioral model proposed may not capture all the possible inputs to an investor's utility function such as the stability of fixed-income cash flows. Second, prior access to the commodity markets may have been limited through investment vehicles such as commodity pools, which typically have high net worth requirements. Finally, because of the perception of commodity futures as a risky investment class, investors may have selected fixed-income products in lieu of commodity futures as a diversifying asset class.

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