Base GARCH Model Comparision

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Introduction

The goal of this document is to describe the thought process behind the base GARCH model specification. Information about the data and the calculation of returns can be found in the data section of the draft. I will not review that, instead beginning with the OLS results. Just to remind of the ols specification:

$$R_t^e = \alpha + \beta R_t^a + \epsilon_t$$

where R^e is the return of the etf and R^a is the return of the asset. We define *tracking error* thus as ϵ_t.

For each ETF I employ the following methology for fitting a base mode. I will go one-by-one through each ETF investigating the tracking error (ϵ_t).

1. Test for ARCH effects by analysing the autocorrelation of the squared tracking error
2. Test for inclusion of unconditional mean in GARCH specification
3. Test for inclusion of ARCH in mean in GARCH specification
4. Consider asymmetric specifications
5. Consider student-t distribution rather than normal
6. Reconsider asymmetric specification with student-t distribution
7. Fit ARMA process using Box-Jenkins Procedure
8. Repeat steps 2:6 including an ARMA process in the model specification
9. Summarize my findings.

Below are the summaries for each ETF. What follows after is the full process for each ETF.

CORN Summary

We find that include ARCH in mean or the unconditional mean does not improve model fit. The assumed distribution is important: the student t distribution is a much better fit than the normal. Regardless of distribution, asymmetric model specifications do not improve model fit. Fitting an ARMA process (in this case an MA1 process) greatly improves model fit.

SOYB Summary

As with CORN, it is likely best to not include ARCH in mean or the unconditional mean in this model specification. Model fit was greatly improved by fitting an ARMA(2,3) process and further greatly improved by using a student-t distribution. Unlike CORN, some of the Asymmetric specifications did have a better fit.

WEAT Summary

As with CORN and SOYB, it is likely best to not include ARCH in mean or the unconditional mean in this model specification as it does not improve model fit or produce significant coefficients. Fitting an ARMA (1,5) process also improved fit significantly. Changing from a normal distribution to a student t distribution *slightly* improved model fit. Asymmetric models did not have better fit.

USO Summary

Again I determine to not include ARCH in mean or the unconditional mean in this model specification as it does not improve model fit or produce significant coefficients. A Student-t distribution fits better than a normal distribution. Asymmetric models are not favorable for normal distributions but *are* preferred under a student-t distributions. Fitting an (3,1) ARMA process improves model fit significant. The best fitting model is an Asymmetric Power ARCH with a student t distribution and an ARCH in mean.

UGA Summary

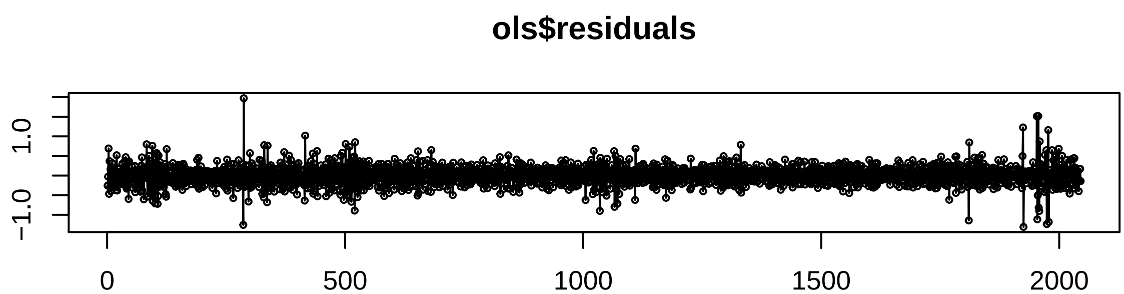
UGA results are very similiar to USO both with and without an ARMA process. Asymmetric models performed better, student t distributions have a better fit, and even ARCH-M produces significant coefficients *but* does not produce a better fit as in USO. The best fit is has a (2,1) ARMA process, is a student t distribution, and is an Exponential Power GARCH.

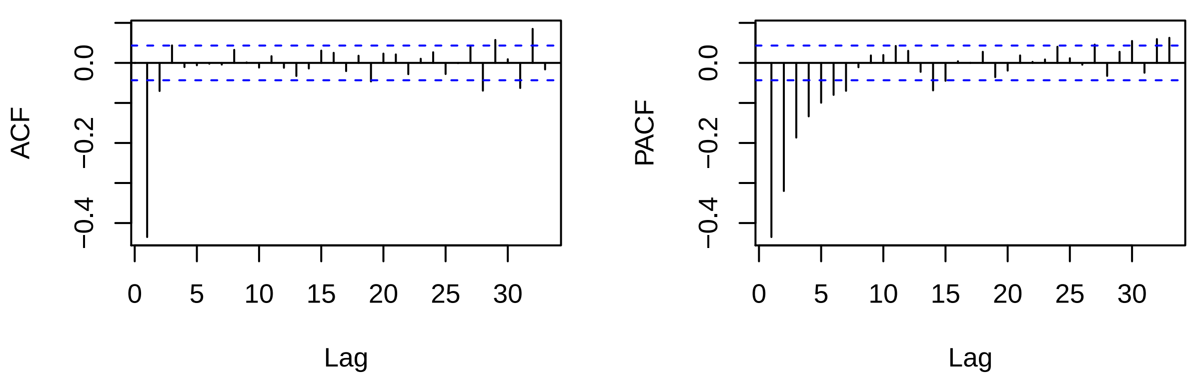
Corn

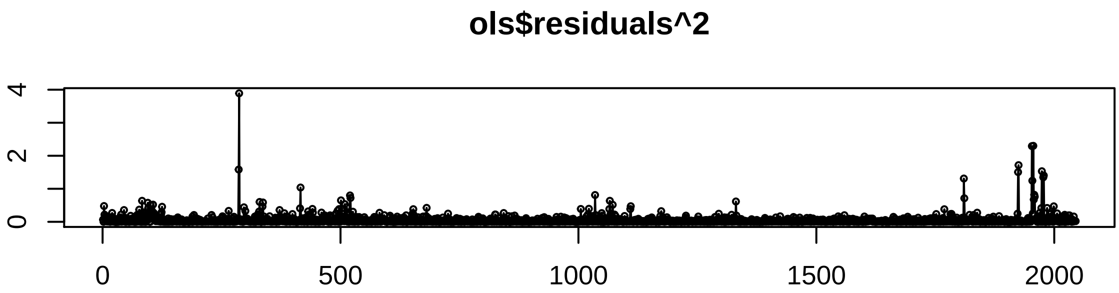
Lets begin by looking at the OLS results for CORN. We would reject both of our null hypothesis ($\alpha = 0$ and $\beta = 1$)

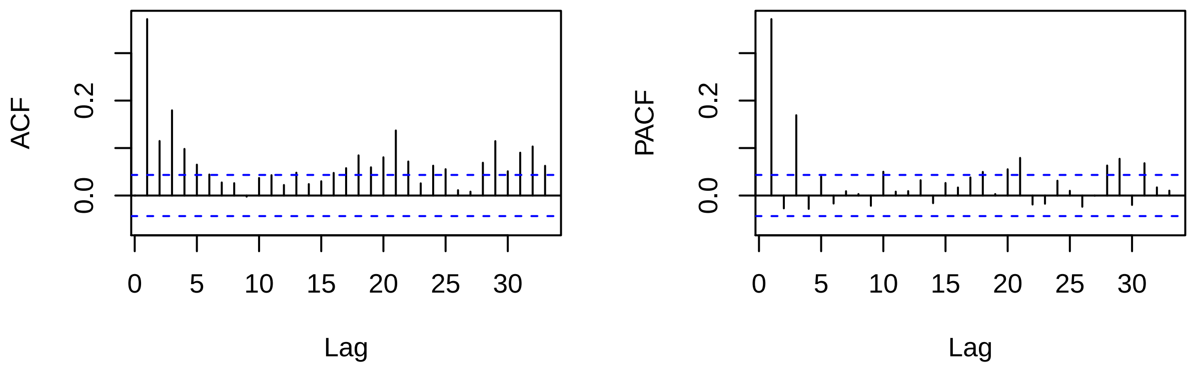
```
##
## Call:
## lm(formula = x$per_ETF_return ~ x$per_asset_return)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.30993 -0.13989  0.00141  0.13233  1.97264
##
## Coefficients:
##                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)        -0.015155   0.005452   -2.78  0.00549 **
## x$per_asset_return  0.981161   0.004487  218.67  < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.2464 on 2043 degrees of freedom
## Multiple R-squared:  0.959,  Adjusted R-squared:  0.959
## F-statistic: 4.782e+04 on 1 and 2043 DF,  p-value: < 2.2e-16
```

Now we investigate the residuals. There is clearly evidence of ARCH effects in the residuals based off the Ljung-Box Test.










```
##
##  Box-Ljung test
##
## data:  ols$residuals^2
## X-squared = 283, df = 1, p-value < 2.2e-16
```

Now a base model without an ARMA process. The simplist GARCH(1,1) model where we do not include the unconditional mean would be produce these robust coefficients

```
## [1] "No unconditional mean Robust Coefficients"
```

```
##         Estimate  Std. Error  t value   Pr(>|t|)
## omega  0.0112759 0.009721723 1.159866 0.24610341
## alpha1 0.2656483 0.108253351 2.453950 0.01412968
## beta1  0.5511842 0.261508184 2.107713 0.03505582
```

```
##
## Akaike       -0.1425799
## Bayes        -0.1343308
## Shibata      -0.1425842
## Hannan-Quinn -0.1395545
```

Including the unconditional mean does not improve model fit.

```
## [1] "Including Unconditional Mean Robust Coefficients"
```

```
##            Estimate   Std. Error      t value     Pr(>|t|)
## mu     4.481055e-16 0.0023576475 1.900647e-13 1.0000000000
## omega  1.028825e-04 0.0002329568 4.416377e-01 0.6587513883
## alpha1 6.896975e-02 0.0196111085 3.516872e+00 0.0004366649
```

```
## beta1  9.270446e-01 0.0240057942 3.861754e+01 0.0000000000
```

```
##
## Akaike       -0.09627133
## Bayes        -0.08527250
## Shibata      -0.09627896
## Hannan-Quinn -0.09223733
```

We can now investigate an ARCH in means. The coefficient estimate for archm is statistically insignificant and inclusion does not improve model fit.

```
## [1] "Robust Coefficients"
```

```
##            Estimate  Std. Error      t value   Pr(>|t|)
## mu     4.481055e-16  0.01533311 2.922469e-14 1.00000000
## archm  1.510646e-02  0.07513125 2.010676e-01 0.84064574
## omega  1.128855e-02  0.00988027 1.142535e+00 0.25323177
## alpha1 2.667130e-01  0.10985332 2.427901e+00 0.01518647
## beta1  5.500118e-01  0.26595629 2.068053e+00 0.03863500
```

```
##
## Akaike       -0.1409083
## Bayes        -0.1271598
## Shibata      -0.1409202
## Hannan-Quinn -0.1358658
```

Currently the best model does not include the unconditional mean or arch in mean. While we have this very basic models, lets try some other flavors or GARCH to see if there are any large difference.

```
## [1] "Asymmetric Power ARCH (apARCH) Robust Coefficients"
```

```
##             Estimate   Std. Error       t value     Pr(>|t|)
## mu     -4.481055e-16 2.585829e-03 -1.732927e-13 1.000000e+00
## omega   1.838797e-04 6.870598e-05  2.676327e+00 7.443389e-03
## alpha1  5.052279e-02 1.978077e-02  2.554137e+00 1.064513e-02
## beta1   8.825704e-01 6.248321e-02  1.412492e+01 0.000000e+00
## gamma1  1.130335e-01 1.090089e-01  1.036920e+00 2.997733e-01
## delta   3.278072e+00 4.735391e-01  6.922496e+00 4.437561e-12
```

```
##
## Akaike       -0.1236779
## Bayes        -0.1071796
## Shibata      -0.1236950
## Hannan-Quinn -0.1176269
```

```
## [1] "GJR ARCH Robust Coefficients"
```

```
##            Estimate   Std. Error      t value  Pr(>|t|)
## mu     4.481055e-16 0.0508884682 8.805640e-15 1.0000000
## omega  6.197297e-05 0.0005366422 1.154828e-01 0.9080624
## alpha1 5.136546e-02 0.0648116364 7.925345e-01 0.4280491
## beta1  8.934909e-01 0.0956493772 9.341314e+00 0.0000000
## gamma1 6.983232e-02 0.1865728519 3.742898e-01 0.7081887
```

```
##
## Akaike       -0.03791434
## Bayes        -0.02416580
## Shibata      -0.03792626
## Hannan-Quinn -0.03287185
```

```
## [1] "eARCH Robust Coefficients"
```

```
##             Estimate  Std. Error       t value     Pr(>|t|)
## mu     -4.481055e-16 0.003834635 -1.168574e-13 1.000000e+00
## omega  -3.169338e-01 0.112811667 -2.809406e+00 4.963296e-03
## alpha1 -5.815674e-02 0.038880754 -1.495772e+00 1.347131e-01
## beta1   8.901923e-01 0.039193793  2.271258e+01 0.000000e+00
## gamma1  4.076816e-01 0.078932002  5.164973e+00 2.404741e-07
```

```
##
## Akaike       -0.1386562
## Bayes        -0.1249076
## Shibata      -0.1386681
## Hannan-Quinn -0.1336137
```

No asymmetric model we have tested has a better fit than the base. We may also try some different distributions. Ramos 2015 used a student-t distribution which introduces a new coefficient *skew*

```
## [1] "Base Robust Coefficients"
```

```
##          Estimate  Std. Error  t value     Pr(>|t|)
## omega  0.01490287  0.00741121 2.010855 4.434082e-02
## alpha1 0.29444887  0.07520812 3.915121 9.035902e-05
## beta1  0.45357217  0.19049446 2.381025 1.726452e-02
## shape  8.36135613  1.66684333 5.016282 5.268100e-07
```

```
##
## Akaike       -0.1802274
## Bayes        -0.1692286
## Shibata      -0.1802350
## Hannan-Quinn -0.1761934
```

```
## [1] "Asymmetric Power ARCH (apARCH) Robust Coefficients"
```

```
##            Estimate   Std. Error      t value     Pr(>|t|)
## mu     4.481055e-16 2.333345e-03 1.920443e-13 1.000000e+00
## omega  7.258694e-05 1.609875e-05 4.508857e+00 6.517791e-06
## alpha1 5.745777e-02 8.353829e-03 6.878015e+00 6.069145e-12
## beta1  8.690594e-01 1.256597e-02 6.915975e+01 0.000000e+00
## gamma1 5.697040e-02 4.538394e-02 1.255299e+00 2.093704e-01
## delta  3.049390e+00 7.139270e-02 4.271292e+01 0.000000e+00
## shape  5.898446e+00 2.321809e-01 2.540452e+01 0.000000e+00
```

```
##
## Akaike       -0.10633334
## Bayes        -0.08708539
## Shibata      -0.10635667
## Hannan-Quinn -0.09927385
```

```
## [1] "GJR ARCH Robust Coefficients"
```

```
##            Estimate   Std. Error      t value    Pr(>|t|)
## mu     4.481055e-16 0.0076684184 5.843520e-14 1.000000000
## omega  6.185732e-05 0.0001771392 3.492017e-01 0.726937883
## alpha1 5.141437e-02 0.0195077253 2.635590e+00 0.008399113
## beta1  8.953275e-01 0.0151002413 5.929227e+01 0.000000000
## gamma1 6.530922e-02 0.0357570349 1.826472e+00 0.067779204
## shape  4.444378e+00 0.1332086437 3.336403e+01 0.000000000
```

```
##
```

```
## Akaike       -0.05551815
## Bayes        -0.03901990
## Shibata      -0.05553530
## Hannan-Quinn -0.04946716
```

```
## [1] "eARCH Robust Coefficients"
```

```
##             Estimate  Std. Error       t value    Pr(>|t|)
## mu     -4.481055e-16 0.003073116 -1.458147e-13 1.000000000
## omega  -3.124815e-01 0.202971162 -1.539537e+00 0.123673352
## alpha1 -3.308027e-02 0.039983194 -8.273544e-01 0.408036194
## beta1   8.908902e-01 0.070971302  1.255282e+01 0.000000000
## gamma1  3.937536e-01 0.152205911  2.586980e+00 0.009682135
## shape   5.036441e+00 0.298372126  1.687973e+01 0.000000000
```

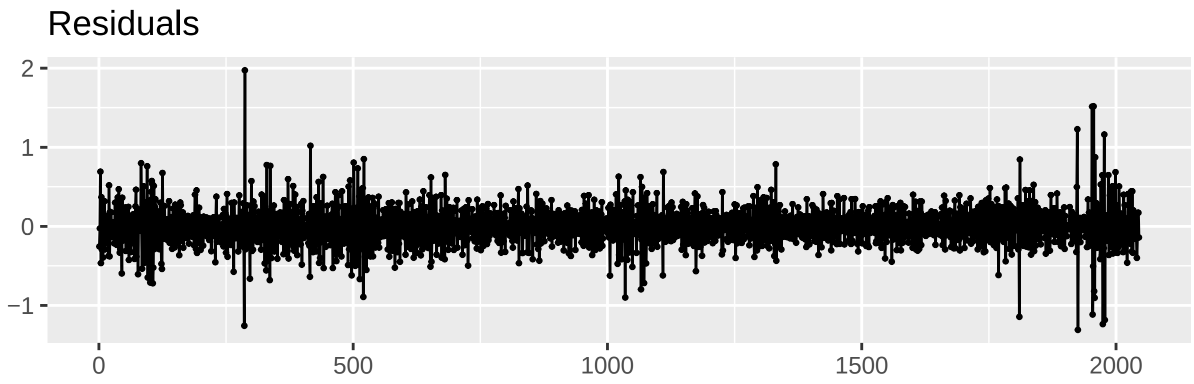
```
##
## Akaike       -0.1648695
## Bayes        -0.1483712
## Shibata      -0.1648866
## Hannan-Quinn -0.1588185
```

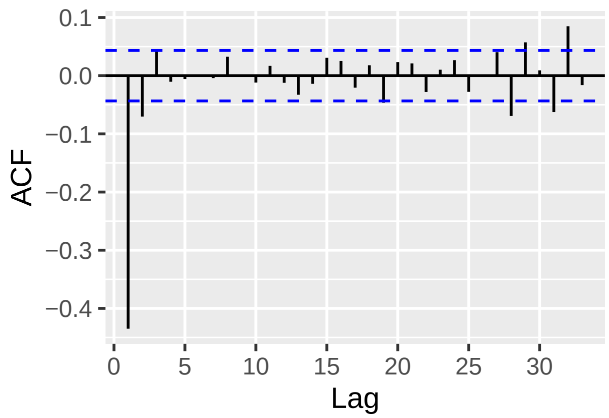
Using a student-t distribution significantly improves model fit. Within the models we test, the base, non asymmetric model still has the best fit.

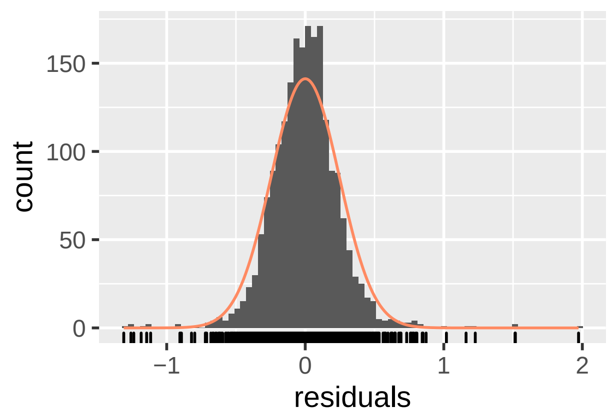
Now lets start over and fit an ARMA process to the residuals. I utilize the Box- Jenkins procedure to fit the model. A good specification would have no autocorrelation in the residuals, but autocorrelation in the squared residuals.

The autocorrelation of the residuals is significant, but dies off very slowly. With a max lag of 10, an ARMA(0,1) model has the characteristics we are searching for.

```
## Warning in modeldf.default(object): Could not find appropriate degrees of
## freedom for this model.
```








```
##
##  Box-Ljung test
##
## data:  a$residuals
## X-squared = 17.28, df = 10, p-value = 0.06839
```

```
##
##  Box-Ljung test
##
## data:  a$residuals^2
## X-squared = 214.72, df = 10, p-value < 2.2e-16
```

Now let's start over from the beginning with out base GARCH model and see if we should include the unconditional mean. Immediately huge improvements in model fit by including an MA process. The fit does not improve when including the unconditional mean, so I will leave it out.

```
## [1] "No Unconditional Mean Robust Coefficients"
```

```
##             Estimate   Std. Error    t value   Pr(>|t|)
## ma1    -0.689967676 0.0180437409 -38.238616 0.00000000
## omega   0.001071474 0.0009854028   1.087346 0.27688391
## alpha1  0.059374009 0.0288876166   2.055345 0.03984572
## beta1   0.915258651 0.0506714579  18.062607 0.00000000
```

```
##
## Akaike       -0.4576778
## Bayes        -0.4466790
## Shibata      -0.4576855
## Hannan-Quinn -0.4536439
```

```
## [1] "Including Unconditional Mean Robust Coefficients"
```

```
##              Estimate   Std. Error       t value   Pr(>|t|)
## mu      4.481055e-16 0.0014181271  3.159840e-13 1.00000000
## ma1    -6.899668e-01 0.0181332874 -3.804974e+01 0.00000000
## omega   1.071499e-03 0.0009923734  1.079734e+00 0.28026087
## alpha1  5.937431e-02 0.0289025981  2.054290e+00 0.03994764
## beta1   9.152580e-01 0.0508357268  1.800423e+01 0.00000000
```

```
##
## Akaike       -0.4566999
## Bayes        -0.4429513
## Shibata      -0.4567118
## Hannan-Quinn -0.4516574
```

As in the non ARMA specification, the archm coefficient is not statistically significant and its inclusion does not help model fit.

```
## [1] "No ARCH in Mean Robust Coefficients"
```

```
##             Estimate   Std. Error    t value   Pr(>|t|)
## ma1    -0.689967676 0.0180437409 -38.238616 0.00000000
## omega   0.001071474 0.0009854028   1.087346 0.27688391
## alpha1  0.059374009 0.0288876166   2.055345 0.03984572
## beta1   0.915258651 0.0506714579  18.062607 0.00000000
```

```
##
## Akaike       -0.4576778
## Bayes        -0.4466790
## Shibata      -0.4576855
## Hannan-Quinn -0.4536439
```

```
## [1] "ARCH in Mean Robust Coefficients"
```

```
##             Estimate   Std. Error     t value   Pr(>|t|)
## ma1    -0.690199601 0.0180740772 -38.1872663 0.00000000
## archm   0.003755637 0.0073522036   0.5108178 0.60947863
## omega   0.001060476 0.0009726352   1.0903120 0.27557573
## alpha1  0.059304715 0.0286944805   2.0667638 0.03875641
## beta1   0.915615298 0.0501233244  18.2672500 0.00000000
```

```
##
## Akaike       -0.4568514
## Bayes        -0.4431028
## Shibata      -0.4568633
## Hannan-Quinn -0.4518089
```

Now lets compare normal distribution with student-t. Again we find student-t distribution to have a much better fit.

```
## [1] "Normal Distribution Robust Coefficients"
```

```
##             Estimate   Std. Error    t value   Pr(>|t|)
## ma1    -0.689967676 0.0180437409 -38.238616 0.00000000
## omega   0.001071474 0.0009854028   1.087346 0.27688391
## alpha1  0.059374009 0.0288876166   2.055345 0.03984572
## beta1   0.915258651 0.0506714579  18.062607 0.00000000
```

```
##
## Akaike       -0.4576778
```

```
## Bayes        -0.4466790
## Shibata      -0.4576855
## Hannan-Quinn -0.4536439
```

```
## [1] "Student t Distribution Robust Coefficients"
```

```
##            Estimate   Std. Error    t value     Pr(>|t|)
## ma1    -0.683252573 0.0169291540 -40.359523 0.000000e+00
## omega   0.001163549 0.0005698446   2.041871 4.116430e-02
## alpha1  0.056790275 0.0187134028   3.034738 2.407447e-03
## beta1   0.912151375 0.0308121555  29.603621 0.000000e+00
## shape   7.103695359 1.2737859446   5.576836 2.449325e-08
```

```
##
## Akaike       -0.5287184
## Bayes        -0.5149699
## Shibata      -0.5287304
## Hannan-Quinn -0.5236760
```

As we saw with the GARCH process with an ARMA process, asymmetric model specifications do not improve model fit

```
## [1] "Standard GARCH Robust Coefficients"
```

```
##            Estimate   Std. Error    t value     Pr(>|t|)
## ma1    -0.683252573 0.0169291540 -40.359523 0.000000e+00
## omega   0.001163549 0.0005698446   2.041871 4.116430e-02
## alpha1  0.056790275 0.0187134028   3.034738 2.407447e-03
## beta1   0.912151375 0.0308121555  29.603621 0.000000e+00
## shape   7.103695359 1.2737859446   5.576836 2.449325e-08
```

```
##
## Akaike       -0.5287184
## Bayes        -0.5149699
## Shibata      -0.5287304
## Hannan-Quinn -0.5236760
```

```
## [1] "Asymmetric Power GARCH (apGARCH) Robust Coefficients"
```

```
##            Estimate  Std. Error     t value     Pr(>|t|)
## ma1    -0.683387193 0.016926248 -40.3744052 0.000000e+00
## omega   0.001879783 0.001720588   1.0925235 2.746031e-01
## alpha1  0.065616071 0.023357684   2.8091857 4.966698e-03
## beta1   0.910388810 0.029079172  31.3072463 0.000000e+00
## gamma1  0.045941000 0.093163167   0.4931241 6.219249e-01
## delta   1.710344561 0.429040575   3.9864401 6.707205e-05
## shape   7.100749767 1.272835354   5.5786868 2.423412e-08
```

```
##
## Akaike       -0.5271518
## Bayes        -0.5079039
## Shibata      -0.5271751
## Hannan-Quinn -0.5200923
```

```
## [1] "GJR GARCH Robust Coefficients"
```

```
##            Estimate   Std. Error     t value     Pr(>|t|)
## ma1    -0.683076046 0.0169762441 -40.2371715 0.000000e+00
## omega   0.001168287 0.0005652598   2.0668136 3.875172e-02
## alpha1  0.053416948 0.0194484770   2.7465877 6.021878e-03
```

```
## beta1   0.911806709 0.0308277861  29.5774307 0.000000e+00
## gamma1  0.007187635 0.0202620128   0.3547345 7.227885e-01
## shape   7.119697365 1.2801230617   5.5617289 2.671151e-08
```

```
##
## Akaike       -0.5278135
## Bayes        -0.5113152
## Shibata      -0.5278306
## Hannan-Quinn -0.5217625
```

```
## [1] "Exponential Power GARCH Robust Coefficients"
```

```
##            Estimate  Std. Error     t value     Pr(>|t|)
## ma1    -0.683661437  0.01636483 -41.7762516 0.000000e+00
## omega  -0.100304501  0.04553832  -2.2026395 2.762016e-02
## alpha1 -0.009002315  0.01347057  -0.6682949 5.039454e-01
## beta1   0.970134770  0.01349122  71.9085809 0.000000e+00
## gamma1  0.148486906  0.03086574   4.8107352 1.503761e-06
## shape   7.022066263  1.25068271   5.6145865 1.970328e-08
```

```
##
## Akaike       -0.5273388
## Bayes        -0.5108406
## Shibata      -0.5273560
## Hannan-Quinn -0.5212878
```

```
##               models        CORN
## 1             Simple -0.14257995
## 2  Unconditional Mean -0.09627133
## 3             ARCH-M -0.14090829
## 4             ApARCH -0.12367789
## 5           GJR ARCH -0.03791434
## 6              eARCH -0.13865616
## 7       t-dist Simple -0.18022740
## 8       t-dist apARCH -0.10633334
## 9     t-dist GJR ARCH -0.05551815
## 10       t-dist eARCH -0.16486947
```

```
##               models  CORN(0,1)
## 1             Simple -0.4576778
## 2  Unconditional Mean -0.4566999
## 3             ARCH-M -0.4568514
## 4             ApARCH -0.4572424
## 5           GJR ARCH -0.4578476
## 6              eARCH -0.4581073
## 7       t-dist Simple -0.5287184
## 8       t-dist apARCH -0.5271518
## 9     t-dist GJR ARCH -0.5278135
## 10       t-dist eARCH -0.5273388
```

CORN Summary

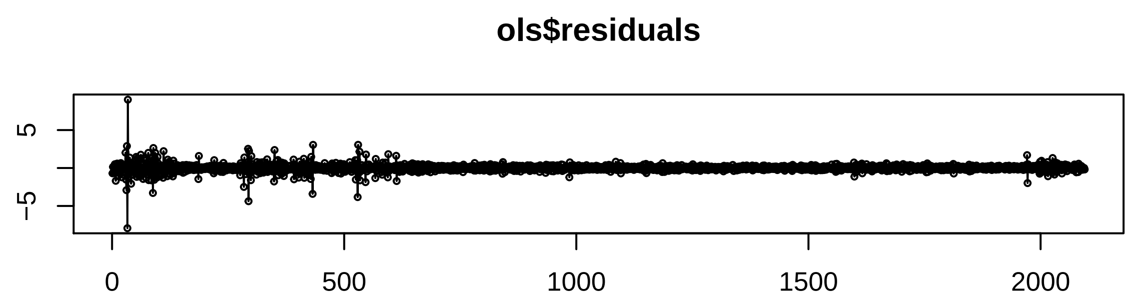
We find that include ARCH in mean or the unconditional mean does not improve model fit. The assumed distribution is important: the student t distribution is a much better fit than the normal. Regardless of distribution, asymmetric model specifications do not improve model fit. Fitting an ARMA process (in this case an MA1 process) greatly improves model fit.

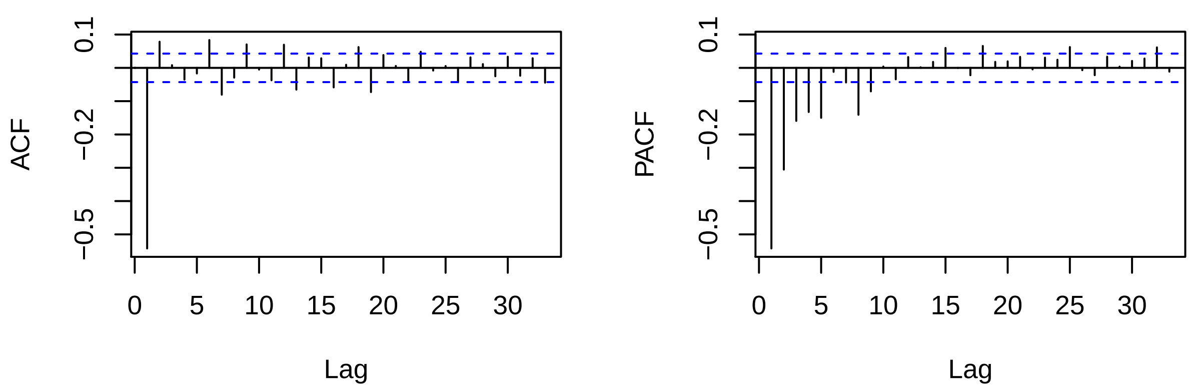
Soybeans

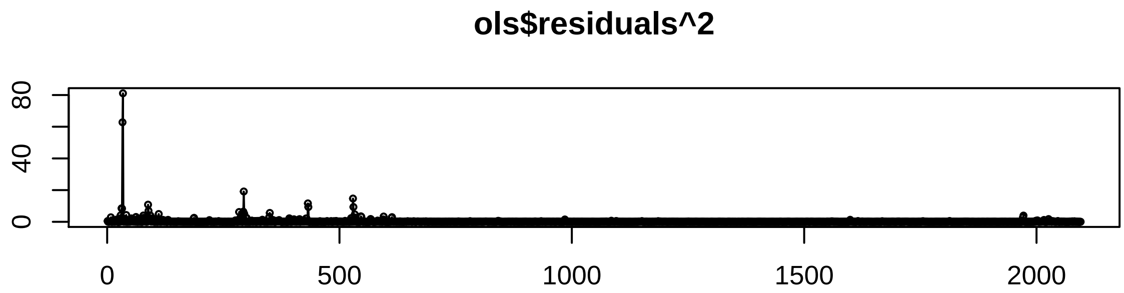
Lets begin by looking at the OLS results for soybeans. Unlike CORN, we fail to reject both of our null hypothesis ($\alpha = 0$ and $\beta = 1$).

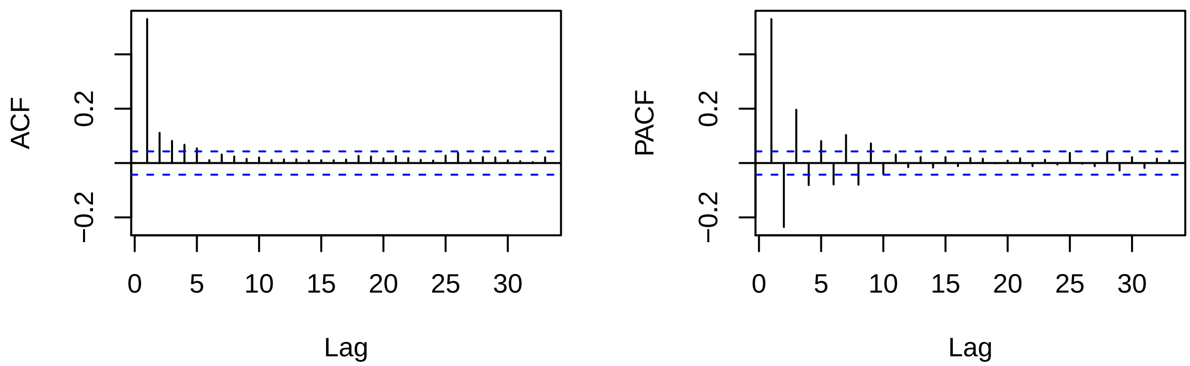
```
##
## Call:
## lm(formula = x$per_ETF_return ~ x$per_asset_return)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -7.9247 -0.1540  0.0003  0.1691  9.0048
##
## Coefficients:
##                    Estimate Std. Error t value Pr(>|t|)
## (Intercept)        -0.01741    0.01126  -1.546    0.122
## x$per_asset_return  0.99141    0.01095  90.552   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5156 on 2093 degrees of freedom
## Multiple R-squared:  0.7967, Adjusted R-squared:  0.7966
## F-statistic:  8200 on 1 and 2093 DF,  p-value: < 2.2e-16
```

Just like CORN, there evidence of ARCH effects in the residuals based off the Ljung-Box Test.










```
##
##  Box-Ljung test
##
## data:  ols$residuals^2
## X-squared = 588.66, df = 1, p-value < 2.2e-16
```

Including the unconditional mean is unimformative.

```
## [1] "No unconditional mean Robust Coefficients"

##           Estimate  Std. Error  t value     Pr(>|t|)
## omega  0.004550509 0.004536879 1.003004 3.158589e-01
## alpha1 0.213888854 0.116692517 1.832927 6.681344e-02
## beta1  0.779941404 0.125291765 6.225001 4.815519e-10

##
## Akaike       0.5464972
## Bayes        0.5545841
## Shibata      0.5464932
## Hannan-Quinn 0.5494596

## [1] "Including Unconditional Mean Robust Coefficients"

##            Estimate   Std. Error      t value    Pr(>|t|)
## mu     9.353571e-17 0.0036536823 2.560039e-14 1.000000000
## omega  3.195054e-04 0.0003706818 8.619398e-01 0.388720655
## alpha1 8.038248e-02 0.0238503629 3.370283e+00 0.000750909
## beta1  9.064098e-01 0.0333808136 2.715362e+01 0.000000000

##
```

```
## Akaike       0.6237427
## Bayes        0.6345252
## Shibata      0.6237355
## Hannan-Quinn 0.6276925
```

The ARCH in mean coefficient is not statistically significant at the 5% level, and including it does not help model fit.

```
## [1] "ARCH in Mean Robust Coefficients"
```

```
##             Estimate  Std. Error       t value     Pr(>|t|)
## mu     -9.353573e-17 0.013008433 -7.190391e-15 1.000000e+00
## archm   1.388458e-02 0.050182189  2.766833e-01 7.820233e-01
## omega   4.567627e-03 0.004606163  9.916339e-01 3.213762e-01
## alpha1  2.144753e-01 0.118622993  1.808042e+00 7.059997e-02
## beta1   7.794020e-01 0.127256344  6.124661e+00 9.087682e-10
```

```
##
## Akaike       0.5481770
## Bayes        0.5616550
## Shibata      0.5481656
## Hannan-Quinn 0.5531142
```

As with CORN, lets compare different asymmetric specifications, still assuming a normal distribution. I find not reason to move to an asymmetric model.

```
## [1] "Standard GARCH Robust Coefficients"
```

```
##           Estimate  Std. Error  t value     Pr(>|t|)
## omega  0.004550509 0.004536879 1.003004 3.158589e-01
## alpha1 0.213888854 0.116692517 1.832927 6.681344e-02
## beta1  0.779941404 0.125291765 6.225001 4.815519e-10
```

```
##
## Akaike       0.5464972
## Bayes        0.5545841
## Shibata      0.5464932
## Hannan-Quinn 0.5494596
```

```
## [1] "Asymmetric Power ARCH (apARCH) Robust Coefficients"
```

```
##             Estimate   Std. Error       t value     Pr(>|t|)
## mu      9.353572e-17 2.979890e-03  3.138899e-14 1.0000000000
## omega   3.371361e-04 8.258468e-05  4.082308e+00 0.0000445907
## alpha1  6.557617e-02 2.610522e-02  2.511994e+00 0.0120051033
## beta1   8.857889e-01 3.844592e-02  2.303986e+01 0.0000000000
## gamma1 -6.408319e-02 1.186269e-01 -5.402078e-01 0.5890537290
## delta   2.857451e+00 7.516140e-01  3.801753e+00 0.0001436757
```

```
##
## Akaike       0.5780570
## Bayes        0.5942307
## Shibata      0.5780407
## Hannan-Quinn 0.5839817
```

```
## [1] "GJR ARCH Robust Coefficients"
```

```
##            Estimate  Std. Error      t value     Pr(>|t|)
## mu     9.353572e-17 0.378435058 2.471645e-16 1.0000000000
## omega  2.708328e-04 0.000609686 4.442168e-01 0.6568858134
```

```
## alpha1 5.128530e-02 0.240628483 2.131306e-01 0.8312250780
## beta1  8.995021e-01 0.238830468 3.766279e+00 0.0001656989
## gamma1 5.119818e-02 0.787090338 6.504741e-02 0.9481362555
```

```
##
## Akaike       0.6941300
## Bayes        0.7076081
## Shibata      0.6941187
## Hannan-Quinn 0.6990673
```

```
## [1] "eARCH Robust Coefficients"
```

```
##             Estimate  Std. Error       t value     Pr(>|t|)
## mu     -9.353564e-17 0.006436382 -1.453233e-14 1.0000000000
## omega  -1.616631e-01 0.191596117 -8.437702e-01 0.3987978252
## alpha1 -2.026829e-02 0.034149086 -5.935237e-01 0.5528307160
## beta1   9.080763e-01 0.090994457  9.979468e+00 0.0000000000
## gamma1  5.335074e-01 0.162110296  3.291015e+00 0.0009982656
```

```
##
## Akaike       0.5612623
## Bayes        0.5747403
## Shibata      0.5612509
## Hannan-Quinn 0.5661995
```

Again, I will replace the normal distribution with the student t distribution. Model fit improves significantly. Notably, an two asymmetric models: the asymmetric power ARCH and GJR ARCH have better fits than the standard model.

```
## [1] "Standard GARCH Robust Coefficients"
```

```
##          Estimate  Std. Error  t value     Pr(>|t|)
## omega  0.01690393 0.004984249 3.391470 6.951866e-04
## alpha1 0.47052374 0.080935371 5.813574 6.115306e-09
## beta1  0.47958691 0.077914278 6.155315 7.492846e-10
## shape  4.29517227 0.524564136 8.188078 2.220446e-16
```

```
##
## Akaike       0.3702626
## Bayes        0.3810450
## Shibata      0.3702553
## Hannan-Quinn 0.3742124
```

```
## [1] "Asymmetric Power ARCH (apARCH) Robust Coefficients"
```

```
##          Estimate  Std. Error  t value     Pr(>|t|)
## omega  0.01216778 0.005118124 2.377391 1.743562e-02
## alpha1 0.44501123 0.072423213 6.144594 8.016836e-10
## beta1  0.45534554 0.088080189 5.169670 2.345073e-07
## gamma1 0.08585973 0.044084119 1.947634 5.145875e-02
## delta  2.28155785 0.287932553 7.923932 2.220446e-15
## shape  4.44709068 0.530265498 8.386536 0.000000e+00
```

```
##
## Akaike       0.3687614
## Bayes        0.3849351
## Shibata      0.3687451
## Hannan-Quinn 0.3746861
```

```
## [1] "GJR ARCH Robust Coefficients"
```

```
##         Estimate  Std. Error  t value     Pr(>|t|)
## omega  0.0166939  0.00499891 3.339509 8.392677e-04
## alpha1 0.3759944  0.08317615 4.520459 6.170578e-06
## beta1  0.4851863  0.07919819 6.126230 8.998562e-10
## gamma1 0.1813732  0.07659533 2.367940 1.788743e-02
## shape  4.2950333  0.52896757 8.119653 4.440892e-16
```

```
##
## Akaike       0.3690870
## Bayes        0.3825651
## Shibata      0.3690756
## Hannan-Quinn 0.3740242
```

```
## [1] "eARCH Robust Coefficients"
```

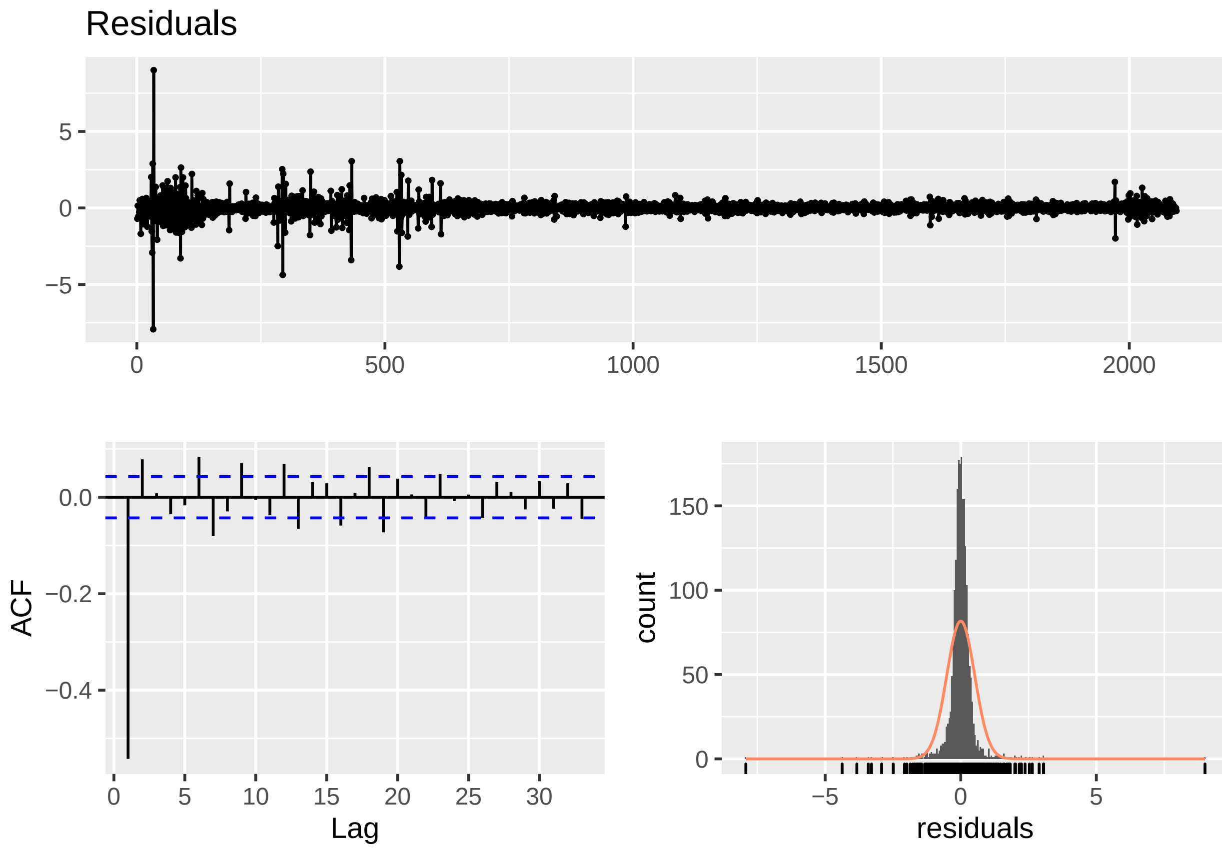
```
##           Estimate  Std. Error   t value     Pr(>|t|)
## omega  -0.44850097  0.20338309 -2.205203 2.743986e-02
## alpha1 -0.03260065  0.02170186 -1.502206 1.330440e-01
## beta1   0.80867613  0.08736300  9.256506 0.000000e+00
## gamma1  0.69427241  0.09938473  6.985705 2.834399e-12
## shape   4.08349350  0.53192374  7.676840 1.620926e-14
```

```
##
## Akaike       0.3733829
## Bayes        0.3868610
## Shibata      0.3733716
## Hannan-Quinn 0.3783202
```

Now lets start over and fit an ARMA process to the residuals. I utilize the Box- Jenkins procedure to fit the model. A good specification would have no autocorrelation in the residuals, but autocorrelation in the squared residuals.

The autocorrelation of the residuals is significant, but dies off very slowly. With a max lag of 10, an ARMA(2,3) is the closest we can get to what we want, but it is not perfect at the 5% level. (3,3) prefers slightly better but the ar3 is not sig.

```
## Warning in modeldf.default(object): Could not find appropriate degrees of
## freedom for this model.
```




```
##
##  Box-Ljung test
##
## data:  a$residuals
## X-squared = 19.933, df = 10, p-value = 0.02989
```

```
##
##  Box-Ljung test
##
## data:  a$residuals^2
## X-squared = 251.57, df = 10, p-value < 2.2e-16
```

Now let's start over from the beginning with out base GARCH model and see if we should include the unconditional mean. The results are challenging as the AIC does improve, but mu is very insignificant. I will leave it out.

```
## [1] "No Unconditional Mean Robust Coefficients"
```

```
##             Estimate   Std. Error      t value   Pr(>|t|)
## ar1      0.718120376 0.0125108542    57.399788 0.00000000
## ar2     -0.984645108 0.0076117561  -129.358468 0.00000000
## ma1     -1.443273802 0.0007000797 -2061.585054 0.00000000
## ma2      1.506917133 0.0009183044  1640.977785 0.00000000
## ma3     -0.720787883 0.0024872147  -289.797210 0.00000000
## omega    0.001161282 0.0010377273     1.119063 0.26311339
## alpha1   0.108404125 0.0658288054     1.646758 0.09960776
## beta1    0.890595875 0.0614461323    14.493929 0.00000000
```

```
##
```

```
## Akaike       0.2120471
## Bayes        0.2336121
## Shibata      0.2120181
## Hannan-Quinn 0.2199467
```

```
## [1] "Including Unconditional Mean Robust Coefficients"
```

```
##              Estimate   Std. Error       t value    Pr(>|t|)
## mu      9.353573e-17 0.0015900297  5.882640e-14 1.000000000
## ar1    -9.171146e-01 0.0027353557 -3.352817e+02 0.000000000
## ar2    -9.940618e-01 0.0031555600 -3.150191e+02 0.000000000
## ma1     1.924108e-01 0.0059148770  3.252998e+01 0.000000000
## ma2     3.225087e-01 0.0078797218  4.092895e+01 0.000000000
## ma3    -7.237283e-01 0.0000812741 -8.904783e+03 0.000000000
## omega   1.030561e-03 0.0006697902  1.538632e+00 0.123894112
## alpha1  1.197659e-01 0.0409071081  2.927753e+00 0.003414216
## beta1   8.848832e-01 0.0326041638  2.714019e+01 0.000000000
```

```
##
## Akaike       0.2033389
## Bayes        0.2275994
## Shibata      0.2033022
## Hannan-Quinn 0.2122260
```

Including the ARCHM process leads to model fit problems.

```
## [1] "No ARCH in Mean Robust Coefficients"
```

```
##             Estimate   Std. Error      t value   Pr(>|t|)
## ar1     0.718120376 0.0125108542    57.399788 0.00000000
## ar2    -0.984645108 0.0076117561  -129.358468 0.00000000
## ma1    -1.443273802 0.0007000797 -2061.585054 0.00000000
## ma2     1.506917133 0.0009183044  1640.977785 0.00000000
## ma3    -0.720787883 0.0024872147  -289.797210 0.00000000
## omega   0.001161282 0.0010377273     1.119063 0.26311339
## alpha1  0.108404125 0.0658288054     1.646758 0.09960776
## beta1   0.890595875 0.0614461323    14.493929 0.00000000
```

```
##
## Akaike       0.2120471
## Bayes        0.2336121
## Shibata      0.2120181
## Hannan-Quinn 0.2199467
```

```
## Warning in arima0(data, order = c(modelinc[2], 0, modelinc[3]), include.mean =
## modelinc[1], : possible convergence problem: optim gave code = 1
```

```
## Warning in .makefitmodel(garchmodel = "sGARCH", f = .sgarchLLH, T = T, m = m, :
## rugarch-->warning: failed to invert hessian
```

```
## [1] "ARCH in Mean Robust Coefficients"
```

```
##              Estimate  Std. Error  t value Pr(>|t|)
## ar1    -0.9459107050          NA       NA       NA
## ar2    -0.7828812588          NA       NA       NA
## ma1    -0.4566485763          NA       NA       NA
## ma2    -0.4318410294          NA       NA       NA
## ma3    -0.9711866295          NA       NA       NA
## archm  -0.0086608344          NA       NA       NA
```

```
## omega   0.0002656958          NA       NA       NA
## alpha1  0.0500000000          NA       NA       NA
## beta1   0.9000000000          NA       NA       NA
```

```
##
## Akaike       0.009642005
## Bayes        0.033902520
## Shibata      0.009605305
## Hannan-Quinn 0.018529054
```

```
base.1.spec <- ugarchspec(variance.model = list(garchOrder = c(1,1), model = "apARCH"),
                        mean.model = list(armaOrder = c(2,3), include.mean = FALSE))
base.1.fit <- ugarchfit(data = x$etf_asset_error, spec = base.1.spec, solver = 'hybrid')

soyb.arma.aic <- append(soyb.arma.aic, infocriteria(base.1.fit)[1])

base.1.spec <- ugarchspec(variance.model = list(garchOrder = c(1,1), model = "gjrGARCH"),
                          mean.model = list(armaOrder = c(2,3), include.mean = FALSE))
base.1.fit <- ugarchfit(data = x$etf_asset_error, spec = base.1.spec, solver = 'hybrid')

soyb.arma.aic <- append(soyb.arma.aic, infocriteria(base.1.fit)[1])

base.1.spec <- ugarchspec(variance.model = list(garchOrder = c(1,1), model = "eGARCH"),
                          mean.model = list(armaOrder = c(2,3), include.mean = FALSE))
base.1.fit <- ugarchfit(data = x$etf_asset_error, spec = base.1.spec, solver = 'hybrid')

soyb.arma.aic <- append(soyb.arma.aic, infocriteria(base.1.fit)[1])
```

Now lets compare normal distribution with student-t. Again we find student-t distribution to have a **much** better fit.

```
## [1] "Normal Distribution Robust Coefficients"
```

```
##            Estimate   Std. Error      t value   Pr(>|t|)
## ar1     0.718120376 0.0125108542    57.399788 0.00000000
## ar2    -0.984645108 0.0076117561  -129.358468 0.00000000
## ma1    -1.443273802 0.0007000797 -2061.585054 0.00000000
## ma2     1.506917133 0.0009183044  1640.977785 0.00000000
## ma3    -0.720787883 0.0024872147  -289.797210 0.00000000
## omega   0.001161282 0.0010377273     1.119063 0.26311339
## alpha1  0.108404125 0.0658288054     1.646758 0.09960776
## beta1   0.890595875 0.0614461323    14.493929 0.00000000
```

```
##
## Akaike       0.2120471
## Bayes        0.2336121
## Shibata      0.2120181
## Hannan-Quinn 0.2199467
```

```
## [1] "Student t Distribution Robust Coefficients"
```

```
##            Estimate   Std. Error       t value   Pr(>|t|)
## ar1    -0.021143719 0.0092694414    -2.2810133 0.02254766
## ar2    -0.956443489 0.0110787322   -86.3314925 0.00000000
## ma1    -0.721886844 0.0005316187 -1357.9034358 0.00000000
## ma2     0.950378295 0.0005480157  1734.2174047 0.00000000
## ma3    -0.716302475 0.0136493610   -52.4788284 0.00000000
## omega   0.002057023 0.0021242106     0.9683705 0.33285935
```

```
## alpha1  0.068087125 0.0696212225     0.9779651 0.32809159
## beta1   0.899887673 0.0907158794     9.9198473 0.00000000
## shape   4.055857264 0.4817281726     8.4193898 0.00000000
```

```
##
## Akaike       -0.014689370
## Bayes         0.009571145
## Shibata      -0.014726070
## Hannan-Quinn -0.005802321
```

Now we fit asymmetric models. Unlike CORN, asymmetric models improve model fit.

```
## [1] "Standard GARCH Robust Coefficients"
```

```
##            Estimate   Std. Error       t value   Pr(>|t|)
## ar1    -0.021143719 0.0092694414    -2.2810133 0.02254766
## ar2    -0.956443489 0.0110787322   -86.3314925 0.00000000
## ma1    -0.721886844 0.0005316187 -1357.9034358 0.00000000
## ma2     0.950378295 0.0005480157  1734.2174047 0.00000000
## ma3    -0.716302475 0.0136493610   -52.4788284 0.00000000
## omega   0.002057023 0.0021242106     0.9683705 0.33285935
## alpha1  0.068087125 0.0696212225     0.9779651 0.32809159
## beta1   0.899887673 0.0907158794     9.9198473 0.00000000
## shape   4.055857264 0.4817281726     8.4193898 0.00000000
```

```
##
## Akaike       -0.014689370
## Bayes         0.009571145
## Shibata      -0.014726070
## Hannan-Quinn -0.005802321
```

```
## [1] "Asymmetric Power GARCH (apGARCH) Robust Coefficients"
```

```
##            Estimate   Std. Error       t value     Pr(>|t|)
## ar1     0.104981890 1.497629e-02  7.009873e+00 2.385425e-12
## ar2     0.772855665 1.227513e-02  6.296109e+01 0.000000e+00
## ma1    -0.878477658 5.896963e-05 -1.489712e+04 0.000000e+00
## ma2    -0.707559818 7.346843e-05 -9.630801e+03 0.000000e+00
## ma3     0.622480421 2.397328e-04  2.596559e+03 0.000000e+00
## omega   0.005015881 4.359465e-03  1.150573e+00 2.499081e-01
## alpha1  0.102163882 4.257613e-02  2.399558e+00 1.641488e-02
## beta1   0.889313162 3.906967e-02  2.276224e+01 0.000000e+00
## gamma1  0.141108982 1.619607e-01  8.712544e-01 3.836152e-01
## delta   1.395704529 4.194455e-01  3.327499e+00 8.762934e-04
## shape   4.030672303 4.861978e-01  8.290190e+00 2.220446e-16
```

```
##
## Akaike       -0.024890198
## Bayes         0.004761543
## Shibata      -0.024944952
## Hannan-Quinn -0.014028248
```

```
## [1] "GJR GARCH Robust Coefficients"
```

```
##             Estimate   Std. Error       t value     Pr(>|t|)
## ar1     6.042264e-02 1.370455e-02  4.408946e+00 1.038748e-05
## ar2     8.380764e-01 1.083399e-02  7.735618e+01 0.000000e+00
## ma1    -8.318861e-01 9.029922e-05 -9.212550e+03 0.000000e+00
## ma2    -8.043443e-01 7.014181e-05 -1.146740e+04 0.000000e+00
```

```
## ma3     6.686245e-01 1.097374e-04  6.092951e+03 0.000000e+00
## omega   1.929601e-03 1.882875e-03  1.024816e+00 3.054498e-01
## alpha1  6.986816e-02 6.399267e-02  1.091815e+00 2.749144e-01
## beta1   9.001173e-01 8.309318e-02  1.083263e+01 0.000000e+00
## gamma1 -6.252425e-05 2.423218e-02 -2.580216e-03 9.979413e-01
## shape   4.131826e+00 5.131431e-01  8.051996e+00 8.881784e-16
```

```
##
## Akaike       -0.023788197
## Bayes         0.003167931
## Shibata      -0.023833477
## Hannan-Quinn -0.013913697
```

```
## [1] "Exponential Power GARCH Robust Coefficients"
```

```
##           Estimate   Std. Error      t value     Pr(>|t|)
## ar1     0.15714845 0.0131594937    11.941831 0.000000e+00
## ar2     0.71634329 0.0134201543    53.378170 0.000000e+00
## ma1    -0.93346057 0.0005170787 -1805.258229 0.000000e+00
## ma2    -0.61356274 0.0001450391 -4230.325790 0.000000e+00
## ma3     0.58396896 0.0004064176  1436.869363 0.000000e+00
## omega  -0.05939593 0.0181470632    -3.273033 1.064002e-03
## alpha1 -0.03593799 0.0177353780    -2.026345 4.272948e-02
## beta1   0.97845015 0.0064306932   152.153136 0.000000e+00
## gamma1  0.16200672 0.0322764796     5.019343 5.184854e-07
## shape   4.04978174 0.4308685330     9.399112 0.000000e+00
```

```
##
## Akaike       -0.028158712
## Bayes        -0.001202584
## Shibata      -0.028203992
## Hannan-Quinn -0.018284212
```

SOYB Summary

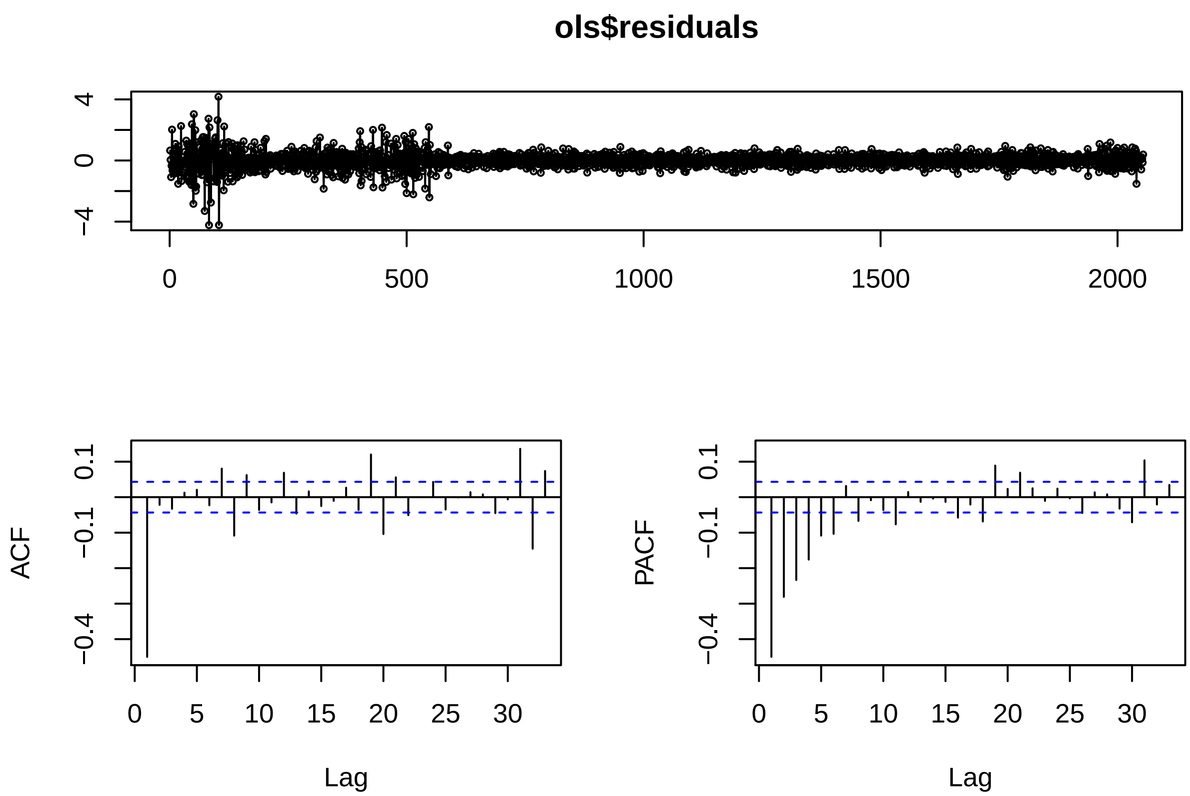
As with CORN, it is likely best to not include ARCH in mean or the unconditional mean in this model specification. Model fit was greatly improved by fitting an ARMA(2,3) process and further greatly improved by using a student-t distribution. Unlike CORN, some of the Asymmetric specifications did have a better fit.

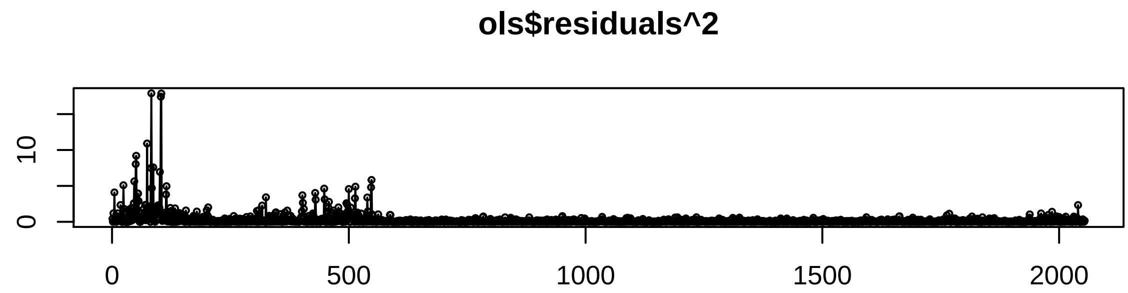
Wheat

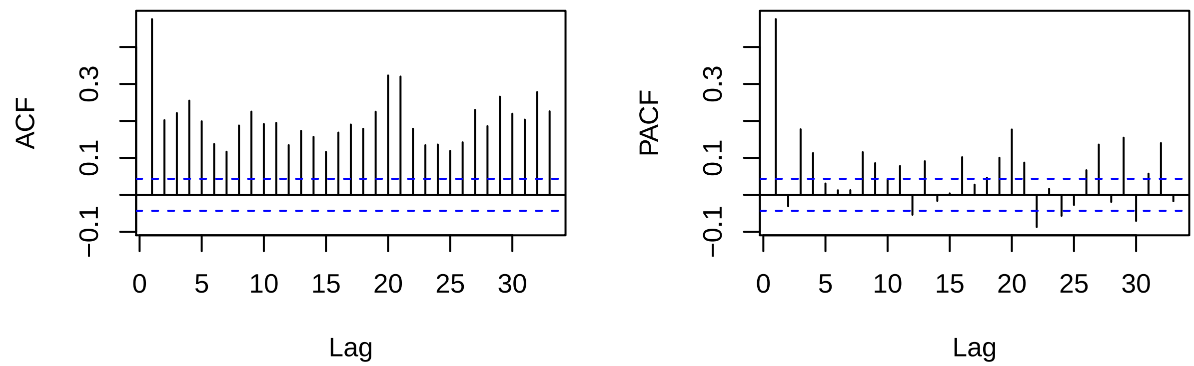
Lets begin by looking at the OLS results. We fail to reject both of the null hypothesis of $\alpha = 0$. $\beta = 1$

```
##
## Call:
## lm(formula = x$per_ETF_return ~ x$per_asset_return)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -4.2303 -0.2278  0.0038  0.2298  4.1735
##
## Coefficients:
##                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)        -0.017761   0.011422  -1.555     0.12
## x$per_asset_return  0.988974   0.008038 123.033   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5173 on 2052 degrees of freedom
## Multiple R-squared:  0.8806, Adjusted R-squared:  0.8806
## F-statistic: 1.514e+04 on 1 and 2052 DF,  p-value: < 2.2e-16
```

There is evidence of ARCH effects in the residuals based off the Ljung-Box Test.








```
##
##  Box-Ljung test
##
## data:  ols$residuals^2
## X-squared = 465.15, df = 1, p-value < 2.2e-16
```

Moving onto GARCH, again including the unconditional mean is unimformative.

```
## [1] "No unconditional mean Robust Coefficients"

##           Estimate  Std. Error   t value     Pr(>|t|)
## omega  0.002141502 0.001129709  1.895622 0.0580100007
## alpha1 0.095690807 0.026781816  3.572977 0.0003529461
## beta1  0.894005537 0.030682980 29.136855 0.0000000000

##
## Akaike       0.9438352
## Bayes        0.9520546
## Shibata      0.9438309
## Hannan-Quinn 0.9468491

## [1] "Including Unconditional Mean Robust Coefficients"

##            Estimate   Std. Error      t value     Pr(>|t|)
## mu     3.204203e-16 0.0034423126 9.308285e-14 1.000000e+00
## omega  3.330144e-04 0.0004387681 7.589758e-01 4.478671e-01
## alpha1 7.631774e-02 0.0155951023 4.893699e+00 9.895810e-07
## beta1  9.126291e-01 0.0226149761 4.035508e+01 0.000000e+00

##
```

```
## Akaike       0.9675119
## Bayes        0.9784711
## Shibata      0.9675044
## Hannan-Quinn 0.9715305
```

The ARCH in mean coefficient is not statistically significant at the 5% level, and including it does not help model fit. For whatever reason, if I try and not include the unconditional mean, the model will not run.

```
## [1] "ARCH in Mean Robust Coefficients"

##            Estimate  Std. Error      t value     Pr(>|t|)
## mu     3.204203e-16 0.009630912 3.326998e-14 1.0000000000
## archm  8.720473e-03 0.029262526 2.980082e-01 0.7656968878
## omega  2.148056e-03 0.001133201 1.895566e+00 0.0580174802
## alpha1 9.581502e-02 0.026913803 3.560070e+00 0.0003707559
## beta1  8.938368e-01 0.030822765 2.899924e+01 0.0000000000

##
## Akaike       0.9456991
## Bayes        0.9593981
## Shibata      0.9456873
## Hannan-Quinn 0.9507223
```

As with CORN, lets compare different asymmetric specifications, still assuming a normal distribution. I find not reason to move to an asymmetric model.

```
## [1] "Standard GARCH Robust Coefficients"

##           Estimate  Std. Error   t value     Pr(>|t|)
## omega  0.002141502 0.001129709  1.895622 0.0580100007
## alpha1 0.095690807 0.026781816  3.572977 0.0003529461
## beta1  0.894005537 0.030682980 29.136855 0.0000000000

##
## Akaike       0.9438352
## Bayes        0.9520546
## Shibata      0.9438309
## Hannan-Quinn 0.9468491

## [1] "Asymmetric Power ARCH (apARCH) Robust Coefficients"

##             Estimate   Std. Error       t value     Pr(>|t|)
## mu      3.204203e-16 1.060568e-02  3.021212e-14 1.000000e+00
## omega   3.004172e-04 6.253434e-05  4.804035e+00 1.554996e-06
## alpha1  5.770663e-02 9.789068e-03  5.895007e+00 3.746647e-09
## beta1   8.838769e-01 9.753083e-03  9.062539e+01 0.000000e+00
## gamma1 -5.185411e-02 4.332758e-01 -1.196792e-01 9.047373e-01
## delta   3.011776e+00 1.466912e-01  2.053141e+01 0.000000e+00

##
## Akaike       0.9673960
## Bayes        0.9838348
## Shibata      0.9673790
## Hannan-Quinn 0.9734239

## [1] "GJR ARCH Robust Coefficients"

##            Estimate  Std. Error      t value     Pr(>|t|)
## mu     3.204202e-16  0.05554348 5.768818e-15 1.000000e+00
## omega  2.802423e-04  0.00123182 2.275027e-01 8.200329e-01
```

```
## alpha1 5.465889e-02  0.03335285 1.638807e+00 1.012534e-01
## beta1  9.027010e-01  0.12160360 7.423308e+00 1.141309e-13
## gamma1 4.600476e-02  0.15698553 2.930510e-01 7.694832e-01
```

```
##
## Akaike       1.007993
## Bayes        1.021692
## Shibata      1.007981
## Hannan-Quinn 1.013016
```

```
## [1] "eARCH Robust Coefficients"
```

```
##             Estimate  Std. Error       t value     Pr(>|t|)
## mu      3.204202e-16 0.008502011  3.768758e-14 1.000000e+00
## omega  -1.456508e-01 0.246418626 -5.910708e-01 5.544730e-01
## alpha1  1.882653e-02 0.044974247  4.186069e-01 6.755034e-01
## beta1   9.149180e-01 0.147904860  6.185855e+00 6.176675e-10
## gamma1  4.191260e-01 0.268864372  1.558875e+00 1.190259e-01
```

```
##
## Akaike       0.9855407
## Bayes        0.9992396
## Shibata      0.9855288
## Hannan-Quinn 0.9905639
```

Again, I will replace the normal distribution with the student t distribution. Model fit improves slightly, certainly not as much as the other ETFs

```
## [1] "Standard GARCH Robust Coefficients"
```

```
##            Estimate  Std. Error   t value     Pr(>|t|)
## omega   0.002811595 0.001588056  1.770463 7.664996e-02
## alpha1  0.096961541 0.034270260  2.829320 4.664699e-03
## beta1   0.887818912 0.040628230 21.852267 0.000000e+00
## shape  10.847160230 2.287432011  4.742069 2.115466e-06
```

```
##
## Akaike       0.9325403
## Bayes        0.9434995
## Shibata      0.9325327
## Hannan-Quinn 0.9365588
```

```
## [1] "Asymmetric Power ARCH (apARCH) Robust Coefficients"
```

```
##            Estimate  Std. Error    t value     Pr(>|t|)
## omega   0.002420834 0.001731991  1.3977168 1.621981e-01
## alpha1  0.088607639 0.035839020  2.4723790 1.342171e-02
## beta1   0.892827266 0.040213090 22.2024037 0.000000e+00
## gamma1 -0.100548282 0.109591835 -0.9174797 3.588913e-01
## delta   2.081768145 0.614017897  3.3904030 6.978994e-04
## shape  10.953032595 2.308662476  4.7443196 2.092083e-06
```

```
##
## Akaike       0.9337804
## Bayes        0.9502192
## Shibata      0.9337634
## Hannan-Quinn 0.9398083
```

```
## [1] "GJR ARCH Robust Coefficients"
```

```
##             Estimate  Std. Error   t value     Pr(>|t|)
## omega   0.002564309 0.001420924  1.804677 7.112519e-02
## alpha1  0.110235596 0.031739136  3.473176 5.143386e-04
## beta1   0.894339543 0.036903119 24.234795 0.000000e+00
## gamma1 -0.037770805 0.032045698 -1.178654 2.385358e-01
## shape  10.926623969 2.276245069  4.800285 1.584404e-06
```

```
##
## Akaike       0.9328237
## Bayes        0.9465227
## Shibata      0.9328119
## Hannan-Quinn 0.9378469
```

```
## [1] "eARCH Robust Coefficients"
```

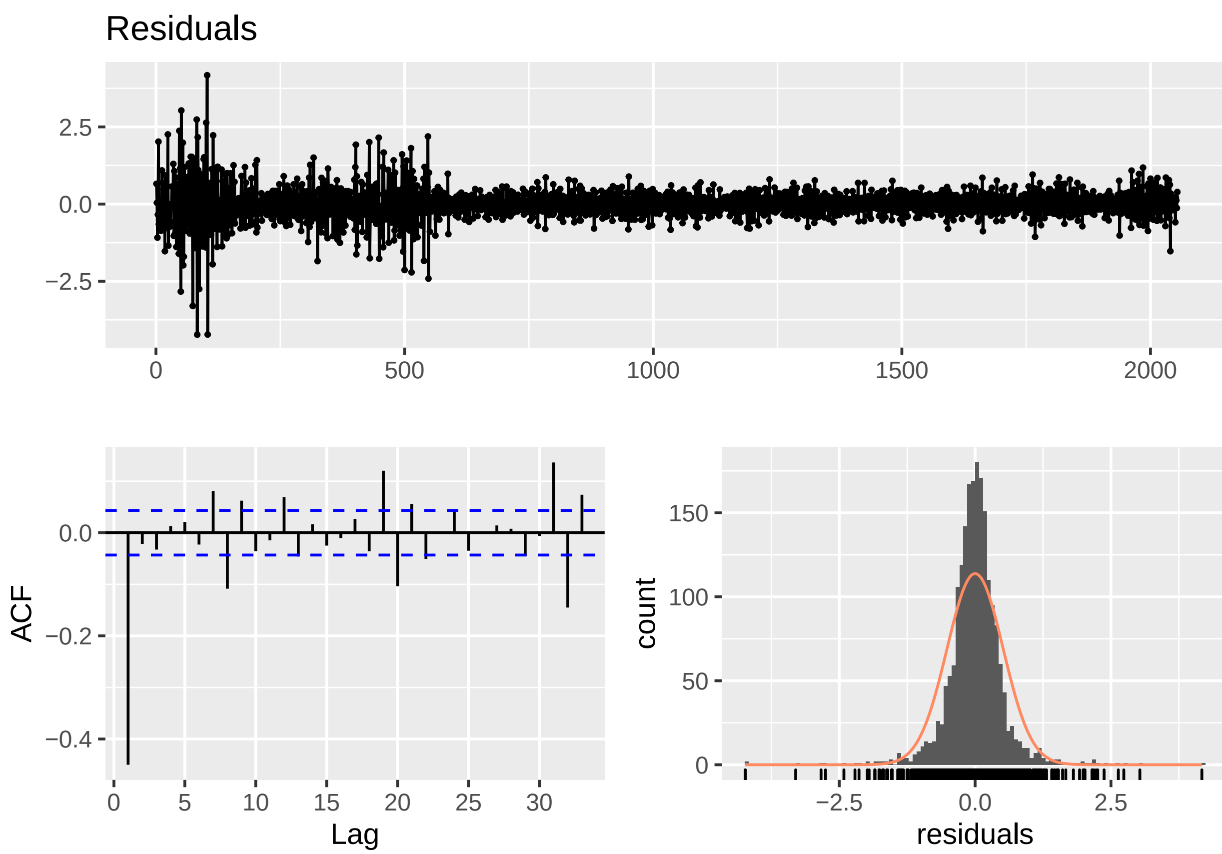
```
##            Estimate  Std. Error    t value     Pr(>|t|)
## omega   -0.01605292 0.009491070  -1.691371 9.076593e-02
## alpha1   0.03309932 0.026445737   1.251594 2.107179e-01
## beta1    0.99200935 0.004530883 218.943940 0.000000e+00
## gamma1   0.17338025 0.032880373   5.273062 1.341661e-07
## shape   10.72441933 2.210808630   4.850904 1.229003e-06
```

```
##
## Akaike       0.9352613
## Bayes        0.9489603
## Shibata      0.9352495
## Hannan-Quinn 0.9402845
```

Now lets start over and fit an ARMA process to the residuals. I utilize the Box- Jenkins procedure to fit the model. A good specification would have no autocorrelation in the residuals, but autocorrelation in the squared residuals.

(1,5) is the best model specification

```
## Warning in modeldf.default(object): Could not find appropriate degrees of
## freedom for this model.
```




```
##
##  Box-Ljung test
##
## data:  a$residuals
## X-squared = 14.507, df = 10, p-value = 0.1511
```

```
##
##  Box-Ljung test
##
## data:  a$residuals^2
## X-squared = 844.99, df = 10, p-value < 2.2e-16
```

Now let's start over from the beginning with out base GARCH model and see if we should include the unconditional mean. The results are challenging as the AIC does improve, but mu is very insignificant. I will leave it out.

```
## [1] "No Unconditional Mean Robust Coefficients"
```

```
##              Estimate   Std. Error       t value     Pr(>|t|)
## ar1     -0.9752251957 9.732634e-03  -100.201570 0.000000e+00
## ma1      0.3001952602 3.120419e-03    96.203504 0.000000e+00
## ma2     -0.7219868726 8.978032e-05 -8041.705467 0.000000e+00
## ma3     -0.0567730800 1.498277e-02    -3.789223 1.511190e-04
## ma4      0.0297574012 3.912621e-03     7.605491 2.842171e-14
## ma5      0.0436989706 1.972010e-02     2.215961 2.669415e-02
## omega    0.0007666242 3.625323e-04     2.114637 3.446088e-02
## alpha1   0.0554082208 9.100132e-03     6.088727 1.138117e-09
## beta1    0.9385474942 1.058731e-02    88.648359 0.000000e+00
```

```
##
## Akaike       0.5916290
## Bayes        0.6162872
## Shibata      0.5915908
## Hannan-Quinn 0.6006708
```

```
## [1] "Including Unconditional Mean Robust Coefficients"
```

```
##             Estimate   Std. Error       t value     Pr(>|t|)
## mu     -3.204203e-16 1.841694e-03 -1.739813e-13 1.000000e+00
## ar1    -9.752258e-01 9.733559e-03 -1.001921e+02 0.000000e+00
## ma1     3.001953e-01 3.124089e-03  9.609051e+01 0.000000e+00
## ma2    -7.219863e-01 8.848695e-05 -8.159240e+03 0.000000e+00
## ma3    -5.677115e-02 1.496368e-02 -3.793930e+00 1.482817e-04
## ma4     2.975901e-02 3.914404e-03  7.602436e+00 2.908784e-14
## ma5     4.369914e-02 1.971030e-02  2.217072e+00 2.661819e-02
## omega   7.666676e-04 3.628354e-04  2.112990e+00 3.460162e-02
## alpha1  5.540881e-02 9.083324e-03  6.100058e+00 1.060300e-09
## beta1   9.385463e-01 1.056125e-02  8.886697e+01 0.000000e+00
```

```
##
## Akaike       0.5926027
## Bayes        0.6200007
## Shibata      0.5925556
## Hannan-Quinn 0.6026491
```

Including the ARCHM process leads to model fit problems.

```
## [1] "No ARCH in Mean Robust Coefficients"
```

```
##             Estimate   Std. Error      t value     Pr(>|t|)
## ar1    -0.9752251957 9.732634e-03  -100.201570 0.000000e+00
## ma1     0.3001952602 3.120419e-03    96.203504 0.000000e+00
## ma2    -0.7219868726 8.978032e-05 -8041.705467 0.000000e+00
## ma3    -0.0567730800 1.498277e-02    -3.789223 1.511190e-04
## ma4     0.0297574012 3.912621e-03     7.605491 2.842171e-14
## ma5     0.0436989706 1.972010e-02     2.215961 2.669415e-02
## omega   0.0007666242 3.625323e-04     2.114637 3.446088e-02
## alpha1  0.0554082208 9.100132e-03     6.088727 1.138117e-09
## beta1   0.9385474942 1.058731e-02    88.648359 0.000000e+00
```

```
##
## Akaike       0.5916290
## Bayes        0.6162872
## Shibata      0.5915908
## Hannan-Quinn 0.6006708
```

```
## Warning in .makefitmodel(garchmodel = "sGARCH", f = .sgarchLLH, T = T, m = m, :
## rugarch-->warning: failed to invert hessian
```

```
## [1] "ARCH in Mean Robust Coefficients"
```

```
##             Estimate  Std. Error  t value Pr(>|t|)
## ar1    -0.4913844154          NA       NA       NA
## ma1    -1.0272981918          NA       NA       NA
## ma2    -0.7596165363          NA       NA       NA
## ma3     0.0342478826          NA       NA       NA
## ma4     0.0980953453          NA       NA       NA
## ma5     0.0822067194          NA       NA       NA
```

```
## archm  -0.0111440039          NA       NA       NA
## omega   0.0002674583          NA       NA       NA
## alpha1  0.0500000000          NA       NA       NA
## beta1   0.9000000000          NA       NA       NA
```

```
##
## Akaike       0.01080818
## Bayes        0.03820616
## Shibata      0.01076108
## Hannan-Quinn 0.02085459
```

Now lets compare normal distribution with student-t. Again we find student-t distribution to have a somewhat better fit.

```
## [1] "Normal Distribution Robust Coefficients"
```

```
##             Estimate   Std. Error      t value     Pr(>|t|)
## ar1    -0.9752251957 9.732634e-03  -100.201570 0.000000e+00
## ma1     0.3001952602 3.120419e-03    96.203504 0.000000e+00
## ma2    -0.7219868726 8.978032e-05 -8041.705467 0.000000e+00
## ma3    -0.0567730800 1.498277e-02    -3.789223 1.511190e-04
## ma4     0.0297574012 3.912621e-03     7.605491 2.842171e-14
## ma5     0.0436989706 1.972010e-02     2.215961 2.669415e-02
## omega   0.0007666242 3.625323e-04     2.114637 3.446088e-02
## alpha1  0.0554082208 9.100132e-03     6.088727 1.138117e-09
## beta1   0.9385474942 1.058731e-02    88.648359 0.000000e+00
```

```
##
## Akaike       0.5916290
## Bayes        0.6162872
## Shibata      0.5915908
## Hannan-Quinn 0.6006708
```

```
## [1] "Student t Distribution Robust Coefficients"
```

```
##            Estimate   Std. Error       t value     Pr(>|t|)
## ar1    -0.973995421 0.0067014244  -145.3415507 0.000000e+00
## ma1     0.290777714 0.0116980636    24.8569100 0.000000e+00
## ma2    -0.714469484 0.0002143742 -3332.8151674 0.000000e+00
## ma3    -0.046635683 0.0288080594    -1.6188415 1.054814e-01
## ma4     0.018533942 0.0218731237     0.8473386 3.968064e-01
## ma5     0.037621540 0.0224971184     1.6722826 9.446861e-02
## omega   0.001244387 0.0004162771     2.9893240 2.795955e-03
## alpha1  0.060123085 0.0101295321     5.9354257 2.930841e-09
## beta1   0.928383263 0.0118331071    78.4564234 0.000000e+00
## shape   8.284378933 1.3621703557     6.0817495 1.188782e-09
```

```
##
## Akaike       0.5594783
## Bayes        0.5868763
## Shibata      0.5594312
## Hannan-Quinn 0.5695247
```

Now we fit asymmetric models. Like CORN and unlike SOYB, asymmetric models do not help model fit

```
## [1] "Standard GARCH Robust Coefficients"
```

```
##            Estimate   Std. Error       t value     Pr(>|t|)
## ar1    -0.973995421 0.0067014244  -145.3415507 0.000000e+00
```

```
## ma1     0.290777714 0.0116980636    24.8569100 0.000000e+00
## ma2    -0.714469484 0.0002143742 -3332.8151674 0.000000e+00
## ma3    -0.046635683 0.0288080594    -1.6188415 1.054814e-01
## ma4     0.018533942 0.0218731237     0.8473386 3.968064e-01
## ma5     0.037621540 0.0224971184     1.6722826 9.446861e-02
## omega   0.001244387 0.0004162771     2.9893240 2.795955e-03
## alpha1  0.060123085 0.0101295321     5.9354257 2.930841e-09
## beta1   0.928383263 0.0118331071    78.4564234 0.000000e+00
## shape   8.284378933 1.3621703557     6.0817495 1.188782e-09

##
## Akaike       0.5594783
## Bayes        0.5868763
## Shibata      0.5594312
## Hannan-Quinn 0.5695247

## [1] "Asymmetric Power GARCH (apGARCH) Robust Coefficients"

##             Estimate   Std. Error      t value     Pr(>|t|)
## ar1    -0.9748394195 0.0631406994  -15.4391609 0.000000e+00
## ma1     0.2919231462 0.1715783722    1.7013983 8.886823e-02
## ma2    -0.7166453197 0.0013920393 -514.8168826 0.000000e+00
## ma3    -0.0475862448 0.2429500765   -0.1958684 8.447132e-01
## ma4     0.0206053305 0.1863604980    0.1105670 9.119597e-01
## ma5     0.0379593895 0.0497003279    0.7637654 4.450071e-01
## omega   0.0007028379 0.0006868765    1.0232376 3.061955e-01
## alpha1  0.0445328106 0.0255811603    1.7408440 8.171092e-02
## beta1   0.9244946913 0.0155109673   59.6026457 0.000000e+00
## gamma1  0.0421762506 0.1230052305    0.3428818 7.316874e-01
## delta   2.6386247454 1.0168511658    2.5948977 9.461908e-03
## shape   8.3162319539 1.4198257728    5.8572200 4.706794e-09

##
## Akaike       0.5604287
## Bayes        0.5933063
## Shibata      0.5603610
## Hannan-Quinn 0.5724844

## [1] "GJR GARCH Robust Coefficients"

##            Estimate   Std. Error       t value     Pr(>|t|)
## ar1    -0.973970525 0.0065503084  -148.6907880 0.000000e+00
## ma1     0.290749210 0.0112542060    25.8347155 0.000000e+00
## ma2    -0.714327802 0.0002128954 -3355.2988761 0.000000e+00
## ma3    -0.046579446 0.0283770822    -1.6414459 1.007049e-01
## ma4     0.018333417 0.0213925133     0.8570016 3.914440e-01
## ma5     0.037512797 0.0224023045     1.6745062 9.403116e-02
## omega   0.001230475 0.0004280004     2.8749391 4.041054e-03
## alpha1  0.060952260 0.0128537804     4.7419715 2.116484e-06
## beta1   0.928604147 0.0117929762    78.7421371 0.000000e+00
## gamma1 -0.001910037 0.0173205033    -0.1102761 9.121904e-01
## shape   8.284592301 1.3637218582     6.0749868 1.239979e-09

##
## Akaike       0.5604464
## Bayes        0.5905842
## Shibata      0.5603895
```

```
## Hannan-Quinn 0.5714975
```

```
## [1] "Exponential Power GARCH Robust Coefficients"
```

```
##           Estimate   Std. Error       t value     Pr(>|t|)
## ar1    -0.97418030 0.0049861791  -195.3761135 0.000000e+00
## ma1     0.29022455 0.0127519142    22.7592929 0.000000e+00
## ma2    -0.71158558 0.0002538714 -2802.9374764 0.000000e+00
## ma3    -0.04332682 0.0722632836    -0.5995689 5.487936e-01
## ma4     0.01459614 0.0147540362     0.9892983 3.225172e-01
## ma5     0.03369983 0.0676718535     0.4979888 6.184919e-01
## omega  -0.01420729 0.0073118985    -1.9430370 5.201168e-02
## alpha1  0.01192758 0.0128371454     0.9291458 3.528136e-01
## beta1   0.99435381 0.0028876944   344.3417690 0.000000e+00
## gamma1  0.13358924 0.0176575090     7.5655768 3.863576e-14
## shape   8.16384915 1.2916608330     6.3204279 2.608400e-10
```

```
##
## Akaike       0.5623342
## Bayes        0.5924720
## Shibata      0.5622773
## Hannan-Quinn 0.5733853
```

WEAT Summary

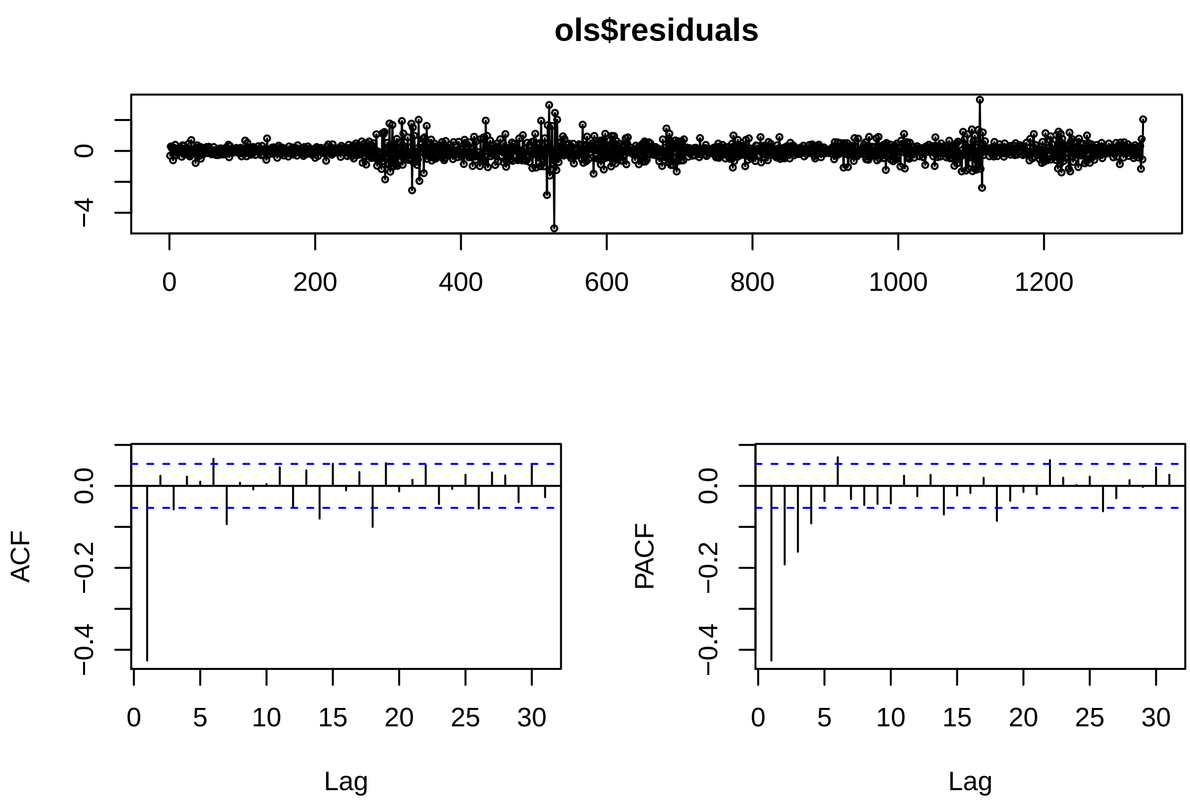
As with CORN and SOYB, it is likely best to not include ARCH in mean or the unconditional mean in this model specification as it does not improve model fit or produce significant coefficients. Fitting an ARMA (1,5) process also improved fit significantly. Changing from a normal distribution to a student t distribution *slightly* improved model fit. Asymmetric models did not have better fit.

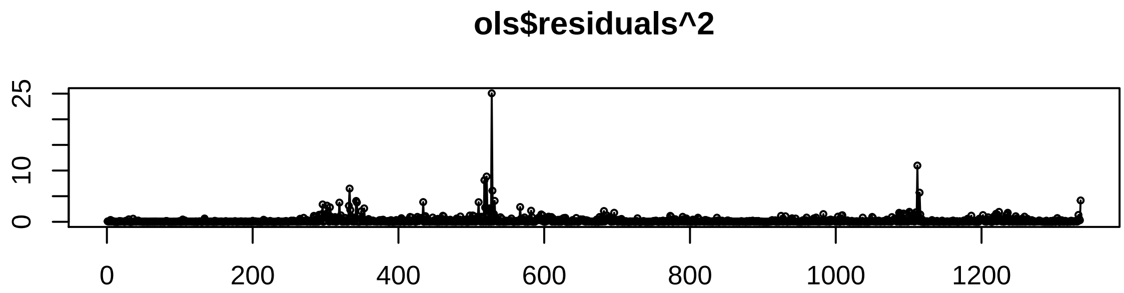
Crude Oil (USO)

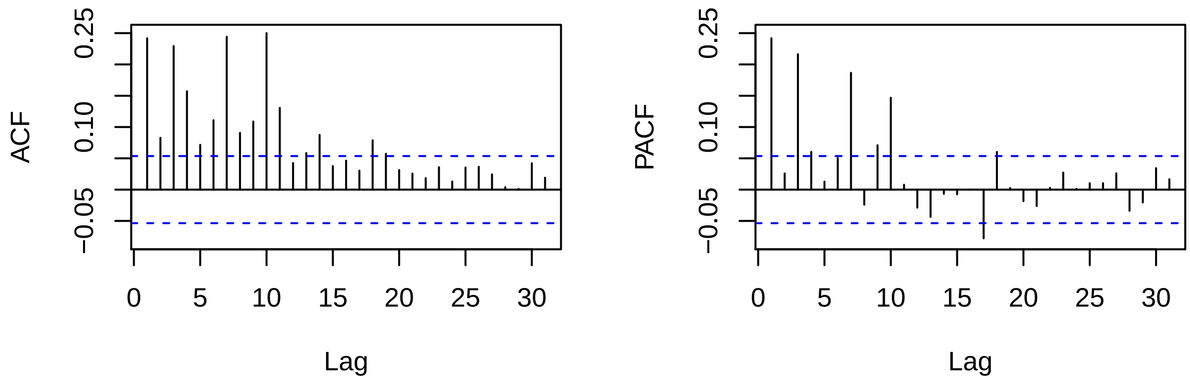
Lets begin by looking at the OLS results. We fail to reject $\alpha = 0$ but reject the second hypothesis of $\beta = 1$ by quite a bit. The β estimate of 0.9 implies that severe disunity.

```
##
## Call:
## lm(formula = x$per_ETF_return ~ x$per_asset_return)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -5.0068 -0.2456  0.0024  0.2392  3.3129
##
## Coefficients:
##                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)        -0.001630   0.014648  -0.111    0.911
## x$per_asset_return  0.903983   0.006778 133.361   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5353 on 1334 degrees of freedom
## Multiple R-squared:  0.9302, Adjusted R-squared:  0.9302
## F-statistic: 1.779e+04 on 1 and 1334 DF,  p-value: < 2.2e-16
```

There is evidence of ARCH effects in the residuals based off the Ljung-Box Test.








```
##
##  Box-Ljung test
##
## data:  ols$residuals^2
## X-squared = 78.303, df = 1, p-value < 2.2e-16
```

Moving onto GARCH, again including the unconditional mean is unimformative.

```
## [1] "No unconditional mean Robust Coefficients"

##           Estimate  Std. Error   t value     Pr(>|t|)
## omega  0.00497121 0.002228314  2.230929 2.568585e-02
## alpha1 0.18189034 0.032367298  5.619571 1.914317e-08
## beta1  0.81531731 0.035897677 22.712258 0.000000e+00

##
## Akaike       1.171617
## Bayes        1.183288
## Shibata      1.171607
## Hannan-Quinn 1.175990

## [1] "Including Unconditional Mean Robust Coefficients"

##             Estimate   Std. Error      t value     Pr(>|t|)
## mu     6.500343e-17 0.0068162922 9.536480e-15 1.000000e+00
## omega  3.638477e-04 0.0005004277 7.270735e-01 4.671809e-01
## alpha1 9.259288e-02 0.0135382043 6.839377e+00 7.953860e-12
## beta1  8.961054e-01 0.0192762447 4.648755e+01 0.000000e+00

##
```

```
## Akaike       1.220585
## Bayes        1.236147
## Shibata      1.220568
## Hannan-Quinn 1.226416
```

The ARCH in mean coefficient is not statistically significant at the 5% level, and including it does not help model fit.

```
## [1] "ARCH in Mean Robust Coefficients"

##             Estimate  Std. Error       t value     Pr(>|t|)
## mu     -6.500344e-17 0.015724484 -4.133900e-15 1.000000e+00
## archm   1.876916e-02 0.044457691  4.221802e-01 6.728935e-01
## omega   4.977873e-03 0.002235011  2.227226e+00 2.593220e-02
## alpha1  1.816601e-01 0.033057814  5.495223e+00 3.902175e-08
## beta1   8.154116e-01 0.036590096  2.228504e+01 0.000000e+00

##
## Akaike       1.174194
## Bayes        1.193645
## Shibata      1.174166
## Hannan-Quinn 1.181482
```

I find not reason to move to an asymmetric model.

```
## [1] "Standard GARCH Robust Coefficients"

##          Estimate  Std. Error   t value     Pr(>|t|)
## omega  0.00497121 0.002228314  2.230929 2.568585e-02
## alpha1 0.18189034 0.032367298  5.619571 1.914317e-08
## beta1  0.81531731 0.035897677 22.712258 0.000000e+00

##
## Akaike       1.171617
## Bayes        1.183288
## Shibata      1.171607
## Hannan-Quinn 1.175990

## [1] "Asymmetric Power ARCH (apARCH) Robust Coefficients"

##            Estimate   Std. Error      t value     Pr(>|t|)
## mu     6.500344e-17 0.0138567875 4.691090e-15 1.000000e+00
## omega  2.978635e-04 0.0001064033 2.799383e+00 5.120036e-03
## alpha1 6.200078e-02 0.0088143567 7.034068e+00 2.005951e-12
## beta1  8.917146e-01 0.0104905549 8.500166e+01 0.000000e+00
## gamma1 1.221163e-01 0.2843080046 4.295212e-01 6.675440e-01
## delta  2.723968e+00 0.2123854926 1.282558e+01 0.000000e+00

##
## Akaike       1.232481
## Bayes        1.255823
## Shibata      1.232441
## Hannan-Quinn 1.241227

## [1] "GJR ARCH Robust Coefficients"

##            Estimate   Std. Error      t value  Pr(>|t|)
## mu     6.500344e-17 0.0659134500 9.861938e-16 1.0000000
## omega  2.892388e-04 0.0006038879 4.789612e-01 0.6319663
## alpha1 5.024106e-02 0.0684567401 7.339096e-01 0.4630038
```

```
## beta1  8.952817e-01 0.0331324533 2.702129e+01 0.0000000
## gamma1 6.209859e-02 0.1265592813 4.906680e-01 0.6236613
```

```
##
## Akaike       1.266789
## Bayes        1.286241
## Shibata      1.266761
## Hannan-Quinn 1.274078
```

```
## [1] "eARCH Robust Coefficients"
```

```
##             Estimate  Std. Error       t value  Pr(>|t|)
## mu     -6.500315e-17    0.269049 -2.416034e-16 1.0000000
## omega  -1.421349e-01    5.223042 -2.721305e-02 0.9782898
## alpha1 -7.167178e-02    1.706896 -4.198954e-02 0.9665070
## beta1   9.027407e-01    3.594377  2.511536e-01 0.8016954
## gamma1  4.323494e-01    6.485791  6.666102e-02 0.9468516
```

```
##
## Akaike       1.196384
## Bayes        1.215836
## Shibata      1.196356
## Hannan-Quinn 1.203673
```

Again, I will replace the normal distribution with the student t distribution. Model fit improves and now the asymmetric models fit better.

```
## [1] "Standard GARCH Robust Coefficients"
```

```
##           Estimate  Std. Error   t value     Pr(>|t|)
## omega  0.006033713 0.002666473  2.262806 2.364763e-02
## alpha1 0.195523192 0.033650409  5.810425 6.231430e-09
## beta1  0.800275474 0.038405892 20.837310 0.000000e+00
## shape  6.823643166 1.052936665  6.480583 9.136913e-11
```

```
##
## Akaike       1.138615
## Bayes        1.154176
## Shibata      1.138597
## Hannan-Quinn 1.144445
```

```
## [1] "Asymmetric Power ARCH (apARCH) Robust Coefficients"
```

```
##          Estimate  Std. Error   t value     Pr(>|t|)
## omega  0.01064767 0.005589118  1.905071 5.677085e-02
## alpha1 0.16616280 0.028727892  5.784024 7.293471e-09
## beta1  0.84170164 0.031383697 26.819710 0.000000e+00
## gamma1 0.36593712 0.178528928  2.049736 4.039023e-02
## delta  1.26826150 0.376547498  3.368132 7.567948e-04
## shape  7.20725841 1.222391787  5.896030 3.723517e-09
```

```
##
## Akaike       1.132893
## Bayes        1.156235
## Shibata      1.132853
## Hannan-Quinn 1.141639
```

```
## [1] "GJR ARCH Robust Coefficients"
```

```
##           Estimate  Std. Error   t value     Pr(>|t|)
```

```
## omega  0.006032686 0.002473456  2.438970 1.472918e-02
## alpha1 0.111899840 0.034033503  3.287932 1.009262e-03
## beta1  0.803633721 0.035874839 22.401041 0.000000e+00
## gamma1 0.158922829 0.054522055  2.914836 3.558759e-03
## shape  6.943865020 1.104838397  6.284960 3.279383e-10
```

```
##
## Akaike       1.134259
## Bayes        1.153710
## Shibata      1.134231
## Hannan-Quinn 1.141547
```

```
## [1] "eARCH Robust Coefficients"
```

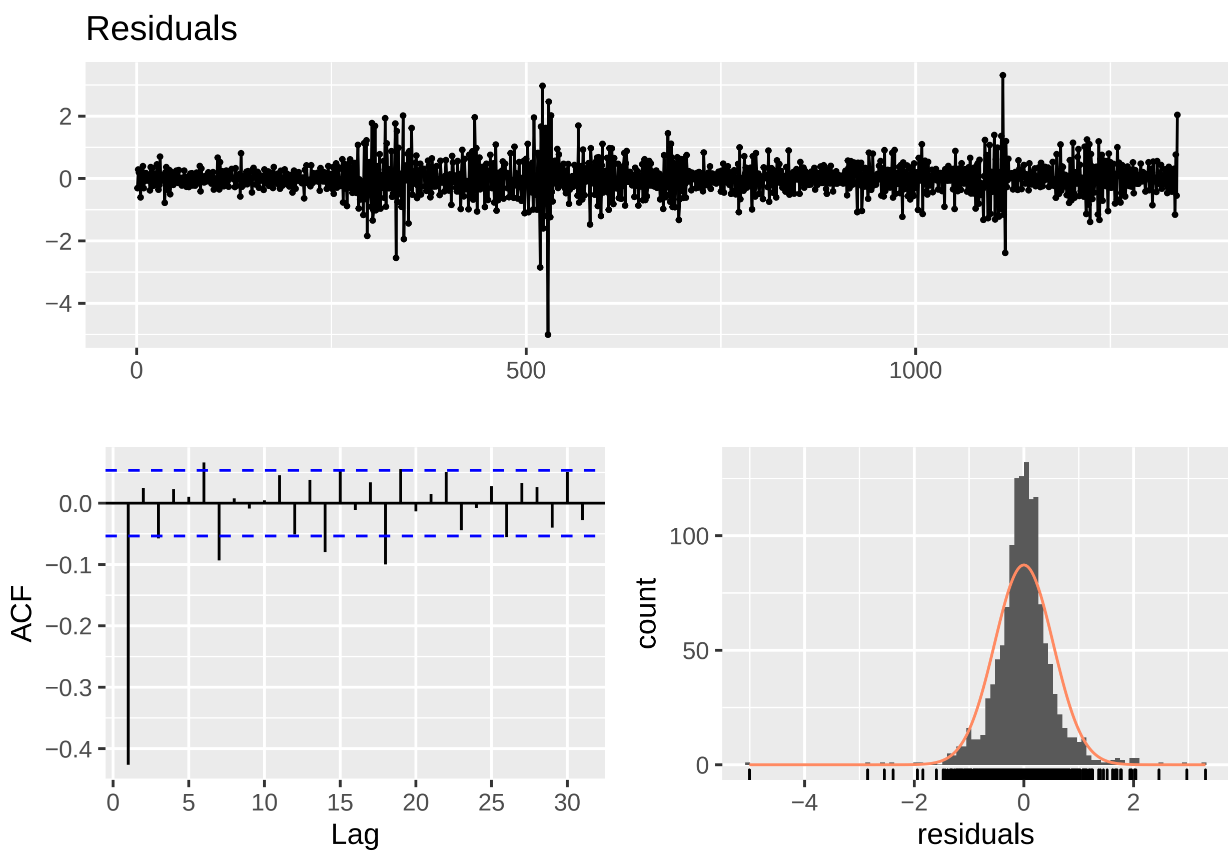
```
##           Estimate  Std. Error   t value     Pr(>|t|)
## omega  -0.05590348  0.02205411 -2.534833 1.125010e-02
## alpha1 -0.10724429  0.03198440 -3.353018 7.993548e-04
## beta1   0.96769557  0.01155956 83.713900 0.000000e+00
## gamma1  0.29346573  0.04288008  6.843870 7.708278e-12
## shape   7.03954078  1.19267348  5.902320 3.584246e-09
```

```
##
## Akaike       1.131547
## Bayes        1.150999
## Shibata      1.131519
## Hannan-Quinn 1.138836
```

Now lets start over and fit an ARMA process to the residuals. I utilize the Box- Jenkins procedure to fit the model. A good specification would have no autocorrelation in the residuals, but autocorrelation in the squared residuals.

(3,2) is the best model specification

```
## Warning in modeldf.default(object): Could not find appropriate degrees of
## freedom for this model.
```




```
##
##  Box-Ljung test
##
## data:  a$residuals
## X-squared = 14.422, df = 10, p-value = 0.1546
```

```
##
##  Box-Ljung test
##
## data:  a$residuals^2
## X-squared = 231.6, df = 10, p-value < 2.2e-16
```

Now let's start over from the beginning with out base GARCH model and see if we should include the unconditional mean. Mu is not significant and does not improve model fit.

```
## [1] "No Unconditional Mean Robust Coefficients"
```

```
##             Estimate  Std. Error    t value     Pr(>|t|)
## ar1      0.048845000 0.250265843  0.1951725 8.452579e-01
## ar2     -0.024525844 0.151302598 -0.1620980 8.712287e-01
## ar3     -0.031736104 0.092996098 -0.3412627 7.329058e-01
## ma1     -0.609073183 0.245632163 -2.4796149 1.315243e-02
## ma2      0.061185595 0.265754454  0.2302336 8.179103e-01
## omega    0.002401725 0.001114749  2.1544980 3.120114e-02
## alpha1   0.137234818 0.020288769  6.7640780 1.341616e-11
## beta1    0.861765182 0.021053889 40.9314013 0.000000e+00
```

```
##
## Akaike       0.9468804
```

```
## Bayes        0.9780028
## Shibata      0.9468093
## Hannan-Quinn 0.9585418
```

```
## [1] "Including Unconditional Mean Robust Coefficients"
```

```
##              Estimate  Std. Error       t value     Pr(>|t|)
## mu     -6.500344e-17 0.004431419 -1.466876e-14 1.000000e+00
## ar1     4.959644e-02 0.262908181  1.886455e-01 8.503707e-01
## ar2    -2.500557e-02 0.156532914 -1.597464e-01 8.730808e-01
## ar3    -3.208637e-02 0.095227657 -3.369438e-01 7.361593e-01
## ma1    -6.101690e-01 0.256676630 -2.377189e+00 1.744513e-02
## ma2     6.260812e-02 0.279523286  2.239818e-01 8.227715e-01
## omega   2.222349e-03 0.001036677  2.143725e+00 3.205496e-02
## alpha1  1.404755e-01 0.021647513  6.489219e+00 8.628231e-11
## beta1   8.620010e-01 0.020984014  4.107894e+01 0.000000e+00
```

```
##
## Akaike       0.9482584
## Bayes        0.9832710
## Shibata      0.9481684
## Hannan-Quinn 0.9613774
```

Including the ARCHM process actually produces a significant coefficient and improves fit! I will come back to this, but for now I leave it out.

```
## [1] "No ARCH in Mean Robust Coefficients"
```

```
##            Estimate  Std. Error    t value     Pr(>|t|)
## ar1     0.048845000 0.250265843  0.1951725 8.452579e-01
## ar2    -0.024525844 0.151302598 -0.1620980 8.712287e-01
## ar3    -0.031736104 0.092996098 -0.3412627 7.329058e-01
## ma1    -0.609073183 0.245632163 -2.4796149 1.315243e-02
## ma2     0.061185595 0.265754454  0.2302336 8.179103e-01
## omega   0.002401725 0.001114749  2.1544980 3.120114e-02
## alpha1  0.137234818 0.020288769  6.7640780 1.341616e-11
## beta1   0.861765182 0.021053889 40.9314013 0.000000e+00
```

```
##
## Akaike       0.9468804
## Bayes        0.9780028
## Shibata      0.9468093
## Hannan-Quinn 0.9585418
```

```
## Warning in arima0(data, order = c(modelinc[2], 0, modelinc[3]), include.mean =
## modelinc[1], : possible convergence problem: optim gave code = 1
```

```
## [1] "ARCH in Mean Robust Coefficients"
```

```
##            Estimate  Std. Error     t value     Pr(>|t|)
## ar1     0.015700695 0.242747286  0.06467918 9.484294e-01
## ar2    -0.014408908 0.155826627 -0.09246756 9.263266e-01
## ar3    -0.025482660 0.093797467 -0.27167749 7.858700e-01
## ma1    -0.577639591 0.238268837 -2.42431868 1.533714e-02
## ma2     0.031679998 0.267213934  0.11855668 9.056266e-01
## archm   0.017496407 0.012217432  1.43208550 1.521194e-01
## omega   0.002429508 0.001116959  2.17510954 2.962191e-02
## alpha1  0.138314808 0.020215020  6.84218026 7.799761e-12
## beta1   0.860685173 0.021142996 40.70781515 0.000000e+00
```

```
##
## Akaike       0.9466442
## Bayes        0.9816569
## Shibata      0.9465543
## Hannan-Quinn 0.9597633
```

Now lets compare normal distribution with student-t. Again we find student-t distribution to have a somewhat better fit.

```
## [1] "Normal Distribution Robust Coefficients"

##             Estimate  Std. Error    t value     Pr(>|t|)
## ar1     0.048845000 0.250265843  0.1951725 8.452579e-01
## ar2    -0.024525844 0.151302598 -0.1620980 8.712287e-01
## ar3    -0.031736104 0.092996098 -0.3412627 7.329058e-01
## ma1    -0.609073183 0.245632163 -2.4796149 1.315243e-02
## ma2     0.061185595 0.265754454  0.2302336 8.179103e-01
## omega   0.002401725 0.001114749  2.1544980 3.120114e-02
## alpha1  0.137234818 0.020288769  6.7640780 1.341616e-11
## beta1   0.861765182 0.021053889 40.9314013 0.000000e+00

##
## Akaike       0.9468804
## Bayes        0.9780028
## Shibata      0.9468093
## Hannan-Quinn 0.9585418

## [1] "Student t Distribution Robust Coefficients"

##             Estimate  Std. Error   t value     Pr(>|t|)
## ar1     0.347626092 0.183602953  1.893358 5.831030e-02
## ar2    -0.132033693 0.099687856 -1.324471 1.853466e-01
## ar3    -0.108257264 0.068392514 -1.582882 1.134484e-01
## ma1    -0.895121659 0.179077155 -4.998525 5.777048e-07
## ma2     0.337121394 0.174471688  1.932241 5.332973e-02
## omega   0.002998159 0.001268104  2.364285 1.806492e-02
## alpha1  0.143659036 0.021499821  6.681871 2.359091e-11
## beta1   0.855340199 0.022978786 37.223038 0.000000e+00
## shape   5.814152394 0.925489863  6.282243 3.337222e-10

##
## Akaike       0.9000732
## Bayes        0.9350858
## Shibata      0.8999832
## Hannan-Quinn 0.9131922
```

Now we fit asymmetric models. Just like the models with an ARMA process, an Asymmetric model has a better fit. The very best model fit based off AIC is Asymmetric Power ARCH (same as WEAT I think)

```
## [1] "Standard GARCH Robust Coefficients"

##             Estimate  Std. Error   t value     Pr(>|t|)
## ar1     0.347626092 0.183602953  1.893358 5.831030e-02
## ar2    -0.132033693 0.099687856 -1.324471 1.853466e-01
## ar3    -0.108257264 0.068392514 -1.582882 1.134484e-01
## ma1    -0.895121659 0.179077155 -4.998525 5.777048e-07
## ma2     0.337121394 0.174471688  1.932241 5.332973e-02
## omega   0.002998159 0.001268104  2.364285 1.806492e-02
## alpha1  0.143659036 0.021499821  6.681871 2.359091e-11
```

```
## beta1   0.855340199 0.022978786 37.223038 0.000000e+00
## shape   5.814152394 0.925489863  6.282243 3.337222e-10
```

```
##
## Akaike       0.9000732
## Bayes        0.9350858
## Shibata      0.8999832
## Hannan-Quinn 0.9131922
```

```
## [1] "Asymmetric Power GARCH (apGARCH) Robust Coefficients"
```

```
##           Estimate  Std. Error     t value     Pr(>|t|)
## ar1     0.36640082 0.043196935    8.482102 0.000000e+00
## ar2    -0.13903280 0.017087964   -8.136300 4.440892e-16
## ar3    -0.08846553 0.011225248   -7.880942 3.330669e-15
## ma1    -0.89290323 0.006352016 -140.570051 0.000000e+00
## ma2     0.33470915 0.029325155   11.413721 0.000000e+00
## omega   0.01286264 0.007141291    1.801165 7.167685e-02
## alpha1  0.10244983 0.029986865    3.416490 6.343398e-04
## beta1   0.90169777 0.020583067   43.807745 0.000000e+00
## gamma1  0.49200971 0.229089551    2.147674 3.173965e-02
## delta   0.60930566 0.431341565    1.412583 1.577784e-01
## shape   6.44830698 1.362546508    4.732541 2.217267e-06
```

```
##
## Akaike       0.8838695
## Bayes        0.9266627
## Shibata      0.8837354
## Hannan-Quinn 0.8999039
```

```
## [1] "GJR GARCH Robust Coefficients"
```

```
##            Estimate  Std. Error   t value     Pr(>|t|)
## ar1     0.333322273  0.20275342  1.643979 1.001806e-01
## ar2    -0.140506565  0.09590837 -1.465008 1.429186e-01
## ar3    -0.110866524  0.06519552 -1.700524 8.903249e-02
## ma1    -0.879716658  0.19826053 -4.437175 9.114726e-06
## ma2     0.334149478  0.17899493  1.866810 6.192816e-02
## omega   0.003517442  0.00141301  2.489326 1.279855e-02
## alpha1  0.089355435  0.02554537  3.497912 4.689163e-04
## beta1   0.851855522  0.02359583 36.101956 0.000000e+00
## gamma1  0.106531882  0.04714834  2.259505 2.385202e-02
## shape   6.025746108  1.01928734  5.911725 3.385441e-09
```

```
##
## Akaike       0.8959139
## Bayes        0.9348168
## Shibata      0.8958029
## Hannan-Quinn 0.9104906
```

```
## [1] "Exponential Power GARCH Robust Coefficients"
```

```
##           Estimate  Std. Error    t value     Pr(>|t|)
## ar1     0.36926774 0.013763794  26.828921 0.000000e+00
## ar2    -0.14451657 0.032777229  -4.409054 1.038232e-05
## ar3    -0.10323048 0.023014657  -4.485423 7.276932e-06
## ma1    -0.90386018 0.020267669 -44.596159 0.000000e+00
## ma2     0.34980837 0.036902158   9.479347 0.000000e+00
```

```
## omega  -0.04887218 0.017026881  -2.870296 4.100882e-03
## alpha1 -0.07262940 0.024918774  -2.914646 3.560925e-03
## beta1   0.97393023 0.008227586 118.373756 0.000000e+00
## gamma1  0.23403151 0.026476213   8.839312 0.000000e+00
## shape   6.20924633 1.172779070   5.294472 1.193606e-07
```

```
##
## Akaike       0.8862982
## Bayes        0.9252011
## Shibata      0.8861872
## Hannan-Quinn 0.9008749
```

Now I want to throw back in ARCH in mean, as we saw that improve model fit.

```
## Warning in arima0(data, order = c(modelinc[2], 0, modelinc[3]), include.mean =
## modelinc[1], : possible convergence problem: optim gave code = 1
```

```
## [1] "Asymmetric Power GARCH (apGARCH) + ARCHM Robust Coefficients"
```

```
##            Estimate  Std. Error    t value     Pr(>|t|)
## ar1     0.361161366 0.026308105  13.728141 0.000000e+00
## ar2    -0.134725638 0.008893386 -15.148970 0.000000e+00
## ar3    -0.085992062 0.007276386 -11.817963 0.000000e+00
## ma1    -0.883024880 0.025577210 -34.523894 0.000000e+00
## ma2     0.323382800 0.018593755  17.392012 0.000000e+00
## archm   0.008815868 0.001299761   6.782681 1.179656e-11
## omega   0.012454437 0.008890163   1.400923 1.612370e-01
## alpha1  0.104286751 0.009401789  11.092224 0.000000e+00
## beta1   0.899999424 0.007651109 117.629933 0.000000e+00
## gamma1  0.462051748 0.141322037   3.269495 1.077395e-03
## delta   0.639299138 0.372861794   1.714574 8.642336e-02
## shape   6.371055232 1.291855850   4.931708 8.151390e-07
```

```
##
## Akaike       0.8849750
## Bayes        0.9316585
## Shibata      0.8848155
## Hannan-Quinn 0.9024670
```

USO Summary

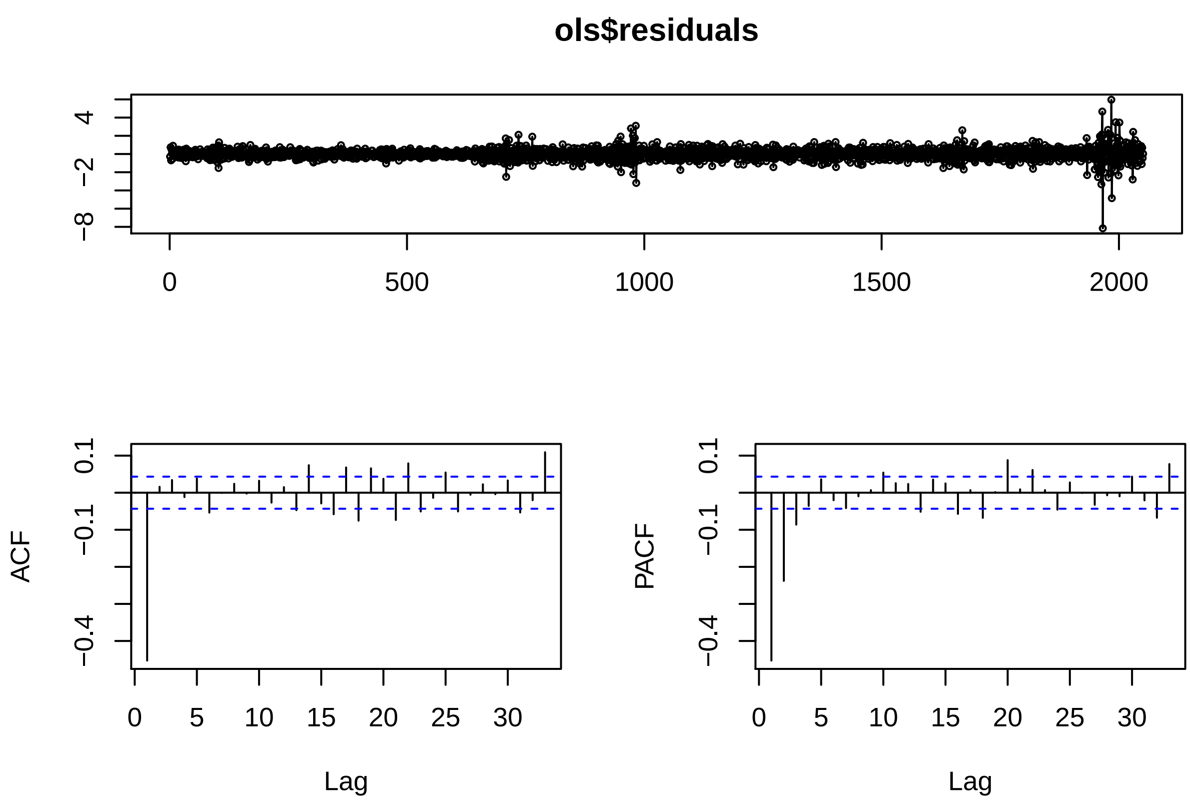
Again I determine to not include ARCH in mean or the unconditional mean in this model specification as it does not improve model fit or produce significant coefficients. A Student-t distribution fits better than a normal distribution. Asymmetric models are not favorable for normal distributions but *are* preferred under a student-t distributions. Fitting an (3,1) ARMA process improves model fit significant. The best fitting model is an Asymmetric Power ARCH with a student t distribution and an ARCH in mean.

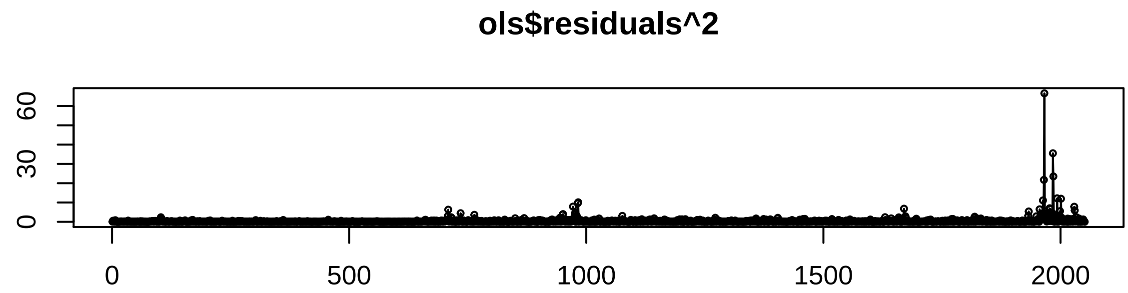
US Gasoline (UGA)

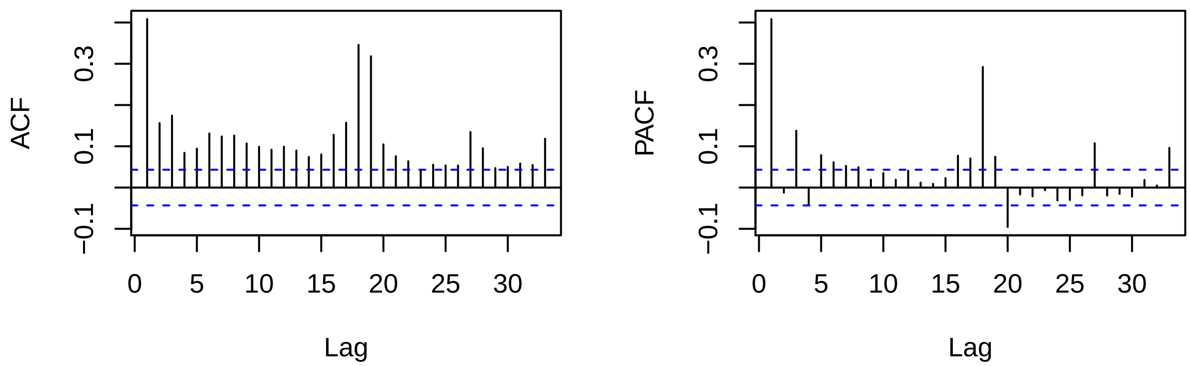
Lets begin by looking at the OLS results. We fail to reject the null hypothesis of $\alpha = 0$ but reject the hypothesis $\beta = 1$. Just like USO, the β coefficient is significantly below 1, inidcating severe disunity.

```
##
## Call:
## lm(formula = x$per_ETF_return ~ x$per_asset_return)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -8.1596 -0.2968  0.0015  0.2916  5.9603
##
## Coefficients:
##                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)        -0.003813   0.013839  -0.276    0.783
## x$per_asset_return  0.887947   0.005651 157.124   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.6265 on 2049 degrees of freedom
## Multiple R-squared:  0.9234, Adjusted R-squared:  0.9233
## F-statistic: 2.469e+04 on 1 and 2049 DF,  p-value: < 2.2e-16
```

There is evidence of ARCH effects in the residuals based off the Ljung-Box Test.








```
##
##  Box-Ljung test
##
## data:  ols$residuals^2
## X-squared = 342.31, df = 1, p-value < 2.2e-16
```

Moving onto GARCH, again including the unconditional mean is unimformative.

```
## [1] "No unconditional mean Robust Coefficients"
```

```
##           Estimate  Std. Error  t value     Pr(>|t|)
## omega  0.01587898  0.01179008 1.346809 1.780419e-01
## alpha1 0.22970300  0.07544087 3.044808 2.328286e-03
## beta1  0.72867000  0.10888954 6.691828 2.204015e-11
```

```
##
## Akaike       1.377805
## Bayes        1.386034
## Shibata      1.377801
## Hannan-Quinn 1.380823
```

```
## [1] "Including Unconditional Mean Robust Coefficients"
```

```
##            Estimate   Std. Error      t value     Pr(>|t|)
## mu     4.702239e-16 0.0056621731 8.304655e-14 1.000000e+00
## omega  9.529938e-04 0.0007686582 1.239815e+00 2.150439e-01
## alpha1 1.012113e-01 0.0206033862 4.912362e+00 8.998570e-07
## beta1  8.948280e-01 0.0233531843 3.831717e+01 0.000000e+00
```

```
##
```

```
## Akaike       1.395052
## Bayes        1.406025
## Shibata      1.395045
## Hannan-Quinn 1.399076
```

The ARCH in mean coefficient is not statistically significant at the 5% level, and including it does not help model fit.

```
## [1] "ARCH in Mean Robust Coefficients"
```

```
##              Estimate  Std. Error       t value     Pr(>|t|)
## mu     -4.702241e-16  0.01920508 -2.448437e-14 1.000000e+00
## archm   1.487075e-02  0.04654492  3.194923e-01 7.493532e-01
## omega   1.604002e-02  0.01178037  1.361589e+00 1.733275e-01
## alpha1  2.309802e-01  0.07479314  3.088253e+00 2.013368e-03
## beta1   7.269084e-01  0.10831435  6.711100e+00 1.931633e-11
```

```
##
## Akaike       1.379488
## Bayes        1.393203
## Shibata      1.379476
## Hannan-Quinn 1.384517
```

No reason to move to an asymmetric model.

```
## [1] "Standard GARCH Robust Coefficients"
```

```
##           Estimate  Std. Error  t value     Pr(>|t|)
## omega  0.01587898  0.01179008 1.346809 1.780419e-01
## alpha1 0.22970300  0.07544087 3.044808 2.328286e-03
## beta1  0.72867000  0.10888954 6.691828 2.204015e-11
```

```
##
## Akaike       1.377805
## Bayes        1.386034
## Shibata      1.377801
## Hannan-Quinn 1.380823
```

```
## [1] "Asymmetric Power ARCH (apARCH) Robust Coefficients"
```

```
##              Estimate  Std. Error       t value     Pr(>|t|)
## mu     -4.702240e-16 0.006707783 -7.010125e-14 1.000000e+00
## omega   4.162271e-03 0.001795785  2.317801e+00 2.046013e-02
## alpha1  1.214449e-01 0.030321737  4.005210e+00 6.196241e-05
## beta1   8.810808e-01 0.074055218  1.189762e+01 0.000000e+00
## gamma1  1.561388e-01 0.197985285  7.886385e-01 4.303233e-01
## delta   1.670407e+00 1.375171029  1.214690e+00 2.244842e-01
```

```
##
## Akaike       1.382204
## Bayes        1.398663
## Shibata      1.382187
## Hannan-Quinn 1.388240
```

```
## [1] "GJR ARCH Robust Coefficients"
```

```
##             Estimate  Std. Error      t value   Pr(>|t|)
## mu     4.702241e-16 0.012810032 3.670749e-14 1.00000000
## omega  4.259009e-04 0.001015367 4.194550e-01 0.67488363
## alpha1 5.572308e-02 0.029043689 1.918595e+00 0.05503557
```

```
## beta1  8.916176e-01 0.051766323 1.722389e+01 0.00000000
## gamma1 7.596485e-02 0.032663886 2.325653e+00 0.02003709
```

```
##
## Akaike       1.428512
## Bayes        1.442228
## Shibata      1.428501
## Hannan-Quinn 1.433542
```

```
## [1] "eARCH Robust Coefficients"
```

```
##             Estimate  Std. Error       t value     Pr(>|t|)
## mu     -4.702216e-16  0.00812111 -5.790116e-14 1.000000e+00
## omega  -9.815418e-02  0.19614853 -5.004074e-01 6.167882e-01
## alpha1 -4.513363e-02  0.03234483 -1.395390e+00 1.628984e-01
## beta1   9.263448e-01  0.13960519  6.635461e+00 3.234901e-11
## gamma1  4.201902e-01  0.30437132  1.380518e+00 1.674271e-01
```

```
##
## Akaike       1.381200
## Bayes        1.394916
## Shibata      1.381188
## Hannan-Quinn 1.386230
```

Again, I will replace the normal distribution with the student t distribution. Model fit improves extremely slightly and suddenly the asymmetric models have a better fit. I will come back to this.

```
## [1] "Standard GARCH Robust Coefficients"
```

```
##           Estimate  Std. Error  t value     Pr(>|t|)
## omega   0.01909105  0.01247659 1.530149 1.259797e-01
## alpha1  0.24988907  0.07515305 3.325069 8.839666e-04
## beta1   0.69886024  0.10917578 6.401239 1.541212e-10
## shape  10.17030176  1.92371868 5.286793 1.244796e-07
```

```
##
## Akaike       1.362240
## Bayes        1.373213
## Shibata      1.362233
## Hannan-Quinn 1.366264
```

```
## [1] "Asymmetric Power ARCH (apARCH) Robust Coefficients"
```

```
##           Estimate  Std. Error  t value     Pr(>|t|)
## omega   0.02036949  0.01674464 1.216479 2.238026e-01
## alpha1  0.22220631  0.12557313 1.769537 7.680429e-02
## beta1   0.75459876  0.18997833 3.972025 7.126413e-05
## gamma1  0.13246652  0.11425898 1.159353 2.463122e-01
## delta   1.59363758  0.57944880 2.750265 5.954715e-03
## shape  10.36800708  2.02341346 5.124018 2.990924e-07
```

```
##
## Akaike       1.361809
## Bayes        1.378268
## Shibata      1.361792
## Hannan-Quinn 1.367845
```

```
## [1] "GJR ARCH Robust Coefficients"
```

```
##           Estimate  Std. Error  t value     Pr(>|t|)
```

```
## omega   0.01898404  0.01340017 1.416701 1.565704e-01
## alpha1  0.19913962  0.07734881 2.574566 1.003660e-02
## beta1   0.69973590  0.11945190 5.857889 4.687889e-09
## gamma1  0.10182018  0.05153095 1.975904 4.816570e-02
## shape  10.12871048  1.88284115 5.379482 7.470037e-08
```

```
##
## Akaike       1.361385
## Bayes        1.375100
## Shibata      1.361373
## Hannan-Quinn 1.366414
```

```
## [1] "eARCH Robust Coefficients"
```

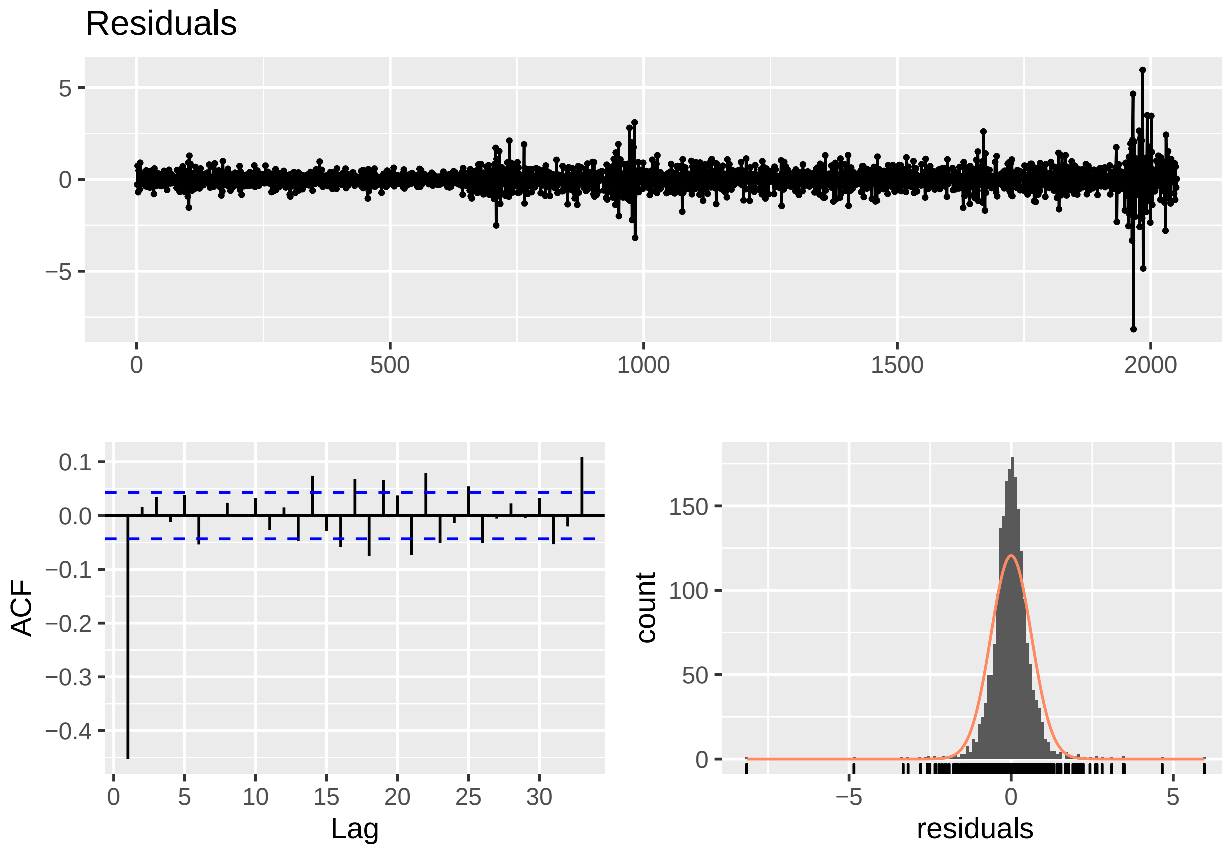
```
##           Estimate  Std. Error    t value     Pr(>|t|)
## omega  -0.05338054  0.05808318 -0.9190361 3.580767e-01
## alpha1 -0.05722820  0.02780446 -2.0582379 3.956730e-02
## beta1   0.96401908  0.03939239 24.4722147 0.000000e+00
## gamma1  0.31124850  0.15720967  1.9798305 4.772258e-02
## shape  10.27629130  1.88309331  5.4571333 4.838830e-08
```

```
##
## Akaike       1.362439
## Bayes        1.376154
## Shibata      1.362427
## Hannan-Quinn 1.367468
```

Now lets start over and fit an ARMA process to the residuals. I utilize the Box- Jenkins procedure to fit the model. A good specification would have no autocorrelation in the residuals, but autocorrelation in the squared residuals.

(2,1) is the best model specification

```
## Warning in modeldf.default(object): Could not find appropriate degrees of
## freedom for this model.
```




```
##
##  Box-Ljung test
##
## data:  a$residuals
## X-squared = 16.141, df = 10, p-value = 0.09567
```

```
##
##  Box-Ljung test
##
## data:  a$residuals^2
## X-squared = 871, df = 10, p-value < 2.2e-16
```

Now let's start over from the beginning with out base GARCH model and see if we should include the unconditional mean. It does not improve fit or produce significant estimates. I will leave it out.

```
## [1] "No Unconditional Mean Robust Coefficients"
```

```
##             Estimate  Std. Error    t value     Pr(>|t|)
## ar1    -0.066254300  0.09817553 -0.6748555 4.997676e-01
## ar2     0.011248619  0.05200512  0.2162983 8.287552e-01
## ma1    -0.446516320  0.09633331 -4.6351188 3.567326e-06
## omega   0.002578852  0.00146366  1.7619208 7.808268e-02
## alpha1  0.096280845  0.02143911  4.4908969 7.092387e-06
## beta1   0.897389804  0.02299112 39.0320156 0.000000e+00
```

```
##
## Akaike       1.178553
## Bayes        1.195011
## Shibata      1.178536
```

```
## Hannan-Quinn 1.184588
```

```
## [1] "Including Unconditional Mean Robust Coefficients"
```

```
##             Estimate  Std. Error       t value     Pr(>|t|)
## mu     -4.702241e-16 0.004429388 -1.061601e-13 1.000000e+00
## ar1    -6.623382e-02 0.098586365 -6.718355e-01 5.016884e-01
## ar2     1.125850e-02 0.052041229  2.163382e-01 8.287242e-01
## ma1    -4.465389e-01 0.097139480 -4.596884e+00 4.288565e-06
## omega   2.578858e-03 0.001467318  1.757532e+00 7.882715e-02
## alpha1  9.628054e-02 0.021564484  4.464773e+00 8.015351e-06
## beta1   8.973899e-01 0.023166751  3.873611e+01 0.000000e+00
```

```
##
## Akaike       1.179528
## Bayes        1.198729
## Shibata      1.179505
## Hannan-Quinn 1.186569
```

Including the ARCHM process does not produce a better fit but does produce a singificant estimate. I will come back.

```
## [1] "No ARCH in Mean Robust Coefficients"
```

```
##            Estimate  Std. Error    t value     Pr(>|t|)
## ar1    -0.066254300  0.09817553 -0.6748555 4.997676e-01
## ar2     0.011248619  0.05200512  0.2162983 8.287552e-01
## ma1    -0.446516320  0.09633331 -4.6351188 3.567326e-06
## omega   0.002578852  0.00146366  1.7619208 7.808268e-02
## alpha1  0.096280845  0.02143911  4.4908969 7.092387e-06
## beta1   0.897389804  0.02299112 39.0320156 0.000000e+00
```

```
##
## Akaike       1.178553
## Bayes        1.195011
## Shibata      1.178536
## Hannan-Quinn 1.184588
```

```
## [1] "ARCH in Mean Robust Coefficients"
```

```
##            Estimate  Std. Error    t value     Pr(>|t|)
## ar1    -0.061410722 0.097955851 -0.6269224 5.307101e-01
## ar2     0.013426593 0.051863623  0.2588827 7.957258e-01
## ma1    -0.452368919 0.096305855 -4.6972110 2.637380e-06
## archm   0.013594062 0.011080711  1.2268222 2.198894e-01
## omega   0.002626971 0.001497754  1.7539399 7.944080e-02
## alpha1  0.097210326 0.021736756  4.4721634 7.743224e-06
## beta1   0.896281786 0.023518310 38.1099569 0.000000e+00
```

```
##
## Akaike       1.178800
## Bayes        1.198001
## Shibata      1.178776
## Hannan-Quinn 1.185841
```

Now lets compare normal distribution with student-t. Again we find student-t distribution to have a better fit.

```
## [1] "Normal Distribution Robust Coefficients"
```

```
##             Estimate  Std. Error    t value     Pr(>|t|)
## ar1    -0.066254300  0.09817553 -0.6748555 4.997676e-01
## ar2     0.011248619  0.05200512  0.2162983 8.287552e-01
## ma1    -0.446516320  0.09633331 -4.6351188 3.567326e-06
## omega   0.002578852  0.00146366  1.7619208 7.808268e-02
## alpha1  0.096280845  0.02143911  4.4908969 7.092387e-06
## beta1   0.897389804  0.02299112 39.0320156 0.000000e+00
```

```
##
## Akaike       1.178553
## Bayes        1.195011
## Shibata      1.178536
## Hannan-Quinn 1.184588
```

```
## [1] "Student t Distribution Robust Coefficients"
```

```
##             Estimate  Std. Error   t value     Pr(>|t|)
## ar1    -0.124180281 0.094976548 -1.307484 1.910485e-01
## ar2    -0.014477019 0.049944356 -0.289863 7.719211e-01
## ma1    -0.391306800 0.092263860 -4.241171 2.223567e-05
## omega   0.003459768 0.001777905  1.945981 5.165703e-02
## alpha1  0.088730994 0.020260038  4.379607 1.188938e-05
## beta1   0.898428579 0.024074194 37.319155 0.000000e+00
## shape   8.106519226 1.192398667  6.798497 1.057154e-11
```

```
##
## Akaike       1.151593
## Bayes        1.170794
## Shibata      1.151569
## Hannan-Quinn 1.158634
```

Now we fit asymmetric models and find that just like the non ARMA models, asymmetrics have a better fit.

```
## [1] "Standard GARCH Robust Coefficients"
```

```
##             Estimate  Std. Error   t value     Pr(>|t|)
## ar1    -0.124180281 0.094976548 -1.307484 1.910485e-01
## ar2    -0.014477019 0.049944356 -0.289863 7.719211e-01
## ma1    -0.391306800 0.092263860 -4.241171 2.223567e-05
## omega   0.003459768 0.001777905  1.945981 5.165703e-02
## alpha1  0.088730994 0.020260038  4.379607 1.188938e-05
## beta1   0.898428579 0.024074194 37.319155 0.000000e+00
## shape   8.106519226 1.192398667  6.798497 1.057154e-11
```

```
##
## Akaike       1.151593
## Bayes        1.170794
## Shibata      1.151569
## Hannan-Quinn 1.158634
```

```
## [1] "Asymmetric Power GARCH (apGARCH) Robust Coefficients"
```

```
##             Estimate  Std. Error    t value     Pr(>|t|)
## ar1    -0.136278845 0.086296779 -1.5791881 1.142929e-01
## ar2    -0.026522178 0.044461857 -0.5965153 5.508310e-01
## ma1    -0.375478770 0.084507835 -4.4431238 8.866212e-06
## omega   0.004512856 0.002200279  2.0510378 4.026326e-02
## alpha1  0.080489166 0.021265232  3.7850124 1.537009e-04
## beta1   0.919254222 0.026235587 35.0384463 0.000000e+00
```

```
## gamma1  0.273778124 0.118886813  2.3028469 2.128746e-02
## delta   1.419438635 0.334988787  4.2372721 2.262519e-05
## shape   8.854861297 1.431519443  6.1856382 6.185175e-10
```

```
##
## Akaike       1.147015
## Bayes        1.171703
## Shibata      1.146977
## Hannan-Quinn 1.156069
```

```
## [1] "GJR GARCH Robust Coefficients"
```

```
##            Estimate  Std. Error    t value     Pr(>|t|)
## ar1    -0.123983613 0.092563421 -1.3394450 1.804258e-01
## ar2    -0.018353095 0.048112157 -0.3814648 7.028584e-01
## ma1    -0.389636469 0.089687932 -4.3443578 1.396837e-05
## omega   0.003194118 0.001697642  1.8815024 5.990361e-02
## alpha1  0.047526765 0.018065058  2.6308670 8.516735e-03
## beta1   0.908100509 0.024549292 36.9909046 0.000000e+00
## gamma1  0.062815328 0.021645562  2.9019957 3.707937e-03
## shape   8.545067830 1.306427119  6.5407918 6.119394e-11
```

```
##
## Akaike       1.147690
## Bayes        1.169634
## Shibata      1.147659
## Hannan-Quinn 1.155737
```

```
## [1] "Exponential Power GARCH Robust Coefficients"
```

```
##           Estimate  Std. Error     t value     Pr(>|t|)
## ar1    -0.14569112 0.108330272  -1.3448791 1.786643e-01
## ar2    -0.03386529 0.056422263  -0.6002114 5.483653e-01
## ma1    -0.36653685 0.107538320  -3.4084301 6.533783e-04
## omega  -0.02173584 0.009459416  -2.2977998 2.157319e-02
## alpha1 -0.04831510 0.014140442  -3.4168026 6.336120e-04
## beta1   0.98712802 0.005202488 189.7415092 0.000000e+00
## gamma1  0.15528439 0.031296418   4.9617306 6.986786e-07
## shape   8.74668021 1.350527246   6.4764930 9.387890e-11
```

```
##
## Akaike       1.146709
## Bayes        1.168654
## Shibata      1.146679
## Hannan-Quinn 1.154756
```

As Exponential GARCH has the best fit, I will try adding back in ARCHM like I did with USO. It does not produce a better fit.

```
## [1] "Exponential Power GARCH Robust Coefficients"
```

```
##            Estimate  Std. Error     t value     Pr(>|t|)
## ar1    -0.143144979 0.023782729  -6.0188627 1.756468e-09
## ar2    -0.032797285 0.020498268  -1.6000028 1.095980e-01
## ma1    -0.368950661 0.024720064 -14.9251500 0.000000e+00
## archm   0.003758879 0.012846320   0.2926036 7.698252e-01
## omega  -0.022090251 0.009650728  -2.2889725 2.208095e-02
## alpha1 -0.047468552 0.014715996  -3.2256432 1.256899e-03
## beta1   0.987107359 0.005210536 189.4445018 0.000000e+00
```

```
## gamma1  0.155520155 0.031229775   4.9798679 6.362769e-07
## shape   8.711070181 1.345577300   6.4738534 9.553469e-11
```

```
##
## Akaike       1.147630
## Bayes        1.172318
## Shibata      1.147592
## Hannan-Quinn 1.156684
```

UGA Summary

UGA results are very similiar to USO both with and without an ARMA process. Asymmetric models performed better, student t distributions have a better fit, and even ARCH-M produces significant coefficients *but* does not produce a better fit as in USO. The best fit is has a (2,1) ARMA process, is a student t distribution, and is an Exponential Power GARCH.

AIC Tables

Table 1: Model Results (no ARMA Process)

models	CORN	SOYB	WEAT	USO	UGA
Simple	-0.1425799	0.5464972	0.9438352	1.171617	1.377805
Unconditional Mean	-0.0962713	0.6237427	0.9675119	1.220585	1.395052
ARCH-M	-0.1409083	0.5481770	0.9456991	1.174194	1.379488
ApARCH	-0.1236779	0.5780570	0.9673960	1.232481	1.382204
GJR ARCH	-0.0379143	0.6941300	1.0079930	1.266789	1.428512
eARCH	-0.1386562	0.5612623	0.9855407	1.196384	1.381200
t-dist Simple	-0.1802274	0.3702626	0.9325403	1.138615	1.362240
t-dist apARCH	-0.1063333	0.3687614	0.9337804	1.132893	1.361809
t-dist GJR ARCH	-0.0555181	0.3690870	0.9328237	1.134259	1.361385
t-dist eARCH	-0.1648695	0.3733829	0.9352613	1.131547	1.362439

Table 2: Model Results (ARMA Process)

models	CORN(0,1)	SOYB(2,3)	WEAT(1,5)	USO(3,1)	UGA(2,1)
Simple	-0.4576778	0.2120471	0.5916290	0.9468804	1.178553
Unconditional Mean	-0.4566999	0.2033389	0.5926027	0.9482584	1.179528
ARCH-M	-0.4568514	NA	NA	0.9466442	1.178800
ApARCH	-0.4572424	0.2121736	0.5604287	0.8838695	1.147015
GJR ARCH	-0.4578476	0.2199670	0.5604464	0.8959139	1.147690
eARCH	-0.4581073	0.2289286	0.5623342	0.8862982	1.146709
t-dist Simple	-0.5287184	-0.0146894	0.5594783	0.9000732	1.151593
t-dist apARCH	-0.5271518	-0.0248902	0.5604287	0.8838695	1.147015
t-dist GJR ARCH	-0.5278135	-0.0237882	0.5604464	0.8959139	1.147690
t-dist eARCH	-0.5273388	-0.0281587	0.5623342	0.8862982	1.146709