



Algorithms: Design  
and Analysis, Part II

# Introduction

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Motivating Application:  
Sequence Alignment

# Motivation

**Sequence alignment:** Fundamental problem in computational genomics.

**Input:** Two strings over the alphabet  $\{A,C,G,T\}$ . [Portions of one or more genomes]

**Problem:** Figure out how similar the two strings are.

Example:

A	G	G	G	C	T
A	G	G	C	A	

**Example applications:**

- Extrapolate function of genome substrings.
- Similar genomes can reflect proximity in evolutionary tree.

# Measuring Similarity

**Question:** What does similar mean?

**Intuition:** AGGGCT, AGGCA are similar because they can be “nicely aligned”.

**Idea:** Measure similarity via quality of “best” alignment.

**Assumption:** Have experimentally determined *penalties* for gaps and the possible matches.

**Example:**

A	G	G	G	C	T
A	G	G	C	A	

**Alignment:**

A	G	G	G	C	T
A	G	G	-	C	A

insert one “gap”

one mismatch

# Problem Statement

**Input:** 2 strings over  $\{A,C,G,T\}$ .

- Penalty  $pen_{gap} \geq 0$  for each gap.
- Penalty  $pen_{AT} \geq 0$  for mismatching A and T.
- etc.

**Output:** Alignment of the strings that minimizes the total penalty  
 $\Rightarrow$  Called the Needleman-Wunsch score [1970].

Small NW score  $\approx$  Similar Strings

# Algorithms are Fundamental

**Note:** This measure of genome similarity would be useless without an efficient algorithm to find the best alignment.

**Brute-force search:** Try all possible alignments, remember the best one.

**Question:** Suppose each string has length 500. Roughly how many possible alignments are there?

- A) # of students in this class  $\approx 10^4 - 10^5$
- B) # of people on earth  $\approx 10^9 - 10^{10}$
- C) # of atoms in known universe  $\approx 10^{80}$
- D) More than any of the above  $\geq 2^{500} \geq 10^{125}$

**Point:** Need a fast, clever algorithm.

**Solution:** Straightforward dynamic programming.



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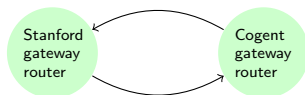
# Introduction

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Motivating Application:  
Distributed Shortest-  
Path Routing

# Graphs and the Internet

**Claim:** The Internet is a graph [vertices = end hosts + routers, directed edges = direct physical or wireless connections].



Other graphs related to the Internet:

Web graph. [vertices = web pages, edges = hyperlinks].



Social networks. [vertices = people, edges = friend/follow relationships].

# Internet Routing

**Suppose:** Stanford gateway router needs to send data to the Cornell gateway router (over multiple hops).

**Question:** Which Stanford→Cornell route to use?

**Obvious idea:** How about the shortest? (e.g. fewest # of hops).

⇒ Need a shortest-path algorithm.

**Recall from Part I:** Dijkstra's algorithm does this (with nonnegative edge lengths).

**Issue:** Stanford gateway router would need to know entire Internet!

⇒ Need a shortest-path algorithm that uses only *local* computation.

**Solution:** the *Bellman-Ford* algorithm (bonus: also handles negative edge costs).