

1 Homework 3

1.1 Boolean Algebra

1. Simplify the following expressions using Boolean algebraic laws. Give each step of your simplification and denote which laws you're using for each step. Do not skip or combine steps!

(a) $A * (\bar{A} + B * B) + (\overline{B + A}) * (\bar{A} + B)$

Work:

$$A * (\bar{A} + B) + (\overline{B + A}) * (\bar{A} + B) // \text{Idempotent law}$$

$$A * B + (\overline{B + A}) * (\bar{A} + B) // \text{Redundancy law}$$

$$A * B + \bar{B} * \bar{A} * (\bar{A} + B) // \text{Demorgan's law}$$

$$\boxed{\text{Answer: } A * B + \bar{B} * \bar{A}}$$

(b) $\overline{C * B} + (A * B * C) + \overline{A + B + \bar{B}}$

Work:

$$\overline{C} + \bar{B} + (A * B * C) + \overline{A + C + \bar{B}} // \text{Demorgan's law}$$

$$\overline{C} + \bar{B} + (A * B * C) + \bar{A} * \bar{C} * B // \text{Involution Law}$$

$$\overline{C} + \bar{B} + A * B * C // \text{Absorption Law}$$

$$\overline{C} + \bar{B} + A * B // \text{Absorption Law}$$

$$\overline{C} + \bar{B} + A // \text{Absorption Law}$$

$$\boxed{\text{Answer: } \overline{C} + \bar{B} + A}$$

(c) $(A + B) * (\bar{A} + C) * (\bar{C} + B)$

Work:

$$(\bar{A} + C) * (\bar{C} + B) * A + (\bar{A} + C) * (\bar{C} + B) * B // \text{Distributive Law}$$

$$(\bar{C} + B) * A * \bar{A} + (\bar{C} + B) * A * C + (\bar{A} + C) * (\bar{C} + B) * B // \text{Distributive Law}$$

$$0 + (\bar{C} + B) * A * C + (\bar{A} + C) * (\bar{C} + B) * B // \text{Complement Law}$$

$$(\bar{C} + B) * A * C + (\bar{A} + C) * (\bar{C} + B) * B // \text{Identity Law}$$

$$A * A * \bar{C} + A * C * B + (\bar{A} + C) * (\bar{C} + B) * B // \text{Distributive Law}$$

$$0 + A * C * B + (\bar{A} + C) * (\bar{C} + B) * B // \text{Complement Law}$$

$$A * C * B + (\bar{A} + C) * (\bar{C} + B) * B // \text{Identity Law}$$

$$A * C * B + B * \bar{A} * \bar{C} + B * \bar{A} * B + (\bar{C} + B) * B * C // \text{Distributive Law}$$

$$A * C * B + B * \bar{A} + (\bar{C} + B) * B * C // \text{Absorption Law}$$

$$B * (A * C + \bar{A}) + (\bar{C} + B) * B * C // \text{Distributive Law}$$

$$B * (C + \bar{A}) + (\bar{C} + B) * B * C // \text{Absorption Law}$$

$$B * C + B * \bar{A} + (\bar{C} + B) * B * C // \text{Distributive Law}$$

$$B * C + B * \bar{A} // \text{Absorption Law}$$

$$\boxed{\text{Answer: } B * C + B * \bar{A}}$$

2. Find all solutions of the following Boolean equations without using the truth tables:

(a) $(\bar{A} + C) * (\bar{B} + D + A) * (D + A * \bar{C}) * (\bar{D} + A) = 1$

Work:

(b) $((\bar{K} * L * N) * (L * M)) + ((\bar{K} + L + N) * (K * \bar{L} * \bar{M})) * (\bar{K} + \bar{N}) = 1$

Work:

3. Simplify the following expression by first constructing a truth table, using that truth table to construct a K-map, and then using that K-map to simplify.

$$Q = \overline{X} * \overline{Y} * Z + X * Y * \overline{Z} + \overline{X} + Y * \overline{Z} + X * \overline{Y} * \overline{Z}$$

Work:

1.2 Logical Circuits

1. Convert the following truth table into its sum of products representation: