Brain Inspired Computing - Problem Set 1

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Exercise 1

a) Let $A := F_u + G_w$, $B := F_u G_w - F_w G_u$ and $C := (F_u + G_w)^2 - 4(F_u G_w - F_w G_u)$. Assume $A > 0$. From the eigenvalue equation

$$\begin{aligned}\lambda_\pm &= \frac{1}{2}(F_u + G_w \pm \sqrt{(F_u + G_w)^2 - 4(F_u G_w - F_w G_u)}) \\ &= \frac{1}{2}(A \pm \sqrt{A^2 - 4B})\end{aligned}$$

we derive that if $C \geq 0$, $\lambda_+ > 0$ as well. If $C < 0$, $\lambda_\pm$ can be expressed through $\lambda_\pm = r \pm i * \omega$ with $r = A/2 > 0$. However, any positive λ leads to growth, which means that the model is not stable under these conditions. In the case of $C > 0$, we have a saddle or an unstable situation, depending on whether λ_- is positive or negative; in the case of $C < 0$ we have a growing spiral. Therefore, $A < 0$ must hold true. In the case of $C < 0$, this is enough, as any solution that satisfies $A < 0$ will lead to $\lambda_\pm = r \pm i * \omega$ with $r < 0$. However, for $C > 0$, we need to make sure that $\lambda_+ < 0$ in any case. To satisfy this condition, $\sqrt{C} < abs(A) = \sqrt{A^2}$ must hold true. Expansion of C leads to $\sqrt{A^2 - 4B} < \sqrt{A^2}$. From this we can easily see that $F_u G_w - F_w G_u = B > 0$ must hold true.

b) The fixpoints are $(-\frac{3}{2}, -\frac{3}{8})$ and $(0, \frac{15}{8})$, respectively. For the case of $I = 0$ we can simply calculate the conditions given above. We have

$$\begin{aligned}F_u &= 1 - u^2 \\ F_w &= -1 \\ G_u &= \epsilon * b \\ G_w &= -\epsilon * w\end{aligned}$$

which gives us

$$\begin{aligned}F_u + G_w &= 1 - u^2 - \epsilon * w \\ &= 1 - \frac{9}{4} + 0.1 * \frac{3}{8} \\ &= \frac{8 - 18 + 0.3}{8} \\ &= -\frac{9.7}{8} < 0\end{aligned}$$

and

$$\begin{aligned}F_u * G_w - F_w * G_u &= (1 - u^2) * (-\epsilon * w) - (-1 * \epsilon * b) \\ &= \epsilon(-\frac{5}{8} * \frac{3}{8} + \frac{3}{2}) \\ &= \epsilon(\frac{3}{2} - \frac{15}{64}) > 0\end{aligned}$$

Therefore the fixpoint for $I = 0$ is stable.
The nullcline of w is given through $G = 0 \Leftrightarrow w = a + b * u$. For $I = 15/8$ lets look at a point on the nullcline of w at coordinates $(u_0 + \delta u, a + b * (u_0 + \delta u))$ with $\delta u > 0, \delta u \ll 1$. As $u_0 = 0$, we get

$$\begin{aligned} F &= \delta u - \frac{\delta u^3}{3} - \frac{15}{8} + \delta u * \frac{3}{2} + \frac{15}{8} \\ &\approx \frac{5}{2}\delta u > 0 \end{aligned}$$

This means that the arrows point away from the fixpoint (at least on the nullcline) and the fixpoint is therefore not stable.

Exercise 2