Brain Inspired Computing (SS 19): Sheet 5

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Exercise 1

a) Assume that N is the number of spikes generated by a Poisson process in the interval $[0, T]$, then $p_T(N = 1) = \nu T e^{-\nu T}$ is the probability for exactly one spike to occur during this time, and $p_{\text{ISI}}(t_1) = \nu e^{-\nu t_1}$ is the probability this spike occuring at t_1. Using this and Bayes theorem we obtain

$$\begin{aligned} p(t_1|N = 1) &= \frac{p(N = 1|t_1)p_{\text{ISI}}(t_1)}{p_T(N = 1)} \\ &= \frac{(p_{T-t_1}(N = 0)) \cdot \nu e^{-\nu t_1}}{\nu T e^{-\nu T}} \\ &= \frac{e^{-\nu(T-t_1)} \cdot e^{\nu(T-t_1)}}{T} = \frac{1}{T}. \end{aligned}$$

b) Given a Poisson process that generates N spikes over an interval T with the ISI distribution $p_{\text{ISI}}(s) = \nu e^{-\nu s}$, the mean of the ISI distribution is

$$\int_0^\infty \nu s e^{-\nu s} ds = \nu^{-1}.$$

Therefore, we have on average $T/\nu^{-1} = \nu T$ spikes over an interval of T. Hence, we have $E[N] = \nu T$ and by virtue of the Poisson distribution $p_T(N) = \frac{(\nu T)^N}{N!} e^{-\nu T}$, i.e., ν is the rate of the Poisson process.

c) Given two random variables N_1 and N_2, that are both Poisson distributed with $p_T(N_1; \nu_1)$

and $p_T(N_2;\nu_2)$, the sum of these variables $N = N_1 + N_2$ is distributed according to

$$
\begin{aligned}
p_T(N=j,\nu) &= \sum_{k=0}^{j} p_T(k;\nu_1)p_T(j-k;\nu_2)\\
&= \sum_{k=0}^{j} \frac{(\nu_1 T)^k}{k!}e^{-\nu_1 T}\frac{(\nu_2 T)^{j-k}}{(j-k)!}e^{-\nu_2 T}\\
&= e^{-(\nu_1+\nu_2)T}\sum_{k=0}^{j}\frac{1}{j!}\frac{j!}{k!(j-k)!}(\nu_1 T)^k(\nu_2 T)^{j-k}\\
&= \frac{(\nu_1+\nu_2 T)^j}{j!}e^{-(\nu_1+\nu_2)T}
\end{aligned}
$$

and we see that $\nu = \nu_1 + \nu_2$, for the last step we used the Binomial theorem, $k, j \in \mathbb{N}$.

d) For the poisson process we have the mean given by

$$
\begin{aligned}
\mathrm{E}[N] &= \sum_{N=0}^{\infty} N\frac{(\nu T)^N}{N!}e^{-\nu T}\\
&= \sum_{N=1}^{\infty} N\frac{(\nu T)^N}{N!}e^{-\nu T}\\
&= \sum_{N=1}^{\infty} \nu T\frac{(\nu T)^{N-1}}{(N-1)!}e^{-\nu T}\\
&= \nu T e^{-\nu T}\sum_{N=0}^{\infty}\frac{(\nu T)^N}{N!}\\
&= \nu T e^{-\nu T}e^{\nu T}\\
&= \nu T.
\end{aligned}
$$

The variance can be derived using $\mathrm{Var}[N] = \mathrm{E}[N^2] - \mathrm{E}[N]^2$, we have

$$
\begin{aligned}
\mathrm{E}[N^2] &= \sum_{N=0}^{\infty} N^2\frac{(\nu T)^N}{N!}e^{-\nu T}\\
&= \sum_{N=1}^{\infty} N^2\frac{(\nu T)^N}{N!}e^{-\nu T}\\
&= \sum_{N=1}^{\infty} \nu T N\frac{(\nu T)^{N-1}}{(N-1)!}e^{-\nu T}\\
&= \nu T e^{-\nu T}\left(\left(\sum_{N=0}^{\infty}\frac{(\nu T)^N}{N!}\right) + \nu T\left(\sum_{N=0}^{\infty}\frac{(\nu T)^N}{N!}\right)\right)\\
&= \nu T e^{-\nu T}\left(e^{\nu T} + \nu T e^{\nu T}\right)\\
&= \nu T + \nu^2 T^2
\end{aligned}
$$

and, therefore, $\mathrm{Var}[N] = \nu T + \nu^2 T^2 - \nu^2 T^2 = \nu T$.

e) The coefficient of variation is $\mathrm{CV}_{\mathrm{ISI}} = \frac{\sqrt{\mathrm{Var}[s]}}{\mathrm{E}[s]}$ and we have

$$\begin{aligned}
\mathrm{E}[s] &= \int_0^\infty \nu s e^{-\nu s} ds \\
&= \int_0^\infty e^{-\nu s} ds \\
&= 1/\nu
\end{aligned}$$

as well as

$$\begin{aligned}
\mathrm{E}[s^2] &= \int_0^\infty \nu s^2 e^{-\nu s} ds \\
&= \int_0^\infty s e^{-\nu s} ds \\
&= \mathrm{E}[s]/\nu \\
&= 1/\nu^2
\end{aligned}$$

and we therefore have $\mathrm{CV}_{\mathrm{ISI}} = \frac{\sqrt{1/\nu^2}}{1/\nu} = 1$.

Exercise 2

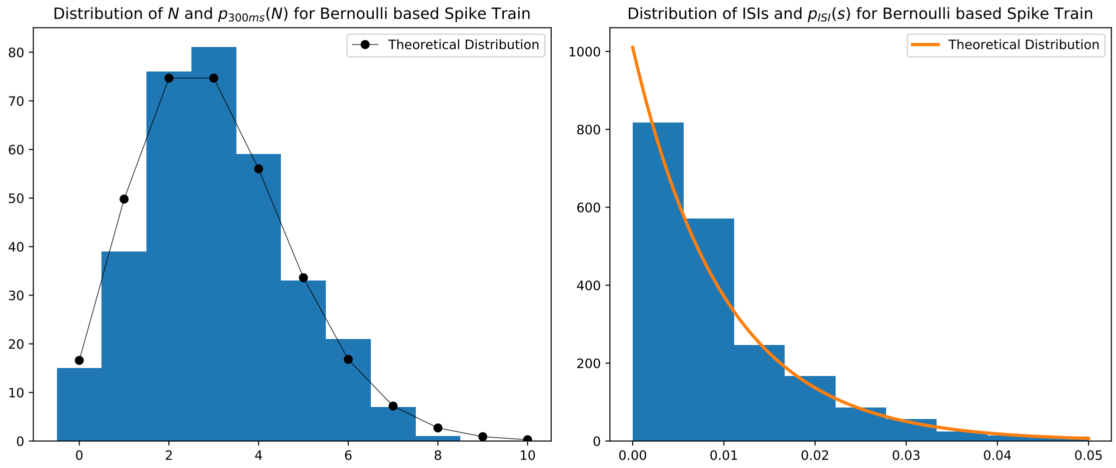

Abbildung 1: Plots of the ISIs distribution and the distribution of spike numbers in 300ms intervals for the approach from a).

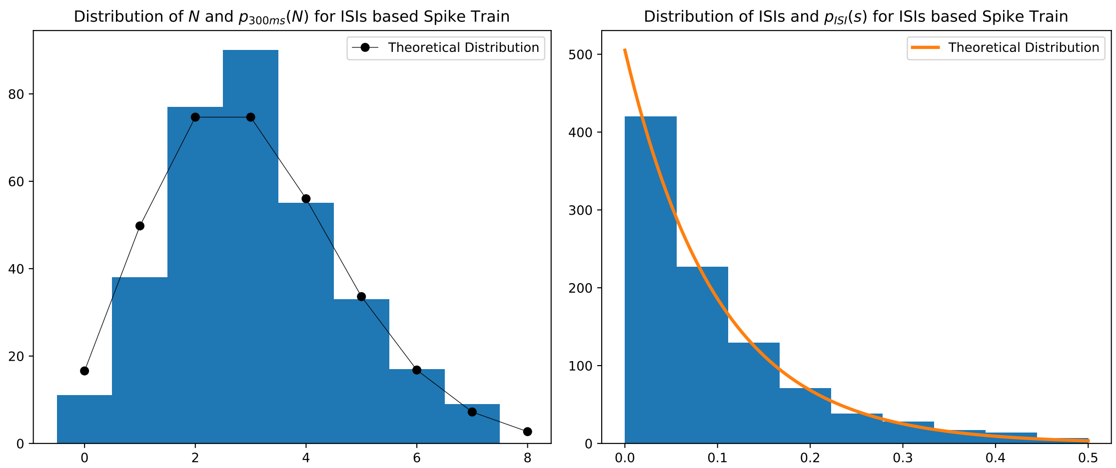

Abbildung 2: Plots of the ISIs distribution and the distribution of spike numbers in 300ms intervals for the approach from b).

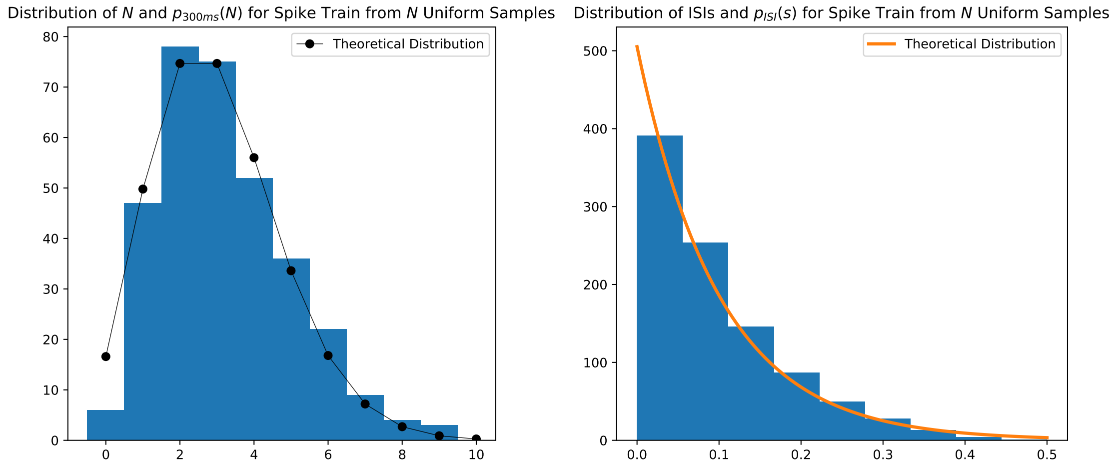

Abbildung 3: Plots of the ISIs distribution and the distribution of spike numbers in 300ms intervals for the approach from c).

Exercise 3

a) Empirically, ν_i is determined to be $\nu_i = 3788Hz$.

b)

$$t_{eff} = \frac{C_m}{g_{tot}} = \frac{C_m}{g_{exc} + g_{inh} + C_m/\tau_m}$$
$$w_{syn}^e = w_0^e * (E_{exc} - u) = 0.45$$
$$w_{syn}^i = w_0^i * (E_{inh} - u) = -0.5$$

c) See figure 5

d) In the HCS case, we see that for the COBA neuron there is a slightly larger count of potentials in the central $(-70mV)$ region as for the CUBA neuron. Furthermore, the bins for the COBA neuron extend further along the x-axis, up to about $-62mV$. The latter can be explained by viewing the very beginning of the experiment, when the potential jumps up rapidly through excitation, without giving the inhibitory neuron a “chance to respond”. In the CUBA case, as everything is based on currents, the jump is much less rapid.

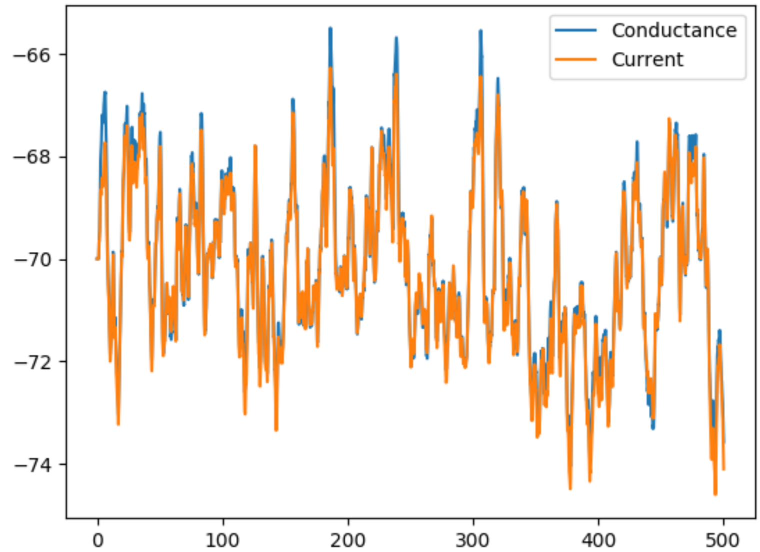

Abbildung 4: Membrane potential trace of COBA and CUBA neurons

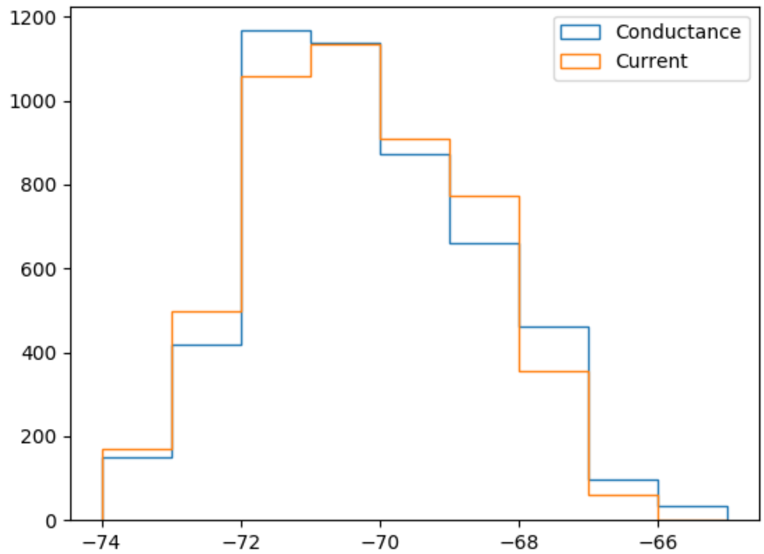

Abbildung 5: Histogram of COBA and CUBA neuron membrane potential

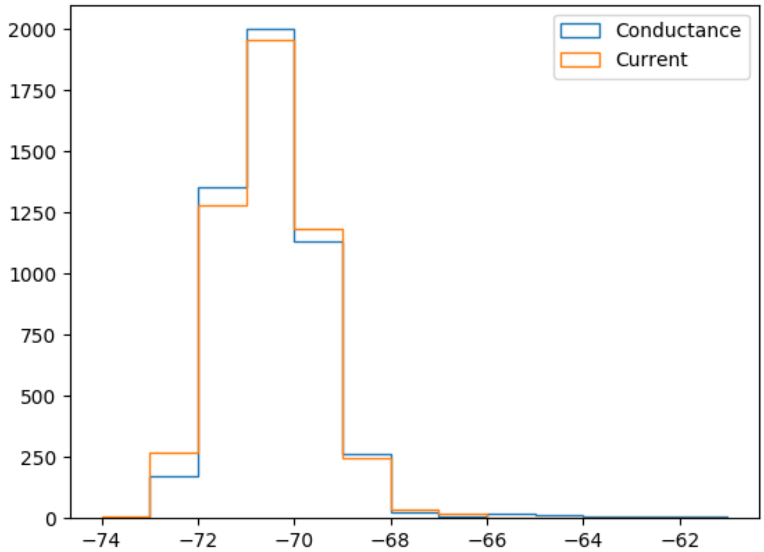

Abbildung 6: Histogram of COBA and CUBA neuron membrane potential in the HCS case