Problems to Turn In

5. I Wish I Could Play Hookey From 201B (ACM Problems 12.25 and 12.26):

This problem examines factors associated with adolescent students being absent from school without a valid excuse. For each of n=252 students, the number of times absent in the last month was recorded (abbreviated nhookey for number of times the student "played hookey") along with possible predictors including age, sex (1 = male and 0 = female) and the degree to which the student liked school (rated from 1 = liked it very much to 5 = disliked it very much). Our goal in this problem is to build a model for school absences.

(a) Fit the Poisson model with no predictors and explain as carefully as you can the meaning of the estimated intercept. (Note-you may find it helpful to exponentiate it!)

 e^{β_0} represents the mean number of times a student was absent in the last month. The Poisson model with no predictors tells us that students were absent on average 2.27 times in the last month.

Analysis Of Maximum Likelihood Parameter Estimates									
Standard Parameter DF Estimate Error Wald 95% Confidence Limits Wald Chi-Square Pr > ChiSq							Pr > ChiSq		
Intercept	1	0.8197	0.0418	0.7378	0.9017	384.34	<.0001		
Scale	0	1.0000	0.0000	1.0000	1.0000				

Figure 1: Poisson model with no predictors

(b) Plot mean number of days absent as a function of (i) likeschool and (ii) age. Do there appear to be significant relationships between these variables and numbers of days absent? Do the relationships appear linear?

There does appear to be a positive monotonic relationship between mean number of days absent and each of likeschool and age. The relationship between mean number of days absent and age looks like it could potentially be curvilinear. The relationship between mean number of days absent and likeschool looks fairly linear.

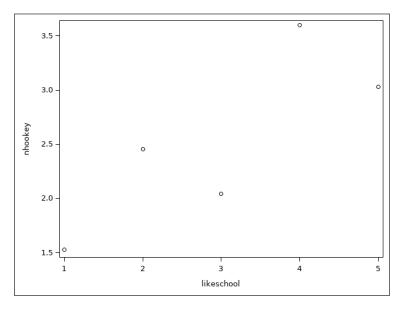


Figure 2: Mean number of days absent as a function of likeschool

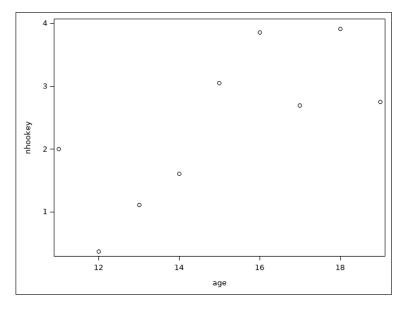


Figure 3: Mean number of days absent as a function of age

(c) Fit a Poisson regression for the number of days the adolescents in the sample were absent from school with sex, age and how much the student liked school as predictors. Obtain two versions—one in which likeschool is treated as continuous and one in which it is treated as categorical. Which version of the model do you think fits better and why?

I think the first model treating likeschool as a categorical variable fits better, but it is not so different from the second model treating likeschool as a continuous variable. While the model treating likeschool as categorical has a higher log likelihood, lower deviance, and a larger Pearson chi-square value, the differences between those goodness of fit criteria in the categorical vs continuous model are minimal. Additionally, we find all variables significant in each of the two models. We see in the categorical model that likeschool = 3 is not statistically significant, which is additional information that we cannot glean from the continuous model, but we sacrifice 3 degrees of freedom to come to a similar conclusion as the continuous model.

	Analysis Of Maximum Likelihood Parameter Estimates										
Parameter	DF	Estimate	Standard Error	Wald 95% Confid	dence Limits	Wald Chi-Square	Pr > ChiSq				
Intercept	1	-3.2817	0.3581	-3.9836	-2.5798	83.97	<.0001				
likeschool	1	0.1095	0.0317	0.0474	0.1717	11.93	0.0006				
age	1	0.2229	0.0221	0.1795	0.2663	101.30	<.0001				
sex	1	0.6814	0.0885	0.5081	0.8548	59.34	<.0001				
Scale	0	1.0000	0.0000	1.0000	1.0000						

Note: The scale parameter was held fixed.

LR Statistics For Type 3 Analysis								
Source	DF Chi-Square		Pr > ChiSq					
likeschool	1	11.71	0.0006					
age	1	105.60	<.0001					
sex	1	62.73	<.0001					

Figure 4: Poisson regression of number of days absent on sex, age, and likeschool, likeschool as continuous

	Analysis Of Maximum Likelihood Parameter Estimates									
Parameter		DF	Estimate	Standard Error	Wald 95% Confid	lence Limits	Wald Chi-Square	Pr > ChiSq		
Intercept		1	-3.2246	0.3659	-3.9418	-2.5073	77.64	<.0001		
likeschool	5	1	0.5159	0.1468	0.2282	0.8037	12.35	0.0004		
likeschool	4	1	0.6719	0.1731	0.3326	1.0112	15.07	0.0001		
likeschool	3	1	0.2245	0.1354	-0.0409	0.4898	2.75	0.0973		
likeschool	2	1	0.3705	0.1284	0.1189	0.6222	8.33	0.0039		
likeschool	1	0	0.0000	0.0000	0.0000	0.0000				
age		1	0.2185	0.0223	0.1747	0.2622	95.82	<.0001		
sex		1	0.6769	0.0890	0.5025	0.8513	57.87	<.0001		
Scale		0	1.0000	0.0000	1.0000	1.0000				

Note: The scale parameter was held fixed.

LR Statistics For Type 3 Analysis								
Source DF Chi-Square Pr > ChiS								
likeschool	4	21.17	0.0003					
age	1	99.73	<.0001					
sex	1	61.06	<.0001					

Figure 5: Poisson regression of number of days absent on sex, age, and likeschool, likeschool as categorical

Criteria For Assessing Goodness Of Fit							
Criterion	DF	Value	Value/DF				
Deviance	248	815.5013	3.2883				
Scaled Deviance	248	815.5013	3.2883				
Pearson Chi-Square	248	885.6149	3.5710				
Scaled Pearson X2	248	885.6149	3.5710				
Log Likelihood		-8.9824					
Full Log Likelihood		-611.8258					
AIC (smaller is better)		1231.6516					
AICC (smaller is better)		1231.8136					
BIC (smaller is better)		1245.7693					

(a) likeschool as continuous	S
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Criteria For Assessing Goodness Of Fit								
Criterion	DF	Value	Value/DF					
Deviance	245	806.0436	3.2900					
Scaled Deviance	245	806.0436	3.2900					
Pearson Chi-Square	245	882.5848	3.6024					
Scaled Pearson X2	245	882.5848	3.6024					
Log Likelihood		-4.2536						
Full Log Likelihood		-607.0970						
AIC (smaller is better)		1228.1940						
AICC (smaller is better)		1228.6530						
BIC (smaller is better)		1252.9000						

(b) likeschool as categorical

Figure 6: Goodness of fit statistics for Poisson regression of number of days absent on sex, age, and likeschool

From here on out, for simplicity, treat the likeschool variable as continuous rather than categorical.

- (d) Give careful interpretations of the coefficients of the age, sex and likeschool variables and their confidence intervals (i) on the log scale and (ii) on the mean scale. Which of these variables appear to be significant predictors?
 - age: The log mean number of days absent increases by 0.2229 units for each additional year in age, all else constant. We are 95% confident that the log mean number of days absent increases between 0.1795 and 0.2663 units for each additional year in age, all else constant. The mean number of days absent is 1.250 times as high for each additional year in age, all else constant. We are 95% confident that the mean number of days absent is anywhere between 1.197 to 1.305 times as high for each additional year in age, all else constant.
 - sex: The log mean number of days absent is 0.6814 units higher for male students compared to female students, all else constant. We are 95% confident that the log mean number of days absent is between 0.5081 and 0.8548

units higher for male students as compared to female students, all else constant. The mean number of days absent is 1.977 times higher for male students as compared to female students, all else constant. We are 95% confident that the mean number of days absent for male students is between 1.662 to 2.351 times higher than for female students.

• likeschool: The log mean number of days absent increases by 0.1095 units for each additional score in how much the student liked school, all else constant. We are 95% confident that the log mean number of days absent increases between 0.0474 and 0.1717 units for each additional score in how much the student liked school, all else constant. The mean number of days absent is 1.116 times as high for each additional score in how much the student liked school, all else constant. We are 95% confident that the mean number of days absent is anywhere between 1.049 to 1.187 times as high for each additional score in how much the student liked school, all else constant.

All three variables appear to be significant predictors in this model (p < .001 for all three predictors).

(e) Now suppose that the number of absences for some of the students were measured over 1 month and some were measured over 3 months as indicated by the variable hmonth. Refit the model for the number of days absent using this variable as the offset. How does this change your answers to part (d)?

The impact of age on log mean number of absences per month/mean number of absences per month is noticeably decreased in this model as compared to the model without the offset term. Additionally, our interpretation of all the coefficients can now be comparable with the same denominator of "per month". Our answers in part (d) can now be about the log mean number of days absent per month/mean number of days absent per month, with the new coefficient values subbed in. We note that the direction and significance of all three predictors are the same in both models, age has a noticeably smaller magnitude (0.1881 unit increase in log mean number of absences per month for each additional year in age), and that the magnitudes of likeschool and sex are quite similar in both models.

	Analysis Of Maximum Likelihood Parameter Estimates										
Parameter	DF	Estimate	Standard Error	Wald 95% Confid	lence Limits	Wald Chi-Square	Pr > ChiSq				
Intercept	1	-3.4546	0.3465	-4.1338	-2.7754	99.38	<.0001				
likeschool	1	0.1114	0.0316	0.0496	0.1733	12.46	0.0004				
age	1	0.1881	0.0221	0.1448	0.2313	72.67	<.0001				
sex	1	0.6867	0.0886	0.5131	0.8604	60.09	<.0001				
Scale	0	1.0000	0.0000	1.0000	1.0000						

Note: The scale parameter was held fixed.

LR Statistics For Type 3 Analysis								
Source DF Chi-Square Pr > ChiSe								
likeschool	1	12.24	0.0005					
age	1	74.78	<.0001					
sex	1	63.51	<.0001					

Figure 7: Poisson regression of number of days absent on sex, age, and likeschool, likeschool as categorical, hmonth as offset

Note: For the rest of the problem revert to thinking of the number of absences as all corresponding to one-month intervals as in parts (a)-(d).

- (f) In this part of the problem we evaluate the issue of over-dispersion. First suggest some plausible reasons why there might be over-dispersion in this data set and then check for it in the following ways:
- (1) Using basic descriptive statistics obtain the mean and variance of the nhookey variable. (i) What do these statistics suggest about over-dispersion and (ii) why might this assessment be too crude to evaluate

the presence of over-dispersion in the model overall?

- (2) Evaluate the goodness of fit of the model using by calculating the Pearson chi-squared goodness of fit statistic and approximating the dispersion factor. Explain what each of these checks tells you.
- (3) Obtain the Pearson residuals for the model and plot them as a function of the fitted values. What does this plot suggest about over-dispersion? Do there appear to be any outliers? Briefly justify your answer.
- (1) The nhookey variable has a mean of 2.270 and a variance of 9.297. The variance is much bigger than the mean, which may suggest over-dispersion in the model. However, this is still a crude assessment because this removes information about the heterogeneity present in the variables included in the model, so we may be calculating the mean and variance of merged groups with very different characteristics.
- (2) The Pearson chi-squared goodness of fit statistic was obtained in part (c), which was $\chi^2_{248} = 885.6149$. Divided by the degrees of freedom we get 885.6149/248 = 3.571. If there is no over-dispersion, the dispersion factor should be equal to 1, but our value is greater than 1. The Pearson chi-squared goodness of fit statistic itself tells us that the model is not well-calibrated and is a poor fit, and the dispersion factor being greater than 1 tells us that there is proof of over-dispersion and that we would need to scale our standard errors accordingly with the square root of the dispersion factor.
- (3) The Pearson residuals vs fitted values plot shows fanning of residuals increasing with larger fitted values, and the residuals are quite large, going up to 7.5 at the most fanned out data points. This suggests over-dispersion, as a model that does not have over-dispersion should have equal spread in the residuals vs fitted values plot.

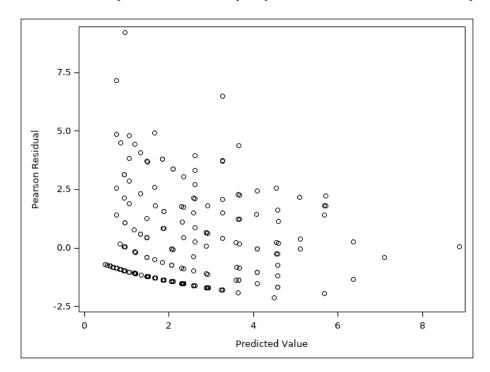


Figure 8: Pearson residuals vs fitted values for the Poisson regression model

(g) Rerun the model using a negative binomial regression. Does this model confirm your conclusions about over-dispersion from part (f)? Explain briefly.

The negative binomial model yields a dispersion parameter of 1.7512, with a 95% confidence interval of [1.3005, 2.3580]. Since this interval is entirely above 1, this model confirms my conclusions about over-dispersion from part (f) since we are 95% confident that the dispersion parameter lies between 1.3005 and 2.3580, a number greater than the desired dispersion parameter of 1.

Criteria For Assessing Goodness Of Fit							
Criterion	DF	Value	Value/DF				
Deviance	248	252.8602	1.0196				
Scaled Deviance	248	252.8602	1.0196				
Pearson Chi-Square	248	223.3531	0.9006				
Scaled Pearson X2	248	223.3531	0.9006				
Log Likelihood		128.7829					
Full Log Likelihood		-474.0605					
AIC (smaller is better)		958.1210					
AICC (smaller is better)		958.3649					
BIC (smaller is better)		975.7681					

Algorithm converged.

Analysis Of Maximum Likelihood Parameter Estimates										
Parameter	DF	Estimate	Standard Error	Wald 95% Confid	lence Limits	Wald Chi-Square	Pr > ChiSq			
Intercept	1	-3.1864	0.7759	-4.7070	-1.6657	16.87	<.0001			
likeschool	1	0.0939	0.0765	-0.0560	0.2439	1.51	0.2194			
age	1	0.2251	0.0519	0.1233	0.3269	18.78	<.0001			
sex	1	0.5329	0.1939	0.1528	0.9130	7.55	0.0060			
Dispersion	1	1.7512	0.2658	1.3005	2.3580					

Note: The negative binomial dispersion parameter was estimated by maximum likelihood.

Figure 9: Negative binomial regression of number of days absent on sex, age, and likeschool, likeschool as continuous

- (h) Give brief interpretations of the coefficients from the negative binomial regression. Are your conclusions (as to magnitude, direction and significance) of the effects of the predictors any different from those in part (d)?
 - age: The mean number of days absent increases by a factor of $e^{0.2251} = 1.252$ for each additional year in age, all else constant.
 - sex: The mean number of days absent is $e^{0.5329} = 1.704$ times higher in male students than in female students, all else equal.
 - likeschool: The mean number of days absent increases by a factor of $e^{0.0939} = 1.098$ for each additional score in how much a student likes school, all else equal.

The direction of the predictors are the same as before; however, the magnitude of the predictor effect of likeschool and age is smaller than before. Additionally, likeschool is no longer significant in this model (p = .2194), while sex is still significant but with a larger p-value (p = .006), indicating that there is less evidence (but still enough evidence to determine a positive association) for the effect of sex on mean number of days absent. This is possibly in part due to the larger standard error estimates in the negative binomial model as compared to the Poisson model.

In the next part of the problem we consider the issue of zero inflation.

(i) Explain intuitively why we might expect zero-inflation in the context of this problem. Of our three predictors, which would you expect to be most associated with a "certain zero"? Explain briefly.

Zero-inflation in the context of this problem may be that some students will never miss a day of school, either because they have always have perfect attendance, parents do not allow them to play hookey, they like attending class, or something else. Of the three predictors, I would most expect likeschool to be associated with a "certain zero" since those that like school would be more inclined to go to class, regardless of gender or age.

(j) Using the estimated rate of unexcused absences from your model in part (a) calculate the expected number of 0's we would see in a sample of 252 students if absences have a Poisson distribution. You may find it helpful to remember that for a Poisson random variable the probability of k events is $P(Y = k) = \mu^k e^{-\mu}/k!$). How many 0's did we actually observe? What does this suggest about whether we have a zero-inflation issue? (Note: This is a fairly crude approximation because we haven't accounted for the covariates but it is rather suggestive. You can of course take this approach further and break it down by sex, age bin and likeschool categories.)

The expected number of 0's we would see in a sample of 252 if absences have a Poisson distribution would be $P(Y=0) = (2.27)^0 e^{-2.27}/0! = 0.103$, suggesting that 10.3% of the sample would be 0's. However, in the sample we observed 121/252 = 48%, suggesting that there is zero-inflation present here, even without the finer breakdown into variable categories.

(k) Fit a zero-inflated Poisson model to the data using age, sex and likeschool as the predictors for both the Poisson component and the zero-inflation component of the model. Does this model seem like an improvement over a standard Poisson regression? Explain briefly.

The model does seem like a better fit over the standard Poisson regression —we note that the log likelihood is significantly higher in the zero-inflated Poisson model as compared to the standard Poisson model, and the Pearson chi-square test statistic is lower in this model vs the standard Poisson model.

Analysis Of Maximum Likelihood Parameter Estimates								
Parameter	DF	Estimate	Standard Error	Wald 95% Confi	dence Limits	Wald Chi-Square	Pr > ChiSq	
Intercept	1	0.8904	0.4072	0.0922	1.6886	4.78	0.0288	
likeschool	1	0.0319	0.0339	-0.0346	0.0985	0.88	0.3472	
age	1	0.0037	0.0268	-0.0488	0.0562	0.02	0.8902	
sex	1	0.6743	0.1022	0.4740	0.8745	43.55	<.0001	
Scale	0	1.0000	0.0000	1.0000	1.0000			

Note: The scale parameter was held fixed.

Analysis Of Maximum Likelihood Zero Inflation Parameter Estimates									
Parameter	DF	Estimate	Standard Error	Wald 95% Confi	dence Limits	Wald Chi-Square	Pr > ChiSq		
Intercept	1	7.7887	1.4080	5.0291	10.5483	30.60	<.0001		
likeschool	1	-0.2503	0.1264	-0.4980	-0.0025	3.92	0.0477		
age	1	-0.5007	0.0963	-0.6894	-0.3120	27.05	<.0001		
sex	1	0.0900	0.3202	-0.5375	0.7175	0.08	0.7787		

Figure 10: Zero-inflated Poisson model of number of days absent on sex, age, and likeschool

Criteria For Assessing Goodness Of Fit								
Criterion	DF	Value	Value/DF					
Deviance		908.1374						
Scaled Deviance		908.1374						
Pearson Chi-Square	244	323.5140	1.3259					
Scaled Pearson X2	244	323.5140	1.3259					
Log Likelihood		148.7747						
Full Log Likelihood		-454.0687						
AIC (smaller is better)		924.1374						
AICC (smaller is better)		924.7300						
BIC (smaller is better)		952.3728						

Figure 11: Goodness of fit criteria for zero-inflated Poisson model of number of days absent on sex, age, and likeschool

(1) Give brief interpretations of the coefficients for each component of the zero-inflated Poisson model. Which variables seem to be significant in each part of the model? Explain as carefully as you can what this model suggests about how these variables affect school absences.

- Count
- age: Among students with absences, the mean number of days absent is $e^{0.0037} = 1.004$ times higher for each additional year in age, all else equal.
- sex: Among students with absences, the mean number of days absent is $e^{0.6743} = 1.963$ times higher in male students than in female students, all else equal.
- likeschool: Among students with absences, the mean number of days absent is $e^{0.0319} = 1.032$ times higher for each additional score in how much a student likes school, all else equal.
- Inflate
- age: All else equal, each additional year in age is associated with a 39.4% reduction in the odds of having no absences (odds ratio $e^{-0.5007} = 0.606$).
- sex: All else equal, male students have 9.4% higher odds of having no absences than female students (odds ratio $e^{0.09} = 1.094$).
- likeschool: All else equal, each additional score in how much a student likes school is associated with a 22.1% reduction in the odds of having no absences (odds ratio $e^{-0.2503} = 0.779$).

In the count component of the ZIP model, sex is the only variable that is significant (p < .001). In the zero-inflated component of the model, both likeschool and age are significant (p = .048 and p < .001), respectively). This model suggests that students that are older or like school more are both more likely to never have absences. Of the students that do have absences, male students are more likely to have higher counts of absences than female students.

(m) Now fit a zero-inflated negative binomial model using the same model set-up as in part (k). Is this model superior to the zero-inflated Poisson model of part (k)? Does it appear to be an improvement over the plain negative binomial model of part (g)? Explain briefly in each instance. What does that tell you about the issues of over-dispersion and zero-inflation? Do any of your conceptual conclusions from part (l) change with this model?

This model is superior to the zero-inflated Poisson model since it has a dispersion parameter that is greater than 0 (dispersion = 0.3711, 95% CI = [0.2091, 0.6585]), so this model better accounts for the over-dispersion. This model appears to be an improvement over the plain negative binomial model, as we see that there is significance observed in the coefficients for age and sex in the inflated component. This tells us that there was over-dispersion and zero-inflation in our data and desired model that the ZINB model is best suited to address. In this model, likeschool is no longer significant in either the count or zero-inflated components, so that would change our conclusions about likeschool being associated with increases in mean number of days absent. Additionally, sex is now significant in both

the count and zero-inflated components. However, the standard error of sex in the zero-inflated component is so large that we cannot make a new conclusion about its role in determining a "certain 0" or not. So, we lose the conclusion about likeschool and keep the same direction and significance for sex in the count portion of the model and age in the zero-inflated portion of the model, as compared to in part (1).

Analysis Of Maximum Likelihood Parameter Estimates								
Parameter	DF	Estimate	Standard Error	Wald 95% Confid	ence Limits	Wald Chi-Square	Pr > ChiSq	
Intercept	1	1.1554	0.6293	-0.0779	2.3888	3.37	0.0663	
likeschool	1	0.0635	0.0531	-0.0405	0.1675	1.43	0.2314	
age	1	-0.0349	0.0396	-0.1126	0.0427	0.78	0.3776	
sex	1	0.9023	0.1413	0.6255	1.1792	40.80	<.0001	
Dispersion	1	0.3711	0.1086	0.2091	0.6585			

Note: The negative binomial dispersion parameter was estimated by maximum likelihood.

Analysis Of Maximum Likelihood Zero Inflation Parameter Estimates									
Parameter	DF	Estimate	Standard Error	Wald 95% Con	fidence Limits	Wald Chi-Square	Pr > ChiSq		
Intercept	1	13.0952	2.5239	8.1485	18.0420	26.92	<.0001		
likeschool	1	-0.2157	0.1601	-0.5295	0.0980	1.82	0.1777		
age	1	-0.9588	0.1951	-1.3411	-0.5765	24.16	<.0001		
sex	1	1.3119	0.5371	0.2593	2.3645	5.97	0.0146		

Figure 12: Zero-inflated negative binomial model of number of days absent on sex, age, and likeschool

Criteria For Assessing Goodness Of Fit								
Criterion	DF	Value	Value/DF					
Deviance		873.8883						
Scaled Deviance		873.8883						
Pearson Chi-Square	244	292.6531	1.1994					
Scaled Pearson X2	244	292.6531	1.1994					
Log Likelihood		-436.9442						
Full Log Likelihood		-436.9442						
AIC (smaller is better)		891.8883						
AICC (smaller is better)		892.6322						
BIC (smaller is better)		923.6532						

Figure 13: Goodness of fit criteria for zero-inflated negative binomial model of number of days absent on sex, age, and likeschool