1. From page 26 in theorybayes15.pdf:

$$y_{ij}|\alpha,\beta_i,\sigma^2 \sim N(x_{ij}'\alpha + z_{ij}'\beta_i, \sigma^2)$$

$$\beta_i|D \sim N(0,D)$$

$$\alpha \sim N(\alpha_0, V_0)$$

$$\sigma^2 \sim \text{InverseGamma}(a/2, b/2)$$

$$D \sim \text{InverseWishart}_q(\cancel{\nu, D_0})$$

(D_0, ν) possible mistake

$$\boxed{\text{assume: } y_j|\alpha,\beta,\sigma^2 \sim N(x'\alpha+\beta,\sigma^2) \qquad q=1, \; z_{ij}=1}$$

β a scalar
α a vector

$$f(\beta|D) = \frac{1}{(2\pi D)^{1/2}} e^{-1/2\left(\frac{\beta}{D^{1/2}}\right)^2}$$

$$= \frac{1}{(2\pi D)^{1/2}} e^{-\frac{\beta^2}{2D}}$$

$$f(D|\beta) \propto f(\beta|D)f(D)$$

$$\propto \left[\frac{1}{(2\pi D)^{1/2}} e^{-\beta^2/2D}\right]\left[\frac{|D_0|^{\nu/2}}{2^{\nu/2}\Gamma(\frac{\nu}{2})} |D|^{-(\nu+1+1)/2} e^{-\frac{1}{2}tr(D_0 D^{-1})}\right]$$

$$\propto \frac{|D_0|^{\nu/2}}{D^{1/2} 2^{\nu/2}\Gamma(\frac{\nu}{2})} e^{-\frac{\beta^2}{2D} - \frac{D_0}{2D}} D^{-(\nu+2)/2}$$

$$\propto D^{-\frac{1}{2}(1+\nu+2)} e^{-\frac{1}{2}(\beta^2+D_0)/D}$$

$$\boxed{D|\beta \sim W^{-1}(\beta^2+D_0, \nu+1)}$$

$$f(\sigma^2|\alpha,\beta,y) \propto L(y|\alpha,\beta,\sigma^2) f(\alpha,\beta,\sigma^2)$$

$$\propto f(y|\alpha,\beta,\sigma^2) f(\sigma^2)$$

$$f(y|\alpha,\beta,\sigma^2) = \prod_{j=1}^{n} f(y_j|\alpha,\beta,\sigma^2)$$

$$= \frac{1}{(2\pi\sigma^2)^{n/2}} e^{-\frac{1}{2\sigma^2}\sum(y_j - x_j'\alpha - (1)\beta)^2}$$

$$f(\sigma^2|\alpha,\beta,y) \propto \frac{1}{(2\pi\sigma^2)^{n/2}} e^{-\frac{1}{2\sigma^2}\sum(y_j-x_j'\alpha-\beta)^2} \frac{(\frac{b}{2})^{a/2}}{\Gamma(\frac{a}{2})} (\sigma^2)^{(-\frac{a}{2}-1)} e^{-\frac{b/2}{\sigma^2}}$$

$$\propto \frac{1}{(2\pi)^{n/2}} \frac{(\frac{b}{2})^{a/2}}{\Gamma(\frac{a}{2})} (\sigma^2)^{(-\frac{a}{2}-\frac{n}{2}-1)} e^{-\frac{1}{2\sigma^2}(\sum(y_j-x_j'\alpha-\beta)+b)}$$

$$\boxed{\sigma^2|\alpha,\beta,y \sim IG\left(\frac{a+n}{2}, \frac{\sum(y_j-x_j'\alpha-\beta)+b}{2}\right)}$$

observations y_{ij} on subject i
$j=1,\ldots,n_i$

known covariate vectors $\vec{x}_i$ and $\vec{z}_i$

subject specific regression coefs β_i

population regression coefs α

conditionally conjugate priors

InverseWishart → distr on positive definite matrices
convenient prior for covariance matrices

positive definite: symmetric w/ all positive eigenvalues
nonsingular

4 unknown $\theta = (\alpha, \{\beta_i\}_{i=1}^n, \sigma^2, D)$

$W^{-1}(\Psi,\nu)$
pdf of InverseWishart:

$$\frac{|\Psi|^{\nu/2}}{2^{\nu p/2}\Gamma_p(\frac{\nu}{2})} |X|^{-(\nu+p+1)/2} e^{-\frac{1}{2}tr(\Psi X^{-1})}$$

det(Ψ)
multivariate gamma function
det(X)

$$f(\beta_i|\alpha,\sigma^2,D,Y_i) \propto L(y_i|\alpha,\beta,\sigma^2)\, f(\beta_i|D) \qquad i=1$$

$$\propto \frac{1}{(2\pi\sigma^2)^{n/2}} e^{-\frac{1}{2\sigma^2}\sum(y_j-x_j'\alpha-\beta)^2} \frac{1}{(2\pi D)^{1/2}} e^{-\frac{\beta^2}{2D}}$$

$$\propto \exp\left(-\frac{1}{2}\left(\frac{1}{\sigma^2}\sum(y_j-x_j'\alpha-\beta)^2+\frac{\beta^2}{D}\right)\right)$$

$$\propto \exp\left(-\frac{1}{2}\left(\frac{1}{\sigma^2}\sum\left(y_j^2-y_jx_j'\alpha-y_j\beta-x_j'\alpha y_j+(x_j'\alpha)^2+x_j'\alpha\beta-\beta y_j+\beta x_j'\alpha+\beta^2\right)+\frac{\beta^2}{D}\right)\right)$$

$$\propto \exp\left(-\frac{1}{2}\left(\frac{1}{\sigma^2}\sum\left(-y_j\beta+x_j'\alpha\beta-\beta y_j+\beta x_j'\alpha+\beta^2\right)+\frac{\beta^2}{D}\right)\right)$$

$$\propto \exp\left(-\frac{1}{2}\left(\frac{1}{\sigma^2}\sum\left(\beta^2+2\beta x_j'\alpha-2\beta y_j\right)+\frac{\beta^2}{D}\right)\right)$$

$$\propto \exp\left(-\frac{1}{2}\left(\frac{1}{\sigma^2}\left(n\beta^2+\sum(2\beta x_j'\alpha-2\beta y_j)\right)+\frac{\beta^2}{D}\right)\right)$$

$$\propto \exp\left(-\frac{1}{2}\left(\frac{n\beta^2}{\sigma^2}+\frac{1}{\sigma^2}\sum(2\beta x_j'\alpha-2\beta y_j)\right)+\frac{\beta^2}{D}\right)$$

$$\propto \exp\left(-\frac{1}{2}\left(\beta^2\left(\frac{n}{\sigma^2}+\frac{1}{D}\right)-\frac{2\beta}{\sigma^2}\sum(y_j-x_j'\alpha)\right)\right)$$

$$\boxed{\beta|\alpha,\sigma^2,D,y_i \sim N\left(\frac{\sum y_j-x_j'\alpha}{\sigma^2\left(\frac{n}{\sigma^2}+\frac{1}{D}\right)},\ \frac{1}{\left(\frac{n}{\sigma^2}+\frac{1}{D}\right)}\right)} \qquad i=1$$

$$f(\alpha|\beta,y,\sigma^2) \propto L(y|\alpha,\beta,\sigma^2)\, f(\alpha)$$

$$\propto \frac{1}{(2\pi\sigma^2)^{n/2}} \exp\left(-\frac{1}{2\sigma^2}\sum(y_j-x_j'\alpha-\beta)^2\right)\frac{1}{(2\pi V_0)^{1/2}}\exp\left(-\frac{1}{2V_0}(\alpha-\alpha_0)^2\right)$$

$$\propto \exp\left(-\frac{1}{2}\left[\frac{1}{\sigma^2}\sum(y_j-x_j'\alpha-\beta)^2+\frac{1}{V_0}(\alpha-\alpha_0)^2\right]\right)$$

$$\propto \exp\left(-\frac{1}{2}\left[\frac{1}{\sigma^2}\sum\left(\cancel{y_j^2}-y_jx_j'\alpha-\cancel{y_j\beta}-x_j'\alpha y_j+(x_j'\alpha)^2+x_j'\alpha\beta-\cancel{\beta y_j}+\beta x_j'\alpha+\cancel{\beta^2}\right)+\frac{1}{V_0}\left(\alpha'\alpha-2\alpha'\alpha_0+\cancel{\alpha_0^2}\right)\right]\right)$$

$$\propto \exp\left(-\frac{1}{2}\left[\frac{1}{\sigma^2}\sum\left(\underline{(x_j'\alpha)}^2-2x_j'\alpha y_j+2x_j'\alpha\beta\right)+\frac{1}{V_0}(\alpha'\alpha-2\alpha'\alpha_0)\right]\right)$$

$$\propto \exp\left(-\frac{1}{2}\left[\frac{1}{\sigma^2}\sum\left(\alpha'x_jx_j'\alpha-2x_j'\alpha y_j+2x_j'\alpha\beta\right)+\frac{1}{V_0}(\alpha'\alpha-2\alpha'\alpha_0)\right]\right)$$

$$\propto \exp\left(-\frac{1}{2}\left[\frac{\sum\alpha'x_jx_j'\alpha}{\sigma^2}+\frac{\alpha'\alpha}{V_0}-\frac{2}{\sigma^2}\sum\alpha'x_j(y_j-\beta)-\frac{1}{V_0}(2\alpha_0'\alpha)\right]\right)$$

$$\propto \exp\left(-\frac{1}{2}\left[\alpha'\left[\frac{\sum x_j x_j'}{\sigma^2}+\frac{1}{V_0}\right]\alpha - 2\left[\frac{\sum y_j x_j' - \beta x_j'}{\sigma^2} - \frac{\alpha_0'}{V_0}\right]\alpha\right]\right)$$

$$\propto \exp\left(-\frac{1}{2}\left[\frac{\sum x_j x_j'}{\sigma^2}+\frac{1}{V_0}\right]\left[\alpha - \frac{\frac{\sum y_j x_j' - \beta x_j'}{\sigma^2} - \frac{\alpha_0'}{V_0}}{\frac{\sum x_j x_j'}{\sigma^2}+\frac{1}{V_0}}\right]^2\right)$$

$$\alpha \mid \beta, y, \sigma^2 \sim N\left(\frac{\frac{\sum y_j x_j' - \beta x_j'}{\sigma^2} - \frac{\alpha_0'}{V_0}}{\frac{\sum x_j x_j'}{\sigma^2}+\frac{1}{V_0}}, \left[\frac{\sum x_j x_j'}{\sigma^2}+\frac{1}{V_0}\right]^{-1}\right)$$

2. a) $f(y_i, \beta \mid \alpha, \sigma^2, D) = L(y_i \mid \alpha, \beta, \sigma^2)\, f(\beta_i \mid D)\, f(D)$

$$= (2\pi\sigma^2)^{-n/2} \exp\left(-\frac{1}{2\sigma^2}\sum_{j=1}^{n}(y_j - x_j'\alpha - \beta)^2\right)(2\pi D)^{-\frac{1}{2}}\exp\left(-\frac{\beta^2}{2D}\right)$$

$$= (2\pi)^{-\left(\frac{n+1}{2}\right)}(\sigma^2)^{-n/2}(D)^{-1/2}\exp\left(-\frac{1}{2}\left[\frac{1}{\sigma^2}\sum_{j=1}^{n}(y_j - x_j'\alpha - \beta)^2 + \frac{\beta^2}{D}\right]\right)$$

$$= (2\pi)^{-\frac{n+1}{2}}(\sigma^2)^{-\frac{n}{2}}(D)^{-\frac{1}{2}}\exp\left(-\frac{1}{2}\left[\frac{1}{\sigma^2}\sum_{j=1}^{n}\left(y_j^2 - y_j x_j'\alpha - y_j\beta - x_j'\alpha y_j + (x_j'\alpha)^2 + x_j'\alpha\beta - \beta y_j + \beta x_j'\alpha + \beta^2\right) + \frac{\beta^2}{D}\right]\right)$$

$$= (2\pi)^{-\frac{n+1}{2}}(\sigma^2)^{-\frac{n}{2}}(D)^{-\frac{1}{2}}\exp\left(-\frac{1}{2}\left[\frac{1}{\sigma^2}\sum_{j=1}^{n}\left(y_j^2 + (x_j'\alpha)^2 - 2y_j x_j'\alpha - 2\beta y_j + 2\beta x_j'\alpha + \beta^2\right) + \frac{\beta^2}{D}\right]\right)$$

b) Integrate out β_i to get $y_i \mid \alpha, \sigma^2, D$

$$f(y_i \mid \alpha, \sigma^2, D) = \int_{-\infty}^{\infty} f(y_i, \beta \mid \alpha, \sigma^2, D)\, d\beta$$

$$= (2\pi)^{-\frac{n+1}{2}}(\sigma^2)^{-\frac{n}{2}}(D)^{-\frac{1}{2}}\exp\left(-\frac{1}{2}\left(\frac{1}{\sigma^2}\sum_{j=1}^{n}\left(y_j^2 + (x_j'\alpha)^2 - 2y_j x_j'\alpha\right)\right)\right)\int_{-\infty}^{\infty}\exp\left(-\frac{1}{2}\left[\sum_{j=1}^{n}\frac{\beta^2 - 2\beta_j y_j + 2\beta x_j'\alpha}{\sigma^2} + \frac{\beta^2}{D}\right]\right)d\beta$$

integrand is proportional to

$$\beta \mid \alpha, \sigma^2, D, y_i \sim N\left(\frac{\sum y_j - x_j'\alpha}{\sigma^2\left(\frac{n}{\sigma^2}+\frac{1}{D}\right)}, \frac{1}{\left(\frac{n}{\sigma^2}+\frac{1}{D}\right)}\right) \quad i=1$$

$$= (2\pi)^{1/2}\left(\frac{n}{\sigma^2}+\frac{1}{D}\right)^{-1/2}(2\pi)^{-\frac{n+1}{2}}(\sigma^2)^{-\frac{n}{2}}(D)^{-\frac{1}{2}}\exp\left(-\frac{1}{2\sigma^2}\sum_{j=1}^{n}\left(y_j^2 + (x_j'\alpha)^2 - 2y_j x_j'\alpha\right)\right)\int_{-\infty}^{\infty}(2\pi)^{-1/2}\left(\frac{n}{\sigma^2}+\frac{1}{D}\right)^{1/2}\exp\left(-\frac{1}{2}\left(\frac{n}{\sigma^2}+\frac{1}{D}\right)\left(\beta - \frac{\sum y_j - x_j'\alpha}{\sigma^2\left(\frac{n}{\sigma^2}+\frac{1}{D}\right)}\right)^2 - \frac{1}{2}\left(\frac{n}{\sigma^2}+\frac{1}{D}\right)\left(\frac{\sum y_j - x_j'\alpha}{\sigma^2\left(\frac{n}{\sigma^2}+\frac{1}{D}\right)}\right)^2\right)d\beta$$

$$= (2\pi)^{1/2}\left(\frac{n}{\sigma^2}+\frac{1}{D}\right)^{-1/2}(2\pi)^{-\frac{n+1}{2}}(\sigma^2)^{-\frac{n}{2}}D^{-\frac{1}{2}}\exp\left(-\frac{1}{2\sigma^2}\sum_{j=1}^{n}\left(y_j^2 + (x_j'\alpha)^2 - 2y_j x_j'\alpha\right)\right)\exp\left(\frac{-1}{2}\left(\frac{n}{\sigma^2}+\frac{1}{D}\right)\left(\frac{\sum y_j - x_j'\alpha}{\sigma^2\left(\frac{n}{\sigma^2}+\frac{1}{D}\right)}\right)^2\right)\underbrace{\int_{-\infty}^{\infty}(2\pi)^{-1/2}\left(\frac{n}{\sigma^2}+\frac{1}{D}\right)^{1/2}\exp\left(-\frac{1}{2}\left(\frac{n}{\sigma^2}+\frac{1}{D}\right)\left(\beta - \frac{\sum y_j - x_j'\alpha}{\sigma^2\left(\frac{n}{\sigma^2}+\frac{1}{D}\right)}\right)^2\right)d\beta}_{\text{integrate to 1}}$$

$$\propto \exp\left(-\frac{1}{2\sigma^2}\left(1 + \frac{1}{\sigma^2\left(\frac{n}{\sigma^2}+\frac{1}{D}\right)}\right)\sum_{j=1}^{n}(y_j - x_j'\alpha)^2\right)$$

$$\boxed{y_i \mid \alpha, \sigma^2, D \sim N\left(x_j'\alpha, \sigma^2\left(1 + \frac{1}{\sigma^2\left(\frac{n}{\sigma^2}+\frac{1}{D}\right)}\right)^{-1}\right)}$$

$$y_{ij} = x_{ij}'\alpha + \beta_i + \epsilon_{ij}$$

3. a) Design 1:

$\frac{n}{2}$ clinics intervention

$\frac{n}{2}$ clinics control

$$x_{ij} = [0 \ 1] \qquad \beta_i \mid D \sim N(0, D) \qquad \epsilon_{ij} \mid \sigma^2 \sim N(0, \sigma^2)$$

$$(\bar{Y}_1 - \bar{Y}_0) = \frac{\left(\sum_{i=1}^{n/2}\sum_{j=1}^{J} y_{ij} - \sum_{i=\frac{n}{2}+1}^{n}\sum_{j=1}^{J} y_{ij}\right)}{\frac{nJ}{2}} = \frac{2}{nJ}\left[\sum_{i=1}^{n/2}\sum_{j=1}^{J} y_{ij} - \sum_{i=\frac{n}{2}+1}^{n}\sum_{j=1}^{J} y_{ij}\right]$$

$$= \frac{2}{nJ}\left[\sum_{i=1}^{n/2}\sum_{j=1}^{J}(x_{ij}'\alpha + \beta_i + \epsilon_{ij}) - \sum_{i=\frac{n}{2}+1}^{n}\sum_{j=1}^{J}(x_{ij}'\alpha + \beta_i + \epsilon_{ij})\right]$$

$$\mathrm{Var}(\bar{Y}_1 - \bar{Y}_0) = \left(\frac{2}{nJ}\right)^2 \mathrm{Var}\left[\sum_{i=1}^{n/2}\sum_{j=1}^{J}(x_{ij}'\alpha + \beta_i + \epsilon_{ij}) - \sum_{i=\frac{n}{2}+1}^{n}\sum_{j=1}^{J}(x_{ij}'\alpha + \beta_i + \epsilon_{ij})\right]$$

$$= \left(\frac{2}{nJ}\right)^2 \mathrm{Var}\left[\cancel{\sum_{i=1}^{n/2}\sum_{j=1}^{J} x_{ij}'\alpha} + \sum_{i=1}^{n/2}\sum_{j=1}^{J}\beta_i + \sum_{i=1}^{n/2}\sum_{j=1}^{J}\epsilon_{ij} - \cancel{\sum_{i=\frac{n}{2}+1}^{n}\sum_{j=1}^{J} x_{ij}'\alpha} - \sum_{i=\frac{n}{2}+1}^{n}\sum_{j=1}^{J}\beta_i - \sum_{i=\frac{n}{2}+1}^{n}\sum_{j=1}^{J}\epsilon_{ij}\right]$$

$$= \left(\frac{2}{nJ}\right)^2 \mathrm{Var}\left[\sum_{i=1}^{n/2} J\beta_i + \sum_{i=1}^{n/2}\sum_{j=1}^{J}\epsilon_{ij} - \sum_{i=\frac{n}{2}+1}^{n} J\beta_i - \sum_{i=\frac{n}{2}+1}^{n}\sum_{j=1}^{J}\epsilon_{ij}\right]$$

$\beta_i \mid D \sim N(0, D) \rightarrow \mathrm{Var}(\beta_i \mid D) = D$

$\epsilon_{ij} \mid \sigma^2 \sim N(0, \sigma^2) \rightarrow \mathrm{Var}(\epsilon_{ij} \mid \sigma^2) = \sigma^2$

$\mathrm{Var}(X - Y) = \mathrm{Var}(X) + \mathrm{Var}(Y)$

$$= \left(\frac{2}{nJ}\right)^2\left(\frac{n}{2}J^2D + \frac{n}{2}J\sigma^2 + \left(\frac{n}{2}\right)J^2D + \left(\frac{n}{2}\right)J\sigma^2\right)$$

$$= \left(\frac{2}{nJ}\right)^2\left(nJ^2D + nJ\sigma^2\right)$$

$$= \left(\frac{4}{n^2J^2}\right)\left(nJ^2D + nJ\sigma^2\right)$$

$$= \frac{4nJ^2D}{n^2J^2} + \frac{4nJ\sigma^2}{n^2J^2}$$

$$= \frac{4}{n}D + \frac{4}{nJ}\sigma^2$$

$$= \frac{4}{n}\left(D + \frac{\sigma^2}{J}\right)$$

Design 2:

for each clinic, $\frac{J}{2}$ patients intervention

$\frac{J}{2}$ patients control

$$(\bar{Y}_1 - \bar{Y}_0) = \frac{\left(\sum_{i=1}^{n}\sum_{j=1}^{J/2} y_{ij} - \sum_{i=1}^{n}\sum_{j=\frac{J}{2}+1}^{J} y_{ij}\right)}{\frac{Jn}{2}} = \frac{2}{nJ}\left[\sum_{i=1}^{n}\sum_{j=1}^{J/2} y_{ij} - \sum_{i=1}^{n}\sum_{j=\frac{J}{2}+1}^{J} y_{ij}\right]$$

$$= \frac{2}{nJ}\left[\sum_{i=1}^{n}\sum_{j=1}^{J/2}(x_{ij}'\alpha + \beta_i + \epsilon_{ij}) - \sum_{i=1}^{n}\sum_{j=\frac{J}{2}+1}^{J}(x_{ij}'\alpha + \beta_i + \epsilon_{ij})\right]$$

$$Var(\bar{Y}_1 - \bar{Y}_0) = \left(\frac{2}{nJ}\right)^2 Var\left[\sum_{i=1}^{n}\sum_{j=1}^{J/2}(x_{ij}'\alpha + \beta_i + \epsilon_{ij}) - \sum_{i=1}^{n}\sum_{j=\frac{J}{2}+1}^{J}(x_{ij}'\alpha + \beta_i + \epsilon_{ij})\right]$$

$$= \left(\frac{2}{nJ}\right)^2 Var\left[\cancel{\sum_{i=1}^{n}\sum_{j=1}^{J/2} x_{ij}'\alpha} + \sum_{i=1}^{n}\sum_{j=1}^{J/2}\beta_i + \sum_{i=1}^{n}\sum_{j=1}^{J/2}\epsilon_{ij} - \cancel{\sum_{i=1}^{n}\sum_{j=\frac{J}{2}+1}^{J} x_{ij}'\alpha} - \sum_{i=1}^{n}\sum_{j=\frac{J}{2}+1}^{J}\beta_i - \sum_{i=1}^{n}\sum_{j=\frac{J}{2}+1}^{J}\epsilon_{ij}\right]$$

$$= \left(\frac{2}{nJ}\right)^2 Var\left[\sum_{i=1}^{n}\sum_{j=1}^{J/2}\beta_i + \sum_{i=1}^{n}\sum_{j=1}^{J/2}\epsilon_{ij} - \sum_{i=1}^{n}\sum_{j=\frac{J}{2}+1}^{J}\beta_i - \sum_{i=1}^{n}\sum_{j=\frac{J}{2}+1}^{J}\epsilon_{ij}\right]$$

$$= \left(\frac{2}{nJ}\right)^2 Var\left[\cancel{\sum_{i=1}^{n}\frac{J}{2}\beta_i} + \sum_{i=1}^{n}\sum_{j=1}^{J/2}\epsilon_{ij} - \cancel{\sum_{i=1}^{n}\frac{J}{2}\beta_i} - \sum_{i=1}^{n}\sum_{j=\frac{J}{2}+1}^{J}\epsilon_{ij}\right]$$

$$= \left(\frac{2}{nJ}\right)^2\left(n\left(\frac{J}{2}\right)\sigma^2 + n\left(\frac{J}{2}\right)\sigma^2\right)$$

$$= \left(\frac{4}{n^2J^2}\right)\left(\frac{2nJ}{2}\sigma^2\right)$$

$$= \frac{4nJ\sigma^2}{n^2J^2}$$

$$= \frac{4}{nJ}\sigma^2$$

b) $$Var_1(\bar{Y}_1 - \bar{Y}_0) - Var_2(\bar{Y}_1 - \bar{Y}_0) = \frac{4}{n}\left(D + \frac{\sigma^2}{J}\right) - \frac{4}{nJ}\sigma^2$$

$$= \frac{4}{n}D + \frac{4\sigma^2}{nJ} - \frac{4\sigma^2}{nJ}$$

$$= \frac{4}{n}D \geq 0 \quad \text{with } J, n > 0$$

$Var(\bar{Y}_1 - \bar{Y}_0)$ for Design 1 is larger

$Var(\bar{Y}_1 - \bar{Y}_0)$ for Design 2 is smaller since we know $D \geq 0$

c) Yes, sometimes randomized block design will be more effective than randomized patient design if there is a possibility of cross contamination on patients affecting trial results — randomized block design is effective for grouping according to known/suspected variation isolated by block design