Recoding variables (2), model validation

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Cleaning messy data

Load messy titanic data

- From terminal: git pull
- In Rstudio: Open project, intermediate_stats

```
# titanic<-read_csv('./slides/data/titanic_messy.csv')
```

Data cleaning lab

Break

Model comparison and validation

Motivation for model comparison

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In the Ordinary Least Squares context

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \varepsilon_i$$

What is the primary characteristic of the vector of β that the OLS method returns? (Hint: it involves the residuals)

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What does RSS mean in plain English?

What’s the most common single measure you use to assess the goodness-of-fit of an OLS model?

Moving from RSS to goodness-of-fit

$$R^2 = 1 - \frac{\sum(y_i - \hat{y})^2}{\sum(y_i - \bar{y})^2}$$

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3. What does this ratio tell us?

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3. What does this ratio tell us?
4. When can we compare R^2 across models?
5. What does RSS look like for logistic models?

For more complex models

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Fun fact: For the normal likelihood model, OLS==MLE

What is a likelihood?

A likelihood function is a function we use to identify parameters for a model given our data. It depends on the probability distribution we use to model our data.

We can think of a likelihood $L(y|\theta)$ as describing the probability of observing our data given a set of parameters.

Options for GoF under MLE

R^2 only describes the proportion of variance explained under a Normal likelihood model.

The likelihood ratio test is similar to comparing R^2. We can directly compare the likelihood of the data conditional on our estimated model for two models as:

$$LR = \frac{L(y|\theta_1)}{L(y|\theta_2)}$$

Conveniently, we can use a χ^2 distribution to perform a significance test on whether model 2 fits better than model 1.

An example: returning to the titanic

```r
titanic <- read_csv("./data/titanic.csv")
m0 <- glm(Survived ~ Sex, data = titanic, family = "binomial")
m1 <- glm(Survived ~ Sex + Age, data = titanic, family = "binomial")
anova(m0, m1, test = "LRT")
```

```
## Analysis of Deviance Table
##
## Model 1: Survived ~ Sex
## Model 2: Survived ~ Sex + Age
##   Resid. Df Resid. Dev Df Deviance Pr(>Chi)
## 1       885     916.12
## 2       884     916.00  1  0.12358   0.7252
```

Likelihood ratio test Example (cont.)

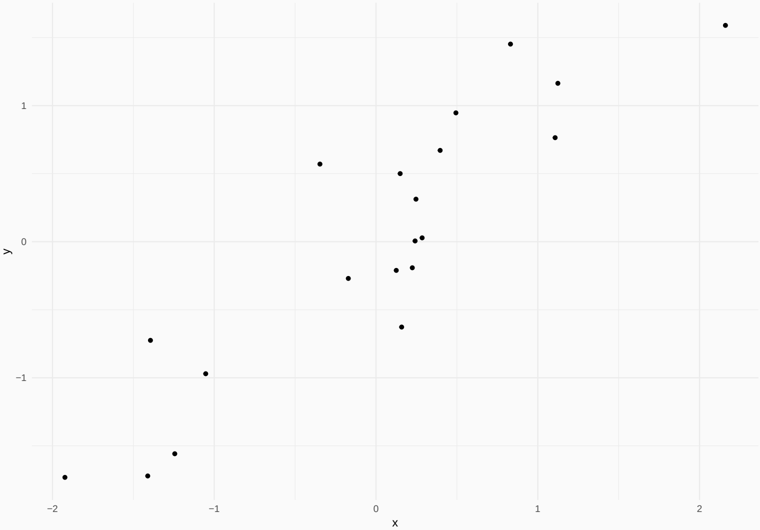
```r
m2 <- glm(Survived ~ Sex + Age + factor(Pclass), data = titanic, family = "binomial")

anova(m0, m2, test = "LRT")
```

```
## Analysis of Deviance Table
##
## Model 1: Survived ~ Sex
## Model 2: Survived ~ Sex + Age + factor(Pclass)
##   Resid. Df Resid. Dev Df Deviance  Pr(>Chi)
## 1       885     916.12
## 2       882     801.59  3   114.53 < 2.2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Conditions for LRT and R^2

- Identical outcome variables
- Nested models (parameters in model A are a subset of parameters in model B)

Dangers of relying exclusively on goodness-of-fit measures



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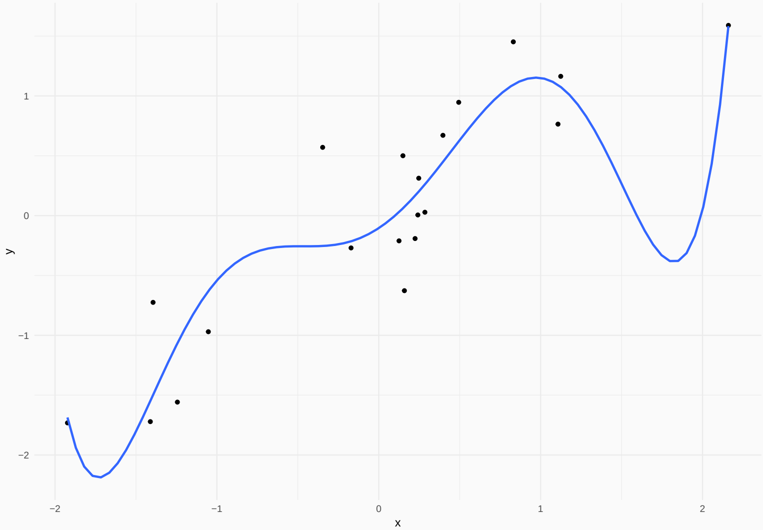

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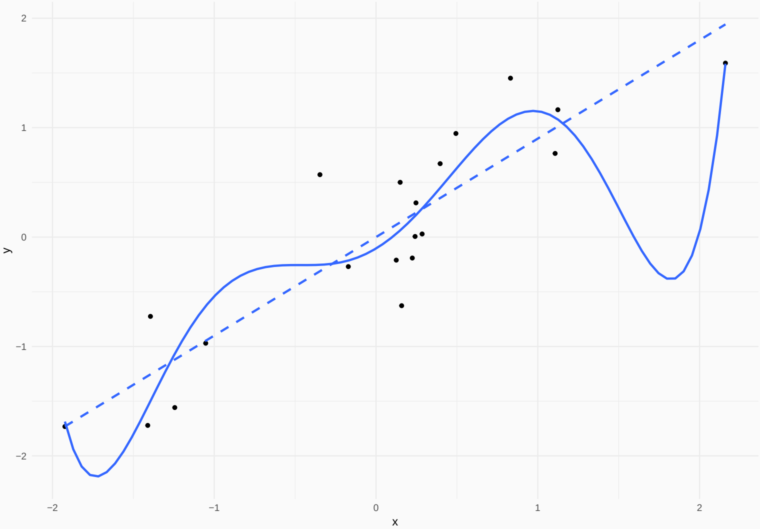

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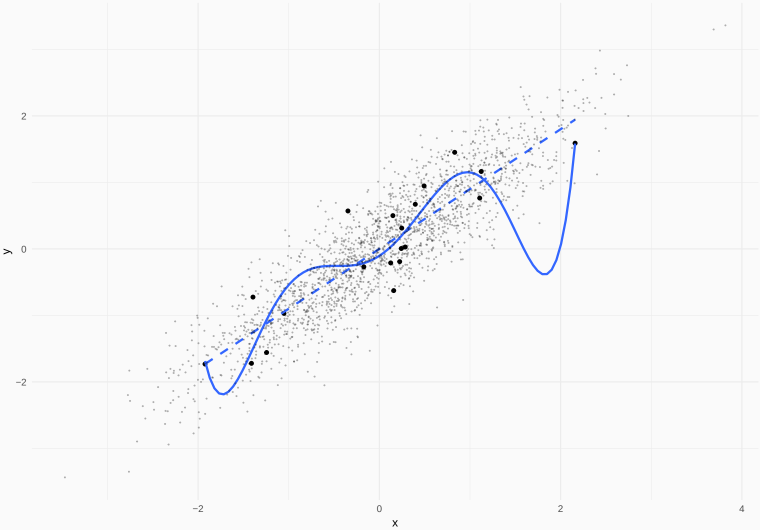

A more general approach: Bayesian Information Criteria

BIC is a general approach to comparing models estimated through MLE that is similar to a likelihood ratio test.

$$BIC = ln(n)k - 2ln(L)$$

Where *n* is the number of observations in our data, *k* is the number of parameters in the model, and *L* is the maximum of the likelihood function of our model.

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How does BIC differ from a likelihood ratio test?

Advantages of BIC

- Directly compares goodness of fit
- Does not require nested models (does require identical outcomes)
- Easy to compare models
- Penalizes models for complexity, helps avoid overfitting

Models with low BIC fit better than models with high BIC

BIC example

```
BIC(m0)
```

```
## [1] 929.6981
```

```
BIC(m1)
```

```
## [1] 936.3624
```

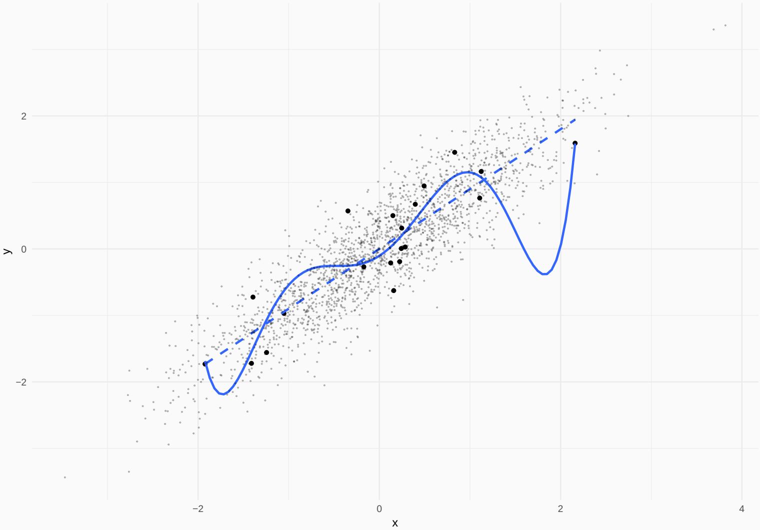
```
BIC(m2)
```

```
## [1] 835.5333
```

So we conclude that m2 is the better fit, because

$$BIC(m0) - BIC(m2) > BIC(m0) - BIC(m1)$$

Returning to our overfit example



Returning to our overfit example

```
formula(m_true)
```

```
## y ~ x
```

```
summary(m_true)$r.squared
```

```
## [1] 0.81
```

```
formula(m_of)
```

```
## y ~ x + I(x^2) + I(x^3) + I(x^4) + I(x^5) + I(x^6)
```

```
summary(m_of)$r.squared
```

```
## [1] 0.8400007
```

```
BIC(m_true)
```

```
## [1] 31.50425
```

```
BIC(m_of)
```

```
## [1] 43.04582
```

General advice

1. Include theoretically sensible predictors
2. Include theoretically sensible interactions
3. Check how your model fits your observed data (i.e. ROC curves, residual/fitted plots, R^2)
4. Use BIC to compare models
5. Think about whether overfitting might be occurring and adjust for parsimony
6. Use cross-validation and other data science-y techniques to check predictive performance