# Intermediate statistics: introduction

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# Introductions: What are you planning to do with statistical models?

# Before we begin

Remember: All models are wrong, some are useful.

# What we will cover

- How to explore, visualize, and model diverse kinds of data with an emphasis on generalized linear models

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- How to explore, visualize, and model diverse kinds of data with an emphasis on generalized linear models
- How to program in R
- Developing a workflow for producing replicable quantitative social science
- Some advanced topics that are relevant for the kinds of data we're dealing with in the course, subject to class interest

## Review the syllabus

https://f-edwards.github.io/intermediate_stats/

# My general approach to data analysis

1. Explore and visualize data

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1. Explore and visualize data
2. Fit models
3. Assess model fit
4. Interpret and describe results through simulation

# The Generalized Linear Model

## The linear model

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Where the likelihood for the outcome conditional on the data takes the form:

$$Y|X \sim Normal(\mu, \sigma^2)$$

# Generalzing the linear model

The linear model:

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Can be written as a more general formulation for a likelihood function $f$

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Now we can extend the (very) useful linear model to data with discrete outcomes

# Generalizing the linear model

A linear predictor $\eta$:

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A linear predictor $\eta$:

$$\eta = \mathsf{X}\beta$$

A link function $g$

$$g(E(Y|X)) = \eta$$

A mean expectation $E(Y|X) = \mu$

$$\mu = g^{-1}(\eta)$$

# From OLS to GLM

OLS:

$$Y|X \sim Normal(\mu, \sigma^2)$$

GLM, for a likelihood function $f$ with parameters $\theta$:

$$Y|X \sim f(\theta)$$

## Diverse likelihood functions

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- Categorical data: Multinomial model
- Count data: Poisson and negative binomial models
- Positive continuous data: Gamma model

# Getting started: software

# Required installations

All software we are using is free and open source.

*Install R*:

`https://cran.r-project.org/`

*Install RStudio*:

`https://www.rstudio.com/products/rstudio/download/`

## Recommended software: Git and GitHub

Git and GitHub are powerful tools for backing up and sharing your research.

All course materials, source code, and most of my research are hosted on GitHub (`https://github.com/f-edwards`).

*Install Git*:

`https://git-scm.com/`

*Set up a GitHub account*:

`https://github.com/`

*Using GitHub for social science*:

`https://happygitwithr.com/`

# Recommended software: LaTeX

LaTeX is a powerful typesetting tool that works well with RMarkdown. It makes very attractive academic papers and slides.

Install it from the console

```
install.packages(”tinytex”)

tinytex::install_tinytex()
```

# Break

# Returning to the linear model

# What do we know about the linear regression model?

$$y = \mathsf{X}\beta + \varepsilon$$

$$\varepsilon \sim Normal(0, \sigma^2)$$

1. What forms can y take?

# Review

1. What forms can y take?
2. What assumptions does the linear regression model require?

# Assumptions of linear regression model

1. Validity of data relative to the research question

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# Let’s analyze some data?

# Two ways to access course data

- All data is accessible through the the course website (see the data link, or data folder on the GitHub page)
- *Recommended approach:* In a terminal (terminal.app on mac, Git Bash on windows):

```
git clone
https://github.com/f-edwards/intermediate_stats.git
```

Before beginning your work each session, pull updates I've pushed to the repo with:

```
git pull
```

Now you have an intermediate_stats folder with all code, slides, and data. Data is in intermediate_stats/data

# Read in

```
#library(tidyverse)
### directly from the web
cj_budgets<-read_csv("./hw/data/revenue_dat.csv")
### from a project directory root
#cj_budgets<-read_csv("./hw/data/revenue_dat.csv")
```

## About the data

Data are for an ongoing research project I'm working on. It's real, and can be a bit messy!

It documents police involved deaths, demographics, and local government budgets at the county-level for two time periods, 2007-11 and 2012-16. Datasets used include Fatal Encounters, American Community Survey, Annual Survey of State and Local Government Finance, and Uniform Crime Reports.

Full code for the project is up at:

`https://github.com/f-edwards/police-mort-revenue`

`merge.r` contains the code to make this merged file from a variety of source files (available if you want the raw data).

# Evaluate the structure of the data

```
head(cj_budgets)
```

```
## # A tibble: 6 x 33
##   year_range fips_st fips_cnty deaths exp_tot exp_correction exp
##   <chr>      <chr>   <chr>      <dbl>   <dbl>          <dbl>
## 1 2007-2011  01      001            3  4.97e7       2101800
## 2 2007-2011  01      005            1  2.86e7       1037880.
## 3 2007-2011  01      007            0  1.30e7         80600
## 4 2007-2011  01      009            0  3.66e7       1703760
## 5 2007-2011  01      011            0  1.09e7             0
## 6 2007-2011  01      013            1  3.05e7        487320
## # ... with 26 more variables: exp_welfare <dbl>, rev_tot <dbl>,
## #   rev_fines <dbl>, rev_gen_ownsource <dbl>, rev_int_gov <dbl>,
## #   rev_prop_tax <dbl>, rev_tax <dbl>, pop_tot <dbl>, pop_pct_me
## #   pop_wht <dbl>, pop_blk <dbl>, pop_ami <dbl>, pop_api <dbl>,
## #   pop_pct_pov <dbl>, pop_pct_deep_pov <dbl>, pop_med_income <d
## #   pop_pc_income <dbl>, violent.yr <dbl>, property.yr <dbl>, mu
```

# Evaluate the structure of the data

```r
nrow(cj_budgets)
```

```
## [1] 4286
```

```r
table(cj_budgets$year_range)
```

```
##
## 2007-2011 2012-2016
##      2308      1978
```

# Evaluate the structure of the data

```
names(cj_budgets)
```

```
##  [1] "year_range"         "fips_st"            "fips_cnty"
##  [4] "deaths"             "exp_tot"            "exp_correction"
##  [7] "exp_police"         "exp_welfare"        "rev_tot"
## [10] "rev_fines"          "rev_gen_ownsource"  "rev_int_gov"
## [13] "rev_prop_tax"       "rev_tax"            "pop_tot"
## [16] "pop_pct_men_15_34"  "pop_wht"            "pop_blk"
## [19] "pop_ami"            "pop_api"            "pop_lat"
## [22] "pop_pct_pov"        "pop_pct_deep_pov"   "pop_med_income"
## [25] "pop_pc_income"      "violent.yr"         "property.yr"
## [28] "murder.yr"          "ft_sworn"           "cbsa"
## [31] "metroname"          "dissim_bw"          "dissim_wl"
```

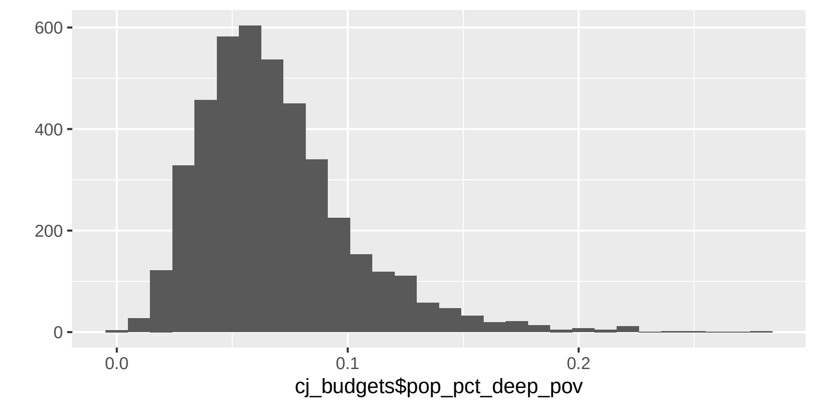
# Descriptives

```
summary(cj_budgets$pop_pct_deep_pov)
```

```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
## 0.00000 0.04553 0.06285 0.06884 0.08442 0.27901
```

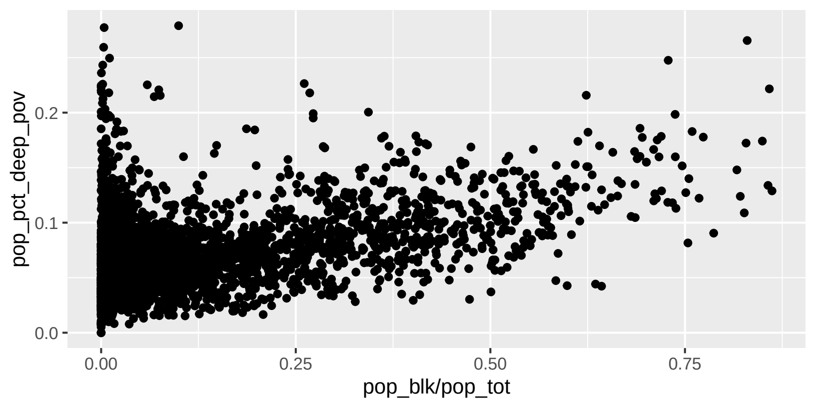
Call individual variables (columns) in a data frame with $, like \texttt{USArrests$Murder}

```
qplot(cj_budgets$pop_pct_deep_pov)
```



# Visualize a bivariate relationship

```
qplot(pop_blk/pop_tot,
      pop_pct_deep_pov,
      data = cj_budgets)
```



# Fitting a linear model

```
model_1<-lm(pop_pct_deep_pov ~
              I(pop_blk/pop_tot),
            data =cj_budgets)
```

# Display the model fit

```
summary(model_1)
```

```
##
## Call:
## lm(formula = pop_pct_deep_pov ~ I(pop_blk/pop_tot), data = cj_bu
##
## Residuals:
##       Min        1Q    Median        3Q       Max
## -0.079709 -0.019343 -0.004579  0.013753  0.217773
##
## Coefficients:
##                     Estimate Std. Error t value Pr(>|t|)
## (Intercept)        0.0591712  0.0005603  105.61   <2e-16 ***
## I(pop_blk/pop_tot) 0.0977188  0.0030884   31.64   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

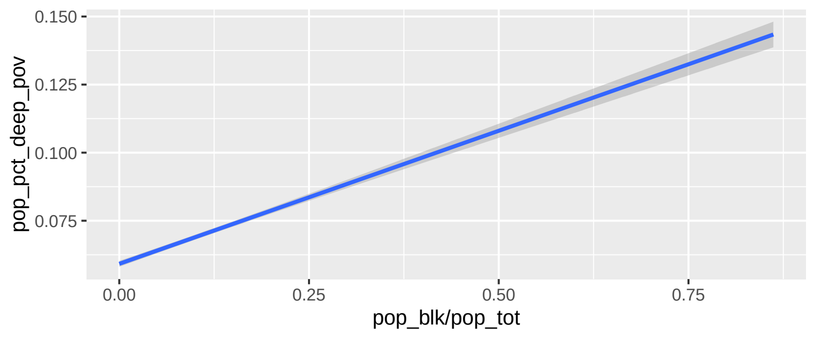
## Display the model fit (nicer)

```
library(broom)
tidy(model_1)
```

```
## # A tibble: 2 x 5
##   term                estimate std.error statistic   p.value
##   <chr>                  <dbl>     <dbl>     <dbl>     <dbl>
## 1 (Intercept)           0.0592  0.000560     106.  0.
## 2 I(pop_blk/pop_tot)    0.0977  0.00309       31.6 1.22e-197
```

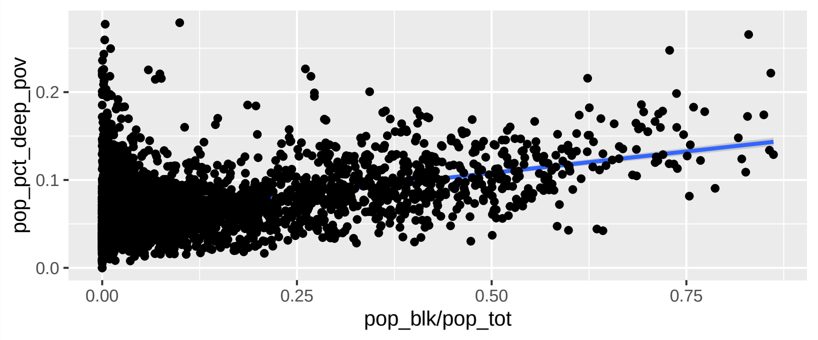
## Visualize the model fit

```
library(ggplot2)
ggplot(cj_budgets,
       aes(x=pop_blk/pop_tot, y=pop_pct_deep_pov))+
  geom_smooth(method = "lm",
              formula = y~x)
```



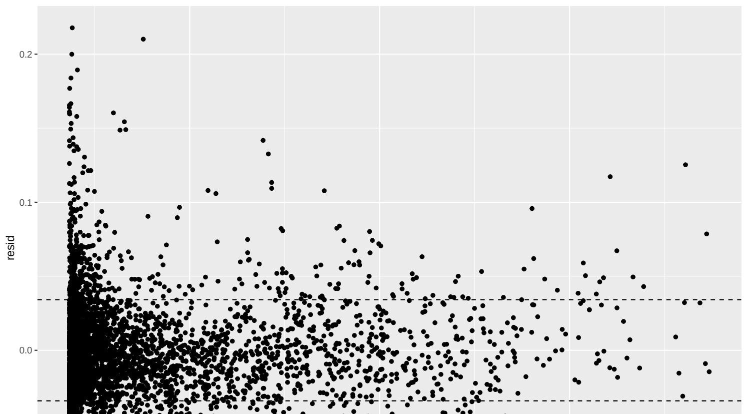
# Visualize the model fit (against the data)

```
library(ggplot2)
ggplot(cj_budgets,
       aes(x=pop_blk/pop_tot, y=pop_pct_deep_pov))+
  geom_smooth(method = "lm",
              formula = y~x) +
  geom_point()
```



# Residuals vs fitted

```
sd_outcome<-sd(cj_budgets$pop_pct_deep_pov)
plot_dat<-data.frame(resid = model_1$residuals, yhat = model_1$fitt
ggplot(plot_dat, aes(y = resid, x = yhat)) +
  geom_point() +
  geom_hline(yintercept = sd_outcome, lty=2) + geom_hline(yintercep
```



# Can we fit a better model?

# Homework programming prep

# Needed concepts / syntax

HW 1 asks you to apply some basic programming, data wrangling, and data visualization to common linear regression challenges.

- Reading data
- Loops
- Lists
- Matrix and data frame indexing
- dplyr::mutate
- ggplot2
- RMarkdown

# HW tips

- Work together!
- Check the Wickham text from the syllabus or other online R courses
- Google it: StackOverflow will become your best friend
- Accept that this is hard and you will probably struggle with it