Entanglement Distillation

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Motivation

Entanglement

- Joint state which cannot be "seperated"
  - i.e $|\phi\rangle$ is seperable $\leftarrow \nexists\ |\phi_1\rangle\,,\ |\phi_2\rangle\ s.t.\ |\phi\rangle = |\phi_1\rangle \otimes |\phi_2\rangle$
- Conditional Von-Neuman entropy quantifies the level of entanglement
  - $S(A|B) - S(B)\langle 0$ implies that ρ_{AB} is entangled
  - $S(A|B) - S(B) = -1$ imples that ρ_{AB} is maximally entangled

Example 1

Conditional entropy of a pure state

$$
\begin{aligned}
\rho_{AB} &= \left|\phi_{00}\right\rangle \left\langle\phi_{00}\right|_{AB} \\
S(A|B) - S(B) &= H(eig(AB)) - H(eig(B)) \\
&= H(1,0,0,0) - H(.5,.5) \\
&= 0 - 1 \\
&= -1
\end{aligned}
$$

Example 2

Conditional entropy of a noisy entangled state

$$
\begin{aligned}
\rho'_{AB} &= .8 \left|\phi_{00}\right\rangle \left\langle\phi_{00}\right|_{AB} + \frac{.2}{4} I \\
S(A|B) - S(B) &= H(eig(AB)) - H(eig(B)) \\
&= H(.85, .05, .05, .05) - H(.5, .5) \\
&= .84 - 1 \\
&= -.16
\end{aligned}
$$

Applications

- Quantum Teleporation
- Quantum Error Correction
- Quantum Key Distribution

Problem

- Necessary to transfer/store entangled particles
- Quantum channels are noisy, storage is decoherrent
- Maximally entangled states degrade into less entangled states
- We want to find a way to increase entanglement at a distance

Entanglement Distillation

Background

- Fidelity
  - "Closeness" of a state to $|\phi_{00}\rangle \langle\phi_{00}|$
  - $F = \langle\phi_{00}| \rho |\phi_{00}\rangle$
- Werner States
  - Invariant under all unitary operators (up to a phase)
  - $\rho = U\rho U^*$
  - For our purposes they take the form $x |\phi_{00}\rangle \langle\phi_{00}| + (1-x)I$
- Depolarization
  - A process of removing off-diagonals/equalizing eigenvalues
  - Can be used to turn an arbitrary state into a Werner state
  - Preserves fidelity of original state

BBPSSW, a Recurrence Purification Protocol (1/5)

- Described in a paper by Bennet et al. in 1996
- Allows the purification of states with arbitrary accuracy
- Can be applied recursively
    - but you need 2^n qubits
- Requires two way classical communication between Alice and Bob

BPPSSW, Three Simple Steps (2/5)

1. Depolarize ρ to a Werner state
   - For example, $x \left|\phi_{00}\right\rangle \left\langle\phi_{00}\right| + (1-x)\frac{1}{4}I$
2. Bilateral CNOTs on the entangled states
3. Measure the target pair. If the measurement is the same keep the pair and repeat. If not, discard the pair.

BPPSSW, Bilateral CNOTS (3/5)

Alice sends Bob qubits 2 and 4. Qubit 1 is entangled with 2 and 3 is entangled with 4.

CNOT	Source	Target
Alice's	qubit 1	qubit 3
Bob's	qubit 2	qubit 4

Construct Alice's CNOT: $|0\rangle\langle 0| \otimes I \otimes I \otimes I + |1\rangle\langle 1| \otimes I \otimes \sigma_x \otimes I$ Construct Bob's CNOT: $I \otimes |0\rangle\langle 0| \otimes I \otimes I + I \otimes |1\rangle\langle 1| \otimes I \otimes \sigma_x$

Alice's CNOT

$$\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}$$

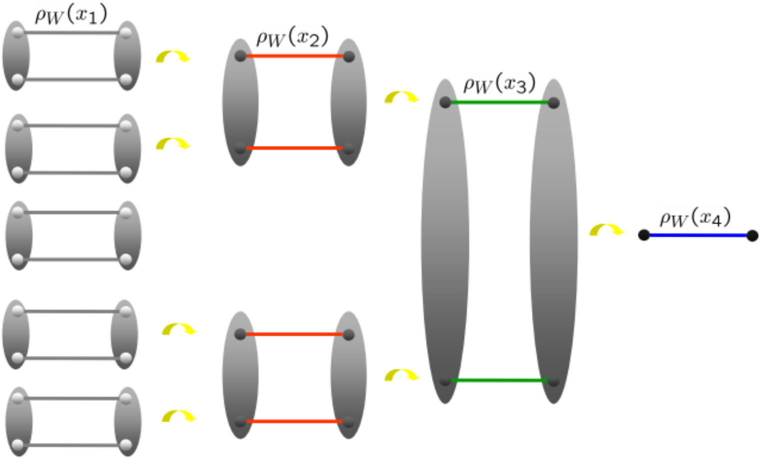
BPPSSW, Deciding to Keep or Discard (4/5)

Alice and Bob choose to measure one of their two pairs in the Z basis. Depending on how pure the entanglement is, it will collapse to a more pure or less pure state.

- If the state is already pure, then measuring a $|1\rangle$ or $|1\rangle$ at one of the entangled qubits will collapse the other qubit into the same state
  - Ex: $|\phi_{00}\rangle$ will collapse to $|00\rangle$ with probability $\frac{1}{2}$ and to $|11\rangle$ with probability $\frac{1}{2}$
  - If $|01\rangle$ or $|10\rangle$ is observed, then the entanglement must have been weakened by noise in the quantum channel
- We don't have any of the $|\psi\rangle$ bell states because the state was depolarized

BPPSSW, Iterations (5/5)

This protocol can be repeated indefinitely.



Entanglement Pumping

- BPPSSW protocol requires exponential number of qubits
  - Not practically feasible
- Entanglment pumping trades spacial requirement for temporal requirement
  - Recieve new “elementary” entangled pairs as protocol advances
  - Probabilistic - but polynomial time

Entanglement Pumping

- Purify one system using another recieved noisily entangled pair
  - Use the two systems for one round of BPPSSW
  - If successfully purify, repeat
  - Else restart (with two new elementary pairs)
- If a purification round fails, whole process restarts
  - But reach arbitrarily high fidelity rather quickly
- Trade-offs:
  - Longer process
  - Can't be parrallelized
  - Requires transmission of qubits
- Can be generalized to $N \mapsto M$ protocols

Questions

References

- Dür, W., & Briegel, H. J. (2007). Entanglement purification and quantum error correction. Reports on Progress in Physics, 70(8), 1381.
- Bennett, C. H., Brassard, G., Popescu, S., Schumacher, B., Smolin, J. A., & Wootters, W. K. (1996). Purification of noisy entanglement and faithful teleportation via noisy channels. Physical review letters, 76(5), 722.