Syllabus Math 513 (Lie theory)

Prof: C. Negron
Time and Location: MWF 12:00–12:50, DMC 207
Text: Humphreys *Introduction to Lie algebras and representation theory*
Office Hours: Wed 2-3 pm or Wed 3-4 pm
Website: c-negron.github.io/513_F24

Prerequisites
Math 510A, 510B.

Course description
The course provides an introduction to the representation theory of semisimple Lie algebras over the complex numbers. Topics include the classification of semisimple Lie algebras via Dynkin diagrams and weight systems, descriptions of semisimple Lie algebras via their Cartan and Borel subalgebras, and the classification of representations for semisimple Lie algebras via dominant weights.

Learning objectives
Students will learn the structure theory and representation theory of complex semisimple Lie algebras. This includes both the internal structure of such algebras (Cartan and Borel subgroups) and the classification of semisimple Lie algebras by root systems and Dynkin diagrams. It also includes the character theory of finite dimensional representations, in particular the classification of irreducible representations by dominant weights. Students will also learn about the relationship between Lie algebras and Lie groups, as well as the universal enveloping algebra of a Lie algebra.

Upon successful completion of this course, students will be able to recognize Lie algebra representations in practice, and use such representations to address structures in cohomology, function algebras, etc. Students will also be able to employ Dynkin diagrams and root systems as generating data for various classification schemas.

Homework and Tests
Homework will be essentially biweekly, and turned in in class. Most problems will be out of Humphrey's text. Homework will be posted on the course website. There will be a single final exam. The date and time for the final exam can be found in the Schedule of Classes.

Grades
The final grade will be calculated as 90% homework and 10% final.

Biweekly outline

Weeks 1 and 2	Ch. I Humphreys, ideals, (auto)morphisms, solvable and nilpotent Lie algebras, Engel's theorem.
Weeks 3 and 4	S.4–6 Lie's Theorem and Cartan's criterion, Killing form, reducibility of representations.
Weeks 5 and 6	S.7–S.9 Simple representations of $\mathfrak{sl}_2$, root systems.
Weeks 7 and 8	S.10 Simple roots and Weyl group.
Weeks 9 and 10	S.11–13 Classification and abstract weights.
Weeks 11 and 12	S.14–16 Varying Cartan subalgebras and Borel subalgebras.
Weeks 13 and 14	S.17 Verma modules, dominant weights, and simple representations.
Week 15	S.22 Character formulas

The final exam will occur at the date and time indicated in the Schedule of Classes.

Statement on Academic Conduct and Support Systems

Academic Conduct
Plagiarism – presenting someone else's ideas as your own, either verbatim or recast in your own words – is a serious academic offense with serious consequences. Please familiarize yourself with the discussion of plagiarism in SCampus in Part B, Section 11, "Behavior Violating University Standards" policy.usc.edu/scampus-part-b. Other forms of academic dishonesty are equally unacceptable. See additional information in SCampus and university policies on scientific misconduct, policy.usc.edu/scientific-misconduct.

Students and Disability Accommodations
The university is committed to the full accessibility of our campus, programs, and activities to individuals with disabilities. USC will make a good faith effort to provide reasonable accommodation for qualified visitors, prospective students, enrolled students, employment applicants, and employees with a disability unless the accommodation requested would cause an undue hardship as defined by the ADA. In compliance with Section 504 of the Rehabilitation Act, the university provides academic adjustments and auxiliary aids for students with disabilities.

For the full text, including university disability services contact information see USC Disability Accommodations Policy at policy.usc.edu/disability-accommodations/. Contacts and basic information regarding accessibility services can be found USC's Office of Student Accessibility Services, osas.usc.edu/.