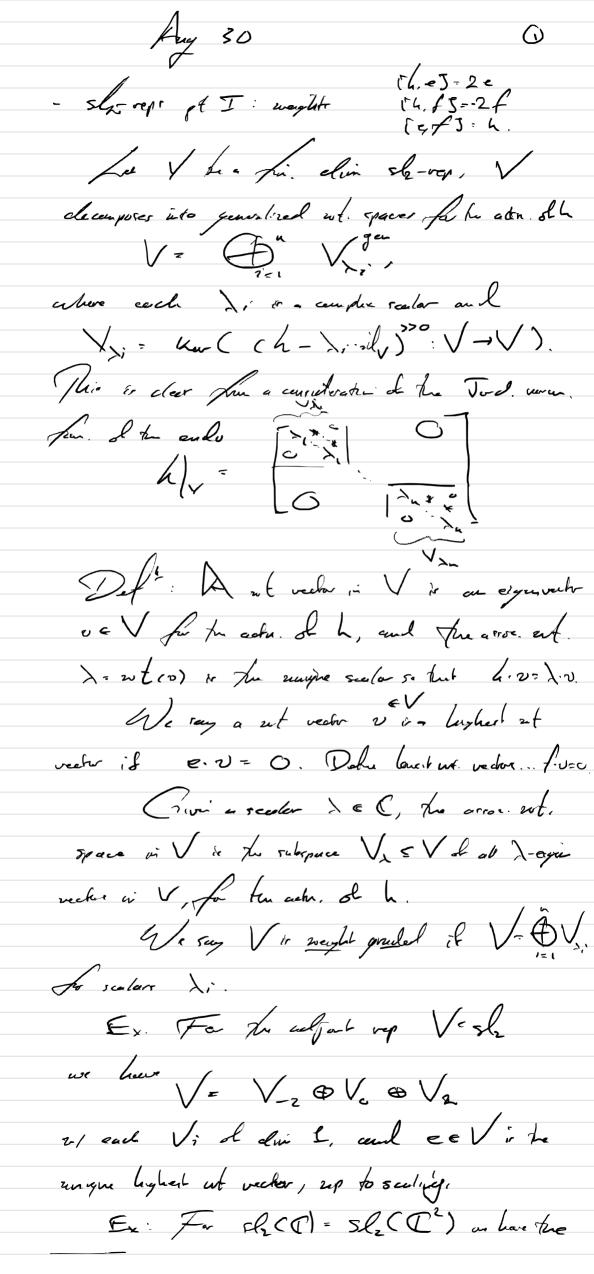
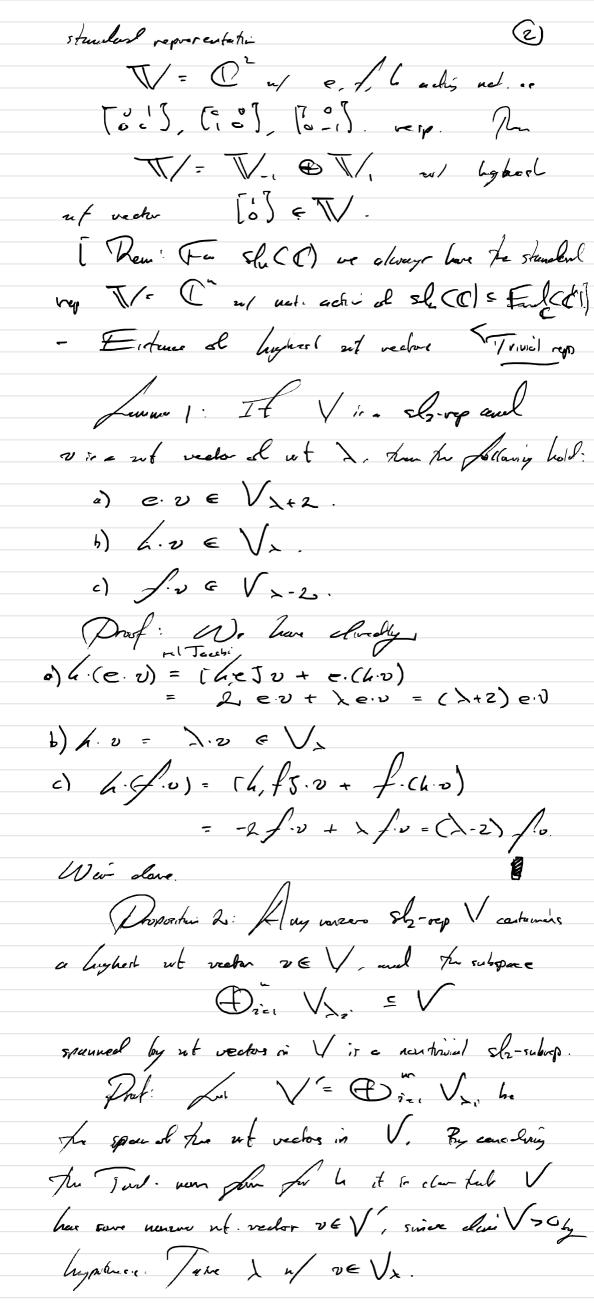
0 Eratt- to Any 28: In order for all dij. in an obelien cat Che here camp series C much be Loth Hatinin and Noetheron . So, in the returned of [Prop 4, Any 285, which char semisurplemen vi ext. of simpler, we should replace Let Che Auturai" with Let C be Artinian and Noelmeni. It is in the Art + North setting That all stipeck have specified length and composition factors. We note that at familiai Aut. cats are already North as well: Ex: The cel Veet of finite-divi vect spaces ir both And, and Noch. Ex 2: The cal A-most of fir-din modules over any C-ely A : both Art and North. Ex 3: The cas R-medfy of five gard med. over an Antinian ming R is both Aut and Noch. Ex 4: The cat rep (of) of fundin of repr. for any his aly of is both Ant. and Noeth. Auti-Ex 5: The opposite cal (CIXI-monly) ir Artiniai but not Noeth. Thee for construints hald for Examples 1,2,4 Se simple dineuria visiono.





By freumy (en. v E / x+2n, and 3) by fire dim nos V1+2n = O file a layen. So her existe som mumal no u/ e'v + 0 and e vot 1 e l'en fire a Leghal wt. vector in V.

The latter closes, that V = V is weather. I

on loop follow by Lanema. 1. Conlay 3: Every rapple slz-veprerentahe V is weight greeled,

- Die Vien Vien for shroups: Let V be a 5 mple 5th - vep. The

o) Vhes a renique highest who weeks v, v to scaling non-new integral (!)

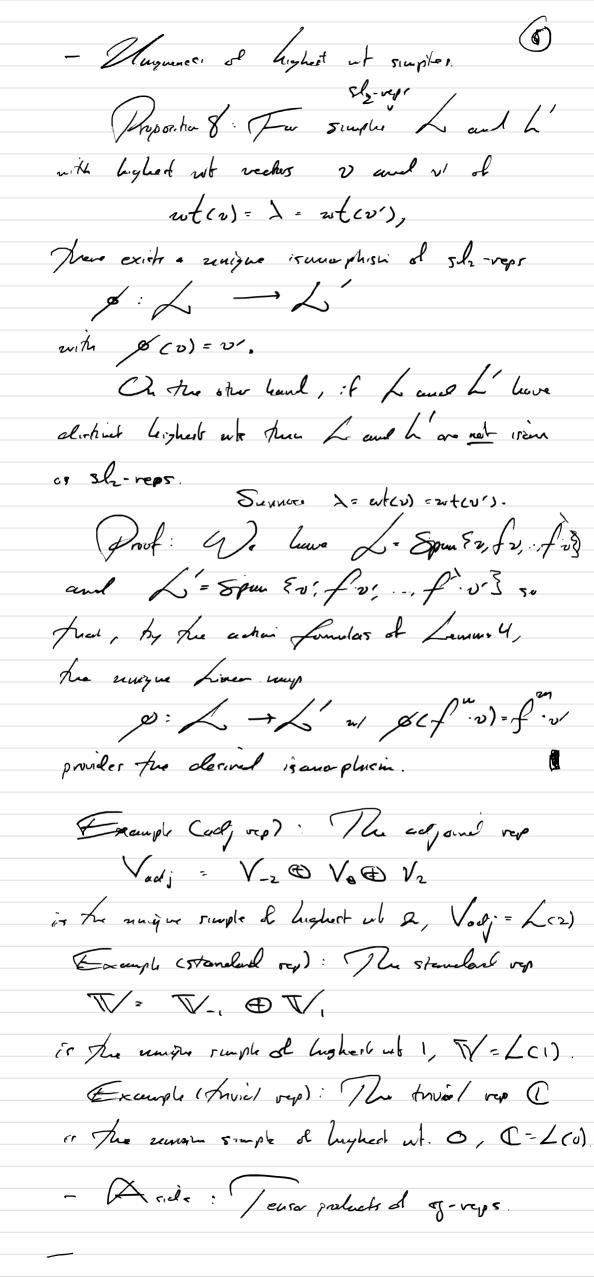
b) The highest at weeker v has v the e) The univer sot spaces is V we precioely V2-2m for OEMEL. d) For each Osmes, Vi-zu is 1-di and spenned by I'm.
We decompres the pref into a say. I Lemmes
and Noir consequences. Lauren 4: If ve V a a lighest weight nedo of weight her, for each M20, and e. ()= m (\ -m+1) f ... e () = [/ k () - k+1)] · 2.

Prof: W. home $e. \int u = \int e_{i} \int u = \int u^{-1-i} \int u^{-1-$ = m(1 - m+1) - fm-1.v. The rescul eg is an immediate convey neace of the fort.

Loveta 5: If V & a varie or, fra-dim solz-vep, Then for any highest ut vector $v \in V$, $wt(v) \in \mathbb{Z}_{\geq 0}$. Brook: Take \= wteo). Since V.x for dui V2-2 m vanisher for m. so. Hence / "2 =0 for 21 >20, and hy remar 4 ne see

(x. () - k+1) = 0 for rome K>0. =) = K-1 for rome k >0 =) > & nonnegative integoral. Proporchio 6: Il Vra findui str-vep. as highest art recker a of at \ =0. The fin v = 0 if and cuts if m > 1, and he realers Ev, fin fin & spen a sumple ste subrep L(1) = V which har muyne highest ut vector v, up to scaling, and te de din din L(X) = x+1.

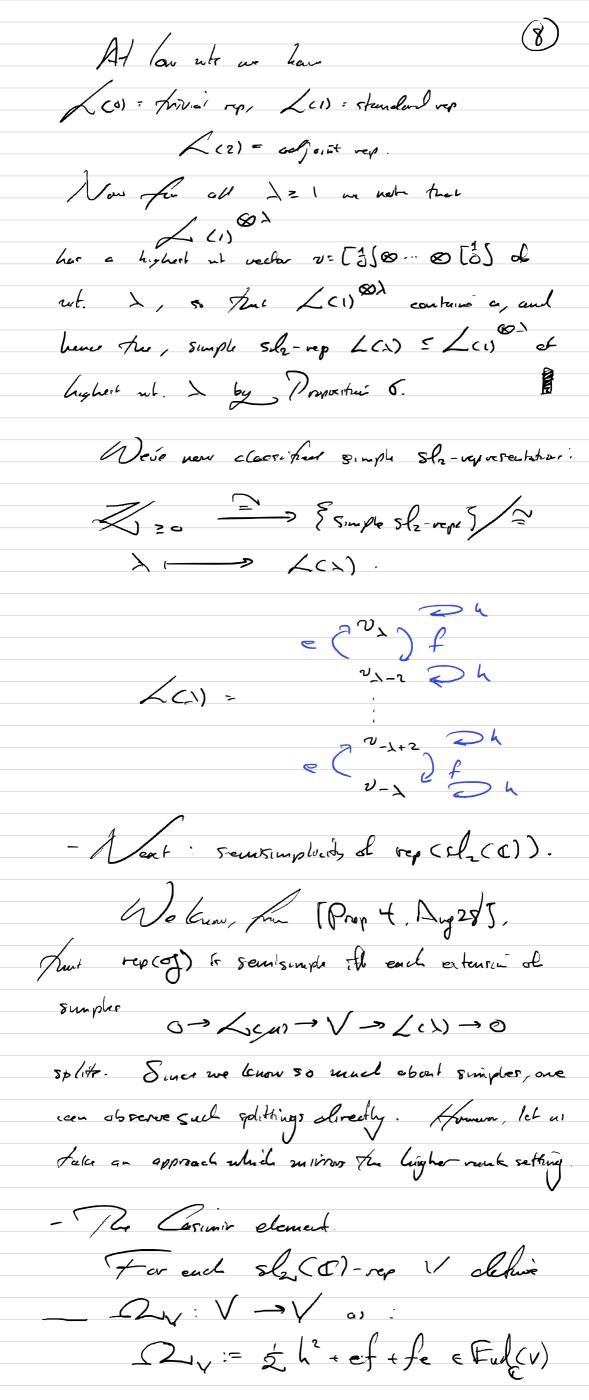
Prost: By L famula from Lewis 4, et fin: W.v for o new wo scalar W, so there for y o whenever m < \. $2 + 1 \quad \text{in have}$ $= (\lambda + 1) (\lambda - \lambda) \cdot \int_{-\infty}^{\lambda} v = (\lambda + 1) (\lambda - \lambda) \cdot \int_{-\infty}^{\lambda} v = 0$ where the flived or fit an hypert ut verto of ut 1-2(1+1)=-1-2 < 0. By Carollany 5 & layhed we western of negative at when m >). The fact Las is a subser ; i.e. so cloud unter for actions of e, f, and h, i's unevolute for Lemme 4. For simplicity, my neuzen rubres < = (A) har a lughert nt vector wed, which it there fore a highest ut week in LCD. But he als he, Let ul- vector in L(X) is 2, up to realing, so that V = C.W for some realor c, VEL, and beene LCA) = Span & for ~3 = L, ro Aut L=L(1). Corlley 7: May simple str-vep L her a serique highest ut rector , up to realing, >= ret(r) is a no-vegetini sutriger, dui L = >+1. Defo: For any simple show L, w/ highert ut vector of nt > 30, ne say Lir - simple of begliech ub).



Leume 9: Let of be on arbitrary Lie algebra. For my kno og-veps V and W The Lencor product VOW = VORW admit a ruighe of -rep structure under the setie X. (ven):= (x.v) ent vex.w). Pref. For each x & of we have he ender $\times_{\mathcal{V}}: \mathcal{V} \to \mathcal{V}$ and $\times_{\mathcal{W}}: \mathcal{W} \to \mathcal{W}$ 50 has in hour for coros. I wan endo Xv & idu + idv & xw: V&W-> VeW, vir netwelly I the tousar graduat. We danie And for cason. line mup

Vow 5 - gl (V&W), Vow = 4,000 d

+,iley, defines a of rep structure or the hence pruland. We deat relative Tacobo climatry on morando 14 V & W, [x,y]. (vow): [x,y].vow + vo 5x,y].w = xyvev + xveyw + y·vex.w + vexyw - y x v & W - y v & x W - x v & y W - v & y x W = $\times \cdot y \cdot (v \otimes w) - y \times (v \otimes w)$. Exemple: Let L' be Simple sole-veps of higher with and & vesp. Let OGL and v'EL' be highert wh Men vov'ir a highest ut rector in LoL'aul 4. (vev') = 4.200° + 066.2' = (x+x) (vev'). So LoL' contains a highest at recker of - Exotence and uniqueness for simple str-verol Theory 10: For each 20, have erite e ringue simple shi-representations L(1) of highert ub. I. Furtherware, for any hishert ut v, we have froto for all med and LC1) = spara & v, f.v, ... f.v3. (4) Prof: Uniquence use covered in Proposition 8, and the structure (follows by Carolley 7. So we need only establish existence.



Leuma 11: a) Far each map 5: V-W of she vers, the diagram b) Each liver endo QV is in fact an Sta-(men endo of V c) For each sumple rep ((1), X= /20. (2) = \frac{1}{2} \lambda (\lambda + 2) \cdot \side \lambda (\lambda). Prof: a) Is clear as at each we Vue have {(= h + e.f + f.e) ·v) = (1 h + e f + f e) {(v), vii starlinenty of F. (b) We went Joshow X. Dy = DVX freed XESle, 1.0. (x RVJ=0 10 of(V)= End(V)Cié Hovever this follows his the cakelating (h, = h2+ef+fe) = 2ef+(-2)ef+(-2)fe+2/e Te, sheteftel = - eh-he + eh +he (f, & h2 + ef + feJ = fh + hf - hf - fh c) By Schuri Lemma Endst. (L(1))=0 St that QL(x) = C. sel for rome Scaler C. We can find the scalar a by evaluating on the highest ut vector ve LCX). We have (= h² + ef + fe). 0 = = = 2 2 0 + ef.0 = = 2 2 2 2 + (ef]. 0 = 2 2 2 2 + 2 2 = \frac{1}{2}\(\hat{1}\)\vartheta\.

Lewerle: De in the activity from element D= = 1/2 + ef + fe in Electe) ar An givin slace)-vep V. This element Er central, by (b) , It is called thre Carmin elevent. - Splitting extension Proposition 12: Any extensión of simple strogs

0 -> Las -> V -> Las -> 0 (*)

5 split. Proof: If I = w her V(X) = (w @ Cw' where w in he wrage of the light wt. vedo ve Las render the giver inclusion and 21 waps to a under the projecti V - 2 (4). By Proposition & we have the surple subvege L, L'EV, L, L'EL(X), with highest ut rectors to and w' respectively. The map L(L) - V is travefore on = anto Land tu map V - L(1) restorate to a- ranophie- L-DV-DL(N). The inner morphine LCD > L co V provider the desired splitting. If le 7 \ She = 2 u (x+2) \$ = 2 \ (\lambda+2). By Lemme 11 the operator Qv: V -V har eigenvalues = escenti) and = >()+1) and the generalized eigen spacer V(u) und V(1) cere nonvenishinis subreps in V w/ V5020 V(2) = V.

(1/) Since Length (V) = 2 we have Val = in La) and the samporite V(X) -V - L(X) it an iranopheni et sle-vept. The inverte The provider The required splitting. (hearen (-semisruptions of reposh)): e) The category rep (Sl2(D)) is remissiple. b) The surples in vep (cl_2 (0)) are clucrificed by Their highest att, c) Every fir-dim str (6) -rep V de comporer runght = unto a sum $V = \bigoplus_{j=1}^{n} u(x_{j}) \cdot L(x_{j})$ with m(X) = dim /femple(L(Xi), V). Prof: I umedicte for Prop 12 and Prop 4, Aug 28/3