HW3_python

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1 Homework 3

1.0.1 Intro to Optimization

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Cade Parkison

U0939163

```
In [1]: import numpy as np
        import matplotlib.pyplot as plt
        from scipy.linalg import hilbert
        from opt_algs import *
```

1.1 Problem 2

Beck Exercise 3.2.
Generate thirty points $(x_i, y_i), i = 1, 2, ..., 30$, by the MATLAB code

```
randn(seed,314);
x = linspace(0,1,30);
y = 2x. 2  3x + 1 + 0.05randn(size(x));
```

Find the quadratic function $y = ax^2 + bx + c$ that best fits the points in the least squares sense. Indicate what are the parameters a, b, c found by the least squares solution and plot the points along with the derived quadratic function. The resulting plot should look like the one in Figure 3.5.

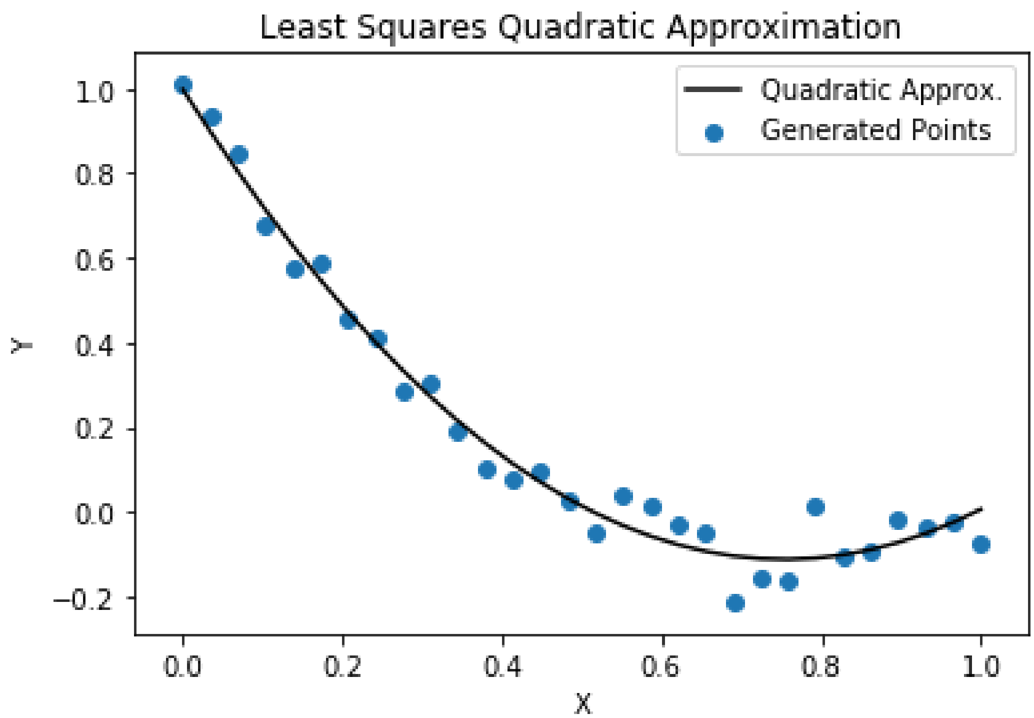
```
In [2]: np.random.seed(314)
        x = np.linspace(0,1,30).T
        y = 2*x**2 - 3*x + 1 + 0.05*np.random.randn(np.size(x))

In [3]: A = np.stack((np.ones(x.shape), x, x**2),axis=-1)
```

```
In [4]: X = np.linalg.solve(np.dot(A.T, A), np.dot(A.T, y))
        X
```

```
Out[4]: array([ 0.99807441, -2.94502388,  1.95315053])
```

Generate Plot:

```
In [5]: plt.scatter(x,y, label='Generated Points')
        plt.plot(x,X[0]+X[1]*x+ X[2]*x**2,c='k', label='Quadratic Approx.')
        plt.title('Least Squares Quadratic Approximation')
        plt.ylabel('Y')
        plt.xlabel('X')
        plt.legend()
        plt.savefig('Problem2.pdf')
        plt.show()
```



1.2 Problem 3

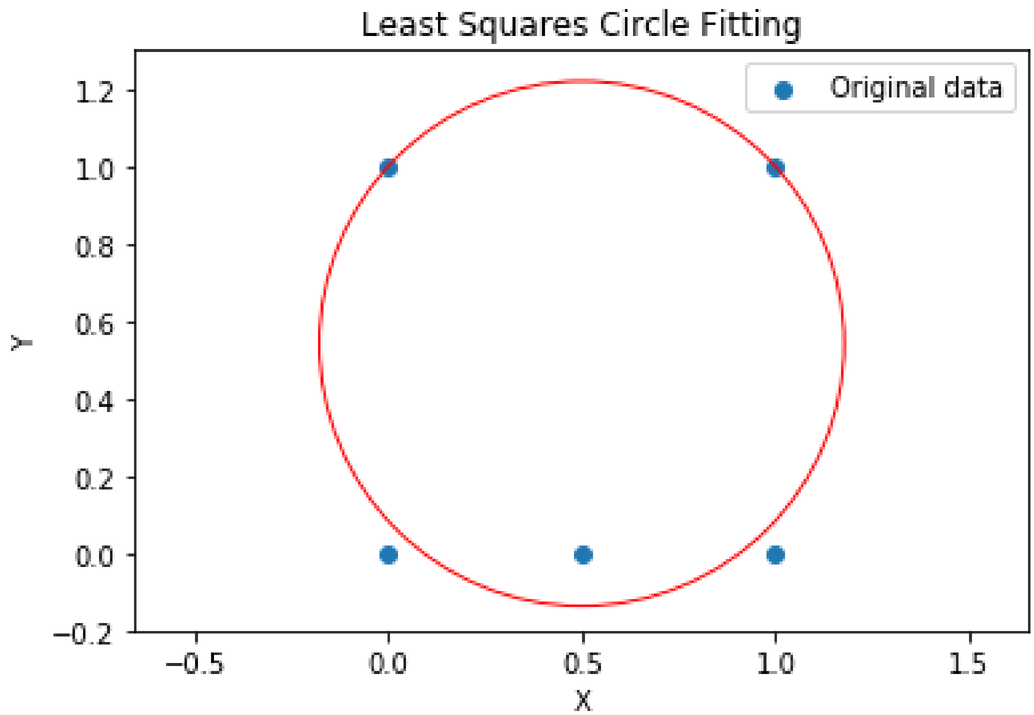
Write a MATLAB function circle_fit whose input is an nm matrix A, the columns of A are the m vectors in R^n to which a circle should be fitted. The call to the function will be of the form:

```
[x, r] = circlefit(A)
```

```
In [6]: # test data
        a1 = np.array([0, 0])
        a2 = np.array([0.5, 0])
        a3 = np.array([1, 0])
        a4 = np.array([1, 1])
        a5 = np.array([0, 1])

        A = np.stack((a1,a2,a3,a4,a5), axis=-1)

        n,m = np.shape(A)
        n,m = int(n), int(m)

        a = np.column_stack((2*A.T, -1*np.ones(m)[:, np.newaxis] ))

        b = np.linalg.norm(A.T, axis=-1)**2

        X = np.dot(np.dot(np.linalg.inv(np.dot(a.T, a)), a.T), b)

        r = np.sqrt(X[0]**2 + X[1]**2 - X[2])
```

Generate Plot:

```
In [7]: ax = plt.gca()
        plt.axis('equal')

        ax.scatter(A[:1], A[1:], label='Original data')
        #ax.scatter(X[0], X[1], c='r')

        circle = plt.Circle((X[0], X[1]), radius=r, fill=False, color='r', label='Fitted Circle
        ax.add_artist(circle)

        plt.title('Least Squares Circle Fitting')
        plt.ylabel('Y')
        plt.xlabel('X')
        plt.legend()
        plt.axis('equal')
        plt.ylim(-.2, 1.3)

        plt.savefig('CircleFit.pdf')
        plt.show()
```



1.3 ## Problem 4

```
In [8]: # Hilbert Matrix
        A = hilbert(5)
        b = np.zeros(5)

        # Quadratic Function
        f = lambda x: x@A@x
        g = lambda x: 2*A@x

        # Initial vector X0
        x0 = np.array([1,2,3,4,5])
        xs = x0

        # convergence tolerance
        epsilon = 1e-4
```

---

gradient method with backtracking stepsize rule and parameters = 0.5, = 0.5, s = 1;

```
In [35]: s = 1
         alpha = 0.5
         beta = 0.5
```

```
In [23]: x, fun_val = gradient_method_backtracking(f, g, x0, s, alpha, beta, epsilon)
```

```
In [24]: iters=3301
```

---

gradient method with backtracking stepsize rule and parameters = 0.1, = 0.5, s = 1;

```
In [25]: alpha = 0.1
         beta = 0.5
         s = 1
```

```
In [26]: x, fun_val = gradient_method_backtracking(f, g, x0, s, alpha, beta, epsilon)
```

```
In [27]: iters=3732
```

---

gradient method with exact line search;

```
In [28]: x, fun_val = gradient_method_quadratic(A, b, x0, epsilon)
```

```
In [29]: iters = 1271
```

---

diagonally scaled gradient method with diagonal elements Dii = 1 Aii , i = 1, 2, 3, 4, 5 and exact line search;

```
In [30]: # Hilbert Matrix
         A = hilbert(5)
         b = np.zeros(5)

         # Quadratic Function
         f = lambda x: x@A@x

         # Initial vector X0
         x0 = np.array([1,2,3,4,5])
         xs = x0

         # convergence tolerance
         epsilon = 1e-4

         d = np.array([])
         for i in range(5):
             d = np.append(d, 1/A[i,i])

         D = np.diag(d)

         x,fun_val = gradient_scaled_quadratic(A,b,D,x0, epsilon)
```

```
In [31]: iters=235
```

Diagonally scaled gradient method with diagonal elements
and backtracking line search with parameters = 0.1, = 0.5, s = 1;

```
In [32]: h = lambda x: 2*A

         alpha = 0.5
         beta = 0.5
         s = 1
```

```
In [33]: x,fun_val = newton_backtracking(f,g,h, x0, alpha, beta, epsilon)
```

```
In [34]: iters=15
```