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Inability to Reason About an Object's Orientation Using an Axis and Angle of Rotation

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The kinematic bases by which humans imagine an object turn from one orientation to another are unknown. The studies reported here show that individuals of high spatial ability are, in most cases, unable to imagine a Shepard–Metzler object rotate about an axis and angle so as to accurately envision its appearance. Nor can they conceive of the axis and angle by which it would rotate in a shortest path between two orientations. Accuracy progressively improves across cases in which neither angle, one of the angles, or both angles between the rotation axis and viewer–environment frame and between the axis and object limb are canonical. When canonical, the angles are more accurately observed from one viewpoint to hold constant in rotation. Such inability, with rare exceptions, is probably true for other kinematic operations requiring fine control of multiple spatial relations. Objects' orientations are not readily represented in terms of shortest path axis and angle.

Many people may have the intuition that they can imagine an object seen at one orientation to be at another. Without deliberation, the object seems to move between orientations by a systematically executed and efficient path. The ability to represent one object at another object's orientation is thought to underlie the judgment of whether the shapes of disoriented objects are the same or different (e.g., Corballis, Zbrodoff, & Roldan, 1976; Hinton & Parsons, 1981, 1988; Jolicoeur, 1988; Parsons, 1987a, 1987b, 1995a; Shepard & Cooper, 1982; Shepard & Metzler, 1971; Ullman, 1989). Although this hypothesis may be supported by experimental results for simple familiar objects, there is some evidence that it is not true for some unfamiliar orientations or complicated objects (Hinton, 1979; Rock, Wheeler, & Tudor, 1989).

There are also findings consistent with the intuition that people can imagine an object move between two orientations via the most efficient path (Carlton & Shepard, 1990; Goebel, 1990; Just & Carpenter, 1985; Shepard, 1984, 1988; Shepard & Metzler, 1971). However, there has been no evidence that, without the aid of a friendly experimental design, people seek, find, or use shortest path rotations. The set of paths capable of passing an object between two orientations is infinite. There are few data on how this set is mapped onto those paths that people are capable of producing or imagining or onto those paths that people prefer to use. Any of a number of procedures implementing different geometrical bases could produce the linear reaction time (RT)–orientation functions described in the literature (Parsons, 1987c, 1988). In addition, the shortest path between two orientations can be represented as a rotation about an axis and angle unique to each pair of orientations, but recent findings indicate that, with some tasks and some individuals, rotating an object about an axis is difficult (Just & Carpenter, 1985; Massironi & Luccio, 1989; Pani, 1993; Parsons, 1987a).

Scientific understanding of which spatial transformations can be performed readily and accurately and which are performed poorly is important for various reasons. It can guide inferences about the processes and representations underlying human spatial transformations in imagination, spatial reasoning, perception, and motor behavior. In addition, a detailed characterization of this capacity will have implications for applied areas such as design, human–computer interaction, and the practice and teaching of mathematics, physics, and engineering (e.g., Benedikt, 1991; Horgan, 1993).

A variety of research has focused on mentally represented spatial transformations. Recent work (e.g., Carlton & Shepard, 1990; Parsons, 1987b, 1987c, 1994, 1995b; Shepard, 1984; Shiffrar & Shepard, 1991) has examined the representations in cognitive and perceptual tasks to determine how they reflect (a) the laws of physics, (b) approaches derived from kinematic geometry, or (c) principles of human biomechanics. On the whole, various phenomena support the idea that mental representations of spatial transformations reflect not the laws of physics per se but aspects of kinematic geometry (in the case of objects that are not parts of one's body), biomechanics (in the case of one's own body), or both. The shortest path rotation method discussed here is one of a variety of geometrical bases from kinematic geometry.

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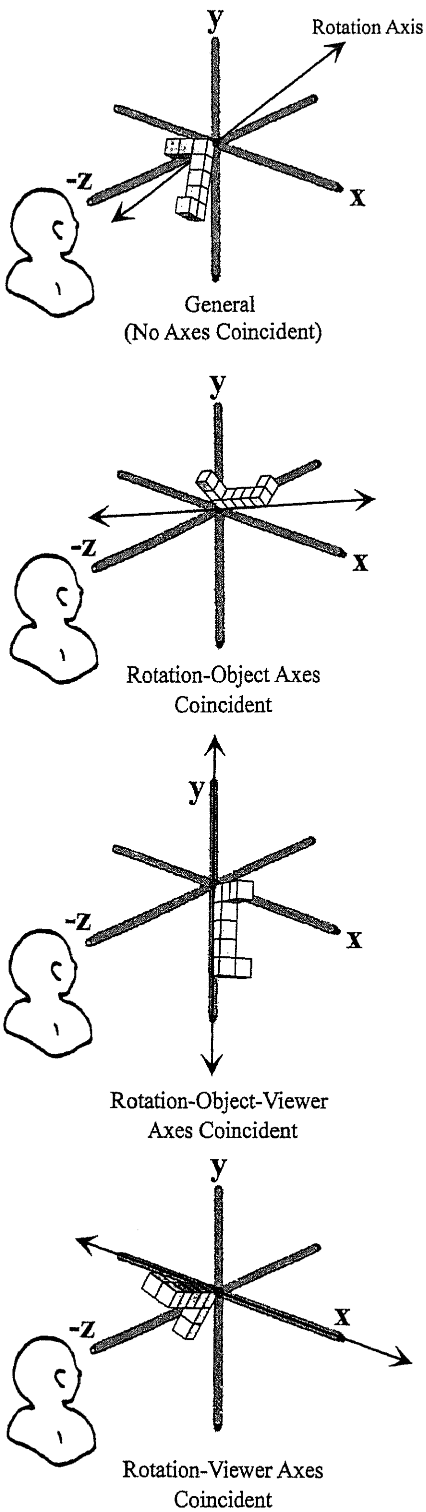

Figure 1. The four stimulus conditions in Experiments 1–4 are defined in terms of the spatial relations among the three principal viewer–environment axes, the major limb of the objects, and the rotation axis.

The studies described here evaluated how well people are able to use an axis and angle of rotation when reasoning about an object's orientation. In the first paradigm, I examined individuals' ability to imagine a simple three-dimensional object perceived at a novel orientation rotate about an axis through an angle and anticipate its new orientation. In the second paradigm, I tested individuals' ability to inspect two novel orientations of an object and conceive of the unique axis and angle used to rotate it in the shortest path from one orientation to the other.

The so-called mental rotation paradigm and the two paradigms just discussed can be considered complementary. Each paradigm requires the computation of a different single term of a basic three-term kinematic relationship. In one paradigm, individuals are presented an object at two orientations and compute a path for the object between them. This computed path may be equivalent to a rotation about an axis and angle unique to each orientation pair (Shepard, 1984, 1988). In another paradigm, individuals are given the object's initial orientation and compute its path about a specific axis through a specific angle to discover its final orientation. In the last paradigm, individuals are given the object's initial and final orientations and compute the unique axis and angle for the path. In spite of these formal similarities, the manner of performance varies across these three paradigms, shading from a well-practiced routine to consciously directed problem solving.

There were four stimulus conditions in each of the two paradigms used here (Figures 1 and 2). They were defined by spatial relations among (a) the object's conspicuous major limb, (b) the shortest path axis of rotation, and (c) a principal axis in the frame of the viewer and environment (i.e., vertical, transverse horizontal, or line of sight). (For brevity, the last axis is referred to here as a principal viewer–environment axis.)[1] Each condition represented a distinct kind of perceptual or dynamic event with probabilities of occurring in general and in specific situations.

In the *general* condition, there was no coincidence among the rotation axis, an object limb, and a principal viewer–environment axis. This is representative of all but a very small percentage of possible cases of the rotation of an object about an axis. The exceptional cases are those in which there are "accidental" congruent spatial relationships among the various frames of reference (Lowe, 1985).

In the *coincident rotation–viewer axes* condition, the rotation axis and a principal axis of the viewer's visual frame coincided, but none of the object's limbs coincided with any of the other axes. This rotation can occur when an object is attached to a vertical or horizontal rod.

[1] The reference frame for the observer and environment was always effectively aligned in the current studies. Findings from several studies of object rotation suggest that the environmental frame is more likely to predominate (Hinton & Parsons, 1988; Pani & Dupree, in press; also see McMullen & Jolicoeur, 1992).

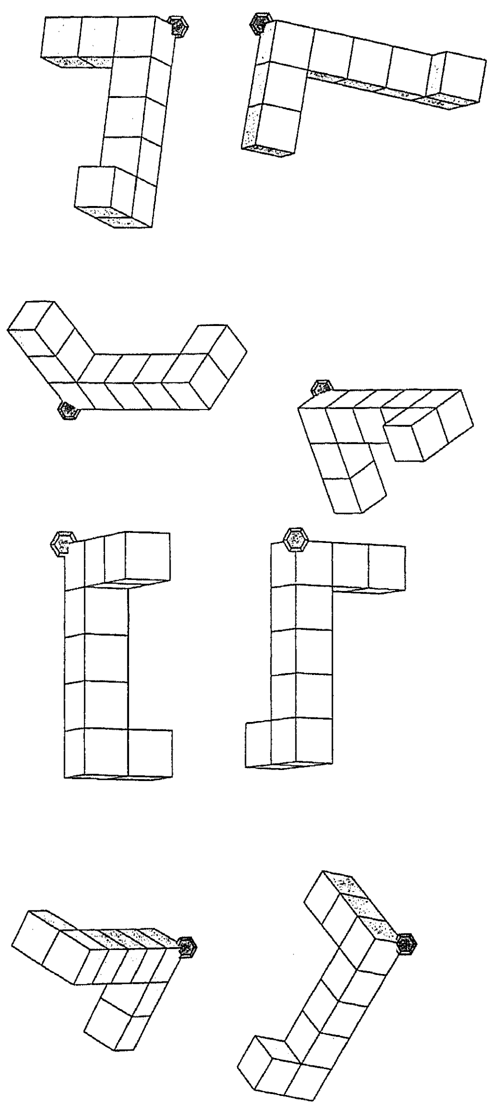

Figure 2. An example of pairs of objects in each condition. The top pair of objects exemplifies the general condition. The second from top of objects is from the rotation–object condition. The third from top pair of objects is from the rotation–object–viewer condition. The bottom pair of objects is from the rotation–viewer condition.

In the *coincident rotation–object axes* condition, the rotation axis and the object's major axis coincided, but neither of these axes coincided with a principal axis of the viewer's frame. This condition often occurs for objects being physically manipulated.

In the most extraordinary condition, *coincident viewer–object–rotation axes,* there was full coincidence among a principal axis of the viewer's visual frame, the object's major limb, and the rotation axis. This condition occurs when an object turns about its axis of elongation while it is aligned in the vertical, line of sight, or transverse horizontal direction.

Using and Finding the Shortest Path Rotation Axis and Angle

Several earlier findings suggested that people may not be very good at using and finding axes and angles of rotation. First, participants have difficulty learning rotations of their own body about any axis not aligned both with a principal axis of their visual environment and with their body (Parsons, 1987a). Second, studies reported that individuals were often poor at envisioning the appearance of a cube, an irregularly twisted wire object, or a two-dimensional rectangle after a rotation (Hinton, 1979; Massironi & Luccio, 1989; Pani, 1993; Rock et al., 1989). Third, in informal tests of many individuals, I found that even those sophisticated in mathematical, physical, and engineering sciences were unable to determine the shortest path for an arbitrary pair of object orientations, commonly mistaking another path, axis, or angle for the shortest path. Fourth, there were no reported experimental chronometric data supporting the hypothesis that, in general, people seek, find, and use shortest path rotations in visual cognition tasks, and there were few reported data that refuted any of the many alternatives.[2]

Various reasons can be hypothesized for why people may lack the ability to find and use shortest path rotations, some of which are as follows (for further commentary, see, e.g., Carlton & Shepard, 1990; Hinton, 1979; Just & Carpenter, 1985; Massironi & Luccio, 1989; Pani, 1993; Pani & Dupree, in press; Parsons, 1988; Rock et al., 1989; Shiffrar & Shepard, 1991).

1. People typically lack experience with rotation about an axis that has an arbitrary spatial relationship to the rotating object's parts.

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[2] One explicit comparison among alternative geometrical methods for the imaginal task found that another procedure for generating a trajectory between two orientations (a procedure termed *spin–precession;* see Footnote 3) showed a slightly better fit than the shortest path procedure to data from a very large and varied sample of object orientations (Parsons, 1987c).

Just and Carpenter (1985) reported finding that participants of high spatial ability spontaneously imagined shortest path rotations of a cube, and participants of low spatial ability imagined a sequence of separate rotations about each of two or three different axes. However, although their data discriminate between use of the latter method and the use of shortest path rotation, the data do not discriminate between use of shortest path and other procedures, such as a spin–precession procedure. In one of the three cases with orientation differences intended to discriminate between different procedures (Just & Carpenter, 1985, Figure 1, Case f), the spin–precession and shortest path procedures produced identical trajectories; in the other two cases (Just & Carpenter, 1985, Figure 1, Cases d and e), the shortest path procedure produced a 120° rotation and the spin–precession procedure produced a 127° rotation (about an axis the orientation of which was instantaneously changing). Given the underlying variability of their RT results, Just and Carpenter did not have the power to discriminate between rotations differing by 7°. It is unclear that their analysis of participants' introspections afforded discrimination between the shortest path and spin–precession procedures in the two cases in which the procedures produced different trajectories.

2. People may rely on other geometrical bases for spatial transformation in most of their perceptual, motor, and cognitive activity.

3. Most people's experiences may be with objects whose motion is not arbitrary but is closely tied to object shape. Motion and shape are associated because the motions of objects people control are related to the orientation of grasped parts and because free motion is dynamically affected by the distribution of an object's mass and shape.

4. The shortest path method may be difficult even for individuals of high spatial ability to induce de novo because there are so many possible methods and trajectories.

5. One's intuitions about the shortest path between two orientations can lead one to assume that the shortest path will move the object's parts in a rectilinear or planar path rather than a curved one.

6. Even when the concept of shortest path angle and axis is explained to individuals, they may not be able to apply it effectively. During rotation, two angles must hold constant: (a) that between the object and the arbitrary rotation axis and (b) that between the rotation axis and the stationary viewer–environment frame. Monitoring these angles from a single viewpoint is difficult.

In spite of these considerations, performance can be expected to improve under some special perceptual conditions. Assuming that people are given instruction and training in the shortest path method and the specific angle and axis for each rotation, the efficiency with which they imagine an object rotate about a shortest path may be affected by four factors: (a) the familiarity and complexity of the shape of the object, (b) the salience of the rotation axis, (c) the ease with which the rotation length or angle can be accurately visualized, and (d) the familiarity or comprehensibility of the path that salient object parts follow relative to one's viewpoint and the environment.

Thus, for example, because the rotation axis has greater saliency, its relationship both to the object and to the viewer–environment frame can be more readily monitored from a single viewpoint to ensure the constancy required for an accurate rotation. When the angle of the object's parts to the rotation axis is simple or canonical, it can be monitored more readily and accurately. When the rotation path of the object's parts is aligned with a significant direction or plane in the scene, deviations from the correct path can be detected more accurately and less effortfully. Similarly, when the rotation axis forms a simple canonical angle to the viewer–environment frame, monitoring of that spatial relationship is easier and more accurate. Finally, when the object's parts are regularly arrayed about its principal visual axes, it will be easier both to monitor the object's path and to mentally represent the object at intermediate orientations. Such aids to monitoring the two spatial relationships should have a significant impact on the efficiency of imagining an object's rotation about an axis. They should help minimize deviations from the rotation path. This is important because errors can be propagated and can result in a final orientation farther from the correct orientation. In addition, imagining an object's rotation requires sustained, focused mental activity, so these aids will reduce demands on attention, short-term memory, and other component information-processing operations. These considerations lead to predictions confirmed in the four studies reported here.

Imagining the Rotation of an Object About an Axis

Some past studies of imagined spatial transformations relied on the ability to efficiently imagine an object after an experimenter-specified rotation in the picture plane (Cooper, 1976). Other studies showed that the mental representation of an object imagined at another orientation may, contrary to one's intuitions, be shockingly inaccurate. Hinton (1979) reported that people were generally incapable of accurately envisioning a cube when it was oriented so that the line through its opposite vertices was vertical. Rock et al. (1989) found that participants were poor at imagining the appearance of an abstract irregularly twisted wire form after a 90° rotation about a vertical axis. Massironi and Luccio (1989) and Pani (1993) reported that participants were often inaccurate at envisioning a rotation of a rectangle or square when the object was not at a right angle to the rotation axis. In some of the studies just mentioned, the authors suggested that their participants were unable to accurately imagine the stimulus at the new orientation and instead based their responses on inferences from higher order organizational properties of the stimulus or its transformation (such as continuity or symmetry).

Experiment 1

In Experiment 1, individuals of high and moderately high spatial ability were instructed to imagine a cube figure perceived at a novel orientation rotate about an axis through a specified angle so as to anticipate its appearance in its new orientation (Figure 1). When they signaled that they had completed the rotation, the object was presented in a second orientation, and participants decided whether it was in the orientation resulting from the rotation on that trial.

A combination of design features distinguished the contribution of Experiment 1 from earlier studies of the ability to imagine an object's rotation about an axis. First, the stimulus was a simple three-dimensional shape (a Shepard–Metzler cube figure) which was used in studies yielding the strongest evidence for the ability to imagine an object at some other orientation. Second, relative to the viewer and the environment, a wide variety of rotation axes were sampled. Third, the object's initial and final orientations and its imagined trajectory on a trial were always unique, providing an assessment of this ability under very general conditions. Fourth, participants performed a very large number of trials and thus attained considerable practice. Fifth, participants were asked to imagine the object at both small and large angles away from its viewed orientation. Sixth, the performances of participants of very high spatial ability were examined and compared with those of participants with moderately high spatial ability. Finally, there was a systematic investigation of the relationship among the rotation axis, object axis, and canonical axes in the frame of the viewer and environment (also see Pani, 1993; Pani & Dupree, in press).

Experiment 1 was specifically designed to optimize participants' preparation for each component of the task, to minimize any extraneous task difficulty, to maximize the number of times participants performed the task, and to comprehensively sample the set of possible spatial properties in the task. Initially, participants were selected for their superior ability to accurately and rapidly imagine an object at another orientation. Then, before each trial, participants became familiar with the structure of the simple object, with the direction of the rotation axis, with the size and direction of rotation angle, and with examples of real rotations about the axis. All of this experience was accumulated with visible real objects, real angles, real rotation axes, and real rotation paths in the same physical position in which experimental trials occurred. This familiarity was extended to two-dimensional computer graphic depictions of the object, initially in interactions on practice trials (wherein participants were given feedback as to the correctness of their responses) and then on test trials.

There may be some concern in terms of two-dimensional computer graphic stimuli yielding deficient perceptual representations and spatial cognitions. The most relevant finding on this issue is not consistent with such a hypothesis. Kaushall and Parsons (1981) studied the discrimination of pairs of identical and mirrored Shepard–Metzler cube figures (exactly like the stimuli used here). They found that the monotonic increase in RT with separation in the depth or picture plane was not affected by enriching the visual stimulus through the use of real objects rather than two-dimensional drawings or through the use of binocular vision rather than monocular vision. These findings suggest that the mental representation of an object's shape and orientation, as well as the representation of its rotation, was not enhanced by greater visual information and not impeded by the two-dimensional stimulus. To further address worries about possibly inadequate stimulus information here, Experiment 2 replicated the basic design of Experiment 1 with participants who performed the task while viewing the real three-dimensional object with an attached rotation axis.

Method

Participants. Twenty-four University of Texas at Austin undergraduates who had not been involved in any related experiments participated in this study. As a means of assessing the human capabilities under study at their best, only students of high and moderately high spatial ability were included. Individuals of low spatial ability may not be capable of envisioning an object rotate through efficient paths (Just & Carpenter, 1985). The 24 students completed two timed pretests of their ability to imagine spatial transformations of objects (Thurstone's Card Rotation and Cube Comparison tests; see Ekstrom, French, Harman, & Dermen, 1976). The near foil and far foil conditions included 7 students of high spatial ability who correctly completed 89% of the items (range = 86% to 93%) with a 5% error rate and 7 students of moderately high spatial ability who correctly completed 78% of the items (range = 71% to 85%) with a 9% error rate. The far foil only conditions included 5 students of high spatial ability who correctly completed 88% of the items (range = 85% to 91%) with a 6% error rate. Five students of moderately high spatial ability correctly completed 83% of the items (range = 81% to 85%) with a 10% error rate.

Stimuli. A single simple object was used throughout (Figure 2). This object had one distinctly colored end to aid in recognizing corresponding object parts at different orientations. The Personal Designer computer program (ComputerVision Corporation) was used in generating the stimuli, which were orthographically projected and printed in black ink on white paper. Photographic slides of those objects with one cube colored yellow were back projected by a slide projector with an attached Gerbrands tachistoscopic shutter to subtend about 5° of visual angle.

Every rotation was about the same object vertex. This vertex was the origin of a fixed Cartesian coordinate system used to describe orientation ($+x$ was rightward transverse horizontal, $+y$ was upward, and $+z$ was along the line of sight into the screen). This fixed point of rotation was always in the same viewer-relative location highlighted by a small sphere. As a means of constructing pairs of orientations, an object was initially (a) rotated so that its major axis was coincident with one of the six rotation axes or (b) rotated into an arbitrary orientation in which none of its limbs coincided with the rotation axis. Then, for each axis and angle, an initial orientation and a target orientation were created by rotating the object about each rotation axis an arbitrary amount and then a counterclockwise 45°, 90°, 135°, or 180°. These angles were sampled because they are familiar, simple, and regularly dispersed. An orientation was replaced if the object's shape was occluded or distorted by pictorial depiction.

Near foil and far foil probes were generated by rotating the object 15° and 30° around a randomly selected axis from its target orientation. In the near and far foil conditions, the probe was correct on half of the trials; on a quarter of the trials, the probe orientation deviated 15° in an arbitrary direction from the correct orientation, and on the remaining trials the probe orientation deviated 30° in an arbitrary direction. In the far foil only conditions, the probe foils were always 30° away from the target orientation. The comparison between the two conditions was made to determine whether more accurate performance occurred when the discrimination between target and foil was easier.

Before a session, each student spent 10 min handling and studying a wooden model of the stimulus to increase his or her familiarity with the object and its appearance at different orientations. On each trial, this model was visible and in front of students, although they were allowed to handle it only between trial blocks.

Six well-dispersed rotation axes were chosen to sample the set of possible axes (Table 1). As a means of facilitating training, a physical model of each rotation axis was constructed on a base with three rods representing the principal axes of the students' visual reference frame. At the origin was a sphere representing the center of rotation, from which each rotation axis originated. A rotation axis was differently colored and longer than the rods representing the principal axes. A cardboard circle on the rotation axis indicated (with an arrow) the rotation direction, which was always anticlockwise with respect to the view down the axis to the origin.

Design. For each rotation axis, there were 16 trials in which the object's major limb coincided with the rotation axis and 16 trials in which none of the object's principal limbs were aligned with the rotation axis. There were an equal number of target and foil trials and an equal number of trials with each rotation angle. Students performed two replications of 16 practice trials and one replication of the unique 192 trials blocked by rotation axis.

Trials were blocked by rotation to allow students to focus on

Table 1
Shortest Path Axes of Rotation Used to Create Pairs of Orientations in Experiment 1

Coordinates[a] (x, y, z)	Description of direction
0, 1, 0	Vertical axis
1, 0, 0	Horizontal axis orthogonal to line of sight
.7, 0, .7	45° between transverse horizontal and line of sight axes
0, .7, −.7	45° between vertical axis and line of sight axis pointing out toward observer
−.58, .58, .58	45° between line of sight axis and negative horizontal axis and 35.5° above horizontal plane
.7, .7, −.7	45° between horizontal axis and line of sight axis pointing out of screen to right of observer and 45° above horizontal plane

[a] Based on a system with $+x$ representing right, $+y$ representing up, and $+z$ representing line of sight into the vertical screen of the observer (or the page of the reader).

accurately imagining the rotation of the object with respect to a single axis rather than having the cognitive load of conceiving of a different axis for each following trial. Within each block, there were four sets of four trials with the same rotation angle. The spatial relation between the rotation axis and the object's major limb was random throughout each block of trials. Each student participated in trials of a unique random order. The practice trials were identical to the test ones; students first performed trials with the vertical axis and then with a (.7, 0, .7) axis. The design for the far foil only condition was identical to the near and far foil conditions except that all trials with a near foil probe were eliminated from the experiment and one target trial was eliminated for each such near foil probe.

Procedure. Students were instructed about a rotation axis by being shown its model and by having its direction described in terms of its angle to the viewer's principal axes. They were told that all rotations were counterclockwise looking down the axis toward the origin. Before the start of each subset of trials, they were shown several rotations illustrating the upcoming rotation. Each student was told the angle for each rotation, and, as well, this information was printed on a card hanging near the stimulus and modeled by the experimenter's hand pivoting through the appropriate angle around the model rotation axis. Physical models of the axis, angle size, and angle direction were visible and directly in front of the students during a trial; however, students were allowed to handle the models only between blocks of trials.

At the start of a trial, when students were prepared to rotate an object about a specified axis through a specified angle, the stimulus appeared at its initial orientation on a screen in front of them. When they had imagined the rotation and had an understanding of the object's new appearance, they pressed a right key to be shown a probe stimulus. The probe followed a 250-ms presentation of ambient light that minimized any apparent motion between the initial stimulus and the probe. If the probe portrayed the object at the orientation they anticipated, students pressed the right key. If the probe portrayed the object at another orientation, they pressed the left key. Students responded as rapidly and accurately as they could and were told not to make any head or body movements. A stimulus was presented until a response was made. A 25-MHz 386 microprocessor personal computer controlled the slide projectors and shutters and recorded response time and accuracy. Students were given feedback as to the accuracy of their responses on the practice trials only.

Results

Separate analyses of variance of errors and response times in the near and far foil conditions and in the far foil only condition showed no significant differences either between the near and far foil participants and the far foil only participants, or between individuals of high and moderately high spatial ability. All further analyses collapsed across these two variables. There was little variation in performance even among the individuals of high spatial ability.

Accuracy. Participants were very often poor at anticipating the object's appearance after a rotation (Figure 3), but accuracy varied with the spatial relationship among the rotation axis, object, and observer, $F(3, 69) = 19.10$, $p < .001$, $MSE = 2.61$. They performed at chance when there was no coincidence among the rotation axis, the object's major limb, and a principal viewer–environment axis. Their d' value was .05, roughly equivalent to a hit rate of .51 and a false alarm rate of .49. In the other conditions, as predicted, accuracy improved, tending to rise above chance. Accuracy improved slightly when the rotation axis coincided with a principal viewer–environment axis but neither of those axes were aligned with an object limb. It increased further when the rotation axis and the object's major limb coincided but were not aligned with a principal viewer–environment axis. Accuracy was highest by far (significantly so, $p < .05$, Tukey's honestly significant difference [HSD] = .33) when the rotation axis coincided with the object's major limb and a principal viewer–environment axis. Roughly, the hit rate was .8 in this condition, and the false alarm rate was .2. Accuracy in this last condition was more than a linear combination of the separate effects of (a) having the rotation axis coincide with a principal viewer–environment axis and (b) having the rotation axis coincide with the object's major limb.

Over all four conditions (Figure 4), participants were more accurate for 45° and 90° rotations (which were not different, $p > .05$, Tukey's HSD = .26), less accurate for 180° ones, and least accurate for 135° ones, the last two of which were significantly different ($p < .05$, Tukey's HSD), $F(3, 69) = 14.33$, $p < .001$, $MSE = 1.59$. A great proportion of the differences in accuracy for different rotation angles occurred when the rotation axis coincided with the object's major limb and a principal viewer–environment axis. Only under this condition was accuracy relatively high for the 180° rotation, producing an interaction between spatial condition and rotation angle, $F(3, 69) = 5.90$, $p < .001$, $MSE = 1.43$ (Figure 5). (For comparisons among means for condition and angle, Tukey's HSD for a significance level of .05 was .33.)

Response times. Over all rotation angles, the time needed to imagine the rotation was about 5.5 s (ranging from 4.5 to 7 s for different combinations of angles and spatial conditions). This was between 2.5 and 5.25 times longer than the time needed to imagine the rotation of one

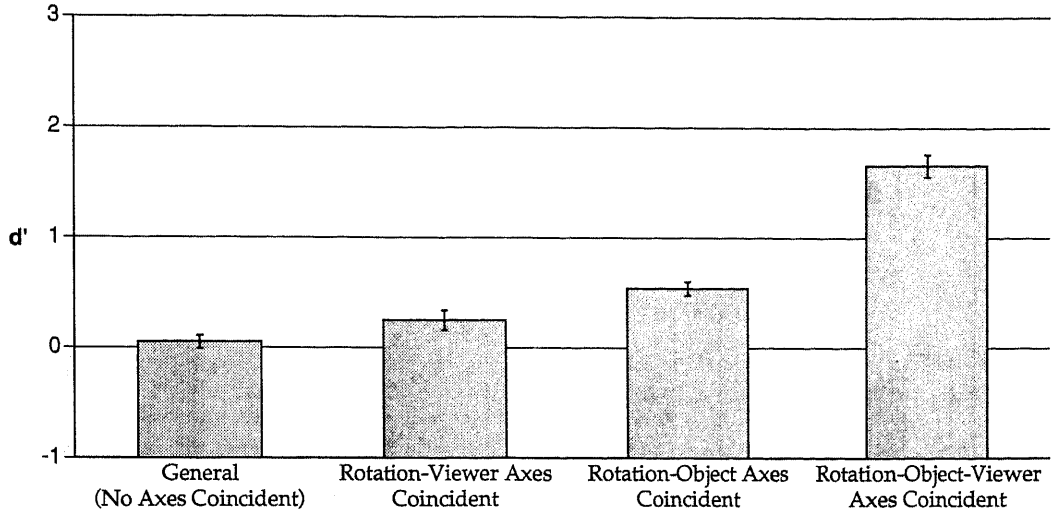

Figure 3. Experiment 1: Mean accuracy (d') for anticipating the appearance of an object after a rotation about an axis as a function of the spatial relationship among the rotation axis, object, viewer, and environment. Error bars show one standard error of the mean.

of two such objects into the orientation of the other so as to discriminate shape (Kaushall & Parsons, 1981; Metzler & Shepard, 1974). Presumably participants took so much longer because they wanted to improve their accuracy in this difficult task.

The time needed to imagine the rotation (Figure 6) depended on the spatial relationship among the rotation axis, the object's major limb, and a principal viewer–environment axis, $F(3, 69) = 17.00$, $p < .001$, $MSE = 2256774$. RT for each condition was significantly different ($p < .05$, Tukey's HSD = 329 ms). It was longest when there was no coincidence among the rotation axis, object, and a principal viewer–environment axis; shortest when there was full coincidence among these structures; and intermediate for the other two conditions. In fact, RT was well correlated with accuracy in the four conditions ($r = -.95$), $F(1, 2) = 19.80$, $p < .05$. This indicates that there was no speed–accuracy trade-off, so participants were sensitive to the actual accuracy in their performances under different conditions.

In addition, the time needed to imagine the rotation generally increased—although not linearly—with angle, $F(3, 69) = 17.50$, $p < .001$, $MSE = 1959825$ (Figure 6). This increase, which was significant in each case ($p < .05$, Tukey's HSD = 306 ms), reflects the classic relationship between rotation angle and rotation time discovered by

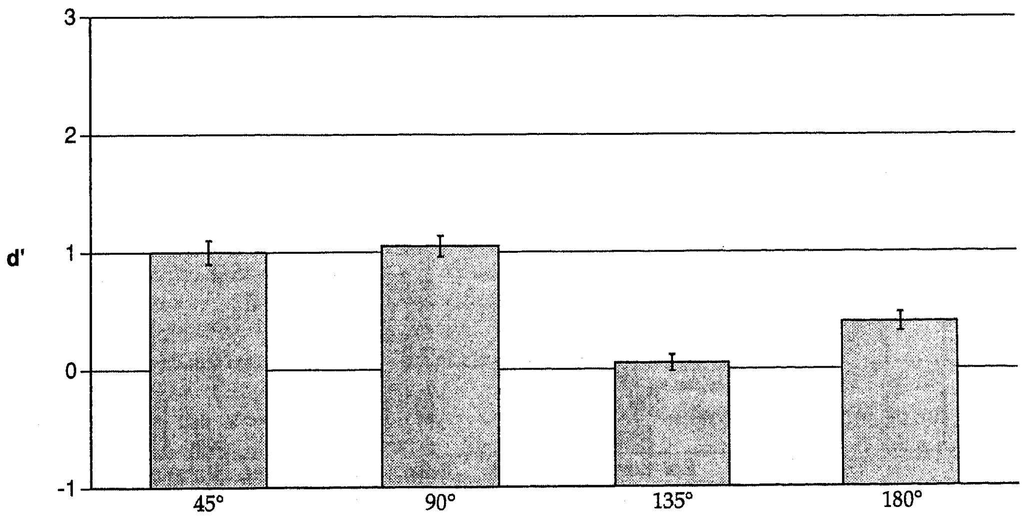

Figure 4. Experiment 1: Mean accuracy (d') for anticipating the appearance of an object after a rotation about an axis as a function of the angle of rotation. Error bars show one standard error of the mean.

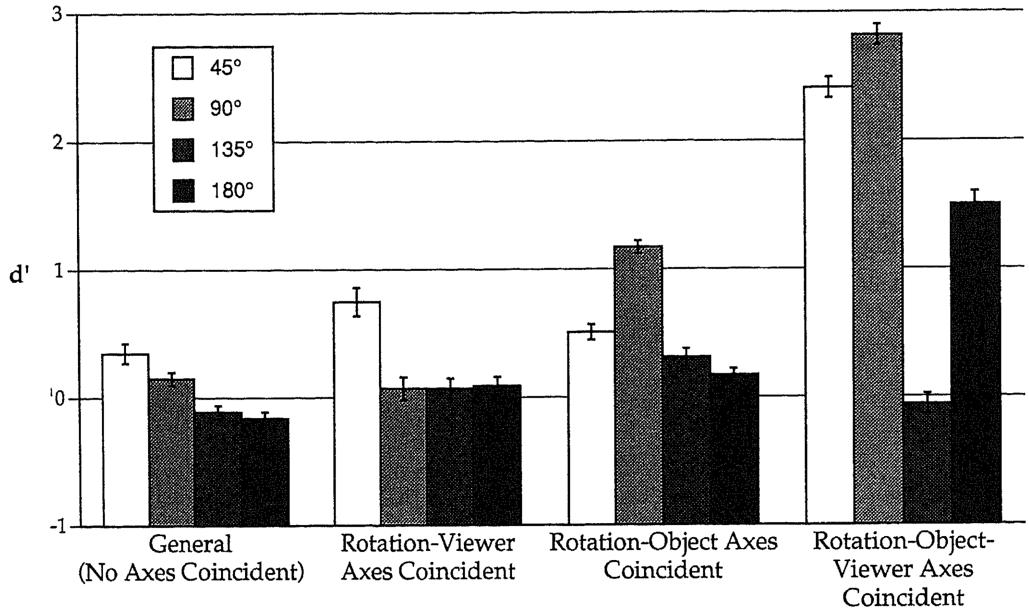

Figure 5. Experiment 1: Mean accuracy (d') for anticipating appearance of an object after a rotation about an axis as a function of both the angle of rotation and the spatial relationship among the rotation axis, object, viewer, and environment. Error bars show one standard error of the mean.

Shepard and Metzler (1971). In fact, the implied rotation rate was 132°/s when these participants were performing very accurately (i.e., on the 45°, 90°, and 180° angles when there was full coincidence among the rotation axis, object, and principal viewer–environment axis), a rate similar to that reported with this stimulus in other studies (Kaushall & Parsons, 1981; Metzler & Shepard, 1974). The disproportionately long RT for the 135° rotation appeared to reflect participants' difficulty using that angle. In fact, the time needed to imagine the rotation depended both on rotation angle and on the spatial relationship among the rotation axis, the object's major limb, and a principal viewer–environment axis, $F(9, 207) = 2.20$, $p < .05$, $MSE = 1{,}185{,}934$.

The overall RT to judge whether the probe showed the stimulus at the correct orientation was 2,245 ms. This time was slightly faster (by 293 ms) when the object's major limb was aligned with the rotation axis, $F(1, 23) = 25.24$, $p < .001$, $MSE = 976{,}586$. No other variables were significant. That the probe RT was independent of the rotation

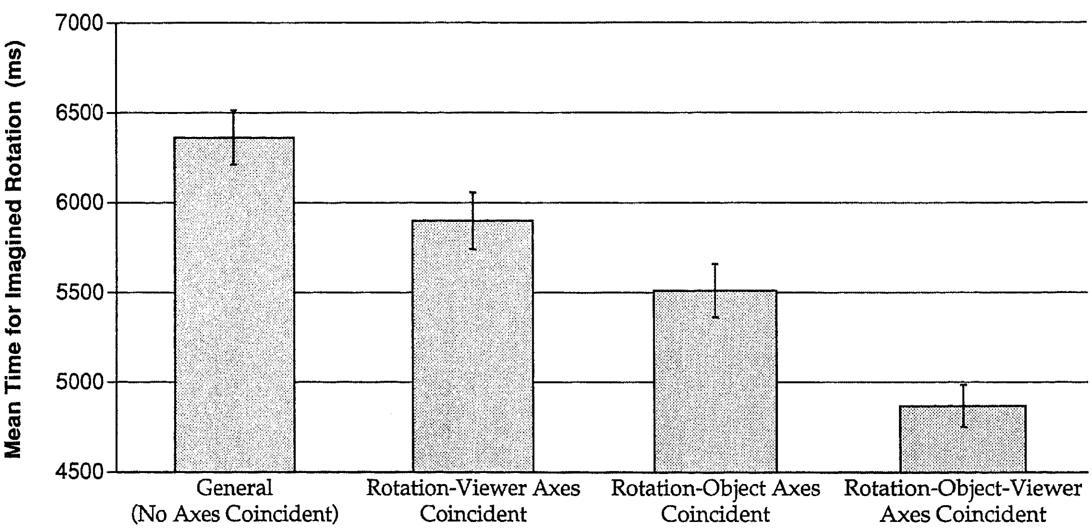

Figure 6. Experiment 1: Mean reaction time for imagining the rotation of an object as a function of the spatial relationship among the rotation axis, object, viewer, and environment. Error bars show one standard error of the mean.

angle indicates that students performed their imagined reorientation before examining the probe. The verification RT was negatively correlated with accuracy ($r = -.91$), $F(1, 2) = 9.74$, $p < .08$. Thus, the difficulty of performing a rotation under different conditions influenced the time needed to make the final verification of the probe without a speed–accuracy trade-off.

Discussion

Even individuals of high and moderately high spatial ability were unable, under most conditions, to imagine a simple three-dimensional object perceived at a novel orientation rotate about an axis through an angle and accurately anticipate its new appearance. When spatial relationships among the object parts, the rotation axis, and significant directions in the frame of the viewer and environment are representative of the vast majority of all possible rotations, performance is especially poor (at chance for all but the smallest rotation angle).

Participants' accuracy improved when there were special spatial relationships among the object's parts, the rotation axis, and significant directions in the viewer–environment frame. These special cases had a simple or canonical angle between the object's salient parts and the rotation axis, between the rotation axis and the viewer–environment frame, or both. As a result, these two angles, which must be accurately maintained during rotation, each are more efficiently monitored from a single viewpoint. These special relationships also made the rotation axis more conspicuous because it was coincident with other features in the scene, allowing for greater efficiency in monitoring the two critical angles, as well as allowing for a reduced load on memory and attention during task performance. Thus, accuracy rose above chance when the rotation axis coincided with a principal viewer–environment axis. It was still limited because there was an arbitrary noncanonical angle between the rotation axis and object's major limb. Performance tended to be better still when the rotation axis coincided with the object's major limb; in that case, the angle between the rotation axis and object parts was readily and accurately monitored, and the rotation axis was even more conspicuous by being aligned with the object.

When the rotation axis coincided with both the object's major limb and a principal viewer–environment axis, participants were quite accurate for the smallest rotation angle of 45° and the canonical rotation angle of 90° (each with a hit rate of about .9 and a false alarm rate of about .1). They were also fairly accurate for the 180° rotation but remained at chance for the 135° angle. This implies that participants were able to use the rotation angles of 45°, 90°, and 180°. It also suggests that other factors were more responsible for inaccurate performance in the other conditions. These results imply that under no conditions, given the training here, can participants perform this task with a 135° rotation angle. This suggests that people will be unable to imagine a simple three-dimensional object perceived at a novel orientation rotate about an axis through any noncanonical angle larger than 45°.

Participants' overall accuracy in performing this task is far lower than that of participants comparing the shape of two disoriented such objects by imagining one at the orientation of the other without instruction as to the path the object follows (Kaushall & Parsons, 1981; Metzler & Shepard, 1974). This low accuracy is probably due to the factors outlined in the introduction, which make it difficult for people to imagine an object rotate about an axis. Accuracy in this task may improve with the presence of feedback on every trial, much more training, constant initial orientation, an explicit axis, and so forth. These conditions have not been present in chronometric studies of imagined rotation in which participants are hypothesized to use shortest path rotation.

Participants were sensitive to the inaccurate performance of this generally difficult task. They commented on its arduousness and their frequent lack of confidence in their responses. The mean time necessary for them to complete the task was about four times longer overall than the time needed to imagine one of two such objects rotate into the orientation of the other so as to compare shape. This is consistent with the overall error rate being about seven times that in the shape comparison task. In addition, the time needed to complete the rotation was proportional to the varying accuracy for the different spatial relations conditions and rotation angles. In those few special cases in which participants were quite accurate (see Results section), the data implied a rate of imagining the rotation (as indicated by the slope of the RT–orientation function) that was comparable to that implied by RTs of participants imagining one such object at the orientation of another to compare shape. This correspondence confirms the efficacy of the experimental conditions here in eliciting imagined rotations of objects in three-dimensional space.

Participants who tested very highly for the ability to rapidly and accurately envision abstract two-dimensional and three-dimensional objects performed this task as well as those who tested only moderately highly. Furthermore, there was little variation in performance among the individuals of high spatial ability. Individuals who test very low on such preexperimental ability tests may be likely to have difficulty imagining an object rotate about an arbitrary, nonconspicuous axis. Just and Carpenter (1985) studied the ability to envision a cube at another orientation in participants of high and quite low spatial ability. They found that only the participants of low spatial ability reported envisioning a sequence of rotations about one conspicuous axis and then another rather than envisioning a more efficient rotation about an axis that was not conspicuous.

Experiment 2: Imagining the Rotation of a Visible Real Three-Dimensional Object About an Attached Axis

Some of the inaccuracy observed in the imagined rotations in Experiment 1 could have resulted if participants had difficulty perceiving accurately an object's three-dimensional spatial structure and orientation from the two-dimensional depictions composed of lines and shaded surfaces. Poor performance could also result if they had difficulty accurately representing the position and orientation of the rotation axis relative to the object and the viewer–environment frame. To examine these possibilities, I replicated the basic design of Experiment 1 in the following study, in which participants performed the task while viewing the real three-dimensional object with an attached rotation axis. A comparison of performance here and in Experiment 1 allowed an assessment of how a richer visual stimulus and an explicit rotation axis affected both overall accuracy and accuracy across spatial conditions.

In addition, this study included not only a group of participants of high and moderately high spatial ability, as in Experiment 1, but a second group of participants of only average spatial ability. The inclusion of the latter group achieved two purposes. First, it allowed the comparison of individuals who differed more in spatial ability than those in Experiment 1. This increased the chances of observing a difference in performance for participants of different spatial ability. Such a finding would be consistent with the observed difference in efficiency between participants of high and low spatial ability in a task requiring the imagined rotation of a cube (Just & Carpenter, 1985). Second, it assessed whether the results in Experiment 1 would generalize to participants whose spatial ability was more typical of the overall population.

Method

Participants. Fourteen University of Texas at Austin undergraduates (7 men and 7 women) participated in this study. None had been involved in any related experiments. They completed a timed pretest of the ability to imagine spatial transformations of an object (Thurstone's Cube Comparison test). Seven students of high or moderately high spatial ability (4 men and 3 women) correctly completed 83% of the items (range = 76% to 93%) with a 13% error rate. Seven students of average spatial ability (3 men and 4 women) correctly completed 64% of the items (range = 52% to 71%) with a 24% error rate.

Stimuli. The stimuli were real object versions of the computer graph stimulus used in Experiment 1. The objects were made of white Styrofoam. Each object's most distant vertices were separated by 135 mm; an object subtended 6° of visual angle when presented on a trial. All objects were identical in shape, size, and color. Each object was pierced through the same vertex by a rod (4 mm in diameter). On a trial, this pierced vertex was the fixed point of rotation and the rod coincided with the rotation axis.

One rotation axis from Experiment 1 was used for each spatial condition. The vertical axis (0, 1, 0) was used for the coincident viewer–object–rotation axes trials; a (−.58, .58, .58) axis was used for the general trials, in which there was no coincidence among the rotation, viewer, and object axes; an (.7, 0, .7) axis was used for the coincident rotation–object axes trials; and the horizontal axis (1, 0, 0) was used for the coincident rotation–viewer axes trials.

In each condition, there were two anticlockwise rotation angles from Experiment 1: 90° and 135°. There were three different initial object orientations for each axis and rotation angle combination. The orientations of the foil probes were generated by rotating the object 30° around a randomly selected axis from its target orientation.

Apparatus. On a trial, the initial and probe objects were each supported by a separate rod inserted into a different hemispherical Styrofoam mount. This mount was attached to a sliding base. The sliding base was set so that, at the start of each trial, the initial object was centered in a window viewed by the participant. During a trial, when the participant said "ready," the base was slid to a new position so that the initial object was withdrawn from view and replaced by the probe. The experimenter who set up the objects was hidden from the participant by a black felt curtain. In a window cut out of the black felt was shown a single object and its supporting rotation axis against a black felt background; no other features of the apparatus were visible.

Design. For each condition, there were 3 trials with a 90° rotation and 3 with a 135° rotation. There were an equal number of target and foil trials for each of the four conditions. There were 24 unique trials, each performed once by each participant. Participants also performed 2 practice trials in each condition. The practice trials were identical to the test ones but with unique initial orientations. The trials were blocked by condition and rotation angle. Each student participated in trials of a unique random order.

Procedure. All aspects of the procedure were identical to those in Experiment 1 except the following. At the start of each trial, an experimenter said "ready" and removed a curtain on a window through which was visible the initial object and rotation axis. The participant imagined this object rotate about the axis and angle and said "ready" when he or she was finished and could judge whether the probe was correct or not. The initial object was then slid out of the window as the probe slid into the window. Participants said "yes" when the probe portrayed the object at the orientation they anticipated and "no" when the probe portrayed the object at another orientation. They were instructed to respond accurately but were not timed. During a session, an experimenter who was unaware of the condition of each trial sat behind the participant and recorded verbal responses.

Results

The accuracy of imagining the rotation of a visible real three-dimensional object about an attached axis was strongly affected by the spatial conditions (see Figure 7), $F(3, 36) = 4.86$, $p < .01$, $MSE = 3.86$. When there was no coincidence among the rotation axis, the object's major limb, and a principal viewer–environment axis, individuals with high and moderately high spatial ability had a d' value that was not different from chance (Figure 6). The individuals of average spatial ability had a d' value that was lower and also not different from chance (Figure 8). When the rotation axis coincided with a principal viewer–environment axis but neither axis was aligned with an object limb, individuals of high and moderately high spatial ability and those of average spatial ability had mean d' values that were not different from chance.

Accuracy rose above chance in perceptually simpler cases when the rotation axis was aligned with an object limb. When the rotation axis and the object's major limb were coincident but were not aligned with a principal viewer–environment axis, the individuals of high and moderately high spatial ability had a d' value different from chance and from their accuracy in the other conditions. Participants of average spatial ability had a d' value different from chance and from their values in the two conditions described earlier.

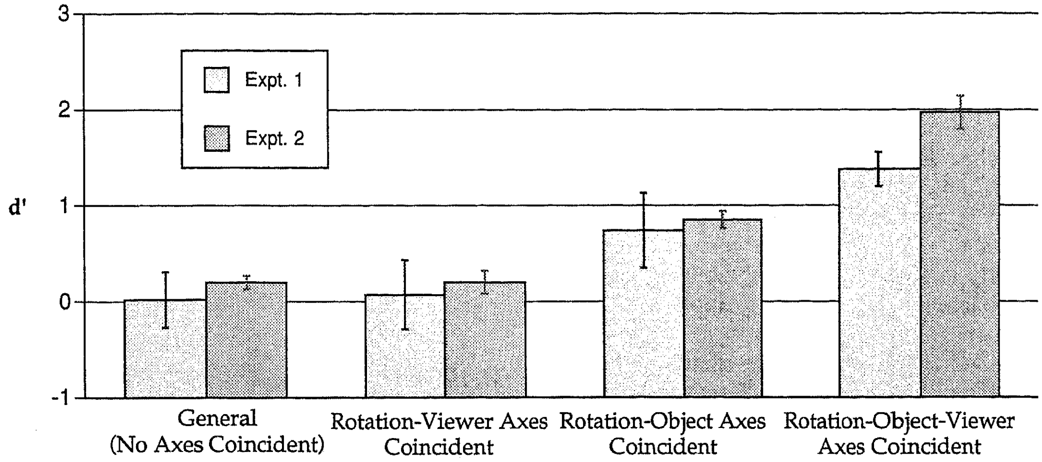

Figure 7. Mean accuracy (d') for anticipating the appearance of an object after a rotation about an axis as a function of spatial relations among the rotation axis, object, viewer, and environment. Data for participants of high and moderately high spatial ability in Experiment 2 performing the task with a visible real object with a rotation axis attached are plotted to the right of those of their counterpart, along with those of their counterparts in Experiment 1 performing the task with a two-dimensional depiction of the object without an attached axis. The means for each group are taken from trials using a 90° and 135° rotation angle. Error bars show one standard error of the mean.

Accuracy was highest when there was full coincidence among the rotation axis, the object's major limb, and a significant direction in the viewer–environment frame. Individuals of high spatial ability had a d' value equivalent to a hit rate of .84 and a false alarm of .16; the value was different both from chance and from these participants' accuracy in all other conditions. The students of average spatial ability had a d' value that was also different from chance.

Overall, there was a trend for the group of participants with high or moderately high spatial ability to be more accurate ($d' = .72$) than the group of participants with average spatial ability ($d' = .45$), $F(1, 12) = 1.13, p < .30$, $MSE = 4.74$. Most of this difference was observed on trials in which there was full coincidence among the rotation axis, the object's major limb, and a principal viewer–environment axis. Furthermore, the overall d' value for individual participants was correlated with their score on the spatial ability pretest ($r = .63$), $F(1, 12) = 7.98, p < .01$.

Over all four conditions and the two rotation angles, these

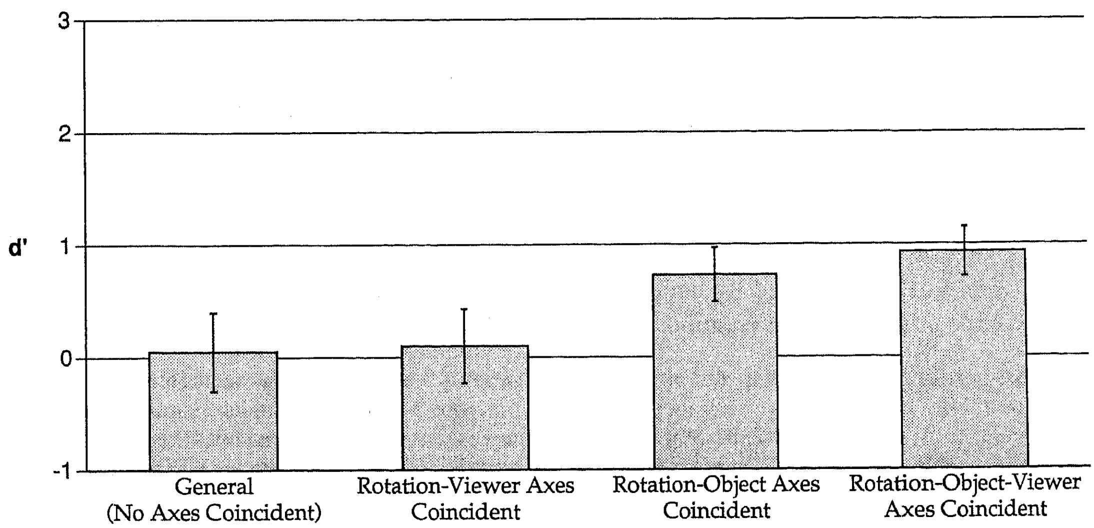

Figure 8. Experiment 2: Mean accuracy (d') of participants of average spatial ability for anticipating appearance of an object after a rotation about an axis. Error bars show one standard error of the mean.

participants tended to be more accurate at imagining a visible real object rotate about an attached visible axis ($d' = .72$) than participants of comparably high spatial ability ($d' = .55$) performing the task with computer graphic stimuli without an attached visible axis. This comparison was significant only when there was full coincidence among the rotation axis, the object's major limb, and a principal viewer–environment axis. Participants of high and moderately high spatial ability here and in Experiment 1 (Figure 6) were very similarly affected by the conditions varying the spatial relations among the object, rotation axes, and viewer–environment ($r = .99$), $F(1, 2) = 73.74$, $p < .01$. Performances by individuals of average spatial ability on this task were also very similar to the performances of those of high and moderately high spatial ability in Experiment 1 ($r = .97$), $F(1, 2) = 35.05$, $p < .02$.

Discussion

These results indicate that the accuracy with which people can imagine an object rotate through an angle about an axis tends to improve somewhat when they view the real three-dimensional object with rotation axis attached rather than looking at a two-dimensional depiction of the object without an explicit attached rotation axis. Performance was still at chance in the two conditions in which the rotation axis and the object's major limb did not coincide. Performance remained poor in the condition most representative of the vast majority of all possible rotations of an object about an axis. Thus, little of the inaccuracy in participants' performances in Experiment 1 can be explained by possible difficulties in comprehending either the object's three-dimensional spatial structure and orientation or the orientation of the rotation axis. Rather, the difficulties appeared to involve the factors discussed earlier.

These data also show that the variation in accuracy with different spatial conditions generalizes to the performance of participants with average spatial ability. However, participants of lesser spatial ability tend to show less overall accuracy in this task than those of greater spatial ability. Thus, these results are consistent with the difference in efficiency observed in participants of high and low spatial ability in a task requiring the imagined rotation of a cube (Just & Carpenter, 1985), as in the spatial ability pretest used in these studies.

Experiment 3: Ability to Perceive and Remember an Object's Appearance at an Orientation

In evaluating the performance of participants in Experiment 1, it was useful to ascertain how well they could perceive and remember the appearance of the object at an orientation under the conditions used. In Experiment 3, a group of participants of high spatial ability viewed an object from Experiment 1 at a target orientation, remembered its appearance for 250 ms, and then judged whether a probe was at the same orientation or not. This task was different from that used in Experiment 1, in which participants retained a mental representation resulting from an imagined spatial transformation. Here participants retained a mental representation resulting from the perception of an external stimulus. Nonetheless, if participants were to perform this task poorly, it would indicate that the poor performances in Experiment 1 were partly due to an inability to perceive and remember an object's orientation. In addition, if there were variation in the accuracy of perceiving and remembering the object's orientation under the different spatial conditions, it would explain some of the variation in accuracy in Experiment 1.

Method

Participants. Ten undergraduate volunteers who had not been involved in any related experiments completed Thurstone's Card Rotation and Cube Comparison tests. Five students of high spatial ability correctly completed 92% of the items (range = 85% to 100%) with a 6% error rate. Five students of moderate spatial ability correctly completed 81% of the items (range = 71% to 85%) with an 8% error rate.

Stimuli, design, and procedure. The stimuli and design were identical to those of Experiment 1 except that the stimuli portraying the object at the initial orientation were eliminated. At the beginning of a session, each participant handled and studied a wooden model of the stimulus shape for 10 min to increase his or her familiarity with the object and its appearance at different orientations. This model was visible and in front of participants on each trial, although they were allowed to handle it only between trial blocks. During a trial, each participant studied the appearance of the object so as to remember its orientation for a short time and then discriminate it from its appearance at another orientation. When they understood its appearance, participants pushed the right key to end its presentation, and the screen was then lit with dim ambient light for 250 ms, after which a probe appeared. On half of the trials, the probe was at the same orientation; on the other half of the trials, it was at an orientation 15° or 30° away. If the probe was at the same orientation, participants pressed the right key; if it was at a foil orientation, they pressed the left key. Participants performed 20 practice trials representing a unique random sample of the test trials for each participant. On practice trials, they were given feedback on their accuracy; the wooden model was used to exemplify stimulus orientation. All other aspects of the procedure and apparatus were identical to those of Experiment 1.

Results

Participants were very accurate at remembering the orientation of this object during a brief period (Figure 9). Over all four conditions, the d' value was 5.49 (the range was 3.12 to 7.83, $SD = 1.43$). Participants' accuracy varied with condition, $F(3, 27) = 9.34$, $p < .001$, $MSE = 5.63$, such that they were extremely accurate when the rotation axis coincided both with the object's major limb and a principal viewer–environment axis but less accurate in the other conditions ($p < .05$, Tukey's HSD = 2.88), in which performance did not vary.

Over all conditions, participants studied the orientation of the initial object for about 2,600 ms and required about 1,300 ms to compare the remembered appearance of the object's initial orientation against a probe. An analysis of

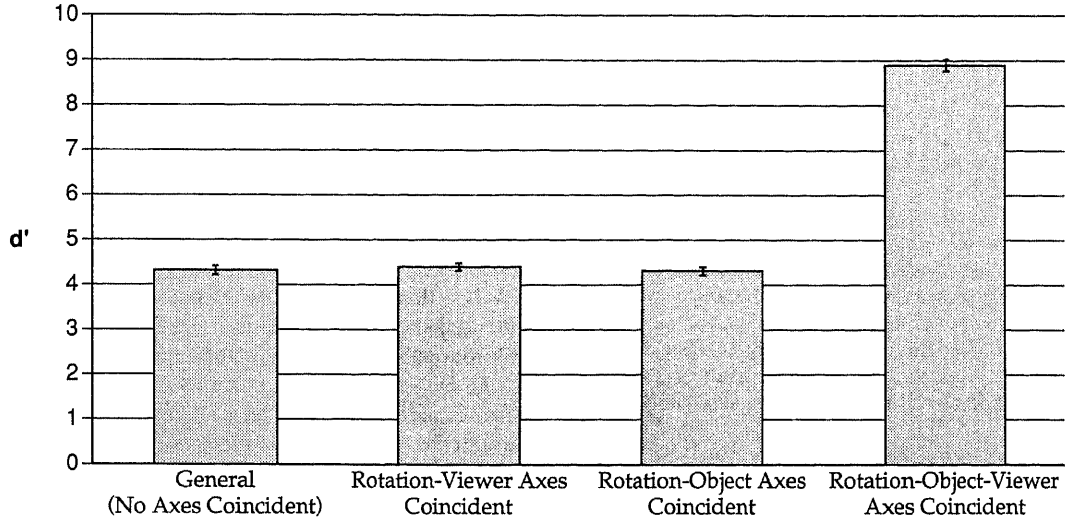

Figure 9. Experiment 3: Mean accuracy (d') for perceiving and remembering the appearance of an object. Error bars show one standard error of the mean.

variance of the two RTs using spatial condition showed no effects.

Discussion

These data suggest that, under conditions much like those of Experiment 1, participants were able to form and maintain a mental representation of the orientation of an object and then very accurately discriminate it from another orientation. Participants were especially accurate at remembering and comparing object orientation when the rotation axis coincided both with the object's major limb and with a principal viewer–environment axis. Thus, some of the superior accuracy in the coincident rotation–object–viewer axes condition of Experiment 1 was probably due to the accuracy with which the appearance of the object at such orientations is remembered and compared with a probe.

Experiment 4: Conceiving the Axis and Angle of Shortest Path Rotation Between Two Orientations of an Object

The next experiment examined, for the first time, the ability to inspect an object seen simultaneously at two novel orientations and conceive of the unique axis and angle that would rotate it from one orientation to the other. In this study, participants from Experiment 1 studied a pair of cube-figure objects used in that experiment. One object was at an initial orientation in Experiment 1, and the other was at a target orientation. When the participants signaled that they had conceived of the shortest path axis and angle that rotated the left object into the orientation of the right object, the objects were withdrawn from view and a set of physical model axes and angles was presented to participants. Participants responded by pointing to the axis and angle in front of them. They were told that the correct axis on a trial was 1 of the 11 axes in front of them and that the correct angle was 45°, 90°, 135°, or 180° anticlockwise (as in Experiment 1).

Another group of participants who had not been involved in any related experiment participated in Experiment 4. Comparison of the two groups allowed assessment of whether prior experience with the objects, rotation axes, and rotation angles could affect accuracy in conceiving of the shortest path angle and axis. If new untrained individuals performed worse than those from Experiment 1, it would suggest the extent to which performance could be improved by both familiarity with the object, axis, angle, and orientations and great practice attempting the rotations of the object about an axis.

I expected participants to perform the task by trying to imagine the object rotate in the shortest path between the two orientations while hypothesizing its rotation axis and angle. Thus, the accuracy of their response in this task was expected to vary, as in Experiment 1, with the spatial relations among the rotation axis, object, viewer, and environment. In addition, it seemed likely that the spatial conditions would similarly affect the accuracy of performing the subtask of conceiving of the shortest path axis and angle from the object's imagined rotation between the two perceived orientations.

Method

Participants. The 14 students in the far and near foil conditions in Experiment 1 participated in this study several days after participating in Experiment 1. Fourteen other undergraduate volunteers completed Thurstone's Card Rotation and Cube Comparison tests. Seven students of high spatial ability correctly completed 92% of the items (range = 86% to 94%) with a 5% error rate. Seven students of moderately high spatial ability correctly completed 81% of the items (range = 71% to 85%) with a 9% error rate.

Stimuli. The stimuli that had been used in Experiment 1 were presented in pairs so that an object at an initial orientation was on the left side of the screen and the object at its final orientation was on the right. The objects were separated by 9° of visual angle. The axes from which the participants chose their responses were divided between two models. The 11 different axes represented the vectors described in Tables 1 and 2. All other aspects of the stimulus and apparatus were the same as in Experiment 1.

Design. Each participant performed 10 practice trials that represented a unique random sample of test trials. Participants were given feedback as to the accuracy of their performance; the wooden stimuli were used in providing such feedback. Then, without feedback, they performed 48 unique trials in a unique random order with respect to all factors. Each student participated individually.

Procedure. As in Experiment 1, participants started the session by studying a wooden object from many different viewpoints and studying the orientation of the 11 candidate model axes, considering rotations about each. Participants pressed a right key to initiate a trial and see a pair of objects. They formed a conception of the axis and angle for turning the left object into the right object's orientation. A stimulus was presented until a response was made. Participants pressed the right key to end the stimulus presentation and used the set of 11 candidate axes to indicate the shortest path axes and angle. A barrier occluded the candidate axes while the stimuli were presented; however, when participants indicated their responses, the model axes were directly before them. Responses were recorded by an experimenter positioned behind the participants.

Results

An analysis of variance of accuracy (in terms of proportion correct) indicated no difference in performance between the individuals of high and moderate spatial ability in either group; all other analyses collapsed across this variable. A second analysis of variance for each group involved the condition of spatial relations among rotation axis, object, viewer, and environment.

Individuals of high and moderately high spatial ability were very often poor at conceiving the unique axis and angle of rotation separating a pair of orientations of an object (the overall accuracy rate was .44; see Figure 10). Participants trained in Experiment 1 tended to be more accurate in performing this task overall than the fresh participants (.48 compared with .40), $F(1, 26) = 2.23, p < .15$, $MSE = 0.0776$. Both groups' accuracy was similarly affected by the spatial conditions, $F(3, 78) = 72.88, p < .001$, $MSE = 0.0371$. Accuracy was significantly different in each condition (at least $p < .05$ by Tukey's HSD, which was .13).

When there was no coincidence among the rotation axis, the object's major limb, and a principal viewer–environment axis, participants had a .13 accuracy rate, which was different from chance ($p < .05$). Participants from Experiment 1 had an accuracy rate of only .17, even though they were quite familiar with the axes and angles of rotation, the object, the specific initial and final object orientations, and the task of imagining the object rotate about each axis.

Participants were more accurate in the special stimulus conditions. They were slightly better when the rotation axis coincided only with a principal viewer–environment axis, better still when the rotation axis coincided only with the object's major limb, and best of all when there was full coincidence among the rotation axis, the object's major limb, and a principal viewer–environment axis. As in Experiment 1, accuracy in this last condition was greater than the linear combination of the effects of (a) having the rotation axis coincide with a principal viewer–environment axis and (b) having the rotation axis coincide with the object's major limb.

Participants often showed false alarms to an axis near the shortest path axis or to an axis with a similar relation to the three principal axes (e.g., if the shortest path axis was outside of the three planes, they showed a false alarm to another axis outside of those planes). As expected, variation of accuracy across the spatial conditions in this task was strongly correlated with the accuracy of imagining the object rotate about an axis: for the trained participants from Experiment 1 ($r = .99$), $F(1, 2) = 95.44, p < .01$, and for the fresh participants ($r = .95$), $F(1, 2) = 18.43, p < .05$.

Accuracy in conceiving the rotation angle was analyzed independently of whether the rotation axis was correctly conceived. For all conditions and individuals, erroneous angle responses were distributed symmetrically about the correct value. The highest accuracy rates were for the full coincidence condition (.82) and for trials in which the rotation axis coincided only with the object's major limb (.76). There was no difference between these conditions, but performance was worse in the other two cases (which involved rates of .51 and .48, $p < .01$).

The absolute accuracy of conceiving the shortest path angle was equal to that of conceiving both axis and angle except in the general condition, in which it was much better for the angle (.51 vs. .13). Thus, in all but the general case, if participants knew the angle, then they probably knew the axis. In the general case, participants probably used some estimate of the distance between orientations to choose among the four available angles. Also, when only the rota-

Table 2
Foil Axes of Rotation for Experiment 4

Coordinates[a] (*x, y, z*)	Description of direction
0, 0, 1	Line of sight axis
−.7, .7, −.7	45° between line of sight axis and negative horizontal axis and 45° above horizontal plane
.7, 0, −.7	45° between horizontal and line of sight axis
0, .7, .7	45° between vertical axis and line of sight axis pointing into screen
.7, .7, .7	45° in between horizontal axis, line of sight axis, and vertical axis; points away from observer

[a] Based on a system with $+x$ representing right, $+y$ representing up, and $+z$ representing line of sight into the vertical screen of the observer (or the page of the reader).

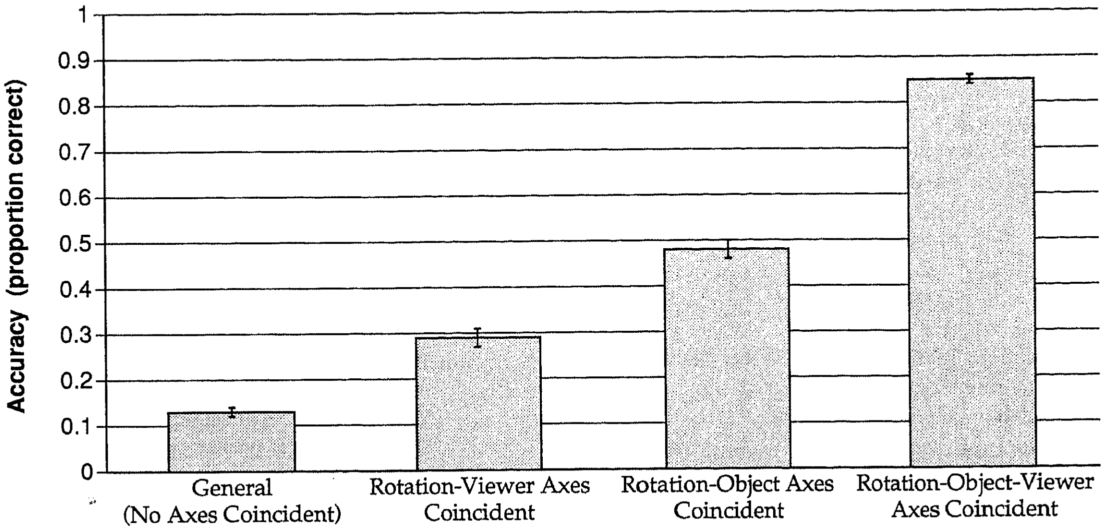

Figure 10. Experiment 4: Mean accuracy of conceiving the shortest path axis and angle of rotation separating two orientations of an object. Error bars show one standard error of the mean.

tion axis and the object's major limb coincided, the accuracy of conceiving the angle was equal to that when there was full coincidence. Thus, angle judgment accuracy was due to the coincidence of the rotation axis and the object's major limb and was not improved by having those two axes aligned with a principal viewer–environment axis.

All participants in both groups typically required between 15 and 25 s to conceive the shortest path axis and angle. Also, all participants reported that the task was extremely difficult, demanding great effort, and that they often lacked confidence in the accuracy of their response. The time needed to perform the task was an order of magnitude longer than the time needed to imagine such an object at the orientation of another so as to compare their shape (Metzler & Shepard, 1974). On completion of the study, introspective reports were elicited from the participants. About 90% of them reported that they performed the task by trying to imagine the object rotate in the shortest path between the two presented orientations; the remaining individuals reported using other strategies.

Discussion

Thus, individuals of high and moderately high spatial ability were very often incapable of conceiving the unique shortest path axis and angle that would rotate a simple three-dimensional object from one novel orientation to another. Half of the participants had moderate familiarity with the axes and angles of rotation and with the object and its orientations, and all participants had various task features assisting performance (see introduction). Yet, they all performed with great and long effort at nearly chance levels in the most general and representative condition of spatial relationships among the object, rotation axis, and viewer–environment frame. These performances contrast sharply with the much faster, less effortful, and much more accurate performances of participants imagining one such object at another's orientation so as to compare shape (Kaushall & Parsons, 1981; Metzler & Shepard, 1974).

Participants with more familiarity and training with the axes and angles of rotation, objects, and orientations tended to be somewhat more accurate at conceiving the shortest path rotation axis and angle than participants with comparable spatial ability but less such experience. This suggests that some of participants' difficulty may have been due to their lack of familiarity with the perceptual situation and with the possibilities for spatial transformation. However, the small improvement associated with training and familiarity does not encourage the idea that performance can be much enhanced by such experience. Perhaps more extensive and explicit training with constant feedback would produce greater enhancement.

The accuracy of conceiving the shortest path angle and axis between two object orientations was improved in special spatial conditions. In these conditions, the angles between the object's major limb and the rotation axis and between the rotation axis and the viewer–environment frame were each simple or canonical, and the rotation axis became more conspicuous by its coincidence with another conspicuous axis.

Most participants performed this task by trying to imagine the object's rotation between the two orientations in the shortest possible path and "figuring out" its axis and angle from this path. This probably explains why the accuracy here was affected by the spatial conditions in the same way as was the accuracy of imagining the rotation of the object about an axis in Experiment 1. As the object's imagined path deviated from the true shortest path under different spatial conditions, the likelihood increased that the axis and angle hypothesized by participants would be incorrect.

General Discussion

Reasoning about an object's orientation with an axis and angle of rotation is a research problem intersected by three lines of investigation. First, there is work on the conditions that affect the efficiency with which people reason about particular spatial relationships and transformations. One relevant issue here is how best to characterize human capacities for finding and using the shortest path axis and angle. Another is how to characterize the principles accounting for the conditions affecting the efficiency with which people perform these tasks. Second, there are studies concerned with understanding the geometrical basis of the trajectory represented spontaneously when one imagines an object move from one orientation to another. The relevant issue here is whether, in such cases, the object is represented to move in the rotation described by the shortest path axis and angle. Third, there are studies on what information about an object's shape is represented when one imagines it turning from one orientation to another. One relevant issue here is whether those representations contain adequate detail to serve as a basis either for object recognition or for the discrimination of similar shape. Another is whether such representations are actually used for those purposes.

These three lines of research are touched on by implications of the following findings from the present studies. Even individuals of the highest spatial ability who are given training and considerable practice are, in the great majority of possible cases, unable to accurately imagine a simple three-dimensional object perceived at a novel orientation rotate through an angle about an axis and accurately anticipate its new orientation. Such individuals are also, in the vast majority of possible cases, unable to inspect two novel orientations of a simple three-dimensional object to accurately conceive of the unique axis and angle that rotate it through a shortest path from one orientation to the other.

The accuracy of finding and using the shortest path axis and angle of rotation rises above chance and progressively improves across a set of perceptual situations that possess fortuitous spatial relationships among the rotation axis, viewer, environment, and salient object parts. The most plausible hypothesis for this pattern of effects is that accurately imagining an object's rotation about an axis requires an effortful and error-prone process of monitoring, from a single viewpoint, two specific spatial relationships to ensure that they hold constant during rotation. These relationships are (a) the angle between the rotation axis and the object and (b) the angle between the rotation axis and the stationary viewer–environment frame of reference. When these two angles are arbitrary or noncanonical, accuracy in the task remains at or near chance. When only the angle between the rotation axis and the stationary viewer–environment frame of reference is simple or canonical (0° or 90°), the accuracy and ease with which it can be monitored are improved, and performance efficiency increases somewhat. When only the angle between the rotation axis and the object is simple or canonical, the accuracy and ease with which it can be monitored are improved further still, and accuracy in the task increases even more. The object's major limb is apparently a more salient and relevant structure for object rotation, and performance is better when the angle between the rotation axis and the object is canonical than when the other angle is canonical. When both angles are simple or canonical, the accuracy and ease with which they can be monitored are especially improved, and performance is by far best of all. Indeed, it is greater than expected from the linear combination of their separate effects, suggesting that other factors may emerge to improve performance. Some improvements may be due to perceptual organizational factors related to the appearance of this object, whose parts join at 90° angles rotating about its natural axis and a canonical environmental axis (see Shiffrar & Shepard, 1991).

Thus, native human ability for the two tasks is very limited, with accurate performance only in a small proportion of all possible cases. Those cases in which participants are more accurate are characterized by simple, coincident, or overlapping spatial relations that simplify the tasks' critical components of generating represented object orientations and checking these orientations for errors. More efficient overall performance may be achieved with sustained and focused training, as suggested by the results of Experiment 2.

This pattern of results is generally consistent with related earlier observations (e.g., Hinton, 1979; Just & Carpenter, 1985; Massironi & Luccio, 1989; Pani, 1993; Pani & Dupree, in press; Parsons, 1987a; Rock et al., 1989; Shiffrar & Shepard, 1991). In the Shiffrar and Shepard study, for instance, the spatial relationship among the object, rotation axis, viewer, and environment affected, in a manner corresponding to that revealed here, the accuracy of deciding whether two successively perceived—not imagined—rotations of a cube were the same or different. This consistency across tasks that require generating and imagining object rotation and tasks that require perceiving and discriminating object rotations suggests that these effects are fairly general and fundamental to spatial representation.

If, even under propitious conditions such as those used here, people cannot, in most cases, use this kinematic procedure in imagining the spatial transformation of a simple object from one novel orientation in 3-dimensional space to another, are there other methods that they can accurately apply? The answer to this question requires further research. However, it seems likely that, except for simple rare cases, participants will not be able to accurately use any kinematic operation that depends on such reasonably fine control of spatial relations among multiple structures. If so, this will be useful in building process models of the mental representation of an object's shape, orientation, and transformation. It will also have significant implications for applied areas such as education, design, and human–computer interaction.

The general inability to perform these tasks is not consistent with the idea that, in general, people explicitly and readily represent object orientations or object trajectories in three-dimensional space with respect to the unique shortest path axis and angle. This is bad news for the hypothesis that when the shapes of disoriented similar objects such as Shepard–Metzler figures are discriminated, the shortest path axis and angle are available as one imagines one object at the other object's orientation. However, a corollary hypothesis remains that implicitly, without conscious intention or guidance, people represent changes in object orientation in terms of shortest path rotations about unique axes. This characterization is consistent with the finding of Parsons, Gabrieli, Yacaitis, and Corkin (1988) that the globally amnesic patient H.M. could improve the rate of imagining the rotation of a letter despite having no conscious memories of seeing the stimulus before or of imagining such rotations. Imagined spatial transformation may be like perceptual–motor skills with respect to the lack of conscious access to performance details; attempts to intercede with conscious attention may disrupt efficient performance.

It is also possible that shortest paths between orientations of an object are represented, but not with explicit reference to an axis and angle of rotation (Carlton & Shepard, 1990; Parsons, 1988). Thus, different orientations of an object could be represented as points of activation in a three-dimensional map or space, and shortest path trajectories between orientations could be planned or produced by a process of spreading activation between such points (Shepard, 1984). Such a representational mechanism does not explicitly represent axis and angle of rotation, and it may be impossible to interrogate or drive that representational process with axis or angle information. Three studies of participants judging whether disoriented objects have the identical or different shape have concluded, on the basis of RT–orientation functions, that participants imagined one object rotate in a shortest path to the orientation of the other (Just & Carpenter, 1985; Parsons, 1987b; Tarr & Pinker, 1989). In each case, the pairs of object orientations were like those used in the general condition here. The participants were apparently not aware of using the shortest path or of the rotation axis and angle that described it. Thus, it will be important to study these issues with the paradigms designed to reveal implicit or nonconscious mental representations and processing.

If, in at least some cases, people base their representation of the orientation space of an object on its perceptually salient principal axis, then they may prefer to use a method of reorientation better suited to such a representation (Carlton & Shepard, 1990), such as a spin–precession procedure (Parsons, 1984, 1987c, 1988).[3] In this procedure, the object precesses between the initial and final orientations of its principal axis while spinning about that axis. This method produces shortest path rotations if orientation space is represented as the product of the two-dimensional space of all possible principal axis positions (i.e., axis direction expressed in spherical coordinates) and the one-dimensional space of all orientations about that particular axis (i.e., spin angle displacement). Further research is necessary to evaluate such possibilities.

Despite appearances, the data here may not add to the recent evidence apparently against the idea that people can imagine one object rotate to another's orientation with enough information about its shape and its appearance at that orientation to accurately discriminate the objects even when they have similar shape. Rock et al. (1989) reported that participants were not very good at knowing what an abstract twisted-wire form perceived at one orientation would look like after being rotated 90° about the vertical. Their participants faced two difficult tasks in addition to imagining an object at an orientation. First, they had to form a structural description of an abstract irregular object that lacked distinct subparts and could be parsed in different ways when seen from different views. When the structural description of an object is unstable and varies with orientation, the accuracy with which it can be used in judgments of shape is limited. Second, the participants had to determine accurately an object's final orientation after it had been rotated about an axis not aligned with the object itself. Findings here confirm that participants have considerable difficulty doing this. In the few special cases in which my participants were accurate in this task, the time needed to imagine the object rotate was roughly linearly proportional to the angle of rotation. The quantitative relationship of that time to angle was comparable to that for shape comparisons of disoriented pairs of this object (see Experiment 1). Thus, my participants' inaccuracy is more aptly interpreted to be difficulty in determining the object's final orientation rather than difficulty or inability in imagining an object at a new orientation. In summary, then, there may be complications in interpreting the poor accuracy of participants in Rock et

[3] The spin–precession procedure (Parsons, 1987c) for reorienting an object yields a rotation about an instantaneously changing axis produced by simultaneous rotations about two orthogonal axes: a significant direction in the object's shape and an axis fixed in the environment, as in the precession of a spinning top or celestial body. The precession angle is formed between the initial and final orientations of the object's major axis (about which the object spins). The spin angle is determined by the amount of rotation necessary to align the object's minor and mean principal axes once it has rotated through its precession angle. Spin–precession motion is roughly like simple planar motion between orientations of the object's major axis (or mass), with some amount of coupled spin. Both components of motion seem relatively simple to conceive in general.

This approach represents orientation space as the product of the two-dimensional space of all possible axis positions (i.e., axis direction expressed in spherical coordinates) and the one-dimensional space of all orientations about a particular axis (i.e., spin angle displacement). For each pair of initial and final orientations, there are two corresponding points in the space of all axis positions and two points in the space of rotations around the axis. Thus, spin and precession represent shortest paths within these two subsets if orientation space is represented in this fashion. Spin is the shortest path in the one-dimensional space of rotations about an axis, and precession is the shortest path between two axis positions.

The rate at which the spin and precession rotations are performed can be varied. For homogeneous motion, the spin rate is made equal to the precession rate, and the resulting total angle of rotation is equal to the precession angle multiplied by the square root of the quantity one plus the square of the ratio of spin to precession angles (Parsons, 1987a, 1987b, 1987c, 1988). A model based on this spin–precession method fits the data on imagined spatial transformations as well as does the shortest path model based on Euclidean orientation space (Parsons, 1987c). Over all possible orientation differences, the path produced by this method is, on average, only 6.5% longer in Euclidean distance than the Euclidean shortest path trajectories (Parsons, 1988).

al. (1989) as an inability to imagine an object at an orientation per se. The findings here indicate a general inability to determine and envision the final orientation of an object, but they also show that response time and accuracy data are consistent with an ability to imagine an object at a new orientation in rare simple cases.

Furthermore, there is a convergence of various kinds of evidence for the claim that people can imagine at least a relatively simple or familiar object at a new orientation. First, there are two independent kinds of evidence that when people report imagining an object at another orientation, they imagine it rotate from its initial orientation to a goal orientation via intermediate orientations (Cooper, 1976; Parsons, 1995a, 1995c, 1995d). Second, when participants are asked to imagine an object rotate more than 180° in the picture plane, the response time continues to increase proportionally (Cooper, 1975; Metzler & Shepard, 1974). Third, the time needed to make a handedness judgment of a single visually presented human hand corresponds closely to (a) the time needed to mentally simulate, on instruction, one moving one's hand to the orientation of that hand (without a left–right judgment) and (b) the time needed for the corresponding real action (without a left–right judgment; Parsons, 1987b, 1994). In addition, the time needed for the left–right judgment, the mental simulation, and the real action for a given stimulus depends in the same way on the actual current kinematic configuration of the observer's body (not a fixed canonical configuration). The complicated variation of RT with stimulus orientation in these different processes is very difficult to explain without recourse to a mechanism similar to that underlying the mental representation of the continuous path of an object other than part of one's body (Parsons et al., 1995; Parsons, Gabrieli, & Gazzaniga, 1995).

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