

Rate-Hardness: A New Performance Metric for Haptic Interfaces

Dale A. Lawrence, *Member, IEEE*, Lucy Y. Pao, *Senior Member, IEEE*, Anne M. Dougherty, Mark A. Salada, and Yiannis Pavlou

Abstract—Rate-hardness is introduced as a quality metric for hard virtual surfaces, and linked to human perception of hardness via a psychophysical study. A 3 degree-of-freedom haptic interface is used to present pairs of virtual walls to users for side-by-side comparison. 49 subjects are tested in a series of three blocks of trials, where different virtual walls are presented in randomly ordered pairs. Results strongly support the use of rate-hardness, as opposed to mechanical stiffness, as the more relevant metric for haptic interface performance in rendering hard virtual surfaces. It is also shown that common techniques of enhancing stability of the rendered surfaces tend to actually enhance performance as measured by rate-hardness.

Index Terms—Control design, haptic interface, haptic perception, hardness, psychophysics, stability, stiffness, virtual surface.

I. INTRODUCTION

HAPTIC interfaces for applications in teleoperation and virtual reality have evolved from simple joysticks and clumsy hand controllers into sophisticated, dextrous user interfaces involving sensitive finger grips [4], [5], [10], [11], [19], [20]. While it is generally accepted that these newer devices are better at rendering objects (in virtual reality) or providing telepresence (in teleoperation), it remains difficult to measure their performance in objective terms.

Performance measures have been examined at two levels. At the lowest level, technical quantities such as sensor resolution, actuator dynamic range, force bandwidth, achievable stiffness, etc., have been proposed as performance metrics [3], [6], [22]. While easy to quantify in technical terms, their perceptual value

to a user of the haptic interface is harder to establish [26]. Hence it is very difficult to design or select a haptic interface for a particular application.

At the highest level, users are often directly involved in overall system performance tests. Task completion times and excess force measures are common in teleoperation [9], [32]. These provide clear comparisons between existing systems, but here it is difficult to correlate these results with the underlying technical capabilities of sensors, actuators, computer controllers, etc.

This paper seeks to provide low level performance metrics which can be easily measured, yet which have clear relevance to the user's perception. We focus on the question: what makes a virtual surface feel hard? We approach this question by asking, more specifically, which physical attributes of these surfaces can users perceive as causing differences in apparent hardness? Only those features that can be distinguished will be relevant as metrics for perceptual performance. Our approach is similar to [26], where perceptual qualities were related to mechanical characteristics (stiffness, damping) of virtual walls. Here, mechanical stiffness and a new concept, rate-hardness, are tested for relevance to the haptic sense. Such metrics are useful in optimizing haptic interface components to improve perceptual performance, or in reducing the cost of haptic interfaces by avoiding technical performance enhancements which are not perceptually relevant.

Section II motivates and poses a specific hypothesis concerning discrimination of hard virtual surfaces, and defines rate-hardness. Section III describes the haptic interface apparatus and the protocol used to test the hypothesis. Results of the test are presented in Section IV. In Section V, we discuss how both stability and performance can be improved in light of our results. Finally, concluding remarks are given in Section VI.

II. HYPOTHESIS

Fig. 1 shows force and position data measured during user contact with four different types of virtual walls. Users comment that some of these surfaces feel harder than others, and some are very difficult to distinguish based on their apparent hardness. The force and position curves differ in several respects, so it is not immediately clear which attributes of the data are responsible for the perceptual judgments of differences in hardness.

Much attention in the recent literature is devoted toward improving the stiffness of haptic interfaces, that is, toward increasing the static forces that can be elicited by penetrating a given distance into a virtual surface, e.g., [30], [6], [8]. The

Manuscript received August 5, 1998; revised December 10, 1999. This paper was recommended for publication by Associate Editor K. Kosuge and Editor S. Salcudean upon evaluation of the reviewers' comments. This work was supported in part by the National Science Foundation (NSF) under Grant IIS-9711936, the Office of Naval Research (ONR) Defense University Research Instrumentation Program through ONR's Cognitive Science Program under Grant N00014-97-1-0354, and the NSF Engineering Research Center on Optoelectronic Computing Systems at the University of Colorado at Boulder under Grant EEC-9015128. This paper was presented in part at the International Mechanical Engineering Congress and Exposition Symposium on Haptic Interfaces for Teleoperation and Virtual Environments, Atlanta, GA, November 1996.

D. A. Lawrence, M. A. Salada, and Y. Pavlou are with the Department of Aerospace Engineering Sciences, University of Colorado, Boulder, CO 80309-0429 USA (e-mail: dale.lawrence@colorado.edu; salada@colorado.edu; pavlou@colorado.edu).

L. Y. Pao is with the Department of Electrical and Computer Engineering, University of Colorado, Boulder, CO 80309-0425 USA (e-mail: pao@colorado.edu).

A. M. Dougherty is with the Department of Applied Mathematics, University of Colorado, Boulder, CO 80309-0526 USA (e-mail: adougher@newton.colorado.edu).

Publisher Item Identifier S 1042-296X(00)06956-1.

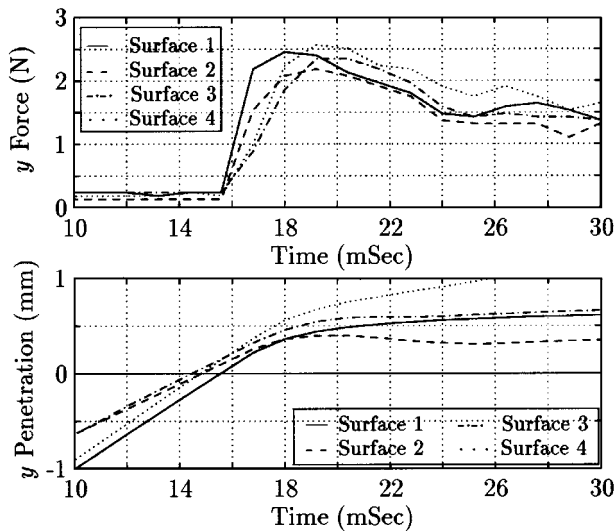


Fig. 1. Plot of forces and positions during a typical encounter with four different virtual walls. Positive y positions indicate penetration inside the virtual wall.

curves in Fig. 1 were measured during interaction with hard virtual walls, using a 3 degree-of-freedom haptic interface (see Section III for details on the device). This interaction consisted of repeated tapping on the virtual surface. Fig. 1 captures four contact events from four different virtual walls. For comparison, all data were post-processed to align them in time, so that wall penetration begins in each case at about the 16 ms mark.¹ Forces were measured (to a resolution of 0.03 N) by strain gauge sensors near the finger grip. Forces increase abruptly as the virtual wall is penetrated. Position inside the virtual wall was measured (to a resolution of 7 μ m) by optical encoders. Negative penetration indicates motion before contact, and positive penetration occurs after contact with the wall. Force and position data were sampled synchronously at a rate of 833 Hz.

The data in Fig. 1 indicate differences in the stiffnesses of the virtual walls. In the region about 10 ms after the initial penetration, where forces and positions begin to settle to constant values, note that all forces are similar but some of the penetration distances differ considerably. In particular the surface described by the dotted line shows a stiffness K which is about half that indicated by the solid line. As one might expect, users remark that the latter surface feels harder as well.

But how are people able to haptically distinguish such high levels of stiffness K (1500–3000 N/m)? If it is the quasistatic relationship between penetration and force, distinguishing the dotted and solid lines in Fig. 1 would require a position sensing resolution (if not accuracy) of better than 0.5 mm derived from hand and arm joint position sensing, which is at the extreme lower end of reported kinesthetic position change detection [1]. Studies using finger-only motion compressing similar stiffnesses through a hard-surface intermediary (analogous to the plastic finger grip used here) also report difficulty in discriminating differences smaller than a factor of 2 [29].

¹The y penetration histories do not appear as aligned as the y force profiles upon wall impact due to the sampled nature of the data. The impact is determined to within the sampling period of 1.2 ms.

When the stiffness is about an order of magnitude smaller, quasistatic discrimination improves to 30% differences (at least using finger/thumb pinch motion [31]). Dominance of quasistatic stiffness discrimination in the present case is further contradicted by the fact that in preliminary tests, the surfaces indicated by the solid and dot-dashed curves have perceptual hardnesses that are easily distinguished by users, yet they have almost identical steady-state forces and penetrations (equal stiffnesses).

Perhaps there are other attributes of this data that are more relevant to the perception of hardness? Notice that during the initial 1–2 ms after penetration that all the rates of position change are similar, but that rates of force change are quite different. The solid and dot-dashed lines in Fig. 1, whose surfaces seem to be easily distinguished by users, differ considerably in the initial rate of force change, but are quite similar in all other respects, e.g., stiffness, peak force, force overshoot, etc. The vibrotactile studies [33], [24] also use very high rate-of-change force profiles, triggered upon virtual surface penetration, to emulate hard surface contact. Similarly, [27] applies force pulses. Discrimination based on force rate of change at these levels (and at 1000–2000 N/s in the present study) would require very high-frequency force sensing in the hand or arm.

Evidence from steady-state vibration testing [2], [15] indicates that psychophysical sensory channels respond to vibration with a sensitivity that increases with frequency up to about 300 Hz, rolling off at higher frequencies. These channels (designated NP-I, P) bear a resemblance to velocity and acceleration sensors [28]. Similarly, physiological channels in the hand have been identified which respond with afferent nerve firing only to changes in stimuli. These channels are designated FA I and FA II due to their fast-adaption to steady stimuli [12], firing during sharp changes in skin deflection. Here, frequency sensitivity should not be confused with reaction time. Extremely short (millisecond) stimuli can be detected and discriminated, albeit after rather long (hundreds of milliseconds) delays. This sensitivity to brief events also plays a role in reflexive reactions to stimuli, e.g., during the maintenance of precision grips [34]. However, motor reactions (voluntary or reflexive) do not play any role in the present study, since the tap event on the virtual surface is quite short compared to reaction time delays.

There is ample evidence that fast rates of change in skin deflection or force can be sensed. But can the rates of force change in Fig. 1 be discriminated, and does the rate-of-change characteristic dominate stiffness in the psychophysical judgment of surface hardness? Although the rates of force change in [33], [24] were not specifically reported, they derive from stiffness models that would produce rates of change similar to those considered here. There, the resulting force waveforms whose “stiffness” differs by more than a factor of 2 were easily discriminated, indicating that discrimination of some of the virtual walls in Fig. 1 is likely on the basis of force rates-of-change. However, the actual stiffness of the walls in [33], [24] were not varied, making it unclear which effect dominates in the perception of hardness. At the same time, tapping on rubber materials where stiffness dominates in the force-motion relationship [17], results in discrimination of stiffnesses when they differ by less than 50% from 2500 N/m. Therefore, if stiffness dominates in the

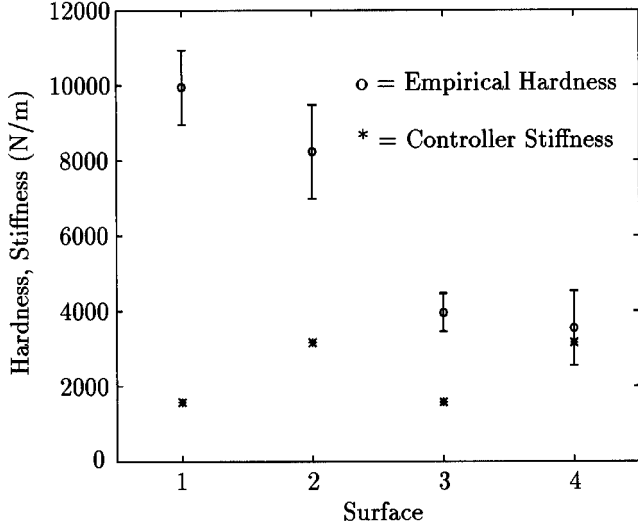


Fig. 2. Rate-hardness and stiffness for the four virtual walls which produced the data in Fig. 1. Rate-hardness estimates were computed for 50 distinct surface encounters by a single user, generating the mean values (circles) and standard deviation (error bars). Stiffnesses (stars) were determined from the DC controller gain relating measured penetration to commanded force.

perception of hardness, the hardness of the virtual walls shown in Fig. 1 should be well-correlated with their stiffness. These questions and preliminary observations lead to the following hypothesis concerning perception of virtual surface hardness:

Hypothesis: Perceptual hardness of the virtual surfaces characterized in Fig. 1 is more closely correlated with the surface *rate-hardness* H_R than with the surface stiffness K , where H_R is defined by

$$H_R = \frac{\text{initial force rate of change (N/s)}}{\text{initial penetration velocity (m/s)}}. \quad (1)$$

Note that the units for rate-hardness simplify to (N/m), but the numerical values may be larger than the surface stiffness, which is also measured in (N/m). Also, rate-hardness differs from material hardness, e.g., as measured by the Rockwell and Brinell numbers, which are related to material yield strength rather than the linear material properties of stiffness or rate-hardness. Fig. 2 shows the measured rate-hardness and stiffness of the virtual walls discussed above. Rate-hardness was measured by recording force and position data while a user produced series of “taps” on each of the four virtual surfaces. As seen in Fig. 1, initial contact rates are observed for only two samples at this sample rate. Ratios between force rate and velocity during the initial 2 samples (i.e., over 2.4 ms) of surface penetration were computed for each tap event. The “o” marks in Fig. 2 show the average of these ratios, and the root mean square (RMS) deviation about the average over the entire data set of 50 taps are represented by the error bars. Stiffness was determined by the static relation between measured position and commanded force, i.e., the DC controller gain. Force control loops on each rod ensure that the DC commanded force equals the force imparted on the plastic finger grip (measured by the white force sensors in the right photo in Fig. 4) to an accuracy of 80 mN. Walls 1 and 2 have a similar rate-hardness, but differ in stiffness by a factor of 2. Walls 3 and 4 have a similar rate-hardness,

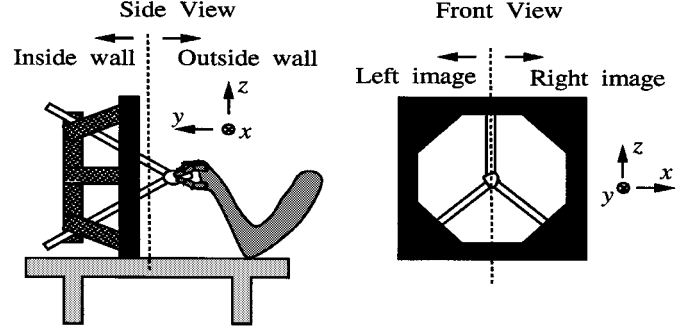


Fig. 3. Sketch of 3-DOF haptic interface configuration showing actuator arrangement, virtual surface location, and nominal operator hand/arm pose.

smaller than Walls 1 and 2, and also have differing levels of stiffness.

The next section discusses a human perception experiment designed to test the above hypothesis using these four virtual surfaces.

III. EXPERIMENT DESCRIPTION

A. Experiment Apparatus

Fig. 3 shows a diagram and Fig. 4 shows photographs of the haptic interface used to obtain the experimental results of this paper. The system provides ranges of motion and forces compatible with human hand motion at the forearm and wrist and is designed to allow exploration of the limits of haptic perception via dextrous, finger-type grips. It consists of three linear actuators arranged on a 40-cm diameter circle, each driving a hollow steel rod. The rods are connected to a plastic molded finger grip via miniature 3-DOF gimbal which are arranged to apply forces to the finger grip that intersect at a point. Thus, no torques are produced to cause confusion, and the operator perceives that the virtual surface is “grasped” at a point between the finger tips. The interface is operated with one’s elbow resting on a padded table top, with the elbow at a nominal 90° angle. The virtual surface is vertically oriented, with the y direction in Fig. 3 normal to the surface. From the operator’s viewpoint, the surface is divided along the vertical midline into a right and left half. Surface properties can be specified separately on each half, allowing the operator to rapidly switch between surfaces by moving slightly to each side of the midline. This reduces the effects of variation in sensory acuity due to changes in finger grip and hand/arm pose.

Fig. 5 describes the dynamics of the haptic interface which relate Cartesian forces and displacements. The system is partitioned into three main elements: human operator dynamics, haptic interface mechanical and electrical dynamics, and digital control dynamics. The operator feels the impedance Z_A at the point where the fingers touch the interface grip. Z_A relates the (x, y, z) forces F_s to the (x, y, z) displacements X via $F_s = Z_A(X)$. The position X is sensed via optical encoders within each actuator to a resolution of 7 μm . Z_h is the equivalent impedance of the human hand/arm, and F_{he} is the equivalent force provided by the operator to cause motion. F_s is the force provided by the haptic interface at the point where the fingers grip. The force F_s is sensed near the rod attachment to the

2 near 100-Hz result in higher rate-hardnesses. If the impedances Z_c are interpreted as a (low pass filtered) stiffness K and damping B with impedance $K + Bs$, the higher rate-hardnesses can be viewed, at least in this case, as resulting from higher values of damping B . Section V discusses this connection in more detail. All four impedances used in this study have ratios between difference to reference values (pair averages) of approximately 0.67, although some only exhibit this difference over a particular frequency range.

In a pairwise comparison, the four impedances shown in Fig. 6 were rendered on right and left halves of the vertical virtual plane shown in Fig. 3, approximately 30 cm in front of the test subject. Subjects probed the pairs of surfaces at will, and were instructed to avoid changing finger grip or arm pose. Subjects were asked to respond with the words “Left” or “Right”, using that word which best described the side that felt harder. Subjects were told that some pairs would not differ, and a response of “Same” was also acceptable. The four different surfaces then yielded a test suite of 16 distinct side-by-side combinations, 12 of which presented different surfaces, and 4 which presented identical surfaces. The presentation of each of these 16 combinations, in a random order, constituted one block of trials. Each subject was given three consecutive blocks of trials, separated by a 5–15-min rest period. Total test duration for each subject averaged 70 min.

There were 49 volunteer subjects, 33 males and 16 females, ranging in age from 18 to 37. Most were engineering students drawn from the authors’ courses, and were offered extra credit toward their course grade for participating in the test. Each subject was given a standardized training session which included information on operating the test apparatus and the general test objectives to judge hardness of virtual surfaces, instructions on arm and hand positioning, and training on several samples of different and identical surface combinations. The training samples were chosen to inform the subjects of the range of wall differences that they would encounter to make them aware of subtleties in the actual test, and to indicate that some pairs of surfaces may not be different at all. No additional feedback on the correctness of any response was provided until all three blocks of trials in the test had been completed. The subjects were not given specific information on what surface characteristics were being tested (e.g., rate-hardness), nor were they informed of the likelihood of being presented different or identical surfaces during each trial. All subjects wore ear protectors to eliminate any audio cues. Visual cues were not believed to be a factor in preliminary testing because the surface penetration differences were too small to reliably observe. Consequently, visual access to the apparatus was permitted. Interestingly, many subjects preferred to close their eyes anyway. Only one subject commented that they believed visual cues were helpful in the test.

IV. ANALYSIS OF RESULTS

We first overview the main results in Section IV-A, and then we discuss a number of additional issues in Section IV-B. Figures containing our raw results and more detailed analyses of our results are presented in the Appendix.

A. Main Results

Each subject was assigned a personal score which quantified his or her ability to correctly identify differences in surfaces. Each of the first six wall combinations was presented twice (in a random order), once as $(a : b)$ (a on the left, b on the right) and once as $(b : a)$. Therefore, the possible personal scores for this combination were 0 (none correct), 1 (one correct answer), or 2 (both correct). The last four wall combinations were presented once with the same surface on each side. Here, the possible scores were 0 (subject could not correctly identify the two surfaces as being the same) or 1 (correct). Notice that wall combinations (2:4) and (1:3) differ only in rate-hardness, wall combinations (3:4) and (1:2) differ only in stiffness, and (1:4) and (2:3) differ in both rate-hardness and stiffness. (Recall Fig. 2). Raw results for all wall combinations in each of the three blocks of trials can be found in Figs. 15–17 in the Appendix.

We begin our analysis of these results by classifying the 16 possible wall combinations into three wall groups. Wall group 1 contains wall pairs that differ significantly in rate-hardness, Wall group 2 contains pairs which differ in stiffness, and Wall group 3 contains pairs of identical walls.

In order to justify combining the personal scores within each wall group (and thus allowing between group analysis), a non-parametric rank F test ([23, p. 1094]) was done for each wall group. The null hypothesis of this test states that if we rank personal scores from each of the wall combinations in a group in each block of trials, the averages of the ranks for each wall combination across the blocks of trials are equal. We accepted the null hypothesis with p -values of 0.27, 0.72, and 0.86 for the first, second, and third wall groups, respectively. Thus, for each subject, we have three wall group measurements for each of three blocks of trials, for a total of nine aggregated measurements.

A repeated measures analysis of variance ([23, Ch. 29]), for all subjects combined, showed that there is a very significant difference between the three wall groups (p -value of $p < 0.0001$). This indicates that the differences between walls (all with impedances that have Weber fractions around 0.67 for at least some frequencies) are perceived very differently among the wall groups. These results are presented as the normalized personal score mean and corresponding 95% confidence intervals² for each wall group in Fig. 7. Note that Wall group 1 contained surface hardness qualities which could be reliably distinguished by the majority of subjects, indicated by normalized personal scores above 80%, compared to the 33% level which would indicate random guessing in subject responses (possible responses are harder surface on the Left, Right, or Same hardness). Even when stiffnesses in the pair contradict rate-hardness levels, [e.g., pairs ((1:2), (1:4))], subjects reliably chose the surface with the higher rate-hardness over that with the higher stiffness.

Wall group 2 contained wall combinations that had similar rate-hardness, but walls differed in each pair by a factor of two in stiffness. The normalized personal score averages near 33% indicate that subjects could not correctly determine which surface had the higher stiffness through their judgement of surface

²The errors indicated in the 95% confidence intervals are theoretical errors. No attempt was made to estimate any additional error resulting from the use of volunteers, as opposed to a random sample, or from potential human error in the recording of each subject’s answers.

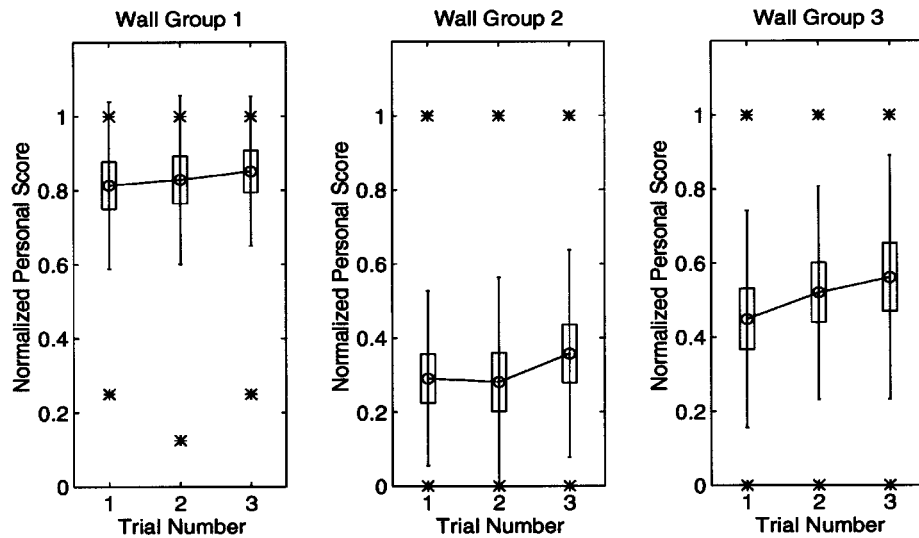


Fig. 7. Normalized personal score averages (circles) over all subjects, aggregated over wall combinations within the three wall groups, versus trial block number. Also shown are standard deviations (error bars), 95% confidence intervals for the means (boxes), and maximum and minimum personal scores (stars).

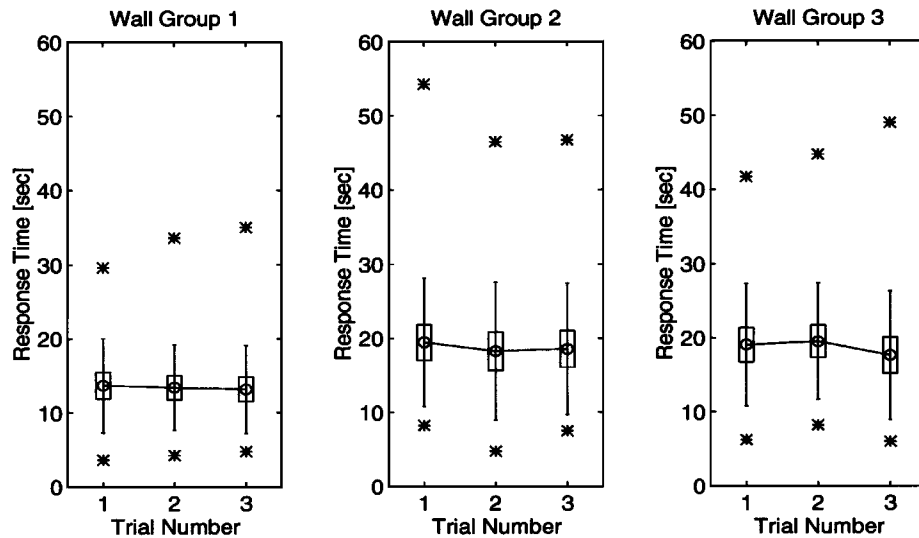


Fig. 8. Time to respond averages (circles) aggregated by wall group, for the population of 49 subjects, with one standard deviation (error bars) and 95% confidence interval for each mean (boxes), and maximum and minimum times to respond (stars).

hardness. The responses for Wall group 2 are similar to the responses one would obtain if the subjects were simply guessing. Also, the two different background levels of rate-hardness produced no significant bias in subjects' inability to discriminate stiffness.

Wall group 3 contained surface pairs that had identical surface properties. Here, correct response frequencies were around 50%—above the guessing level, but below the highly accurate responses obtained from wall group 1. Since the subjects knew (from the training session) that some of the differences between the wall combinations (i.e., those in Wall group 2) would be very subtle, they may have been trying to detect differences when there weren't any. The rank test between surfaces in this group indicates that the level of rate-hardness or stiffness did not have a significant effect on the difficulty of identifying identical surfaces.

For each wall group, the repeated measures analysis of variance also indicated a small, but significant, change in the aver-

ages between the three blocks of trials (p -value of $p = 0.0292$). We interpret this change as indicating an increase in familiarity with the equipment, an increase in subjects' haptic acuity, or improved ability (learning) to recognize the four individual surfaces presented. Regarding learning, it is interesting to note that, on average, the subjects did not respond to each wall combination faster in later trial blocks compared to earlier ones. See Fig. 8. There was a significant difference in the time to respond between wall groups, with Wall group 1 responses being consistently faster. This is an added indication that Wall group 1 surfaces were easier to distinguish—subjects took less time and achieved more correct responses than the other two wall combinations.

Fig. 9 plots the responses averaged over all 3 trial blocks, with a separate point ($a : b$) for each wall pair. Rate-hardness versus percent correct response is also plotted separately from stiffness versus percent correct. Linear regressions are shown, as well as correlation coefficients for each case. In the top plot, the 8 points

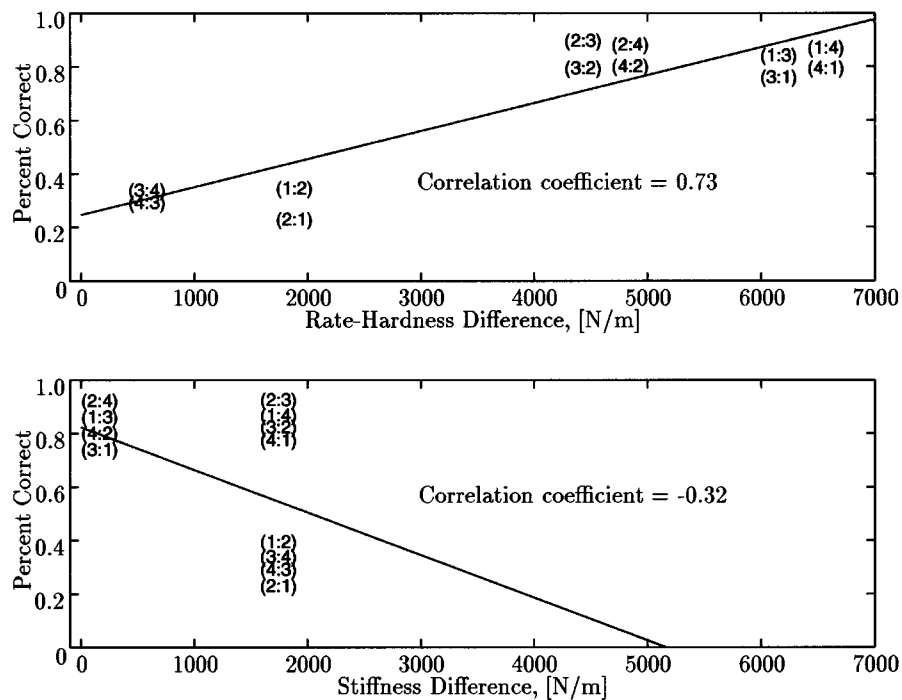


Fig. 9. Average percent correct responses over all subjects and all trial blocks, computed for individual wall combinations (combinations with identical walls are not shown), plotted versus pairwise differences in measured rate-hardness (top) and differences in controller stiffness (bottom).

clustered in the top right-hand corner form wall group 1, with wall group 2 formed by the four pairs clustered in the bottom left corner, showing a strong, positive correlation between percent correct response and wall pairs that differ in rate-hardness. The bottom plot shows that hardness discrimination ability has a weaker, negative correlation with larger stiffness differences in the presence of equal or larger rate-hardnesses. This data presentation underscores the comments above, but also suggests that rate-hardness differences for wall pairs (1:2), (2:1), (3:4), and (4:3) are below the discrimination threshold. These pairs have Weber fractions (computed as the difference over the pair average) of 0.19 and 0.11, showing that the just noticeable differences must have larger Weber fractions at these levels of rate-hardness (pair averages of 9090 and 3960 N/m, respectively). On the other hand, the above-threshold results from wall group 1 show that the Weber fraction must be below 0.79 at 5880 N/m, 0.70 at 6090 N/m, 0.95 at 6750 N/m, and 0.86 at 6950 N/m.

B. Additional Issues

Despite the clear aggregate results for the three wall groups for the test population as a whole, there are very wide variations in individual subject personal scores. This can be seen by looking at the differences between subjects in raw results shown in Figs. 15–17 in the Appendix, and by the wide error bars in Fig. 7. In particular, of the seven subjects who had 50% or fewer correct responses for the combinations in Wall group 1 on trial block 1, only two improved to greater than 50% by the third trial block. Thus, approximately 10% of the subjects had persistent difficulty distinguishing between wall combinations with the largest difference. This finding implies that similar haptic perception studies with very small numbers of subjects may be misleading for the larger population of haptic interface users.

Why are the differences between individuals so great? Initial insights were obtained from further analysis of subgroups of the population, taking into consideration differences in gender, subject right/left handedness, right/left bias among responses, and time to respond to each new wall combination.

1) *Gender Differences:* Since it is expected that haptic interfaces will be used by both men and women, it is important to understand any differences in haptic perception between them so that the design of haptic interfaces can be made to accommodate both men and women users.

A repeated measures analysis of variance classifying subjects based on gender indicated, with a p -value of 0.0024, that there were significant differences based on the sex of the subject. See Fig. 10. In Wall groups 1 and 3, men had higher personal scores, on average, compared to women. Note also that mens' scores tended to stop increasing after the second trial block, but womens' scores show continued improvement through the third trial block. The curves in Fig. 10 are not inconsistent with an hypothesis that success rates between men and women in correctly identifying surfaces would converge with more experience on the haptic interface, although more data would be clearly needed to verify this hypothesis. Also, among the ten subjects who had 50% or fewer correct responses in Wall group 1 in at least one trial block, six were women and four were men, consistent with the lower average for women versus men overall. Gender alone, however, does not account for the large variation in personal scores.

2) *Right-Handed Versus Left-Handed Users:* Because the equipment is designed for use in the right hand, we were curious whether there are differences in the responses of left-handed or right-handed subjects. However, only 2 of our 49 subjects were left-handed, too small of a number to yield any statistically significant results. However, no irregularities

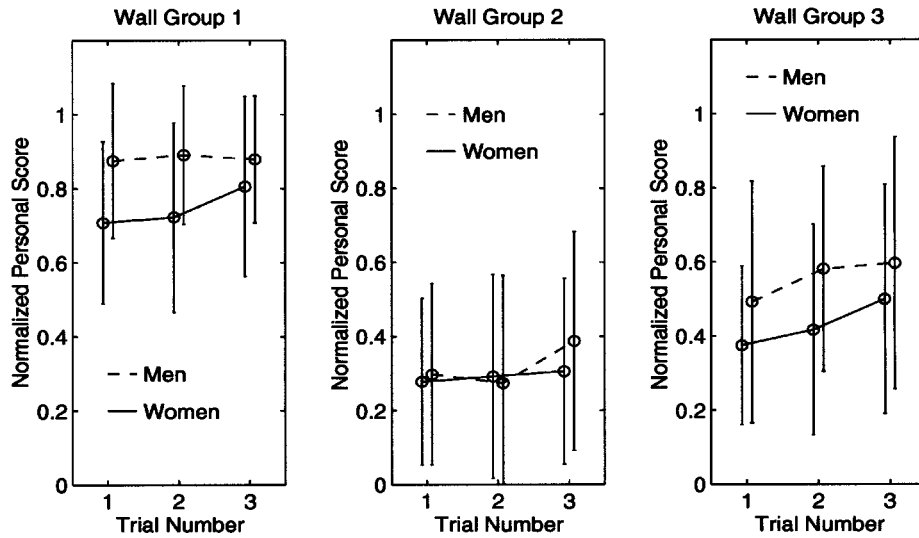


Fig. 10. Normalized personal score averages over men and women subjects separately, aggregated over wall combinations among the three wall groups, versus trial block number. Error bars indicate one standard deviation.

or inconsistencies were found upon inspection of the responses of these two subjects. In particular, both left handed subjects' personal scores in Wall group 1 were within one standard deviation of the population average on all three blocks of trials. In future studies, a greater attempt will be made to obtain a larger number of left-handed subjects.

3) *Left/Right Bias*: Another issue which we explored is whether users more easily perceive a harder surface when it is presented on the left or right. Overall, we noticed a very slight bias toward responses indicating the harder surface was perceived on the left side of the pair of surfaces. Detailed results indicating this are shown in Figs. 18–19 in the Appendix.

If we look at the same data limited to the ten subjects who had 50% or fewer correct responses in Wall group 1 in at least one of the three trial blocks, we see a much stronger bias to the left. Details can be seen in Figs. 20–21 in the Appendix. Thus, much of the variation in scores toward the low end seems to be due to significant left-right bias in a small number of subjects. This indicates that these subjects were either operating the apparatus differently on the left and right (e.g., changing the hand pose), or they are more susceptible to subtle differences in stimuli as the hand is moved from the left to the right surface during the test.

4) *Time to Respond*: Another possible explanation for variation in personal scores among individuals is that some subjects were not carefully considering the wall properties before deciding on a response, i.e., responding too quickly. Is there a correlation between correctness and time-to-respond? Fig. 11 shows that, in fact, the faster responses tend to come from those subjects who were more correct! Some subjects, therefore, seem to be genuinely unable to discriminate surfaces which are quite distinct for the majority of the test population.

V. STABILITY VERSUS PERFORMANCE

A surface with a given rate-hardness H_R can be implemented in many ways, since there are a variety of dynamic systems relating position to force which can produce the required force

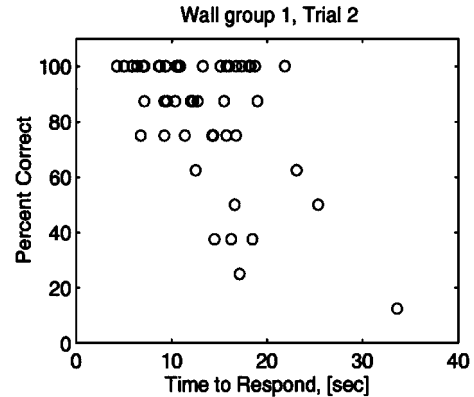


Fig. 11. Plot of percent correct responses for each subject in Wall group 1 versus average time to respond. Correlation coefficient is -0.53 .

rate-of-change per position rate-of-change on initial contact. Perhaps the simplest approach is to emulate hard natural surfaces, such as metal, glass, etc., whose constituent materials have very large elastic constants and minimal damping, i.e., let

$$f(t) = K(x(t) - x_o) \quad (2)$$

during surface contact, where K is the surface stiffness and x_o is the location of the boundary of the surface. As a mechanical impedance Z relating displacement to force, a large stiffness produces a flat frequency response with correspondingly large magnitude K . For a given initial penetration velocity v_o , the initial force rate of change is given by

$$\dot{f}(0) = K v_o \quad (3)$$

hence the surface stiffness K could be selected equal to H_R to provide the desired rate-hardness. For example, this would have required a stiffness larger than 8000 N/m to supply a rate-hardness equivalent to wall 1 (see Fig. 2).

Another approach is to provide a surface which adds significant damping so that

$$f(t) = B\dot{x}(t) + K(x(t) - x_o) \quad (4)$$

during surface contact. Presuming the force is zero before contact, the initial rate-of-change of force due to penetration with initial velocity v_o is infinite, since f is discontinuous on the surface boundary. In practice, this surface (control law) is not implementable since the derivative cannot be exactly computed. Even if the damping B is provided mechanically, and cancelled outside the surface as in [6] and [7], there are computation delays and finite bandwidth limitations on actuators which result in continuous forces on boundary contact. To gain some insight into the role of B in specifying desired rate-hardnesses H_R in practice, consider the following low pass filtered version of the control law above:

$$\begin{aligned} f(s) &= \frac{(Bs + K)}{(s/a + 1)} \delta x(s) \\ \dot{f}(t) &= -af(t) + aB\dot{x}(t) + aK(\delta x(t)) \end{aligned} \quad (5)$$

where a models the bandwidth with which forces can be supplied via the actuator, power amplifier, and computer electronics, and $\delta x(t) = x(t) - x_o$. Here, an initial penetration velocity v_o produces the initial force rate-of-change

$$\dot{f}(0) = aBv_o. \quad (6)$$

Here the rate-hardness is given by $H_R = aB$, essentially the *damping-bandwidth product* of the haptic interface. Note that when $K/B = a$, $f(s) = K\delta x(s)$, and perceptual hardness is equal to K as above. Hence we need $B \geq K/a$ to cause a significant damping effect in the surface, in which case the rate-hardness $H_R = aB$ is greater than the stiffness K . This shows that damping can be used to substitute for large values of stiffness to achieve a desired level of rate-hardness. In fact, this argument indicates that any K satisfying $K \leq aB$ produces equivalent rate-hardness H_R . However, a lower bound on K must be enforced, since $K = 0$ produces a surface which is initially hard, but allows arbitrary penetration under static force by the user. Such a surface could be distinguished from surfaces with nonzero K , and hence would not be perceptually equivalent. See [26] for a convincing experiment in this regard. In our tests with walls 1, 2, and 3 (see Fig. 2), K was substantially smaller than H_R , but still large enough that users could not discriminate surfaces based on differences only in K (recall the wall group 2 results in Section IV). How small K can become for a given B before rate-hardness H_R ceases to be the primary factor in discriminating surfaces is an interesting topic for further research.

Limits on the stiffness and damping parameters are always present due to bounds on the achievable feedback gains in the control system. Large gains can cause amplification of sensor noise and dynamic instability. Sensor noise issues are straightforward to analyze by applying the controller gain to the resolution of the sensor. For example, in the case of wall 1 an equivalent B value of 16.9 Ns/m was used, so that a single bit change in position over one sampling period in the approximation $(d/dt)x(t) \approx (x(t) - x(t-T))/T = \Delta/T$ produces a damping force change of $B\Delta/T$ N. Using $\Delta = 7\mu\text{m}$ and $T = 1.2$ ms, quantization noise in the force is predicted to be 0.098 N (about 5% of the full scale force of 2.0 N), showing that substantial increases in B cannot be obtained without the

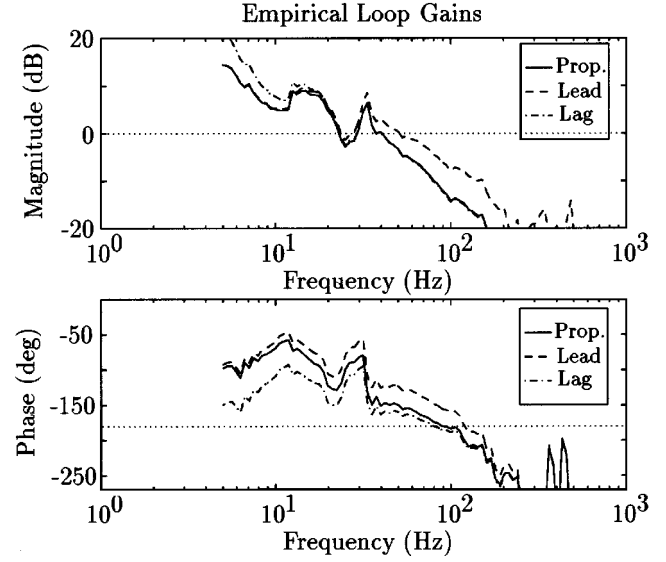


Fig. 12. Measured loop gains in combination with three simple control laws (proportional, lead, and lag), showing stability implications of these compensation techniques.

risk of inducing noticeable quantization noise in the forces presented to the operator.

Stability limits on the achievable virtual stiffness can be quantified by modeling (or measuring) the loop transfer function relating the forces F and F_c in Fig. 5. This was accomplished in our case by supplying excitation forces via F_{ce} while the haptic interface was held in the nominal position by the operator, converting the y -direction Cartesian forces F and F_c to analog form, and processing this data into transfer function form using a commercial servo analyzer (HP 3562A). The control law Z_c was a simple gain $K_0 = 1500$ N/m for this test. Fig. 12 shows this measured loop gain in conventional Bode plot format (solid line), using the maximum stiffness (proportional gain) that can be achieved with reasonable stability margins (about 40° phase margin and 10 dB gain margin). An attempt to produce a larger virtual stiffness by supplying a larger proportional gain in the controller Z_c , raising the loop gain magnitude in Fig. 12, is limited by loss of phase margin near 100 Hz, which is due primarily to the delay effect of the sampling process. Higher low frequency loop gain (higher stiffness) can be achieved by traditional lag compensation (dot-dashed line in Fig. 12). But this introduces objectionable dynamics which make the interface feel sluggish and “alive”, allowing easy penetration into the surface followed by a slow push of the operator’s hand back out of the surface. Thus the stiffness is technically larger, but the surface does not feel “harder”. Another stability compensation approach is to add phase lead near the phase crossover frequency (dashed line in Fig. 12) to enhance the stability margin for the maximum loop gain determined without compensation. This phase lead compensation produces a controller in the form

$$f(s) = K \frac{a(s+c)}{c(s+a)} \delta x(s) \quad (7)$$

which is in the same form as (5), with $c = K/B$. This lead may not increase the maximum achievable stiffness, but can, according to our test results, make the resulting virtual surface feel

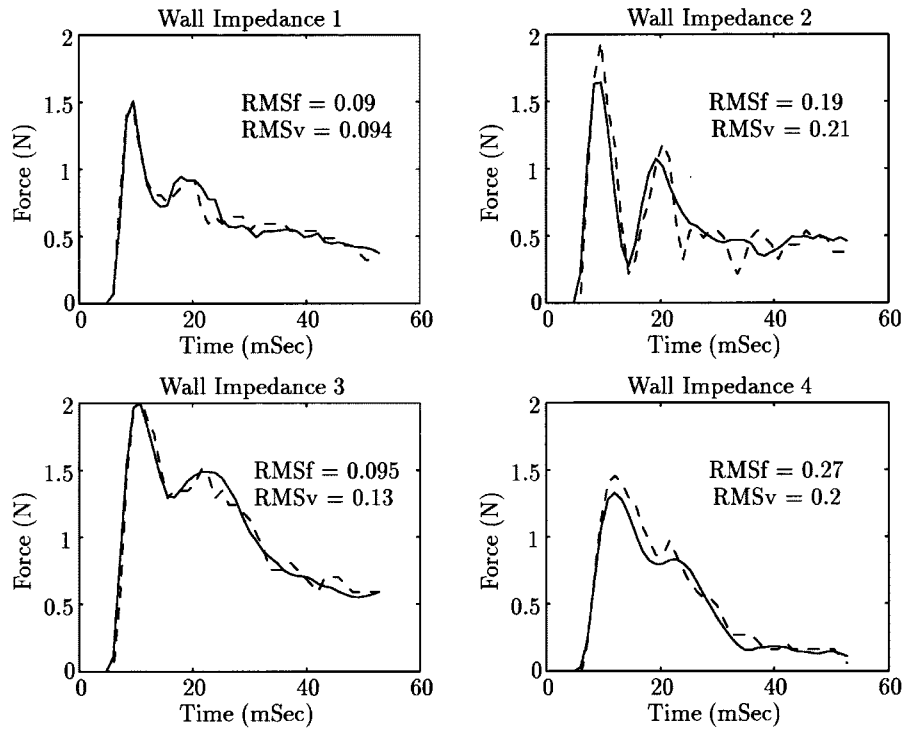


Fig. 13. Typical individual force data model fits and overall RMS fit errors for the four surface impedances rendered in the experimental tests.

much harder via the added effective damping factor $B = K/c$. Shifting the focus from high stiffness K (with corresponding damping added for stability) to sufficiently high rate-hardness H_R may allow significantly reduced feedback gains for perceptually hard surfaces, since lower levels of stiffness are possible. This makes mechanical design on the haptic interface less demanding, since lower gains are less prone to destabilization of mechanical resonances.

Stability-enhancing compensation can actually increase the perceptual fidelity of hard surface rendering—a somewhat surprising result from the traditional control system stability/performance trade-off point of view based on technical measures of performance (high stiffness in this case). Since a haptic interface is only as good as users perceive it to be, technical measures of performance must be based on capabilities and limitations of human haptic perception. In particular, we have shown that stiffness may play a minor role in hardness perception, e.g., when rate-hardness is dominated by other factors such as damping.

The control laws in Fig 6 are not necessarily what is actually presented to the operator via the overall combination of control law, electronics interface, and mechanical hardware of the haptic interface. Users perceive the rendered impedance Z_A (see Fig. 5), which may differ substantially from the control law impedance Z_C , particularly at high frequencies where mechanical dynamics and sampling effects can dominate.

The actual impedance Z_A was quantified by measuring forces and positions at the finger grip, during wall contact, as well as during free (no contact) motion outside the wall. Fifth-order transfer function models were fit to the captured force and position data using the recursive “output error” identification procedure from the MATLAB System Identification Toolbox. Fig. 13 shows typical actual (dashed line) and mod-

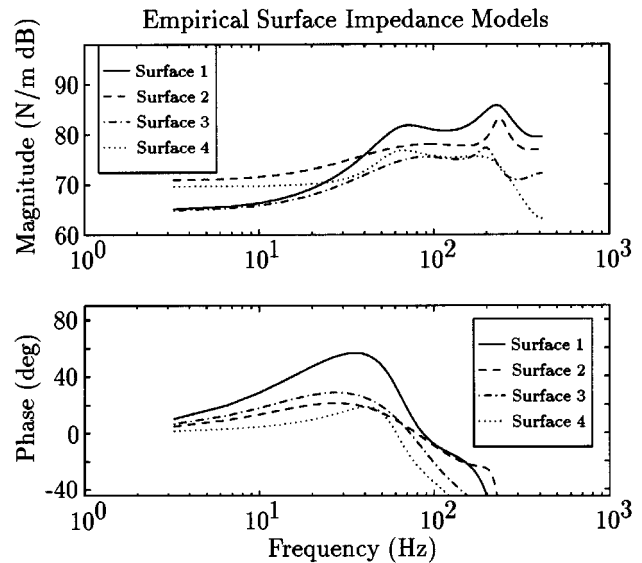


Fig. 14. Frequency responses for models fit to measured force and position data for the four rendered impedances.

eled (solid line) forces resulting from an operator “tapping” on the virtual surface. RMS fit errors were computed over the 30 such data records used to form the models (“RMSf”), as well as over the remaining 40 validation records not part of the fitting process (“RMSv”). Note that good fits to the overall shape of the forces resulting from “tapping” are indicated by the small RMS error values relative to the amplitude of the measured forces. As indicated in Fig. 13, fits are especially good during the initial rise of force as the wall is penetrated.

The frequency responses resulting from these fitted transfer functions are shown in Fig. 14. These empirical impedances

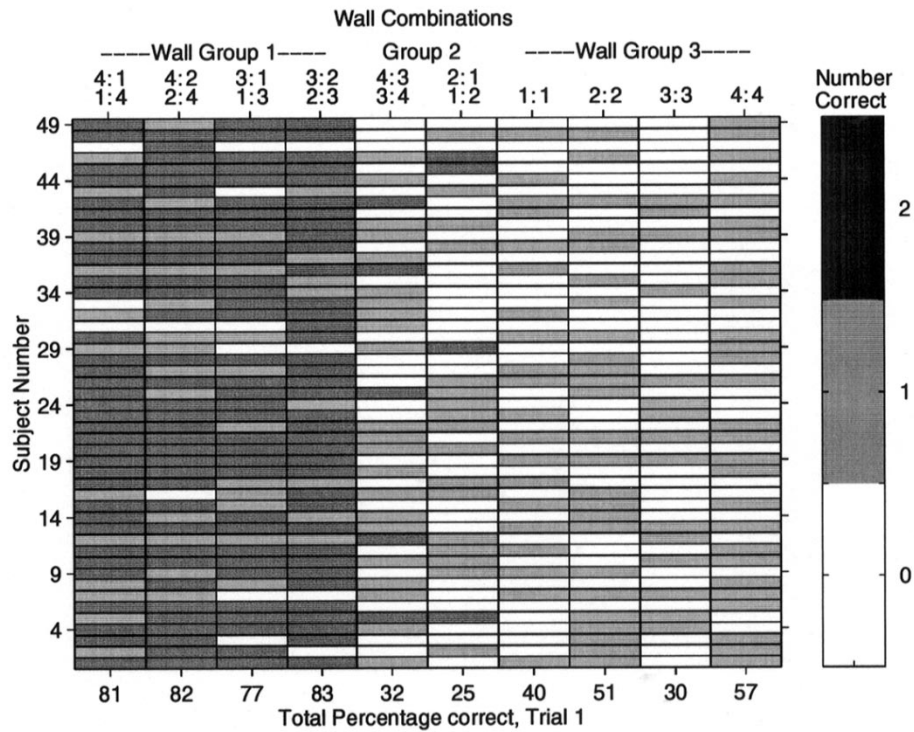


Fig. 15. Raw results for all subjects and all wall combinations during trial 1.

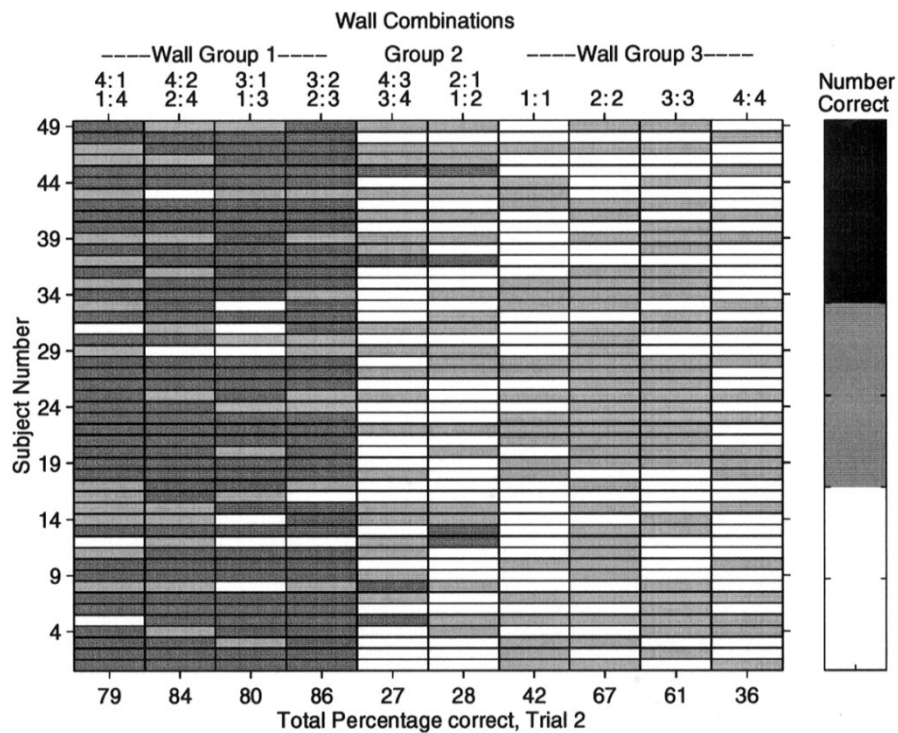


Fig. 16. Raw results for all subjects and all wall combinations during trial 2.

differ from the desired impedances shown in Fig. 6 mostly at high frequencies, but show the same general shape and levels. This correspondence between Z_C and Z_A makes it possible to use the simpler Z_C as a basis for intuition about the cause of differences in perception, e.g., that damping or

lead compensation plays a substantial role in the perception of hardness. This can only be used as a guide however, and the actual impedance Z_A , or its manifestation as relations between forces and positions as in Fig. 1, must be used as the basis for detailed understanding.

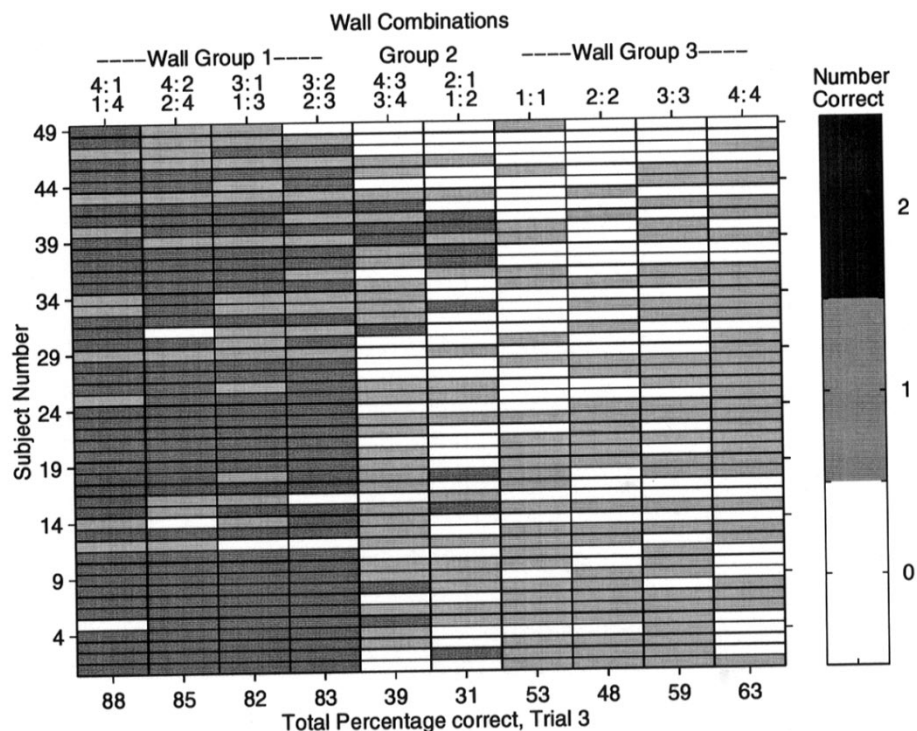


Fig. 17. Raw results for all subjects and all wall combinations during trial 3.

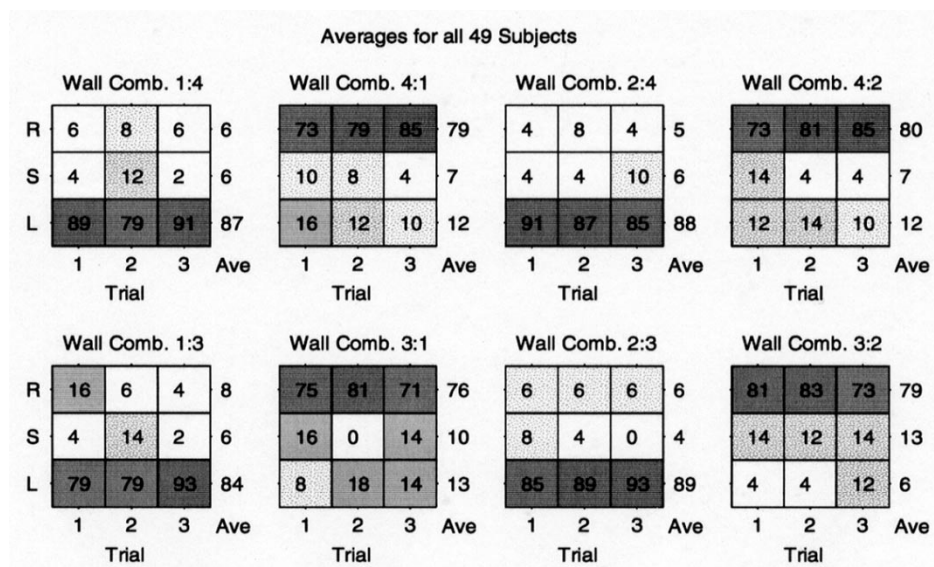


Fig. 18. Percent of responses in Wall group 1, segregated according to the three possible subject responses Right, Same, Left. Responses for each trial block are shown, together with the average across trial blocks.

VI. CONCLUSION

The results of this experimental study indicate that hard virtual walls are not necessarily the result of surfaces with high stiffness. We find that the ratio of initial rate of change of force versus initial velocity upon penetrating the surface, defined here as rate-hardness, can effectively substitute for large stiffnesses in people's perception of wall hardness. We believe this is due to the relatively poor accuracy of the human kinesthetic sense, which seems to be unable to distinguish stiffnesses as a ratio of static position to force at the levels considered here

(1700–3400 N/m). These and much harder surfaces can be discriminated, however. This seems to be enabled by dynamic tapping of one-sided surfaces, which elicits force differences at much higher frequencies than can be achieved by voluntary motion in linear media such as in [13], [14], [16], [29], and [31]. A similar reliance on “edges” was found in studies of damping discrimination [21], in “braking pulse” experiments [27], and in vibrotactile substitution for stiff surfaces [33], [24]. These high-frequency forces bring subtle surface qualities into the perceptual range of sensory receptors in the fingers [2], [12], [28].

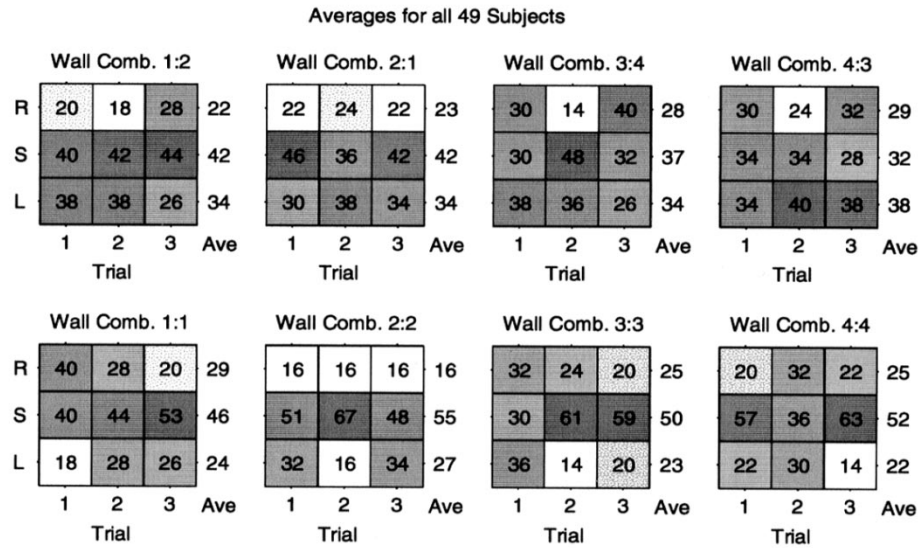


Fig. 19. Percent of responses in Wall groups 2 and 3, segregated according to the three possible subject responses Right, Same, Left. Responses for each trial block are shown, together with the average across trial blocks.

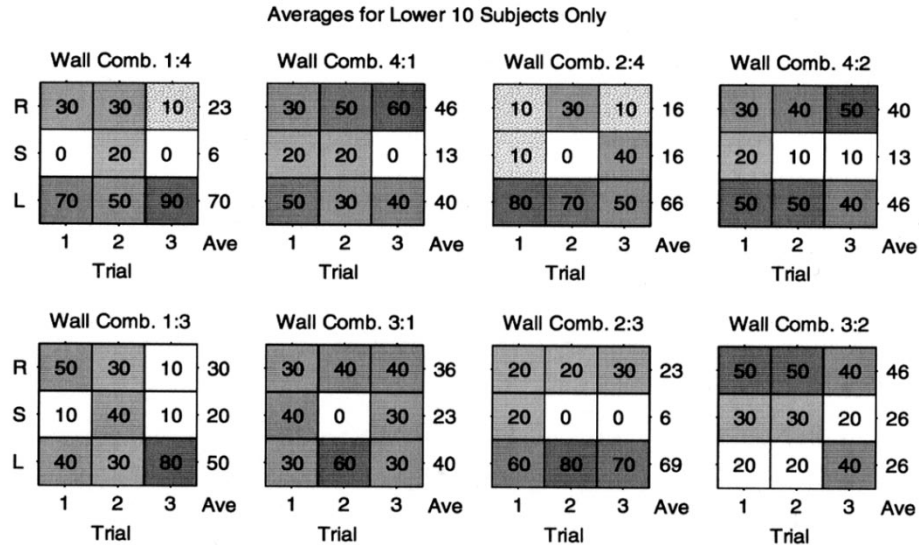


Fig. 20. Percent of responses in Wall group 1 for the 10 subjects who had 50% or fewer correct responses. The results are segregated according to the three possible subject responses Right, Same, Left. Responses for each trial block are shown, together with the average across trial blocks.

For the population of 49 subjects studied, the users were generally able to distinguish surfaces of different rate-hardnesses H_R , even if the stiffnesses were the same. Further, for differing stiffnesses but equal rate-hardnesses H_R , users had difficulty distinguishing between walls. We found a persistent, wide variation in the discrimination abilities of subjects. Approximately 10% of the population, a group containing both men and women, were unable to distinguish surfaces with the greatest differences in rate-hardness more than 50% of the time, while the majority could discriminate these surfaces at rates of 80% or better. A bias toward the left virtual surface was slight for the majority, but pronounced for the lower 10% group. Time to respond to each wall combination was inversely correlated to discrimination ability: those with the shorter times were apparently more sure of the difference between walls; those with longer times were trying longer, but just could not accurately perceive the difference.

High rate-hardness was achieved in this study by lead-type compensation in the control system relating measured hand position to commanded force. This compensation can be interpreted as band-limited damping added to a moderately stiff virtual wall, so that controller impedance magnitudes at high frequency are larger than at lower frequency. This accentuates the rate of force change that can be obtained by a given surface penetration velocity over that provided by the level of stiffness. This has the effect of producing a kind of haptic illusion, making the surface seem harder than the stiffness alone would predict. Moreover, we found that improvement of stability margins in the impedances rendered here actually improved rate-hardness (and perceived hardness), which was quite surprising given the conventional wisdom that stability and transparency are competing objectives [18], [25]. This has very important implications for design of haptic interfaces to meet requirements for perceptually hard virtual surfaces: large virtual stiffnesses are difficult

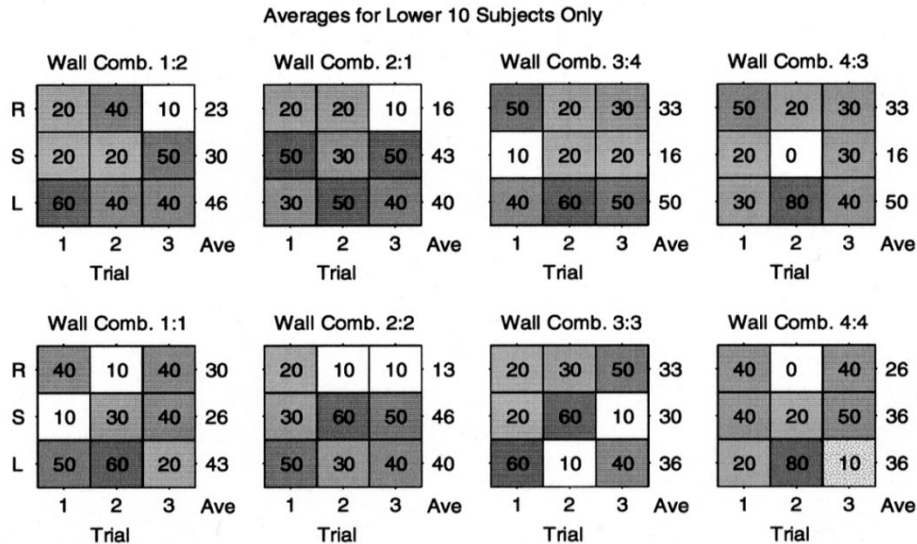


Fig. 21. Percent of responses in wall groups 2 and 3 for the 10 subjects who had 50% or fewer correct responses. The results are segregated according to the three possible subject responses Right, Same, Left. Responses for each trial block are shown, together with the average across trial blocks.

to achieve, and are not, apparently, the only pertinent design objective. Careful consideration should be paid to the rendered rate-hardness H_R as defined here.

These results also lead to two general conclusions. First, human haptic abilities vary greatly, suggesting caution in drawing general inferences from studies involving small numbers of subjects. Second, one must be careful in predicting in-use performance of haptic interfaces, unless the performance measures used are compatible with the nuances of human haptic perception.

APPENDIX A RAW RESULTS

The raw results of the three blocks of trials, for all 49 subjects, are presented in Figs. 15–17. Wall group 1 consists of the first four columns of Figs. 15–17, Wall group 2 refers to the next two columns, and Wall group 3 consists of the last four columns.

A. Left/Right Bias Results

Fig. 18 shows the percent of responses in the Right, Same, Left categories for each block of trials, along with averages over trial blocks, for the combinations in Wall group 1. This figure shows the emphasis on the side yielding the larger rate-hardness, as indicated earlier. However, note the slightly larger number of opposite (incorrect) choices on the Left than on the Right in all wall combinations except (2:3) and (3:2). Similar results are seen in Wall group 2 (see Fig. 19). For instance, in wall combinations (1:2) and (2:1), the number of Left and Right responses are nearly identical across trial blocks, but in each trial block, the number of Left responses is larger. However, in Wall group 3 (see Fig. 19), while there was bias to the left in wall pair (2:2), there was not a strong bias either to the left or right in wall combinations (3:3) and (4:4), and a slight bias to the right in wall pair (1:1). Over the test population as a whole, therefore, the left bias is present but weak.

Figs. 20 and 21 show the percent of responses in the Right, Same, Left categories for each trial block, along with averages

over trial blocks, for the ten subjects that consistently had difficulty correctly determining the harder surface. Note that the sub-population of ten had almost equal response rates for Left and Right on wall combinations (4:1) and (4:2), where the harder surface was on the right. Yet when the harder surface was on the left, combinations (1:4) and (2:4), the responses were correct with a frequency corresponding to the population as a whole.

REFERENCES

- [1] *Handbook of Perception and Human Performance*, vol. I, K. R. Boff, L. Kaufman, and J. P. Thomas, Eds., 1986, pp. 13-6–13-9.
- [2] S. J. Bolanowski, G. A. Gescheider, R. T. Verrillo, and C. M. Checkosky, "Four channels mediate the mechanical aspects of touch," *J. Acoust. Soc. Amer.*, vol. 84, no. 5, pp. 1680–1694, 1988.
- [3] T. L. Brooks, "Telerobotic response requirements," in *Proc. IEEE Conf. on Systems, Man, Cybernetics*, Los Angeles, CA, 1990.
- [4] G. Burdette and J. Zhuang, "Dextrous telerobotics with force feedback—An overview," *Robotica*, vol. 9, pp. 171–298, 1991.
- [5] P. Buttolo and B. Hannaford, "Pen-based force display for precision manipulation in virtual environments," in *Proc. IEEE Virtual Reality Symp.*, 1995, pp. 217–224.
- [6] J. E. Colgate and J. M. Brown, "Factors affecting the Z-width of a haptic display," in *Proc. IEEE Int. Conf. on Robotics and Automation*, San Diego, CA, 1994, pp. 3205–3210.
- [7] J. E. Colgate and G. Schenkel, "Passivity of a class of sampled-data systems: Application to haptic interfaces," in *Proc. IEEE American Control Conf.*, Baltimore, MD, 1994.
- [8] R. B. Gillespie and M. R. Cutkosky, "Stable user-specific haptic rendering of the virtual wall," in *Proc. ASME Int. Mechanical Engineering Congress and Exposition*, DSC 58, Atlanta, GA, Nov. 1996, pp. 397–406.
- [9] B. Hannaford, L. Wood, D. A. McAfee, and H. Zak, "Performance evaluation of a six-axis generalized force-reflecting teleoperator," *IEEE Trans. Syst. Man Cybern.*, vol. 21, pp. 620–633, 1991.
- [10] V. Hayward, "Toward a seven axis haptic device," in *Proc. Int. Conf. on Intelligent Robots and Systems*, Pittsburgh, PA, 1995, pp. 133–139.
- [11] H. Iwata, "Artificial reality with force feedback: Development of desktop virtual space with compact master manipulator," *Comput. Graph.*, vol. 24, no. 4, pp. 165–170, 1990.
- [12] R. S. Johansson and A. B. Valbo, "Tactile sensory coding in the glabrous skin of the human hand," *Trends Neurosci.*, vol. 6, pp. 27–31, 1983.
- [13] L. A. Jones and I. W. Hunter, "A perceptual analysis of stiffness," *Exp. Brain Res.*, vol. 79, pp. 150–156, 1990.
- [14] —, "A perceptual analysis of viscosity," *Exp. Brain Res.*, vol. 94, pp. 343–351, 1993.

- [15] K. A. Kaczmarek, J. G. Webster, P. Bach-y-Rita, and W. J. Tompkins, "Electrotactile and vibrotactile displays for sensory substitution systems," *IEEE Trans. Biomed. Eng.*, vol. 38, pp. 1–16, 1991.
- [16] J. G. Kreifeldt and M. C. Chuang, "Moment of inertia: Psychophysical study of an overlooked sensation," *Science*, vol. 206, pp. 588–590, 1979.
- [17] R. H. LaMotte, "Softness discrimination with a tool," *J. Neurophysiology*, to be published.
- [18] D. A. Lawrence, "Stability and transparency in bilateral teleoperation," *IEEE Trans. Robot. Automat.*, vol. 9, pp. 624–637, Oct. 1993.
- [19] D. A. Lawrence, L. Y. Pao, M. A. Salada, and A. M. Dougherty, "Quantitative experimental analysis of transparency and stability in haptic interfaces," in *Proc. ASME Int. Mechanical Engineering Congress and Exposition, DSC 58*, Atlanta, GA, Nov. 1996, pp. 441–449.
- [20] T. H. Massie and J. K. Salisbury, "The phantom haptic interface: A device for probing virtual objects," *Proc. ASME Dynamics Systems and Control*, vol. 1, pp. 295–301, 1994.
- [21] P. A. Millman and J. E. Colgate, "Effects of nonuniform environment damping on haptic perception and performance of aimed movements," in *Proc. Int. Mechanical Engineering Congress and Exposition, DSC 57-2*, San Francisco, CA, Nov. 1995, pp. 703–712.
- [22] J. B. Morrell and J. E. Colgate, "Performance measurements for robotic actuators," in *Proc. ASME Int. Mechanical Engineering Congress and Exposition, DSC 58*, Atlanta, GA, Nov. 1996, pp. 531–528.
- [23] J. Neter, M. Kutner, C. Nachtsheim, and W. Wasserman, *Applied Linear Statistical Models*, 4th ed. Chicago, IL: Irwin, 1996.
- [24] A. M. Okamura, J. T. Dennerlein, and R. D. Howe, "Vibration feedback models for virtual environments," in *Proc. IEEE Int. Conf. on Robotics and Automation*, Leuven, Belgium, 1998, pp. 674–679.
- [25] G. J. Raju, G. C. Verghese, and T. B. Sheridan, "Design issues in 2-port network models of bilateral remote teleoperation," in *Proc. IEEE Int. Conf. on Robotics and Automation*, Scottsdale, AZ, May 1989, pp. 1317–1321.
- [26] L. B. Rosenberg and B. D. Adelstein, "Perceptual decomposition of virtual haptic surfaces," in *Proc. IEEE Symp. on Research Frontiers in Virtual Reality*, San Jose, CA, Oct. 1993, pp. 46–53.
- [27] S. E. Salcudean and T. D. Vlaar, "On the emulation of stiff walls and static friction with a magnetically levitated input/output device," *ASME Dyn. Syst. Contr.*, vol. 55-1, pp. 303–309, 1994.
- [28] R. F. Schmidt, Ed., *Fundamentals of Sensory Physiology*, 3rd ed. New York: Springer-Verlag, 1986.
- [29] M. A. Srinivasan and R. H. LaMotte, "Tactile discrimination of softness," *J. Neurophysiology*, vol. 73, no. 1, pp. 88–101, Jan. 1995.
- [30] H. Z. Tan, M. A. Srinivasan, B. Eberman, and B. Cheng, "Human Factors for the design of force-reflecting haptic interfaces," *ASME Dyn. Syst. Contr.*, DSC 55-1, pp. 353–359, 1994.
- [31] H. Z. Tan, N. I. Durlach, G. L. Beauregard, and M. A. Srinivasan, "Manual discrimination of compliance using active pinch grasp: The roles of force and work cues," *Perception Psychophys.*, vol. 57, no. 4, pp. 495–510, 1995.
- [32] J. Vertut and P. Coiffet, *Teleoperation and Robotics*. Englewood Cliffs, NJ: Prentice-Hall, 1996, vol. 3.
- [33] P. Wellman and R. D. Howe, "Toward realistic vibrotactile display in virtual environments," in *Proc. ASME Int. Mechanical Engineering Congress and Exposition, DSC 57-2*, San Francisco, CA, Nov. 1996, pp. 713–718.
- [34] G. Westling and R. S. Johansson, "Responses in glabrous skin mechanoreceptors during precision grip in humans," *Exp. Brain Res.*, vol. 66, pp. 128–140, 1987.



Dale A. Lawrence (S'83–M'85) received the B.S. degree in electrical engineering from Colorado State University, Fort Collins, in 1980, and the M.S. and Ph.D. degrees in electrical engineering from Cornell University, Ithaca, NY, in 1982 and 1985, respectively.

He has held positions with ADR Ultrasound, Technicare Corporation, and Martin Marietta Astronautics. He joined the Department of Electrical and Computer Engineering at the University of Cincinnati in 1988, and the Department of Aerospace Engineering Sciences at the University of Colorado at Boulder in 1991, where he is currently an Associate Professor. His research interests include the areas of haptic interfaces, teleoperation, spacecraft attitude control, and control systems for data storage devices.



Lucy Y. Pao (S'89–M'92–SM'98) was born in Washington, DC, in 1968. She received the B.S., M.S., and Ph.D. degrees in electrical engineering from Stanford University, Stanford, CA, in 1987, 1988, and 1992, respectively.

From 1991 to 1993, she worked at The MITRE Corporation, and from 1993 to 1995, she was an Assistant Professor in the Electrical Engineering and Computer Science Department at Northwestern University, Evanston, IL. Since 1995, she has been with the Electrical and Computer Engineering

Department at the University of Colorado at Boulder, where she is now an Associate Professor. Her research interests include haptic and visual/haptic interfaces, control of flexible structures, and multisensor data fusion.

Dr. Pao has served on the IEEE Control Systems Society Conference Editorial Board (1995–1997), on Program Committees of the American Control Conference (1995, 1997, 2000, 2001) and the IEEE Conference on Decision and Control (2000), and as the American Automatic Control Council Newsletter Editor (1995–). She is the recipient of the 1996 IFAC World Congress Young Author Prize, a National Science Foundation Early Faculty CAREER Development Award (1996–2001), and an Office of Naval Research Young Investigator Award (1997–2000).



Anne M. Dougherty received B.S. degrees in chemistry and mathematics from Texas Christian University, Fort Worth, TX, in 1981, the M.S. degree in mathematics from Oregon State University, Corvallis, in 1984, and the Ph.D. degree in mathematics from the University of Wisconsin at Madison in 1994.

She is currently a Senior Instructor in Applied Mathematics at the University of Colorado at Boulder. Her research interests include the areas of probability and statistics, specifically limit theorems

of stochastic processes, and extreme value theory.



Mark A. Salada received the B.S. and M.S. degrees in aerospace engineering in 1994 and 1996, respectively, from the University of Colorado at Boulder.

After working at the Johns Hopkins Applied Physics Laboratory Space Department for two and a half years, Mr. Salada is working toward the Ph.D. degree in mechanical engineering at Northwestern University, Evanston, IL.



Yiannis Pavlou received the B.S. and M.S. degrees in aerospace engineering from the University of Colorado at Boulder in 1996 and 1998, respectively.

He is currently a Data Acquisition Product Manager at National Instruments, Austin, TX. His research interests include the areas of control system simulation and implementation in general, and haptic interface control systems in particular.