10.1098/rsta.2001.0860

THE ROYAL SOCIETY

Laboratory testing of structures under dynamic loads: an introductory review

By M. S. Williams and A. Blakeborough
Department of Engineering Science, University of Oxford, Parks Road, Oxford OX1 3PJ, UK

This paper introduces and reviews the theme of laboratory testing of structures under dynamic loads. The emphasis is on the simulation of earthquake effects, for which three principle methods are discussed: shaking tables, pseudo-dynamic testing and real-time testing. The latest developments in these areas are discussed in depth in the subsequent papers in this issue. While shaking tables and pseudo-dynamic methods are quite well established, both techniques have undergone significant advances in recent years, including improvements in control to ensure accurate reproduction of dynamic loads, and the construction of very large facilities aimed at eliminating the significant scaling problems. Development of the substructuring method has enabled large-scale pseudo-dynamic tests of parts of structures, coupled to numerical models of the remainder. Attempts are now being made to extend this approach to shaking tables. Recently, considerable efforts have been devoted to methods of testing both at large scale and in real time. Two approaches are discussed: the real-time substructure method, in which a physical test and a numerical model interact in real time; and effective force testing, in which equivalent seismic forces are applied by actuators operating under force control. Both methods have been shown to be feasible, but require further development. Although the techniques described have been developed primarily for seismic testing of structures, there is considerable potential for their application to other load types in the fields of civil and mechanical engineering.

Keywords: structural dynamics; earthquake engineering; shaking tables; pseudo-dynamic testing; substructuring; real-time testing

1. Introduction

Although methods of dynamic analysis have advanced in recent years, there remains a strong need for experimental evaluation of structural performance. This is particularly true when the structure responds inelastically and/or includes elements whose behaviour is strongly rate dependent. The aim of this paper is to provide a brief overview of recent developments in experimental methods of dynamic testing, as a prelude to the more detailed accounts presented in the remainder of this issue.

There is a wide variety of methods for dynamic testing of structures. When only the modal properties of a structure are required, very low levels of excitation are generally sufficient. These may be provided by ambient sources such as wind or traffic (Brownjohn *et al.* 1987) or by devices which give a greater degree of control, such as eccentric mass exciters (Severn *et al.* 1980) or electrodynamic shakers (Pavic 1999). When the structural response to a particular type of dynamic loading is sought,

 © 2001 The Royal Society

then more sophisticated equipment is likely to be needed. In the field of earthquake engineering the most widely used experimental methods are as follows (Booth 1998; Carvalho 1998).

(i) Static cyclic testing is widely used for determining the performance of materials and elements under the repeated load reversals that occur during earthquakes, though it does not model dynamic behaviour.

(ii) Shaking-table testing provides true dynamic loading. It generally requires the use of reduced-scale models, though a few tables exist which have the capacity to apply seismic base motions to full-scale structures.

(iii) Pseudo-dynamic testing, in which loads are applied over a greatly expanded time-scale with the dynamic behaviour of the structure accounted for computationally, is a widely used alternative to shaking-table testing.

(iv) More recently, a variety of real-time methods has been developed. Like pseudo-dynamic testing, these use a combination of physical testing and numerical modelling, but the experimental part of the process is performed at the correct rate so that the test specimen can respond dynamically.

(v) Arrays of explosive charges are occasionally used to simulate earthquake ground motions at large scale (Kitada *et al*. 2000).

(vi) Dynamic centrifuge testing is used to model geotechnical and soil-structure interaction problems.

Of these methods, (i) does not involve dynamics, (v) is a rather infrequently used approach, and (vi) is primarily used in the testing of soils. These approaches will therefore not be discussed further here. The remainder of this paper gives a brief introduction to shaking-table testing, pseudo-dynamic testing and real-time dynamic test methods, and reviews recent developments in all three areas.

2. Shaking tables

Shaking tables were first used for simulating seismic loads on structures in the 1940s and their use has been widespread since the 1960s. The basic principle is very simple: a model of a structure is mounted on a stiff platform, which is shaken so as to apply the appropriate base motion. Assuming that this can be done accurately, the correct inertia forces are then generated throughout the structure and the response to these forces can be measured. In the earliest tables profiled cams were used to provide the excitation, but modern tables all use servo-hydraulic actuators.

There are several key challenges to be overcome in performing an accurate shaking-table test. Faithful reproduction of the desired motion requires high-quality equipment and sophisticated control engineering. The issue of scaling also needs to be carefully addressed, since it is not normally possible to test full-scale structures.

(*a*) *Shaking-table hardware*

Shaking tables come in a wide variety of sizes and configurations. The majority of tables aim to provide economical and reasonably realistic testing of reduced-scale models. An example of a relatively small table is the one at Bristol University, UK (Blakeborough *et al.* 1986), which has a plan area of 3 m × 3 m and can carry a maximum payload of 10 t. Larger tables typically have platform areas of *ca.* 6 m × 6 m and capacities in the range of 50 t (e.g. Berkeley, USA) to 100 t (e.g. CEA Saclay, France). Recently in Japan some very large and expensive facilities have been constructed, in an attempt to avoid the problems associated with scaling of nonlinear dynamic responses. To date this has been achieved at the expense of reducing the number of controlled degrees of freedom. For example, the table at Tsukuba (Minowa *et al.* 1996) has a plan area of 15 m × 14.5 m and a capacity of 500 t, but allows controlled motion only in one horizontal direction. However, the 1200 t table currently under construction at Miki City will allow three-dimensional tests to be performed at full scale (Ogawa *et al.* 2000, 2001).

The main elements of the table system are the platform itself and the actuators that drive it. The platform must be sufficiently rigid that it does not itself respond dynamically, so that it transmits the input motion to the structure with as little modification as possible. A variety of designs has been used: the Berkeley table is a ribbed, post-tensioned concrete slab, while the Bristol table is a cast aluminium inverted pyramid (Blakeborough *et al.* 1986). High-capacity servo-hydraulic equipment is required in order to drive the large mass of the table and test specimen at the required rate. The size of earthquake that can be reproduced is normally governed by its velocity content, since this is directly related to the oil flow rate that can be provided by the pumping system and servovalves. For conventional tables the actuators and servovalves required are well within current technology. However, the very large Miki City table has required the development of new bearing and pressure seal systems, as well as servovalves able to provide flows of 15 000 $\mathrm{l\,min^{-1}}$ (Ogawa *et al.* 2000).

Where active control is not provided on all axes, physical restraints are required to prevent motion in the unwanted degrees of freedom. However, there is a risk that such restraint systems will affect the motion in the actively controlled directions. The current trend, therefore, is to provide active control of all six degrees of freedom, even though in many cases the aim may be to control the motion in several of the directions to be zero. A notable exception is the table at LNEC Lisbon, which provides active control only of the three translational axes, with a system of large torque tubes used to prevent rotational movements (Bairrao & Vaz 2000).

(*b*) *Similitude*

Unless the table is very large a reduced-scale structural model must be used, which causes difficulties in ensuring correct dynamic scaling. Dynamic similitude can be conveniently expressed using two parameters. The Cauchy number is the ratio between the dynamic inertia forces F_i and the elastic restoring forces F_e,

$$\frac{F_\mathrm{i}}{F_\mathrm{e}} = \frac{\rho v^2}{E}, \tag{2.1}$$

where ρ is density, v is velocity and E is Young's modulus. The Froude number is the ratio between the inertia and gravity forces,

$$\frac{F_\mathrm{i}}{F_\mathrm{g}} = \frac{v^2}{Lg}, \tag{2.2}$$

where L is length and g is the acceleration due to gravity. In most cases it is desirable that both the Cauchy and Froude numbers of the model match the values for the prototype (Severn 1997). The most important consequences of this matching are that the mass-scale factor should be the inverse of the length-scale factor and that the time-scale factor should be the square root of the length-scale factor. For example, a $\frac{1}{4}$-scale model will require a fourfold increase in specific mass and a halving of the time-scale. The compression of the time-scale results in an increase in the frequency content of the input earthquake which may be difficult to achieve, so that some compromise is often necessary.

Even when equations (2.1) and (2.2) can be satisfied, it is often difficult to have confidence in the extrapolation of nonlinear dynamic response to full scale, especially for scale-sensitive problems such as connections in steel structures or bond and anchorage in reinforced concrete. This lack of confidence has provided the motivation for the development of the very large Japanese shaking tables mentioned earlier.

(c) *Control issues*

Whatever mechanical equipment is used, it is bound to cause some modification of the input signal. Sophisticated control is therefore required to compensate for this modification, so as to ensure that the table accurately reproduces the desired input motion.

The traditional control strategy is shown in figure 1*a* (Severn 1997). An inverse kinematic solver converts the desired table displacements into n actuator drive signals. Normally eight actuators are used for a six-axis table, so that the solver must carry out a transformation from six degrees of freedom to eight. Each actuator is then controlled by a linear controller using feedback of actuator position, velocity or acceleration. These operations are carried out in real time as the test proceeds.

In recent years some modifications have been made to this approach. For instance, many control systems now provide closed-loop control directly on the table degrees of freedom, rather than on the individual actuators. Also, many controllers now use *three-variable control*, in which the feedback parameter is a weighted combination of position, velocity and acceleration. These changes have improved the fidelity with which the desired motion is reproduced. However, the use of a linear controller with fixed gains remains a major limitation for two reasons.

First, the linear controller requires knowledge of the properties of the system being controlled. However, the properties of the shaking-table system (of which the test specimen itself forms a substantial part) are generally not known to a high degree of accuracy. To overcome this problem, an iterative matching procedure must be carried out prior to the start of a test, in which the demand signal is adjusted while keeping the test specimen in the linear regime, until the desired table motion is achieved. This typically takes several minutes.

The second, more serious, problem is that the linear controller is based on the assumption that the system being controlled can be represented by a linear equation which does not change with time. The shaking-table system is likely to be nonlinear, especially if the test specimen undergoes damage during the test. Any damage to the specimen will result in the chosen control parameters ceasing to be optimal, so that the desired earthquake motion is not accurately reproduced.

One possible way of overcoming these limitations is to use adaptive controllers, in which the gains can be updated at every sampling interval to account for dynamic

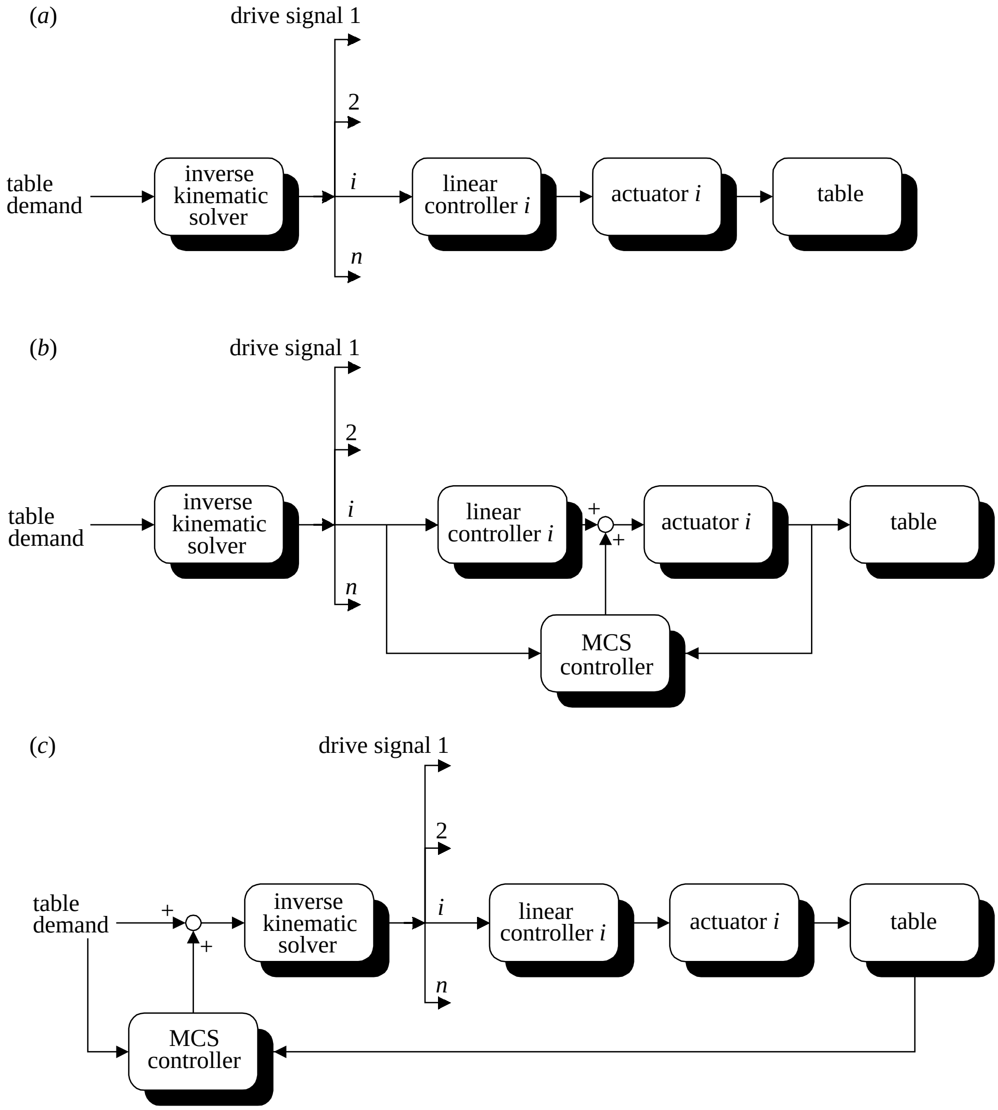

Figure 1. Shaking-table control strategies.

changes in the system being controlled. Recent attempts to apply adaptive control technology to shaking tables have centred on the minimal control synthesis (MCS) algorithm (Stoten & Benchoubane 1990*a*, *b*). This is a particularly attractive form of adaptive controller, since it requires no identification of the dynamics of the system being controlled (either prior to commencing a test or online), and it is well-suited for use as a retrofit strategy around existing controllers. A fuller description of the MCS algorithm is given elsewhere in this issue (Stoten & Gomez 2001).

MCS can be used to control a shaking table in one of two ways. Figure 1*b* shows the inner loop implementation in which MCS control is applied directly to each actuator. The actuator drive signal and output are fed to the MCS algorithm, which generates a control signal that is added to the normal control signal. Figure 1*c* shows the alternative outer-loop approach, which is generally easier to implement on existing tables, where direct access to the actuators is not available. In this case the MCS algorithm compares the overall table demand with the actual table motion and outputs a control signal that is added to the demand signal. This is similar to the matching procedure used for conventionally controlled tables, but it is now carried out in real time, with no need for iteration at the start of a test. Stoten & Gomez (2001) have shown that both inner- and outer-loop configurations result in substantial improvements in the accuracy with which the table motion follows the demand signal.

3. Pseudo-dynamic testing

The pseudo-dynamic (PsD) test method, also known as online testing, was developed under the US–Japan Cooperative Earthquake Programme in the early 1980s (Mahin & Shing 1985; Takanashi & Nakashima 1987). PsD testing is a hybrid method, in which the structural displacements due to the earthquake are calculated computationally using a stepwise integration procedure and applied quasi-statically to the test specimen. The resulting resistance forces are measured and fed back to the computational model as part of the input for the next calculation step. Tests normally run on an expanded time-scale of the order of 100 times the actual time-scale (Mahin *et al*. 1989). This is advantageous in that it simplifies the equipment needed and it allows for inspection of the test structure between load steps. A major potential drawback, however, is that any time-dependent behaviour in the test specimen is not included.

Because the PsD method allows realistic dynamic testing without the need for dynamically rated actuators or very high oil flow rates, it makes full-scale testing feasible, so long as a sufficiently large strong floor and reaction wall are available. An example of such a facility is the ELSA laboratory at the European Commission's Joint Research Centre (JRC) at Ispra, which includes a 16 m high reaction wall capable of resisting a base shear of 20 MN and a bending moment of 200 MN m (Donea *et al*. 1996).

Most PsD testing to date has been unidirectional. However, recent work has demonstrated the feasibility of performing multidimensional tests (Thewalt & Mahin 1995*b*; Molina *et al*. 1999).

(*a*) *Outline of method*

The basic principles of the PsD approach will be illustrated using the central difference method (CDM) of stepwise integration. Other numerical integration procedures will be discussed later. If the structure to be tested is modelled using a numerical method such as finite elements, its equations of motion may be written in matrix form as

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{C}\dot{\boldsymbol{u}} + \boldsymbol{R} = \boldsymbol{F}, \tag{3.1}$$

where $\boldsymbol{M}$ is the mass matrix, $\boldsymbol{C}$ the damping matrix, $\boldsymbol{u}$ the vector of nodal displacements, $\boldsymbol{R}$ the restoring force vector, $\boldsymbol{F}$ the vector of external excitation forces, and dots represent differentiation with respect to time. This system of equations is solved over a series of time-steps Δt apart. At the $(i+1)$th step,

$$\boldsymbol{M}\ddot{\boldsymbol{u}}_{i+1} + \boldsymbol{C}\dot{\boldsymbol{u}}_{i+1} + \boldsymbol{R}_{i+1} = \boldsymbol{F}_{i+1}. \tag{3.2}$$

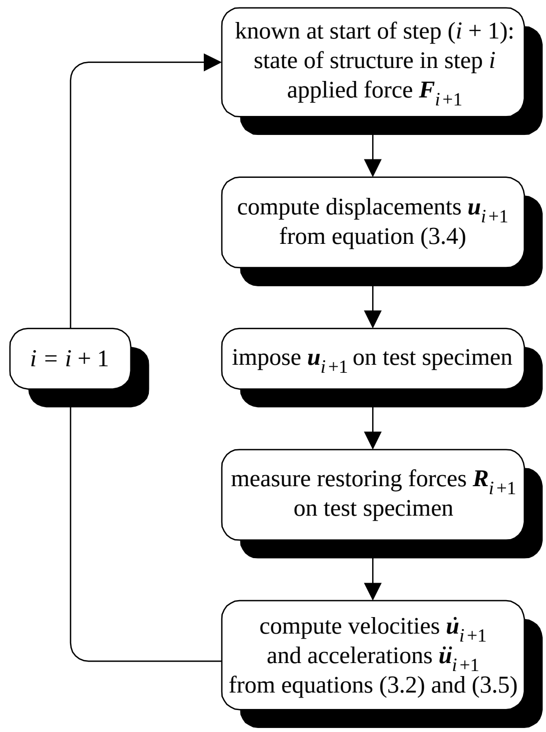

Figure 2. Flow chart for PsD test using central difference method.

Using the CDM, the velocity and acceleration in step i can be approximated by

$$\dot{\boldsymbol{u}}_i = \frac{\boldsymbol{u}_{i+1} - \boldsymbol{u}_{i-1}}{2\Delta t}, \qquad \ddot{\boldsymbol{u}}_i = \frac{\boldsymbol{u}_{i+1} - 2\boldsymbol{u}_i + \boldsymbol{u}_{i-1}}{\Delta t^2}, \tag{3.3}$$

from which it follows that

$$\boldsymbol{u}_{i+1} = \boldsymbol{u}_i + \Delta t \dot{\boldsymbol{u}}_i + \tfrac{1}{2}\Delta t^2 \ddot{\boldsymbol{u}}_i \tag{3.4}$$

and

$$\dot{\boldsymbol{u}}_{i+1} = \dot{\boldsymbol{u}}_i + \tfrac{1}{2}\Delta t(\ddot{\boldsymbol{u}}_i + \ddot{\boldsymbol{u}}_{i+1}). \tag{3.5}$$

The PsD test now proceeds as shown in the flowchart in figure 2. Because the restoring forces are measured experimentally, there is no uncertainty over the non-linear stiffness characteristics of the structure. Also, hysteretic damping due to non-linear behaviour is modelled experimentally, so that the damping matrix $\boldsymbol{C}$ need only account for the viscous damping forces.

(*b*) *Experimental errors*

It has been found that the PsD method is particularly sensitive to small errors in applying the computed displacements to the test specimen (Shing & Mahin 1987*a*; Yamazaki *et al*. 1989). While random errors can generally be kept at acceptable levels by choice of an appropriate integration time-step, systematic errors can have severe effects on the overall energy content. Systematic overshooting by the actuators results in unrealistically high apparent damping of the structure, while systematic undershooting has an effect equivalent to negative damping. Because the resulting forces are fed back and used in the calculation of the next displacement increment, the errors tend to build up quickly and may result in complete failure of the test. This effect is most pronounced for the higher frequency modes of the structure, and makes PsD testing of very stiff structures particularly problematic.

Several approaches have been used to minimize the displacement error effect. Shing & Mahin (1987*b*) showed that, while viscous damping can be used to reduce the spurious higher mode responses, it also has undesirable effects on the lower mode responses. Shing & Mahin (1990) observed that the numerical integration should always be based on previous computed displacements rather than measured values. This prevents direct build-up of displacement errors, but does not prevent inaccuracies in the applied displacements causing errors in the fed-back forces. Seible *et al*. (1996) used a soft coupling system between the actuators and the test structure to improve the displacement control at low amplitudes. Chang (2001) showed that error amplification can be significantly reduced by basing the PsD algorithm on the integrated form of the equations of motion. Kabayama (2000) proposed a method in which the target displacement is deliberately overshot, with the restoring force at the correct displacement then deduced by linear interpolation. While this introduces a deliberate overshooting error, the interpolation procedure prevents the build-up of errors in the energy levels of the system. Many researchers have used a dissipative numerical integration scheme to damp out the higher frequency modes, which are most prone to rapid error accumulation. This is discussed further in the following section.

Another potential source of error is the stepwise nature of a PsD test. Conventionally, a displacement increment is applied over a short *ramp* period and the structure is then held stationary for a *wait* period while measurements are taken and damage observations made. If the structure is yielding, then significant force reductions may occur during the wait period. This problem may be overcome by minimizing the wait period and by measuring forces at intervals during the ramp period. At the limit, the wait period may be reduced to zero, resulting in a continuous PsD test (Casciati & Magonette 1999).

(*c*) *Numerical integration algorithms*

A key element of the PsD method is the numerical algorithm that is used to perform the stepwise integration of the equations of motion.

Most integration methods used in PsD testing are based on the Newmark algorithm, in which the displacement and velocity are given by

$$\boldsymbol{u}_{i+1} = \boldsymbol{u}_i + \Delta t\dot{\boldsymbol{u}}_i + \Delta t^2[(\tfrac{1}{2} - \beta)\ddot{\boldsymbol{u}}_i + \beta\ddot{\boldsymbol{u}}_{i+1}], \tag{3.6}$$

$$\dot{\boldsymbol{u}}_{i+1} = \dot{\boldsymbol{u}}_i + \Delta t[(1 - \gamma)\ddot{\boldsymbol{u}}_i + \gamma\ddot{\boldsymbol{u}}_{i+1}]. \tag{3.7}$$

Numerical integration techniques are known as explicit if the displacement solution at each step is calculated entirely in terms of the solutions in previous steps. If knowledge of the state of the structure in the current step is required, then the method is implicit. The Newmark scheme may be implicit or explicit, depending on the values assigned to the parameters β and γ. For example, if we put $\beta = 0$ and $\gamma = \frac{1}{2}$, then the central difference method (CDM) is obtained, which is explicit.

For PsD testing, both explicit and implicit methods have advantages and drawbacks. Explicit schemes have the advantage that the required displacement increment can be computed directly from the results of the previous time-step. Implicit methods, on the other hand, require knowledge of the acceleration at the end of the current time-step, which can only be achieved by some form of iterative procedure. This is undesirable in structural testing because of the risk of overshooting, which may have a significant effect on the response of the structure.

The drawback of explicit methods is that they are only conditionally stable and so require the use of quite a short time-step. Stability is ensured only if $\Delta t \leqslant T_{\min}/\pi$, where $T_{\min}$ is the shortest period of the structure. This may be particularly problematic for stiff structures with low natural periods or for structures having a large number of degrees of freedom, with widely dispersed natural periods. PsD testing based on explicit methods is therefore best suited to structures with a high mass-to-stiffness ratio and relatively few degrees of freedom.

Because early PsD tests were performed on comparatively simple structures, explicit methods such as the CDM were generally preferred. When the test method came to be applied to stiffer and/or more complex structures, the need for an unconditionally stable integration scheme became paramount. As a result, implicit schemes have been widely used (see, for example, Mahin *et al.* 1989; Thewalt & Mahin 1995*a*; Shing *et al.* 1996; Seible *et al.* 1996).

Another important aspect of the Newmark algorithm is that it is dissipative when $\gamma > \frac{1}{2}$. This can be useful as a way of reducing the effects of displacement-control errors, as discussed earlier. Since these errors are most problematic at high frequencies, it is particularly useful to have a way of numerically damping the higher modes while minimizing the damping effects on the lower modes. Hilber *et al.* (1977) introduced a modification to the conventional Newmark scheme that achieves this. Equation (3.2) is replaced by the α-shifted equation:

$$\boldsymbol{M}\ddot{\boldsymbol{u}}_{i+1} + (1+\alpha)\boldsymbol{C}\dot{\boldsymbol{u}}_{i+1} - \alpha\boldsymbol{C}\dot{\boldsymbol{u}}_i + (1+\alpha)\boldsymbol{R}_{i+1} - \alpha\boldsymbol{R}_i = (1+\alpha)\boldsymbol{F}_{i+1} - \alpha\boldsymbol{F}_i. \quad (3.8)$$

This is then solved using equations (3.6) and (3.7), with the parameters α, β and γ given by

$$\left.\begin{aligned} \beta &= \tfrac{1}{4}(1-\alpha^2), \\ \gamma &= \tfrac{1}{2}(1-2\alpha), \end{aligned}\right\} \qquad -\tfrac{1}{3} \leqslant \alpha \leqslant 0. \quad (3.9)$$

When $\alpha = 0$ this reduces to the well-known implicit Newmark scheme. As α becomes increasingly negative, the level of numerical damping increases. For all values of α in the stated range, the scheme is implicit and therefore requires an iterative solution procedure.

Nakashima *et al.* (1990) and Combescure & Pegon (1997) have developed the operator splitting approach, which combines the positive attributes of both implicit and explicit schemes. If equations (3.6) and (3.7) are each written as the combination of a predictor step,

$$\boldsymbol{u}^*_{i+1} = \boldsymbol{u}_i + \Delta t\dot{\boldsymbol{u}}_i + \Delta t^2(\tfrac{1}{2} - \beta)\ddot{\boldsymbol{u}}_i, \quad (3.10)$$

$$\dot{\boldsymbol{u}}^*_{i+1} = \dot{\boldsymbol{u}}_i + \Delta t(1-\gamma)\ddot{\boldsymbol{u}}_i, \quad (3.11)$$

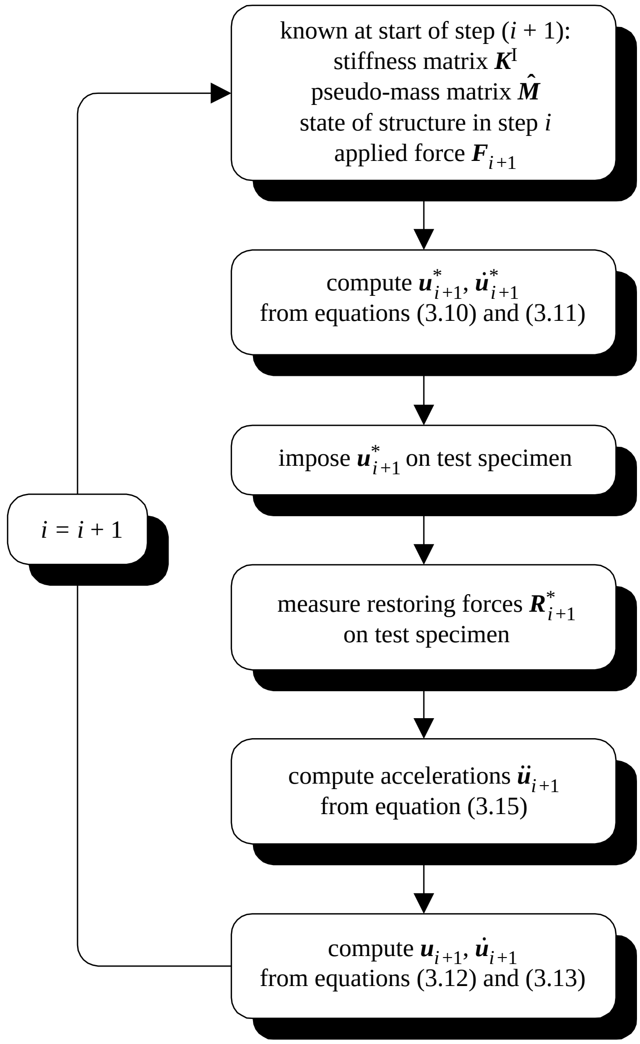

Figure 3. Flow chart for PsD test using α-operator splitting method.

and a corrector step,

$$\boldsymbol{u}_{i+1} = \boldsymbol{u}^*_{i+1} + \Delta t^2 \beta \ddot{\boldsymbol{u}}_{i+1}, \tag{3.12}$$

$$\dot{\boldsymbol{u}}_{i+1} = \dot{\boldsymbol{u}}^*_{i+1} + \Delta t \gamma \ddot{\boldsymbol{u}}_{i+1}, \tag{3.13}$$

then the restoring force can be approximated by the sum of an elastic and a nonlinear term:

$$\boldsymbol{R}_{i+1} \approx \boldsymbol{K}^{\mathrm{I}} \boldsymbol{u}_{i+1} + (\boldsymbol{R}^*_{i+1} - \boldsymbol{K}^{\mathrm{I}} \boldsymbol{u}^*_{i+1}), \tag{3.14}$$

where $\boldsymbol{R}^*_{i+1}$ is the measured restoring force corresponding to $\boldsymbol{u}^*_{i+1}$, and $\boldsymbol{K}^{\mathrm{I}}$ is a stiffness matrix, normally taken to be the initial elastic one. Substituting equations (3.12)–(3.14) into (3.8) gives an equation of the form

$$\hat{\boldsymbol{M}} \ddot{\boldsymbol{u}}_{i+1} = \hat{\boldsymbol{F}}_{i+1}, \tag{3.15}$$

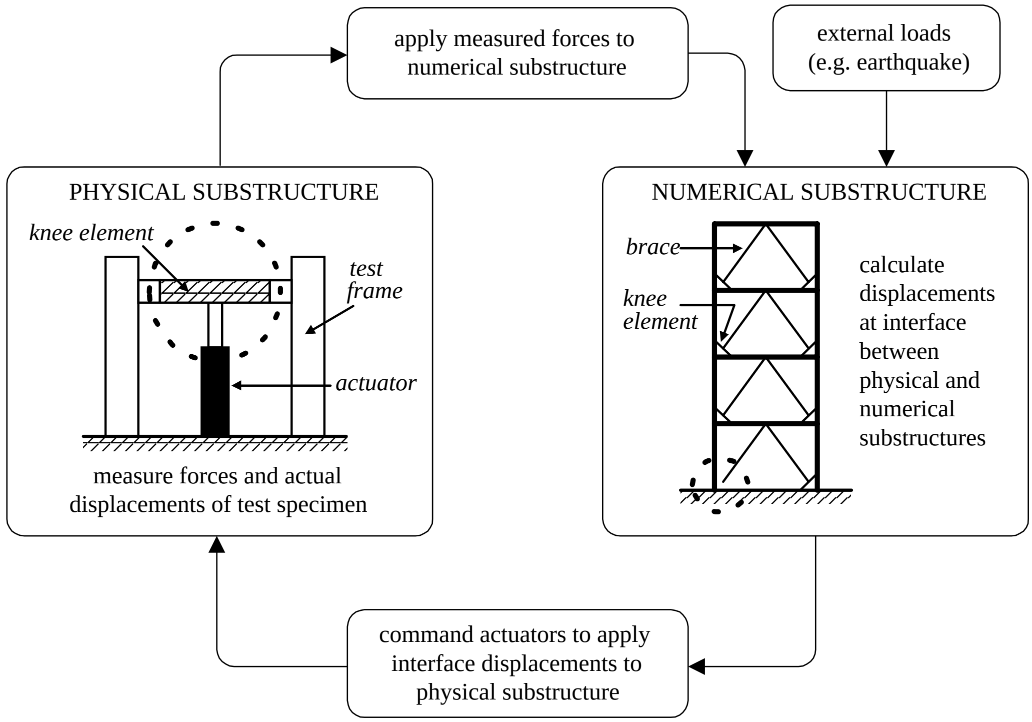

Figure 4. Substructure test loop.

where the pseudo-mass matrix $\hat{\boldsymbol{M}}$ contains only constant terms and the pseudo-force vector $\hat{\boldsymbol{F}}_{i+1}$ contains only quantities from step i and from the predictor step ($\boldsymbol{u}^*_{i+1}$, $\boldsymbol{R}^*_{i+1}$, etc.). A PsD test based on these equations can now be performed as set out in figure 3. The resulting algorithm is implicit (and therefore unconditionally stable) for the elastic part of the response, but explicit for the nonlinear part, so that no iteration is required.

(*d*) *PsD with substructuring*

In many structures, the unpredictable nonlinear behaviour that provides the motivation for laboratory testing is quite localized. In these circumstances a far more economical test can be performed using the substructuring approach. This method was initially developed by Takanashi & Nakashima (1987) and Mahin *et al*. (1989), and has been considerably extended by researchers at the JRC, Ispra (Buchet & Pegon 1994).

In substructuring, a physical model is built only of the part or parts where nonlinearity is expected (the physical substructure), with the remainder modelled computationally (the numerical substructure). As an example, figure 4 shows a schematic control loop for a substructure test on an earthquake-resistant knee-braced frame. The novel feature of this structural form is that the diagonal braces connect into short 'knee elements' spanning diagonally across the beam–column joints. The nonlinear behaviour of these knee elements dominates the structural response during an earthquake. In the example shown a physical test is performed on a single knee element taken from the bottom storey of the structure. This is coupled via a control loop to a numerical substructure, which comprises a finite element model of the entire frame minus the physically tested knee element. The test commences by analysing the response of the numerical substructure to the first element of the earthquake time history. The displacement at the interface between the physical and numerical substructures is output and this is applied to the test specimen by hydraulic actuators. The resulting resistance force is measured by a load cell and fed back to the numerical model, together with the next increment of earthquake ground motion. A new interface displacement is then calculated and applied to the test specimen, and the loop is repeated until the test is complete.

In a substructure test the number of degrees of freedom of the structure is likely to be quite large, so that an explicit integration scheme would require a very short time-step, which may make the test impracticable. The use of a mixed implicit–explicit scheme such as the operator splitting approach is therefore recommended (Casciati & Magonette 1999). A further potential problem is the determination of the viscous damping to be used in the numerical substructure. A simple strategy is to use Rayleigh damping based on the initial stiffness and mass matrices. However, Pegon (1996*a*) has shown that the inclusion of a mass-proportional term does not give the correct equivalence between relative and absolute motions of a base-excited structure. This is likely to be particularly problematic when asynchronous input motions are considered (e.g. for long-span bridges). In such cases more elaborate damping formulations are recommended (Pegon 1996*b*).

4. Real-time test methods

The need to perform structural testing in real time is paramount when rate-dependent effects are important. Such effects are often significant for concrete structures, and to a lesser extent for steel structures, and they are crucial to the performance of many of the dissipative devices such as dampers, rubber bearings and frictional elements, which have been developed to reduce seismic structural response.

In these instances the expanded time-scales used in PsD testing are problematic. These difficulties have been recognized and partly addressed by the use of fast/continuous PsD testing (Takanashi & Nakashima 1987; Casciati & Magonette 1999), in which the loading rate is increased above that of a normal PsD test and the hold period eliminated. However, this does not fully account for rate-dependent behaviour, since the correct structural velocities are still not achieved. Shaking tables can provide real-time loading; however, this advantage is often offset by the associated scaling problems. Considerable attention has therefore been focused on the development of real-time test methods for full or large-scale structures, or structural elements.

Two possible approaches have been attempted. Real-time substructure testing is essentially a fast version of the substructure approach to PsD testing, while effective force testing uses actuators operating under force control to apply the appropriate seismic forces in real time.

(*a*) *Real-time substructure testing*

Real-time substructure (RTS) testing is an extension of the PsD substructuring approach described earlier. Again, the structure is divided into a test specimen (the physical substructure) and a surrounding numerical substructure. The test proceeds according to the substructure control loop in figure 4. However, instead of operating at an expanded time-scale, it is now necessary for the test to proceed in real time. Thus each cycle through the control loop must be completed within the time interval between load increments, so that the loading and structural response occur at the same rate in the test as in a real dynamic loading event on a prototype structure. For an earthquake load, this means that each cycle through the loop must be completed in a time-scale of a few milliseconds. Testing in real time thus requires very rapid computation and communication between the two substructures, as well as robust control. Whereas in a PsD test only the static restoring force is fed back from the physical to the numerical substructure, in a real-time test the fed-back force will also include damping and inertia components, which therefore do not need to be included in the numerical substructure.

RTS testing generally makes use of explicit numerical integration methods such as the central difference method, for which the computations are very simple and quick. However, Darby *et al*. (2000) used a more complex algorithm based on a first-order hold approximation, which appears to offer improved accuracy and stability.

The first reported RTS test (Nakashima *et al*. 1992) was performed on a viscous damper located at the base of a multi-storey building. Only the damper was tested physically, with the building modelled as a linear single-degree-of-freedom (SDOF) system, so that the computations involved were very simple. Darby *et al*. (1999) have also performed real-time tests using a linear SDOF numerical substructure, with the physical test specimen being a stiffness, damping or inertia element.

A problem with real-time testing is the finite response time of the hydraulic actuators; there is an unavoidable delay between a command signal being sent to an actuator and it moving to the desired position. The force fed back from the experiment to the numerical model is therefore incorrect, since it is measured before the actuator has reached its target position.

Horiuchi *et al*. (1996) have shown that, for a linear system, the effect of this error is to introduce additional energy into the system, equivalent to negative damping. This can distort the results and, if the negative damping exceeds the inherent structural damping, cause the test to become unstable. It is therefore necessary to compensate for the delay by extrapolating the displacements to be applied to the test specimen one or two steps ahead. This compensation does itself cause some modification of the apparent stiffness and damping of the system, but this can be kept at acceptable levels so long as the extrapolation is reasonably accurate. For example, Horiuchi *et al*. (1999) investigated the use of simple polynomial curve fits and found that stable and accurate results could be achieved using a third-order function.

Recently, efforts have been made to perform RTS tests for multi-degree-of-freedom (MDOF) systems. Nakashima & Masaoka (1999) and Darby *et al*. (2000) have reported tests using a linear numerical substructure with many degrees of freedom. However, in both cases only a single degree of freedom was passed between the numerical substructure and the physical test specimen, which was loaded by a single actuator. With current computing capabilities, there is a limit on the number of degrees of freedom that can be included in the numerical model, since a large model will require a long computation time. It may then become necessary to extrapolate several steps ahead in order to provide the correct driving signal. With such a long extrapolation it becomes increasingly difficult to ensure the test remains stable.

Under a large earthquake load it is quite likely that yielding will occur in several locations. It is therefore desirable to be able to perform tests in which nonlinearities are permitted in both the physical and numerical substructures. However, as with MDOF systems, nonlinear analysis requires long computation times so that considerable extrapolation may again be necessary (Nakashima & Masaoka 1999). To minimize this problem, Blakeborough *et al*. (2001) have proposed a fast, approximate analysis method using a set of basis vectors derived from the static deformed shapes of the structure as it undergoes successive yield events.

All of the tests described above have involved passing only one degree of freedom between the physical and numerical substructures. Very few tests in which the two substructures are connected by more than one degree of freedom have been attempted. Williams *et al*. (1999) have successfully performed tests in which both a deflection and a rotation were applied to the test specimen, with the corresponding force and moment fed back to the numerical model. However, the interaction between the two degrees of freedom caused the tests to be stable only within quite limited parameter ranges. New control strategies are currently being investigated in order to develop a more robust MDOF test procedure.

(*b*) *Effective force testing (EFT)*

Although the idea of running a real-time test under force control was proposed some time ago (Mahin *et al*. 1989), its experimental implementation has only been attempted very recently (Dimig *et al*. 1999). The underlying idea is attractively simple. For a structure subjected to an earthquake ground displacement u_{g} the equations of motion take the form

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{C}\dot{\boldsymbol{v}} + \boldsymbol{R}(\boldsymbol{v}) = \boldsymbol{0}, \tag{4.1}$$

where $\boldsymbol{u}$ is the vector of absolute structural displacements, $\boldsymbol{v}$ is the vector of displacements relative to the ground,

$$\boldsymbol{v} = \boldsymbol{u} - \boldsymbol{u}_{\mathrm{g}}, \tag{4.2}$$

and $\boldsymbol{u}_{\mathrm{g}}$ is a column vector all of whose elements are equal to u_{g}. Differentiating (4.2) twice and substituting into (4.1) gives

$$\boldsymbol{M}\ddot{\boldsymbol{v}} + \boldsymbol{C}\dot{\boldsymbol{v}} + \boldsymbol{R}(\boldsymbol{v}) = -\boldsymbol{M}\ddot{\boldsymbol{u}}_{\mathrm{g}} = \boldsymbol{F}_{\mathrm{eff}}, \tag{4.3}$$

where $\boldsymbol{F}_{\mathrm{eff}}$ is the effective force vector, whose jth element is obtained by multiplying the jth structural mass by the ground acceleration $\ddot{u}_{\mathrm{g}}$. Equation (4.3) indicates that a structure subjected to a base motion can be replaced by an equivalent fixed-base structure with the effective forces $\boldsymbol{F}_{\mathrm{eff}}$ applied to the structural masses.

This is the basis of the EFT method: the effective forces are applied directly to a fixed-base model of the structure using actuators operating under force control. The principle is illustrated in figure 5 using the example of a planar, multi-storey frame. The prototype structure in figure 5*a* is subjected to an earthquake base motion. The absolute displacement u_j of the jth storey is the sum of its displacement v_j relative to the ground and the ground displacement u_{g}. The effective force test for this structure is shown in figure 5*b*. The ground motion is removed from the system and its effect on the structure is replaced by actuators. The force applied to the jth mass is simply $-m_j\ddot{u}_{\mathrm{g}}$ and the resulting motions are equal to the relative motions in the prototype system.

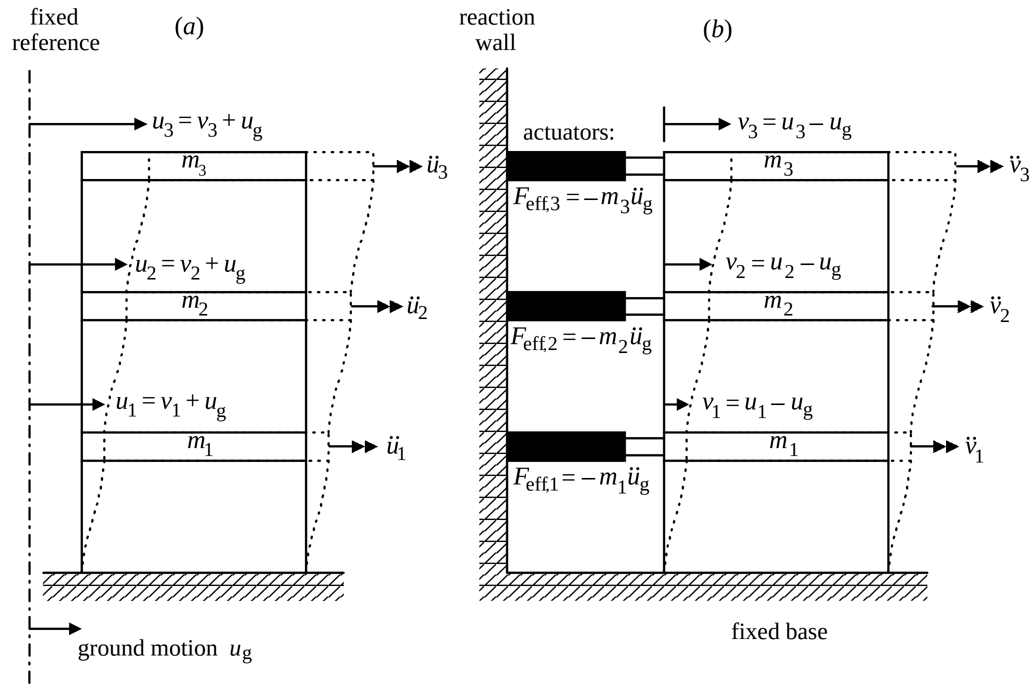

Figure 5. Effective force testing: (*a*) prototype structure and (*b*) laboratory test set-up.

The key advantage of this approach is that, since the effective forces depend only on the ground acceleration record and the structural masses, they are independent of any nonlinear behaviour of the structure. They can therefore be calculated in advance of the test and the need for online computations is eliminated. A disadvantage compared with RTS testing is that, to ensure that equation (4.3) is satisfied, the full structural mass must be included in the test set-up. This may be difficult to achieve in all but the largest laboratories.

A major problem with the EFT approach is that accurate force control can be difficult to maintain due to the natural velocity feedback that exists between the structure and the driving hydraulic actuator. As the structure moves under the applied force, the actuator piston moves with it, and the resulting changes in the internal volumes of the actuator require additional oil flow in order to maintain the prescribed force. This problem is particularly acute close to the natural frequency of lightly damped structures, when the structural motions are greatly amplified by resonance. Dimig *et al.* (1999) compensated for this effect by providing a velocity-related feedback term to the controller in addition to the force feedback. This was successful for a linear SDOF system, but has not yet been attempted for MDOF or nonlinear systems.

5. Summary of current status and future directions

This paper has provided a brief introduction to the basic techniques of dynamic laboratory testing and has highlighted some recent developments. The sector is undergoing a period of rapid development, driven both by catastrophic events such as the recent Japanese and Turkish earthquakes and by the availability of enhanced computer power, which has greatly increased the capacity for online control and computation.

In shaking tables, advances in adaptive control are offering great improvements in the fidelity with which earthquake motions can be reproduced. Significant hardware developments are also being made to enable the development of very large shaking tables capable of performing full-scale testing.

Since its introduction in the 1980s, PsD testing has steadily increased in sophistication. Early problems with experimental errors have been greatly reduced both by the use of high-quality hardware and by the development of improved numerical integration techniques. Recent developments include the construction of very large facilities, the increased use and sophistication of substructuring techniques and the performance of multidimensional tests.

Whereas both shaking tables and PsD testing are mature technologies, real-time test methods have so far been implemented in only a few laboratories and require significant further development. Nevertheless, both real-time substructuring and effective force testing have been shown to be feasible and their use is likely to grow rapidly in the near future. Both techniques have their advantages and drawbacks, and it is not yet clear whether one or the other will prove more useful in the long term.

Future development of the real-time substructure method is likely to focus on the size and complexity of numerical substructure that can be analysed online, and the stability problems that arise when more than one degree of freedom is passed between the two substructures. A more detailed discussion of these issues is given elsewhere in this issue (Nakashima 2001). Effective force testing also requires further refinement of control strategies to address the stability problems when attempting to provide accurate force control at the natural frequency of the test specimen.

The substructuring technique has played an important part in the development of both PsD and real-time test methods, making it possible to test at full or large scale without the need for exceptionally large laboratory facilities. As a result, attempts are now being made to implement substructuring on shaking tables. This would enable seismic testing of, for example, a single storey within a multi-storey frame. Except in very simply cases, this is likely to involve the use of hydraulic actuators exciting points on the test specimen directly in addition to the base excitation provided by the table.

Although the techniques described have been developed primarily for seismic testing of structures, there is considerable potential for their use to cross over into other areas. For instance, shaking tables have found applications in the aerospace industry (Füllekrug 2001). Real-time substructuring has great potential for use in the automobile industry, where rapid product development schedules make it desirable to test components of vehicles rather than full prototypes. It would be possible to perform a physical test of a wheel or a suspension system coupled to a numerical model of the remainder of the vehicle. The frequencies involved in such a test will be significantly higher than for seismic loading. Nevertheless, it is possible to perform such a test in real time, since a linear numerical substructure would normally be adequate for this application. The RTS method is also being taken up by the aerospace industry (Bayer *et al.* 2000), with a physical test of a satellite coupled in real time to a numerical simulation of the launch vehicle in which it is loaded.

References

Bairrao, R. & Vaz, C. T. 2000 Shaking table testing of civil engineering structures—the LNEC 3D simulator experience. *Proc. 12th World Conf. on Earthquake Engineering, Auckland*, paper 2129. Auckland, NZ: New Zealand Society for Earthquake Engineering.

Bayer, V., Dorka, U. E. & Füllekrug, U. 2000 A new algorithm for real-time substructure pseudo-dynamic tests. *Proc. 12th World Conf. on Earthquake Engineering, Auckland*, paper 0401. Auckland, NZ: New Zealand Society for Earthquake Engineering.

Blakeborough, A., Severn, R. T. & Taylor, C. A. 1986 The new UK six-axis shaking table. *Proc. 8th Eur. Conf. on Earthquake Engineering, Lisbon*, vol. 4, pp. 97–100. Lisbon: Laboratorio Nacional de Engenharia Civil.

Blakeborough, A., Williams, M. S., Darby, A. P. & Williams, D. M. 2001 The development of real-time substructure testing. *Phil. Trans. R. Soc. Lond.* A **359**, 1869–1891.

Booth, E. D. 1998 Earthquake engineering in the 1990s: achievements, concerns and future directions. *Proc. Inst. Civ. Engrs* **128**, 154–166.

Brownjohn, J. M. W., Dumanoglu, A. A., Severn, R. T. & Taylor, C. A. 1987 Ambient vibration measurements of the Humber suspension bridge and comparison with calculated characteristics. *Proc. Inst. Civ. Engrs* **83**, 561–600.

Buchet, P. & Pegon, P. 1994 PSD testing with substructuring—implementation and use. Special publication 1.94.25, JRC Ispra.

Carvalho, E. 1998 Seismic testing of structures. *Proc. 11th Eur. Conf. on Earthquake Engineering, Paris*, pp. 53–64. Rotterdam: Balkema.

Casciati, F. & Magonette, G. 1999 Testing facilities and laboratory validation. In *Advances in structural control* (ed. J. Rodellar, A. H. Barbat & F. Casciati), pp. 1–23. Barcelona: CINME.

Chang, S.-Y. 2001 Application of the momentum equation of motion to pseudo-dynamic testing. *Phil. Trans. R. Soc. Lond.* A **359**, 1801–1827.

Combescure, D. & Pegon, P. 1997 α-Operator splitting time integration technique for pseudo-dynamic testing: error propagation analysis. *Soil Dynam. Earthquake Engng* **16**, 427–443.

Darby, A. P., Blakeborough, A. & Williams, M. S. 1999 Real-time substructure tests using hydraulic actuator. *J. Engng Mech.* **125**, 1133–1139.

Darby, A. P., Blakeborough, A. & Williams, M. S. 2000 Improved control algorithm for real-time substructure testing. *Earthquake Engng Struct. Dynam.* **30**, 431–448.

Dimig, J., Shield, C., French, C., Bailey, F. & Clark, A. 1999 Effective force testing: a method of seismic simulation for structural testing. *J. Struct. Engng* **125**, 1028–1037.

Donea, J., Magonette, G., Negro, P., Pegon, P., Pinto, A. & Verzeletti, G. 1996 Pseudodynamic capabilities of the ELSA laboratory for earthquake testing of large structures. *Earthquake Spectra* **12**, 163–180.

Füllekrug, U. 2001 Utilization of multi-axial shaking tables for the modal identification of structures. *Phil. Trans. R. Soc. Lond.* A **359**, 1753–1770.

Hilber, H. M., Hughes, T. J. R. & Taylor, R. L. 1977 Improved numerical dissipation for time integration in structural dynamics. *Earthquake Engng Struct. Dynam.* **5**, 283–292.

Horiuchi, T., Nakagawa, M., Sugano, M. & Konno, T. 1996 Development of a real-time hybrid experimental system with actuator delay compensation. *Proc. 11th World Conf. on Earthquake Engineering, Acapulco*, paper 660. Oxford: Pergamon.

Horiuchi, T., Inoue, M., Konno, T. & Namita, Y. 1999 Real-time hybrid experimental system with actuator delay compensation and its application to a piping system with energy absorber. *Earthquake Engng Struct. Dynam.* **28**, 1121–1141.

Kabayama, K. 2000 Study on the improvement of on-line test applying the linear interpolation method. *Proc. 12th World Conf. on Earthquake Engineering, Auckland*, paper 623. Auckland, NZ: New Zealand Society for Earthquake Engineering.

Kitada, Y., Kinoshita, M., Kubo, T., Seo, K. & Konno, T. 2000 The test methodology to evaluate ultimate earthquake response of an NPP building using earthquake ground motion by blasting. *Proc. 12th World Conf. on Earthquake Engineering, Auckland*, paper 900. Auckland, NZ: New Zealand Society for Earthquake Engineering.

Mahin, S. A. & Shing, P. B. 1985 Pseudodynamic method for seismic testing. *J. Struct. Engng* **111**, 1482–1503.

Mahin, S. A., Shing, P. B., Thewalt, C. R. & Hanson, R. D. 1989 Pseudodynamic test method: current status and future directions. *J. Struct. Engng* **115**, 2113–2128.

Minowa, C., Hayashida, T., Abe, I., Kida, T. & Okada, T. 1996 A shaking table damage test of actual size RC frame. *Proc. 11th World Conf. on Earthquake Engineering, Acapulco*, paper 747. Oxford: Pergamon.

Molina, F. J., Verzeletti, G., Magonette, G., Buchet, P. & Geradin, M. 1999 Bi-directional pseudodynamic test of a full-size three-storey building. *Earthquake Engng Struct. Dynam.* **28**, 1541–1566.

Nakashima, M. 2001 Development, potential and limitations of real-time online (pseudo-dynamic) test. *Phil. Trans. R. Soc. Lond.* A **359**, 1851–1867.

Nakashima, M. & Masaoka, N. 1999 Real-time on-line test for MDOF systems. *Earthquake Engng Struct. Dynam.* **28**, 393–420.

Nakashima, M., Kaminosono, T., Ishida, M. & Ando, K. 1990 Integration technique for substructure pseudodynamic test. *Proc. 4th US Natl Conf. on Earthquake Engineering, Palm Springs*, pp. 515–524.

Nakashima, M., Kato, H. & Takaoka, E. 1992 Development of real-time pseudo dynamic testing. *Earthquake Engng Struct. Dynam.* **21**, 79–92.

Ogawa, N., Ohtani, K., Nakamura, I., Sato, E. & Nagasaki, T. 2000 Development of core technology for 3D 1200 tonne large shaking table. *Proc. 12th World Conf. on Earthquake Engineering, Auckland*, paper 2156. Auckland, NZ: New Zealand Society for Earthquake Engineering.

Ogawa, N., Ohtani, K., Katayama, T. & Shibata, H. 2001 Construction of a three-dimensional large-scale shaking table and development of core technology. *Phil. Trans. R. Soc. Lond.* A **359**, 1725–1751.

Pavic, A. 1999 Vibration serviceability of long-span cast in-situ concrete floors. PhD thesis, University of Sheffield.

Pegon, P. 1996*a* PSD testing with substructuring: the case of asynchronous motion. Special publication 1.96.42, JRC Ispra.

Pegon, P. 1996*b* Derivation of consistent proportional viscous damping matrices. Special publication 1.96.49, JRC Ispra.

Seible, F., Hegemier, G. & Igarashi, A. 1996 Simulated seismic laboratory load testing of full-scale buildings. *Earthquake Spectra* **12**, 57–86.

Severn, R. T. 1997 *Structural response prediction using experimental data.* 6th Mallet–Milne Lecture. Rotterdam: A. A. Balkema.

Severn, R. T., Jeary, A. P. & Ellis, B. R. 1980 Forced vibration tests on dams. *Proc. Inst. Civ. Engrs* **69**, 605–634.

Shing, P. B. & Mahin, S. A. 1987*a* Cumulative experimental errors in pseudodynamic tests. *Earthquake Engng Struct. Dynam.* **15**, 409–424.

Shing, P. B. & Mahin, S. A. 1987*b* Elimination of spurious higher-mode response in pseudodynamic tests. *Earthquake Engng Struct. Dynam.* **15**, 425–445.

Shing, P. B. & Mahin, S. A. 1990 Experimental error effects in pseudodynamic testing. *J. Engng Mech.* **116**, 805–821.

Shing, P., Nakashima, M. & Bursi, O. S. 1996 Application of pseudodynamic test method to structural research. *Earthquake Spectra* **12**, 29–56.

Stoten, D. P. & Benchoubane, H. 1990*a* Empirical studies of an MRAC algorithm with minimal controller synthesis. *Int. J. Control* **51**, 823–849.

Stoten, D. P. & Benchoubane, H. 1990*b* Robustness of a minimal controller synthesis algorithm. *Int. J. Control* **51**, 851–861.

Stoten, D. P. & Gómez, E. G. 2001 Adaptive control of shaking tables using the minimal control synthesis algorithm. *Phil. Trans. R. Soc. Lond.* A **359**, 1697–1723.

Takanashi, K. & Nakashima, M. 1987 Japanese activities on online testing. *J. Engng Mech.* **113**, 1014–1032.

Thewalt, C. R. & Mahin, S. A. 1995*a* Unconditionally stable hybrid pseudodynamic algorithm. *Earthquake Engng Struct. Dynam.* **24**, 723–731.

Thewalt, C. R. & Mahin, S. A. 1995*b* Non-planar pseudodynamic testing. *Earthquake Engng Struct. Dynam.* **24**, 733–746.

Williams, D. M., Darby, A. P., Williams, M. S. & Blakeborough, A. 1999 Real-time dynamic testing system for structures with delay compensation. *Proc. 8th Canadian Conf. on Earthquake Engineering, Vancouver*, pp. 603–608.

Yamazaki, Y., Nakashima, M. & Kaminosono, T. 1989 Reliability of pseudodynamic test in earthquake response simulation. *J. Struct. Engng* **115**, 2098–2112.