J. Proc. Cont. Vol. 7, No. 6, pp. 425–438, 1997

Printed in Great Britain
0959-1524/97 $17.00 + 0.00

PII: S0959-1524(97)00019–X

Closed-loop identification: a two step approach

Biao Huang and Sirish L. Shah*

Department of Chemical Engineering, University of Alberta, Edmonton, AB, Canada T6G 2G6

Received 17 May 1996; revised 24 February 1997

The accuracy aspects of identification (with respect to both variance and bias of estimates) and the role of filtering in closed-loop identification is discussed in this paper. It is shown that the key difference between closed-loop and open-loop identification is the existence of the sensitivity function. A closed-loop identification algorithm which asymptotically yields the same expressions as open-loop identification, in both variance and bias errors, is proposed. The proposed algorithm is evaluated by simulated examples as well as experiments performed on a computer-interfaced pilot-scale process. © 1997 Elsevier Science Ltd

Keywords: closed-loop identification; process identification; prediction error methods; reduced-complexity models

A necessary prerequisite for model-based control is a model of the process. Such certainty-equivalence, model-based control schemes rely on an off-line estimated model of the process, i.e. the process is 'probed' or excited by a carefully designed input-signal under open-loop conditions and the input–output data are used to generate a suitable model of the process. In a majority of model-based control schemes used in the chemical process industry, the models are generated with little regard for their ultimate end-use, e.g. as in model-predictive control. Almost always in such cases, reduced-complexity models are generated to capture the most dominant dynamics of the process. Such batch or off-line identification methods represent a major effort and may require anywhere from several hours to several weeks of open-loop tests.

In contrast with this, the objective in closed-loop identification is to use routine operating data with dither signal excitation to develop a dynamic model of the process. It is practically a very appealing idea. In this mode, process identification can commence with the process in its natural closed-loop state. In some cases, the plant has to run under closed-loop conditions due to safety reasons. In other cases, if a linearized dynamic model around a nominal operating point is desired, then this can be achieved by closed-loop identification, since otherwise under open-loop conditions the process variables may drift away from the nominal operating point.

The method of closed-loop identification has been in the development stage over the last 20 years. Important issues such as identifiability under closed-loop conditions have received attention from many researchers[1–4]. A number of identification strategies have been developed[4,5]. In traditional identification literature, the quality of identification and identifiability issues are mainly addressed under the assumption that the model set contains the plant, i.e. the model can describe true process dynamics.

The more typical case is that of under-modelling or identification of reduced-complexity models when the plant is not in the model set. This is the focus of the present paper and is a more realistic situation since a plant is generally of relatively high order and the model structure used to approximate such process is almost always lower order. Stated simply, identification is an exercise in model-reduction. Under such circumstances, the model-plant error consists of two terms, the bias error (due to under-modelling) and the random error (or variance error) due to noise and disturbance effects. This implies that the bias term cannot be zero even under asymptotic open- or closed-loop conditions. Several general expressions for the asymptotic variance and bias errors have been given by Ljung[5]. The relationship between the variance and bias errors has been recently addressed by Guo and Ljung (1994)[6] and Ljung (1994)[7]. These general expressions for the bias distribution have also been extended to closed-loop identification[8–10].

Closed-loop identification has also attracted much interest due to the emerging research area of joint identification and control. The key idea in the joint identification and control strategy (as opposed to a 'disjoint' or separate identification and control) is to

This paper was not presented at any IFAC meeting.

*To whom all correspondence should be addressed. E-mail: Sirish.Shah@UAlberta.CA identify and control with the objective of minimizing a joint global control performance criterion. This topic has received attention under such headings as: control-relevant identification, iterative identification and control, etc. Readers are referred to Kosut *et al.* (1992)[11], Shook *et al.* (1992)[12], Bitmead (1993)[13], Gevers (1993)[14] and Van den Hof and Schrama (1995)[15] for detailed discussions on these topics. The study of control-relevant identification requires that the best identification strategy is to identify the process under feedback with the intended controller in use. For example, a model intended for the design of minimum variance control is best identified under minimum variance feedback control[16], and similarly for linear quadratic Gaussian (LQG) control[17,18], model reference control[19], etc.

The purpose of this paper, however, is to focus attention only on the identification of the process model under closed-loop conditions. The estimated model is shown to have asymptotically identical expressions for the bias and variance terms regardless of how the identification run is conducted, i.e. irrespective of open-loop or closed-loop conditions. The estimated model can then be subsequently used for improving existing controller design, controller re-design, control-loop performance assessment, general analysis, etc. For example, a model obtained from closed-loop data under proportional-integral-derivative (PID) control may be used for the design of a dynamic matrix control (DMC) controller. Control loop performance assessment techniques as discussed in Harris (1989)[20] do not require an explicit process model when minimum variance control is used as a benchmark. However, if a more practical benchmark standard such as LQG is used for evaluating existing control loop performance, then more information about the process is required[21].

The main contribution of this paper is the development of a two-step closed-loop identification algorithm which asymptotically yields the same expressions for both variance and bias errors as in open-loop identification. These results obviate the need to conduct expensive open-loop tests when simple closed-loop tests with dither signal excitation can suffice. This paper illustrates the point that a suitable model of the process can be estimated from closed-loop data with appropriate filtering. For the reduced-order identification case, it is demonstrated by a simulation example that direct closed-loop identification based on the classical time-series approach that relies on residual tests may give erroneous results, whereas the two-step approach is able to detect the model-plant mismatch.

The paper is organized as follows. Section 2 gives comparison between open-loop and closed-loop identification in terms of variance and bias errors. In Section 3, a two-step closed-loop identification scheme is proposed, which asymptotically yields the same expressions of variance and bias errors as open-loop identification. Extension to multivariate systems is discussed in Section A. The proposed algorithm is evaluated on simulation examples in Section 5, and a computer-interfaced pilot-scale plant is evaluated in Section 6.

Accuracy aspects of closed-loop identification

Consider a linear SISO plant, schematically illustrated in *Figure 1*, and described by

$$y = Tx + Na$$

where a is a white noise sequence, x is the input to the process, T is the process transfer function, and N is the noise transfer function. Let a model of the form

$$\hat{y} = \hat{T}x + \hat{N}a$$

be used to approximate the process dynamics. The prediction error is defined as

$$\hat{a} = \frac{1}{\hat{N}}(y - \hat{T}x)$$

The commonly used objective function for parameter identification is to minimise the sum of squares of the prediction error:

$$V = \frac{1}{M}\sum_{t=1}^{M}\hat{a}^2(t)$$

This method is denoted as the prediction error method or PEM[5]. The total error of the estimates can be attributed to variance and bias errors[5] and may be conceptually written as a sum of the variance error and the bias error[7]:

$$V_T = V_V + V_B$$

In this section, we deal with both the bias and variance estimation errors and compare them between the open-loop and closed-loop cases. We show that the issue of variance and bias error of the parameter estimates is common to both open- and closed-loop identification. The PEM is a general and efficient method for system identification. The variance of the PEM estimator asymptotically reaches the Cramer–Rao lower bound[5]. This asymptotic variance expressed in the frequency domain has been given by Ljung (1987)[5].

Theorem 1:[5] For input and output data x and y obtained from the process shown in *Figure 1*, where

$$y = Tx + v = Tx + Na \tag{1}$$

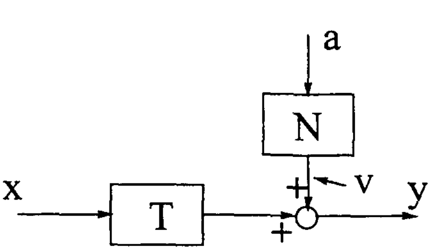

Figure 1 Process model block diagram

the following result holds for large sample size M, large model order n and small n/M:

$$Cov\begin{bmatrix}\hat{T}(e^{j\omega})\\ \hat{N}(e^{j\omega})\end{bmatrix} \sim \frac{n}{M}\Phi_v(\omega)\Phi^{-1}(\omega) \tag{2}$$

where

$$\Phi(\omega) = \begin{bmatrix}\Phi_x(\omega) & \Phi_{xa}(\omega)\\ \Phi_{ax}(\omega) & \sigma_a^2\end{bmatrix} \text{ and } \Phi_v(\omega) = |N(e^{j\omega})|^2\sigma_a^2$$

and where $\Phi * (\omega)$ denotes the spectrum of the corresponding signal*.

Corollary 2: For an open-loop system, with the input $x = w$ (w is input excitation signal and is independent of noise a), and output y, Equation (1) can be written as

$$y = Tw + v = Tw + Na \tag{3}$$

The asymptotic variance of estimates using the PEM is given by

$$Var[\hat{T}(e^{j\omega})] = \frac{n}{M}\frac{|N(e^{j\omega})|^2\sigma_a^2}{\Phi_w(\omega)} = \frac{n}{M}\frac{\Phi_v(\omega)}{\Phi_w(\omega)} \tag{4}$$

$$\frac{Var\hat{N}(e^{j\omega})}{|N(e^{j\omega})|^2} = \frac{n}{M} \tag{5}$$

Proof follows directly from Equation (2).

The results of Theorem 1 are more general than its statement may indicate. These results are also applicable to the closed-loop case (direct identification†)[16] since in this case the correlation between x and a, $\Phi_{xa}(\omega)$, is considered in the expression for $\Phi(\omega)$ (see Equation (2)). In this way, Theorem 1 can also be extended to find the asymptotic variance of estimates $\hat{T}$ [9] and $\hat{N}$ under closed-loop conditions, i.e. $\Phi_{xa}(\omega) \neq 0$ (for the closed-loop ease).

Corollary 3: Under closed-loop control, as illustrated in *Figure 2*, the asymptotic variance of the estimates is given by

$$Var[\hat{T}(e^{j\omega})] = \frac{n}{M}\frac{\Phi_v(\omega)}{\Phi_w(\omega)}\frac{1}{|S(e^{j\omega})|^2} \tag{6}$$

$$Var[\hat{N}(e^{j\omega})] = \frac{n}{M}|N(e^{j\omega})|^2\left(1 + |Q(e^{j\omega})|^2\frac{\Phi_v(\omega)}{\Phi_w(\omega)}\right) \tag{7}$$

where $S = 1/(1 + QT)$ is the sensitivity function; w is the dither signal and is independent of the noise sequence, a.

†Direct identification — identification of a plant model by directly using the input and output data regardless of the feedback effect.

Proof: Under closed-loop conditions, the manipulated variable x can be written as

$$x = Sw - SQNa$$

Therefore

$$\begin{aligned}\Phi_x(\omega) = {} & |S(e^{j\omega})|^2\Phi_w(\omega)\\ & + |S(e^{j\omega})|^2|Q(e^{j\omega})|^2|N(e^{j\omega})|^2\sigma_a^2\end{aligned} \tag{8}$$

$$\Phi_{xa}(\omega) = -S(e^{j\omega})Q(e^{j\omega})N(e^{j\omega})\sigma_a^2 \tag{9}$$

The corollary follows after substituting Equations (8) and (9) into Equation (2).

Thus the asymptotic variance of $\hat{T}$ under closed-loop conditions depends on the sample size M, the signal to noise ratio (SNR), $\Phi_w(\omega)/\Phi_v(\omega)$, and the sensitivity function $S(e^{j\omega})$. Increasing sample size tends to improve the estimate. However, the sensitivity function affects the accuracy inversely. In process control, the asymptotic regulatory property under closed-loop control is the main term of interest, i.e. it is desired to have asymptotic disturbance rejection or $\lim_{\omega \to 0} S(e^{j\omega}) = 0$. Typically, the disturbances are step-type and therefore such asymptotic disturbance rejection (for step-type disturbances) is achieved by incorporating integral action. The estimate is consequently poor at these low frequencies if a white noise dither signal is used. However, to offset the effect of the small value of the sensitivity at low frequency one can use a dither signal which has more power at low frequency. In such cases, the closed-loop estimates at low frequencies will not necessarily be poorer. For other control strategies such as regulation of stochastic disturbances, the sensitivity function may have a small value in the middle or high frequency range, and therefore poor estimates in middle or high frequency range would be expected. The main difference in the asymptotic variance between open-loop and closed-loop identification is the presence of the sensitivity function S (cf. Equations (4) and (6)).

In addition to the variance of the estimates, another important measure of identification quality is the bias error. It exists whenever the process dynamics are not contained in the model set, as in reduced-complexity model identification. This in fact is almost always the case in practice. The distribution of bias errors in the frequency domain has been considered by Ljung (1987)[15] through spectral characterization of the identification problem.

Theorem 4:[5] For an open-loop process shown in Equation (3), the estimation of model parameters in the limit is given by the following optimization problem:

$$\begin{aligned}\theta_M \overset{M\to\infty}{\to} \arg_\theta\min\int_{-\pi}^{\pi} & [|T(e^{j\omega}) - \hat{T}(e^{j\omega})|^2\Phi_w(\omega)\\ & + \Phi_v(\omega)]\frac{1}{|\hat{N}(e^{j\omega})|^2}d\omega\end{aligned} \tag{10}$$

where θ_M is the estimated model parameters. If the noise model $\hat{N}$ is chosen as being fixed such as $\hat{N} = \overline{N}$, then the second term on the right hand side of Equation (10) is constant, and the optimization problem simplifies to

$$\theta_M \overset{M\to\infty}{\Rightarrow} \arg_\theta \min \int_{-\pi}^{\pi} | T(e^{j\omega}) - \hat{T}(e^{j\omega}) |^2 \Phi_w(\omega) \frac{1}{| \overline{N}(e^{j\omega}) |^2} d\omega \tag{11}$$

These results clearly indicate that the bias distribution of $| T(e^{j\omega}) - \hat{T}(e^{j\omega}) |$ in the frequency domain is weighted by the dither spectrum, $\Phi_w(\omega)$, and the inverse of the noise spectrum, i.e. $1/| \hat{N}(e^{j\omega}) |^2$ (also regarded as a noise filter), or simply the signal to noise ratio (SNR), $\Phi_w(\omega)/ | \hat{N}(e^{j\omega}) |^2$. If a unity noise model is considered, i.e. $\overline{N}=1$, then the identification algorithm can be characterized as the output error method (OEM)[5] which does not depend on the noise spectrum:

$$\theta_M \overset{M\to\infty}{\Rightarrow} \arg_\theta \min \int_{-\pi}^{\pi} | T(e^{j\omega}) - \hat{T}(e^{j\omega}) |^2 \Phi_w(\omega) d\omega \tag{12}$$

Theorem 4 can also be extended to the closed-loop case.

Corollary 5:[10] For the closed-loop process shown in *Figure 2*, the estimation of model parameters in the limit is given by the following optimization problem:

$$\theta_M \overset{M\to\infty}{\Rightarrow} \arg_\theta \min \int_{-\pi}^{\pi} [| T(e^{j\omega}) - \hat{T}(e^{j\omega}) |^2 \frac{| S(e^{j\omega}) |^2}{| \hat{N}(e^{j\omega}) |^2} \Phi_w(\omega) + \frac{| S(e^{j\omega}) |^2 | N(e^{j\omega}) |^2}{| \hat{S}(e^{j\omega}) |^2 | \hat{N}(e^{j\omega}) |^2} \sigma_a^2] d\omega \tag{13}$$

where S is the sensitivity function and $\hat{S} = \frac{1}{1+Q\hat{T}}$.

Proof: See the proofs in MacGregor and Fogal (1995)[10] and Bitmead *et al.* (1990)[8] and also the fundamental theory in Ljung (1987)[5].

If w is a sufficiently high order persistently exciting signal, and the set of $\hat{T}$ contains T, the set of $\hat{N}$ contains N, then both estimates are consistent by using the direct identification method[4], i.e. $\hat{T} \overset{M\to\infty}{\Rightarrow} T$ and $\hat{N} \overset{M\to\infty}{\Rightarrow} N$. When the model set does not contain the process dynamics, which is generally the case and is of the interest in this paper, a bias in estimation results, which is weighted once again not only by the SNR but also by the sensitivity function S. Thus the presence of the sensitivity function is the key difference between open-loop and closed-loop identification (cf. *Table 1*). To summarize, the expressions for the asymptotic variance and bias errors under open-loop and closed-loop conditions are listed in *Table 1*.

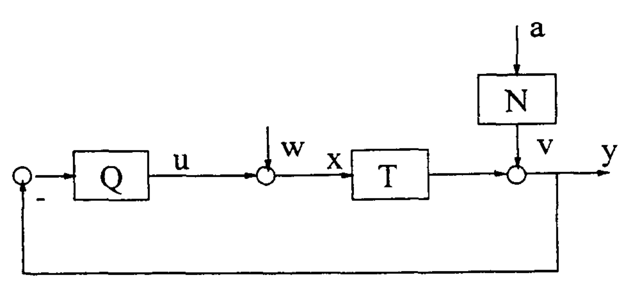

Figure 2 Feedback control loop block diagram

Remark 6: As pointed out by Schrama (1992)[22], Gevers (1993)[14] and Hjalmarsson *et al.* (1994)[19], the 'best' model for the joint identification and control design is not necessarily the 'best' open-loop model. In fact the 'best' model for such design should have the bias error distribution weighted by the desired sensitivity function. This desired sensitivity function is precisely the sensitivity function that one wishes to 'design' through the choice of suitable model-based controller once a suitable model is available. In terms of identification and control, this represents a 'catch-22' situation, since an optimal controller cannot be designed if a control-compatible model is not available, and such a model cannot be estimated via closed-loop identification if its bias spectrum is not weighted by the appropriate sensitivity function. This is the main justification for iterative identification and control. However, in the majority of the design of model-based controllers, the estimation will not have the appropriate sensitivity function as a weighting term. Throughout this paper, we do not assume that the feedback controller under which the closed-loop data are collected is the intended or the ideal controller of choice. In such cases, the dependence on the sensitivity function in place, based on existing control, should be decoupled in the first place. Therefore, one can decouple the effect of the current sensitivity function on the variance and bias errors and also shape the bias distribution through the choice of appropriate data filters and the spectrum of the signal (for closed-loop identification) or the input signal (for open-loop identification). In this way, a model obtained via open- or closed-loop identification can truly serve the purpose of improved control law design, analysis or control-loop performance assessment.

Two-step closed-loop identification

An effective way to reduce variance of estimates is to increase sample size, but this may not have the desired effect on the reduction of the bias error. Depending on the application, smaller errors in some frequency, range, e.g. around the cross-over frequency, may be desired, while larger errors at other frequencies may be tolerated. Data prefiltering can change the distribution of the bias error over the frequency range of interest[5,8].

Table 1 Expressions of the asymptotic variance and bias errors

	Open-loop	Closed-loop
Variance	$\frac{n}{M}\frac{\Phi_v(\omega)}{\Phi_w(\omega)}$	$\frac{n}{M}\frac{\Phi_v(\omega)}{\Phi_w(\omega)}\frac{1}{\vert S(e^{j\omega})\vert^2}$
Bias distribution	$\int_{-\pi}^{\pi}[\vert T(e^{j\omega})-\hat{T}(e^{j\omega})\vert^2\frac{\Phi_w(\omega)}{\vert\hat{N}(e^{j\omega})\vert^2}+$ $+\frac{\Phi_v(\omega)}{\vert\hat{N}(e^{j\omega})\vert^2}]d\omega$	$\int_{-\pi}^{\pi}[\vert T(e^{j\omega})-\hat{T}(e^{j\omega})\vert^2\frac{\vert S(e^{j\omega})\vert^2}{\vert\hat{N}(e^{j\omega})\vert^2}\Phi_w(\omega)+$ $+\frac{\vert S(e^{j\omega})\vert^2}{\vert\hat{S}(e^{j\omega})\vert^2}\frac{\vert N(e^{j\omega})\vert^2}{\vert\hat{N}(e^{j\omega})\vert^2}\sigma_a^2]d\omega$

Bitmead *et al.* (1990)[8] and MacGregor and Fogal (1995)[10] have also shown that data prefilters and the noise model have significant effect on the bias error and identifiability for closed-loop identification. Under closed-loop conditions, the design of data filter is complicated by the presence of the sensitivity function. In this section, we propose a two-step closed-loop identification method which can asymptotically decouple closed-loop parameter estimation from the effect of the undesired sensitivity function. In so doing, this work provides a closed-loop identification method which asymptotically retains the accuracy of open-loop identification. Thus many of the available open-loop experimental design techniques and data prefilters can be applied to closed-loop data. The most recent two-step identification algorithm proposed by Van den Hof and Schrama (1993)[23], which has a similar procedure but has a different objective, is also summarized and compared with the two-step approach proposed here.

Estimation of the sensitivity function — Step 1

For a closed-loop system shown in *Figure 2*, the closed-loop response can be written as

$$y = TSw + NSa \tag{14}$$

and

$$x = Sw - SQNa \tag{15}$$

The sensitivity function, S, can be estimated from Equation (15), i.e. Equation (15) presents a simple open-loop identification problem where the correlation between w and x yields $\hat{S}$. In order to apply Corollary 2 to analyze the variance error of the estimate, $\hat{S}$, the corresponding terms between Equations (3) and (15) should be identified. The one-to-one correspondence between different terms in Equations (3) and (15) is summarized in *Table 2*.

Using *Table 2*, the variance of the estimate of S can be found by applying Corollary 2 to Equation (15) as

$$Var[\hat{S}(e^{j\omega})] = \frac{n}{M}\frac{\Phi_a(\omega)\mid N(e^{j\omega})\mid^2}{\Phi_w(\omega)}\mid S(e^{j\omega})\mid^2\mid Q(e^{j\omega})\mid^2$$
$$= \frac{n}{M}\frac{\Phi_v(\omega)}{\Phi_w(\omega)}\mid S(e^{j\omega})\mid^2\mid Q(e^{j\omega})\mid^2$$

and its relative variance as

$$\frac{Var[\hat{S}(e^{j\omega})]}{\mid S(e^{j\omega})\mid^2} = \frac{n}{M}\frac{\Phi_v(\omega)}{\Phi_w(\omega)}\mid Q(e^{j\omega})\mid^2 \tag{16}$$

which depends on controller dynamics Q in addition to the sample size, the model order and SNR. In subsequent applications we will show that only the relative accuracy of the sensitivity function is important.

Using *Table 2*, the bias distribution of the sensitivity function over the frequency range can be found by applying Theorem 4 to Equation (15). This yields the bias distribution in the frequency domain as

$$\theta_{SM} \overset{M\to\infty}{\to} \arg_\theta \min \int_{-\pi}^{\pi}[\mid S(e^{j\omega}) - \hat{S}(e^{j\omega})\mid^2 \Phi_w(\omega) + \mid H(e^{j\omega})\mid^2 \sigma_a^2]\frac{1}{\mid\hat{H}(e^{j\omega})\mid^2}d\omega \tag{17}$$

where $\hat{H}$ is the closed-loop noise model with $H = -SQN$. Since the sensitivity function serves as the first or intermediate result for subsequent identification of the process dynamics, its order can be selected to be fairly large, i.e. the model set, $\hat{S}$, should then be able to capture most of the dynamics of the actual sensitivity function, S. The total error (bias plus variance) would then be dominated by the variance error[6]. The variance error is then the main issue of concern here. To achieve this, PEM may be used for estimation of the sensitivity function, which has asymptotic minimum variance. However, the distribution of the asymptotic minimum variance error over the frequency range cannot be controlled by pre-filtering of the input–output data since both the process model and the noise model (or filter) are jointly parameterized in the PEM algorithm to yield asymptotic minimum variance estimates[5]. Therefore the main 'tuning knob' or 'control parameter' to adjust the relative variance of the sensitivity function (see

Table 2 Term for term correspondence list

Equation (3)	Equation (15)
y	x
w	w
T	S
N	$-SQN$
v	$-SQNa$

Equation (16)) is the spectrum of the dither signal, which has to be designed carefully in order to control the variance error over the frequency range of interest. However, the variance error can also be reduced by increasing the number of data points. Since it is not difficult to collect a relatively large number of data points under closed-loop operation, a relatively accurate estimate of the sensitivity function can be expected. In the following discussion we therefore assume that $\hat{S} \rightarrow S$.

Estimation of the process model — Step 2

Once the sensitivity function is available, the process dynamics can be estimated by filtering output data with the inverse of the sensitivity function.

Proposition 7: If one filters y by the inverse of the sensitivity function, and then applies PEM (joint parameterization of process and noise models) to Equation (14), the asymptotic variance of the estimates is given by

$$Var[\hat{T}(e^{j\omega})] = \frac{n}{M}\frac{\Phi_v(\omega)}{\Phi_w(\omega)} \tag{18}$$

and

$$\frac{Var[\hat{N}(e^{j\omega})]}{|N(e^{j\omega})|^2} = \frac{n}{M} \tag{19}$$

Thus the variance of $\hat{T}$ and $\hat{N}/N$ is independent of closed-loop dynamics, i.e. both open-loop and closed-loop estimates have the same expression for accuracy with respect to variance (see Corollary 2).

Proof: Using $1/S$ to filter y yields the following relationship between y and w from Equation (14):

$$y/S = y^f = Tw + Na \tag{20}$$

Identification of T from Equation (20) is an open-loop problem. This equation has the same form as Equation (3). Using w as input data and y^f as output data by applying Corollary 2, the proposition follows.

The bias distribution of the estimate in the frequency domain is also asymptotically independent of the closed-loop dynamics as shown in the following proposition.

Proposition 8: From Equation (20), the asymptotic estimates of T and N by using the filtered data y^f and w are given by the following optimization problem:

$$\theta_{SM} \overset{M \to \infty}{\rightarrow} \arg_\theta \min \int_{-\pi}^{\pi} [|T(e^{j\omega}) - \hat{T}(e^{j\omega})|^2 \Phi_w(\omega) + \Phi_v(\omega)] \frac{1}{|\hat{N}(e^{j\omega})|^2}] \, d\omega \tag{21}$$

Again this yields the same bias distribution as under open-loop condition (see Theorem 4). If both model sets, i.e. the process, $\hat{T}$, and the noise, $\hat{N}$, contain the true process dynamics, and w is a sufficiently high-order persistently exciting signal, then the parameter estimates as per Equation (21) will converge to the true values.

Proof: Follows by applying Theorem 4 to Equation (20).

Remark 9: In the proof of Propositions 7 and 8, it is assumed that the true sensitivity function S is used to filter y. If this sensitivity function is substituted by its estimate, $\hat{S}$, then Equation (20) should be written as

$$y/\hat{S} = y^f = \frac{S}{\hat{S}}Tw + \frac{S}{\hat{S}}Na$$

The validity of Propositions 7 and 8 will then depend on the relative accuracy of the sensitivity function, $S/\hat{S}$. Therefore, the relative accuracy of the estimated sensitivity is of main concern in the first step. However, provided that $\hat{S}$ is sufficiently close to S (there is no model order or other structural limitations for the estimate of S), this relative error of the estimate of S should have a negligible effect on the bias expression in the estimates of T and N, but the effect on the variance expression remains a future research topic.

If only the process model T is to be estimated, then the output error method can be applied in this two-step identification approach. Estimation of both, the sensitivity function, S, and the process model, T, are open-loop identification problems. The consistency of the estimates $\hat{S}$ and $\hat{T}$ is independent of the noise model, as long as the noise model is fixed[16] as in the output error method.

Pre-filtering of data is important in identification. In particular, the choice of the data pre-filter can allow one to shape the spectral distribution or composition of the bias errors. The choice of the shaping filter should take into account the intended end-use of the model. This topic overlaps with the area of joint identification and control and has received much attention in the literature. The design and application of shaping filters for control-loop performance assessment is the subject of a future study. The point is that the sensitivity function decoupling filter provides a good or fair starting point for the design of the shaping filter under closed-loop conditions. In the following proposition, we show that the bias error under the two-step identification strategy can be freely shaped.

Proposition 10: (Shaping Filter) For the two-step identification, based on the output error method, if output data y is filtered by $F = G_f/S$, and the input data w is filtered by G_f only, where G_f is a shaping filter, then the asymptotic bias distribution is given by

$$\theta_M \overset{M \to \infty}{\rightarrow} \arg_\theta \min \int_{-\pi}^{\pi} [|T(e^{j\omega}) - \hat{T}(e^{j\omega})|^2 \Phi_w(\omega) \, |G_f(e^{j\omega})|^2] \, d\omega$$

Therefore the bias distribution in the limit is independent of the closed-loop sensitivity function and can be shaped by the user-specified filter G_f to meet accuracy requirement over frequencies of interest.

Proof: Using filter $F = G_f/S$, the relationship between y and w from Equation (14) is now written as

$$y^f = Tw^f + G_f Na \tag{22}$$

where

$$y^f = \frac{G_f}{S} y$$

$$w^f = G_f w$$

Therefore

$$\Phi_{w^f} = | G_f(e^{j\omega}) |^2 \Phi_w(\omega) \tag{23}$$

By applying Theorem 4 with fixed noise model of unit value (i.e. $\overline{N} = 1$) as in OEM yields

$$\theta_M \overset{M \to \infty}{\to} \arg_\theta \min \int_{-\pi}^{\pi} | T(e^{j\omega}) - \hat{T}(e^{j\omega}) |^2 \Phi_{w^f}(\omega) \mathrm{d}\omega \tag{24}$$

The proposition follows on substituting Equation (23) into Equation (24).

The significance of this result is that the estimate obtained under closed-loop conditions can be shaped in the frequency domain if the model does not contain the true dynamics, while the estimator still maintains the property of consistency should the plant model ($\hat{T}$ only) contain the true dynamics. The classic closed-loop direct identification does not have such a property. All available methods for the design of the shaping filter for open-loop identification can therefore be applied in this closed-loop case. For example, Shook *et al.*'s (1992)[12] open-loop long range predictive identification prefilter and Rivera *et al.*'s (1992)[24] systematic design of the control-relevant shaping filter can be applied. The choice of the shaping filter is analogous to selecting frequency weighting of the bias error function. Tighter weighting at some frequencies would result in expected corresponding reduction in bias errors at these frequencies but at the cost of perhaps larger bias errors at other frequencies. The effect of the shaping filter will be briefly shown in the experimental study of a pilot-scale process.

The aforementioned two-step identification algorithm is summarized in *Tables 3* and *4*:

Remark 11: If S contains non-minimum phase zeros, then $1/S$ cannot be used as an unstable decoupling filter. In this case, factorize S as

$$S = \frac{N^+ N^-}{D}$$

where the polynomial N^+ contains all non-minimum phase or unstable zeros. Let polynomial N^{+*} be the reciprocal polynomial of N^+, i.e. all roots of N^{+*} are reciprocal roots of N^+ and therefore are inside the unit circle. Instead of using $1/S$ as the sensitivity function decoupling filter, $1/S'$ should be used as the decoupling filter to filter y_t, where $S' = (N^{+*}N^-)/D$. At the same time, w_t should also be filtered by N^+/N^{+*}. This will yield the same asymptotic properties (variance and bias) as using $1/S$ to filter y_t. However, when S contains the unit-value zeros, the decoupling filter $1/S$ will have an integral term which in some cases may cause numerical problems. In this case, the probing or excitation signal is preferably inserted at the setpoint as discussed in the following sections.

Among many other two-step closed-loop indirect identification strategies[4,25–27], one of the most recent two-step identification strategies with a different objective has been proposed by Van den Hof and Schrama (1993)[23] whose approach is summarized below.

Lemma 12: Assume that the consistent estimate of the sensitivity function, $\hat{S} \to S$, is obtained in the first step. If the input data w is filtered by S, i.e. $w^f = Sw$, before applying OEM, then $\hat{T}$ can be directly estimated from the filtered data and is a consistent estimate.

This is clearly seen from Equation (14), where

$$y = TSw + NSa = Tw^f + NSa \tag{25}$$

whereas the approach proposed in this paper considers filtering y by $1/S$ as follows:

$$y/S = Tw + Na$$

Table 3 The procedure for two-step identification

(i) Fit x to w by using the PEM or OEM, and obtain an estimate, $\hat{S}$, of the sensitivity function, S.
(ii) Filter y by $F = 1/\hat{S}$ and then fit y^f to w by applying the PEM. Then obtain estimates of T and N whose variance and bias expressions are asymptotically the same as open-loop identification.

Table 4 The procedure for two-step identification plus shaping

(i) Fit x to w by using the PEM or OEM, and obtain an estimate, $\hat{S}$, of the sensitivity function S.
(ii) Filter y by $F = G_f/\hat{S}$ and w by G_f and then fit y^f to w^f by using the OEM. Obtain an estimate of T whose bias is asymptotically independent of the closed-loop sensitivity function and is shaped by the filter G_f.

which is

$$y^f = Tw + Na$$

For brevity, the approach proposed by Van den Hof and Schrama is denoted as w-filtering method, while the approach proposed in this paper is denoted as y-filtering method. In Van den Hof and Schrama (1993)[23], 'The sensitivity function is used to simulate a noise free input signal for an open loop identification of the plant to be identified. Using the output error method, an explicit approximation criterion can be formulated, characterizing the bias of identified models in the case of undermodelling'.

Lemma 13: By using the OEM, the w-filter approach yields the asymptotic frequency bias distribution as

$$\theta_M \overset{M\to\infty}{\to} \arg_\theta \min \int_{-\pi}^{\pi} [| T(e^{j\omega}) - \hat{T}(e^{j\omega}) |^2 \\ | S(e^{j\omega}) |^2 \Phi_w(\omega)\, d\omega$$

Thus the frequency weighting on the bias $| T(e^{j\omega}) - \hat{T}(e^{j\omega}) |^2$ depends on the sensitivity function, S. In the approach proposed in this paper, the frequency weighting on the bias is independent of the sensitivity function.

This can be proved by applying Equation (12) in Theorem 4 to Equation (25). Both w and y filtering methods would have the identical asymptotic variance expression should the prediction error method be used for w-filtering method. However, by using the output error method as the identification engine for the w-filtering method, the asymptotic variance expression for the w-filtering method is very different[5].

It should be pointed out that, under the framework of joint identification and control, the dependency of the bias error on the sensitivity function is not undesired provided that the desired sensitivity function or the intended feedback controller is running during the data collection.

Remark 14: One of the main differences between the w-filtering approach and the y-filtering approach is whether w or y should be filtered by the sensitivity function or the inverse of the sensitivity function before carrying out the second step of identification. These two approaches result in different identification objectives. The y-filtering approach as proposed in this paper aims at (1) achievement of the same 'accuracy' expressions with respect to bias and variance errors under closed-loop and open-loop conditions (including consistency of the estimates if the model set contains the plant dynamics); this result is achieved by decoupling the closed-loop sensitivity function from closed-loop data, and (2) obtaining explicit expressions for both asymptotic variance and bias errors. This approach is obtained by comparison of the asymptotic variance and bias errors for open-loop and closed-loop conditions. The w-filtering approach as proposed by Van den Hof and Schrama provides (1) a consistent estimate of the input–output transfer function if the model contains the plant dynamics, and (2) an explicit expression for the asymptotic bias distribution only. The following illustrations show that these differences have important implications in closed-loop identification.

Other practical considerations

Until now we have mainly considered the case where the dither signal is injected from w as shown in *Figure 2.* We will illustrate that the general result can be extended to the case where the dither signal is injected from any point, for example via the setpoint r. *Figure 3* shows an equivalent transformation of the block diagrams. The closed-loop response is now written as

$$y = STQr + SNa$$

This can be transformed to

$$\frac{1}{SQ} y = Tr + \frac{N}{Q} a$$

Therefore if y is filtered by $1/SQ$ before applying the PEM or OEM, the relationship between r and y^f is

$$y^f = Tr + \frac{N}{Q} a$$

It is clear that both variance and bias distribution of estimates by using the PEM or OEM will be independent of the sensitivity function. Since it is again an open-loop identification problem, a shaping filter G_f can also be cascaded to the decoupling filter to shape the bias distribution in the frequency domain, as illustrated in the foregoing discussion.

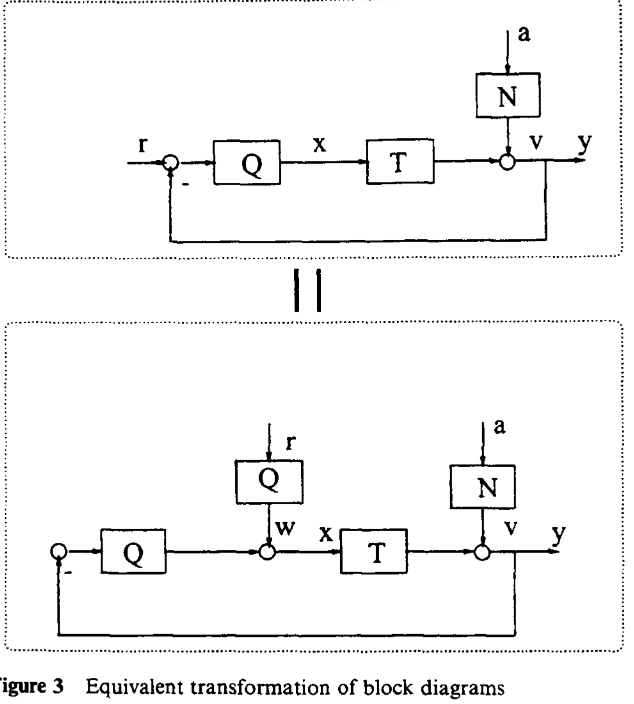

Figure 3 Equivalent transformation of block diagrams

In this case, instead of estimating the sensitivity function S during the first step as in the foregoing section, QS should be estimated jointly. This can be obtained by noticing the following relationship between r and x:

$$x = QSr - NQSa$$

Identification of QS using data r and x is an open-loop identification problem. Therefore, it can be shown that the relative accuracy of the estimate of QS is independent of the sensitivity function. Since the inverse of QS does not contain the unit-value zeros introduced by integral control, it is the preferred sensitivity function decoupling filter when integral action exists in the controller.

Extension to MIMO systems

The two-step closed-loop identification can be extended to MIMO systems.

Proposition 15: Under closed-loop condition, the transfer function matrix T can be estimated via two steps. The sensitivity function is estimated from closed-loop data in the first step. The transfer function matrix T from the sensitivity-filtered closed-loop data is then estimated in the second step.

Proof: From *Figure 2*, we have

$$\begin{aligned} Y_t &= (I+TQ)^{-1}Tw_t + (I+TQ)^{-1}Na_t \\ &= STw_t + SNa_t \end{aligned} \tag{26}$$

where $S=(I+TQ)^{-1}$ is defined as the sensitivity matrix. Filtering both sides of Equation (26) by the inverse sensitivity matrix $S^{-1} = (I+TQ)$ (or the return difference matrix[2]) gives

$$Y_t^f = S^{-1}Y_t = Tw_t + Na_t \tag{27}$$

This is clearly an open-loop identification problem. We also have

$$\begin{aligned} x_t &= (I+QT)^{-1}w_t - (I+QT)^{-1}QNa_{2t} \\ &= S_x w_t - S_x QNa_{2t} \end{aligned} \tag{28}$$

where $S_x \triangleq (I+QT)^{-1}$. The sensitivity function S can be written as

$$S = Q^{-1}(I+QT)^{-1}Q = Q^{-1}S_xQ \tag{29}$$

where the controller transfer function matrix Q either is known as *a priori* knowledge or can be identified from closed-loop data. Clearly, estimation of S_x via Equation (28) is also an open-loop identification problem. Therefore, the two-step identification can be achieved by (1) estimation of the sensitivity function S via Equations (28) and (29), and (2) identification of the transfer function matrix T via Equation (27).

Simulation

Example 16: Consider a second order ARMAX model with the transfer function given by

$$(1-0.7859q^{-1}+0.3679q^{-2})y_t = (0.3403+0.2417q^{-1})$$
$$u_{t-1} + (1-0.8q^{-1}+0.12q^{-2})a(t)$$

A unity feedback control law is implemented in this simulation. The proposed y-filtering approach is compared with the direct identification method. The white noise a_t and the white-noise dither signal w_t are independent with $Var(a_t)=2.25$ and $Var(w_t)=1$, respectively. The number of data points in the simulation is $M=5000$.

In general, identifiability under direct closed-loop identification requires that both the plant and disturbance dynamics lie in the set of plant and disturbance models[4]. However, the w-filtering and y-filtering approaches do not have such a restriction in the choice of the noise model. The difficulty with the direct identification method is the choice (or tradeoff) of the plant model and the noise model, i.e. the choices of the plant model and the noise model are strongly coupled. One may choose high-order models for both the plant and the noise, but this may violate the parsimony principle and also increase the variance error of the estimates, as discussed in the previous sections. Therefore, an incorrect choice of the noise model may yield an erroneous plant model and *vice versa*. In this example, we show that a first-order model, that is identified using the direct identification method and passes all residual tests, deviates significantly from the true dynamics. On the other hand, the y-filtering approach transforms the closed-loop identification to an open-loop identification problem and successfully detects the lack of fit when the first-order plant model is used.

Both the direct identification and y-filtering methods begin with a model of the first-order plant and second-order disturbance. There is clearly a model-order mismatch for such a choice of plant model. The objective in this exercise is to show which identification method can detect such a mismatch. Both methods use the Box–Jenkins model structure, i.e. the *BJ* function in the System Identification toolbox in Matlab. Residual tests for the models identified from both methods detect the correlation between residuals and inputs and thus indicate a lack of fit or a model-plant mismatch. This indicates that one may either increase the order of the noise model or increase the order of the plant model for the next trial.

To see the effect of the noise model, the noise models are increased to order three. The residual test for the direct identification is shown in *Figure 4*. The upper part of the figure shows the autocorrelation of the residuals and clearly indicates 'whiteness' of the residuals. The lower part of the figure is the cross correlation between the residuals and past inputs, i.e $E[\hat{a}_t u_{t-\tau}]/\sigma_{\hat{a}}\sigma_u$ for $\tau > 0$, where τ is the lag of the cross-correlation function. This cross-correlation test clearly indicates a good

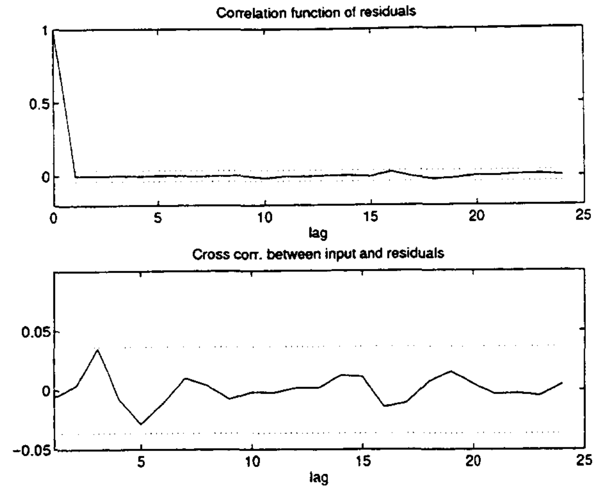

Figure 4 Residual test for the model identified by using direct identification method (first-order plant and third-order noise model)

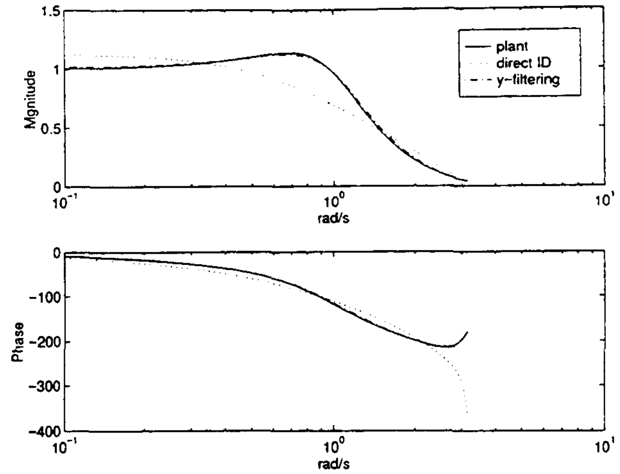

Figure 6 Comparison between direct identification and the y-filtering methods

fit of the data, i.e. no regions outside the 99% confidence intervals. Therefore, the model obtained from direct identification passes the residual test, but the Bode diagram of the model shown in *Figure 6* clearly demonstrates a lack of fit. The residual test of the y-filtering identification is shown in *Figure 5*. The residuals also pass the 'whiteness' test, but the cross-correlation between the residuals and the inputs shows 'spikes' outside the 99% confidence intervals and fails the test. Note that since the y-filtering approach transforms the closed-loop identification problem into the open-loop identification problem, the cross-correlation test has to be carried out for both positive and negative lags (including the zero lag), i.e. the cross correlations between the residuals and all inputs (both past and future inputs)[4,5]. Now we try to further increase the order of the noise model to a higher order (e.g. fifth-order) for the y-filtering method while keeping the first-order plant model. The residual test is shown in *Figure 7*, and the model again fails the cross-correlation test. This indicates that one has to increase the plant model order. Consequently, the plant model is increased to second order. The residual test is shown in *Figure 8*, and this model clearly passes the residual test. Therefore, the y-filtering method is able to find the correct model of the plant despite the error in the choice of the noise model. The Bode plot of the final estimate is shown in *Figure 6* and clearly demonstrates an excellent fit.

The asymptotic variance of the estimate $Var(\hat{T})$ using the y-filtering approach is given in Equation (18). This equation is valid when the exact sensitivity function S is used as the decoupling filter, $1/S$, as shown in Proposition 7 and Remark 9. The predicted variance can be calculated from Equation (18) and is denoted by the solid line in *Figure 9*. To test validity of this predicted variance, 50 Monte-Carlo simulation runs are performed for this example. $\hat{T}$ is calculated from the two-step y-filtering approach using the exact sensitivity function as the decoupling filter. The variance of the estimate, $\hat{T}$, from 50 runs is calculated and also plotted in *Figure 9* as the dash-dotted line. Note that $Var(\hat{T})$ is defined by the variance of complex-valued random variables as[5]:

$$Var(\hat{T}) = E(\hat{T} - E\hat{T})(\hat{T} - E\hat{T})^*$$

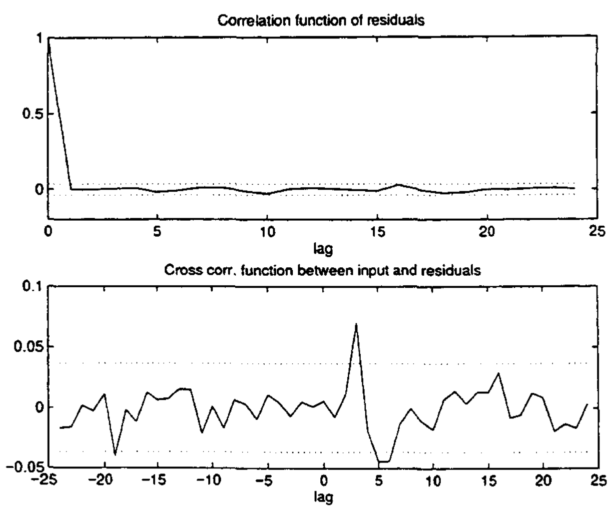

Figure 5 Residual test for the model identified by using the y-filtering method (first-order plant and third-order noise model)

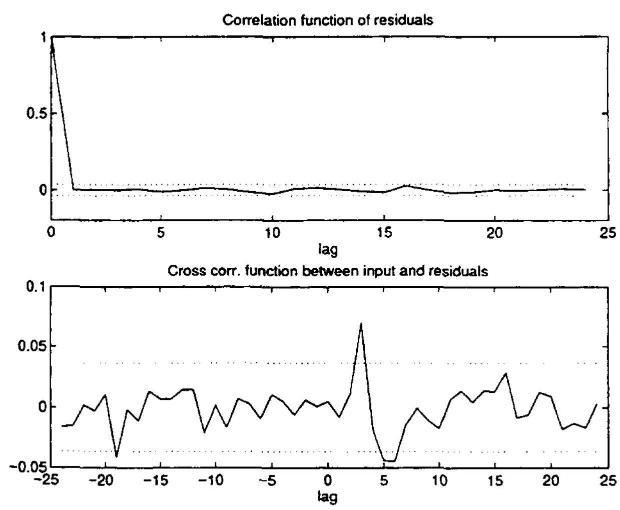

Figure 7 Residual test for the model identified by using the y-filtering method (first-order plant and fifth-order noise model)

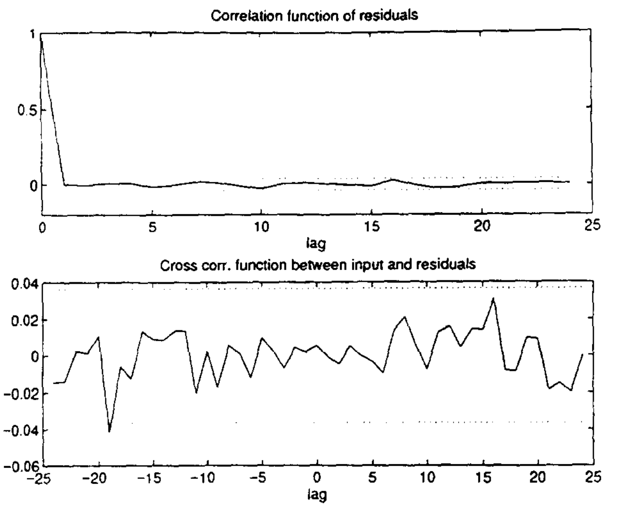

Figure 8 Residual test for the model identified by using the y-filtering method (second-order plant and second-order noise model)

where * means complex conjugate. From *Figure 9*, one can see that a good match in the low to medium frequency range is obtained in this example with mismatch in the high frequency range. The reason could be that the variance as given by Ljung (1987)[5] is an asymptotic and approximate result, and should not be regarded as an exact expression. Nevertheless, this expression does give us a good indication on how disturbanes, dither signals and the sensitivity function affect the accuracy of the estimates.

Example 17: Consider an example in Schrama (1991)[28]. The plant under consideration consists of a transfer function, which is a discrete-time model of a laboratory set-up with some artificial noise contribution. The plant transfer function is given by

$$T = \frac{10^{-3}(0.98q^{-1} + 12.99q^{-2} + 18.59q^{-3} + 3.30q^{-4} - 0.02q^{-5})}{1 - 4.40q^{-1} + 8.09q^{-2} - 7.83q^{-3} + 4.00q^{-4} - 0.86q^{-5}}$$

In order to state a non-trivial case-study, according to Schrama (1991)[28], noise contributions are assumed to additively affect the input u and output y. The additive input noise is white noise with variance 1/9. The output noise is a white noise that is filtered by

$$N = \frac{0.01(2.89 + 11.13q^{-1} + 2.74q^{-2})}{1 - 2.70q^{-1} + 2.61q^{-2} - 0.90q^{-3}}$$

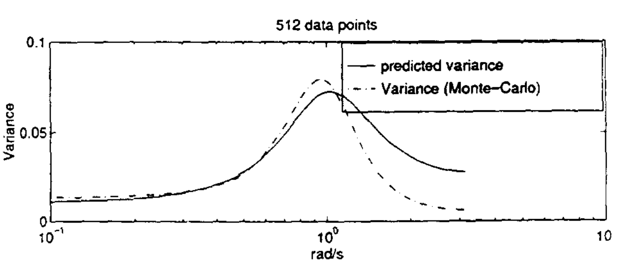

Figure 9 Variance of the estimate calculated from Monte-Carlo simulation (second-order plant and second-order noise model). The number of data points for each simulation is 512

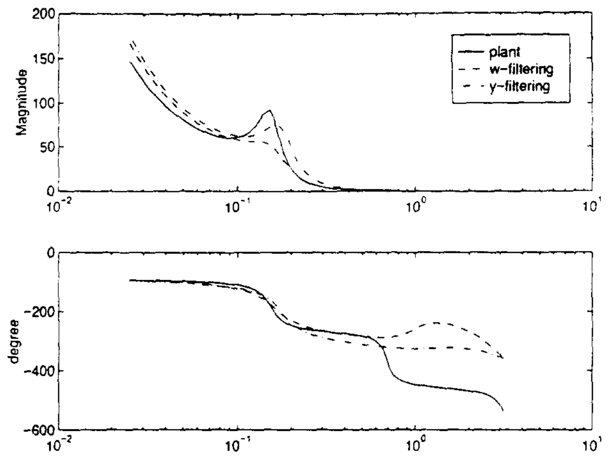

Figure 10 Comparison between y-filtering and w-filtering approaches

A control law

$$Q = \frac{0.61 - 2.03q^{-1} + 2.76q^{-2} - 1.83q^{-3} + 0.49q^{-4}}{1 - 2.65q^{-1} + 3.11q^{-2} - 1.75q^{-3} + 0.39q^{-4}}$$

is implemented on this plant. A dither signal with variance 1 is injected in the process in order to perform closed-loop identification. To demonstrate the effect of under-modelling, a fourth-order plant model (the original plant is fifth-order) is used for the identification.

The w-filtering approach and y-filtering approach are applied to the process and the results presented in the Bode diagrams shown in *Figure 10*. Since the interest in this example is the plant model and the output error method is used for parameter estimation, the most relevant model validation is the cross-correlation test between residuals and inputs[5]. The cross-correlation tests are performed with results shown in *Figures 11* and *12*. Since both w-filtering and y-filtering transform

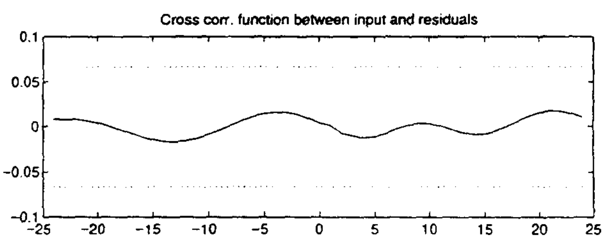

Figure 11 Cross-correlation test for w-filtering

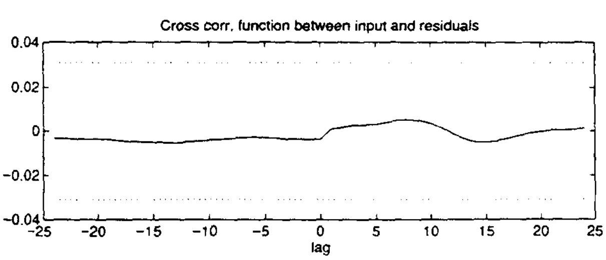

Figure 12 Cross-correlation test for y-filtering

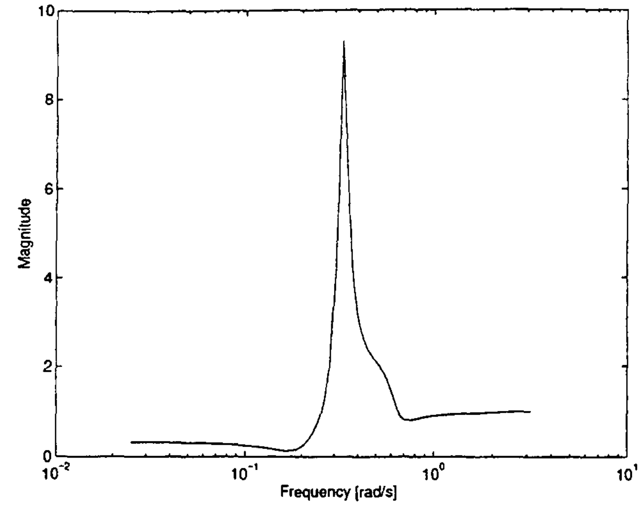

Figure 13 Estimate of the sensitivity function

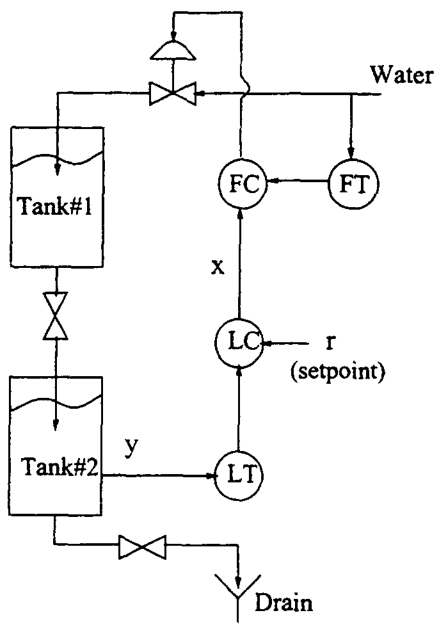

Figure 14 Schematic of the computer-interfaced pilot-scale process

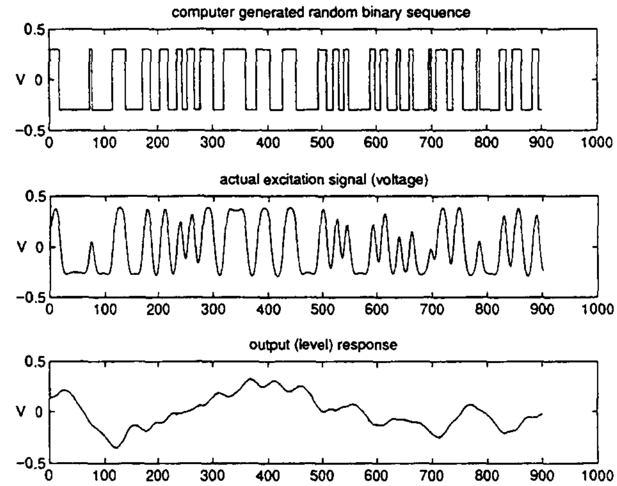

Figure 16 Excitation signal and response. The physical units are voltage in the pilot scale plant where $-2V$ to $+2V$ correspond to 0 to 100%. The time scale is in terms of sampling intervals

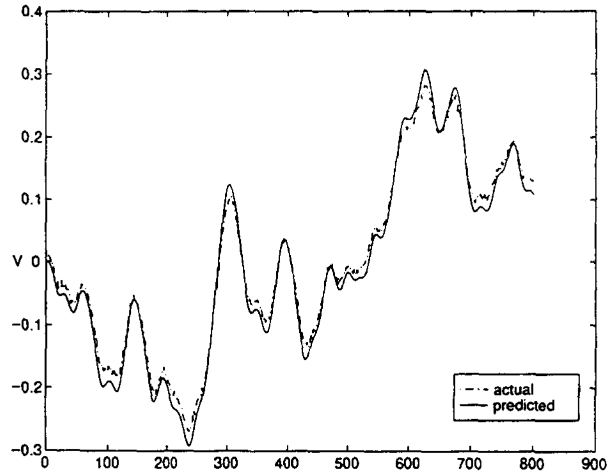

Figure 17 Simulated versus actual data from another open-loop test. The time scale is the sampling intervals

the closed-loop identification to open-loop identification, the cross-correlation test should be conducted over the whole graph (i.e. including both negative and positive legs). Clearly, the models obtained under w-filtering and y-filtering both pass the residual test.

Although both models have passed the time-domain test, the qualities of the models are significantly different in the frequency domain. If we look at the estimated sensitivity function shown in *Figure 13*, we can see the smaller magnitude of the sensitivity function in the medium frequency range with the minimum occurring around the frequency $\omega = 0.17 rad/s$. This shape of the sensitivity function is expected to affect the identification result. This is confirmed in *Figure 10*. The w-filtering

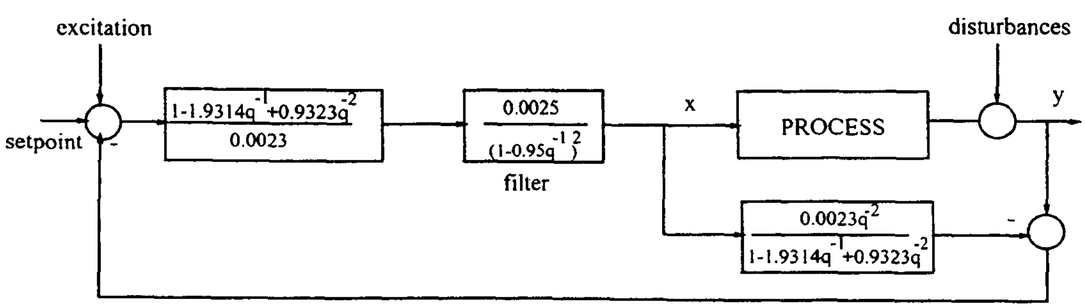

Figure 15 Block diagram for implementation of IMC control using the real-time Simulink Workshop

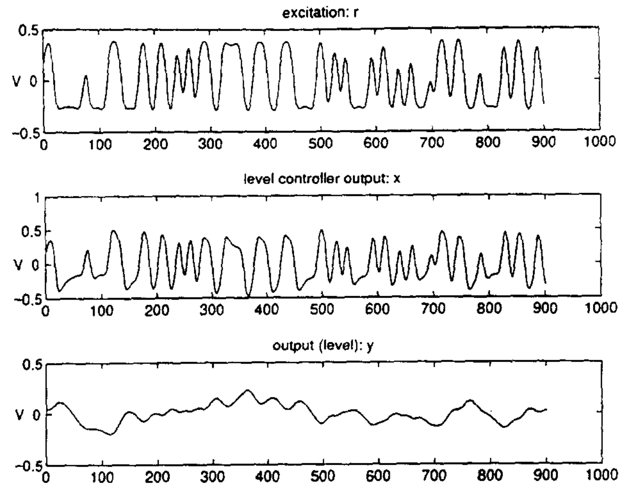

Figure 18 Excitation signal and response under the closed-loop condition. All physical units are voltage in the plot where −2V to +2V correspond to 0 to 100%. The time scale is in terms of sampling intervals

approach gives a poor match in the medium frequency range including the cross-over frequency, particularly around the frequency $\omega = 0.17 rad/s$. The y-filtering, on the other hand, matches the true plant relatively well in the medium frequency range including the cross-over frequency, although this improvement is at the cost of the high frequency mismatch.

Experimental evaluation on a pilot-scale process

In real practical situations, it is difficult to validate the model, $\hat{T}$, estimated under closed-loop conditions with the real process, T, since the latter is unknown. For the purpose of practical evaluation, in the following experimental study, separate identification tests under open-loop and closed-loop conditions are performed. The model estimated under closed-loop condition can be considered a suitably adequate and validated model if it matches the model estimated under carefully designed open-loop conditions.

Example 18: The proposed algorithm is evaluated on a pilot-scale process shown in *Figure 14*. Each tank is a double-walled glass tank 50 cm high with an inside diameter of 14.5 cm. The level of the second tank is the output or controlled variable. The water flow to the first tank is manipulated in order to control the level of the second tank. A PID controller (with T_s = 1 sec) is implemented on the inner loop (flow loop). An IMC controller (T_s = 5 sec) is implemented on the outer loop (level loop). The block diagram of the real-time Simulink Workshop implementation of the IMC controller is shown in *Figure 15*. A second-order model was obtained from open-loop test. This model was validated by checking it with a separate input–output data set. Closed-loop tests were then conducted. Using the proposed method and other closed-loop identification methods, several process models were obtained. These models were compared with the model obtained from the open-loop tests.

Figure 16 shows the computer-generated random binary sequence as used in the open-loop test. The step-type random binary sequence was smoothed by a second-order Butterworth filter with the cutoff frequency significantly larger than the bandwidth of the process. The bandwidth of the process was estimated from previous open-loop tests. A second-order model was estimated by using the prediction error method and found to be

$$\hat{T} = \frac{0.0023q^{-2}}{1 - 1.9314q^{-1} + 0.9323q^{-2}}$$

where the two-step time delay is due to a zero order hold and an additional artificially introduced unit-step time delay. The simulated versus actual data (using a separate validation or test data with different setpoint excitation inputs) are shown in *Figure 17*. Clearly the open-loop model is a good representation of the real process.

Since there is integral action in the IMC control, it is preferable to insert an excitation signal via the setpoint to avoid a pole on the unit circle in the sensitivity function decoupling filter. *Figure 18* shows the excitation signal and the output under the closed-loop test. The

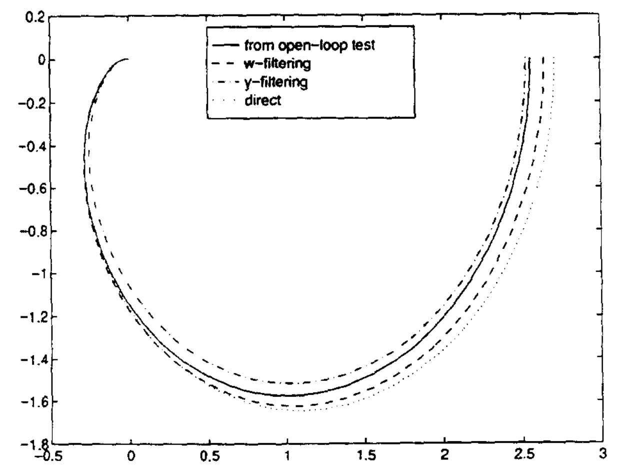

Figure 19 Comparison of the identified process models using different methods when a second-order model is used

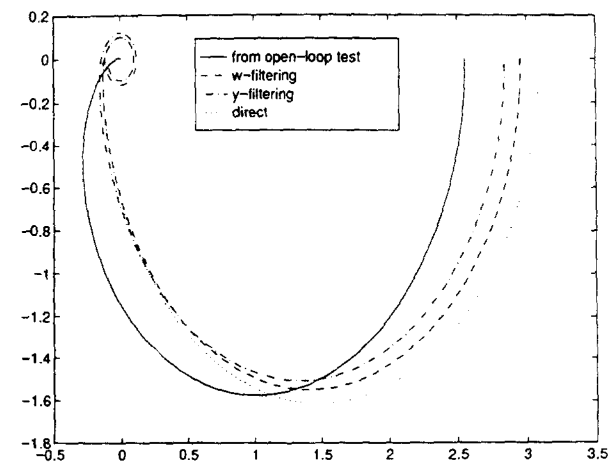

Figure 20 Comparison of the identified process models using different methods when a first-order model is used

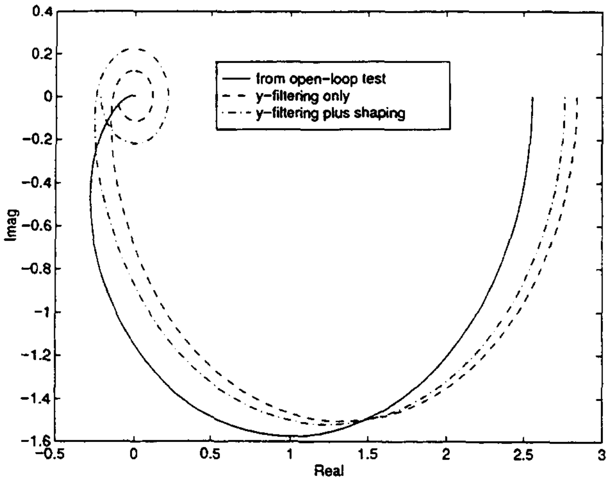

Figure 21 Effect of the shaping filter for the first-order model

y-filtering, w-filtering and direct closed-loop identification methods were used to estimate the process model. Second-order models are identified and the resulting Nyquist plots are shown in *Figure 19*. If there is no model-plant mismatch, all the Nyquist plots should converge to one plot when the sample size increases.

If a first-order process model is assumed, then a model-plant mismatch is indeed present. A larger bias error would be expected at lower frequencies if w-filtering or direct closed-loop identification is used. Nyquist plots of the identified models shown in *Figure 20* confirm this. Since this is an over-damped second-order plant, the model-plant mismatch by using a first-order model to represent a second-order over-damped plant is not severe. The direct identification does not fail in this example. The effect of the shaping filter on the identification is illustrated by using a fourth-order low-pass Butterworth filter cascaded to the decoupling filter, as shown by the results displayed in *Figure 21*. Clear improvement of the estimate at low to middle frequencies is obtained by cascading the shaping filter to the decoupling filter. Depending on the application, different shaping filters at different frequencies can be designed for the control algorithm of choice.

Conclusions

The accuracy aspects of closed-loop identification have been discussed. It has been shown that the key difference between closed-loop and open-loop identification is the sensitivity function. The sensitivity function inversely affects the variance and bias errors of the estimate under closed-loop conditions. A two-step closed-loop identification has been proposed, which yields identical asymptotic properties as under open-loop identification. The proposed algorithm has been evaluated by simulated examples as well as by pilot-scale experiments. These results affirm the strategy that a suitable model commensurate with its intended end use can always be identified under closed-loop conditions through the choice of appropriate data prefilters.

References

1. Box, G. E. P. and MacGregor, J. F., Parameter estimation with closed-loop operating data. *Technometrics*, 1976, **18**, 371–380.
2. Goodwin, G. C. and Payne, R. L., *Dynamic System Identification: Experiment Design and Data Analysis*. Academic Press, New York, 1977.
3. Gustavsson, I., Ljung, L. and Soderstrom, T., Identification of processes in closed loop — identifiability and accuracy aspects. *Identification and System Parameter Estimation*, 1978, pp. 41–77.
4. Soderstrom, T. and Stoica, P., *System Identification*. Prentice Hall International, UK 1989.
5. Ljung, L., *System Identification*. Prentice-Hall, Englewood Cliffs, 1987.
6. Guo, L. and Ljung, L., The role of model validation for assessing the size of the unmodelled dynamics. In *Proceedings of CDC*, Florida, pp. 3894–3899 1994.
7. Ljung, L., System identification in a mic perspective. *Modelling, Identification and Control*, 1994, **15**(3), 153–159.
8. Bitmead, R. R., Gevers, M. and Wertz, V., *Adaptive Optimal Control*. Prentice Hall, Englewood Cliffs, 1990.
9. Zhu, Y. and Backe, T., *Identification of Multivariable Industrial Processes*. Springer-Verlag, 1993.
10. MacGregor, J. F. and Fogal, D. T., Closed-loop identification: the role of the noise model and prefilters. *Journal of Process Control*, 1995, **5**(3), 163–171.
11. Kosut, R. L., Goodwin, G. C. and Polis, M. P., Introduction to Special issue on system identification for robust control design. *IEEE Trans Automatica Control*, 1992, **37**(7), 899.
12. Shook, D. S., Mohtadi, C. and Shah, S. L., A control-relevant identification strategy for gpc. *IEEE Trans. AC*, 1992, **37**(7), 975–980.
13. Bitmead, R. R, Iterative control design approaches. In *Proceedings of the 12th IFAC World Congress*, Vol. 9, pp. 381–384, Sydney 1993.
14. Gevers, M., Towards a joint design of identification and control. In *2nd European Control Conference*, Holland, June 1993.
15. Van den Hof, P. M. J. and Schrama, R. J. P., Identification and control — closed-loop issues. *Automatica*, 1995, **31**(12), 1751–1770.
16. Gevers, M. and Ljung, L., Optimal experiment designs with respect to the intended model application. *Automatica*, 1986, **22**(5), 543–554.
17. Zang, Z., Bitmead, R. R. and Gevers, M., Iterative weighted least-squares identification and weighted lqg control design. *Automatica*, 1995, **31**(11), 1577–1594.
18. Hakvoort, R. G., Schrama, R. J. P. and Van den Hof, P. M. J., Approximate identification with closed-loop performance criterion and application to lqg feedback design. *Automatica*, 1994, **30**(4), 679–690.
19. Hjalmarsson, H., Gevers, M., Bruyne, F. D. and Leblond, J., Identification for control: closing the loop gives more accurate controllers. In *Proceedings of 1994* CDC, pp. 4150–4155, Lake Buena Vista, FL, December 1994.
20. Harris, T., Assessment of closed loop performance. *Can. J. Chem. Eng.*, 1989, **67**, 856–861.
21. Huang, B., Multivariate statistical methods for control loop performance assessment. PhD thesis, Department of Chemical Engineering, University of Alberta, Edmonton, Alberta, Canada, 1997.
22. Schrama, R., Approximate identification and control design. PhD thesis, Delft University of Technology, 1992.
23. Van den Hof, P. M. J. and Schrama, R. J. P., An indirect method for transfer function estimation from closed loop data. *Automatica*, 1993, **29**(6), 1523–1527.
24. Rivera, D. E., Pollard, J. F. and Garcia, C. E., Control-Relevant Prefiltering: A Systematic Design Approach and Case Study. *IEEE Trans. AC*, 1992, **37**(7), 964–974.
25. Caines, P. E. and Chan, C. W., Feedback between stationary stochastic processes. *IEEE Trans AC*, 1975, **20**, 498–508.
26. Phadke, M. S. and Wu, S. M., Identification of multi-input-multi-output transfer function and noise model of a blast furnace from closed-loop data. *IEEE Trans AC*, 1974, **19**, 944–951.
27. Defalque, B., Gevers, M. and Installe, M., Combined identification of the input–output and noise dynamics of a closed-loop controlled linear system. *Int. J. Control*, 1976, **24**, 345–360.
28. Schrama, R. J. P., An open-loop solution to the approximate closed-loop approximation problem. In *Proceedings of IFAC identification and system parameter estimation*, pp. 761–766, Budapest, Hungary, 1991.