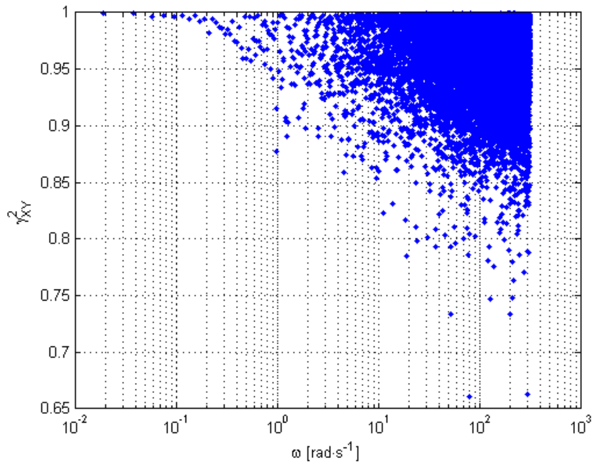

Figure 39.5: Coherence function of Example 39.3.

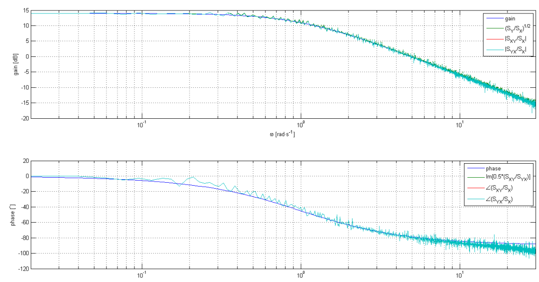

Figure 39.6: Improved Bode diagrams of the plant of Example 39.3.

Fixing the phase and neglecting the last decade, the Bode diagram in Figure 39.6 is obtained. □

In the previous example we used white noise as input. Why? Are not (39.35)–(39.43) true irrespective of what the input is? The answer is yes, but notice that the PSD of the input $S_X(\omega)$ always appears in the denominator. If input X is not white noise, there will be frequencies for which $S_X(\omega)$ will be small, even close to zero. When applying (39.35)–(39.43), at those frequencies we will be dividing by that small value, and numerical errors will certainly be very large.

(Actually, the denominator of (39.38) does not include $S_X(\omega)$, but a similar reasoning is valid just the same: if X is not white noise, the CSD $S_{YX}(\omega)$ will be small, even close to zero, at some frequencies, resulting in significant numerical errors.)

Use white noise as input for identification

For this reason, (39.35)–(39.43) should be applied when the input of the system we want to identify is as close to white noise as possible. (Hence the name of this chapter.)

39.3 Checking identification results

Because of unavoidable noise in measurements, the output $Y(s)$ of an identified model never perfectly matches the measured output $\tilde{Y}(s)$ for the same input $U(s)$. We already know that we can model this mismatch using additive output noise as

$$Y(s) = G(s)X(s) + E(s) \tag{39.48}$$

where $E(s) = \tilde{Y}(s) - Y(s)$. This noise $E(s)$ can often be assumed to be white noise.

We know, too, that noise is not the only reason for output mismatch: there may be unmodelled dynamics, i.e. our model $G(s)$ will differ from the real model of the plant $\tilde{G}(s)$. In that case,

$$E(s) = \tilde{G}(s)X(s) - G(s)X(s) = \left(\tilde{G}(s) - G(s)\right) X(s) \tag{39.49}$$

This shows that a good model for a plant should verify the two following properties:

- the input $X(s)$ and the error $E(s) = \tilde{Y}(s) - Y(s)$ are not correlated, i.e. the correlation coefficient of these two variables verifies $R_{XE}(\tau) = 0$ for all lags τ;
- the error $E(s)$ is white noise, i.e. its autocorrelation coefficient is given by $R_E(0) = 1$ and $R_E(\tau) = 0,\ \forall \tau \neq 0$.

Example 39.4. Consider a second order overdamped plant, with poles at 10 rad/s and 100 rad/s, and unit gain at low frequencies. Suppose that its output to white noise is corrupted by white noise with a standard deviation which is $\frac{1}{10}$ of that of the output.

```
Ts = 0.01; tfinal = 30; t = 0 : Ts : tfinal;
X = randn(size(t));
Greal = zpk([],[-10 -100],1e3);
Yreal = lsim(Greal, X, t);
noise = randn(size(Yreal))*0.1*std(Yreal);
Ymeasured = Yreal + noise;
```

Imagine that a model was obtained for this plant that missed the highest frequency zero. The same input is provided to the model. The error between its output and the measured output has the autocorrelation coefficient and the correlation coefficient with the input shown in Figure 39.7:

```
Gmodel = zpk([],[-10],10);
Ymodel = lsim(Gmodel, X, t);
E = Ymodel - Ymeasured;
figure, autocorr(E)
R_XE = xcorr(E,X, 'coeff');
figure, plot(-tfinal : Ts : tfinal, R_XE)
title('Correlation coefficient of noise and input')
```

The same Figure compares the results obtained with the correct model:

```
figure, autocorr(noise)
R_XE = xcorr(noise,X, 'coeff');
figure, plot(-tfinal : Ts : tfinal, R_XE)
title('Correlation coefficient of noise and input')
```

It is clear from the Figure that, when the model has a missing pole, the autocorrelation of the error lies, for many lags, outside the confidence limits where it can be assumed to be zero; and the correlation coefficient of input and error is not zero, particularly when there is no lag ($\tau = 0$). This strongly indicates that the model does not explain the dynamic behaviour of the plant completely, and that a better model could be got.

On the other hand, if the model has both poles, the autocorrelation of the error is that of white noise, and the correlation coefficient of input and error is practically zero. □

Remark 39.1. Verifying if $E = \tilde{Y} - Y$ is white noise, and if the cross-correlation of X and E is zero, makes sense also when the model is discrete in time, a situation addressed below in Chapter 41.

Glossary

> The political dialects to be found in pamphlets, leading articles, manifestos, White Papers and the speeches of Under-Secretaries do,

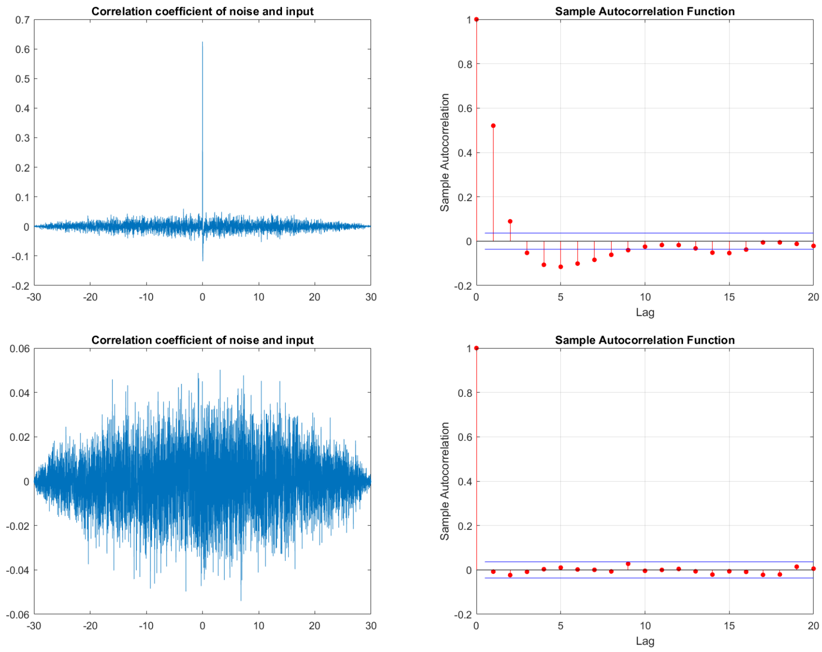

Figure 39.7: Plots from Example 39.4. Top: what happens when the model has a missing pole. Bottom: what happens when the model is correct. Left: correlation coefficient of input and error. Right: autocorrelation of error.

> of course, vary from party to party, but they are all alike in that one almost never finds in them a fresh, vivid, home-made turn of speech. When one watches some tired hack on the platform mechanically repeating the familiar phrases — *bestial atrocities*, *iron heel*, *blood-stained tyranny*, *free peoples of the world*, *stand shoulder to shoulder* — one often has a curious feeling that one is not watching a live human being but some kind of dummy: a feeling which suddenly becomes stronger at moments when the light catches the speaker's spectacles and turns them into blank discs which seem to have no eyes behind them. And this is not altogether fanciful. A speaker who uses that kind of phraseology has gone some distance toward turning himself into a machine. The appropriate noises are coming out of his larynx, but his brain is not involved as it would be if he were choosing his words for himself. If the speech he is making is one that he is accustomed to make over and over again, he may be almost unconscious of what he is saying, as one is when one utters the responses in church. And this reduced state of consciousness, if not indispensable, is at any rate favourable to political conformity.

George Orwell (1903 — †1950), *Politics and the English Language* (1946)

coherence function função de coerência

Exercises

1. Prove (39.19).