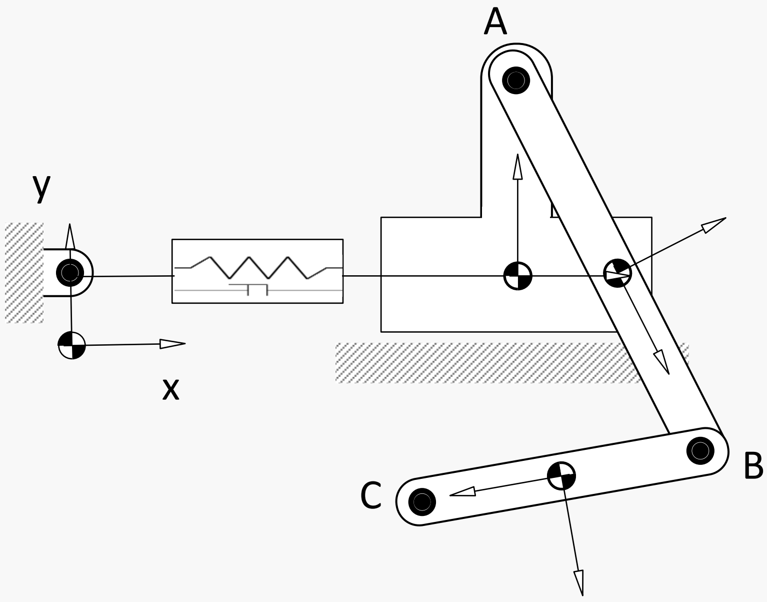
Dynamics of Mechanical Systems

2023/24

Summary

Dynamic analysis of mechanical systems using Cartesian Coordinates.

Application to a sliding double-pendulum using MuboDAP.



Dynamics of Mechanical Systems

Input structure and Pre-Processor

Like for MuboKAP, the modeling data and analysis profile is supplied to MuboDAP pre-processor via an ASCII text file (with the extension '.txt'). The input file follows a similar structure as that of MuboKAP with a few additional options:

Model dimensions (1st line): Information on the dimensions of the multibody system (data separated by space, tab, or comma), which includes, by order:

- NBody – Number of rigid bodies in the model
- NRevolute – Number of revolute joints
- NTranslation – Number of translation joints
- NRevRev – Number of composite revolute-revolute joints
- NTraRev – Number of composite revolute-translation joints
- NCam – Number of cam joints
- NRigid – Number of rigid joints
- NSimple – Number of simple joints
- NDriver – Number of driving constraints
- NPointsOfInt – Number of points of interest for reporting
- NForceAppl – Number of externally applied forces
- NSprDamp – Number of spring-damper-actuator systems

Input structure and Pre-Processor

Data regarding the rigid bodies requires additional information besides the position and orientation of the body.

Rigid bodies data (From lines 2 to NBody + 2): Information on the position and orientation of each of the rigid bodies of the model, which is constituted by:

- x_i – Position along X in the body fixed coordinate system
- y_i – Position along Y in the body fixed coordinate system
- θ_i – Angular orientation of the rigid body (in radians)
- $\dot{x}_i$ – Velocity along X in the body fixed coordinate system
- $\dot{y}_i$ – Velocity along Y in the body fixed coordinate system
- $\dot{\theta}_i$ – Angular velocity of the rigid body (in radians / s)
- m_i – Mass of the rigid body
- J_i – Moment of inertia of the rigid body

Note: The positions and velocities are not just estimates. They should be consistent with the mechanical system being modeled.

Input structure and Pre-Processor

The next set of data concerns the information regarding the revolute joints. The required modelling data to be provided includes:

$$\mathbf{\Phi}^{(Rev,2)} = \mathbf{r}_i + \mathbf{A}_i \mathbf{s}_i'^P - \mathbf{r}_j - \mathbf{A}_j \mathbf{s}_j'^P$$

Revolute joints data (From lines Nbody + 3 to NBody + NRevolute + 3): Information on the rigid bodies of the model connected by the joint and location of the required geometric features:

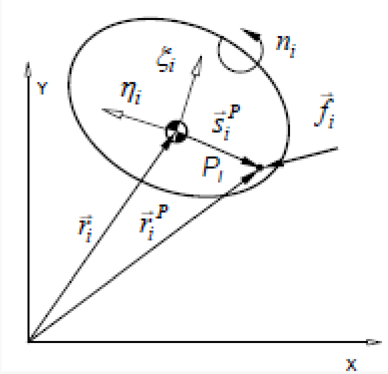
- i – Number of the 1st body connected by the revolute joint
- j – Number of the 2nd body connected by the revolute joint
- ξ_i^P – ξ coordinate of point P in body i
- η_i^P – η coordinate of point P in body i
- ξ_j^P – ξ coordinate of point P in body j
- η_j^P – η coordinate of point P in body j

Note: Data required for this and remaining kinematic joints are similar to those of MuboKAP.

<u>Input structure and Pre-Processor</u>

…

After the definition of the points of interest, if any exist, the next set of data concerns the information regarding applied forces and moments, if any exist. The required modelling data to be provided includes:



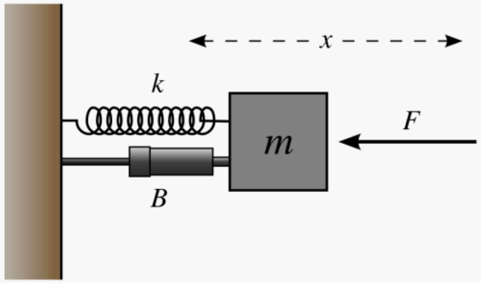
Applied force data: Information on the forces and moments applied to a given rigid body:

- i – Number of the body to which the force and moment are to be applied
- f_x – x-component of the force to be applied
- f_y – y-component of the force to be applied
- n – moment to be applied
- ξ_i – ξ coordinate of the force application point in body i
- η_i – η coordinate of the force application point in body i

Dynamics of Mechanical Systems

<u>Input structure and Pre-Processor</u>

The next set of data concerns the information regarding the spring-damper-actuator systems, if any exist. The required modelling data to be provided includes:



Spring-damper-actuator data: Information on the rigid bodies connected by the spring-damper-actuator system and the required properties of the system:

- i – Number of the 1st body connected by the system
- j – Number of the 2nd body connected by the system
- ξ_i^P – ξ coordinate of point P in body i
- η_i^P – η coordinate of point P in body i
- ξ_j^P – ξ coordinate of point P in body j
- η_j^P – η coordinate of point P in body j
- k – Stiffness of the spring
- l_0 – Resting length of the spring
- C – Damping coefficient
- a – Actuator force

Dynamics of Mechanical Systems

Input structure and Pre-Processor

NBody NRevolute NTranslation … NPointsInt NForceAppl NSprDamp

…

After the description of all parameters from the 1st line, the last lines continue to require additional information regarding the simulation. For MuboDAP, this information concerns the gravitational acceleration, numerical methods and time profile.

Gravitational acceleration (Line…):

- x-ag – x-component of the gravitational acceleration vector
- y-ag – x-component of the gravitational acceleration vector

Numerical methods data (Line …):

- Ode – Number of the ode solver to use (1 – ode15i; 2 – ode23tb; 3 – ode23t; 4 – ode23s; 5 – ode15s; 6 – ode113; 7 – ode23; 8 – ode45)
- EquationSolver – If 1, it uses the backslash solver of Matlab.
- α – Parameter α of the Baumgarte stabilization
- β – Parameter β of the Baumgarte stabilization

MuboDAP

Input structure and Pre-Processor

...

Time analysis profile data (Line ...):

- t_0 – Starting time for the dynamic analysis
- t_{dt} – Time step
- t_{end} – Ending time for the dynamic analysis
- t_{rep} – Time step for reporting results

Output

Once the simulation is finished, the post-processing function implemented in MuboDAP generates the following output files:

- **.out* – Time history of the positions, velocities, and accelerations of all bodies
- **.poi* – Time history of the positions, velocities, and accelerations of the points of interest
- **.jnt* – Time history of the joint reaction forces and moments for all joints

Output

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The *.out file should contain 9NBody columns:

	Body 1									Body 2					
t	x_1	y_1	θ_1	$\dot{x}_1$	$\dot{y}_1$	$\dot{\theta}_1$	$\ddot{x}_1$	$\ddot{y}_1$	$\ddot{\theta}_1$	x_2	y_2	θ_2	$\dot{x}_2$	$\dot{y}_2$	…
t_0															…
t_1															…
…															…

Dynamics of Mechanical Systems

Output

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- **.poi* – Time history of the positions, velocities, and accelerations of the points of interest
- **.jnt* – Time history of the joint reaction forces and moments for all joints

The *.poi file should contain 6NPointsOfInt columns:

	Poi 1						Poi 2				
t	x_1	y_1	$\dot{x}_1$	$\dot{y}_1$	$\ddot{x}_1$	$\ddot{y}_1$	x_2	y_2	$\dot{x}_2$	$\dot{y}_2$	…
t_0											…
t_1											…
…											…

Output

Once the simulation is finished, the post-processing function implemented in MuboDAP generates the following output files:

- **.out* – Time history of the positions, velocities, and accelerations of all bodies
- **.poi* – Time history of the positions, velocities, and accelerations of the points of interest
- **.jnt* – Time history of the joint reaction forces and moments for all joints

The *.jnt file should contain the forces and moments for each body involved in a joint:

	Jnt 1						Jnt 2				
	Body *i*			Body *j*			Body *i*			…	
t	f_x	f_y	n	f_x	f_y	n	f_x	f_y	n	f_x	…
t_0											…
t_1											…
…											…

<u>Exercise</u>:

For the sliding double-pendulum mechanism shown in the figure, simulate its dynamic response using MuboDAP considering the following data:

Body	Mass (kg)	Inertia (Kgm^2)
1	0.300	0.0500
2	0.200	0.0010
3	0.150	0.0008

	Spring-damper
l_0 (m)	0.08
k (N/m)	100
C (Ns/m)	5

$$\mathbf{r}^D = \begin{Bmatrix} 0.00 \\ 0.01 \end{Bmatrix}$$

$$\mathbf{s}_1'^A = \begin{Bmatrix} 0.00 \\ 0.03 \end{Bmatrix}$$

$$\mathbf{s}_2'^A = \begin{Bmatrix} -0.03 \\ 0.00 \end{Bmatrix}$$

$$\mathbf{s}_2'^B = \begin{Bmatrix} 0.03 \\ 0.00 \end{Bmatrix}$$

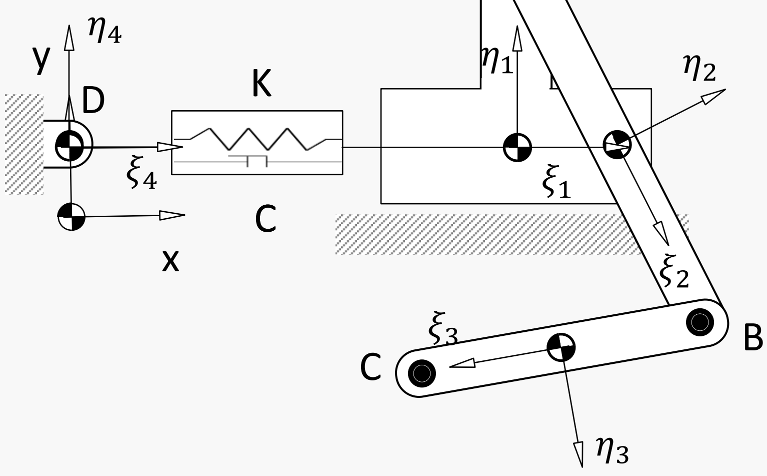
$$\mathbf{s}_3'^C = \begin{Bmatrix} 0.02 \\ 0.00 \end{Bmatrix}$$

$$\mathbf{s}_3'^B = \begin{Bmatrix} -0.02 \\ 0.00 \end{Bmatrix}$$

$$\mathbf{y}_0 = \begin{Bmatrix} 0.0700 \\ 0.0100 \\ 0.0000 \\ 0.0836 \\ 0.0133 \\ 5.1840 \\ 0.0776 \\ -0.0169 \\ 3.3160 \\ 0.0000 \\ 0.0100 \\ 0.0000 \\ \mathbf{0}_{12\times1} \end{Bmatrix}$$

$$l_{AB} = 0.06$$

$$l_{BC} = 0.04$$



Dynamics of Mechanical Systems

Exercise:

Input

NBody – 4

NRevolute –

NTranslation –

NRevRev –

NTraRev –

NCam –

NRigid –

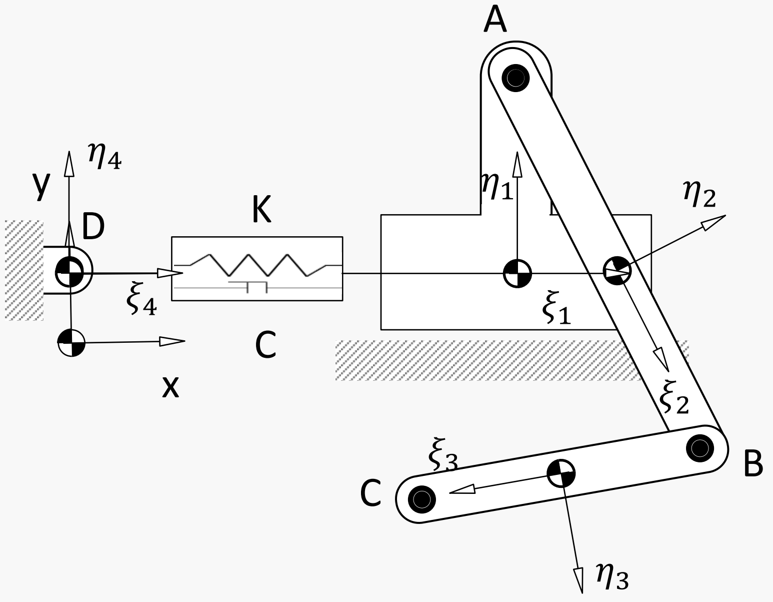
NSimple –

NDriver –

NPointsOfInt –

NForceAppl –

NSprDamp –



Exercise:

Input

NBody – 4

NRevolute – 2

NTranslation –

NRevRev –

NTraRev –

NCam –

NRigid –

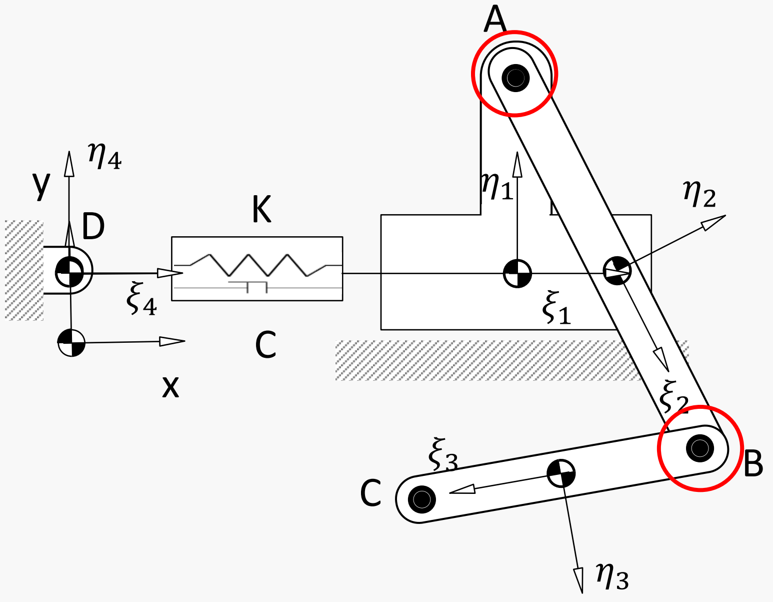
NSimple –

NDriver –

NPointsOfInt –

NForceAppl –

NSprDamp –



Exercise:

Input

NBody – 4

NRevolute – 2

NTranslation – 1

NRevRev –

NTraRev –

NCam –

NRigid –

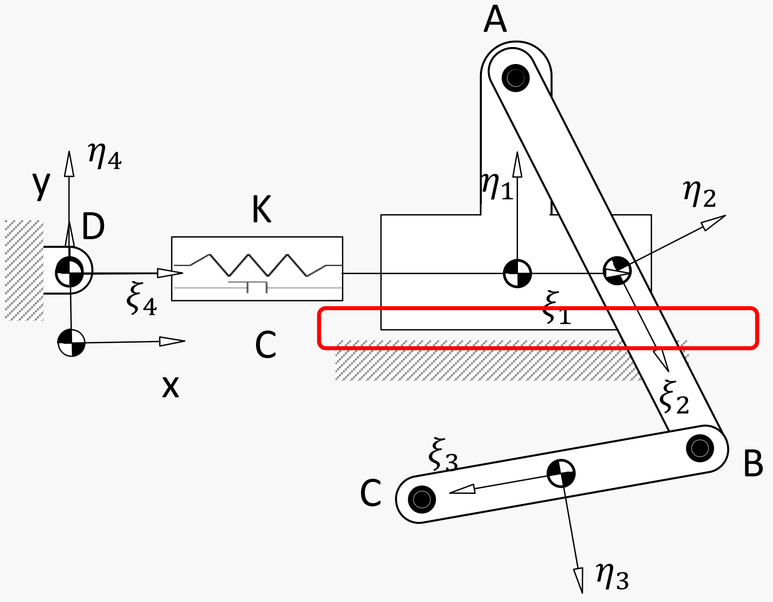
NSimple –

NDriver –

NPointsOfInt –

NForceAppl –

NSprDamp –



MuboDAP: Application case

Exercise:

Input

NBody – 4

NRevolute – 2

NTranslation – 1

NRevRev – 0

NTraRev – 0

NCam – 0

NRigid – 0

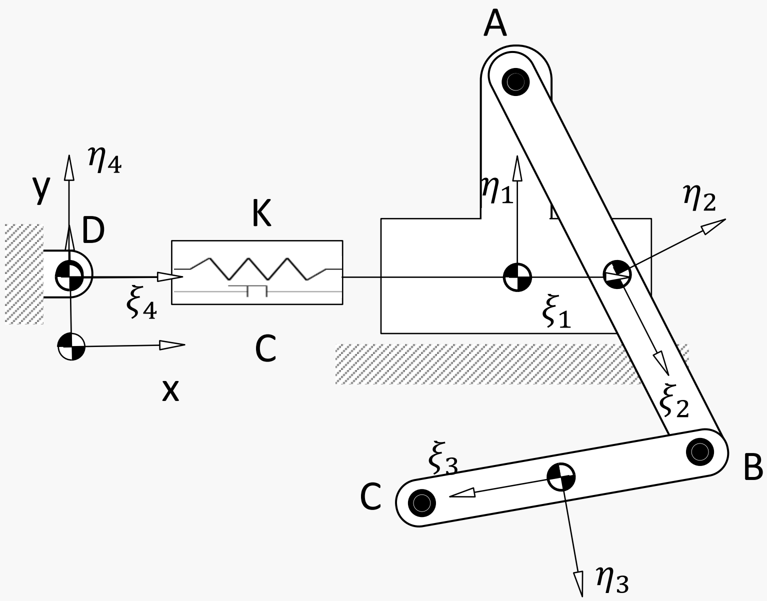
NSimple –

NDriver –

NPointsOfInt –

NForceAppl –

NSprDamp –



Dynamics of Mechanical Systems

Exercise:

Input

NBody – 4

NRevolute – 2

NTranslation – 1

NRevRev – 0

NTraRev – 0

NCam – 0

NRigid – 0

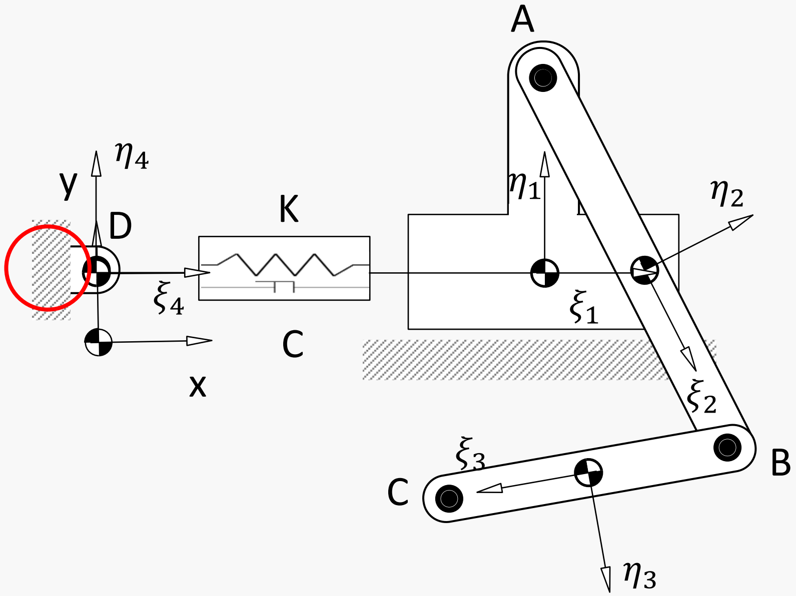
NSimple – 3

NDriver –

NPointsOfInt –

NForceAppl –

NSprDamp –



Dynamics of Mechanical Systems

MuboDAP: Application case

Exercise:

Input

NBody – 4

NRevolute – 2

NTranslation – 1

NRevRev – 0

NTraRev – 0

NCam – 0

NRigid – 0

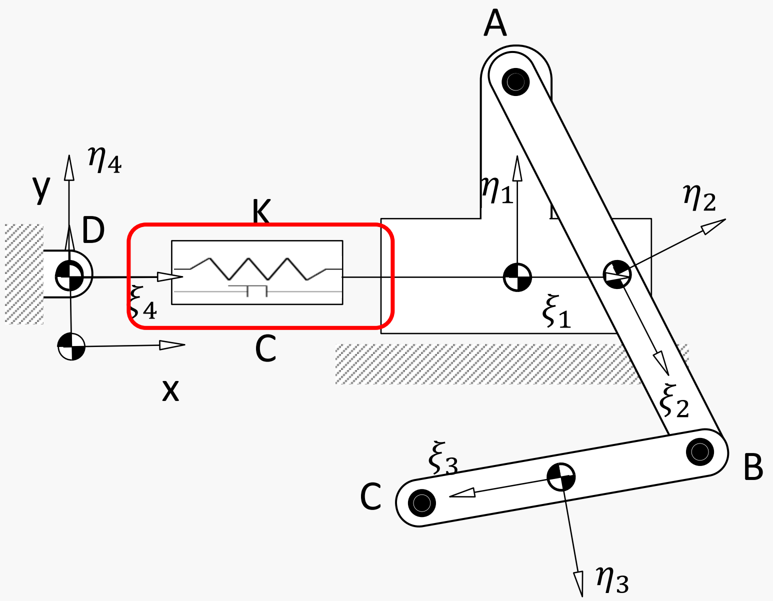
NSimple – 3

NDriver – 0

NPointsOfInt – 0

NForceAppl – 0

NSprDamp – 1



Dynamics of Mechanical Systems

Exercise:

Input

4 2 1 0 0 0 0 3 0 0 0 1

0.0700	0.0100	0.0000	0.0000	0.0000	0.0000	0.3000	0.0500
0.0836	0.0133	5.1840	0.0000	0.0000	0.0000	0.2000	0.0010
0.0776	-0.0169	3.3160	0.0000	0.0000	0.0000	0.1500	0.0008
0.0000	0.0100	0.0000	0.0000	0.0000	0.0000	1.0000	1.0000

$$\mathbf{y}_0 = \begin{Bmatrix} 0.0700 \\ 0.0100 \\ 0.0000 \\ 0.0836 \\ 0.0133 \\ 5.1840 \\ 0.0776 \\ -0.0169 \\ 3.3160 \\ 0.0000 \\ 0.0100 \\ 0.0000 \\ \mathbf{0}_{12\times 1} \end{Bmatrix}$$

Body	Mass (kg)	Inertia (Kgm^2)
1	0.300	0.0500
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MuboDAP: Application case

Exercise:

Input

4 2 1 0 0 0 0 3 0 0 0 1

0.0700	0.0100	0.0000	0.0000	0.0000	0.0000	0.3000	0.0500
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0.0776	-0.0169	3.3160	0.0000	0.0000	0.0000	0.1500	0.0008
0.0000	0.0100	0.0000	0.0000	0.0000	0.0000	1.0000	1.0000

1	2	0.0000	0.0300	-0.0300	0.0000				
2	3	0.0300	0.0000	-0.0200	0.0000				
1	4	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1.0000	0.0000

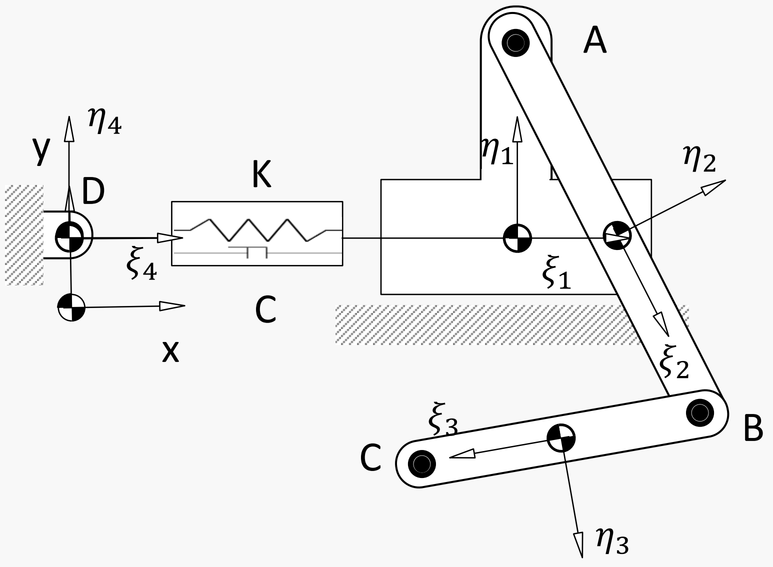
$$\mathbf{s}'^{A}_{1} = \begin{Bmatrix} 0.00 \\ 0.03 \end{Bmatrix}$$

$$\mathbf{s}'^{A}_{2} = \begin{Bmatrix} -0.03 \\ 0.00 \end{Bmatrix}$$

$$\mathbf{s}'^{B}_{2} = \begin{Bmatrix} 0.03 \\ 0.00 \end{Bmatrix}$$

$$\mathbf{s}'^{C}_{3} = \begin{Bmatrix} 0.02 \\ 0.00 \end{Bmatrix}$$

$$\mathbf{s}'^{B}_{3} = \begin{Bmatrix} -0.02 \\ 0.00 \end{Bmatrix}$$



Dynamics of Mechanical Systems

Exercise:

Input

4 2 1 0 0 0 0 3 0 0 0 1

…

4 1 0.0000

4 2 0.0100

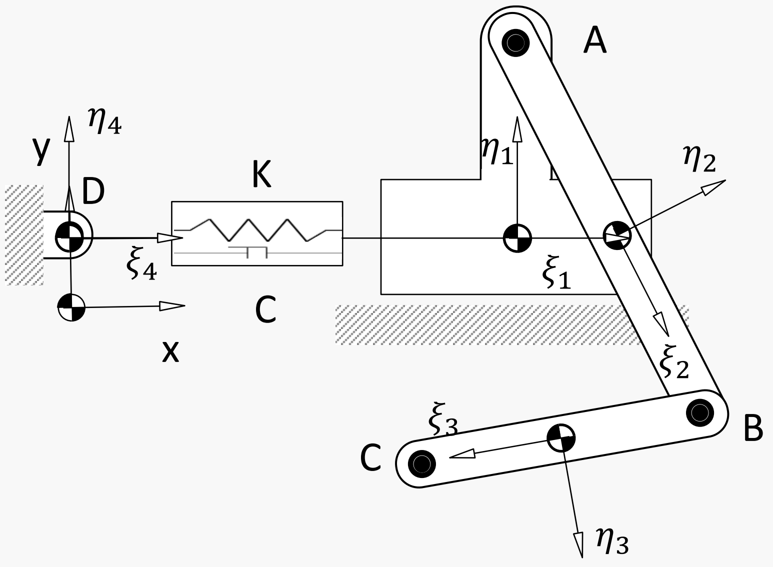
4 3 0.0000

4 1 0.0000 0.0000 0.0000 0.0000 100.00 0.0800 5.0000 0.0000

0.0000 -9.8000

8 1 5.0000 5.0000

0.0000 0.0100 1.2500 0.0100



MuboDAP: Application case

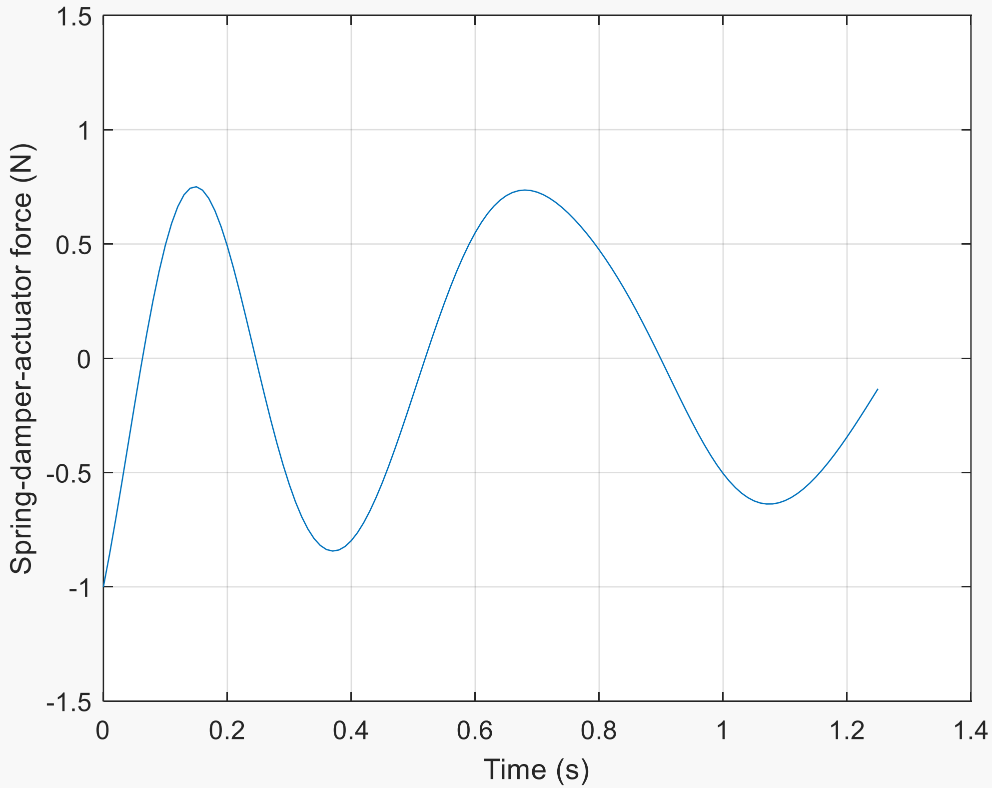
Exercise:

Input

```
4 2 1 0 0 0 0 3 0 0 0 1
 0.0700   0.0100   0.0000   0.0000   0.0000   0.0000   0.3000   0.0500
 0.0836   0.0133   5.1840   0.0000   0.0000   0.0000   0.2000   0.0010
 0.0776  -0.0169   3.3160   0.0000   0.0000   0.0000   0.1500   0.0008
 0.0000   0.0100   0.0000   0.0000   0.0000   0.0000   1.0000   1.0000
 1   2   0.0000   0.0300  -0.0300   0.0000
 2   3   0.0300   0.0000  -0.0200   0.0000
 1   4   0.0000   0.0000   1.0000   0.0000   0.0000   0.0000   1.0000   0.0000
 4   1   0.0000
 4   2   0.0100
 4   3   0.0000
 4   1   0.0000   0.0000   0.0000   0.0000   100.00   0.0800   5.0000   0.0000
 0.0000  -9.8000
 8   1   5.0000   5.0000
 0.0000   0.0100   1.2500   0.0100
```

Exercise:

For the sliding double-pendulum mechanism shown in the figure, simulate its dynamic response using MuboDAP considering the following data:



Dynamics of Mechanical Systems

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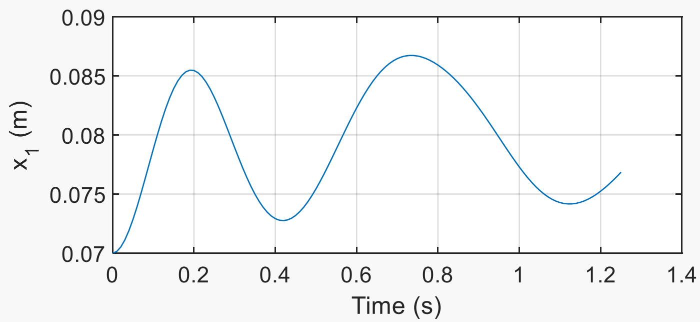
<u>Exercise</u>:

The spring-damper system is substituted by a time-varying actuator, whose properties are unknown, but the position x of body 1 throughout time is known. Considering this, simulate its dynamic response using MuboDAP and compute the actuator force.



Body	Mass (kg)	Inertia (Kgm²)
1	0.300	0.0500
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$$\mathbf{r}^D = \begin{Bmatrix} 0.00 \\ 0.01 \end{Bmatrix}$$

$$\mathbf{s}_1'^A = \begin{Bmatrix} 0.00 \\ 0.03 \end{Bmatrix}$$

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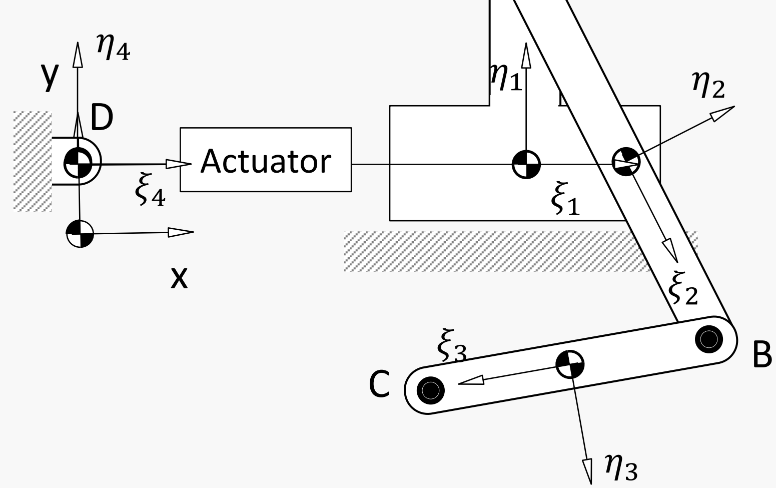
$$\mathbf{s}_3'^C = \begin{Bmatrix} 0.02 \\ 0.00 \end{Bmatrix}$$

$$\mathbf{s}_3'^B = \begin{Bmatrix} -0.02 \\ 0.00 \end{Bmatrix}$$

$$\mathbf{y}_0 = \begin{Bmatrix} 0.0700 \\ 0.0100 \\ 0.0000 \\ 0.0836 \\ 0.0133 \\ 5.1840 \\ 0.0776 \\ -0.0169 \\ 3.3160 \\ 0.0000 \\ 0.0100 \\ 0.0000 \\ \mathbf{0}_{12\times1} \end{Bmatrix}$$

$$l_{AB} = 0.06$$

$$l_{BC} = 0.04$$



MuboDAP: Application case

Exercise:

Input

4 2 1 0 0 0 0 3 1 0 0 0

0.0700	0.0100	0.0000	0.0000	0.0000	0.0000	0.3000	0.0500		
0.0836	0.0133	5.1840	0.0000	0.0000	0.0000	0.2000	0.0010		
0.0776	-0.0169	3.3160	0.0000	0.0000	0.0000	0.1500	0.0008		
0.0000	0.0100	0.0000	0.0000	0.0000	0.0000	1.0000	1.0000		
1	2	0.0000	0.0300	-0.0300	0.0000				
2	3	0.0300	0.0000	-0.0200	0.0000				
1	4	0.0000	0.0000	1.0000	0.0000	0.0000	0.0000	1.0000	0.0000
4	1	0.0000							
4	2	0.0100							
4	3	0.0000							
5	1	0.0000	4	0.0000	0.0000	0.000	0.000	4	1
0.0000	-9.8000								
8	1	5.0000	5.0000						
0.0000	0.0100	1.2500	0.0100						

Dynamics of Mechanical Systems

Note:

The ode outputs only time and the positions and velocities of the system throughout time:

```
%%... Integration of the equations of motion
[t, y]  = feval(solver,@FuncEval,tspan,y_init);
```

To obtain the accelerations and Lagrange multipliers, the equations of motion need to be solved again in the post-processing of the results.

Solve for $\ddot{\mathbf{q}}$ and λ

$$\boxed{\begin{bmatrix} \mathbf{M} & \mathbf{\Phi}_{\mathbf{q}}^{T} \\ \mathbf{\Phi}_{\mathbf{q}} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}} \\ \boldsymbol{\lambda} \end{Bmatrix} = \begin{Bmatrix} \mathbf{g} \\ \boldsymbol{\gamma} \end{Bmatrix}}$$

Exercise:

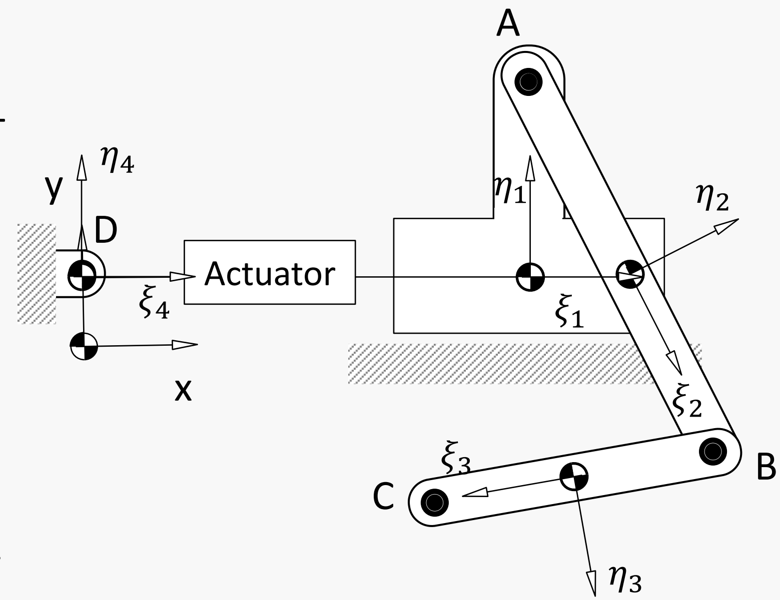
The spring-damper system is substituted by a time-varying actuator, whose properties are unknown, but the position x of body 1 throughout time is known. Considering this, simulate its dynamic response using MuboDAP and compute the actuator force.

If the actuator is modeled as a point2point driver, its force is the reaction force produced by the driver.

$$\mathbf{g}_{actuator} = -\mathbf{\Phi}_{\mathbf{q}}^{(dist,1)^{\mathrm{T}}} \lambda^{(dist,1)}$$

For the initial time:

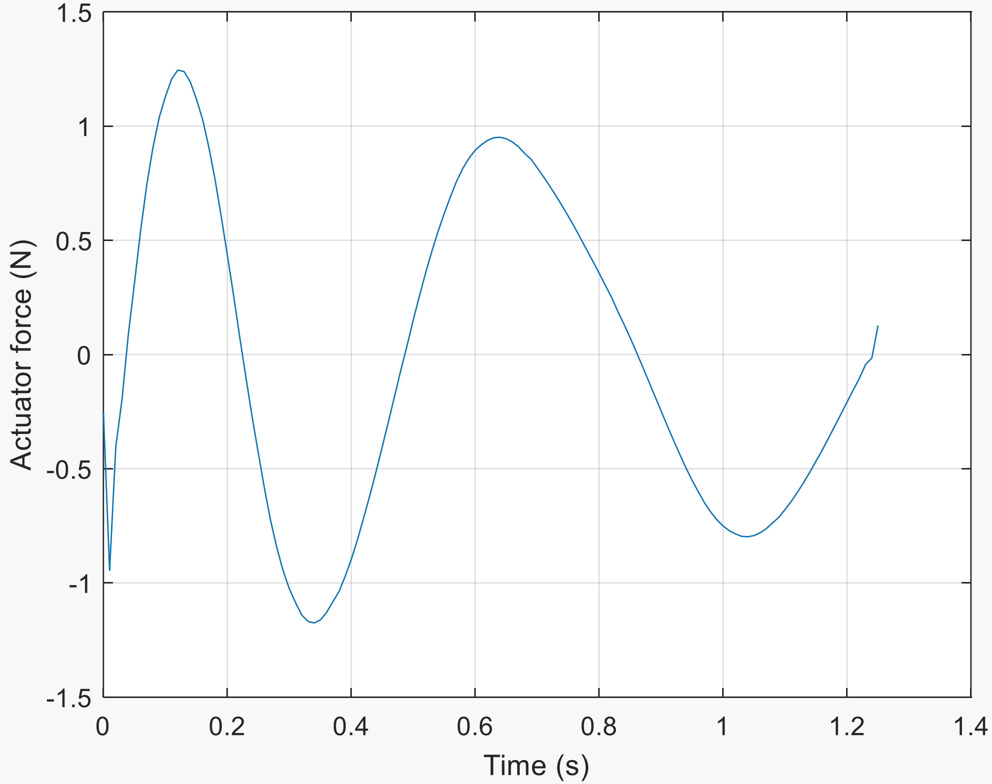
$$\mathbf{g}_{actuator}^{0} = \begin{array}{c} \left\{ \begin{array}{c} 0.2556 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ 0.0000 \\ -0.2556 \\ 0.0000 \\ 0.0000 \end{array} \right\} \end{array} \begin{array}{l} \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} \text{Body 1} \\ \\ \\ \\ \\ \\ \\ \left.\begin{array}{l} \\ \\ \\ \end{array}\right\} \text{Body 4} \end{array}$$



Dynamics of Mechanical Systems

Exercise:

The spring-damper system is substituted by a time-varying actuator, whose properties are unknown, but the position x of body 1 throughout time is known. Considering this, simulate its dynamic response using MuboDAP and compute the actuator force.



MuboDAP: Application case

Comparison between the two simulations: