Dynamics of Mechanical Systems

Summary

Exercises on the kinematic and dynamic analysis of mechanical systems using Cartesian Coordinates.

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Exam 2014 Part I

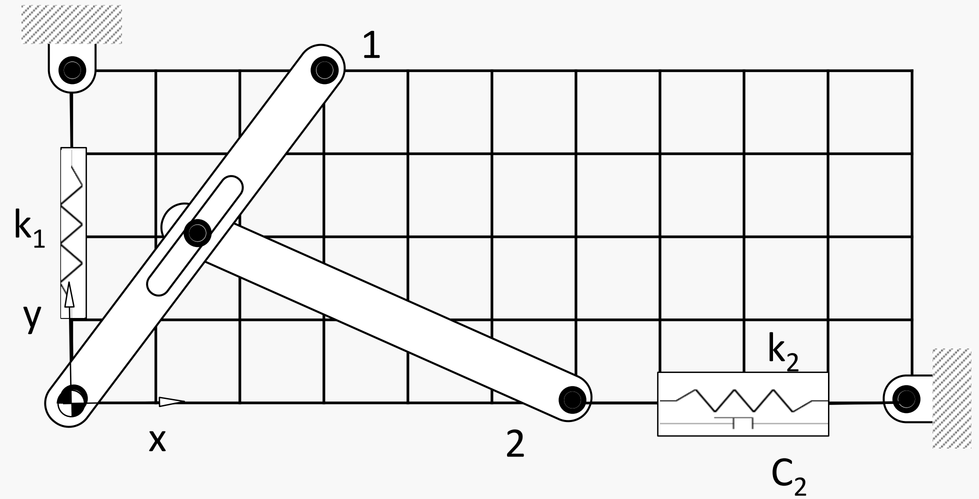
The two bodies shown in the figure are connected by a pin in a slot and are acted upon by springs and dampers. All data are supplied, though you are supposed to answer all questions without substituting the variables by their numerical value or performing any numerical calculations (except for f)).

$m_1 = 5kg$ $m_2 = 6kg$ $k_1 = 20N/m$ $k_2 = 30N/m$ $C_2 = 6Ns/m$

$J_1 = 10kgm^2$ $J_2 = 12.5kgm^2$ $l_1^0 = 5m$ $l_2^0 = 4.5m$

$l_{b1} = 5m$

$l_{b2} = 4.9m$



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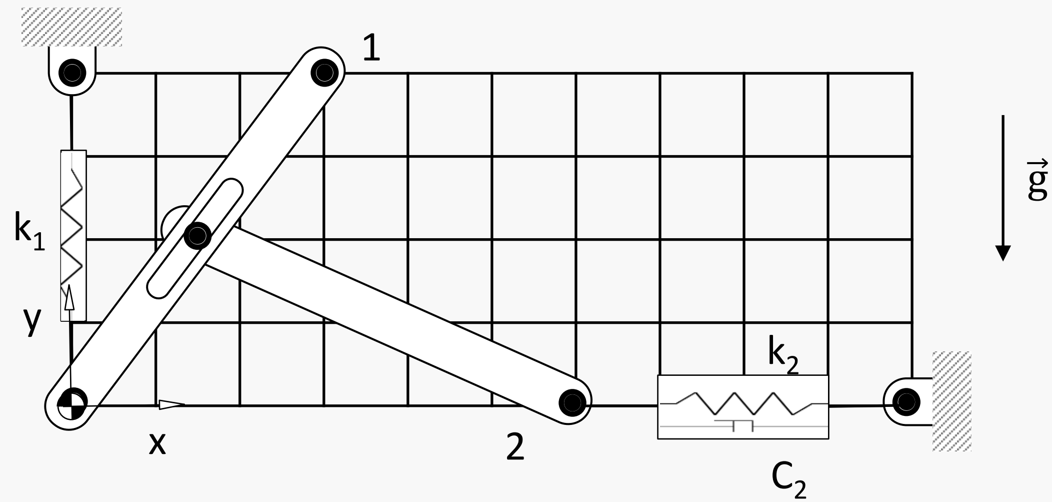

a) Identify all bodies, locate the body-fixed coordinate frames and identify the coordinates of all points required to formulate the kinematic joints (1 pt).

b) Write the equations of the constraints for the kinematic joints. For the kinematic constraints identified, formulate the Jacobian matrix and the right-hand side vectors of the velocity and acceleration equations (2 pts).

c) Write the equations of motion of the system, identifying, in the process, the force vectors (1 pt).

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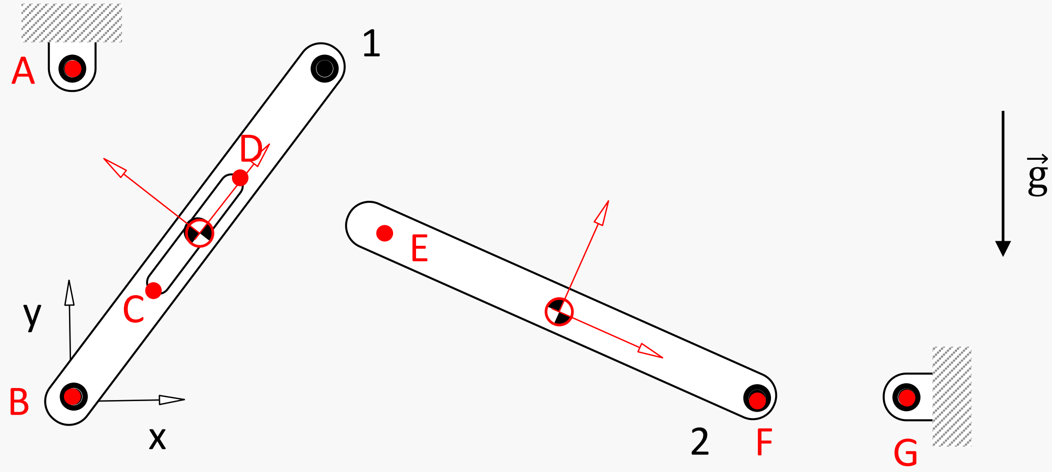

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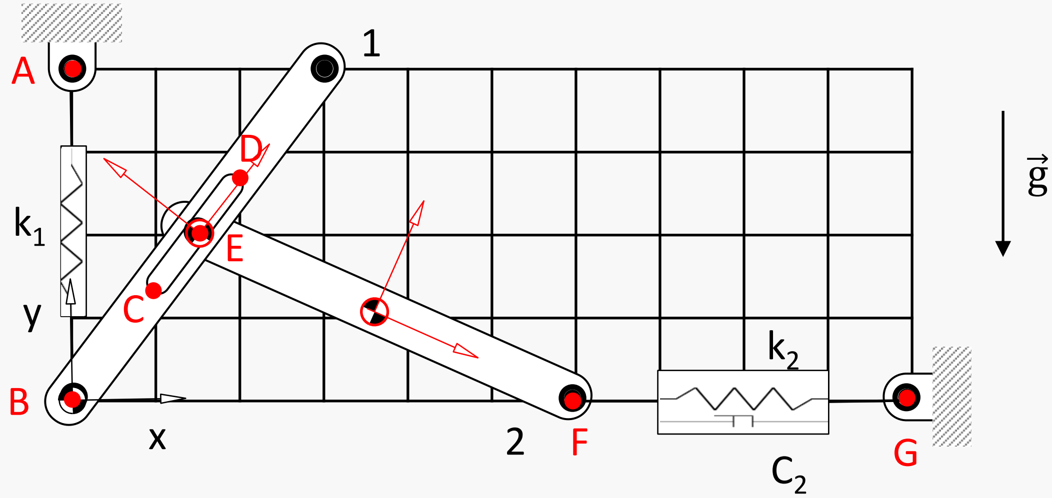

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Exam 2014 Part I

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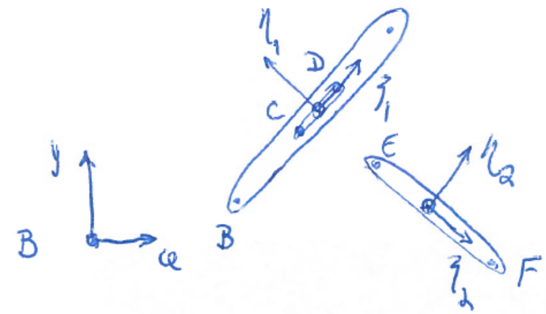

$n_b = 2$

$n_{tra-rev} = 1$

$n_{dof} = 6 - 1 = 5$

$$\underline{q} = \{ 1.5,\ 2.0,\ 0.9250,\ 3.75,\ 1.0,\ 5.8643 \}^T$$

$$\underline{r}^A = \begin{Bmatrix} 0 \\ 4 \end{Bmatrix} \qquad \underline{r}^G = \begin{Bmatrix} 10 \\ 0 \end{Bmatrix}$$

$$\underline{s}_1'^B = \begin{Bmatrix} -2.5 \\ 0.0 \end{Bmatrix} \qquad \underline{s}_1'^C = \begin{Bmatrix} -1.0 \\ 0.0 \end{Bmatrix} \qquad \underline{s}_1'^D = \begin{Bmatrix} 1.0 \\ 0.0 \end{Bmatrix}$$

$$\underline{s}_2'^E = \begin{Bmatrix} -2.45 \\ 0.00 \end{Bmatrix} \qquad \underline{s}_2'^F = \begin{Bmatrix} 2.45 \\ 0.00 \end{Bmatrix}$$

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b) Write the equations of the constraints for the kinematic joints. For the kinematic constraints identified, formulate the Jacobian matrix and the right-hand side vectors of the velocity and acceleration equations (2 pts).

b) Vectors for the formulation of the translation-revolute joint

- Translation axis in body 1 $\Rightarrow$ $\underline{s}_1 = \underline{r}_1^D - \underline{r}_1^C = \underline{A}_1 (\underline{s}_1'^D - \underline{s}_1'^C)$

- Vector perpendicular to the translation vector $\Rightarrow$ $\underline{h}_1 = \underline{A}_{90} \cdot \underline{s}_1$, where $\underline{A}_{90} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

- Vector from a point in translation axis in body 1 to point in translation axis in body 2 $\Rightarrow$ $\underline{d} = \underline{r}_1^D - \underline{r}_2^E = \underline{r}_1 + \underline{A}_1 \underline{s}_1'^D - \underline{r}_2 - \underline{A}_2 \underline{s}_2'^E$

The constraint for the translation-revolute joint is given by

$$\underline{\Phi}^{(t-r,1)} = \underline{d}^T \cdot \underline{h}_1 = 0$$

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b) Write the equations of the constraints for the kinematic joints. For the kinematic constraints identified, formulate the Jacobian matrix and the right-hand side vectors of the velocity and acceleration equations (2 pts).

The velocity equations are obtained from the first derivative of the constraints with respect to time:

$$\dot{\underline{\Phi}} = 0 \Rightarrow \underline{d}^T \cdot \underline{h}_1 + \underline{h}_1^T \cdot \dot{\underline{d}} = 0 \Leftrightarrow$$

$$\Leftrightarrow \underline{d}^T \left(\underline{A}_{90} \cdot \underline{B}_1 \left(\underline{s}_1'^{D} - \underline{s}_1'^{C} \right) \dot{\theta}_1 \right) + \underline{h}_1^T \left(\dot{\underline{r}}_1 + \underline{B}_1 \underline{s}_1'^{D} \dot{\theta}_1 - \dot{\underline{r}}_2 - \underline{B}_2 \underline{s}_2'^{E} \dot{\theta}_2 \right) = 0 \Leftrightarrow$$

$$\Leftrightarrow \underbrace{\begin{bmatrix} \underline{h}_1^T & \underline{d}^T \underline{A}_{90} \underline{B}_1 \left(\underline{s}_1'^{D} - \underline{s}_1'^{C} \right) + \underline{h}_1^T \underline{B}_1 \underline{s}_1'^{D} & -\underline{h}_1^T & -\underline{h}_1^T \underline{B}_2 \underline{s}_2'^{E} \end{bmatrix}}_{\underline{\underline{\Phi}}_q} \begin{Bmatrix} \dot{\underline{r}}_1 \\ \dot{\theta}_1 \\ \dot{\underline{r}}_2 \\ \dot{\theta}_2 \end{Bmatrix} = \underbrace{0}_{\underline{\nu}}$$

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b) Write the equations of the constraints for the kinematic joints. For the kinematic constraints identified, formulate the Jacobian matrix and the right-hand side vectors of the velocity and acceleration equations (2 pts).

The acceleration equations are:

$$\ddot{\mathbf{\Phi}} = 0$$

$$\Rightarrow \mathbf{d}^T \ddot{\mathbf{h}}_1 + \mathbf{h}_1^T \ddot{\mathbf{d}} + \dot{\mathbf{h}}_1^T \dot{\mathbf{d}} + \dot{\mathbf{d}}^T \dot{\mathbf{h}}_1 = 0 \Rightarrow$$

$$\Rightarrow \mathbf{d}^T \ddot{\mathbf{h}}_1 + \mathbf{h}_1^T \ddot{\mathbf{d}} = -2\dot{\mathbf{d}}^T \dot{\mathbf{h}}_1 \Rightarrow$$

$$\Rightarrow \mathbf{d}^T \left[-\mathbf{A}_{90} \mathbf{A}_1 (\mathbf{s}_1'^D - \mathbf{s}_1'^C) \dot{\theta}_1^2 + \mathbf{A}_{90} \mathbf{B}_1 (\mathbf{s}_1'^D - \mathbf{s}_1'^C) \ddot{\theta}_1 \right] + \mathbf{h}_1^T \left[\ddot{\mathbf{r}}_1 - \mathbf{A}_1 \mathbf{s}_1'^D \dot{\theta}_1^2 + \mathbf{B}_1 \mathbf{s}_1'^D \ddot{\theta}_1 - \ddot{\mathbf{r}}_2 + \mathbf{A}_2 \mathbf{s}_2'^E \dot{\theta}_2^2 - \mathbf{B}_2 \mathbf{s}_2'^E \ddot{\theta}_2 \right] = -2 \dot{\mathbf{d}}^T \dot{\mathbf{h}}_1 \Rightarrow$$

$$\Leftrightarrow \begin{bmatrix} \mathbf{h}_1^T & \mathbf{d}^T \mathbf{A}_{90} \mathbf{B}_1 (\mathbf{s}_1'^D - \mathbf{s}_1'^C) + \mathbf{h}_1^T \mathbf{B}_1 \mathbf{s}_1'^D & -\mathbf{h}_1^T & -\mathbf{h}_1^T \mathbf{B}_2 \mathbf{s}_2'^E \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{r}}_1 \\ \ddot{\theta}_1 \\ \ddot{\mathbf{r}}_2 \\ \ddot{\theta}_2 \end{Bmatrix} = \boldsymbol{\gamma},$$

where $\boldsymbol{\gamma} = -2 \dot{\mathbf{d}}^T \dot{\mathbf{h}}_1 + \mathbf{d}^T \mathbf{A}_{90} \mathbf{A}_1 (\mathbf{s}_1'^D - \mathbf{s}_1'^C) \dot{\theta}_1^2 + \mathbf{h}_1^T (\mathbf{A}_1 \mathbf{s}_1'^D \dot{\theta}_1^2 - \mathbf{A}_2 \mathbf{s}_2'^E \dot{\theta}_2^2)$

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c) Write the equations of motion of the system, identifying, in the process, the force vectors (1 pt).

c) The equations of motion for the system are:

$$\begin{bmatrix} \underline{M} & \underline{\Phi}_{\underline{q}}^T \\ \underline{\Phi}_{\underline{q}} & \underline{0} \end{bmatrix} \begin{Bmatrix} \ddot{\underline{q}} \\ \underline{\lambda} \end{Bmatrix} = \begin{Bmatrix} \underline{g} \\ \underline{\gamma} \end{Bmatrix}$$

The Jacobian matrix $\underline{\Phi}_{\underline{q}}$ and $\underline{\gamma}$ are known. they were computed in b).

The mass matrix is given by:

$$\underline{M} = \begin{bmatrix} m_1\underline{I} & 0 & 0 & 0 \\ 0 & J_1 & 0 & 0 \\ 0 & 0 & m_2\underline{I} & 0 \\ 0 & 0 & 0 & J_2 \end{bmatrix}$$

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c) Write the equations of motion of the system, identifying, in the process, the force vectors (1 pt).

The force vector for each body includes the gravitational force and the spring-damper forces:

$$\mathbf{g}_i = \mathbf{g}_{grav_i} + \mathbf{g}_{sda_i} \quad , \quad \text{where} \quad \mathbf{g}_{grav_i} = \begin{Bmatrix} m_i a_g \\ 0 \end{Bmatrix}$$

For a generic spring damper system

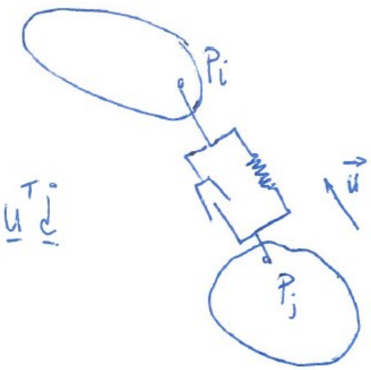
$$\mathbf{d} = \mathbf{r}_i + \mathbf{A}_i \mathbf{s}_i'^{P_i} - \mathbf{r}_j - \mathbf{A}_j \mathbf{s}_j'^{P_j}$$

$$\mathbf{u} = \frac{\mathbf{d}}{l}$$

$$l = (\mathbf{d}^T \mathbf{d})^{1/2} \qquad \dot{l} = (\mathbf{d}^T \mathbf{d})^{-1/2} \cdot (\mathbf{d}^T \dot{\mathbf{d}}) = \mathbf{u}^T \dot{\mathbf{d}}$$

$$\Downarrow$$

$$\mathbf{f}_{sda_i} = k\,(l - l^0)\,\mathbf{u} + c\,\dot{l}\,\mathbf{u}$$



$\vec{u}$ → unit vector defining the direction of the force

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c) Write the equations of motion of the system, identifying, in the process, the force vectors (1 pt).

For the spring system 1

$$\underline{f}_{sda_1} = k_1 \left(l_1 - l_1^0 \right) \underline{u}_1 \quad , \quad \text{where} \quad \underline{d}_1 = \underline{r}^A - \underline{r}_1 - \underline{A}_1 \underline{s}_1'^{B}$$

$$l_1 = \left(\underline{d}_1^T \underline{d}_1 \right)^{1/2}$$

$$\underline{u}_1 = \frac{\underline{d}_1}{l_1}$$

The force vector of body 1 is $\underline{g}_1 = \begin{Bmatrix} m_1 a g + \underline{f}_{sda_1} \\ \underline{n}_{sda_1} \end{Bmatrix}$. Since the force of the spring is not acting on the center of mass, the equivalent system of the center of mass is the force itself plus a transport moment. This transport moment is given by

$$\underline{n}_{sda_1} = \underline{s}_1^B \times \underline{f}_{sda_1} =$$

$$= \underline{f}_{sda_1}^T \underline{A}_{90} \cdot \underline{s}_1^B$$

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c) Write the equations of motion of the system, identifying, in the process, the force vectors (1 pt).

For the spring - damper system 2

$$\underline{f}_{sd_2} = K_2 (l_2 - l_2^0)\underline{u}_2 + C_2 \dot{l}_2 \underline{u}_2 , \quad \text{where} \quad \underline{d}_2 = \underline{r}_2 + \underline{A}_2 \underline{s}_2'^F - \underline{r}^G$$

$$l_2 = (\underline{d}_2^T \cdot \underline{d}_2)^{1/2}$$

$$\dot{l}_2 = \underline{u}_2^T \cdot \dot{\underline{d}}_2$$

$$\dot{\underline{d}}_2 = \dot{\underline{r}}_2 + \underline{B}_2 \underline{s}_2'^F \dot{\theta}_2$$

$$\underline{u}_2 = \frac{\underline{d}_2}{l_2}$$

The force vector of body 2 is

$$\underline{g}_2 = \begin{Bmatrix} m_2 \underline{g} - \underline{f}_{sd_2} \\ -\underline{f}_{sd_2}^T \cdot \underline{A}_{90} \cdot \underline{s}_2^F \end{Bmatrix}$$

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c) Write the equations of motion of the system, identifying, in the process, the force vectors (1 pt).

The vector of externally applied forces is given by:

$$\underline{g} = \begin{Bmatrix} m_1 \underline{a}_g + \underline{f}_{sda_1} \\ \underline{n}_{sda_1} \\ m_2 \underline{g} - \underline{f}_{sda_2} \\ -\underline{f}_{sda_2}^{T} \cdot \underline{A}_{SB} \cdot \underline{s}_2^{F} \end{Bmatrix}$$

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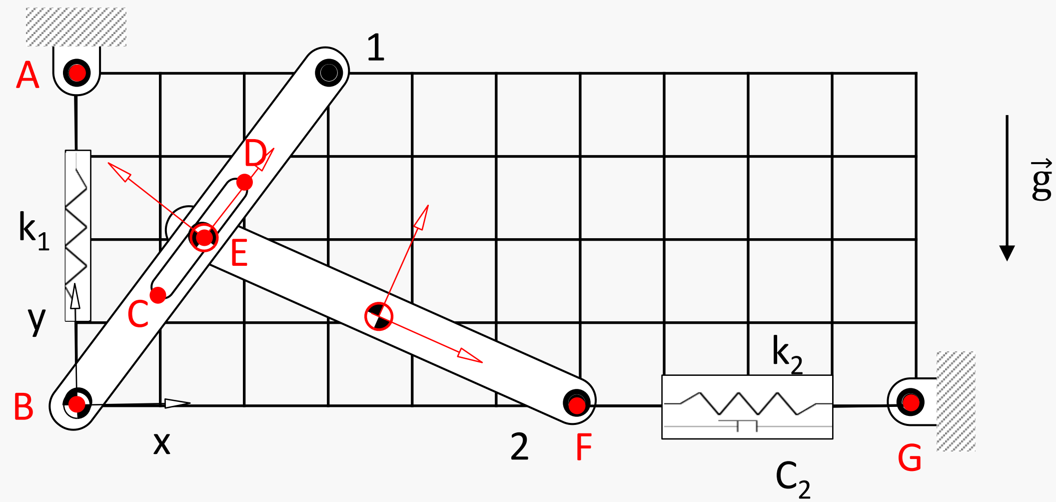

$$\mathbf{q} = \begin{Bmatrix} 1.5000 \\ 2.0000 \\ 0.9250 \\ 3.7500 \\ 1.0000 \\ 5.8643 \end{Bmatrix}$$

d) Show how to calculate the accelerations and the joint reaction forces (1 pt).

e) Substitute the spring and damper between body 2 and ground by a massless link (l = 4m). Write the equations of motion of the system (2 pts).

f) Provided that you can measure the initial positions in the figure, and if the initial velocities of the system, formulated in b), are $\dot{x}_1 = 0.5\, m/s$, $\dot{y}_1 = -0.2\, m/s$, $\dot{x}_2 = 0.5\, m/s$, $\dot{y}_2 = -0.2\, m/s$, $\dot{\theta}_1 = -0.2\, rad/s$, and $\dot{\theta}_2 = -0.1\, rad/s$, show if the velocity constraint equations of the system are satisfied (2 pts).

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d) Show how to calculate the accelerations and the joint reaction forces (1 pt).

d)
$$\begin{cases} \mathbf{M}\ddot{\mathbf{q}} + \boldsymbol{\Phi}_{\mathbf{q}}^T \boldsymbol{\lambda} = \mathbf{g} \\ \boldsymbol{\Phi}_{\mathbf{q}}\ddot{\mathbf{q}} = \boldsymbol{\gamma} \end{cases} \Rightarrow \begin{cases} \mathbf{M}\ddot{\mathbf{q}} = \mathbf{g} - \boldsymbol{\Phi}_{\mathbf{q}}^T \boldsymbol{\lambda} \\ \sim \end{cases} \Rightarrow \begin{cases} \ddot{\mathbf{q}} = \mathbf{M}^{-1}(\mathbf{g} - \boldsymbol{\Phi}_{\mathbf{q}}^T \boldsymbol{\lambda}) \\ \boldsymbol{\Phi}_{\mathbf{q}}\left[\mathbf{M}^{-1}(\mathbf{g} - \boldsymbol{\Phi}_{\mathbf{q}}^T \boldsymbol{\lambda})\right] = \boldsymbol{\gamma} \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} \sim \\ \boldsymbol{\Phi}_{\mathbf{q}}\mathbf{M}^{-1}\mathbf{g} - \boldsymbol{\Phi}_{\mathbf{q}}\mathbf{M}^{-1}\boldsymbol{\Phi}_{\mathbf{q}}^T \boldsymbol{\lambda} = \boldsymbol{\gamma} \end{cases} \Rightarrow \begin{cases} \sim \\ -\boldsymbol{\Phi}_{\mathbf{q}}\mathbf{M}^{-1}\boldsymbol{\Phi}_{\mathbf{q}}^T \boldsymbol{\lambda} = \boldsymbol{\gamma} - \boldsymbol{\Phi}_{\mathbf{q}}\mathbf{M}^{-1}\mathbf{g} \end{cases} \Rightarrow$$

$$\begin{cases} \ddot{\mathbf{q}} = \mathbf{M}^{-1}(\mathbf{g} - \boldsymbol{\Phi}_{\mathbf{q}}^T \boldsymbol{\lambda}) \\ \boldsymbol{\lambda} = \left(\boldsymbol{\Phi}_{\mathbf{q}}\mathbf{M}^{-1}\boldsymbol{\Phi}_{\mathbf{q}}^T\right)^{-1}\left(\boldsymbol{\Phi}_{\mathbf{q}}\mathbf{M}^{-1}\mathbf{g} - \boldsymbol{\gamma}\right) \end{cases}$$

Once $\ddot{\mathbf{q}}$ and $\boldsymbol{\lambda}$ are known, the joint reaction force can be computed by

$$\mathbf{g}_{int} = -\boldsymbol{\Phi}_{\mathbf{q}}^T \boldsymbol{\lambda}$$

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e) Substitute the spring and damper between body 2 and ground by a massless link (l = 4m). Write the equations of motion of the system (2 pts).

e) $\underline{\Phi}^{(rev-rev,1)} = \underline{d}_2^T \underline{d}_2 - l^2 = 0$, where $\underline{d}_2 = \underline{r}^G - \underline{r}_2 - \underline{A}_2 \underline{s}_2'^F$

The velocity equations are given by

$$\dot{\underline{\Phi}} = 0 \Rightarrow 2 \underline{d}_2^T \dot{\underline{d}}_2 = 0 \Leftrightarrow$$

$$\Leftrightarrow 2 \underline{d}_2^T \left(-\dot{\underline{r}}_2 - \underline{B}_2 \underline{s}_2'^F \dot{\theta}_2 \right) = 0 \Leftrightarrow$$

$$\Leftrightarrow \underbrace{\begin{bmatrix} \underline{0} & 0 & -2\underline{d}_2^T & -2\underline{d}_2^T \underline{B}_2 \underline{s}_2'^F \end{bmatrix}}_{\underline{\Phi}_q^{(rev-rev,1)}} \begin{Bmatrix} \dot{\underline{r}}_1 \\ \dot{\theta}_1 \\ \dot{\underline{r}}_2 \\ \dot{\theta}_2 \end{Bmatrix} = \underbrace{0}_{\underline{\nu}^{(rev-rev,1)}}$$

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Exam 2014 Part I

e) Substitute the spring and damper between body 2 and ground by a massless link (l = 4m). Write the equations of motion of the system (2 pts).

The acceleration equations are given by

$$\ddot{\underline{\Phi}} = 0 \Rightarrow 2\,\dot{\underline{d}}_2^T \dot{\underline{d}}_2 + 2\,\underline{d}_2^T \ddot{\underline{d}}_2 = 0 \Rightarrow$$

$$\Rightarrow 2\,\underline{d}_2^T \left(-\ddot{\underline{r}}_2 - \underline{B}_2 \underline{s}_2'^F \ddot{\theta}_2 + \underline{A}_2 \underline{s}_2'^F \dot{\theta}_2^2 \right) = -2\,\dot{\underline{d}}_2^T \dot{\underline{d}}_2 \Rightarrow$$

$$\Rightarrow \begin{bmatrix} \underline{0} & 0 & -2\underline{d}_2^T & -2\underline{d}_2^T \underline{B}_2 \underline{s}_2'^F \end{bmatrix} \begin{Bmatrix} \ddot{\underline{r}}_1 \\ \ddot{\theta}_1 \\ \ddot{\underline{r}}_2 \\ \ddot{\theta}_2 \end{Bmatrix} = \underbrace{-2\,\dot{\underline{d}}_2^T \dot{\underline{d}}_2 - 2\,\underline{d}_2^T \underline{A}_2 \underline{s}_2'^F \dot{\theta}_2^2}_{\underline{\gamma}^{(nh-nc,\,1)}}$$

Exam 2014 Part I

e) Substitute the spring and damper between body 2 and ground by a massless link (l = 4m). Write the equations of motion of the system (2 pts).

The equations of motion are

$$\begin{bmatrix} \underline{M} & \underline{\Phi}_q^T \\ \underline{\Phi}_q & \underline{0} \end{bmatrix} \begin{Bmatrix} \ddot{\underline{q}} \\ \lambda \end{Bmatrix} = \begin{Bmatrix} \underline{g} \\ \underline{\gamma} \end{Bmatrix}, \quad \text{where}$$

$$\underline{\Phi}_q = \begin{bmatrix} \underline{h}_1^T & \underline{d}^T \underline{A}_{90} (\underline{s}_1'^D - \underline{s}_1'^C) + \underline{h}_1^T \underline{B}_1 \underline{s}_1'^D & -\underline{h}_1^T & -\underline{h}_1^T \underline{B}_2 \underline{s}_2'^E \\ \underline{0} & 0 & -2\underline{d}_2^T & -2\underline{d}_2^T \underline{B}_2 \underline{s}_2'^F \end{bmatrix}$$

$$\underline{\gamma} = \begin{Bmatrix} -2\dot{\underline{d}}^T \dot{\underline{h}}_1 + \underline{d}^T \underline{A}_{90} \underline{A}_1 (\underline{s}_1'^D - \underline{s}_1'^C) \dot{\theta}_1^2 + \underline{h}_1^T (\underline{A}_1 \underline{s}_1'^D \dot{\theta}_1^2 - \underline{A}_2 \underline{s}_2'^E \dot{\theta}_2^2) \\ -2\dot{\underline{d}}_2^T \dot{\underline{d}}_2 - 2\underline{d}_2^T \underline{A}_2 \underline{s}_2'^F \dot{\theta}_2^2 \end{Bmatrix}$$

$$\underline{g} = \begin{Bmatrix} m_1 \underline{a}_g + \underline{f}_{sda_1} \\ \underline{f}_{sda_1}^T \underline{A}_{90} \underline{s}_1^B \\ \underline{0} \\ 0 \end{Bmatrix}$$

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f) Provided that you can measure the initial positions in the figure, and if the initial velocities of the system, formulated in b), are $\dot{x}_1 = 0.5\, m/s$, $\dot{y}_1 = -0.2\, m/s$, $\dot{x}_2 = 0.5\, m/s$, $\dot{y}_2 = -0.2\, m/s$, $\dot{\theta}_1 = -0.2\, rad/s$, and $\dot{\theta}_2 = -0.1\, rad/s$, show if the velocity constraint equations of the system are satisfied (2 pts).

f) $\underline{\underline{\Phi}}_q \, \underline{\dot{q}} = \underline{\nu}$

$$\Rightarrow \begin{bmatrix} \underline{h}_1^T & \underline{d}^T \underline{A}_{90} \underline{B}_1 (\underline{s}_1'^D - \underline{s}_1'^C) + \underline{h}_1^T \underline{B}_1 \underline{s}_1'^D & -\underline{h}_1^T & -\underline{h}_1^T \underline{B}_2 \underline{s}_2'^E \\ \underline{0} & 0 & -2\underline{d}_2^T & -2\underline{d}_2^T \underline{B}_2 \underline{s}_2'^F \end{bmatrix} \begin{Bmatrix} \dot{\underline{r}}_1 \\ \dot{\theta}_1 \\ \dot{\underline{r}}_2 \\ \dot{\theta}_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\underline{h}_1 = \begin{Bmatrix} -1.5972 \\ 1.2037 \end{Bmatrix} \qquad \underline{d}^T = \begin{Bmatrix} 0.5900 \\ 0.8021 \end{Bmatrix} \qquad \underline{B}_1 = \begin{bmatrix} -0.7986 & -0.6018 \\ 0.6018 & -0.7986 \end{bmatrix}$$

$$\underline{B}_2 = \begin{bmatrix} 0.4067 & -0.9135 \\ 0.9135 & 0.4067 \end{bmatrix} \qquad \underline{d}_2 = \begin{Bmatrix} 4.0118 \\ -0.0035 \end{Bmatrix}$$

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f) Provided that you can measure the initial positions in the figure, and if the initial velocities of the system, formulated in b), are $\dot{x}_1 = 0.5\,m/s$, $\dot{y}_1 = -0.2\,m/s$, $\dot{x}_2 = 0.5\,m/s$, $\dot{y}_2 = -0.2\,m/s$, $\dot{\theta}_1 = -0.2\,rad/s$, and $\dot{\theta}_2 = -0.1\,rad/s$, show if the velocity constraint equations of the system are satisfied (2 pts).

$$\Rightarrow \begin{bmatrix} -1.5972 & 1.2037 & 0.0087 & 1.5972 & -1.2037 & 1.1023 \\ 0 & 0 & 0 & -8.0236 & 0.0070 & -7.9801 \end{bmatrix} \begin{Bmatrix} 0.5 \\ -0.2 \\ -0.2 \\ 0.5 \\ -0.2 \\ -0.1 \end{Bmatrix} \stackrel{?}{=} \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} -0.1120 \\ -3.2152 \end{Bmatrix} \neq \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

The velocity constraint equations are not satisfied.