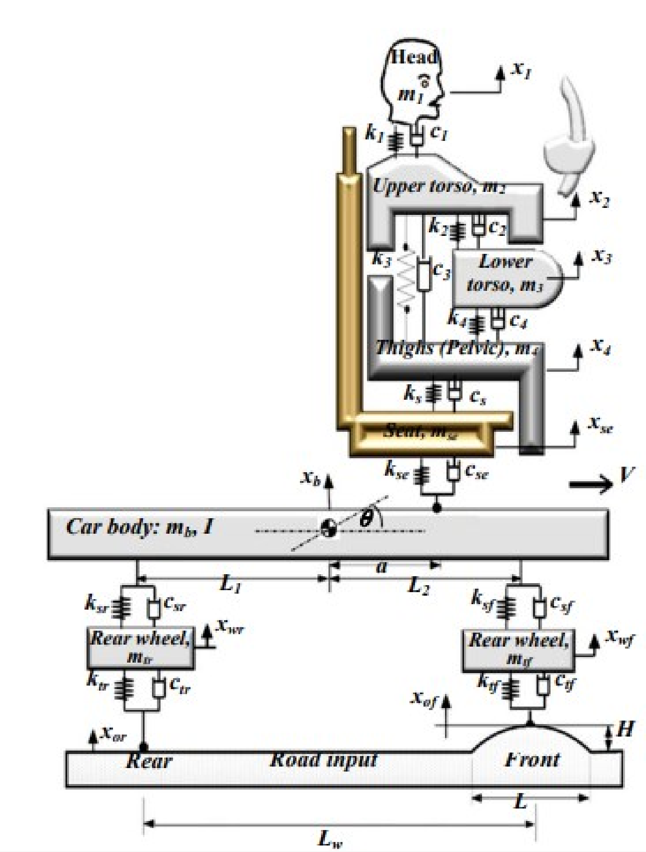
Dynamics of Mechanical Systems

2023/24

Summary

First dynamic analysis program.

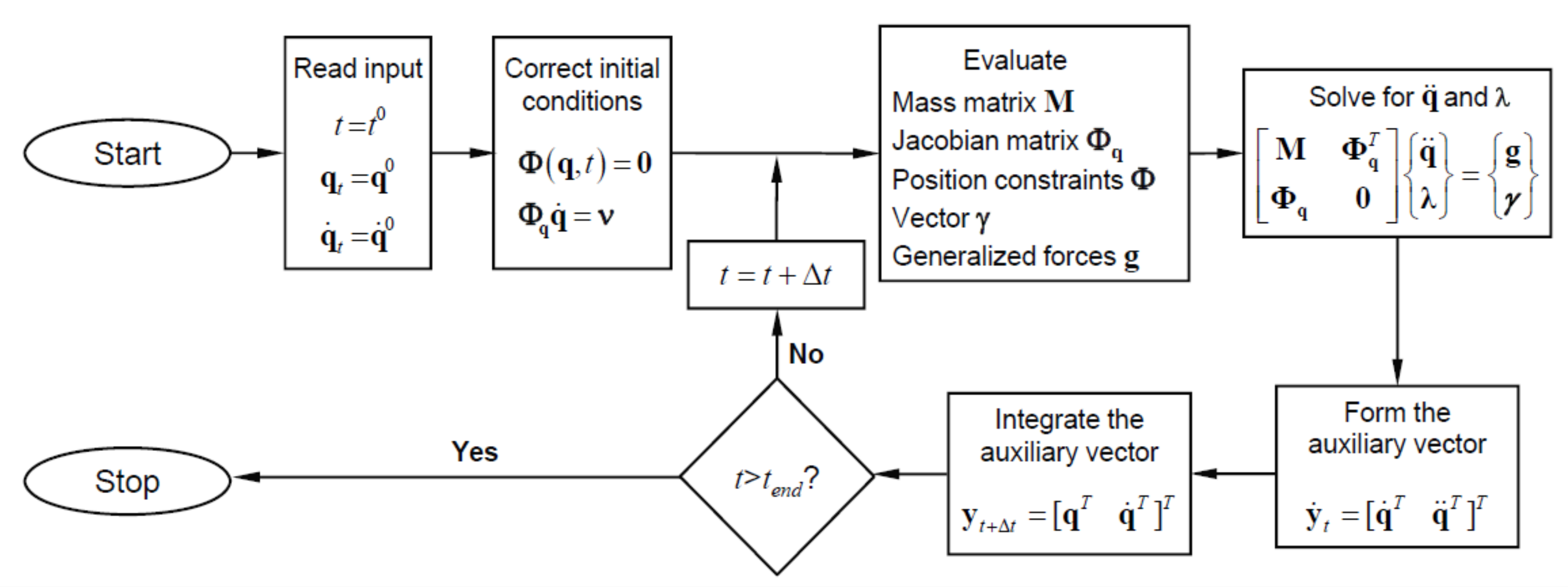
Evaluation of the dynamic response of the human body to a vibration environment.



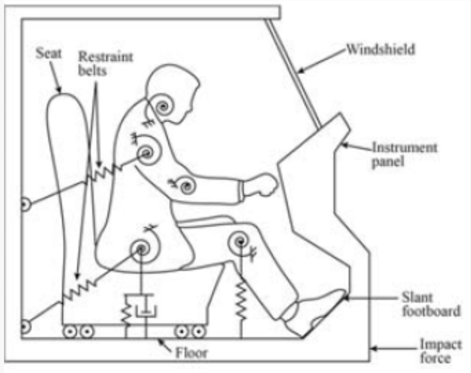
Source: W. Abbas et al., 2013. Optimal seat and suspension design for a half-car with driver model using genetic algorithm. Intelligent Control and Automation 4:199-205.

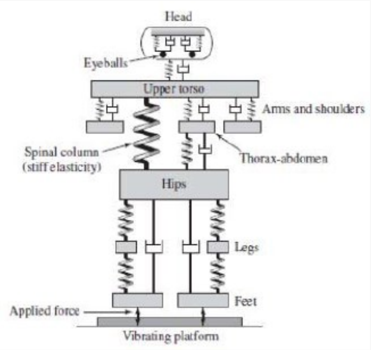
Dynamic analysis

Flowchart for the forward dynamic analysis:



Simple biomechanical models are used to study the response of the human body to different types of vibratory excitations. One example in which the control of the human exposure to vibrations is fundamental is in transportation systems.





Notes:

- In transportation systems, vibrations are transferred through the floor of the vehicle into a person's body. Vehicle design and type of seat, including suspension and seat cushions, can affect exposure to body vibrations.

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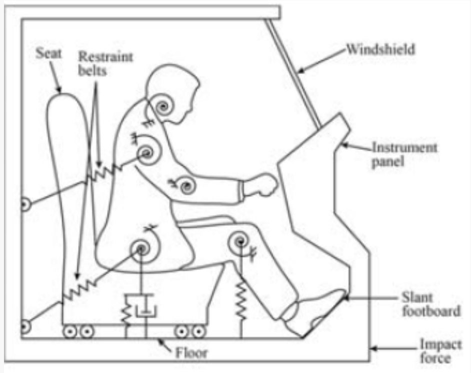
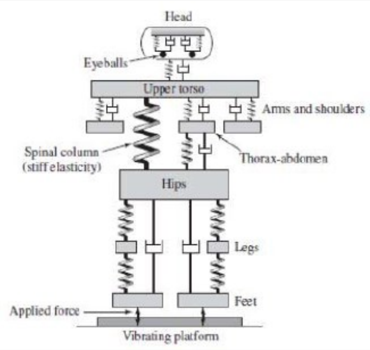


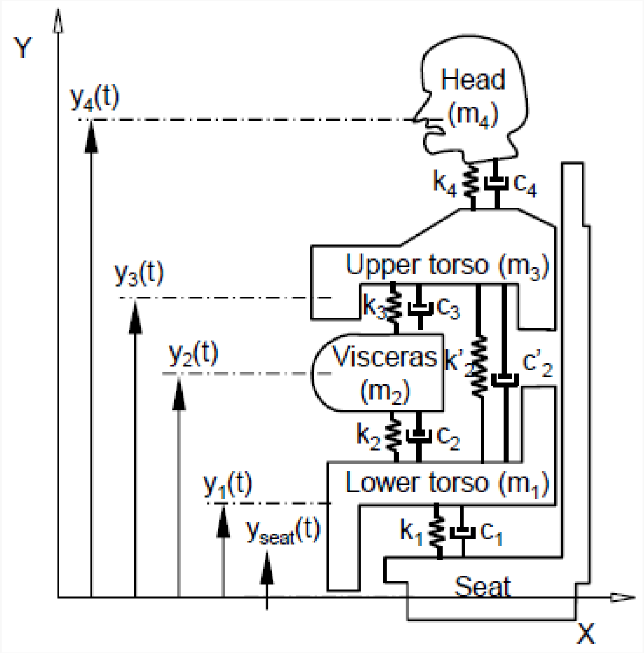


Notes:

- The most relevant impact of whole-body vibrations is the development of musculoskeletal disorders, most commonly lower back pain, but research has also shown that they contribute to other negative health effects, including cardiovascular, gastrointestinal, nervous and urological disorders.
- Among other factors, the vibration acceleration (in m/s^2) is important to assess the exposure to vibrations. Health research data shows that the degree of harm is related to the magnitude of acceleration.

Exercise:

Considering $y_{seat}(t) = sin(20t)$, find the dynamic response of the 2D biomechanical model described in the figure for a period of 2s. Note that the motion is unidimensional, i.e., it is described only along y.



	Mass (kg)
1 – Lower torso	36
2 – Visceras	5.5
3 – Upper torso	15
4 - Head	4.17

	Resting length (m)	Stiffness (N/m)	Damping (Ns/m)
1	0.462	49342	2475
2	0.125	20000	330
3	0.285	10000	200
4	0.205	134400	250
5 = 2’	0.405	192000	910

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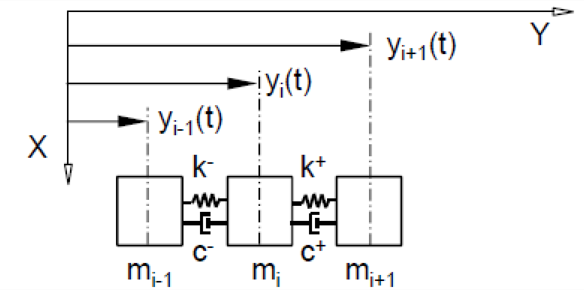
Exercise: Remark 1

For an isolated mass-spring system,



$$f_{spring} = k\Delta x$$

$$f_{damper} = c\Delta\dot{x}$$

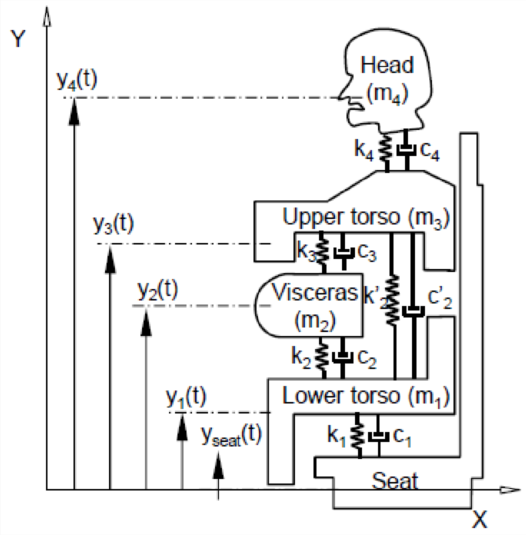
the equations of motion for a mass *i* are given by,

$$m_i\ddot{y}_i = -k^-(y_i - y_{i-1} - l_0^-) + k^+(y_{i+1} - y_i - l_0^+) - \; c^-(\dot{y}_i - \dot{y}_{i-1}) + c^+(\dot{y}_{i+1} - \dot{y}_i)$$

Write the equations of motion for the entire system.

Initial condition:

$$\mathbf{y}_0 = \begin{Bmatrix} 0.45 \\ 0.57 \\ 0.85 \\ 1.05 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \end{Bmatrix}$$



Dynamics of Mechanical Systems

First dynamic analysis program

Exercise: First dynamic analysis program

```
%%
%... Access global memory
global Model Time
%
%... Read (load) the model input data and analysis profile
filename = 'HumanBodyVibrations.mat';
load(filename);
CPUStart = cputime;
%
%% ... Perform the dynamic analysis by calling Matlab function to integrate
%       the ode (time integration)
[t, y] = ode45(Model.Function, Time.tspan, y0);
%
%.... Report analysis CPU time
CPUTime = cputime - CPUStart;
disp(['CPU Time = ', num2str(CPUTime)]);
%
%% ... Post-Process the analysis results
First_Post_Process_Results(t, y);
```

Dynamics of Mechanical Systems

Exercise: Model data

The model data script includes information about body masses, spring-damper systems, time analysis parameters, and the initial condition of the system.

```
%... Model & Simulation scenario filename
filename = 'HumanBodyVibrations.mat';
%
%... Model masses
Model.Body(1).mass = 36.00;
Model.Body(2).mass = 5.500;
Model.Body(3).mass = 15.00;
Model.Body(4).mass = 4.170;

%
%... Spring-damper characteristics
Model.SpringDamper(1).k  = 49342.00;
Model.SpringDamper(1).c  = 2475.000;
Model.SpringDamper(1).l0 = 0.462000;
Model.SpringDamper(2).k  = 20000.00;
Model.SpringDamper(2).c  = 330.0000;
Model.SpringDamper(2).l0 = 0.125000;
Model.SpringDamper(3).k  = 10000.00;
Model.SpringDamper(3).c  = 200.0000;
Model.SpringDamper(3).l0 = 0.285000;
Model.SpringDamper(4).k  = 134400.0;
Model.SpringDamper(4).c  = 250.0000;
Model.SpringDamper(4).l0 = 0.205000;
Model.SpringDamper(5).k  = 192000.0;
Model.SpringDamper(5).c  = 910.0000;
Model.SpringDamper(5).l0 = 0.405000;
```

Dynamics of Mechanical Systems

Exercise: Model data

The model data script includes information about body masses, spring-damper systems, time analysis parameters, and the initial condition of the system.

```
%... Function for evaluating equations
Model.Function     = @HumanBodyResponse;
%
%... Time analysis parameters
Time.start = 0.0;
Time.end   = 2.0;
Time.step  = 0.01;
Time.tspan = Time.start : Time.step : Time.end;
%
%... Initial condition (position and velocities)
Model.Body(1).y  = 0.45;
Model.Body(1).yd = 0.00;
Model.Body(2).y  = 0.57;
Model.Body(2).yd = 0.00;
Model.Body(3).y  = 0.85;
Model.Body(3).yd = 0.00;
Model.Body(4).y  = 1.05;
Model.Body(4).yd = 0.00;
%
%... Initial condition placed in vector y0
y0 = [Model.Body(1).y;  Model.Body(2).y;  Model.Body(3).y;  Model.Body(4).y;
      Model.Body(1).yd; Model.Body(2).yd; Model.Body(3).yd; Model.Body(4).yd];
%
%... Save the data to a 'mat' file
save(filename);
```

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Exercise: FuncEval (HumanBodyResponse)

This function computes the accelerations of the system and builds vector yd.

```
function yd = HumanBodyResponse(t, y)
%
% Summary: This function contructs the yd vector, containing the velocities
%          and accelerations for time t, for the biomechanical model of the
%          human body for the analysis of vibrations.
%
% Input:   t      - Current time of the analysis
%          y      - Vector containing position and velocities of the system
%                   for time t
%
% Output:  yd     - Vector containing velocities and accelerations of the
%                   system for time t
%
% Shared:  Flag   - Flags for the function evaluation
%          Model  - Moedl parameters
%
%%
%
%... Access global memory
global Model;
%
%%
%... Transfer coordinates from vector y to the body structure
Model.Body(1).y  = y(1); Model.Body(2).y  = y(2);
Model.Body(3).y  = y(3); Model.Body(4).y  = y(4);
Model.Body(1).yd = y(5); Model.Body(2).yd = y(6);
Model.Body(3).yd = y(7); Model.Body(4).yd = y(8);
```

First dynamic analysis program

Exercise: FuncEval (HumanBodyResponse)

This function computes the accelerations of the system and builds vector yd.

```
%... Builds the mass matrix for the system
Mass = [Model.Body(1).mass, 0.00              , 0.00              , 0.00;
        0.00              , Model.Body(2).mass, 0.00              , 0.00;
        0.00              , 0.00              , Model.Body(3).mass, 0.00;
        0.00              , 0.00              , 0.00              , Model.Body(4).mass];
%
%%
%... Evaluate the seat motion
yseat  = sin(20 * t);
ydseat = 20 * cos(20 * t);
```

```
%... Create the force vector
g = [ - Model.SpringDamper(1).k * (Model.Body(1).y - yseat - Model.SpringDamper(1).l0) +...
        Model.SpringDamper(2).k * (Model.Body(2).y - Model.Body(1).y - Model.SpringDamper(2).l0) +...
        Model.SpringDamper(5).k * (Model.Body(3).y - Model.Body(1).y - Model.SpringDamper(5).l0) -...
        Model.SpringDamper(1).c * (Model.Body(1).yd - ydseat) +...
        Model.SpringDamper(2).c * (Model.Body(2).yd - Model.Body(1).yd) +...
        Model.SpringDamper(5).c * (Model.Body(3).yd - Model.Body(1).yd) -...
        Model.Body(1).mass * 9.8;
      - Model.SpringDamper(2).k * (Model.Body(2).y - Model.Body(1).y - Model.SpringDamper(2).l0) +...
        Model.SpringDamper(3).k * (Model.Body(3).y - Model.Body(2).y - Model.SpringDamper(3).l0) -...
        Model.SpringDamper(2).c * (Model.Body(2).yd - Model.Body(1).yd) +...
        Model.SpringDamper(3).c * (Model.Body(3).yd - Model.Body(2).yd) -...
        Model.Body(2).mass * 9.8;
      - Model.SpringDamper(3).k * (Model.Body(3).y - Model.Body(2).y - Model.SpringDamper(3).l0) +...
        Model.SpringDamper(4).k * (Model.Body(4).y - Model.Body(3).y - Model.SpringDamper(4).l0) -...
        Model.SpringDamper(5).k * (Model.Body(3).y - Model.Body(1).y - Model.SpringDamper(5).l0) -...
        Model.SpringDamper(3).c * (Model.Body(3).yd - Model.Body(2).yd) +...
        Model.SpringDamper(4).c * (Model.Body(4).yd - Model.Body(3).yd) -...
        Model.SpringDamper(5).c * (Model.Body(3).yd - Model.Body(1).yd) -...
        Model.Body(3).mass * 9.8;
      - Model.SpringDamper(4).k * (Model.Body(4).y - Model.Body(3).y - Model.SpringDamper(4).l0) -...
        Model.SpringDamper(4).c * (Model.Body(4).yd - Model.Body(3).yd) -...
        Model.Body(4).mass * 9.8];
```

Dynamics of Mechanical Systems

Exercise: FuncEval (HumanBodyResponse)

This function computes the accelerations of the system and builds vector yd.

```
%%
%... Calculate the system accelerations
qdd = Mass \ g;
%
%%
%... Build the yd vector
yd = [y(5:8); qdd];
```

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Note: Animation

```
%%... Makes an animation of the human body
% Colors for the bodies
colmap = ['b', 'c', 'r', 'k'];
for i = 1 : NFrames
    for j = 1 : 4
        ybody = y(i, j);

        % Plots the body as a square
        P1 = [-0.05, ybody - 0.025];
        P2 = [ 0.05, ybody - 0.025];
        P3 = [ 0.05, ybody + 0.025];
        P4 = [-0.05, ybody + 0.025];
        plot([P1(1) P2(1) P3(1) P4(1)], [P1(2) P2(2) P3(2) P4(2)], colmap(j));
        text(0, ybody, num2str(j));
        hold on;
    end
    axis([-0.1 0.1 -3 3]);
    pbaspect([1 1 1]);
    hold off;
    pause(0.1);
end
```