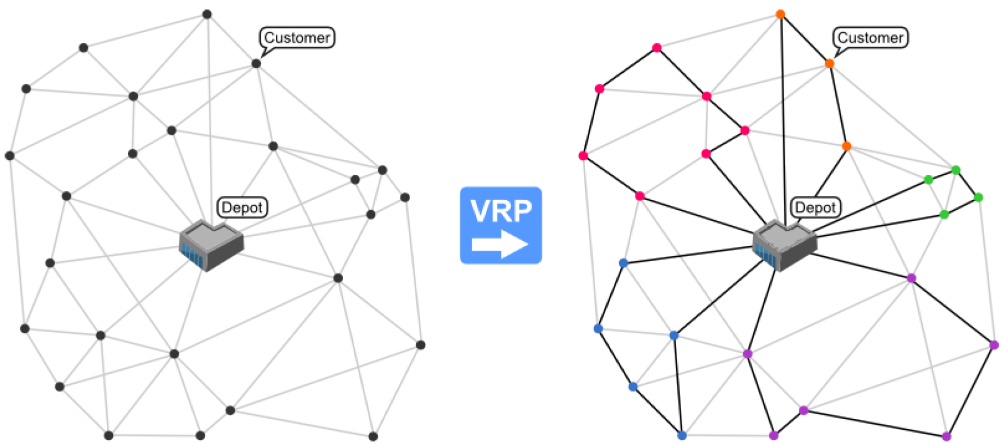
4. Vehicle Routing Problem with Soft Time Windows and Stochastic Travel Times

(VRPsTW-STT)



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1 Introduction

One of the big challenges companies face today is improving efficiency. Costs should be reduced while the quality is maintained. Making planning decisions is often computationally hard. Therefore, advanced planning methods are needed to determine the best strategy for the near and distant future. One of those hard planning problems is known as the Vehicle Routing Problem with Time Windows and Stochastic Travel Times (VRPsTW-STT). A fleet of identical vehicles with a fixed capacity is located at a central depot. Furthermore, there is a set of customers. Every customer has to be visited exactly once within a predefined time window by one of the vehicles. A time window consists of a release date and a due date. In transportation problems two types of time window handling can occur. Time windows are called soft when service can start before the release date, or after the due date. However, this induces a violation cost, which penalises early or late service. When service must start within the time windows, time windows are called hard. When a vehicle arrives at a customer before its release date it is required to wait until the release date before service can start. A vehicle is not allowed to arrive after the due date. Each vehicle starts and ends at the depot.

1st Part: The mathematical formulation of the problem should be done. In this first part, the problem will be solved using simplex, and for that the model should be simplified and formulated as a LP problem. Report all the necessary simplifications of the model and the impact on the results and the solution of the real problem.

2 Dataset

The dataset used for the following model of the Vehicle distribution problem was taken from the VRPTW Benchmark Problems from Professor Marius M. Solomon's page. [3] These datasets contained several parameters that permit simulating the behaviour of routing and scheduling problems. The datasets varied in geographical data, number of customers serviced by a vehicle, vehicle capacity, fraction of time-constrained customers, and tightness and positioning of the time windows. The datasets are euclidean problems where travel times equal the corresponding distances.

The dataset contains 7 columns:

- CUST NO: the customer number
- XCOORD: the x coordinate of the costumer location
- YCOORD: the y coordinate of the costumer location
- DEMAND: the costumer demand
- READY TIME: the time from which the costumer is ready to receive
- DUE TIME: the time from which the costumer isn't ready to receive
- SERVICE TIME: service time: the time the vehicle shall remain in the costumer location before leaving

The problem chosen was R101.25. This dataset has randomly generated geographical data; a short scheduling horizon, allowing only a few customers per route (approximately 5 to 10); and, due to the complexity of the problem, only a smaller subset of 25 customers was considered, allowing comparison with the best known solution. The vehicle capacity for this problem is 200, and service time is 10, equal for all customers.

3 Problem Formulation

To formulate the Vehicle Routing Problem with Soft Time Windows - Stochastic Travel Times (VRPsTW-STT) as a linear programming problem is necessary to define decision variables, constraints, and objective function.

3.1 Rules for the problem:

- There's a set number of customers, n, each with a specified ready time, t_i^r, due time, t_i^d
- Vehicles are permitted to arrive earlier than t_i^r to customer i, in which case it'll wait until service begins at time t_i^r. Arriving earlier than t_i has no additional cost
- Vehicles cannot arrive after t_i^d
- t_{ij} is the travel time between customers i and j, and equals the distance between the customers
- The depot has a time window $[t_0^r; t_0^d]$ within which the routes must begin/end at the depot
- All vehicles have the same set capacity q
- All customers have set demand d_i
- Vehicles can only service as much customers as their capacity and demands of the route allow and all customers are serviced exactly once (meaning split deliveries are not allowed)

3.2 Decision Variables:

- x_{ij}^k is a binary decision variable that equals 1 if a vehicle k travels from customer i to customer j, and zero other wise
- s_i^k is variable that equals the time at which a vehicle k starts the service at customer i. If a vehicle k doesn't service customer i than $s_i^k = 0$

3.3 Constraints:

1. Ensures each vehicle starts at the depot:

$$\sum_{k=1}^{K} x_{1i}^k = 1 \quad \forall i \in \{2, 3, ..., n\} \tag{1}$$

2. Ensures each vehicle ends at the depot:

$$\sum_{k=1}^{K} x_{i1}^k = 1 \quad \forall i \in \{1, 2, 3, ..., n\} \tag{2}$$

3. Ensure that each customer is visited exactly once by exactly one vehicle, except for the depot:

$$\sum_{k=1}^{K} \sum_{i=1, j\neq i}^{n} x_{ij}^k = 1 \quad \forall i \in \{2, 3, ..., n\}, \quad \forall k \in \{1, 2, ..., K\} \tag{3}$$

4. Vehicles can only leave a node after entering it:

$$\sum_{i=1}^{n} x_{ih}^{k} = \sum_{j=1}^{n} x_{hj}^{k}, \quad \forall\, i, j, h \in \{2, 3, ..., n\}, \quad \forall k \in \{1, 2, ..., K\} \tag{4}$$

5. Service can only start within the time window:

$$t_i^r \leq s_i^k \leq t_i^d, \quad \forall i \in \{2, 3, ..., n\} \tag{5}$$

6. Arrival time at customer j of a vehicle k leaving customer i:

$$s_i^k + \text{ServiceTime} + t_{ij} - M(1 - x_{ij}^k) \leq s_j^k, \quad \text{where } M \text{ is a large constant} \tag{6}$$

7. The total delivered by vehicle k in it's route cannot exceed it's capacity

$$\sum_{j=1}^{n} x_{ij}^{k} d_i \leq q \quad \forall k \in \{1, 2, ..., K\} \tag{7}$$

3.4 Objective Function:

Minimise the total cost, which consists on the time travelling between customers based on expected travel times and the penalty for each customer visited outside its time window.

$$\text{Minimize} \sum_{k=1}^{K} \sum_{i=1}^{n} \sum_{j=1}^{n} \left(x_{ij}^{k} t_{ij} + \mathrm{P}_i^k \right) \tag{8}$$

Where P_i^k is the penalty for vehicle k visiting customer i outside its time window.

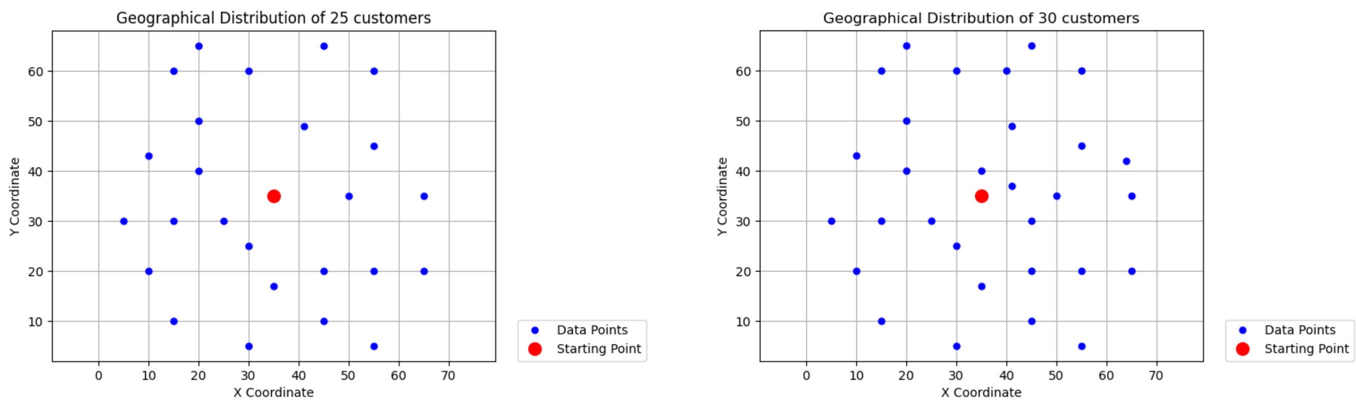
The penalty term is calculated based on the difference between the arrival time and the time window and weighted by a penalty factor w:

$$\mathrm{P}_i^k = \mathrm{w} \cdot \left[\max(0, s_i^k - t_i^r) \right] \tag{9}$$

This formulation allows for the consideration of soft time windows and stochastic travel times in the vehicle routing problem. Adjustments might be needed based on specific requirements and assumptions of the problem.

4 Results

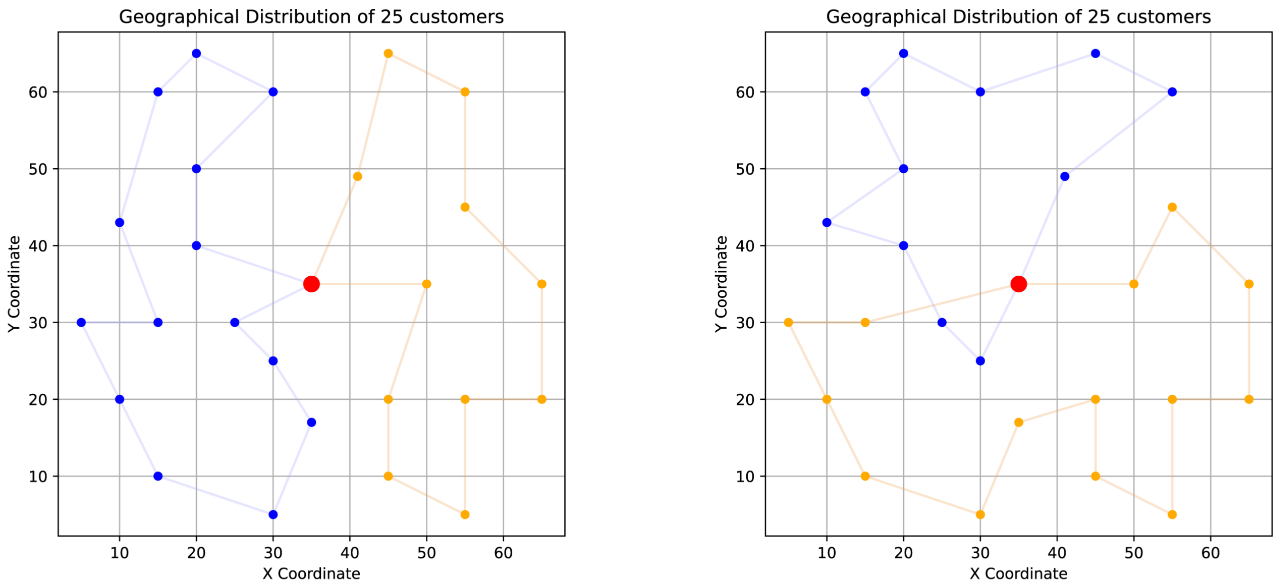
In this section we'll compare the results found using sets with 25 and 30 customers. We were unable to find results for bigger datasets due to the limitations of the Python libraries used (particularly cplex) in the implementation. We'll also compare the results found with different time limits for the solver.



(a) Geographical distribution of set with 25 customers.

(b) Geographical distribution of set with 30 customers.

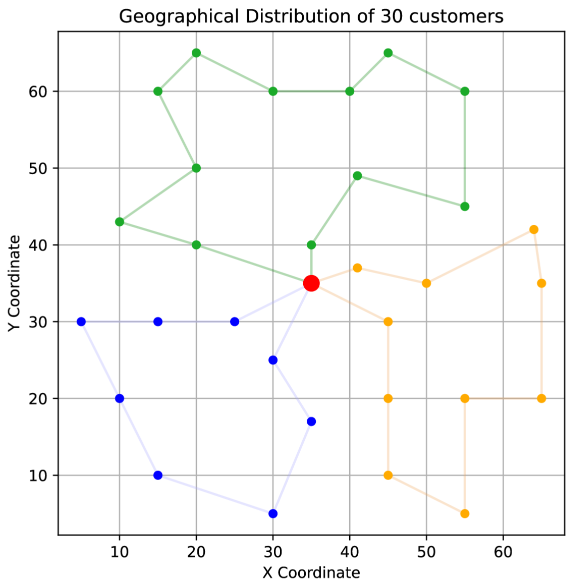
Figure 1: Geographical distribution of customers in both sets.



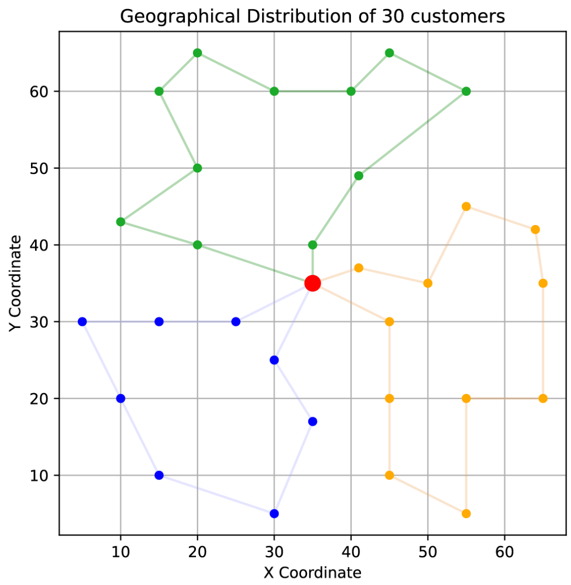
(a) Routes for set with 25 customers and solver time of 5 seconds. The routes had a total cost of 595.85

(b) Routes for set with 25 customers and solver time of 45 seconds. The routes had a total cost of 585.27

Figure 2: Results for set with 25 customers



(a) Routes for set with 30 customers and solver time of 5 seconds. The routes had a total cost of 698.72



(b) Routes for set with 30 customers and solver time of 30 seconds. The routes had a total cost of 658.37

Figure 3: Results for set with 30 customers.

5 Conclusion

In summary, while our project successfully tackled the capacitated vehicle routing problem using linear formulation and the simplex method in Python, it's evident that scalability is a challenge for larger datasets. This emphasises the ongoing need for more efficient optimisation methods to address real-world logistics issues effectively. Moving forward, our focus remains on exploring time windows (which will also allow us to compare results with the benchmarks set in [1], and stochastic travel times using more complex programming methods capable of handling modern logistical complexities with greater efficacy. In the future we will also try to address the computational challenges related to the library used.

Number of Customers	Solver Run Time (s)	Total Cost	Number of Vehicles
25	5	595.85	2
25	15	585.27	2
30	5	698.72	3
30	30	658.37	3

References

[1] Niklas Kohl, Jacques Desrosiers, Oli B. G. Madsen, Marius M. Solomon, and François Soumis. 2-path cuts for the vehicle routing problem with time windows. *Transportation Science*, 33(1):101–116, February 1999.

[2] Jens Lysgaard. Reachability cuts for the vehicle routing problem with time windows. *Department of Accounting, Finance and Logistics, Aarhus School of Business*, November 2004. Email: lys@asb.dk.

[3] M. M. Solomon. Vehicle routing problem instances. http://w.cba.neu.edu/~msolomon/r101.htm.