

More about Tests and Intervals

Recall: The definition of a _____ is the probability of getting the observed test statistic (whether it is z or t) or one that is more extreme if the null hypothesis (H_o) is true.

We note, that a p-values _____ the probability that the null hypothesis is true.

Relationship between H_o and H_a

To demonstrate the relationship between the null and the alternative hypothesis, we might compare a hypothesis test to a criminal trial. In the U.S judicial system, we assume that the defendant is innocent until proven otherwise. Therefore, we might say the null hypothesis in this case is:

H_o : the defendant is innocent

Therefore the prosecutor must convince the jury of the defendant's guilt by presenting evidence of a crime. We might say that this is the alternative hypothesis, it is the claim that we are trying to find evidence to support.

H_a : the defendant is guilty

- If **enough evidence** is presented to suggest that the defendant is guilty ...

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- If **not enough evidence** is presented to suggest that the defendant is guilty ...

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But a question might arise, how much evidence is enough evidence.

Practical vs. Statistical Significance

A test is _____ if ...

A test is _____ if ...

For large samples, even small deviations from the null hypothesis could be statistically significant. But even if these differences are statistically significant, they may not be

_____.

For small samples, small but impactful differences may not end up being statistically significant. Hypothesis tests can only detect a very large difference between H_o and the true value of the parameter. But in these cases, lack of statistically significant evidence does not mean that a significant relationship does not exist.

Errors in Hypothesis Testing

Recall from Chapter 17 handouts part two that there are four different options for hypothesis tests:

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But just like how courts can sometimes wrongfully convict an innocent person or let a guilty person walk free, we can make errors in our hypothesis testing. For this class we will focus on two different types of hypothesis errors.

Type I and Type II Errors

	Evidence Against H_o	No Evidence Against H_o
H_o is Actually True	Type I Error	Correct
H_a is Actually True	Correct	Type II Error

The probabilities of Type I and Type II Errors are inversely related:

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So ultimately the goal of hypothesis testing is to minimize both the probability of a Type I Error and the probability of a Type II Error. But because these two concepts are inversely related, these goals conflict. So often we choose to minimize our Type I error at the expense of our Type II error

Hypothesis Tests & Confidence Intervals for μ

Consider a two-sided hypothesis test:

$$H_o : \mu = \mu_o \quad H_a : \mu \neq \mu_o$$

This test has a direct relationship with a confidence interval for μ :

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Example A study was done to determine the average commute time to work in Atlanta, Georgia. A random sample of 500 residents of metropolitan Atlanta was taken. The sample mean was $\bar{y} = 29.11$ minutes with a standard deviation of $s = 20.7$ minutes. A 90% confidence interval is computed to be (27.58 minutes, 30.64 minutes). Consider the following hypotheses:

$$H_o : \mu = 31 \text{ minutes} \quad H_a : \mu \neq 31 \text{ minutes}$$

Based on the above confidence interval, what can we say about the strength of the evidence against the null hypothesis.

Important Take-Aways

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