ComSS 331 Fall 2017 Name:____

HW 3 Due: 15 sep 2017

1. Prove or disprove: every finite language is recognized by some FA. 50

2. Define a DFA, simplified to the best of your abilities, that recognizes the language $L = \{w \in \{a,b\}^* : w \text{ does not contain the substring } abba\}$. 50

3. Consider the n-bit binary representation of a natural number x:

$$\text{the binary representation of } x \text{ is } (x_{n-1}x_{n-2}\cdots x_1x_0)_2 \iff x = \sum_{i=0}^{n-1} x_i 2^i$$

   where each bit x_i is a binary digit, either zero or one. For example, $(00000101)_2$ is the 8-bit binary representation of the number 5, since $0\cdot2^7+0\cdot2^6+0\cdot2^5+0\cdot2^4+0\cdot2^3+1\cdot2^2+0\cdot2^1+1\cdot2^0 = 4+1 = 5$. This is the format normally employed by digital computers to store nonnegative integers.

   Consider the language

$$\begin{aligned} L \;=\; \{&a_0b_0c_0\cdots a_{n-1}b_{n-1}c_{n-1} \;:\; n \in \mathbb{N} \wedge \forall i, 0 \le i < n, a_i \in \{0,1\}, b_i \in \{0,1\}, c_i \in \{0,1\} \wedge \\ &(a_{n-1}\cdots a_0)_2 + (b_{n-1}\cdots b_0)_2 = (c_{n-1}\cdots c_0)_2\} \end{aligned}$$

   For example, since $5+3=8$, $5 = (000101)_2$, $3 = (000011)_2$, and $8 = (001000)_2$, then

$$110\ 010\ 100\ 001\ 000\ 000 \in L$$

   (the string is spaced every three digits for readability's sake only).

   Define a DFA that accepts L. 100