COMS 331
Homework 0

Christian Shinkle

August 28, 2017

1. This is an inline equation: $x + y = 3$
This is a displayed equation:

$$x + \frac{y}{z - \sqrt{3}} = 2.$$

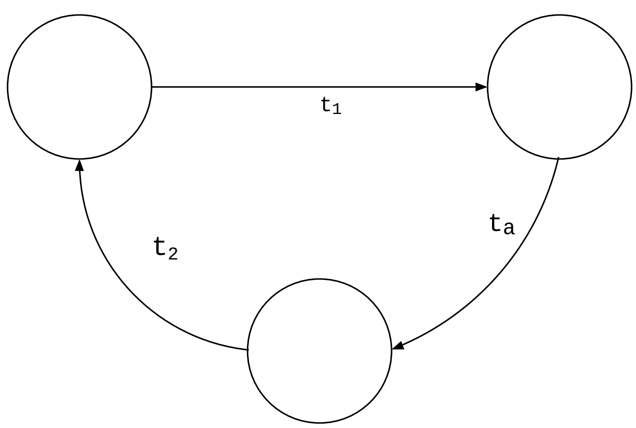
This is how you can define a piece-wise linear function:

$$f(x) = \begin{cases} 3x + 2 & \text{if } x < 0 \\ 7x + 2 & \text{if } x \geq 0 \text{ and } x < 10 \\ 5x + 22 & \text{otherwise} \end{cases}$$

This is a matrix:

9	9	9	9
6	6	6	
3		3	3

This is a figure incorporated in a LaTeX file



2. In order to show that $\mathbb{N}$ and $\mathbb{Z}$ are equinumerous, we must define a bijection

between them. Note the function f : $\mathbb{N} \to \mathbb{Z}$ where

$$f(n) = \begin{cases} \text{-(n/2)} & \text{if } n \text{ is even} \\ \text{(n+1)/2} & \text{if } n \text{ is odd} \end{cases}$$

First, we prove f is one-to-one. Let $\forall n, m \in \mathbb{N},\ f(n) = f(m)$. Then we must prove 2 cases.
Case 1: n is even

$$f(n) = f(m)$$
$$-(n/2) = -(m/2)$$
$$n/2 = m/2$$
$$n = m$$

Case 2: n is odd

$$f(n) = f(m)$$
$$(n+1)/2 = (m+1)/2$$
$$n + 1 = m + 1$$
$$n = m$$

Both cases hold, therefore $f(n)$ is one-to-one.

The next step is to prove $f(n)$ is onto.
Then we must prove 2 cases:
Case 1: n is even
Let $\forall m \in \mathbb{Z}^- \cup \{0\},\ \exists n \in \mathbb{N}\ f(n) = m$.

$$f(n) = m$$
$$-(n/2) = m$$
$$n/2 = -m$$
$$n = -2m$$

By definition of even numbers, every n can be reached.

Case 2: n is odd
Let $\forall m \in \mathbb{Z}^+, \exists n \in \mathbb{N}\ f(n) = m$.

$$f(n) = m$$
$$(n+1)/2 = m$$
$$n + 1 = 2m$$
$$n = 2m + 1$$

By definition of odd numbers, every n can be reached.
Therefore $\mathbb{Z}^- \cup \{0\} \cup \mathbb{Z}^+ = \mathbb{Z}$ and all value in the codomain can be reached.

Both cases are satisfied, so $f(x)$ is a bijective function. By definition of equinumerous, $\mathbb{N}$ and $\mathbb{Z}$ are equinumerous.

3. $f(x)$ is one-to-one, but not onto for $f(x) = x^2$.
   $f(x)$ is onto, but is not one-to-one for

$$f(x) = \begin{cases} \sqrt{x} & \text{if } x \geq 0 \\ -\sqrt{x} & \text{if } x < 0 \end{cases}$$

4. To prove R is an equivalence relation, we must prove it is reflexive, symmetric, and transitive.
   Reflexive: $\forall a \in \mathbb{N}, (a, a) \in R = (a - a)\ mod\ 3 = 0\ mod\ 3 = 0$.

   Symmetric: $\forall a, b \in \mathbb{N}$, let $(a - b) \in R$. Then $(a - b)\ mod\ 3$ or $\exists x \in \mathbb{N}\ , a - b = 3x$. Note the equation can be rewritten as $b - a = -3x$. $-3x\ mod\ 3 = 0$ therefore $b - a\ mod\ 3 = 0$ and $(b, a) \in R$.

   Transitive: Suppose $a, b, c \in \mathbb{N}$ and $(a, b) \in R$ and $(b, c) \in R$. Let $a - b = 3x$ and $b - c = 3y$. Then $a - c = 3x + 3y$. Note $3x + 3y = 3(x + y)$. Therefore $(a, c) \in R$.

   Relations R has three equivalence classes:

   - $\{0, 3, 6, ...\}$
   - $\{1, 4, 7, ...\}$
   - $\{2, 5, 8, ...\}$

5. For this proof, we will be inducting n from the formula.
   Base case: Note $\Sigma_{i=1}^{1} i^2 = (2 * 1 + 1)(1 + 1)1/6 = 1$.

   Inductive Step: $\Sigma_{i=1}^{n+1} i^2 = \Sigma_{i=1}^{n} i^2 + (n + 1)^2$. The inductive hypothesis is $\Sigma_{i=1}^{n} i^2 = (2n + 1)(n + 1)n/6$. Using the I.H.,

$$\Sigma_{i=1}^{n+1} i^2 =$$
$$(2n + 1)(n + 1)n/6 + (n + 1)^2 =$$
$$\frac{n^3}{3} + \frac{3n^2}{2} + \frac{13n}{6} + 1 =$$
$$(2n^3 + 9n^2 + 13n + 6) \div 6 =$$
$$(2n^2 + 7n + 6)(n + 1) \div 6 =$$
$$(2(n + 1) + 1)((n + 1) + 1)(n + 1) \div 6$$

   It follows by inductions that $\Sigma_{i=1}^{n+1} i^2 = (2 * 1 + 1)(1 + 1)1/6 = 1$