# COM S 331

Homework 4

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1. Proof by Contradiction: Assume $L$ is regular, then $\exists m \in \mathbb{N}, \forall x \in L$, $|x| > m, \exists a, b, c \in w^*, x = abc, |ab| \leq m, |b| > 0$, $\forall k \in \mathbb{N}, ab^k c \in L$. Consider $ww$ and denote each substring $w$ as $w_1$ and $w_2$. Then, decompose it in any possible way into $abc$ satisfying $|ab| \leq m$, $|b| > 0$. Then, set $k = 0$ for any $b$ that was chosen. Three cases can arise:

   (a) b is entirely in $w_1$

   (b) b is entirely in $w_2$

   (c) b is partially in $w_1$ and $w_2$

   The first two cases are trivial because under pumping $b^0$ will make either $w_1$ or $w_2$ no longer equal to the original $w$. If the thrid cases arises, then both $w_1$ and $w_2$ will both not be equal to the $w$. In all three cases, a contradiction has occured.

2. Proof by Contradiction: Assume $L$ is regular. Then, using the Closure property of Homomorphism, we can say $L^{'} = \{w \in \{0,1\}^* : |w|_0 \neq |w|_1\}$ is obtained from the homomorphism $a \to 0$ and $b \to 1$. Then, using the Closure property of Complementation, we say $L^{''} = \{w \in \{0,1\}^* : |w|_0 = |w|_1\}$. Then, using the Closure property of Concatenation, we say $L^{''}(M_1) * L^{''}(M_2) = L^{'''} = \{w \in \{0,1\}^* : ww\}$. However, note that $L^{'''}$ is regular, but using the proof from problem one, $L^{'''}$ is nonregular, a contradiction.