

Change-making problem

You have an unlimited number of pennies, dimes, and quarters. Given an integer $x \geq 0$, what's the minimum number of coins you can choose to sum to x exactly?

Recursive algorithm:

```
opt(i) {  
    if i < 0:  
        return ∞;  
    if i = 0:  
        return 0;  
    return min(  
        opt(i - 1) + 1,  
        opt(i - 10) + 1,  
        opt(i - 25) + 1  
    );  
}
```

Dynamic Programming algorithm:

```
opt-DP(x) {  
    d = [1, 10, 25];  
    arr = new int[x + 1];  
    arr[0] = 0;  
    for i = 1 to x:  
        arr[i] = ∞;  
        for j = 1 to 3:  
            if i - dj ≥ 0 and 1 + arr[i - dj] < arr[i]:  
                arr[i] = 1 + arr[i - dj];  
    return arr[x];  
}
```