COM S 311

Homework 1

Recitation 5, 1-2pm, Marios Tsekitsidis

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1. Base Case: Note

$$F_1^2 = F_1 * F_1$$

$$1^2 = 1 * 1$$

$$1 = 1$$

Inductive Hypothesis: $\forall n \leq 1, F_1^2 + F_2^2 + F_3^2 + ... + F_n^2 = F_n * F_{n+1}$
Goal: Prove $F_1^2 + F_2^2 + F_3^2 + ... + F_n^2 + F_{n+1}^2 = F_{n+1} * F_{n+2}$
Inductive Case: By I.H., $F_1^2 + F_2^2 + F_3^2 + ... + F_n^2 + F_{n+1}^2 = F_n * F_{n+1} + F_{n+1}^2$
$= F_{n+1}(F_n + F_{n+1})$
$= F_{n+1} * F_{n+2}$ by definition of Fibonacci Numbers

2. (a)

(b) Base Case: A T which is a single node is a leaf, so

$$i(T) = (n(T) - 1)2 = (1 - 1)/2 = 0$$

Inductive Hypothesis: Let X and Y be two FBT's with $i(X) = (n(x) - 1)/2$ and $i(Y) = (n(Y) - 1)/2$.
Goal: For tree R, $i(R) = (n(R) - 1)/2$.
Inductive Case: Let R be a FBT with root r and left child FBT X and right child FBT Y. Then, $i(R) = i(X) + i(Y) + 1$ because R consists of all nodes of X and Y plus the root r. By I.H.,
$i(R) = (n(X) - 1)/2 + (n(Y) - 1)/2) + 1$. Then, $i(R)$
$= \frac{(n(X)-1+n(Y)-1}{2} + 1$
$= \frac{(n(X)+n(Y)-2}{2} + 1$
$= \frac{(n(R)-1)-2}{2} + 1$ by definition of FBT
$= (n(R) - 1)/2 - 1 + 1$
$= (n(R) - 1)/2$

3. Base Case: Note $x = a, m = n, y = 1$. Therefore,
   $a^n = x_0^{m_0} * y_0$
   $= a^n * 1$
   Inductive Hypothesis: $\forall i, a^n = x_i^{m_i} * y_i$ where is the number of iterations through the loop.
   Inductive Case: At the begining of the $i + 1th$ iteration through the loop, one of two cases can arise, m is even or odd.
   Case 1: m is even. Then,
   $x_{i+1} = x_i^2$, $m_{i+1} = m_i/2$, and $y_{i+1} = y_i$. So,
   $x_{i+1}^{m_{i+1}} * y_{i+1}$
   $= x_i^{2*m_i/2} * y_i$
   $= x_i^{m_i} * y_i$
   By I.H., $a^n = x_i^{m_i} * y_i$, therefore the property holds.
   Case 2: m is odd. Then,
   $y_{i+1} = x_i * y_i$, $x_{i+1} = x_i^2$, and $m_{i+1} = (m_i - 1)/2$. So,
   $x_{i+1}^{m_{i+1}} * y_{i+1}$
   $= x_i^{2*(m_i-1)/2} * x_i * y_i$
   $= x_i^{m_i-1} * x_i * y_i$
   $= x_i^{m_i} * y_i$
   By I.H., $a^n = x_i^{m_i} * y_i$, therefore the property holds.
   Both cases hold, so the property holds for the $i + 1th$ iteration.

4.