

Homework 4
Com S 311
Due: Mar 25, 11:59PM

Late Submissions are NOT accepted.

There are 4 problems and each problem is worth 50 points. When asked to design an algorithm, please write *pseudo code*. If you write Java/C code, it will not be graded. You are not allowed to use *Master Theorem* in this homework.

1. Solve the following recurrences

(a) $T(n) = 3T(n/2) + n; T(1) = 1$

(b) $T(n) = T(n/8) + n; T(1) = 1.$

2. Give a recursive algorithm that gets a sorted array (of integers) as input and constructs a Balanced BST. Your algorithm should not use balancing ideas such as AVL balancing Trees. Your algorithm must be a recursive algorithm. Write the recurrence relation for the run-time and solve the recurrence. You will receive zero credit, if your algorithm is not recursive. Your grade depends on the run-time of the algorithm.
3. Given two 2-dimensional points $A = \langle x_1, y_1 \rangle$ and $B = \langle x_2, y_2 \rangle$, we say that $A < B$ if $x_1 < x_2$ and $y_1 < y_2$. Given a set of $S = \{A_1, A_2, \dots, A_n\}$ of 2-dimensional points, the goal is to output a *maximal* set $S' \subseteq S$ such that for every pair of points C and D in S' , neither $C < D$ nor $D < C$. Give a divide and conquer algorithm to solve this problem. Write the recurrence relation for the run-time and solve the recurrence. Your grade depends on the run-time. You will receive zero credit, if you do not use divide-and-conquer. A subset S' of S maximal if the following holds: For every $D \in S - S'$, there exists C in S' such that either $C < D$ or $D < C$.
4. Given a directed graph $G = (V, E)$, let $G^2 = (V', E')$ be defined as follows: $V' = V$; $\langle u, v \rangle \in E'$ if there is a path of length 2 between u and v in G . Suppose that a directed graph G is given as adjacency list. Give an algorithm to compute G^2 . Derive the time bound of your algorithm. Your grade depends on the run-time. Here m is number of edges in G and n is number of vertices in G .

GUIDE LINES:

- It is important to know whether you really know! For each problem, if you write the statement "I do not know how to solve this problem" (and nothing else), you will receive 20%

credit for that problem. If you do write a solution, then your grade could be anywhere between 0% to 100%. To receive this 20% credit, you must explicitly state that you do not know how to solve the problem.

- You must work on the homework problems on your own. You should write the final solutions alone, without consulting any one. Your writing should demonstrate that you understand the proofs completely.
- When proofs are required, you should make them both clear and rigorous. Do not hand waive.
- If your hand writing is not legible, then your homework will not be graded.
- Any concerns about grading should be made within one week of returning the homework.
- **Please submit your HW via Canvas. If you type your solutions, then please submit pdf version. If you hand-write your solutions, then please scan your solutions and submit a pdf version. Please make sure that the quality of the scan is good, and your hand writing is legible. HW's submitted in incorrect format (non pdf) will incur a penalty of 20%**