

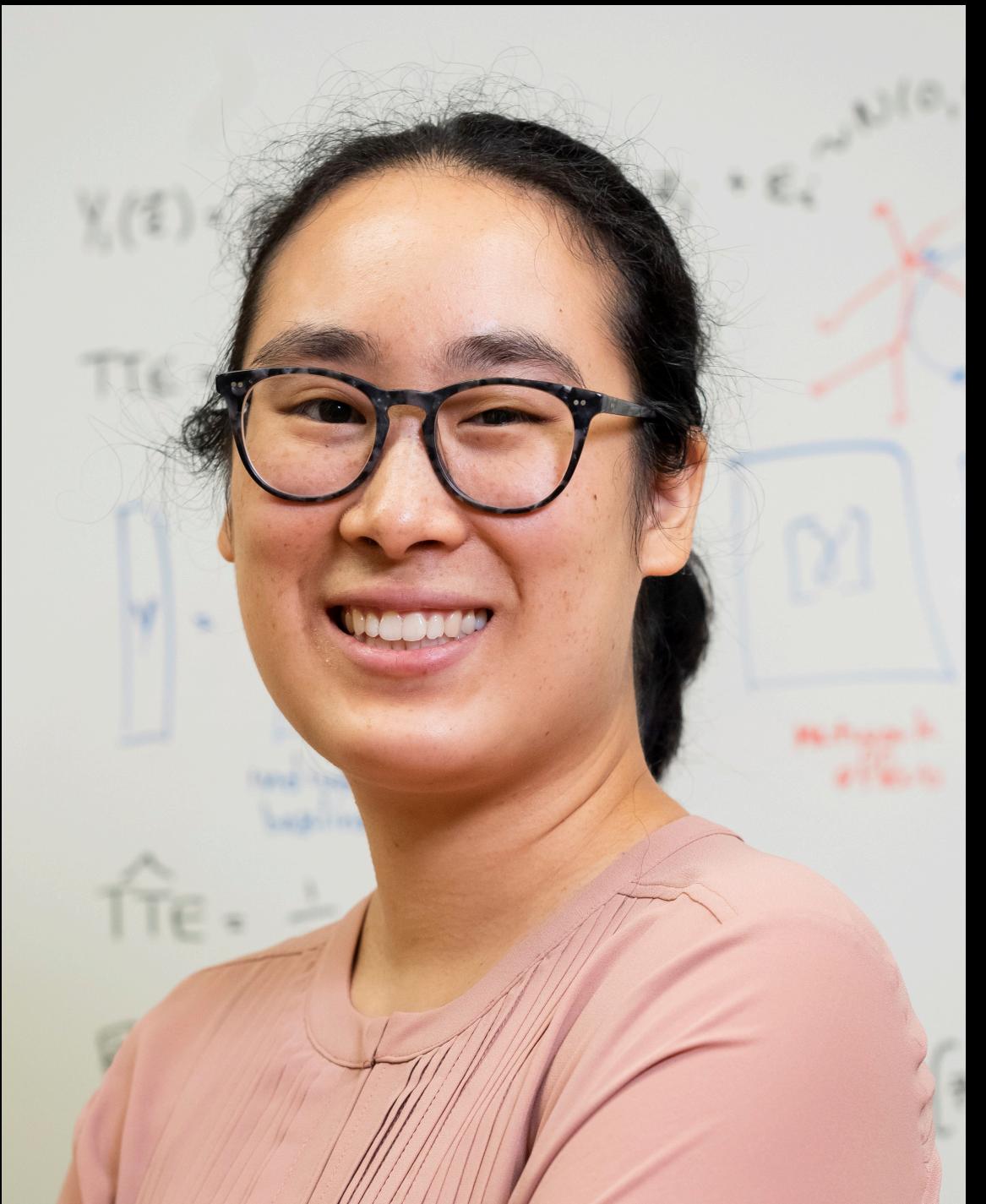
Preferential Attachment and Homology

Evolution of Higher Dimensional Interactions

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Collaborators



Christina Lee Yu



Gennady Samorodnitsky



Caroline He

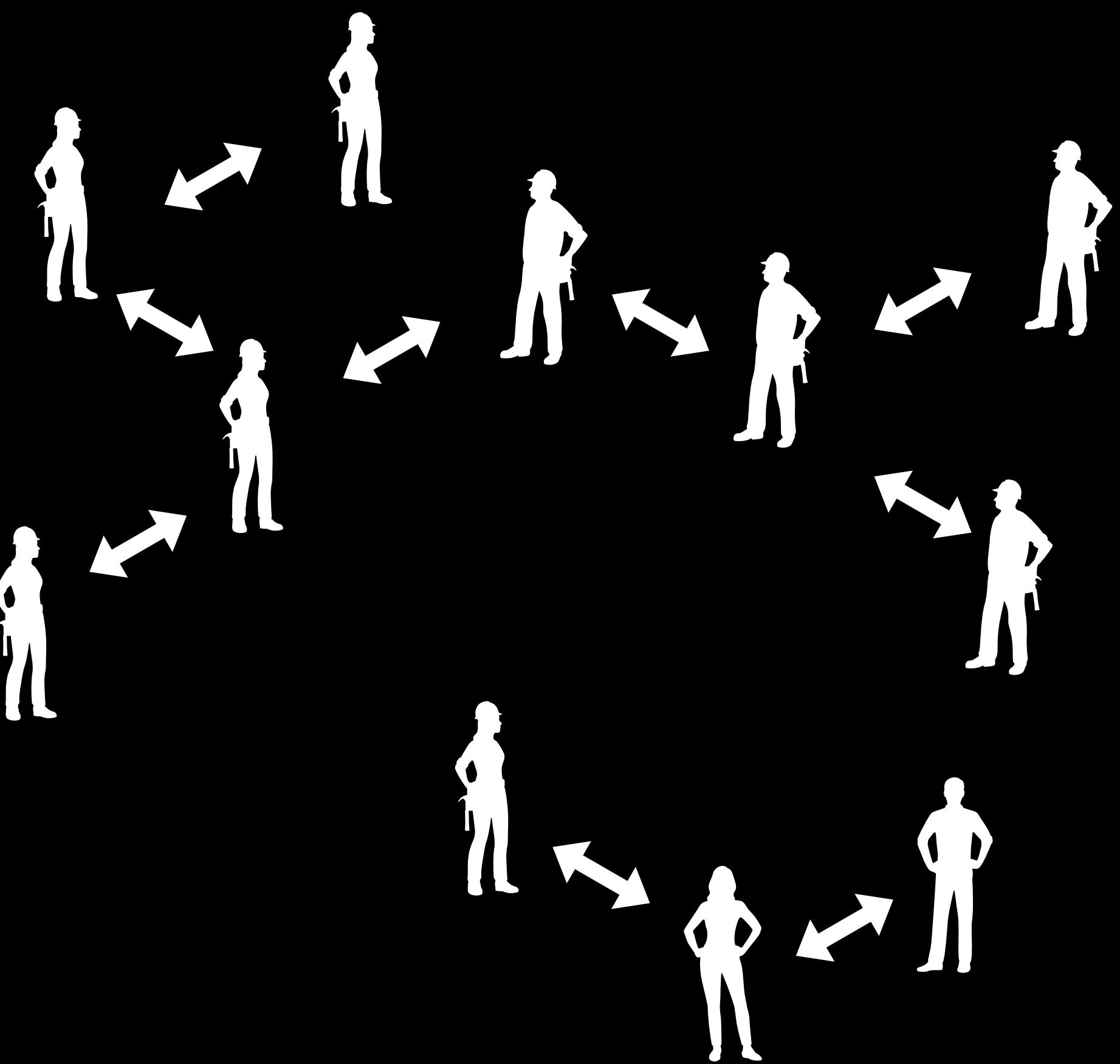
Agenda

- What is preferential attachment?
- What is homology?
 - Why people care?
- What we know about the homology of preferential attachment complexes

Preferential Attachment

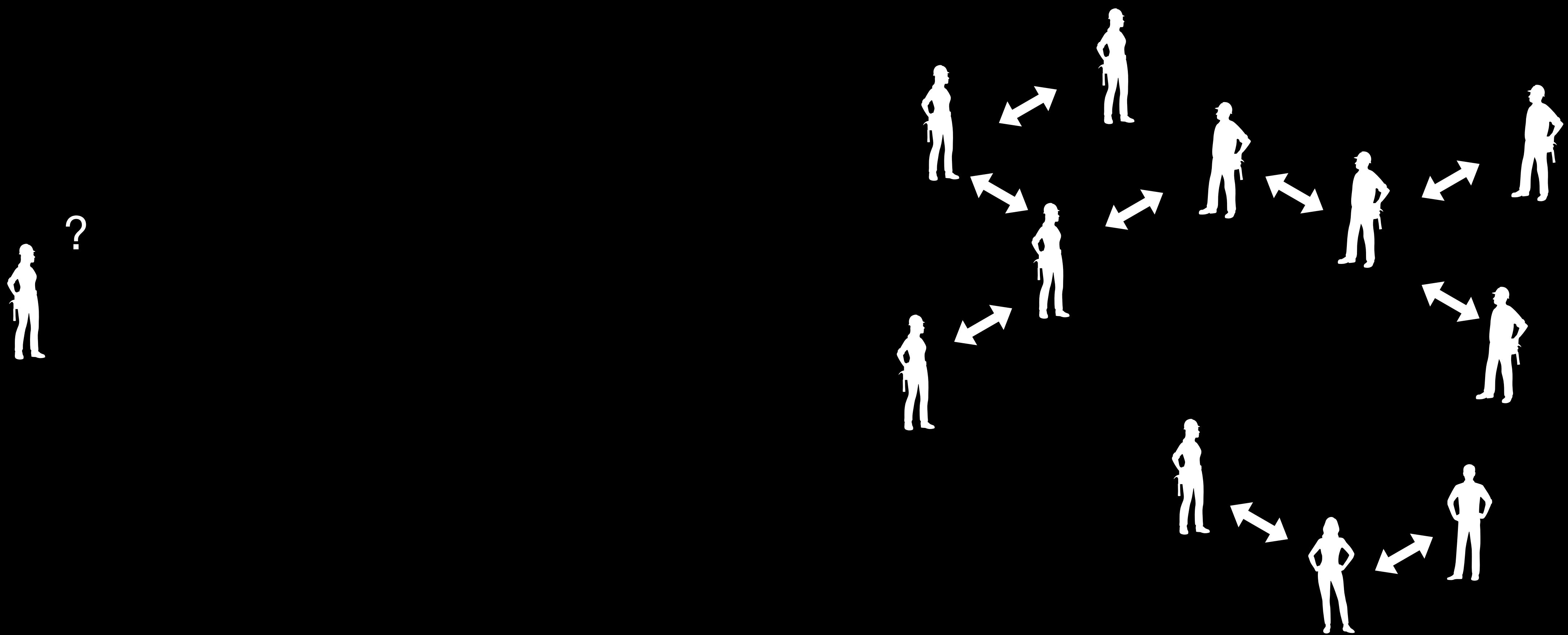
Preferential Attachment

[Albert and Barabasi 1999]



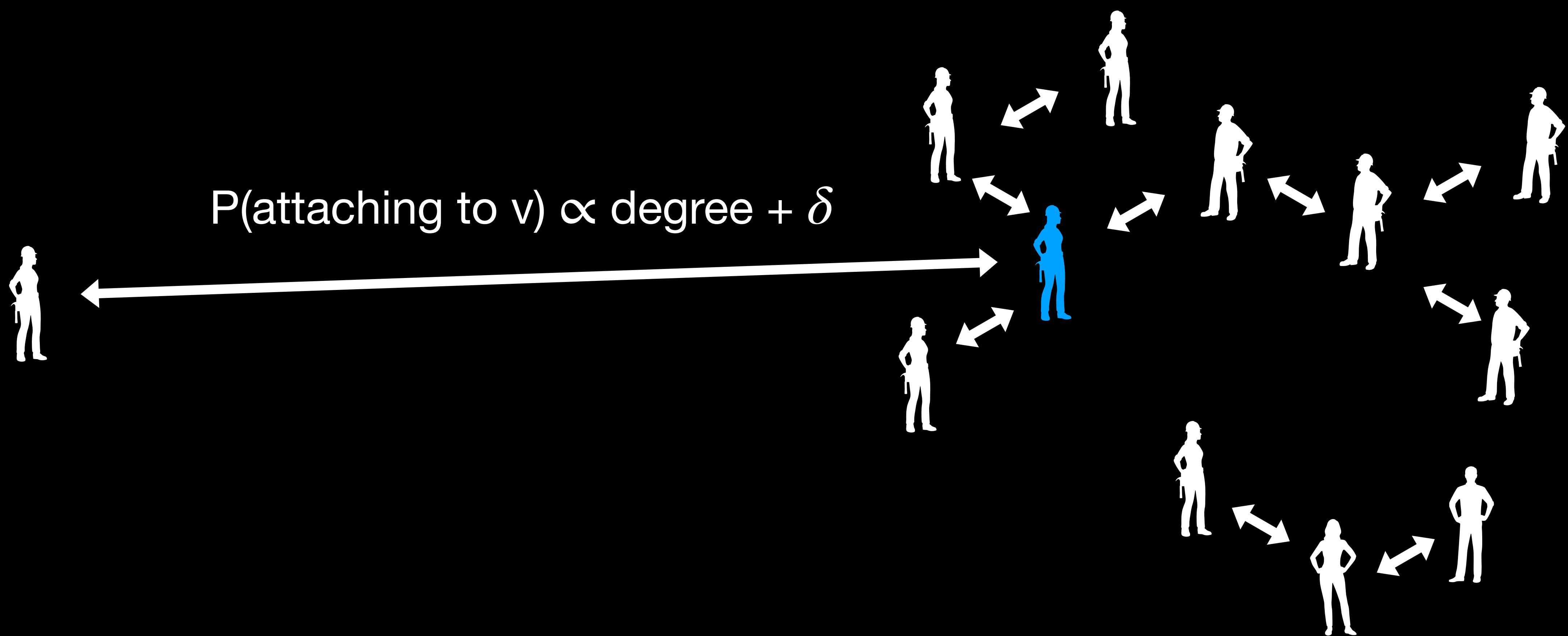
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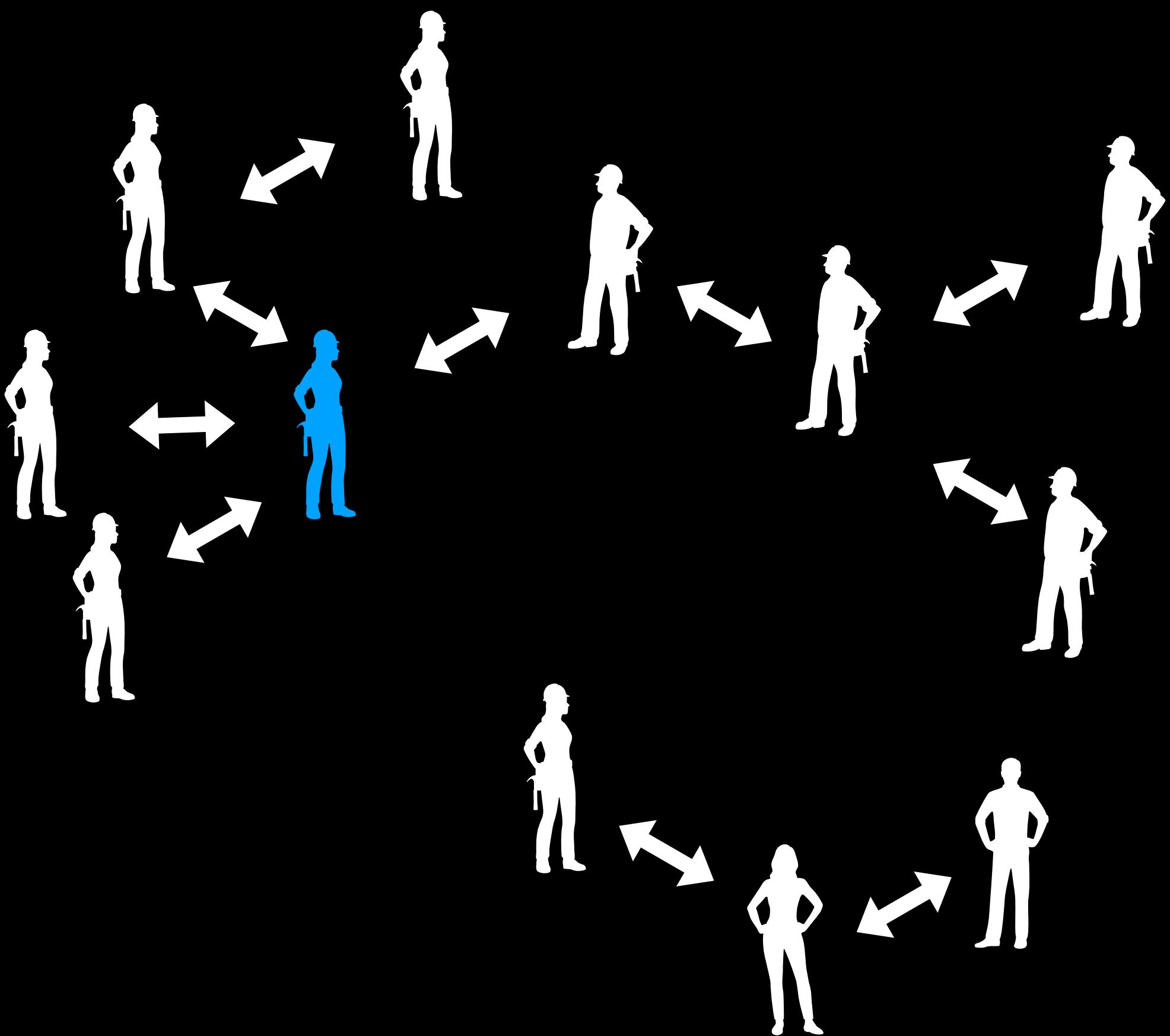
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Preferential Attachment

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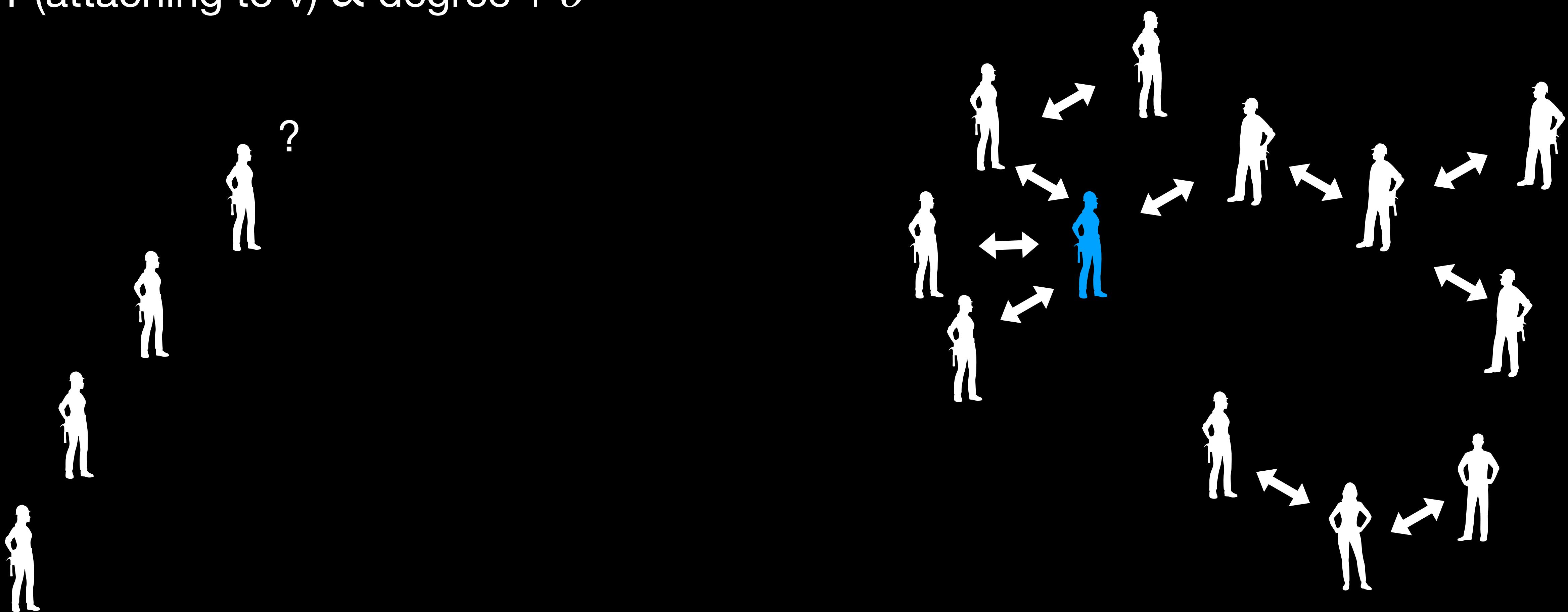
$$P(\text{attaching to } v) \propto \text{degree} + \delta$$



Preferential Attachment

[Albert and Barabasi 1999]

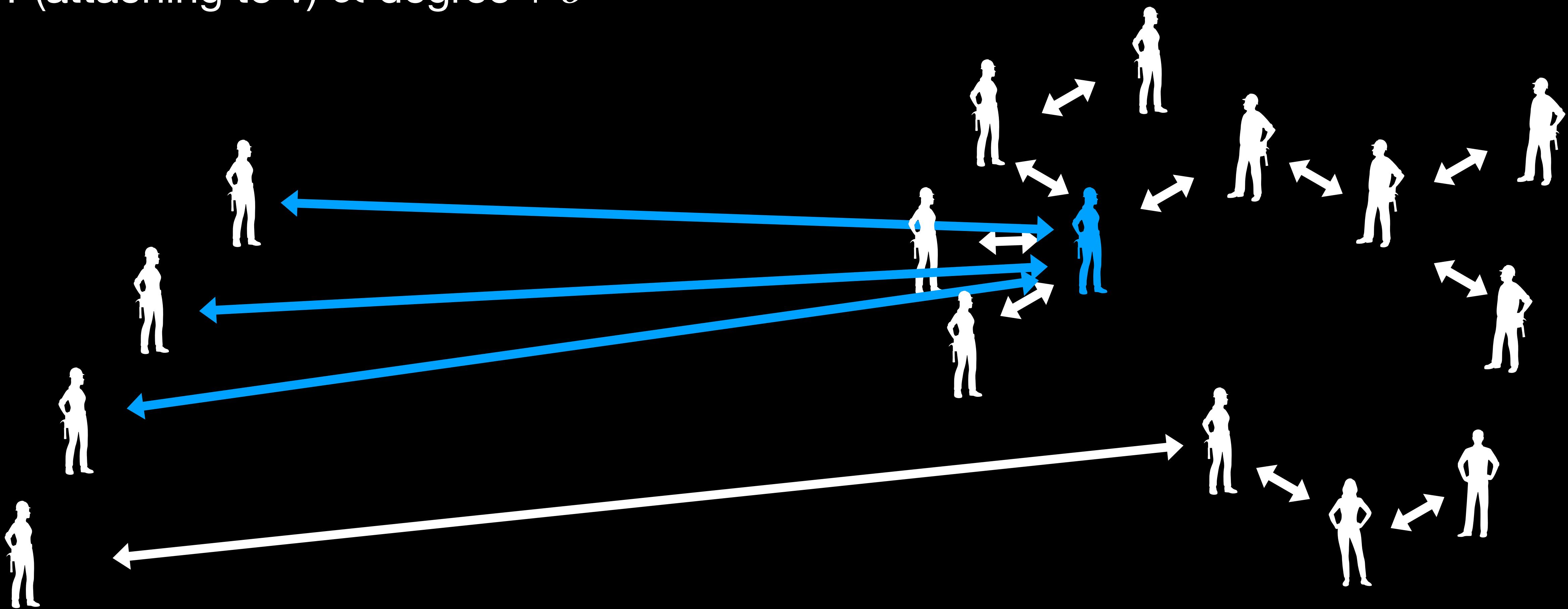
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Preferential Attachment

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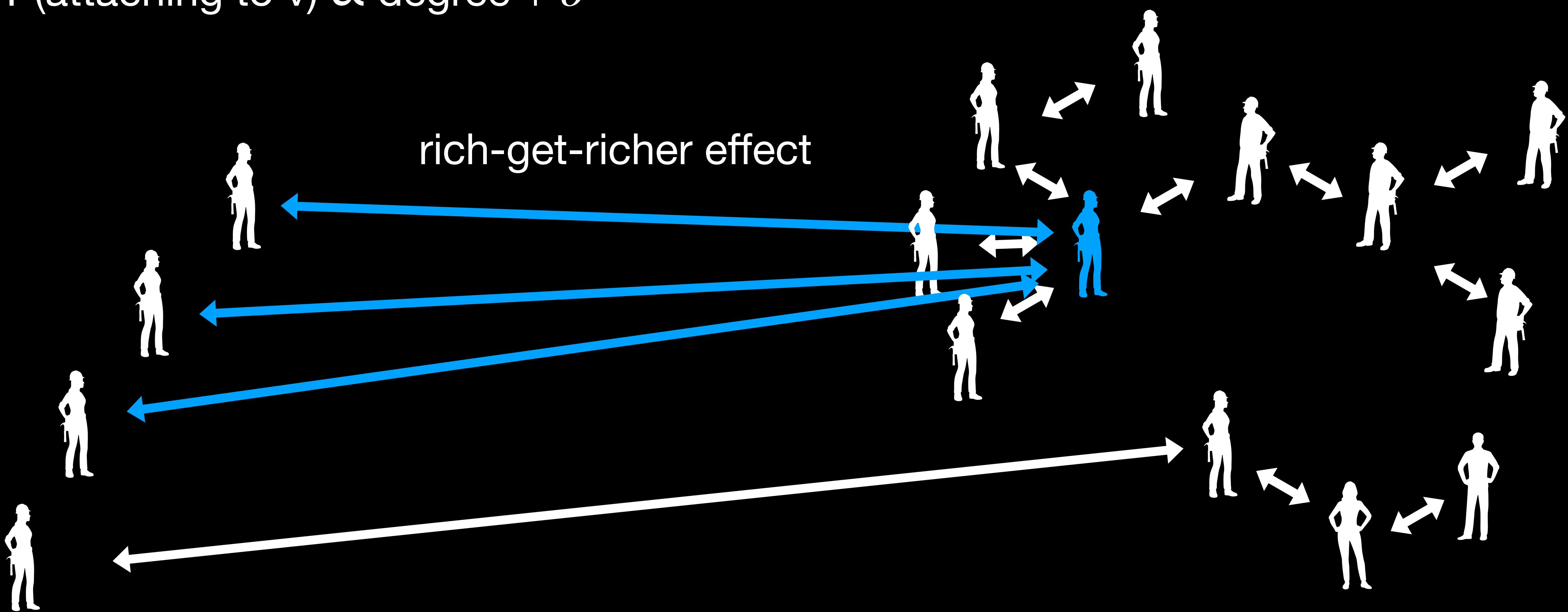
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Preferential Attachment

[Albert and Barabasi 1999]

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What do we know?

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- heavy tailed degree distribution [Albert and Barabasi 1999]

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- triangle counts and clustering coefficient [Bollobas and Ridden 2002, Prokhorenkova et al 2013]

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- triangle counts and clustering coefficient [Bollobas and Ridden 2002, Prokhorenkova et al 2013]
- subgraph counts [Garavaglia and Steghuis 2019]
- and more...

Triangles, Tetrahedra and Topology

Triangles, Tetrahedra and Topology

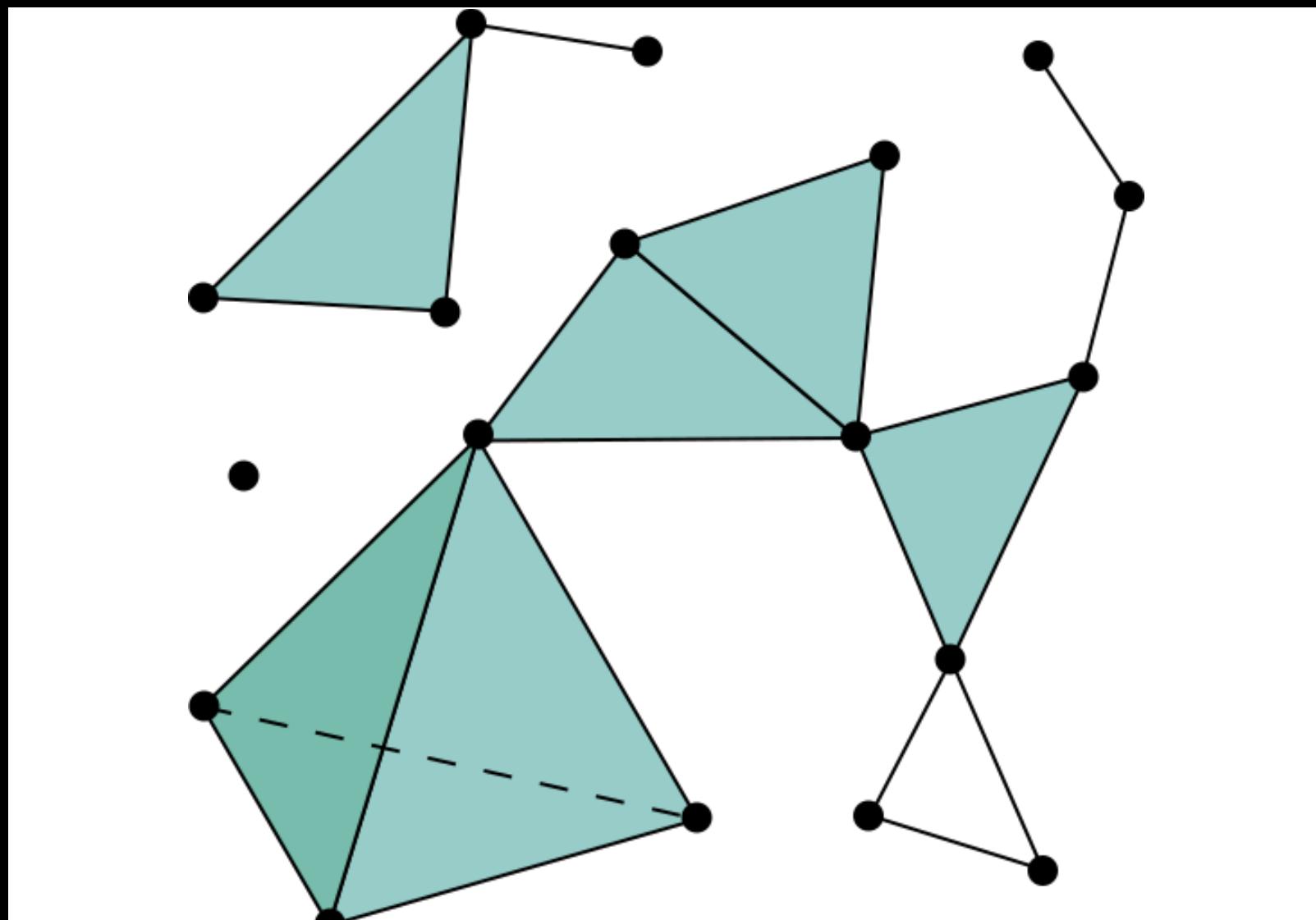


image credit: calm

Triangles, Tetrahedra and Topology

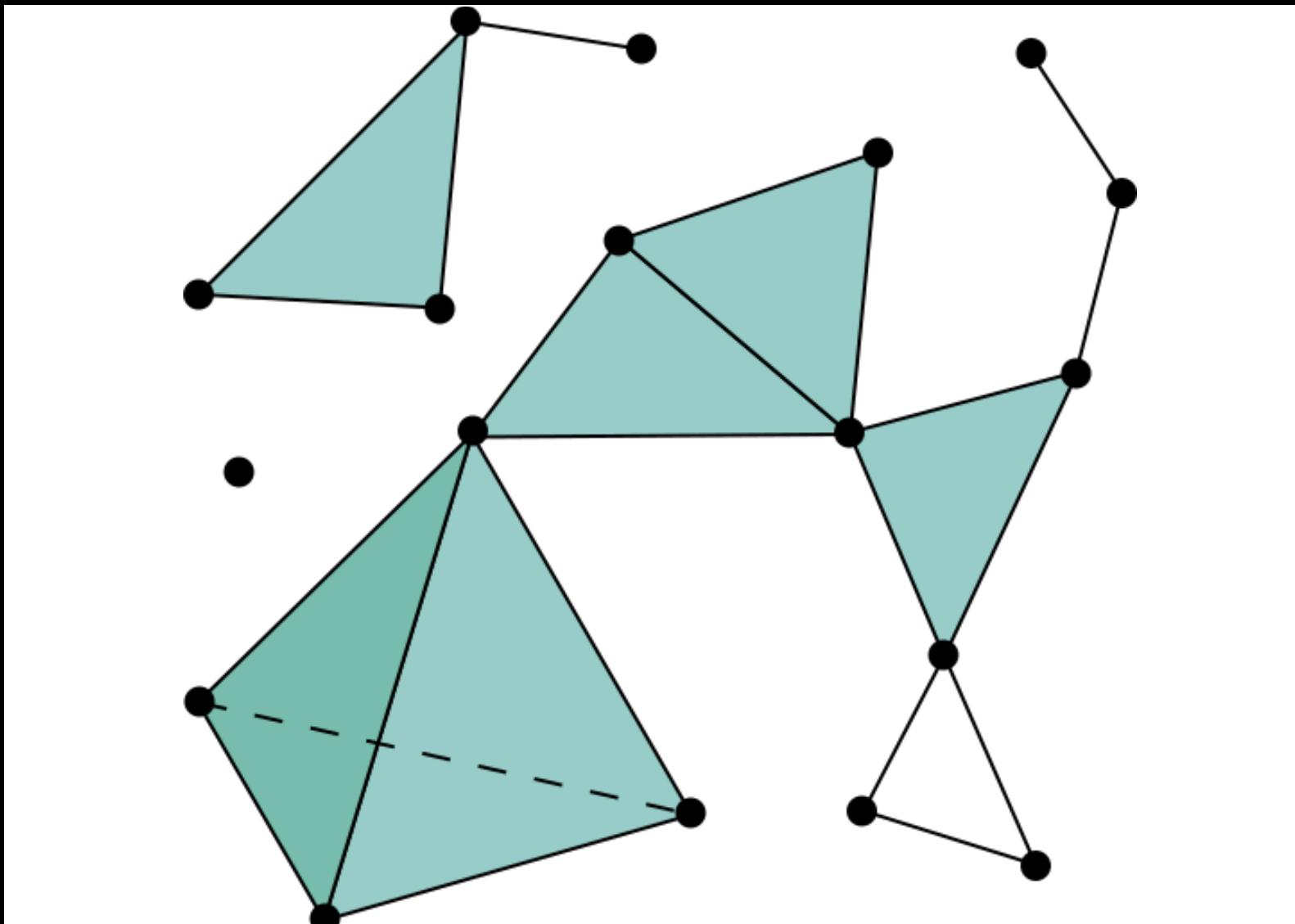


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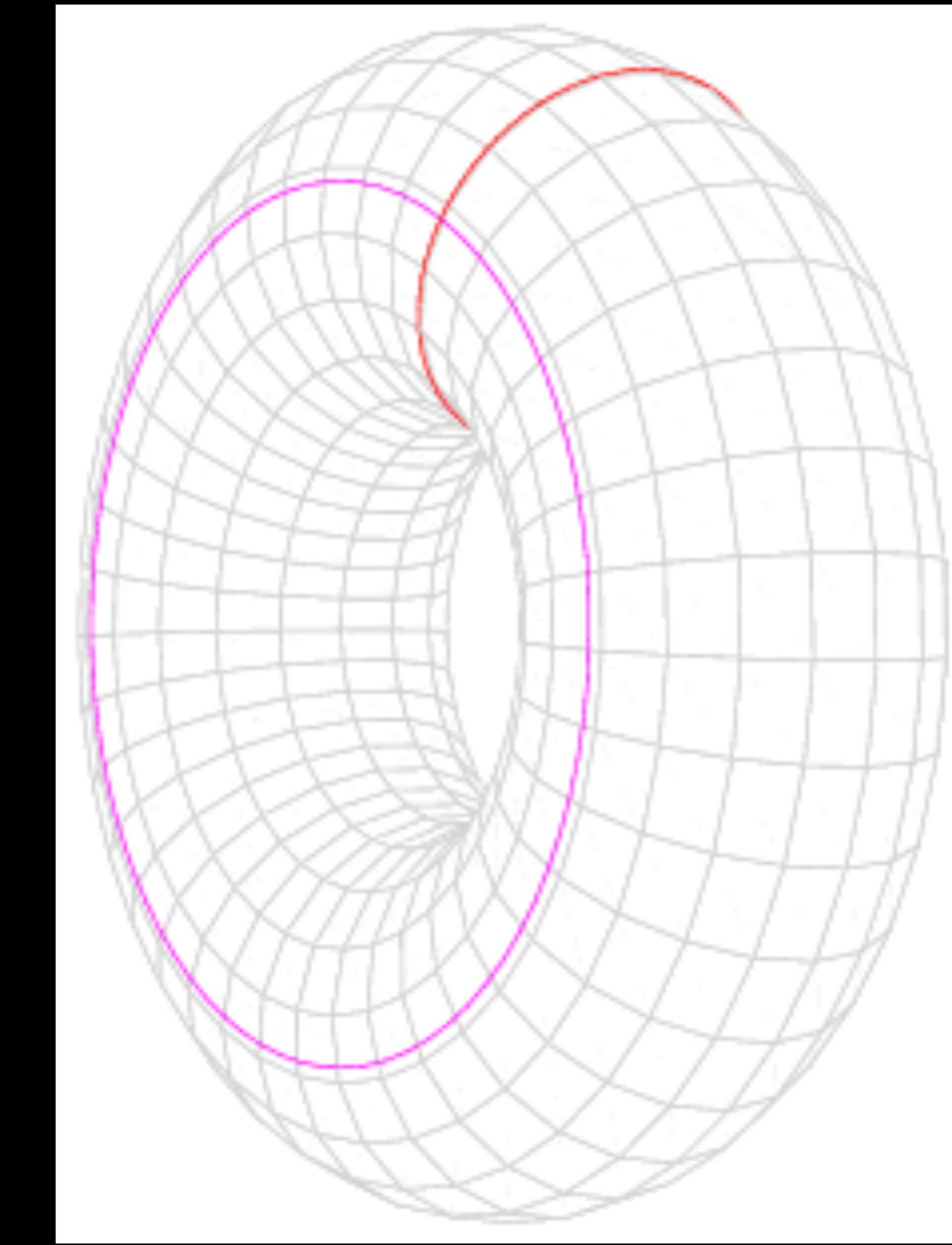


image credit: Krishnavedala

Who cares?

Examples of academic networks

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Examples of academic networks

- Holes are repeated pathways. [Patania, Petri and Vaccarino 2017]

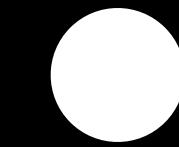
Who cares?

Examples of academic networks

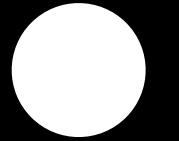
- Holes are repeated pathways. [Patania, Petri and Vaccarino 2017]
- Unifying concepts fill holes. [Salnikov et al 2018]

The Topology of the Citation Network

The story of Venus



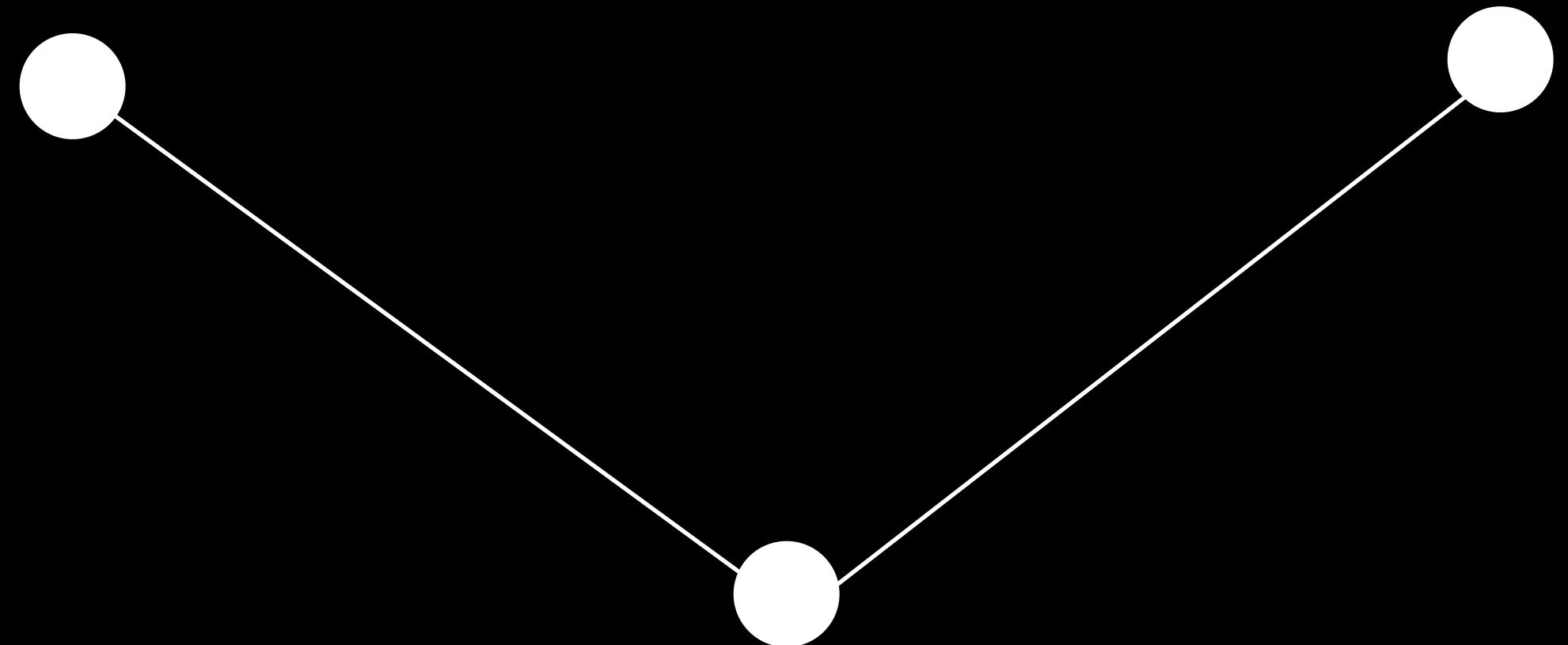
morning star paper



evening star paper

Unification merges components.

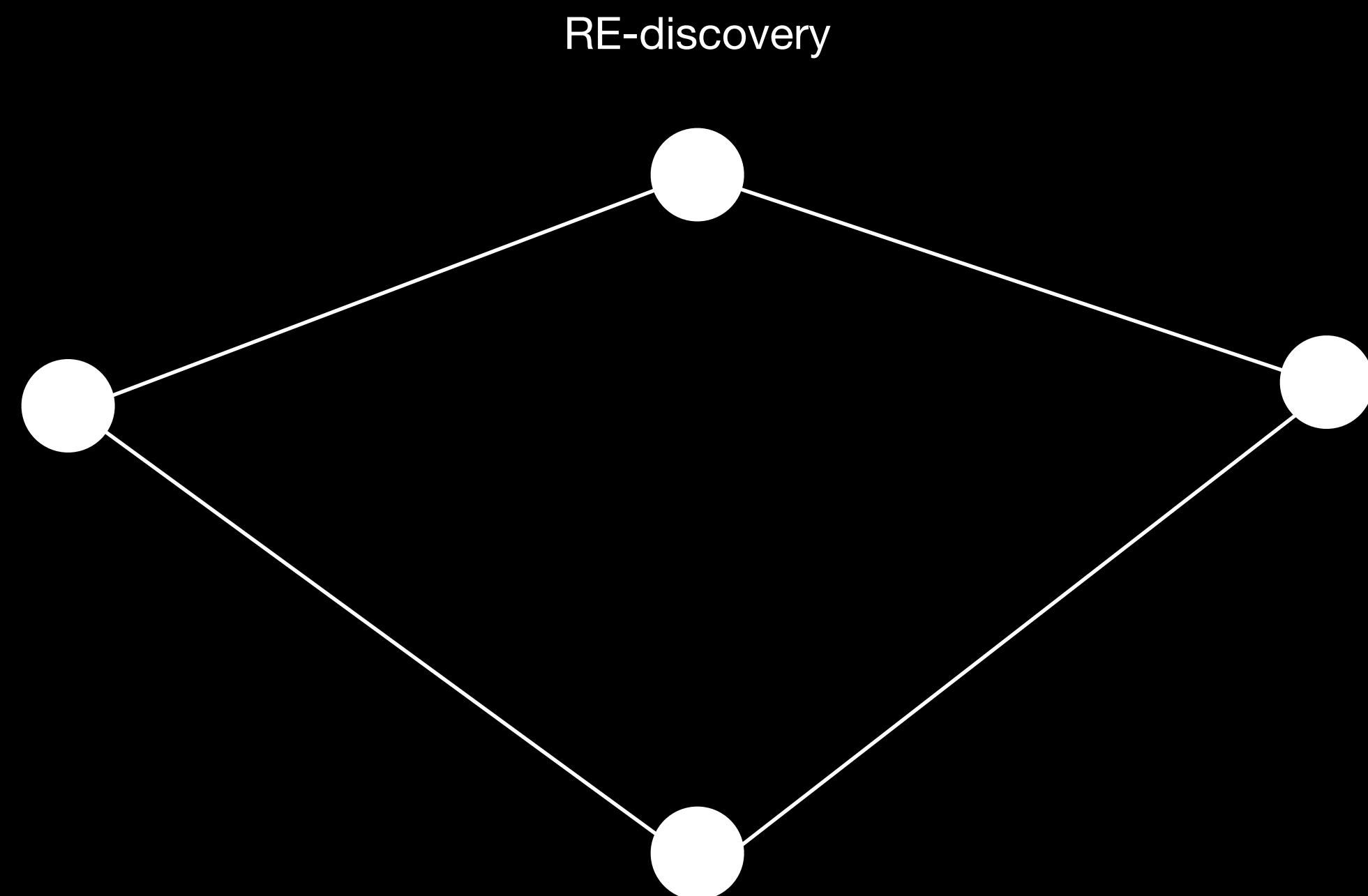
Venus' citation network



discovery: both are Venus

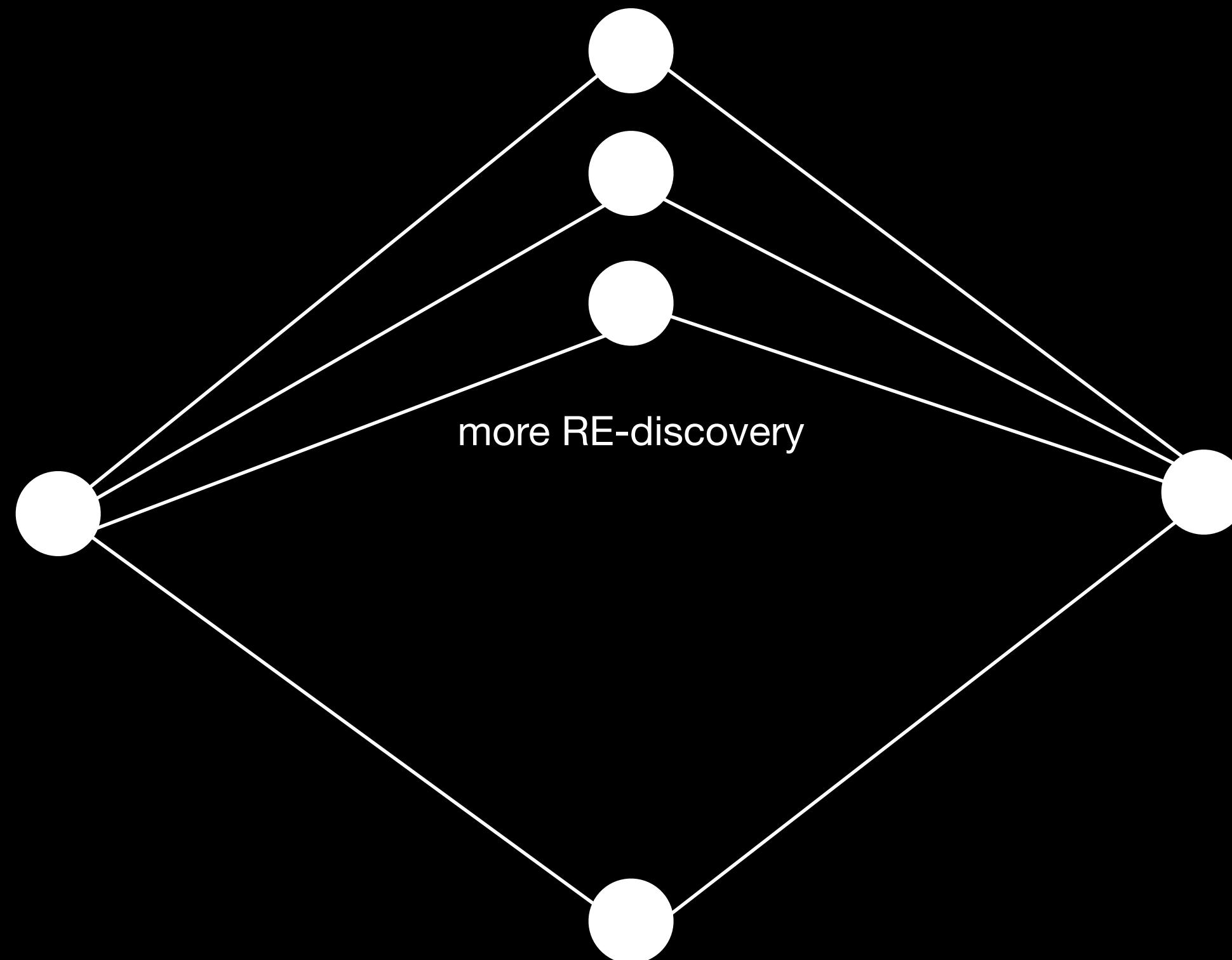
Re-unification creates a loop.

Venus' citation network



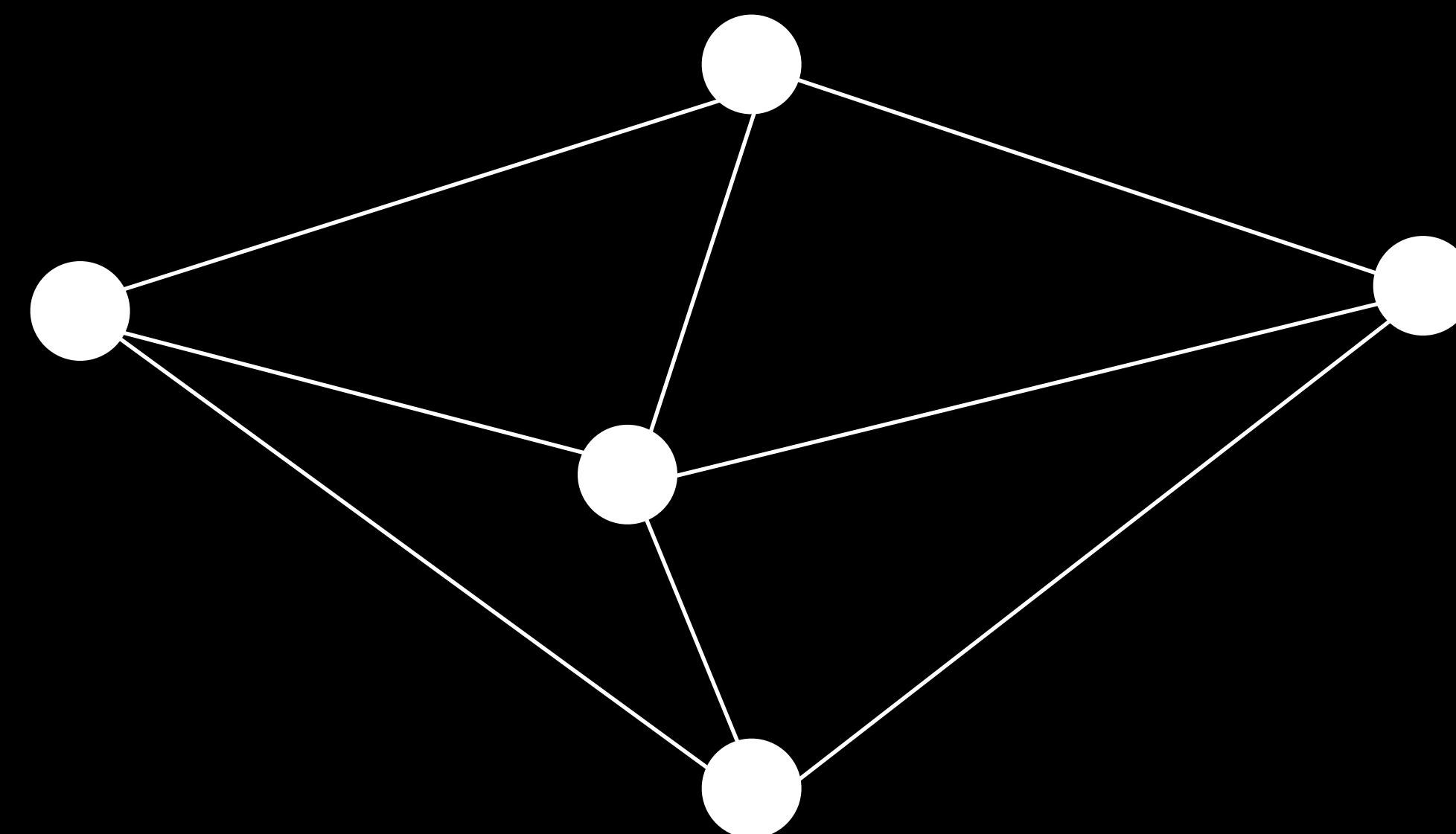
More re-unification creates more loops.

Venus' citation network



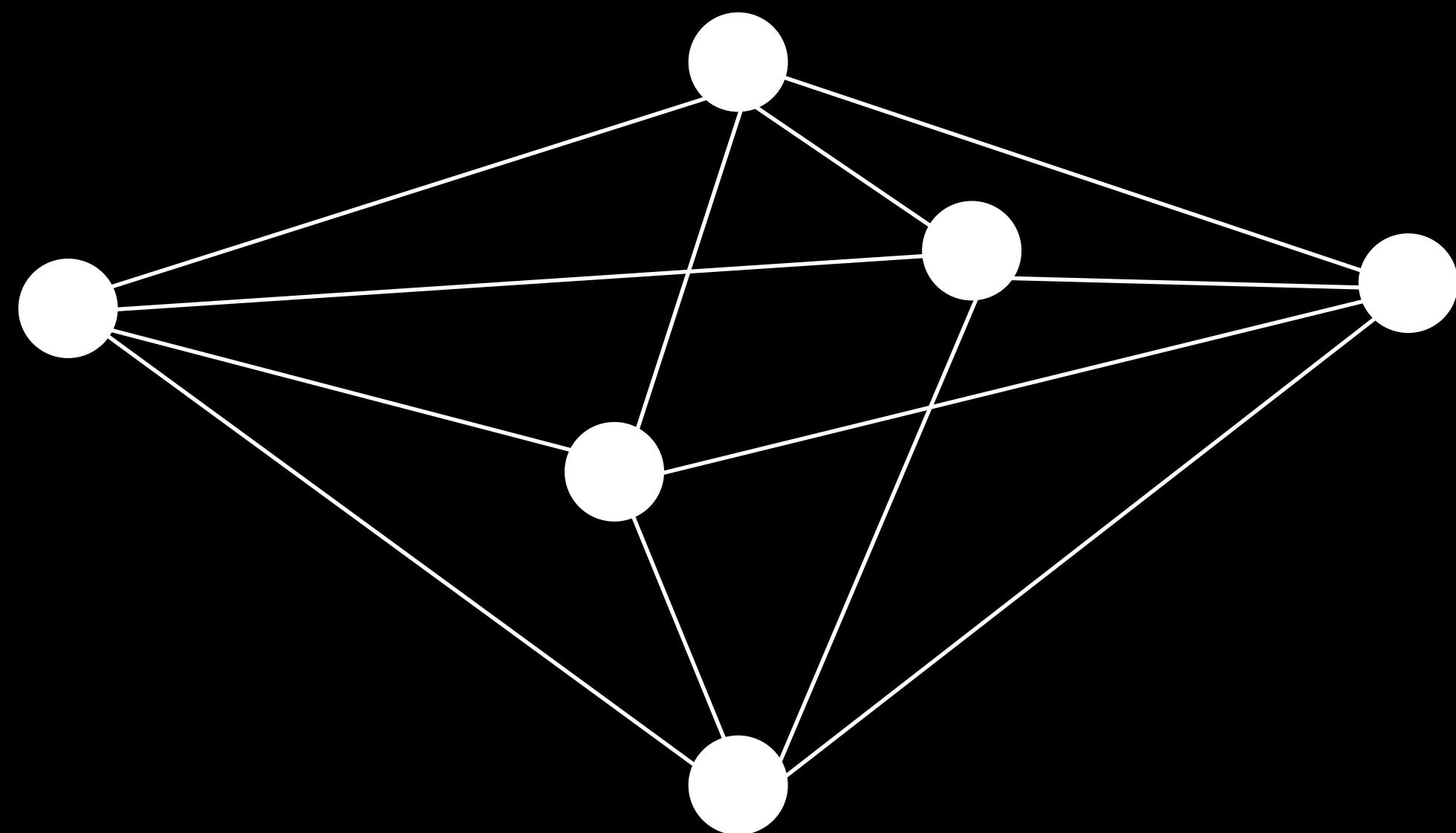
Hyper-unification reduces the number of loops.

Venus' citation network



Re-hyper-unification creates a cavity.

Venus' citation network



betti numbers

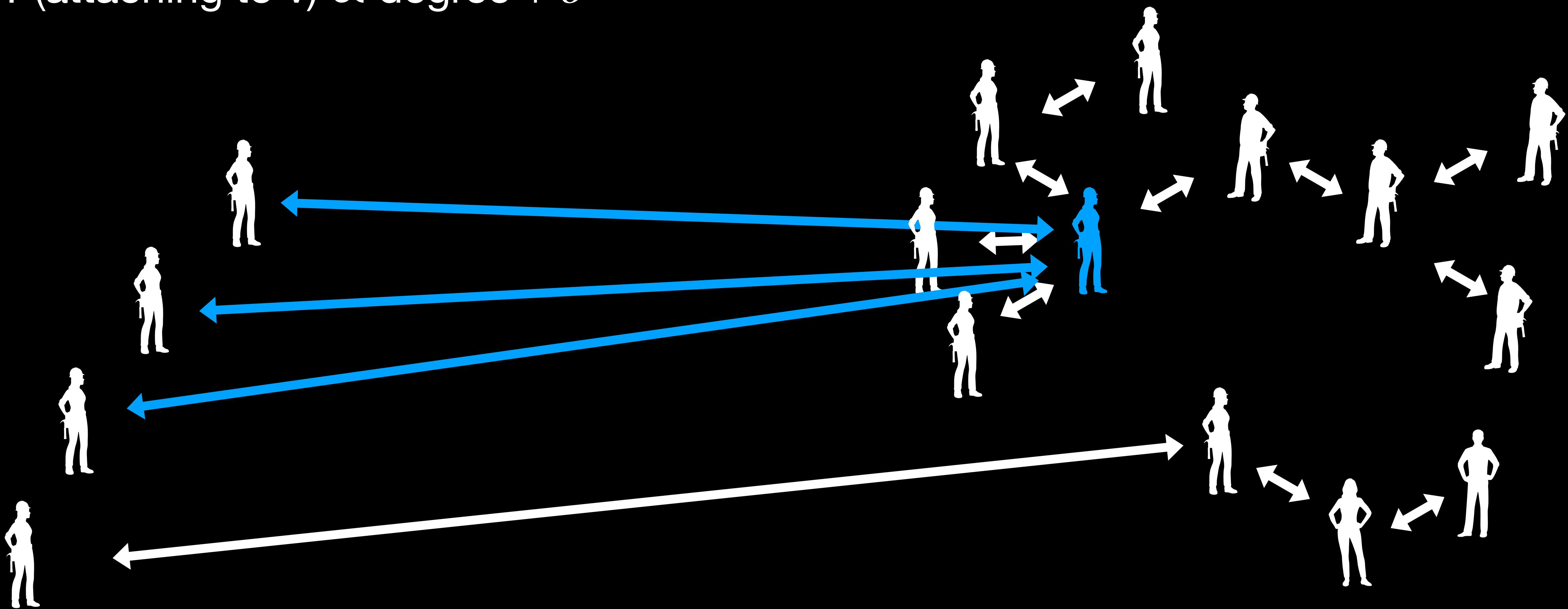
- = connected component and holes
- = repeated higher-order connections

Betti numbers and preferential attachment

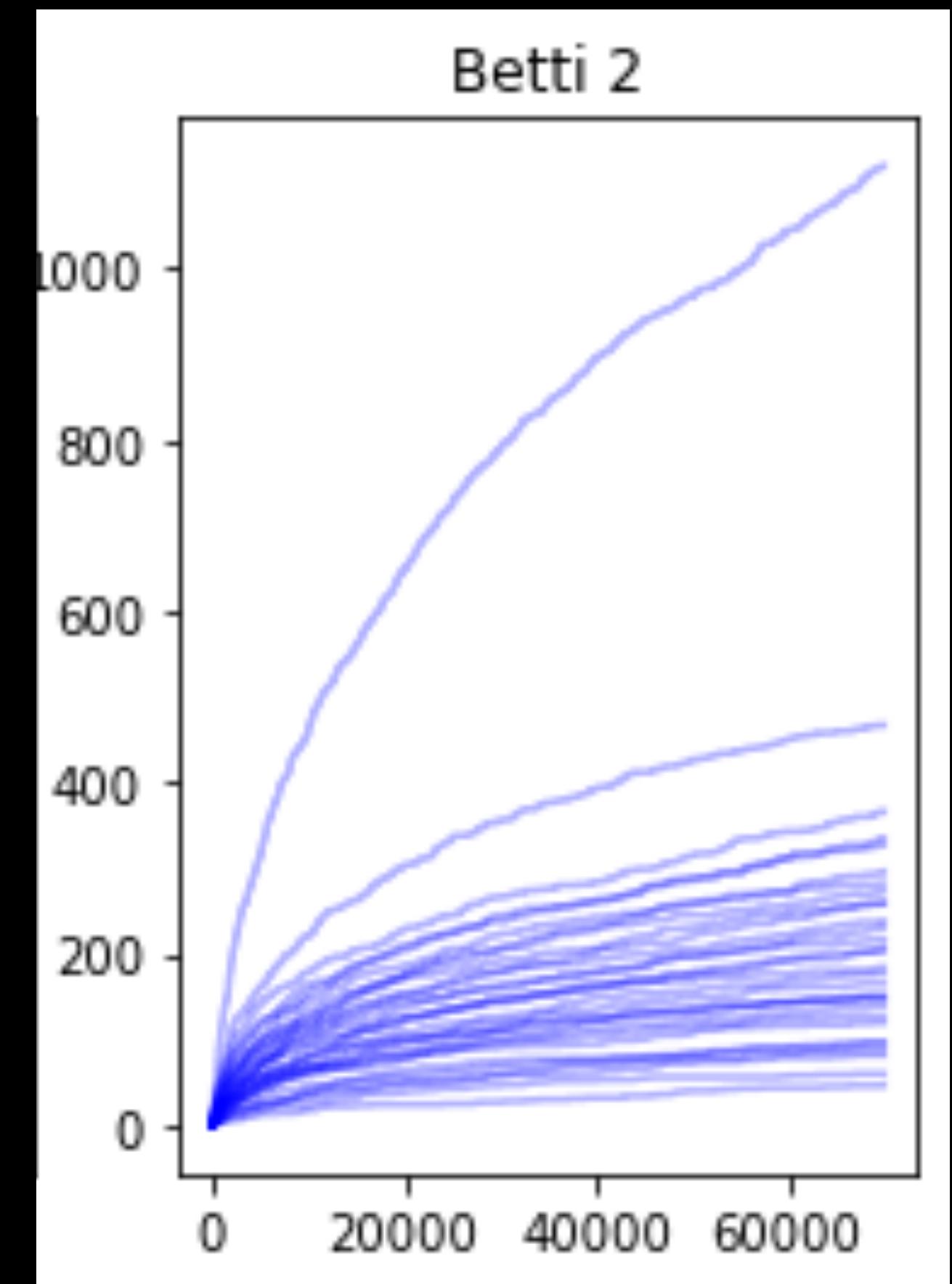
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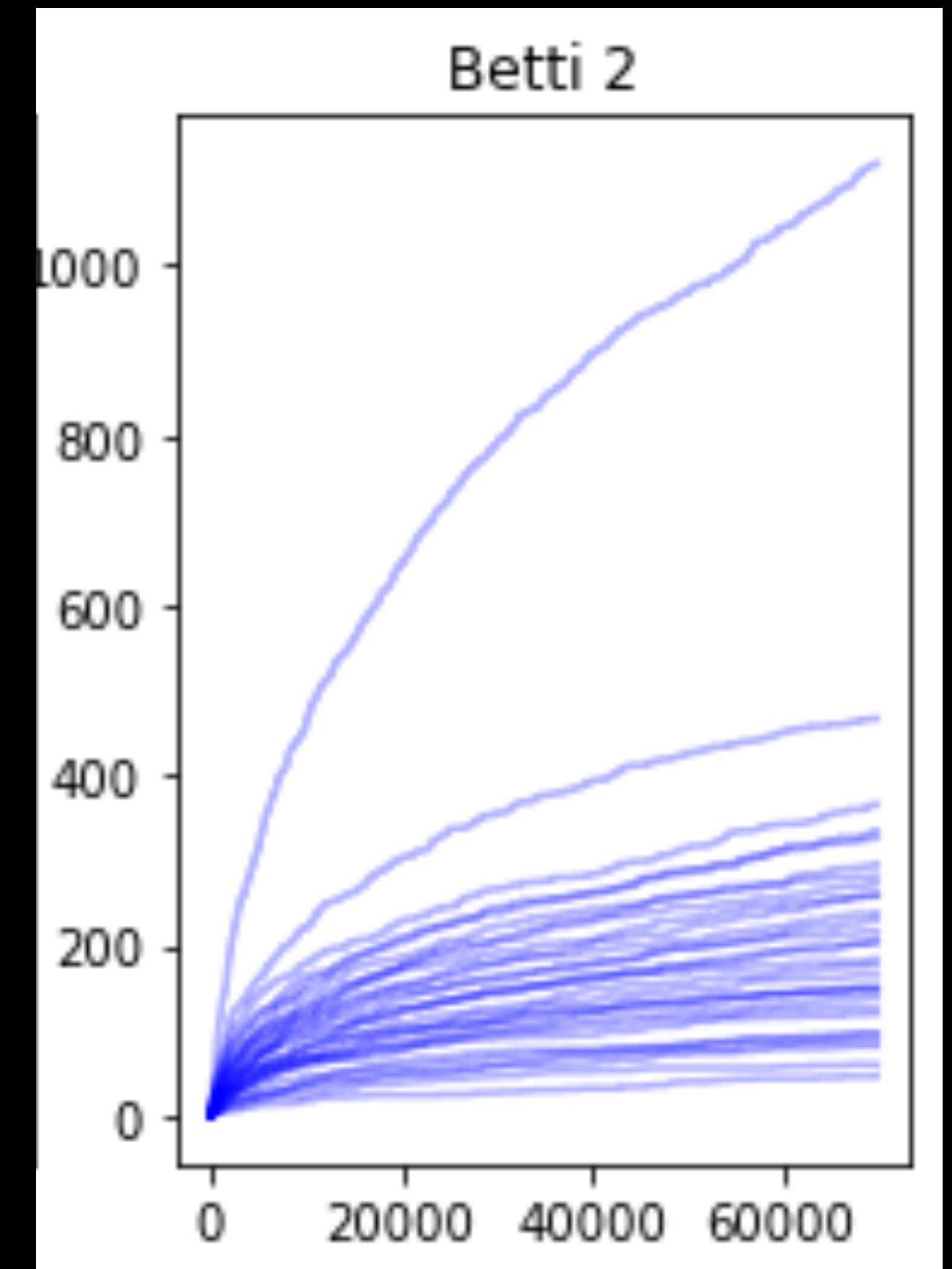


Expected Betti Number $E[\beta_q]$



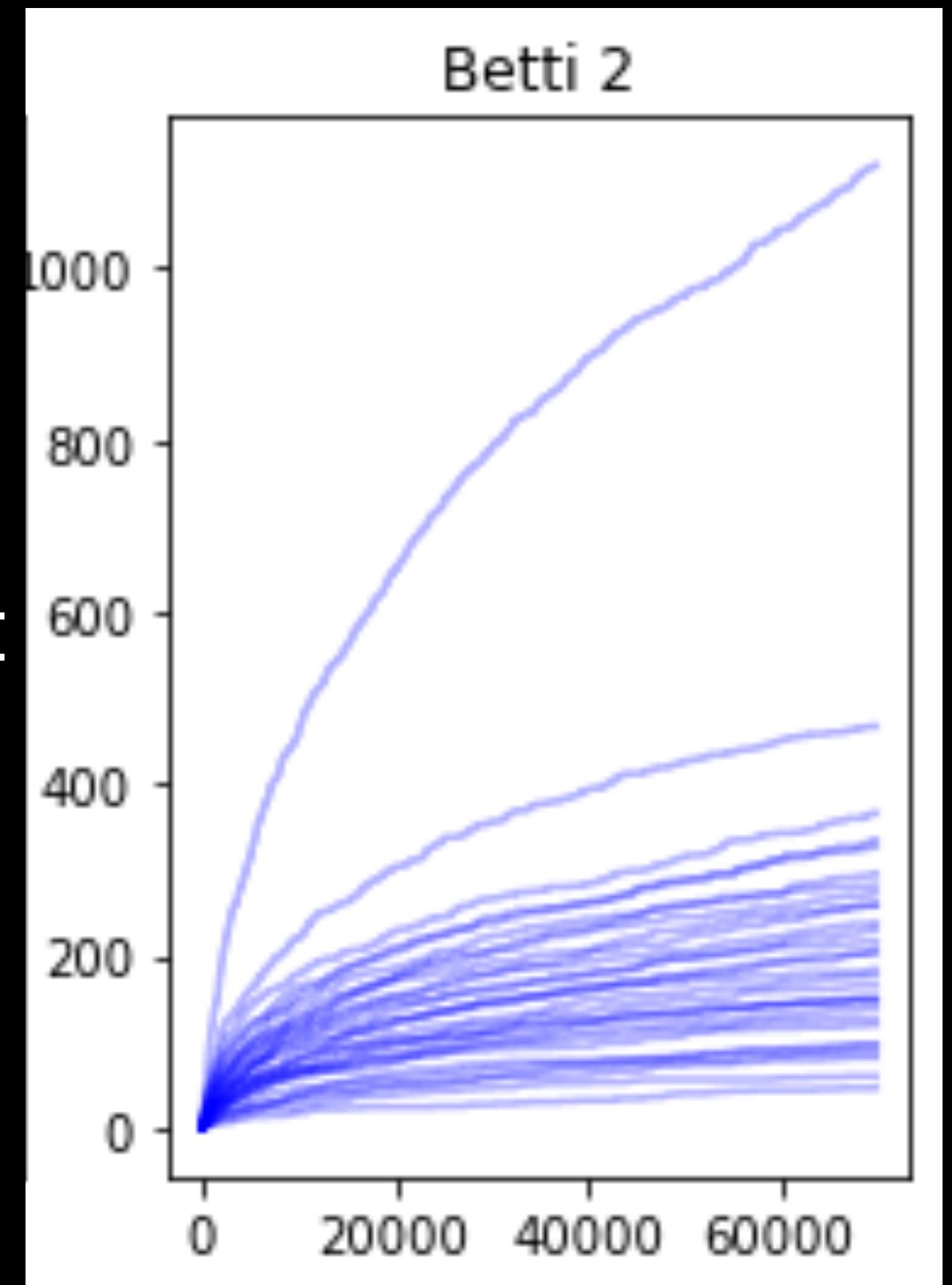
Expected Betti Number $E[\beta_q]$

- $E[\beta_2] = \Theta(\text{num of nodes}^{1-4\chi})$ under mild assumptions
- $\chi = 1 - \frac{1}{2 + \delta/m} \in (0, 1/2)$
- small χ :
 - heavier degree tail
 - stronger rich-get-richer effect



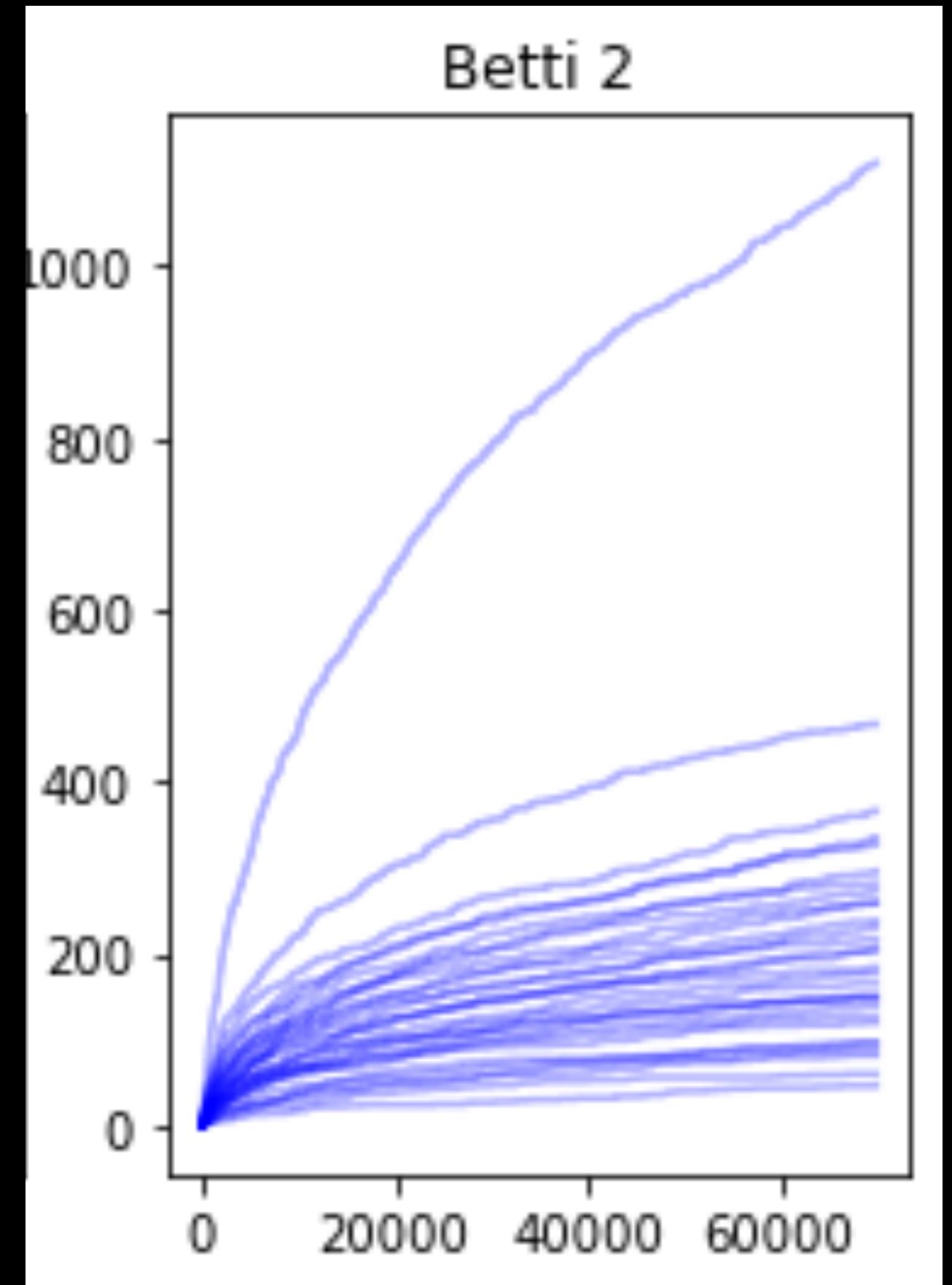
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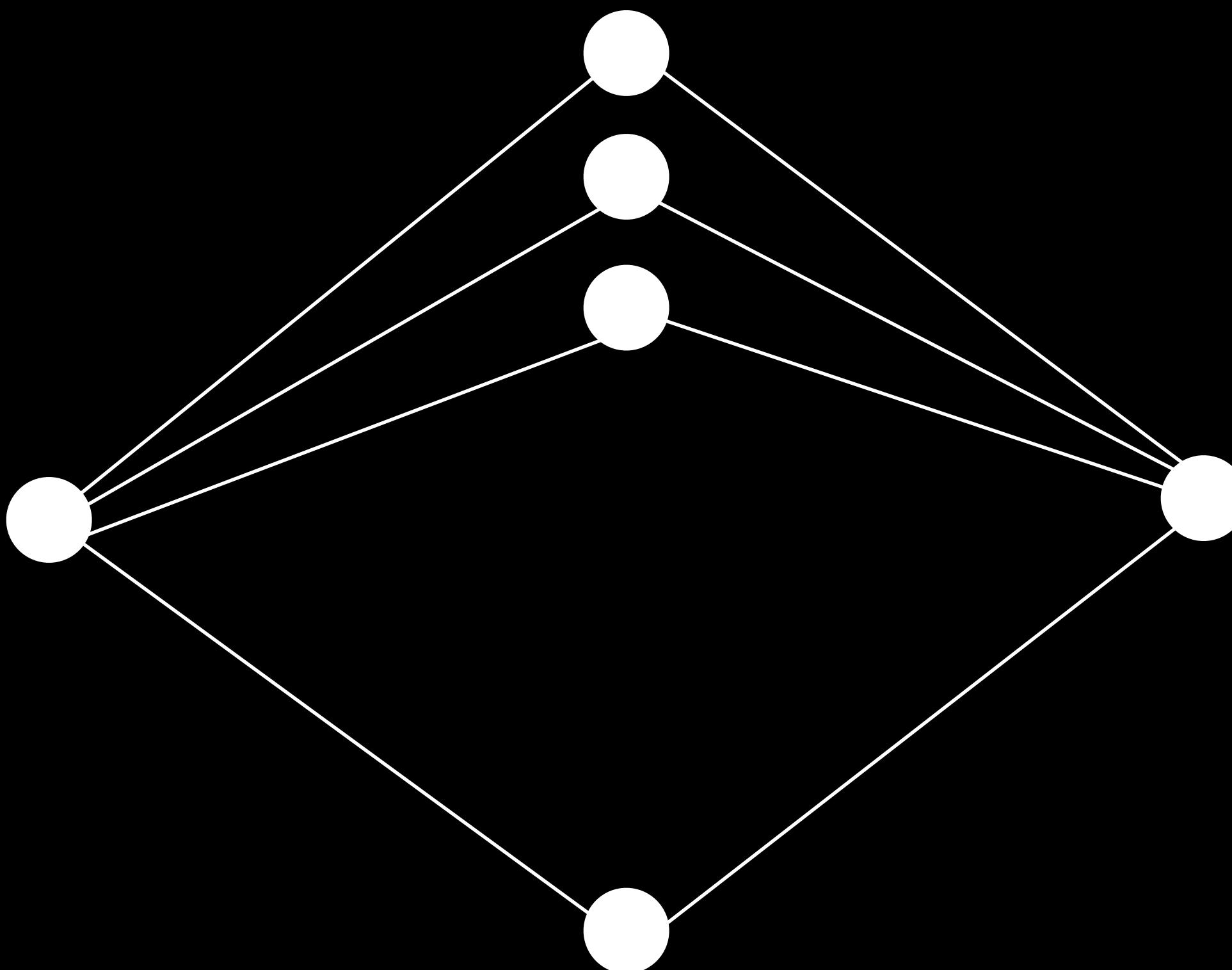


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 - small χ : heavier tail and stronger rich-get-richer effect
- $E[\beta_q] = \Theta(\text{num of nodes}^{1-2q\chi})$ for $q \geq 2$
- $E[\beta_q]$ decreases as dimension q increases
- $E[\beta_q]$ increases with the rich-get-richer effect



Main Idea



What's next?

- Tail of betti numbers?
- robustness and betti numbers?

Thank you

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