Betti Numbers of Preferential Attachment Complexes

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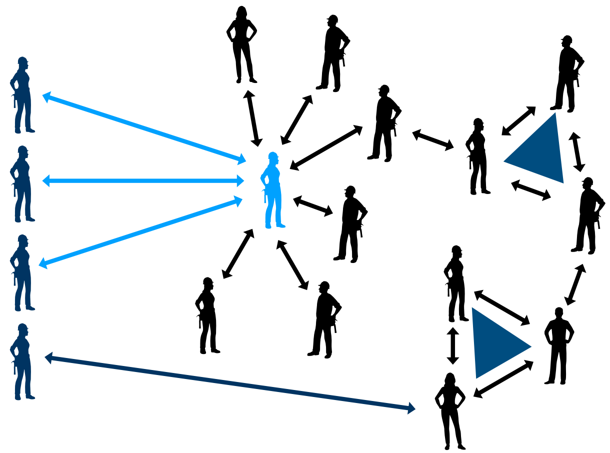
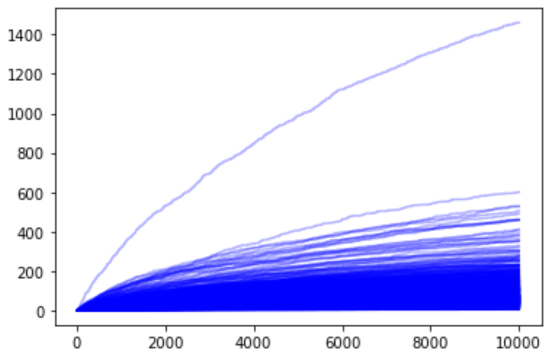
joint work with Gennady Samorodnitsky, Christina Yu and Caroline He

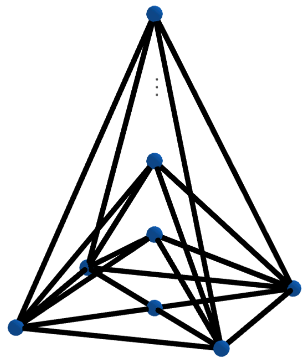
Preferential Attachment Clique Complexes

- inductively built **random graph** with ***T* nodes**
- Each node *v* is connected to ***m* previous nodes**
- $P(v \to j) \propto$ deg $j + \delta$, with **tuning parameter** $\delta \in (-m,0)$
- Collapse repeated edges and build **clique complex**





evolution of β_2 as the number of nodes increases



repeatedly coned squares

Evolution of Betti Numbers

Dimension 1 $E[\beta_1] = (m-1)T + o(T)$

Higher-dimension Let $\chi = 1 - (2 + \delta/m)^{-1}$. For $q \geq 2$,

$$cT^{1-2q\chi} \leq E[\beta_q] \leq CT^{1-2q\chi}$$

for some constants *c*, *C* > 0 if $1 - 2q\chi > 0$ and $m \geq 2q$.

Intuition?

- **Coned squares** dominate. So do their higher-dimensional analogues.
- Boundaries are **rare** because they are more **complicated**.

How to Show?

- Localize the computation with a **mapping cone** argument.
- Characterize cycles by a **minimal-cycle** [Kahle 2009] argument.
- Apply **graph-counting** [Garavaglia and Steghuis 2019] arguments.

In Human Language?

- **sublinear** growth
- gradually decreasing **topological complexity**
- Complexity increases with the **rich-get-richer** effect.

What's Next?

- Tail behavior?
- Computable local invariants?

Cited Works on this Poster

- Garavaglia A. and Steghuis C.: Subgraphs in preferential attachment models. *Advances in Applied Probability*,51(3), 898 — 926 (2019).
- Kahle M.: Topology of random clique complexes. *Discrete Mathematics*, 309(6): 1658 — 1671.