

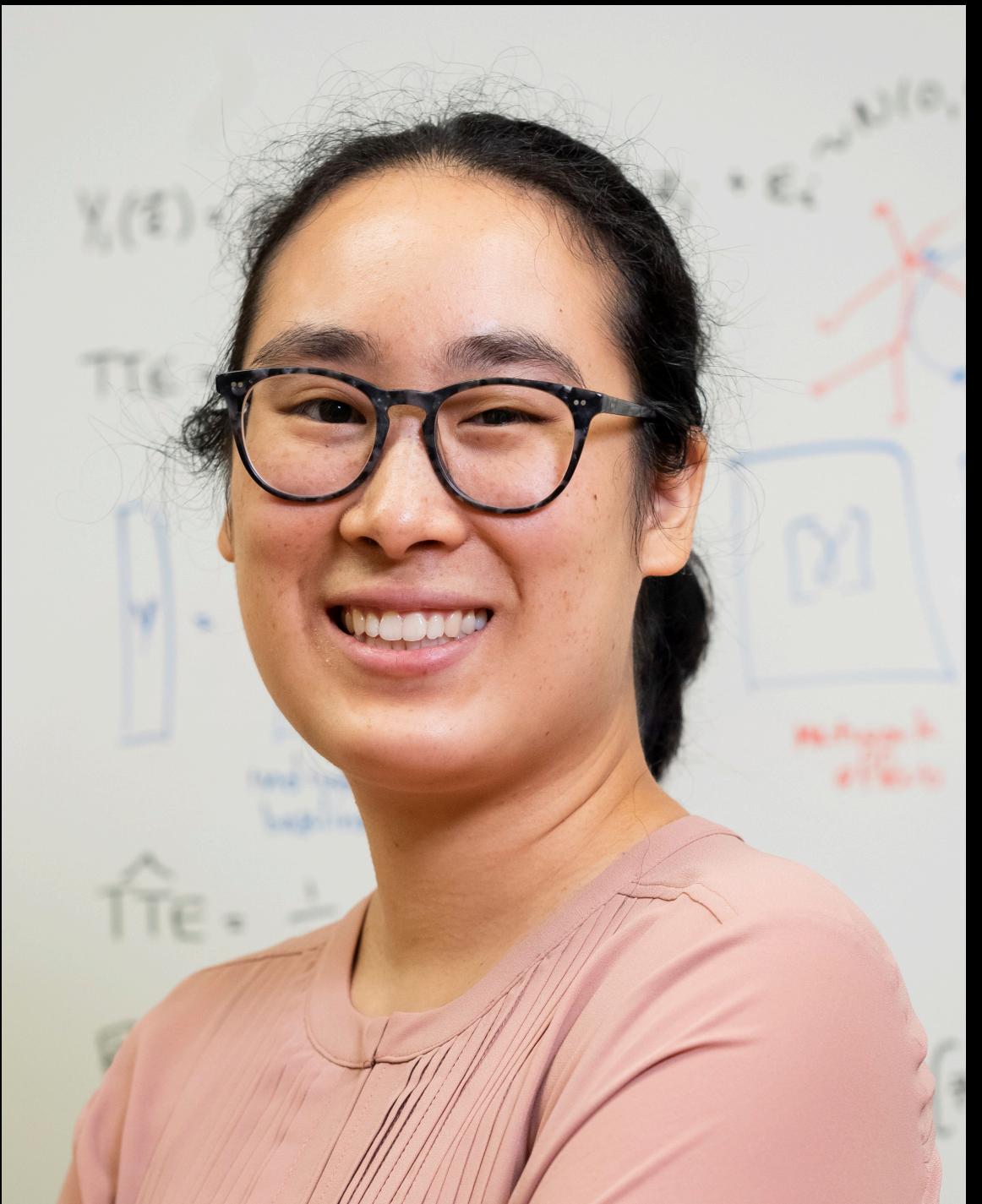
# Preferential Attachment and Homology

Evolution of Higher Dimensional Interactions

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Cornell University  
[cs2323@cornell.edu](mailto:cs2323@cornell.edu)



# Collaborators



Christina Lee Yu



Gennady Samorodnitsky



Caroline He

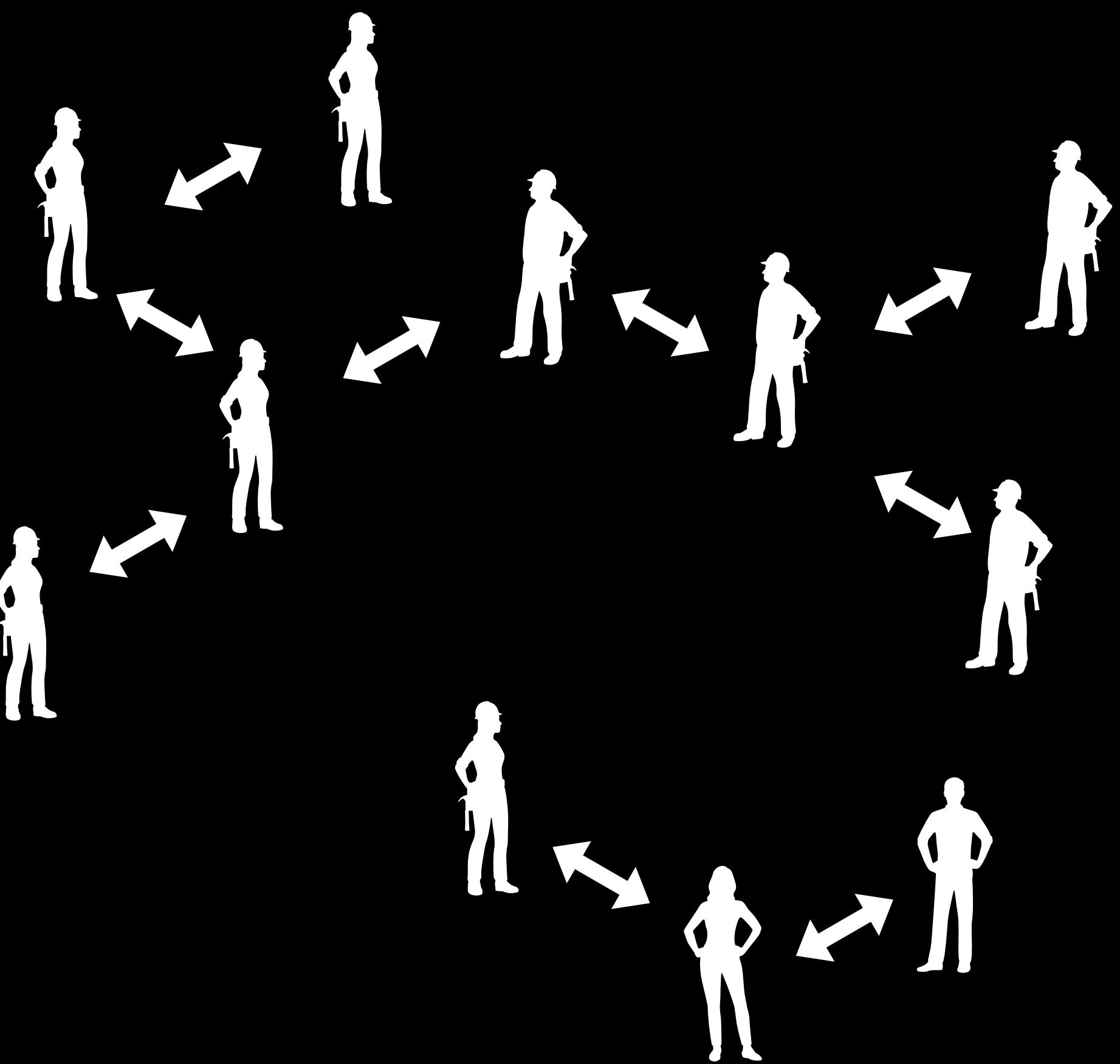
# Agenda

- What is preferential attachment?
- What is homology?
  - Why people care?
- What we know about the homology of preferential attachment complexes

# Preferential Attachment

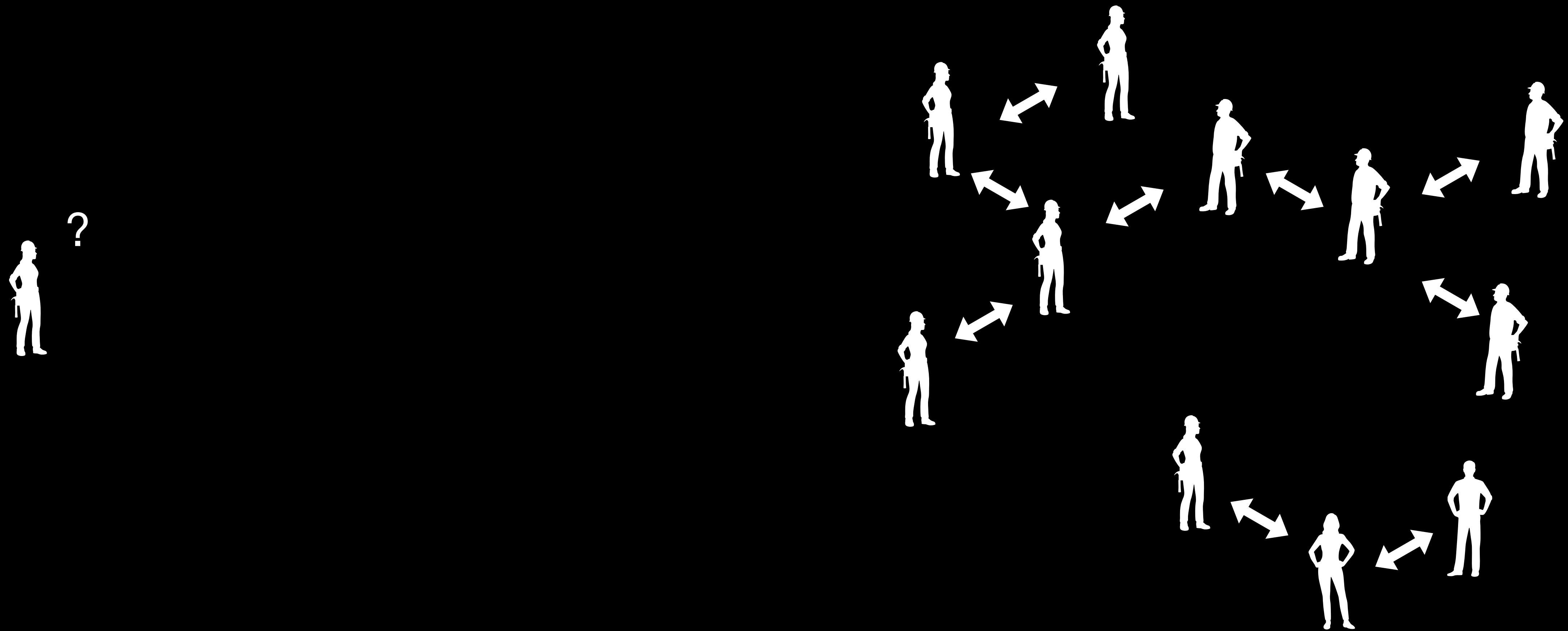
# Preferential Attachment

[Albert and Barabasi 1999]



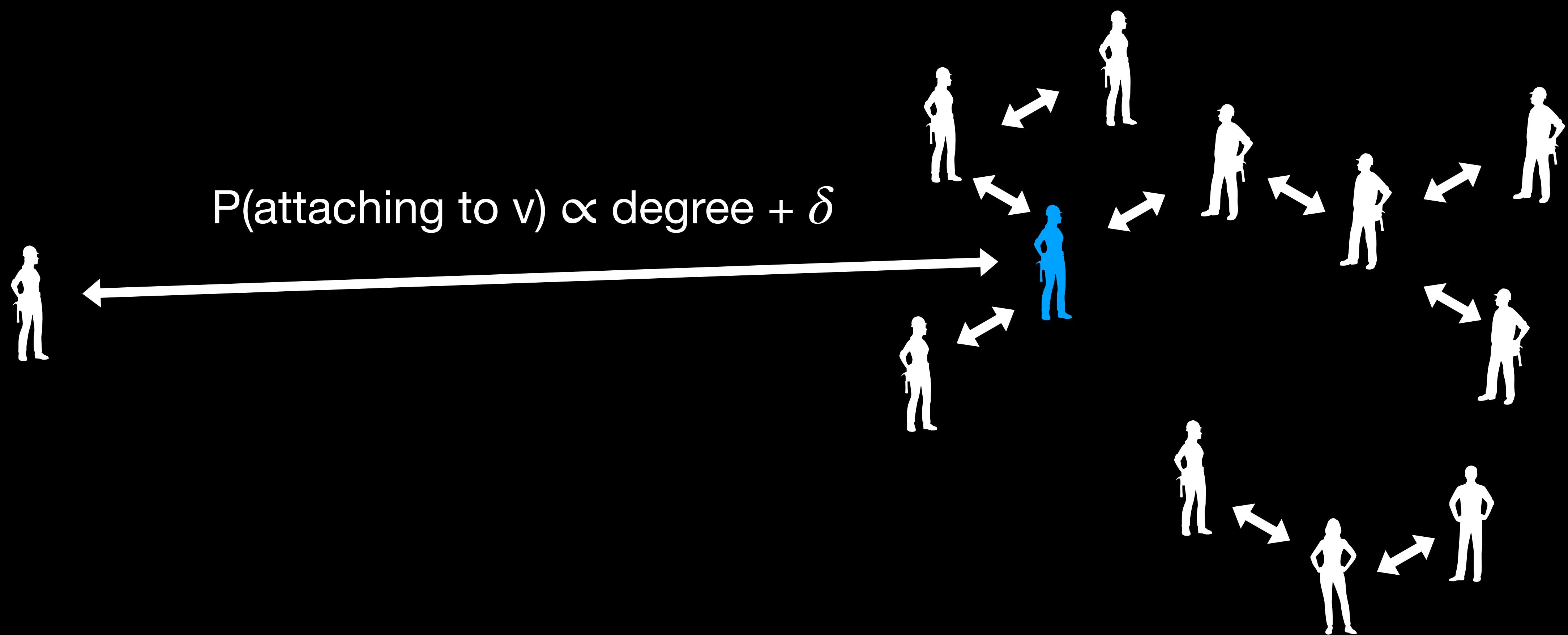
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[Albert and Barabasi 1999]



# Preferential Attachment

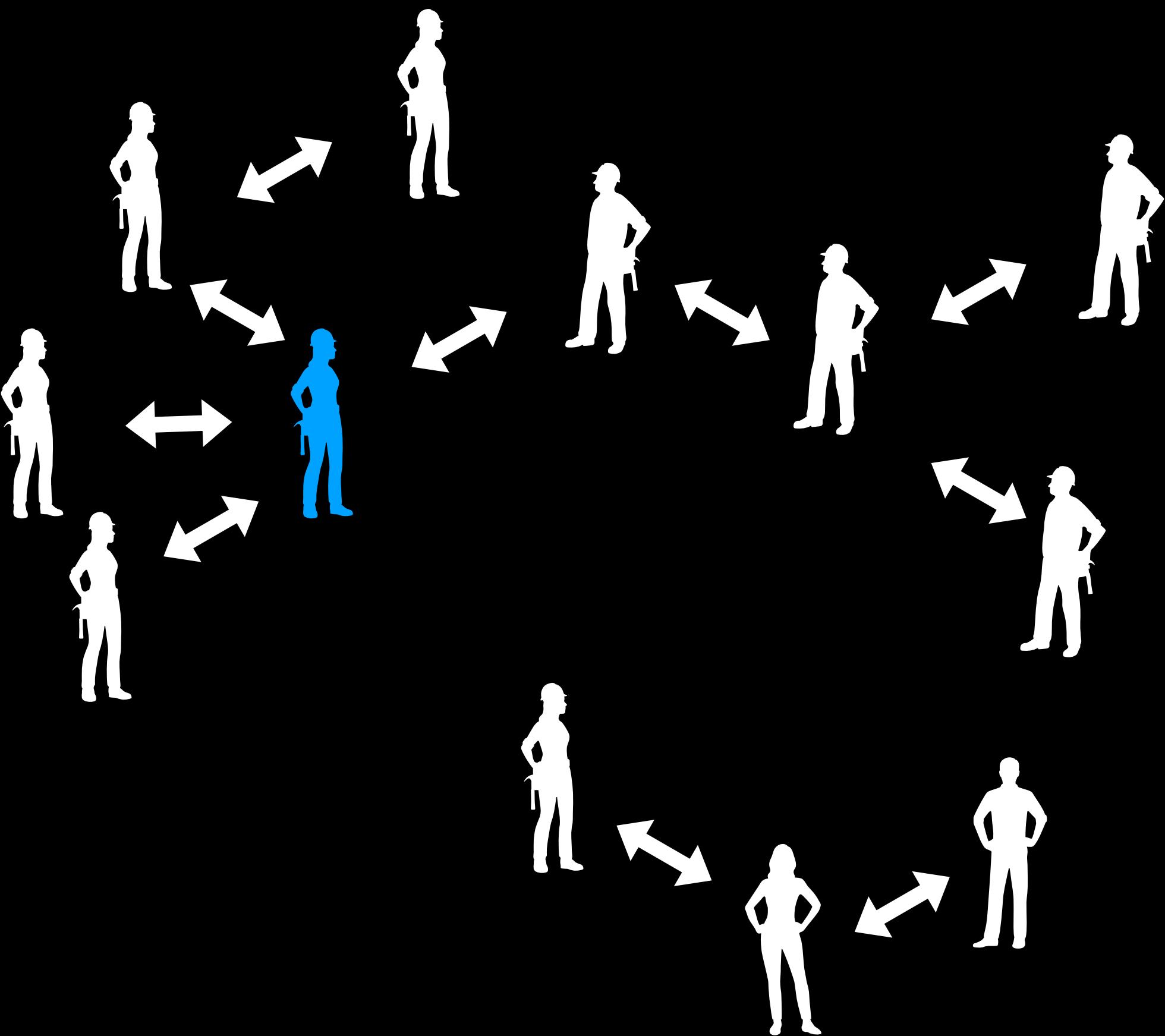
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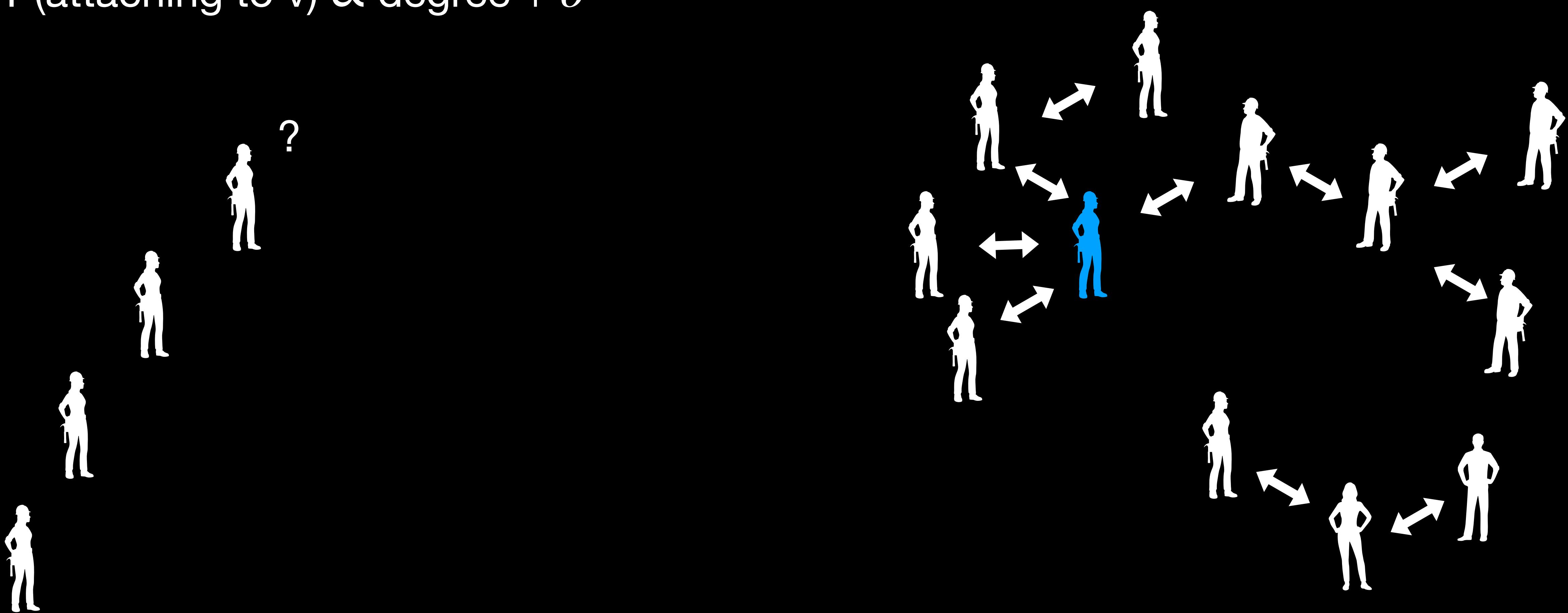
$$P(\text{attaching to } v) \propto \text{degree} + \delta$$



# Preferential Attachment

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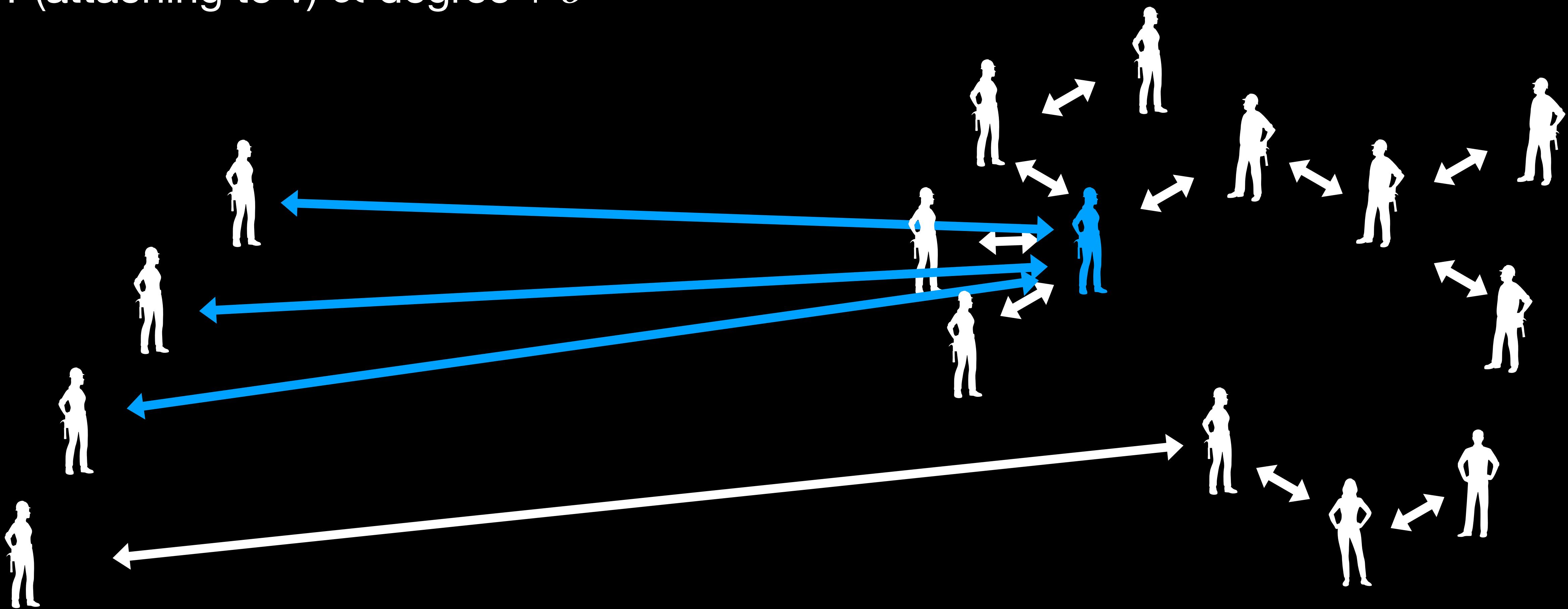
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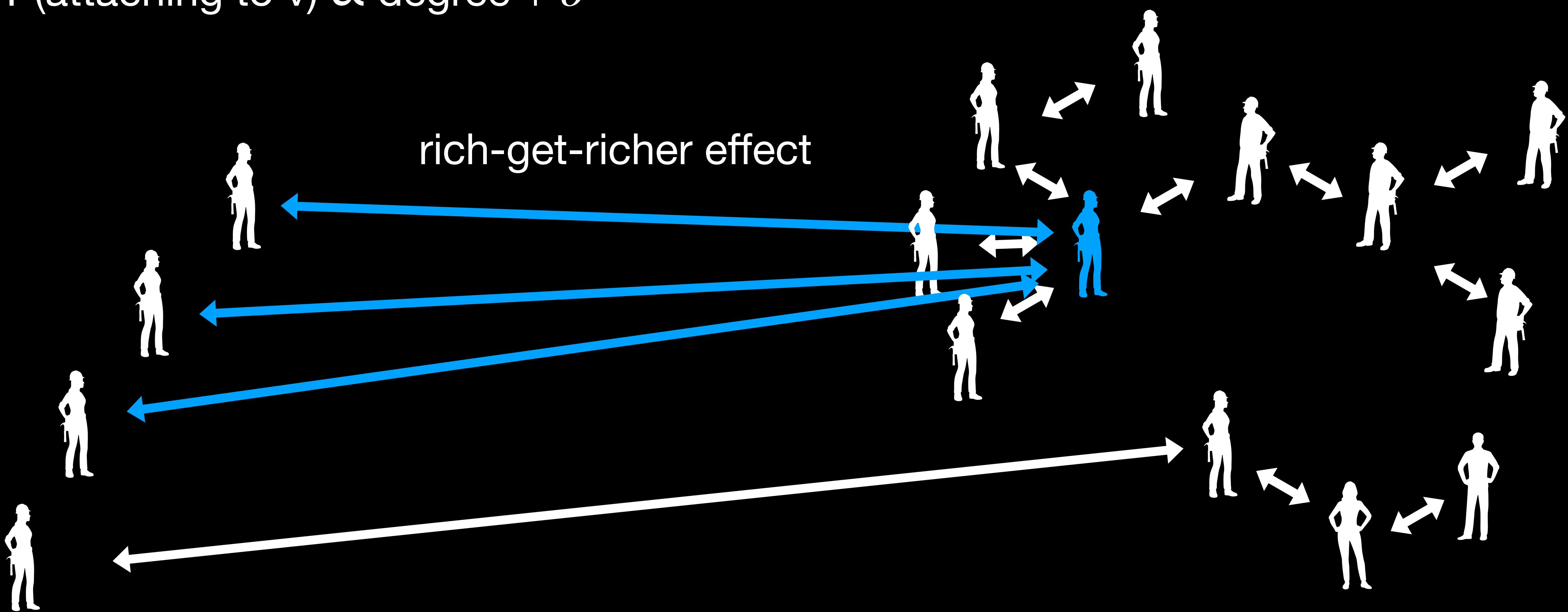
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# Preferential Attachment

[Albert and Barabasi 1999]

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# What do we know?

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- heavy tailed degree distribution [Albert and Barabasi 1999]

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- heavy tailed degree distribution [Albert and Barabasi 1999]
- triangle counts and clustering coefficient [Bollobas and Ridden 2002, Prokhorenkova et al 2013]
- subgraph counts [Garavaglia and Steghuis 2019]
- and more...

# Triangles, Tetrahedra and Topology

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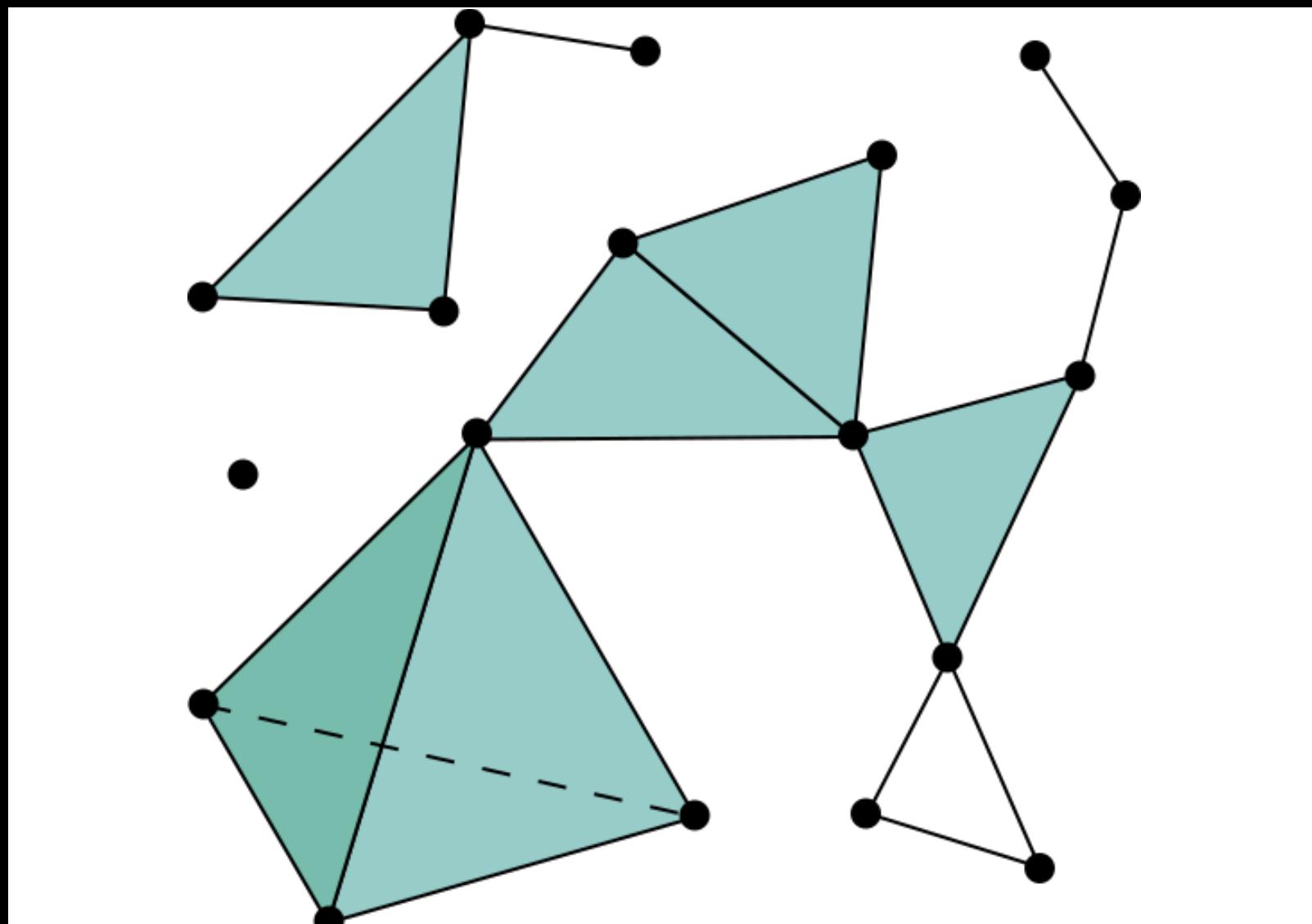


image credit: calm

# Triangles, Tetrahedra and Topology

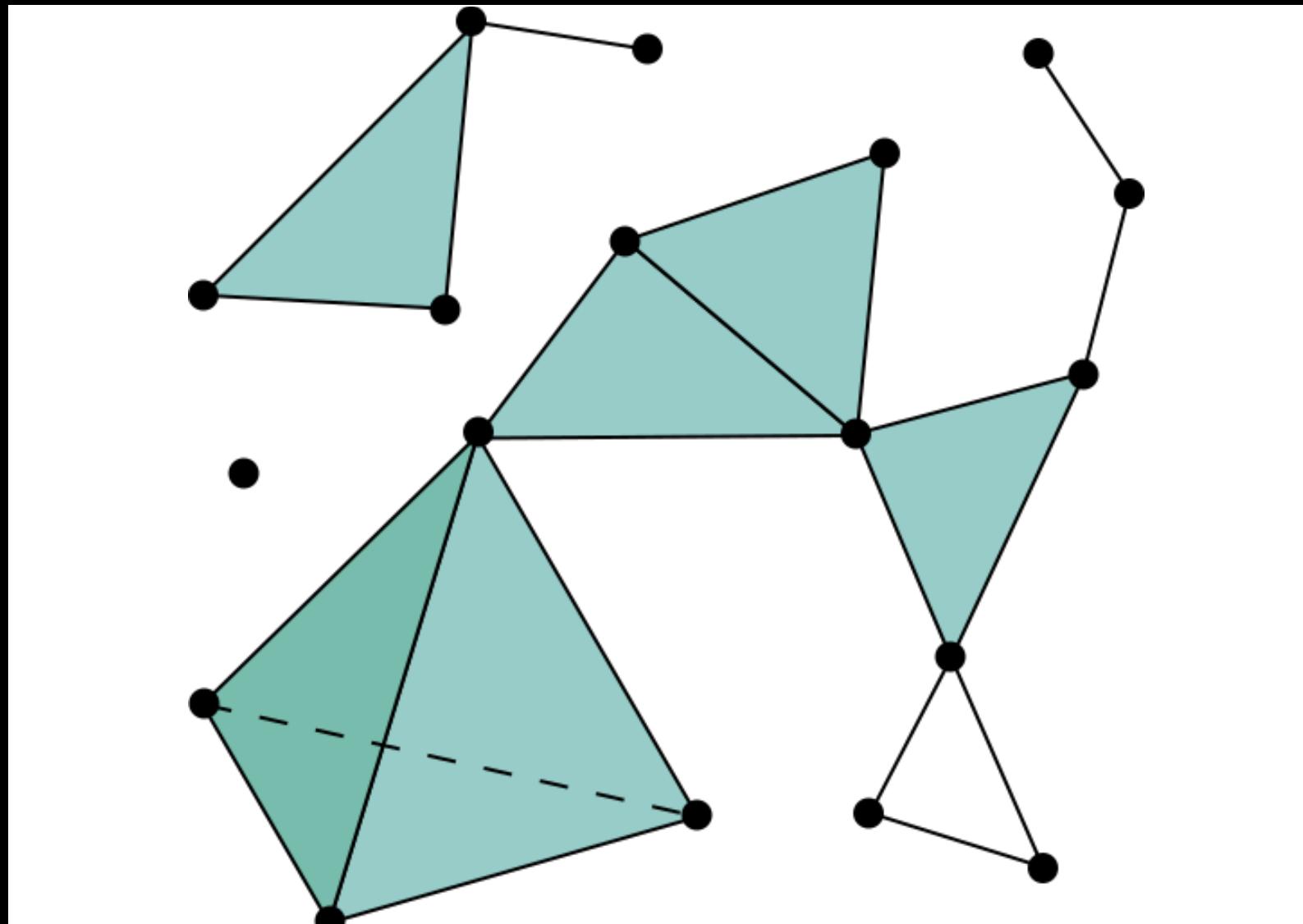


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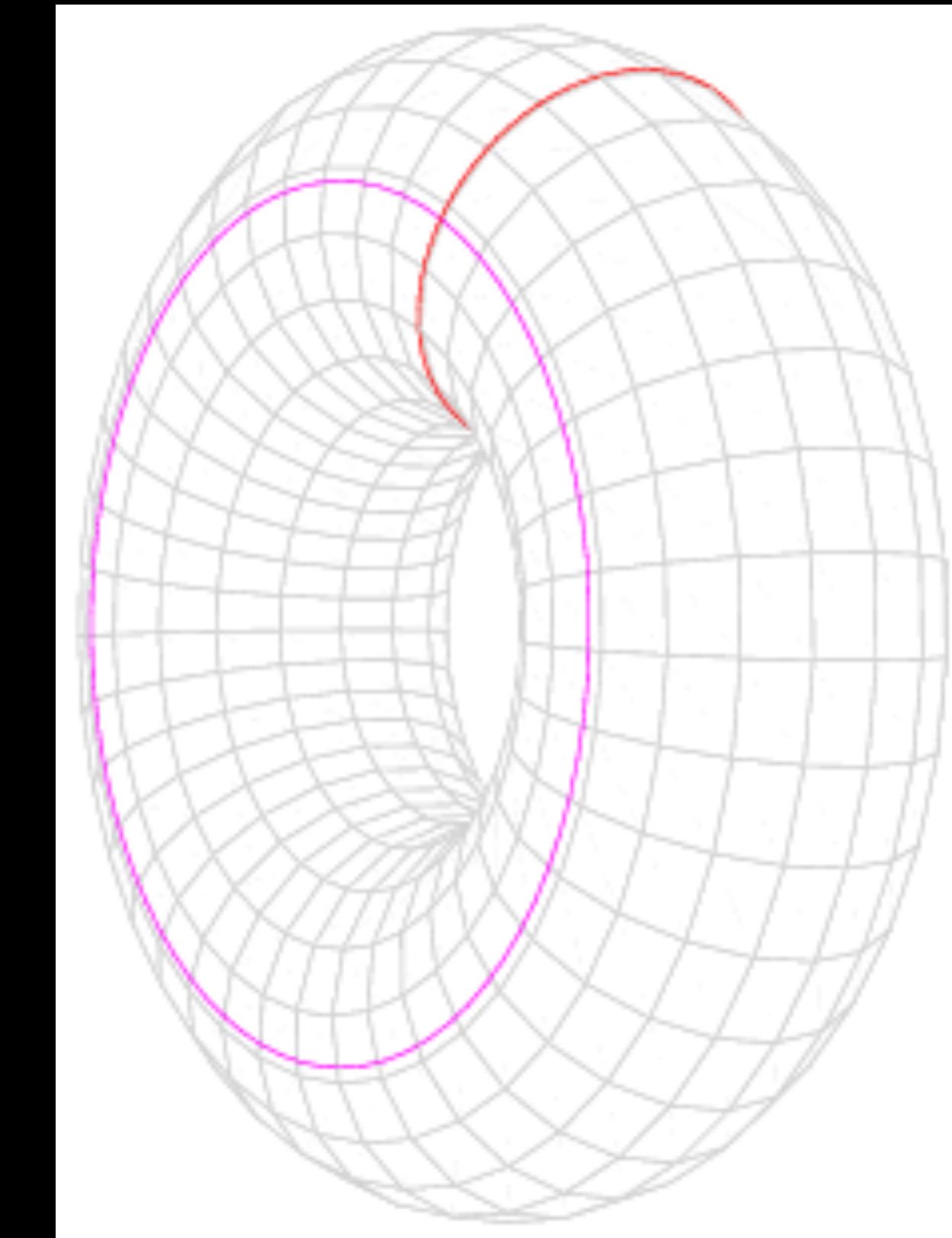


image credit: Krishnavedala

# Who cares?

## Examples of academic networks

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## Examples of academic networks

- Holes are repeated pathways. [Patania, Petri and Vaccarino 2017]

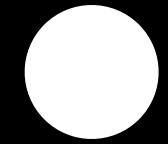
# Who cares?

## Examples of academic networks

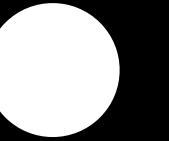
- Holes are repeated pathways. [Patania, Petri and Vaccarino 2017]
- Unifying concepts fill holes. [Salnikov et al 2018]

# The Story of Venus

## Knowledge



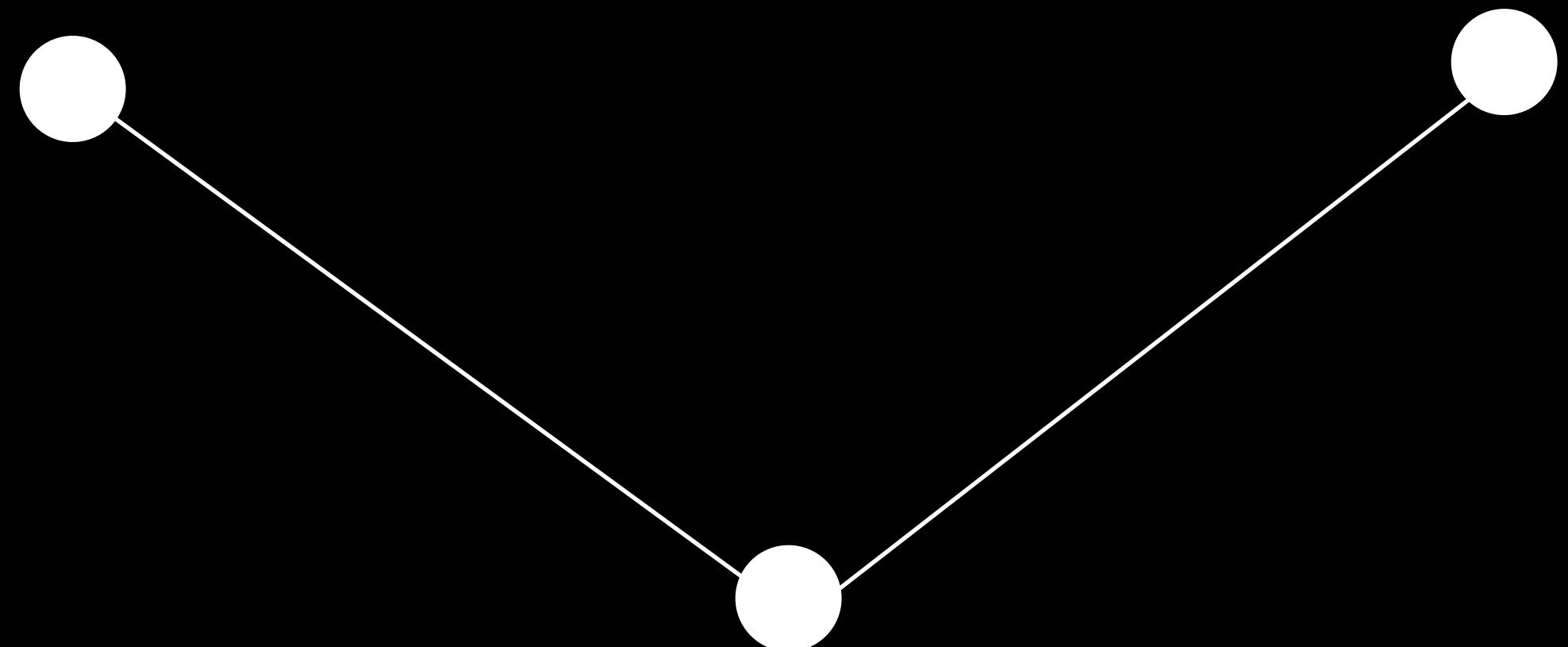
morning star paper



evening star paper

# The Story of Venus

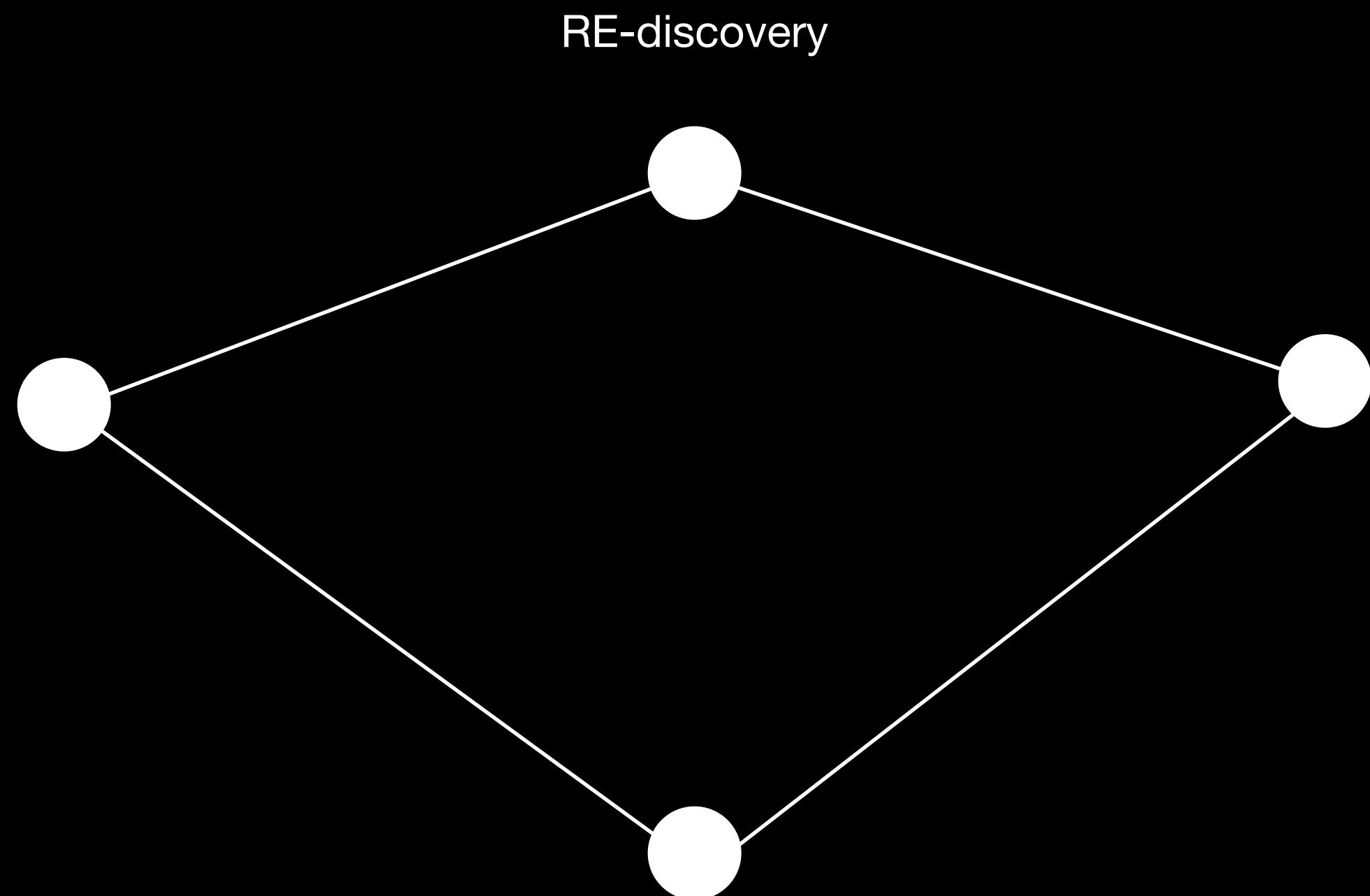
## Knowledge of knowledge



discovery: both are Venus

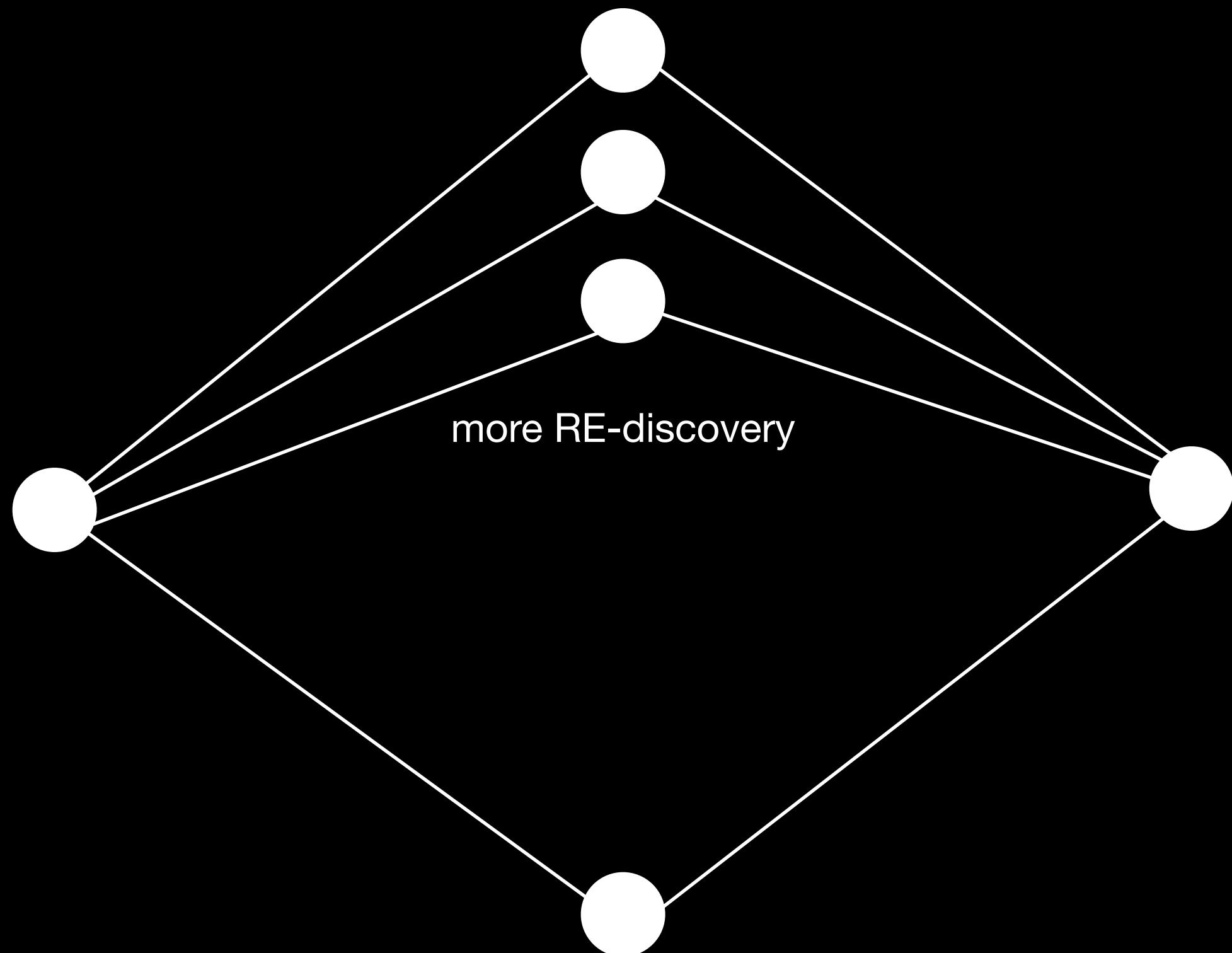
# The Story of Venus

## Repeated knowledge of knowledge



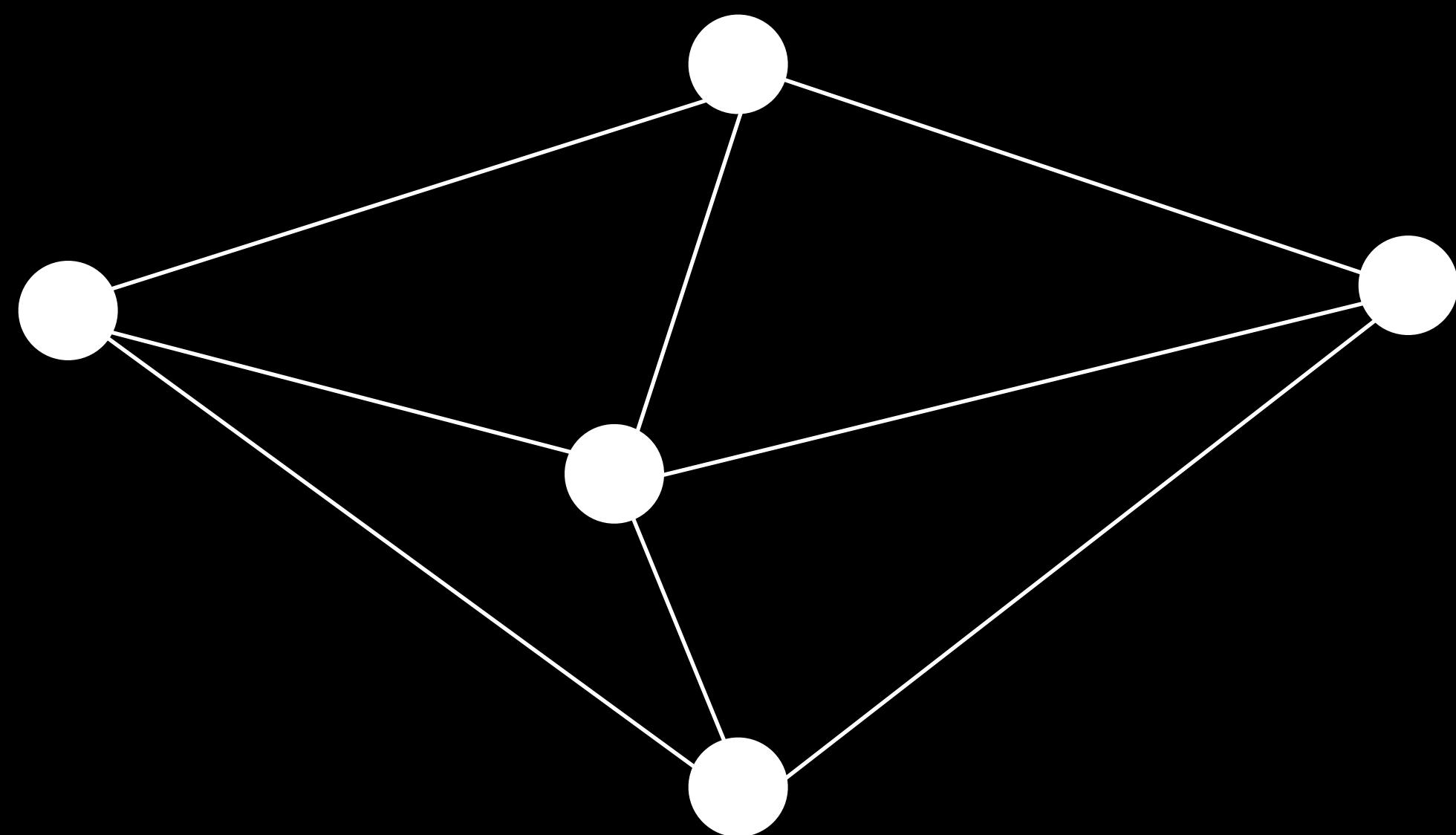
# The Story of Venus

## Repeated knowledge of knowledge



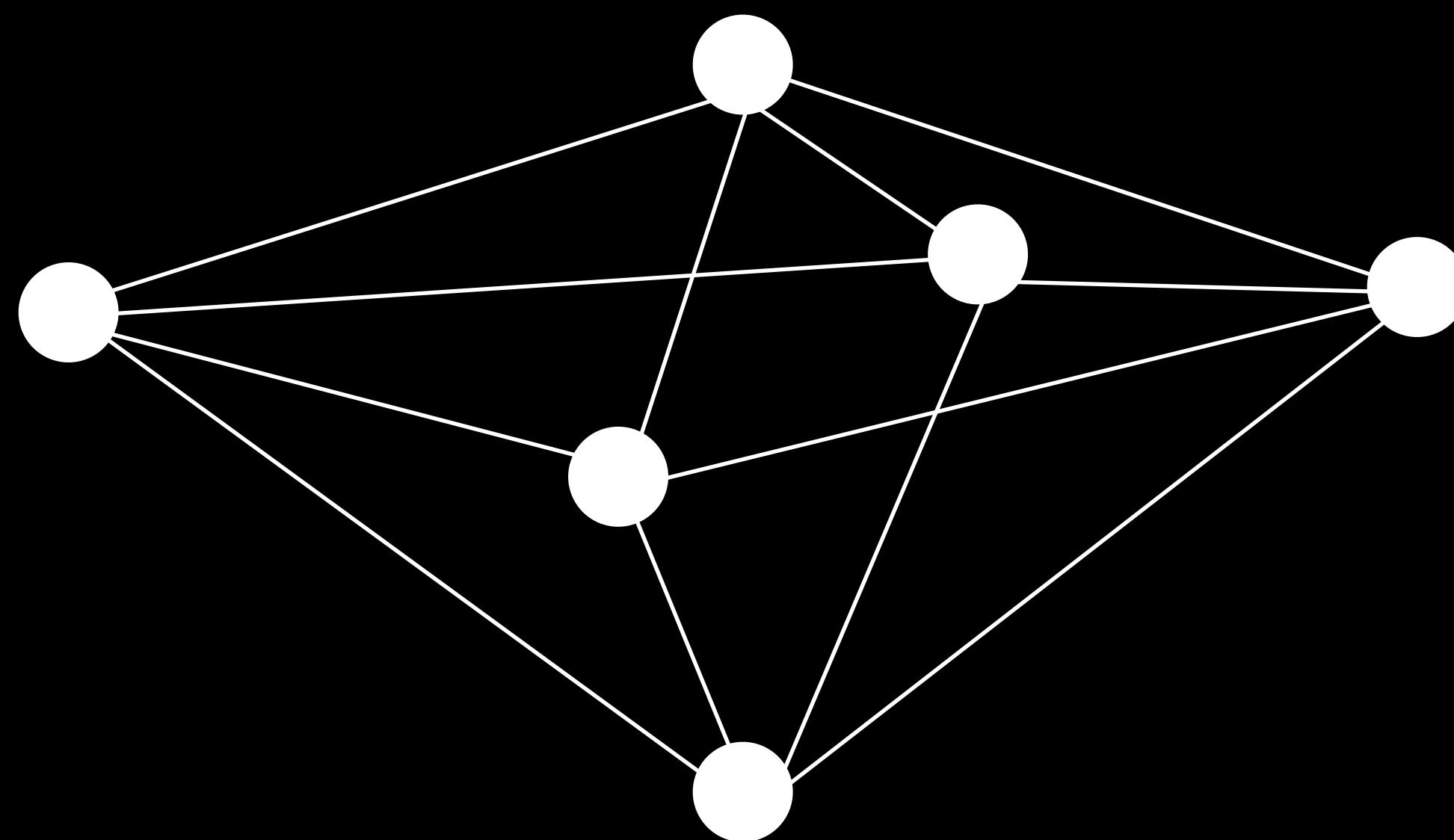
# The Story of Venus

Knowledge of knowledge of knowledge



# The Story of Venus

Repeated Knowledge of knowledge of knowledge



betti numbers

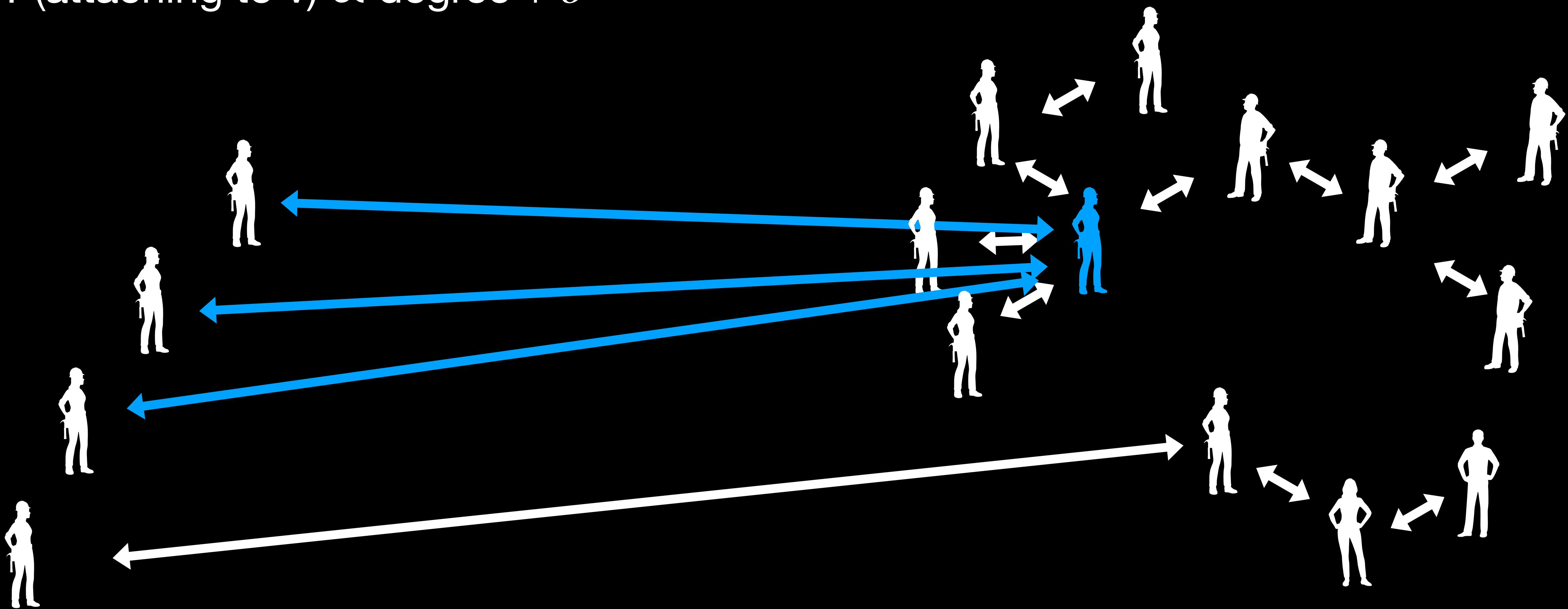
- = connected component and holes
- = repeated higher-order connections

# Betti numbers and preferential attachment

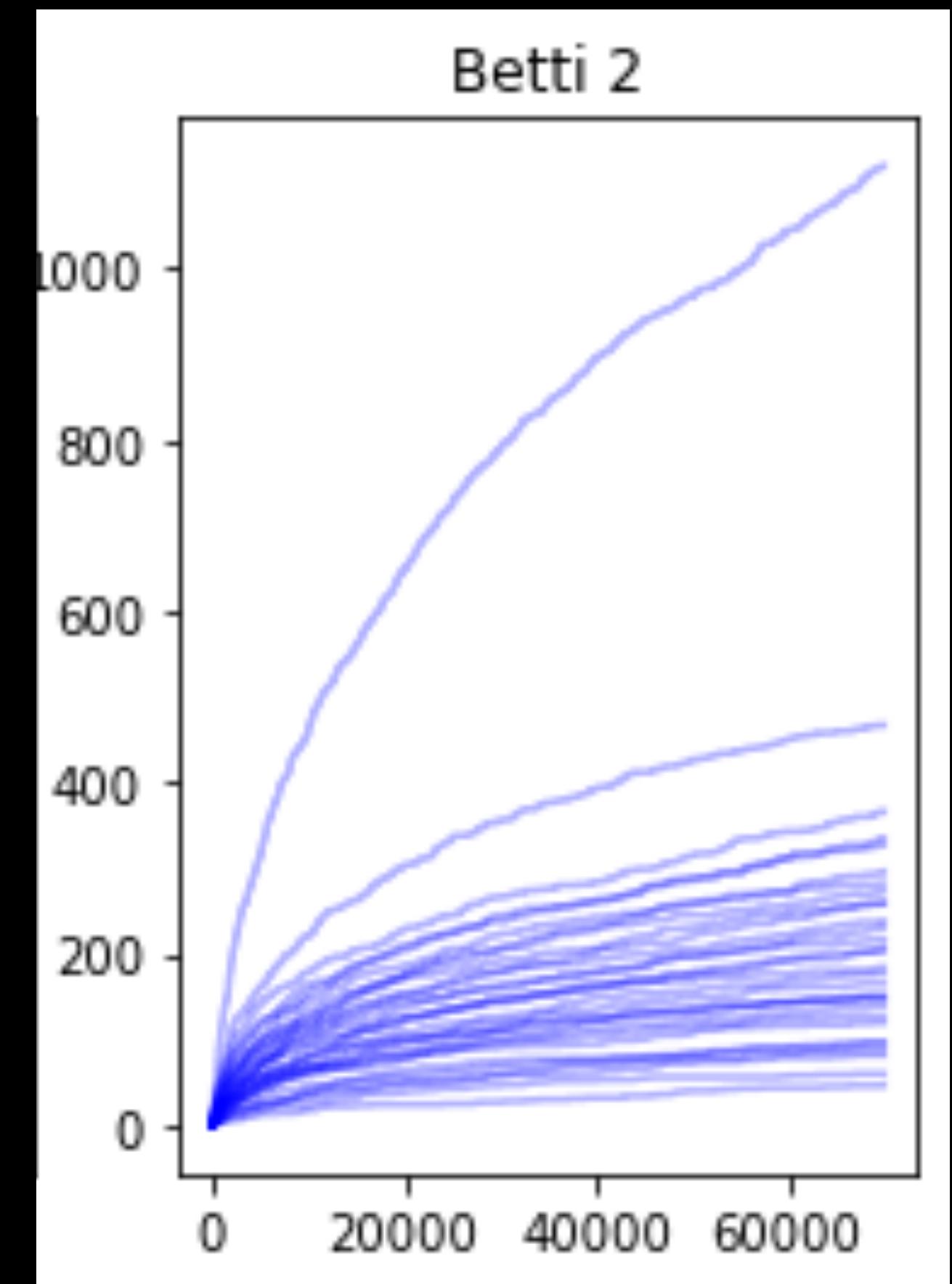
# Preferential Attachment

[Albert and Barabasi 1999]

$$P(\text{attaching to } v) \propto \text{degree} + \delta$$

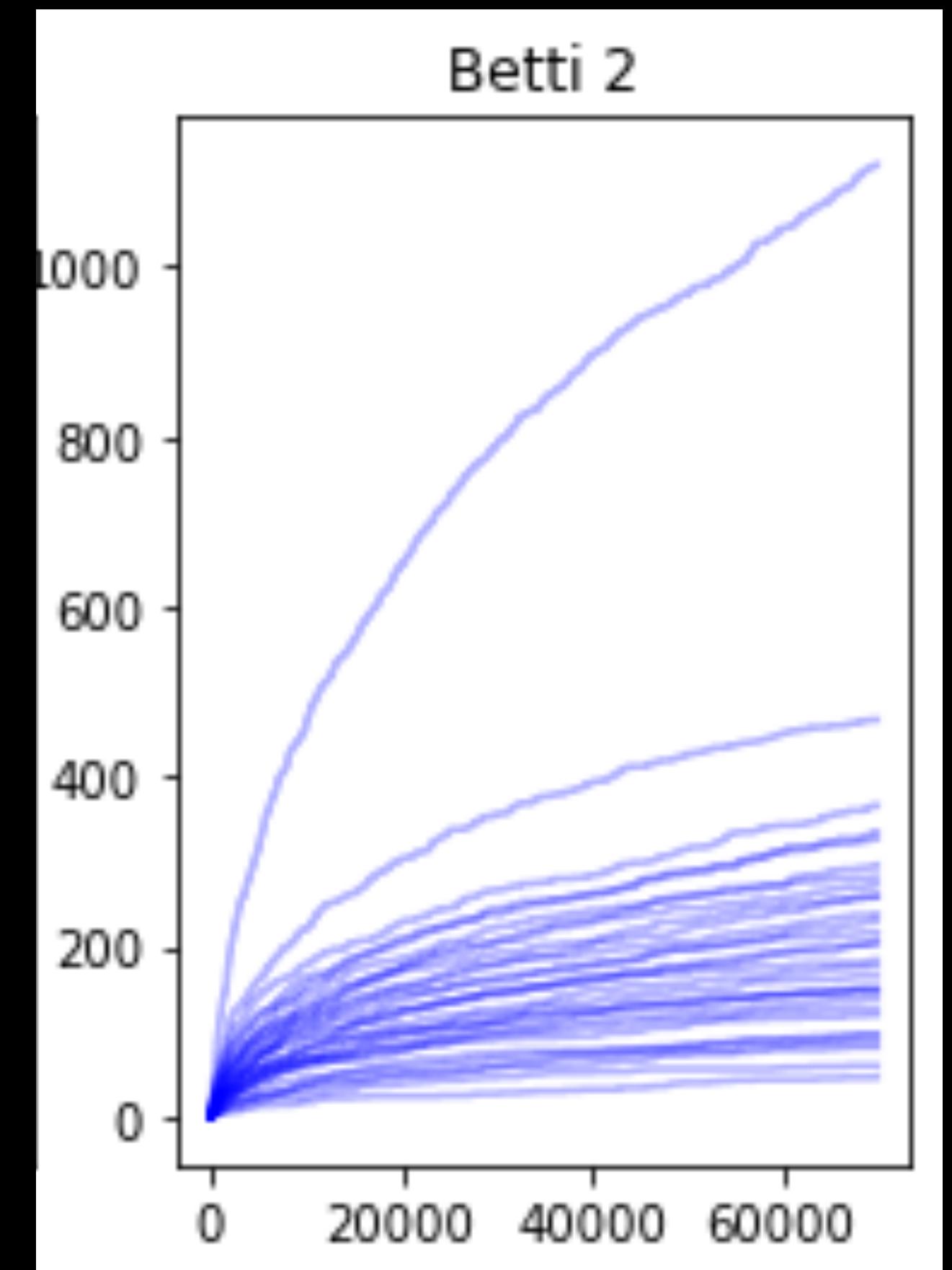


# Expected Betti Number $E[\beta_q]$



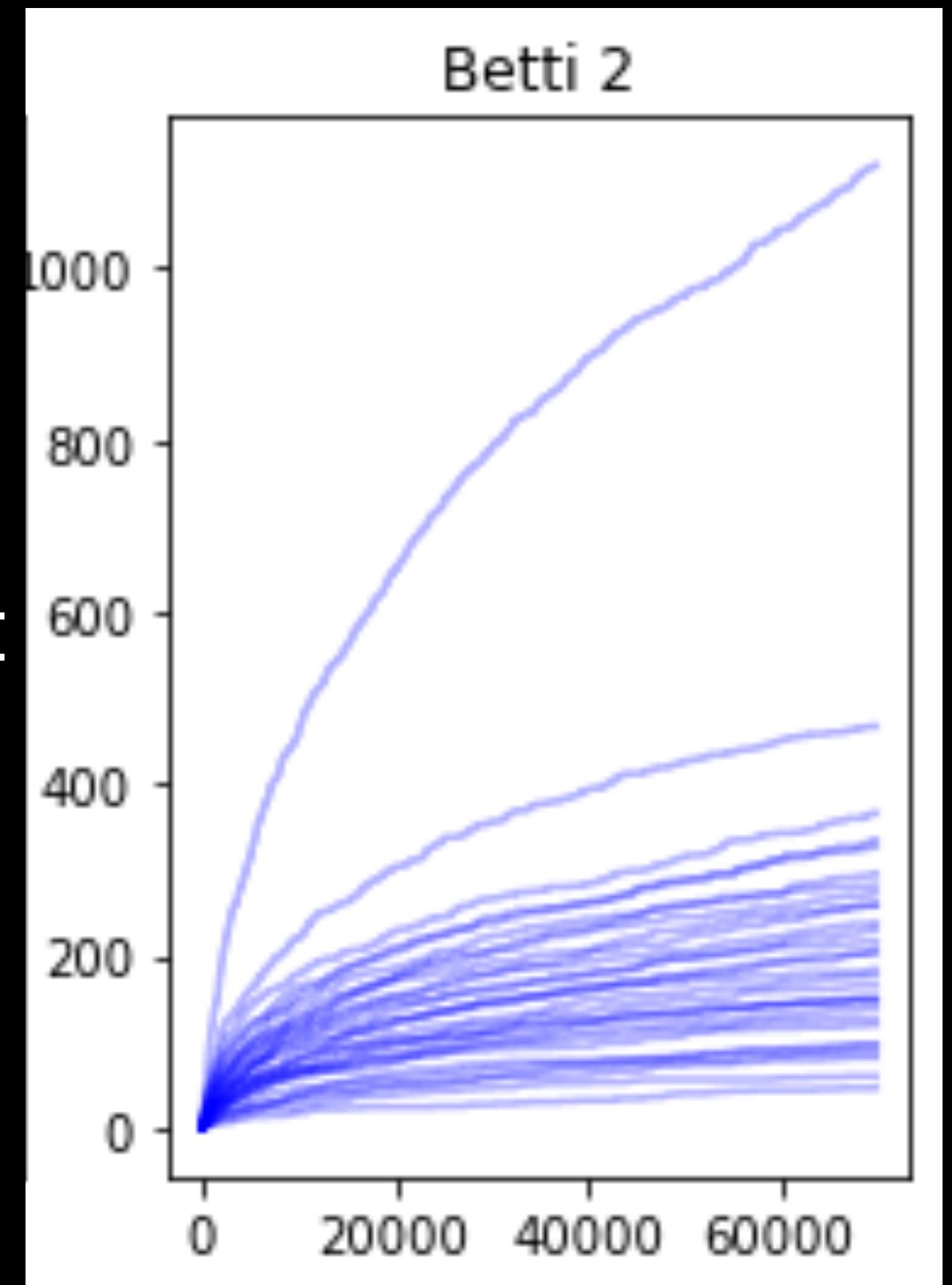
# Expected Betti Number $E[\beta_q]$

- $E[\beta_2] = \Theta(\text{num of nodes}^{1-4\chi})$  under mild assumptions
- $\chi = 1 - \frac{1}{2 + \delta/m} \in (0, 1/2)$
- small  $\chi$ :
  - heavier degree tail
  - stronger rich-get-richer effect



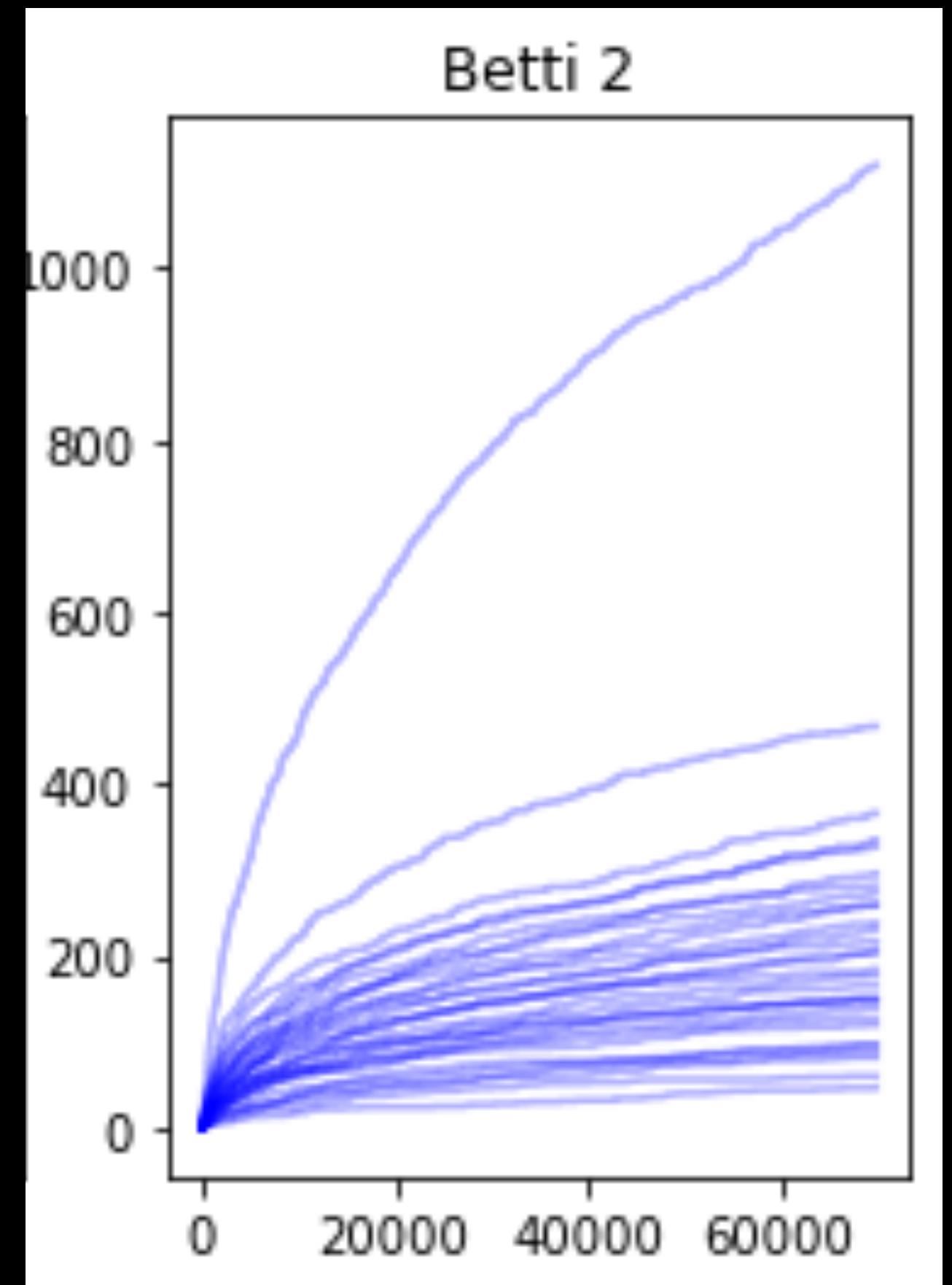
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- $E[\beta_q] = \Theta(\text{num of nodes}^{1-2q\chi})$  for  $q \geq 2$

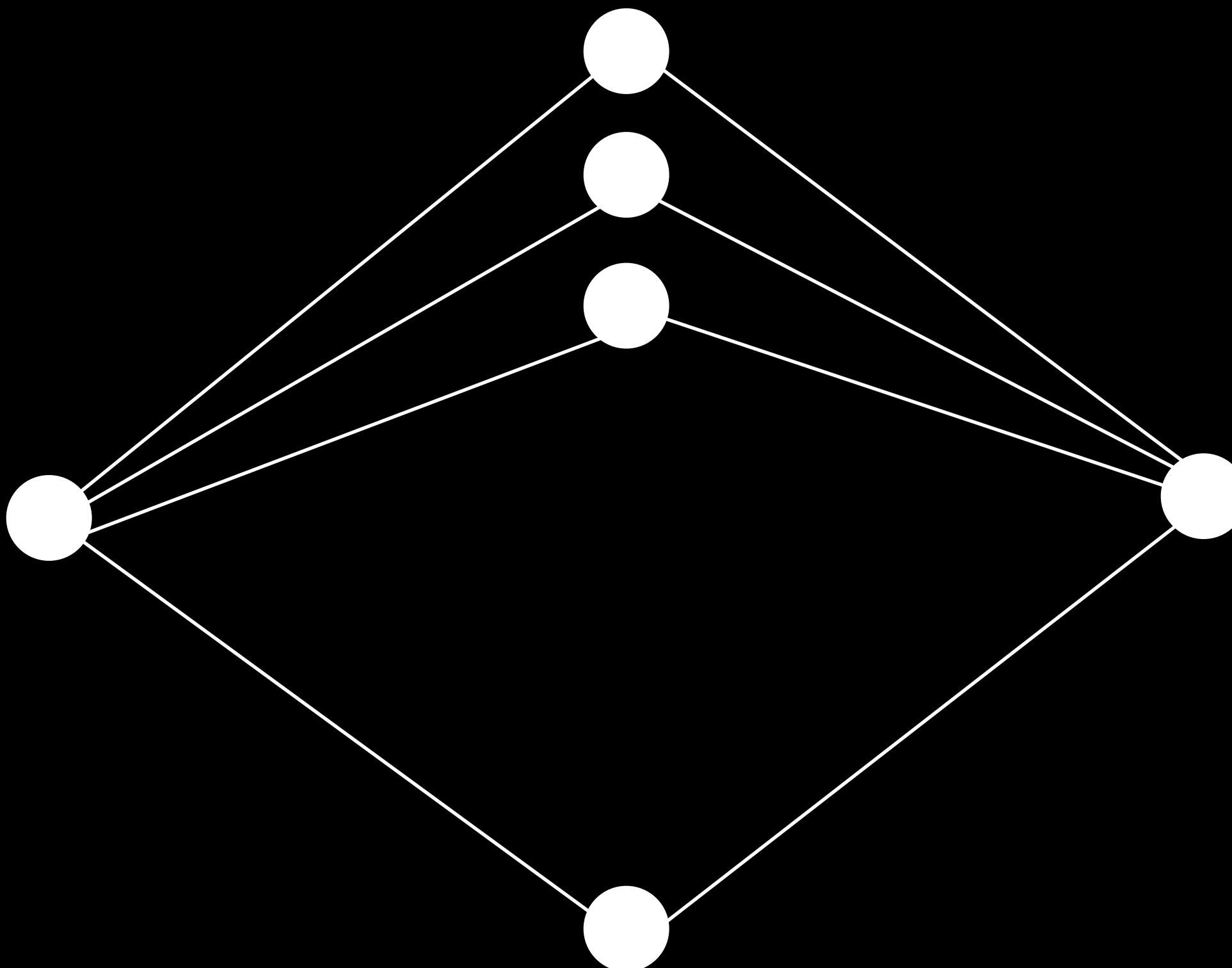


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- $E[\beta_q] = \Theta(\text{num of nodes}^{1-2q\chi})$  for  $q \geq 2$
- $E[\beta_q]$  decreases as dimension  $q$  increases
- $E[\beta_q]$  increases with the rich-get-richer effect



# Main Idea



# What's next?

- Tail of betti numbers?
- robustness and betti numbers?

# Thank you

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