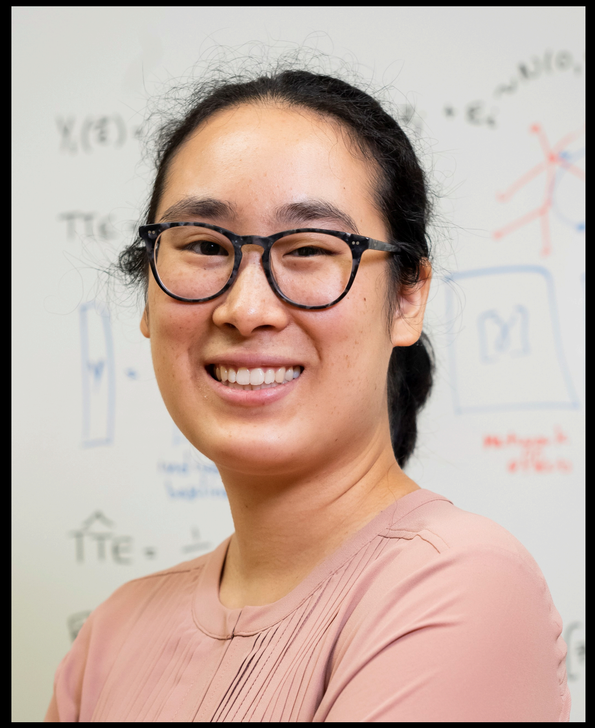
Preferential Attachment and Homology

Evolution of Higher Dimensional Interactions

Chunyin Siu
Cornell University
cs2323@cornell.edu

Collaborators



Christina Lee Yu

Gennady Samorodnitsky

Caroline He

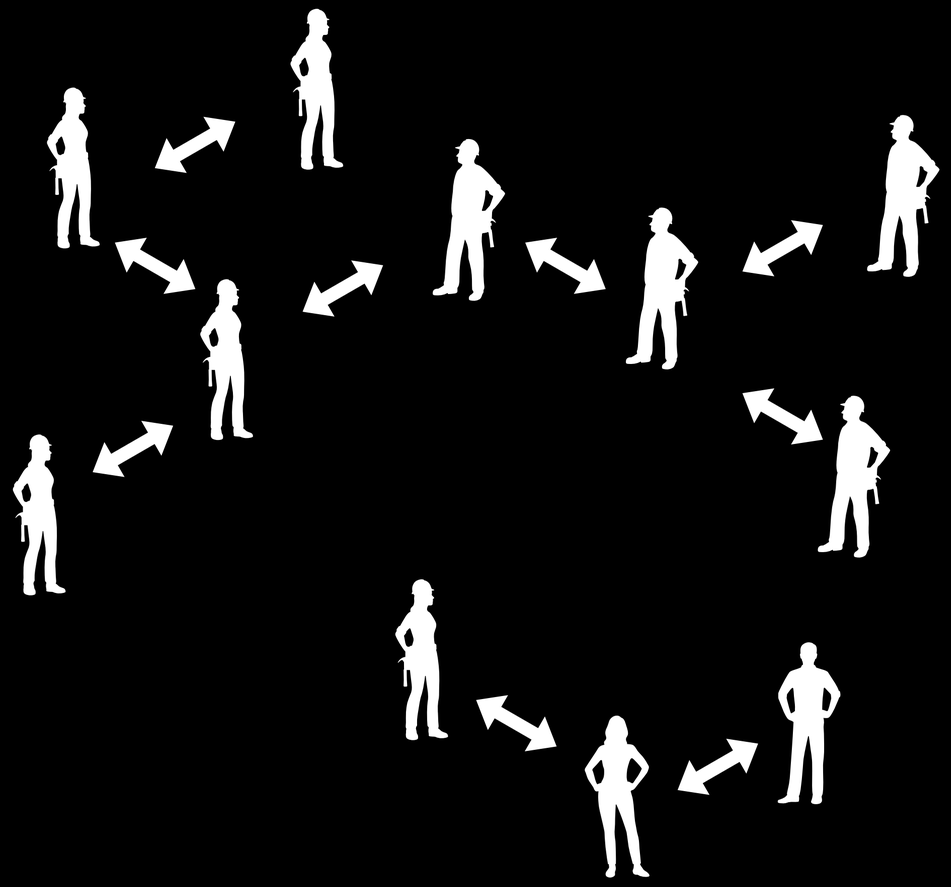
Agenda

- What is preferential attachment?
- What is homology?
  - Why people care?
- What we know about the homology of preferential attachment complexes

Preferential Attachment

Preferential Attachment

[Albert and Barabasi 1999]



Preferential Attachment

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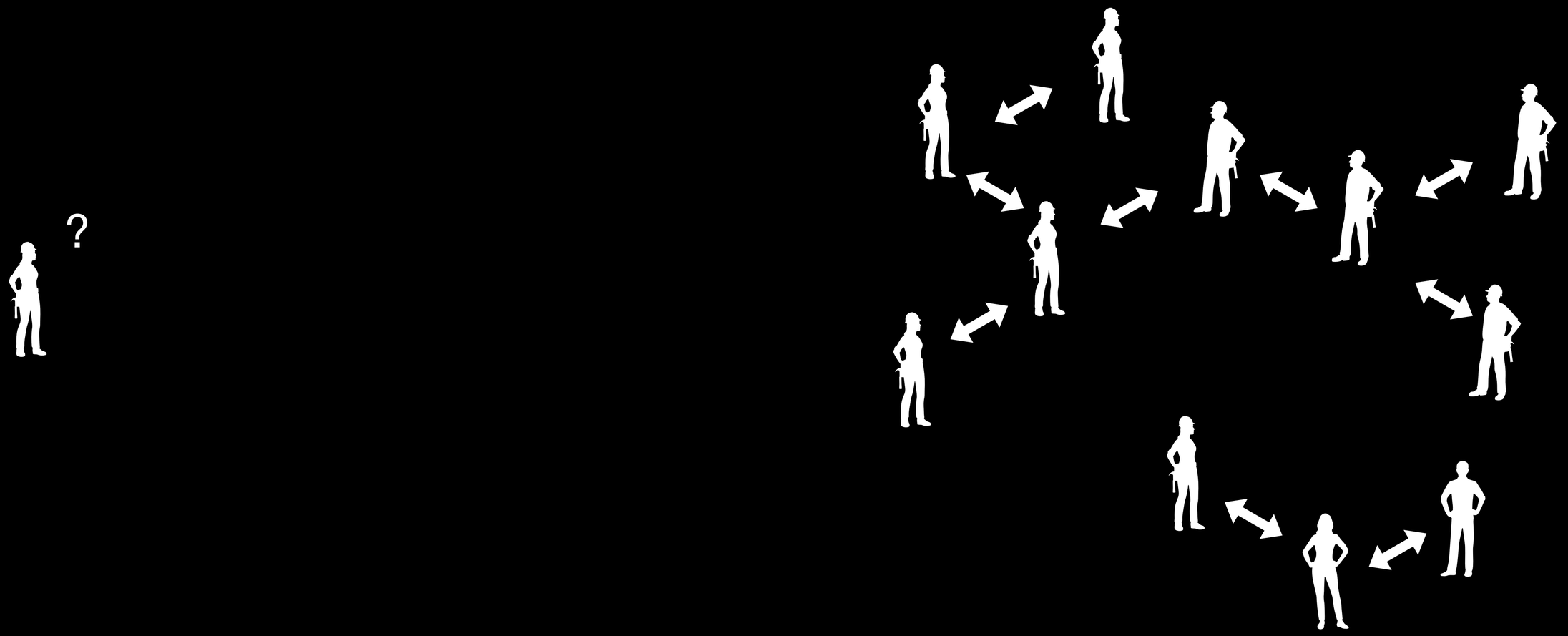

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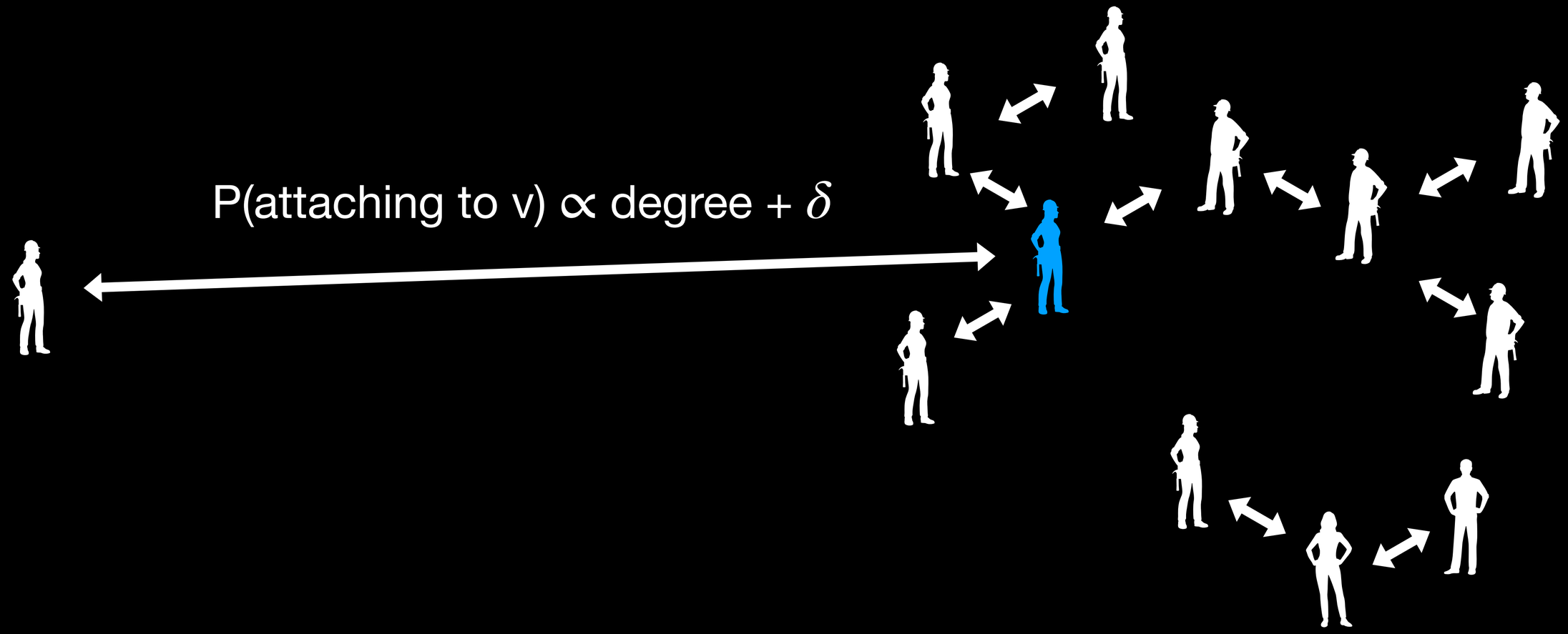
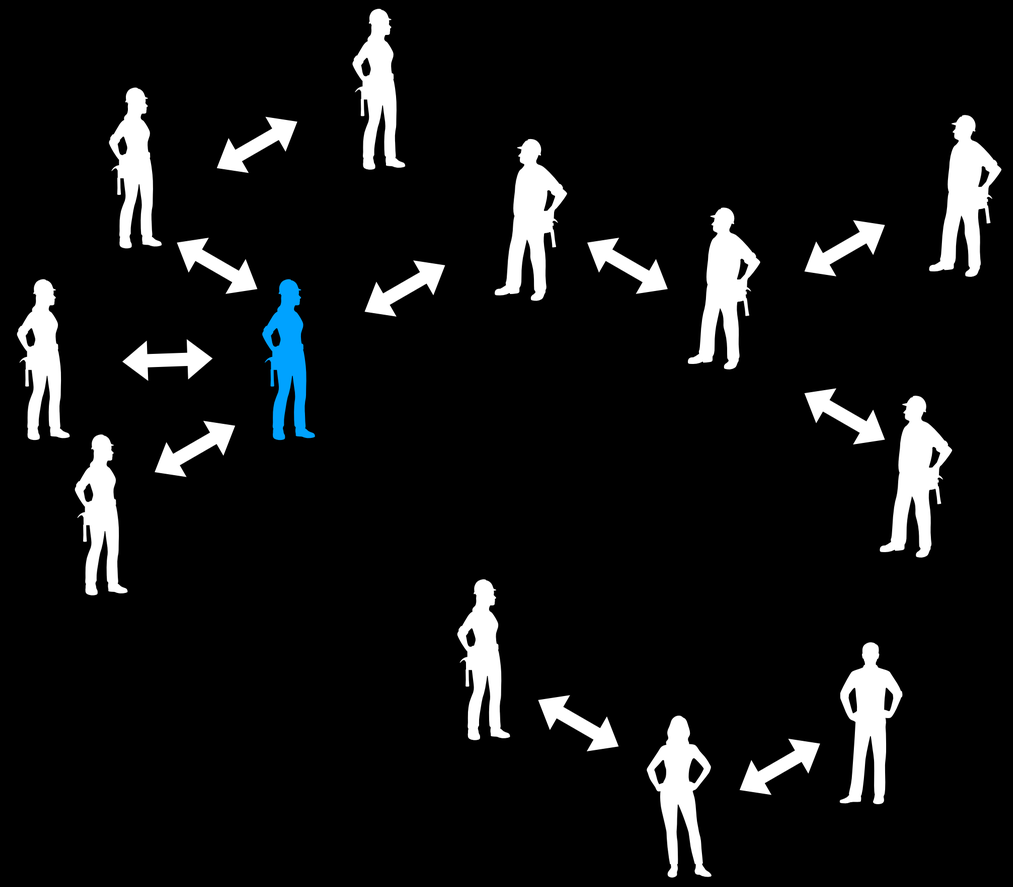


Preferential Attachment

[Albert and Barabasi 1999]

P(attaching to v) $\propto$ degree + δ



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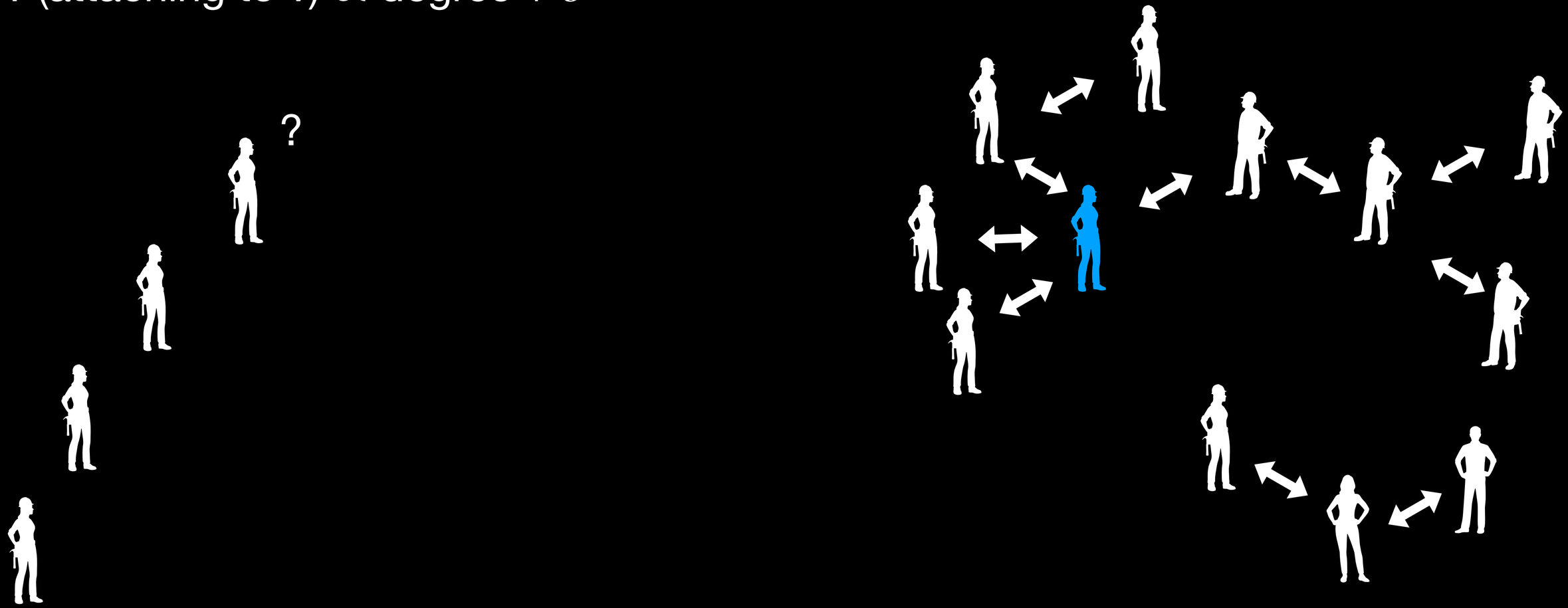

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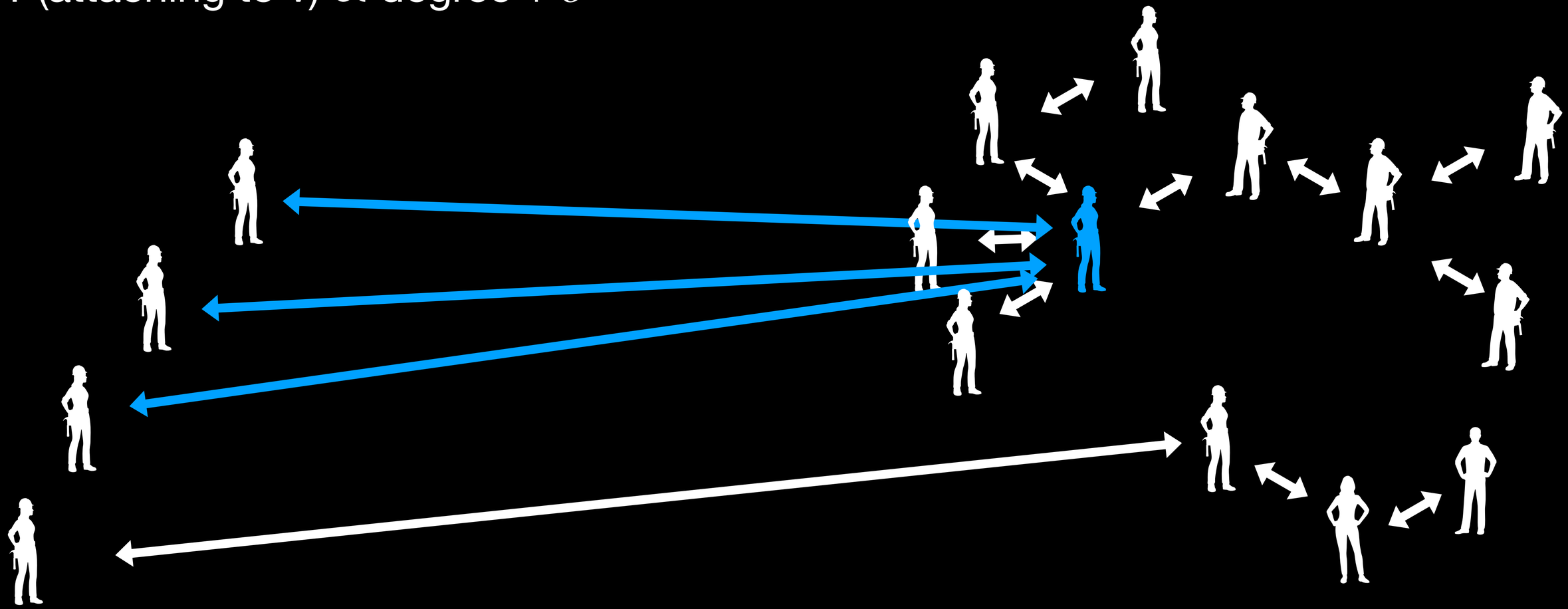

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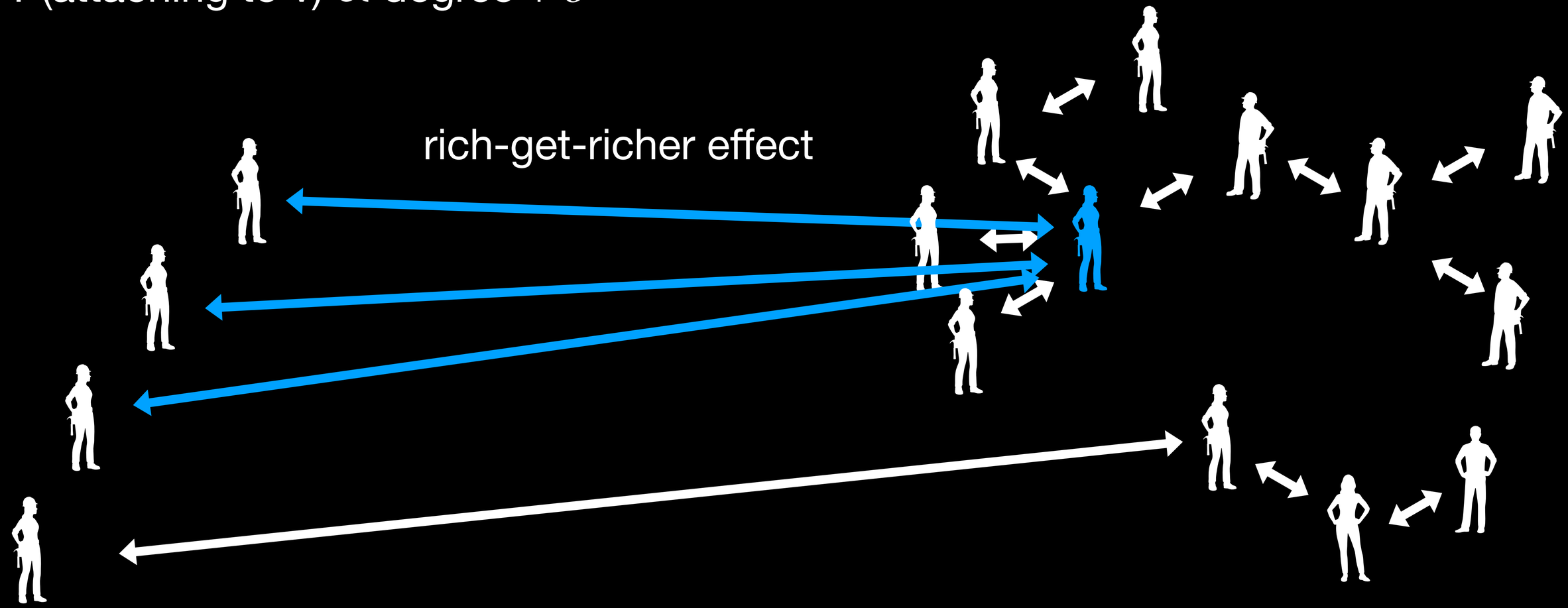

What do we know?

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- heavy tailed degree distribution [Albert and Barabasi 1999]

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- heavy tailed degree distribution [Albert and Barabasi 1999]
- triangle counts and clustering coefficient [Bollobas and Ridden 2002, Prokhorenkova et al 2013]
- subgraph counts [Garavaglia and Steghuis 2019]
- and more…

Triangles, Tetrahedra and Topology

Triangles, Tetrahedra and Topology

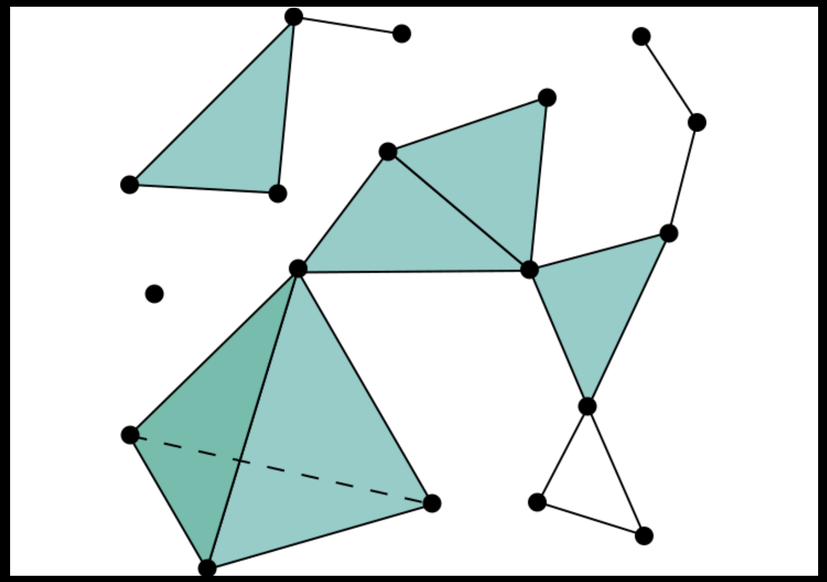
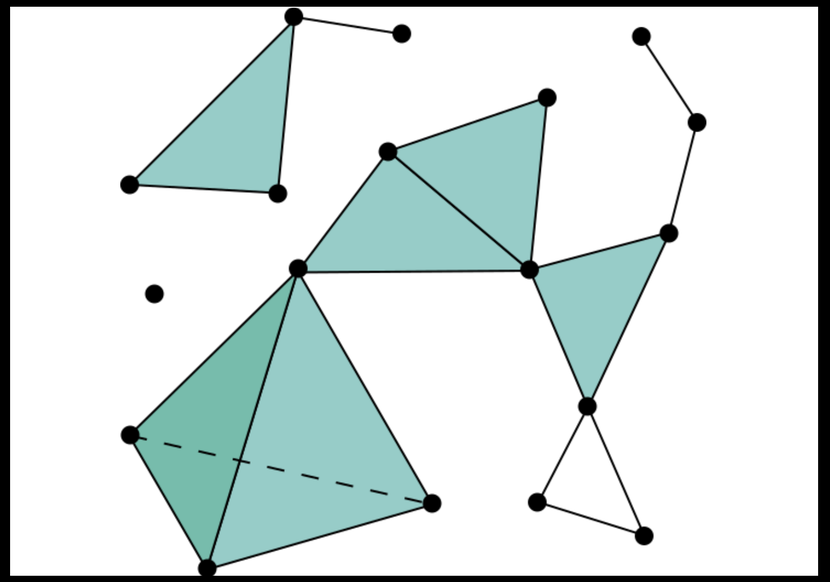


image credit: calm

Triangles, Tetrahedra and Topology



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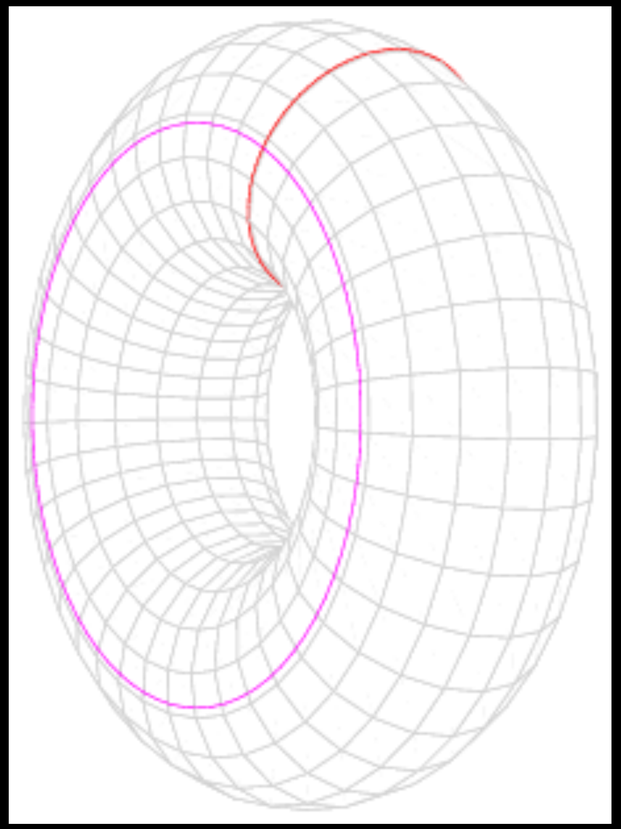

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Who cares?

Examples of academic networks

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- Holes are repeated pathways. [Patania, Petri and Vaccarino 2017]

Who cares?

Examples of academic networks

- Holes are repeated pathways. [Patania, Petri and Vaccarino 2017]
- Unifying concepts fill holes. [Salnikov et al 2018]

The Story of Venus

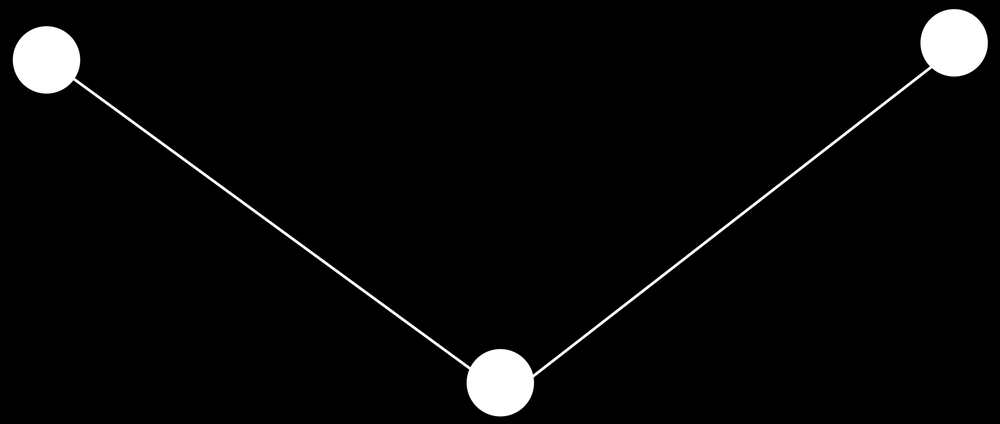
Knowledge

morning star paper

evening star paper

The Story of Venus

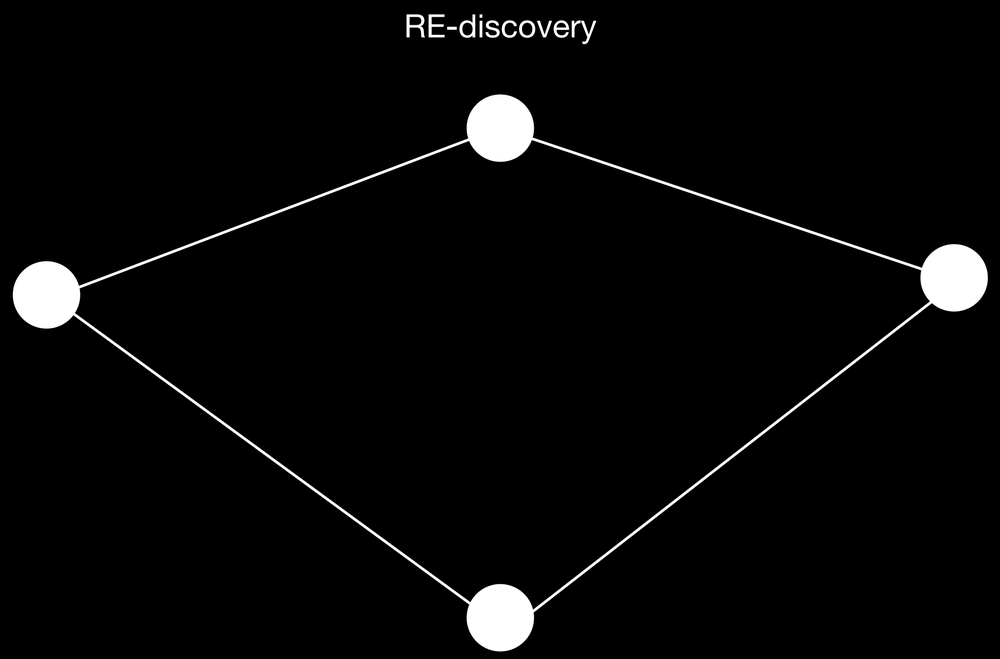
Knowledge of knowledge



discovery: both are Venus

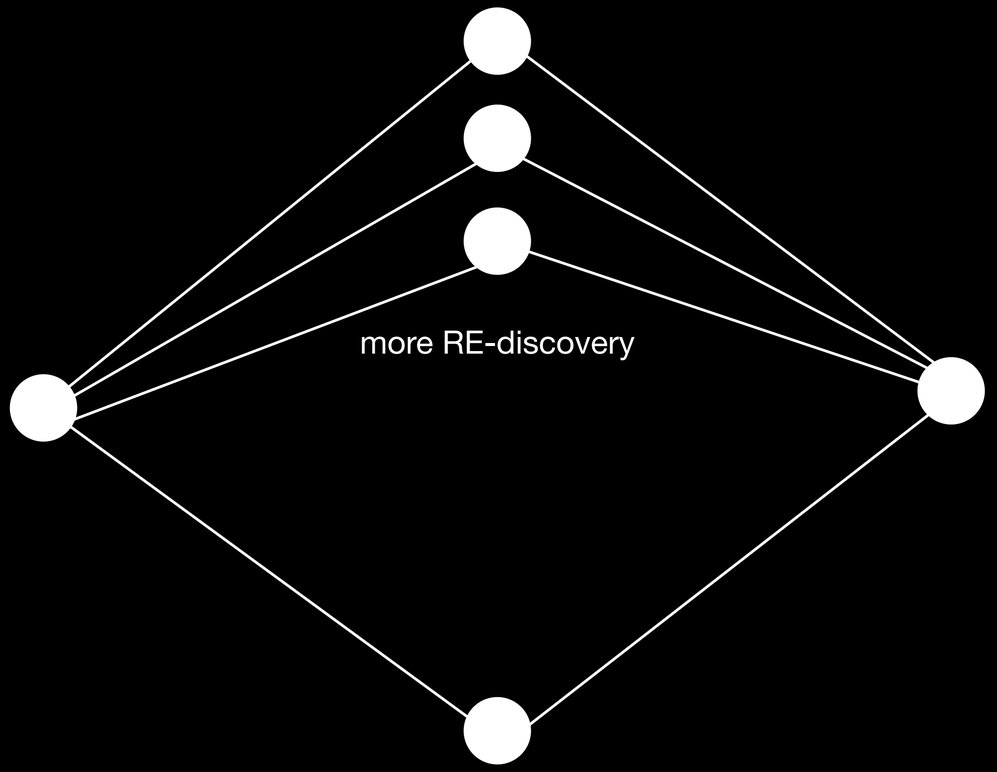
The Story of Venus

Repeated knowledge of knowledge



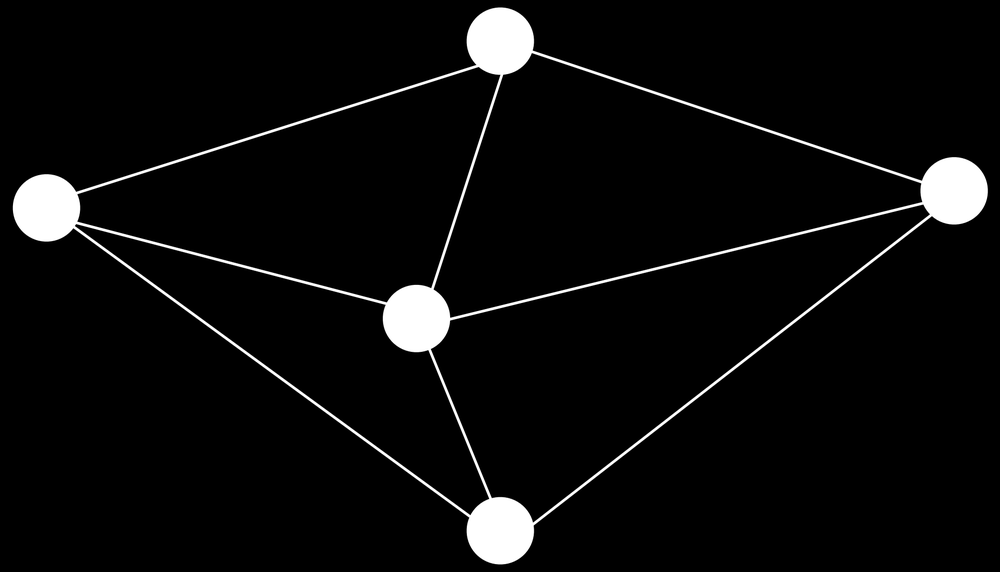
The Story of Venus

Repeated knowledge of knowledge



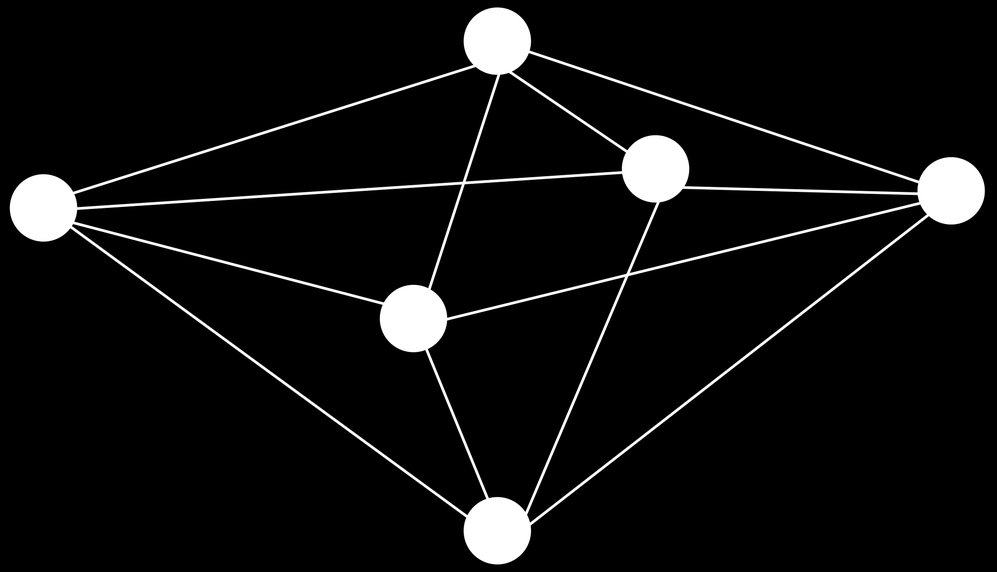
The Story of Venus

Knowledge of knowledge of knowledge



The Story of Venus

Repeated Knowledge of knowledge of knowledge



betti numbers
= connected component and holes
= repeated higher-order connections

Betti numbers and preferential attachment

Preferential Attachment

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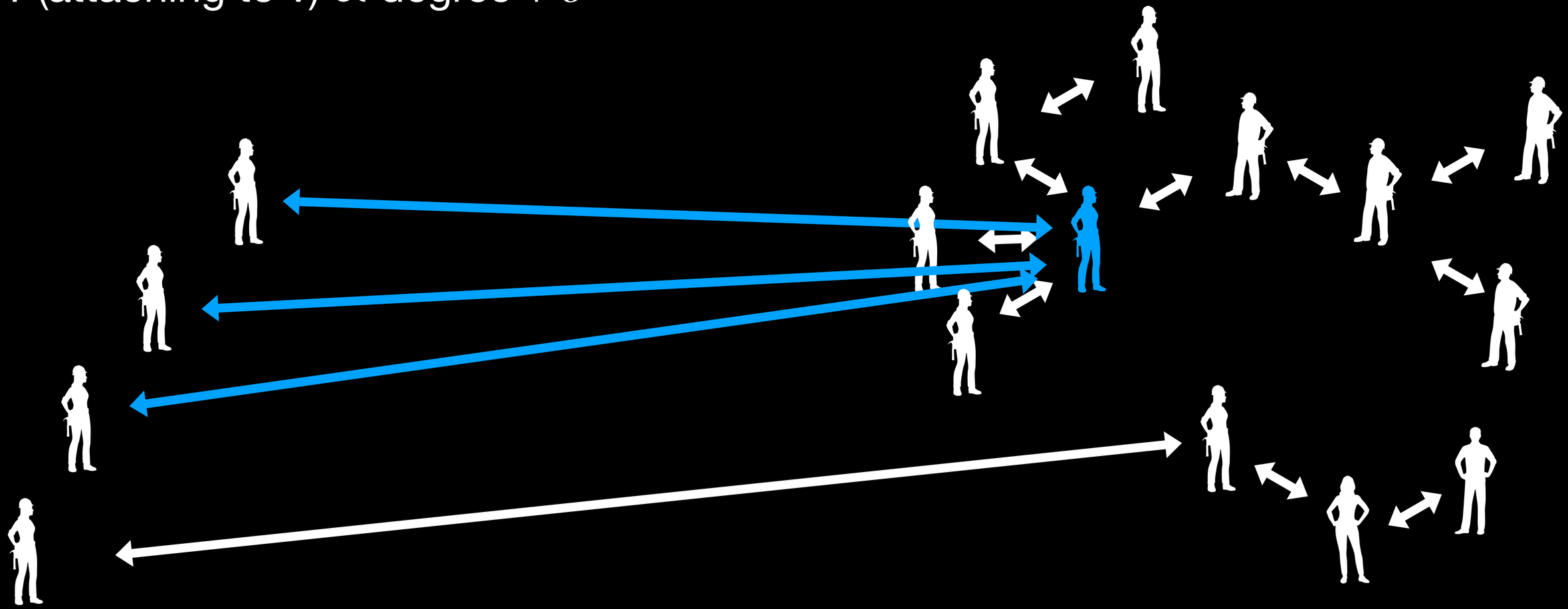
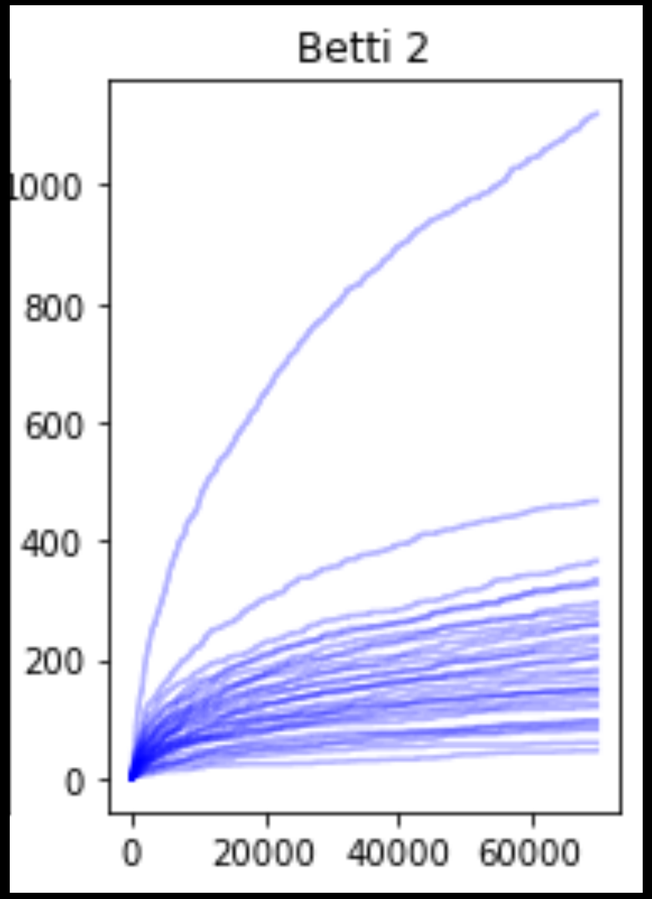
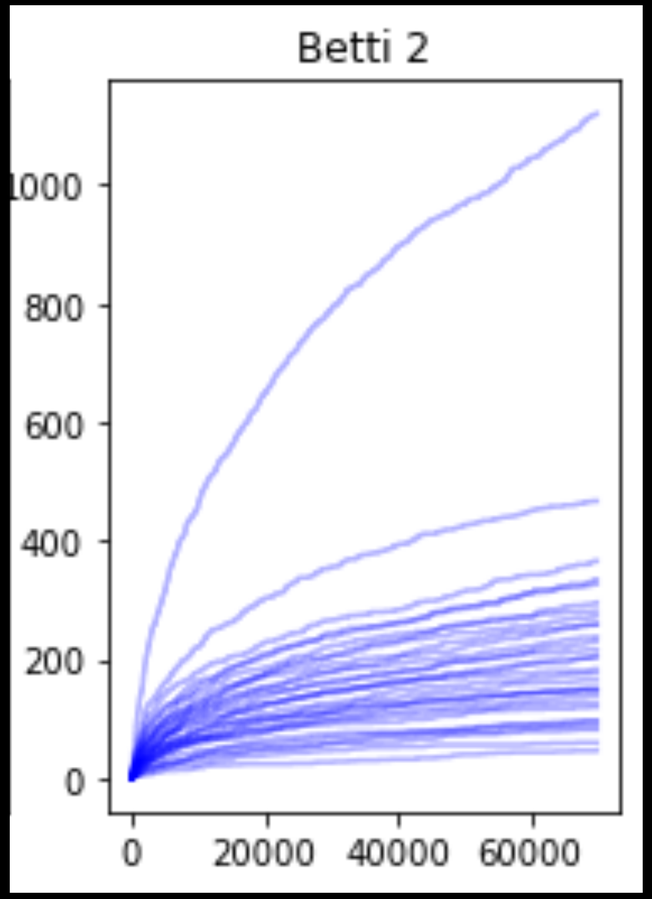


Expected Betti Number $E[\beta_q]$



Expected Betti Number $E[\beta_q]$

- $E[\beta_2] = \Theta(\text{num of nodes}^{1-4\chi})$ under mild assumptions
  - $\chi = 1 - \frac{1}{2 + \delta/m} \in (0,1/2)$
  - small χ:
    - heavier degree tail
    - stronger rich-get-richer effect



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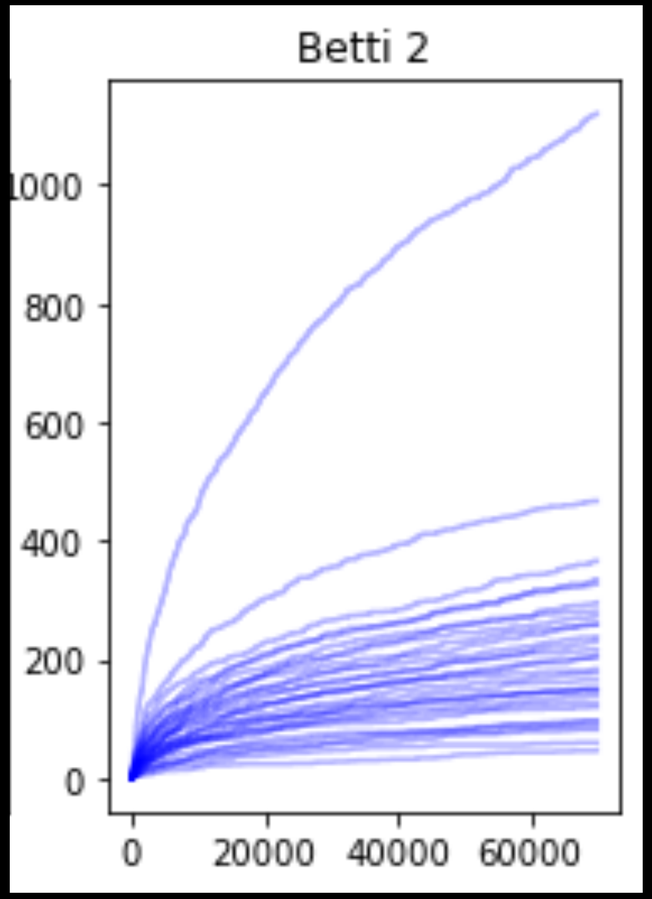

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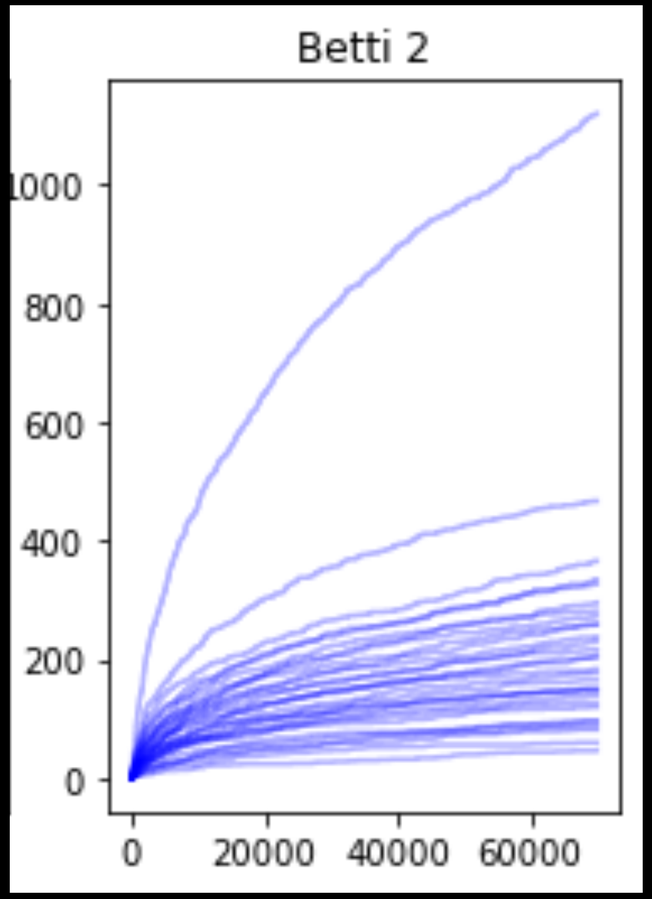
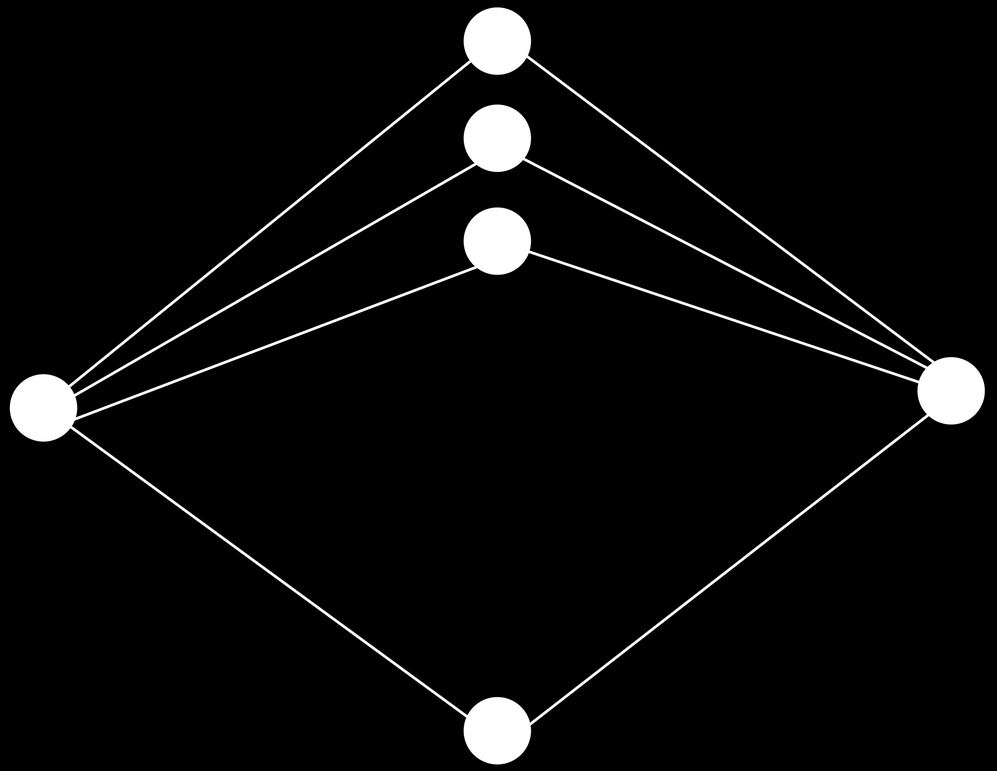
- $E[\beta_q] = \Theta(\text{num of nodes}^{1-2q\chi})$ for $q \geq 2$
- $E[\beta_q]$ decreases as dimension q increases
- $E[\beta_q]$ increases with the rich-get-richer effect



Main Idea



What’s next?

- Tail of betti numbers?
- robustness and betti numbers?

Thank you

- Chunyin Siu
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